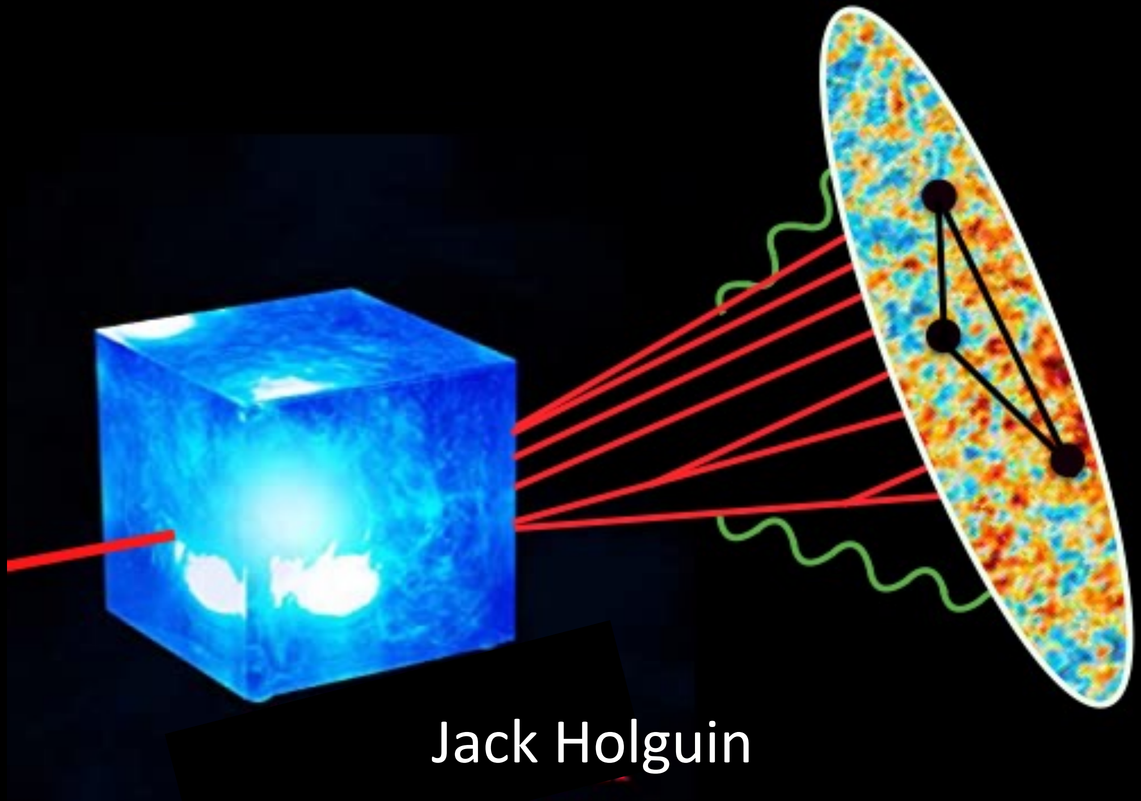
Energy Correlators on HI collisions



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Based on 2303.03413 and work to come.

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Introduction

Very quickly, what is $\mathcal{E}(\vec{n})$?

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\ r^2 n^i T_{0i}(t, r\vec{n})$$

$\mathcal{E}(\vec{n}) =$ Idealised calorimeter output at a solid angle labelled by $\vec{n}$.

Correlation functions of $\mathcal{E}(\vec{n})$ quantify the correlations between the average calorimeter outputs at different points across the celestial sphere from a particular process.

They are functions of the angles between the ‘idealised’ calorimeters.

Introduction

$$\frac{\langle \mathcal{E}^n(\boldsymbol{n}_1)\mathcal{E}^n(\boldsymbol{n}_2)\rangle}{Q^{2n}} = \frac{1}{\sigma}\sum_{ij}\int \frac{\mathrm{d}\sigma_{ij}}{\mathrm{d}\boldsymbol{n}_i \mathrm{d}\boldsymbol{n}_j}\frac{E_i^n E_j^n}{Q^{2n}}\delta^{(2)}(\boldsymbol{n}_i - \boldsymbol{n}_1)\delta^{(2)}(\boldsymbol{n}_j - \boldsymbol{n}_2)$$

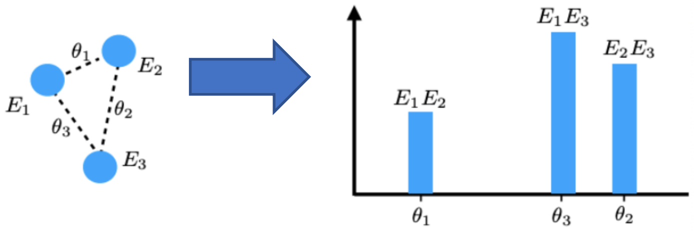
Where i, j are final state hadrons and σ_{ij} is the inclusive cross section to produce i, j with a hard scale Q.

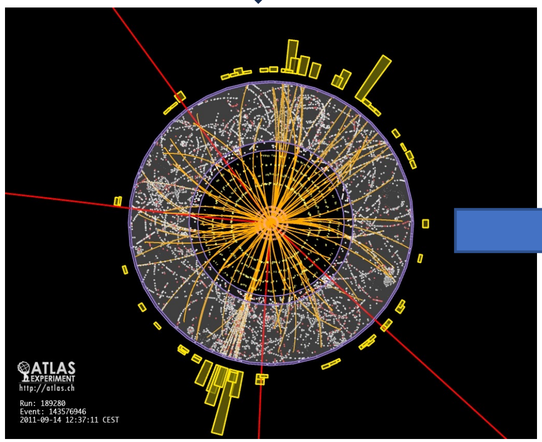
We integrate out the global $\mathrm{SO}(3)$ symmetry to find the distribution we're interested in.

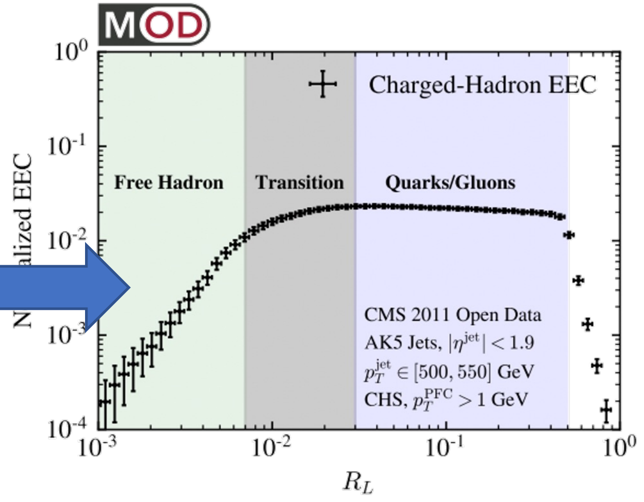
$$\frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\theta} = \int \mathrm{d}\boldsymbol{n}_{1,2}\frac{\langle \mathcal{E}^n(\boldsymbol{n}_1)\mathcal{E}^n(\boldsymbol{n}_2)\rangle}{Q^{2n}}\delta(\boldsymbol{n}_2 \cdot \boldsymbol{n}_1 - \cos\theta)$$

Introduction

Important difference between correlators and more ‘typical’ observables.

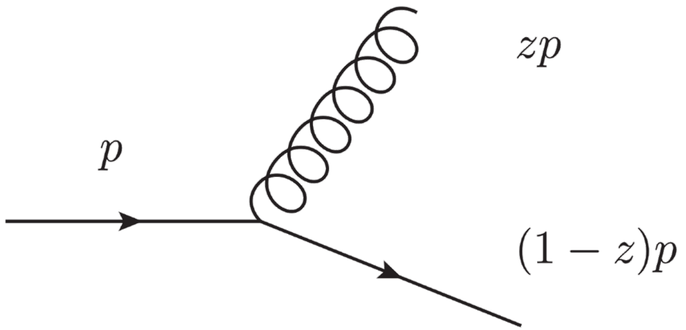






Introduction

Energy correlators are very good at isolating parts of multiscale dynamics.

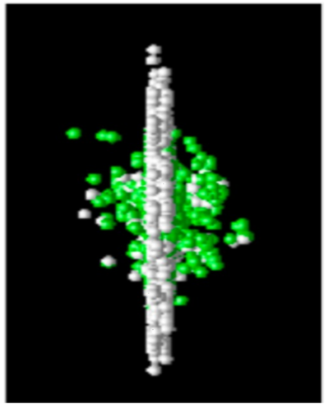


$$t_f = \frac{2}{z(1-z)p_0\theta^2} \xrightarrow{z \approx \frac{1}{2}} \theta^2 \sim t_f^{-1} \sim p_{\text{exchanged}}^2$$

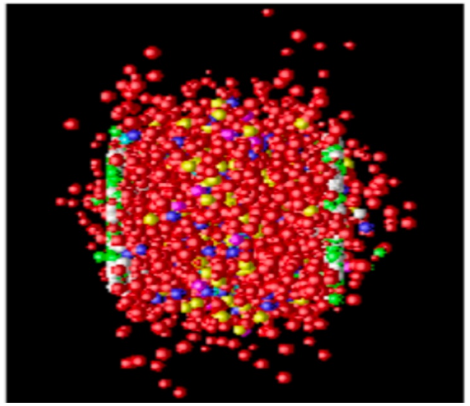
The angular size of a correlation often can be interpreted as a time parameter for the physics inducing the correlations.

Introduction

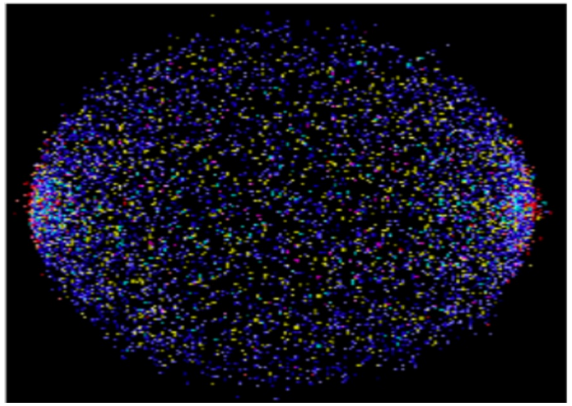
Time



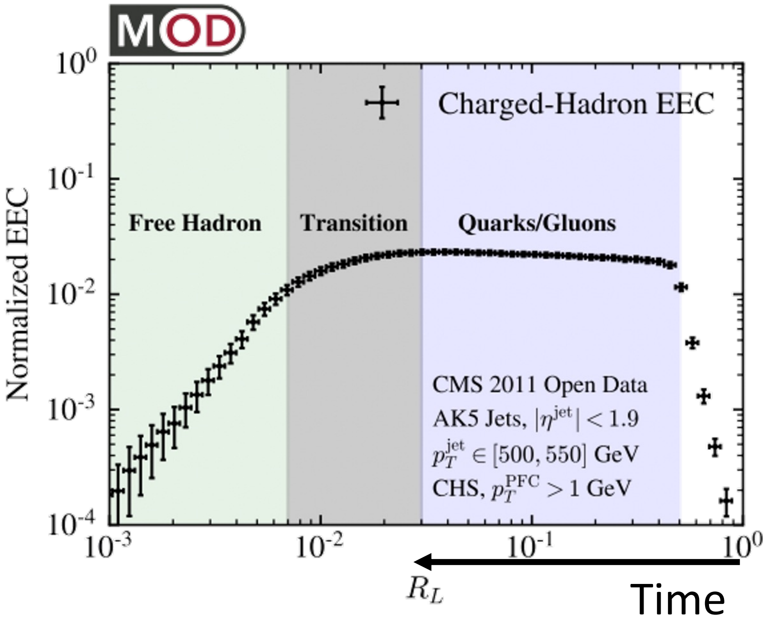
t=0 to 1 fm/c c



t=1 to 10 fm/c



t>10 fm/c



Time

$$\theta^2 \sim t_f^{-1} \sim p_{\mathrm{exchanged}}^2$$

Advertising

I won't have time today to discuss our recent HI deadcone studies.

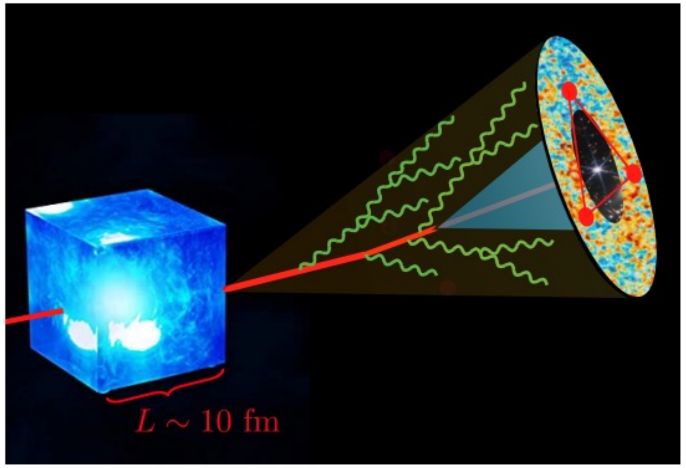


FIG. 1: A heavy-flavor jet propagating through the QGP forms a complicated energy pattern due to an interplay of two characteristic angular scales: the dead-cone angle θ_0 and the onset angle $\theta_{\rm on}$. These scales can be extracted from the asymptotic energy flux using energy correlators.

[2307.15110](2307.15110)

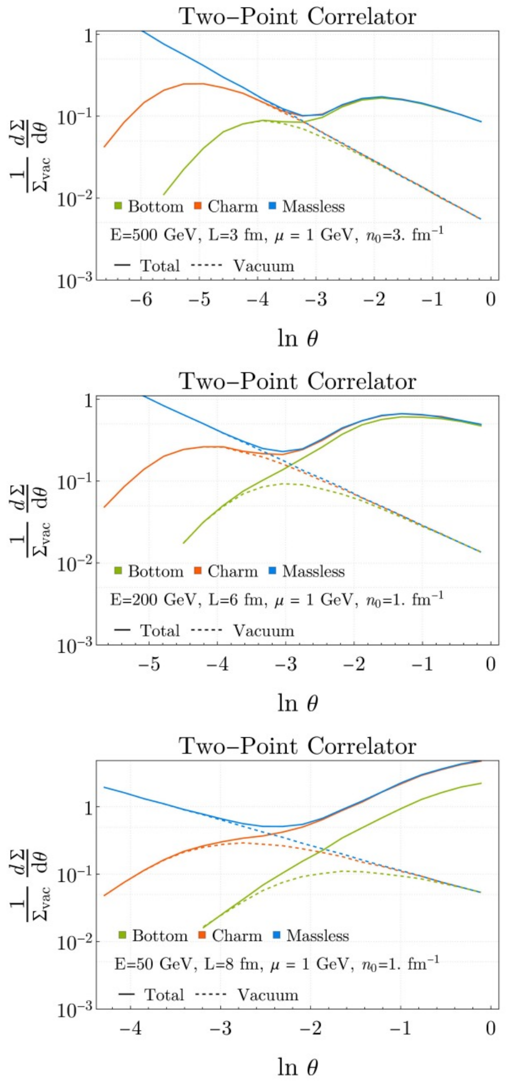
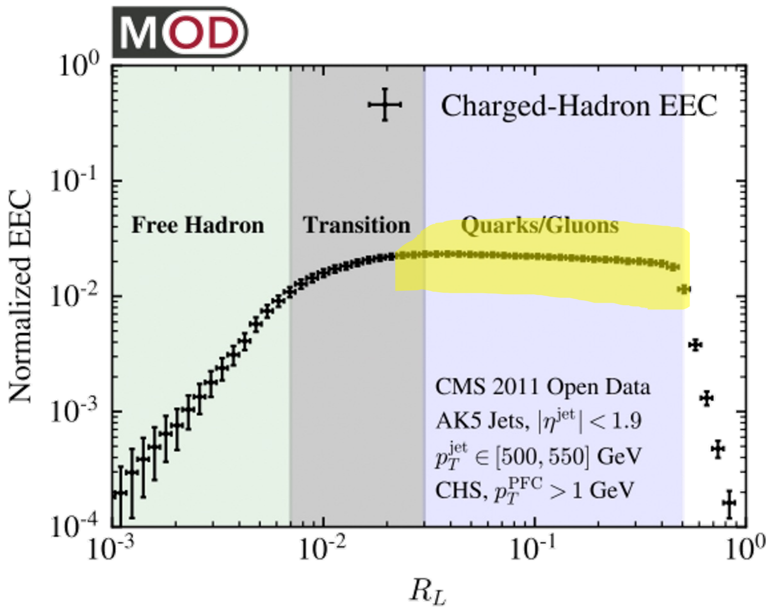


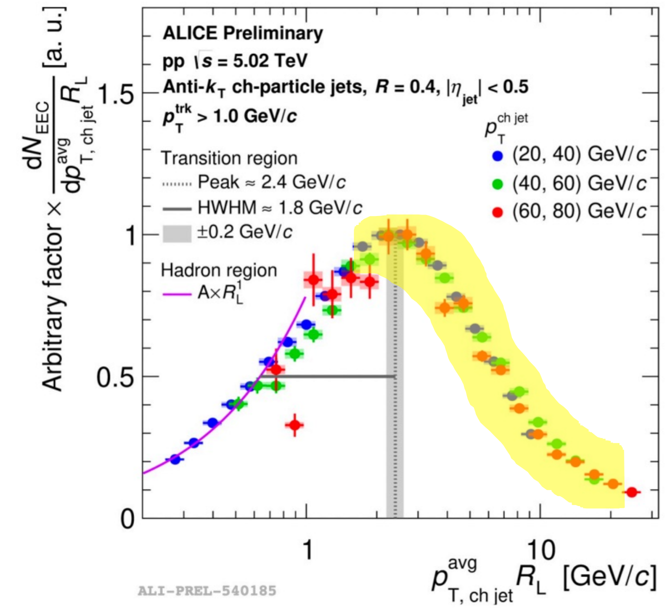
FIG. 2: EEC of a light-quark (blue), c-quark (orange), and b-quark (green) jet in p-p (dashed) and heavy-ion (solid) collisions. Different panels correspond to different jet energies and medium parameters. All curves are normalised by the integrated vacuum result $\Sigma_{\rm vac}$.

Computing Correlation Functions of $\mathcal{E}(\vec{n})$

$$\frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\theta} = \int \mathrm{d}\boldsymbol{n}_{1,2} \frac{\langle \mathcal{E}^n(\boldsymbol{n}_1)\mathcal{E}^n(\boldsymbol{n}_2)\rangle}{Q^{2n}} \delta(\boldsymbol{n}_2 \cdot \boldsymbol{n}_1 - \cos\theta)$$

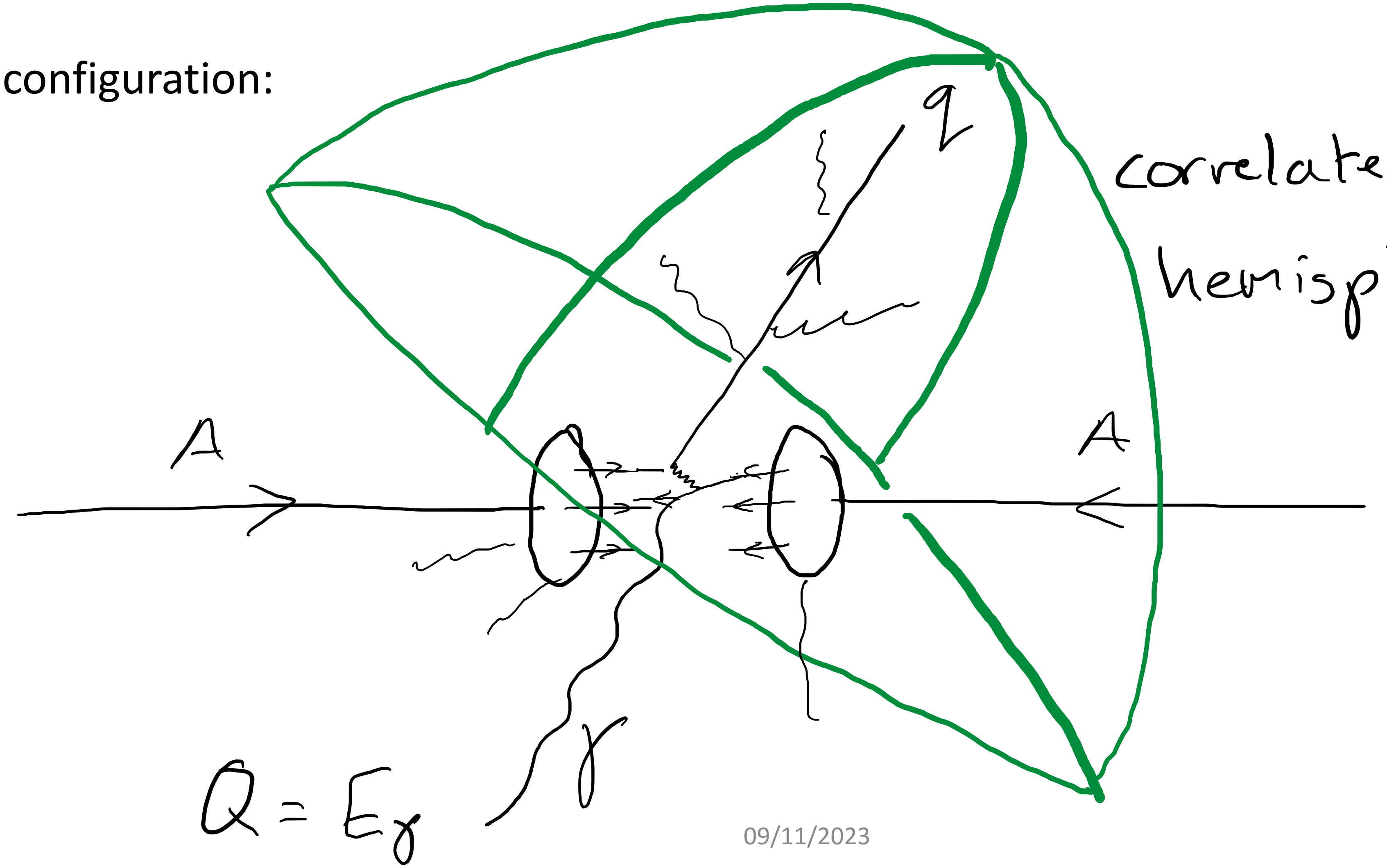
Let me now set up the perturbative calculation we perform.





Computing Correlation Functions of $\mathcal{E}(\vec{n})$

The configuration:



correlate the hemisphere jet.

Computing Correlation Functions of $\mathcal{E}(\vec{n})$

The average momentum exchange between the two correlator points goes as $\sim \theta Q$, the small angle region (where $\theta Q \gg \Lambda_{\mathrm{QCD}}$) is largely determined by perturbative physics. We therefore write the observable as a sum over inclusive partonic cross-sections:

$$\begin{aligned}\frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\theta} =& \frac{1}{\sigma}\int \mathrm{d}E_{q,g}\,\frac{\mathrm{d}\hat{\sigma}_{qg}}{\mathrm{d}\theta\mathrm{d}E_q\mathrm{d}E_g}\,\frac{E_g^n E_q^n}{Q^{2n}} + \frac{1}{\sigma}\int \mathrm{d}E_{g_1,g_2}\,\frac{\mathrm{d}\hat{\sigma}_{g_1 g_2}}{\mathrm{d}\theta\mathrm{d}E_{g_1}\mathrm{d}E_{g_2}}\,\frac{E_{g_1}^n E_{g_2}^n}{Q^{2n}} \\ &+ \frac{1}{\sigma}\int \mathrm{d}E_{q_1,q_2}\,\frac{\mathrm{d}\hat{\sigma}_{q_1 q_2}}{\mathrm{d}\theta\mathrm{d}E_{q_1}\mathrm{d}E_{q_2}}\,\frac{E_{q_1}^n E_{q_2}^n}{Q^{2n}} + (\text{perm. } q \leftrightarrow \bar{q}) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{\theta\, Q}\right)\end{aligned}$$

In pp collisions this is a simple application of CSS inclusive factorisation and can be convoluted with fragmentation or track functions. We must assume this also holds in AA collisions.

Note the finite number of terms in the sum over partonic cross sections!

Computing Correlation Functions of $\mathcal{E}(\vec{n})$

We now re-parameterise the medium contribution to each partonic cross section. This is not a factorisation, just a parameterisation.

$$\frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}\theta \mathrm{d}E_i \mathrm{d}E_j} = \left(1 + F_{\mathrm{med}}^{(ij)}(E_i, E_j, \theta)\right) \frac{\mathrm{d}\hat{\sigma}_{ij}^{\mathrm{vac}}}{\mathrm{d}\theta \mathrm{d}E_i \mathrm{d}E_j}$$

And using this parameterisation we can now compute terms not dependent on $F_{med}^{(ij)}$ using the well developed frameworks from pp physics (pick your favourite between the celestial OPE, SCET, or jet calculus).

I'll show results at LO+NLL for the pp-like terms later on. NLO+NNLL is available in the literature. This part is well understood and not the focus of my talk.

Computing Correlation Functions of $\mathcal{E}(\vec{n})$

Now we must focus on the 'medium' terms that contain the physics intrinsic to HI collisions. So far, we've not approximated anything other than assuming perturbative factorisation. Let's introduce some new helpful variables:

$$\int \mathrm{d}E_{q,g}\, F_{\mathrm{med}}^{(qg)} \frac{\mathrm{d}\hat{\sigma}_{qg}^{\mathrm{vac}}}{\mathrm{d}\theta \mathrm{d}E_q \mathrm{d}E_g} \frac{E_q^n E_g^n}{Q^{2n}} = \int \mathrm{d}z\, \mathrm{d}\mu_{\mathrm{s}}\, F_{\mathrm{med}}^{(qg)} \frac{\mathrm{d}\hat{\sigma}_{qg}^{\mathrm{vac}}}{\mathrm{d}\theta \mathrm{d}z \mathrm{d}\mu_{\mathrm{s}}} z^n (1 - z - \mu_{\mathrm{s}}/Q)^n$$

where $z = E_q/Q$ and $\mu_s = Q - E_q - E_g > 0$ is the energy scale of the radiation over which the perturbative cross sections are inclusive.

With this parameterisation and assuming we are measuring quark jets:

$$\sum_{ij \in \{g,q,\bar{q}\}} \int \mathrm{d}E_{i,j}\, F_{\mathrm{med}}^{(ij)} \frac{\mathrm{d}\hat{\sigma}_{ij}^{\mathrm{vac}}}{\mathrm{d}\theta \mathrm{d}E_i \mathrm{d}E_j} \frac{E_i^n E_j^n}{Q^{2n}}$$

$$= \int \mathrm{d}z\, F_{\mathrm{med}}^{(qg)} \frac{\mathrm{d}\hat{\sigma}_{qg}^{\mathrm{vac}}}{\mathrm{d}\theta \mathrm{d}z} z^n (1-z)^n \left(1 + \mathcal{O}\left(\frac{\bar{\mu}_{\mathrm{s}}}{Q}\right) + \mathcal{O}\left(\alpha_{\mathrm{s}}(\theta Q) \ln \theta \frac{\bar{\mu}_{\mathrm{s}}^n}{Q^n}\right)\right)$$

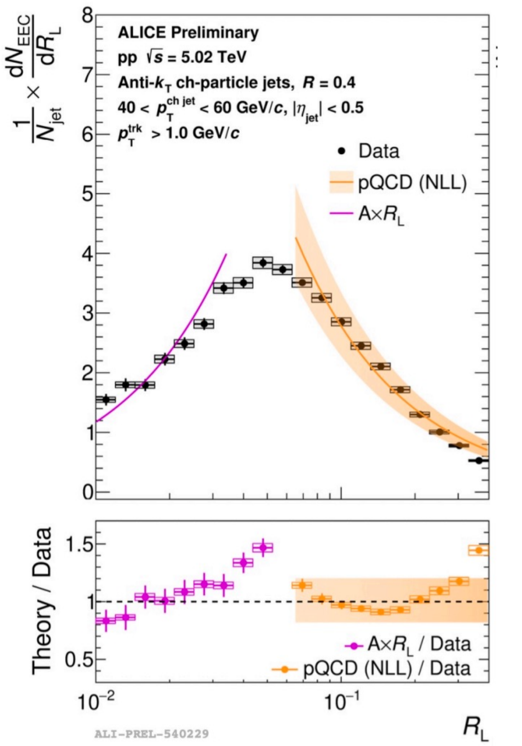
$$\bar{\mu}_{\mathrm{s}}/Q \sim \sqrt{\mu/Q}$$

Debye mass

Computing Correlation Functions of $\mathcal{E}(\vec{n})$

Thus we will compute our observable from the master formula:

$$\frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\theta} = \frac{1}{\sigma}\int \mathrm{d}z \left(g^{(n)}(\theta,\alpha_{\mathrm{s}}) + F_{\mathrm{med}}(z,\theta)\right)\frac{\mathrm{d}\sigma^{\mathrm{vac}}_{qg}}{\mathrm{d}\theta\mathrm{d}z} z^n(1-z)^n\left(1+\mathcal{O}\left(\frac{\bar{\mu}_{\mathrm{s}}}{Q}\right)\right) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{\theta\ Q}\right)$$



As promised, the pp-like part at LO+NLL:

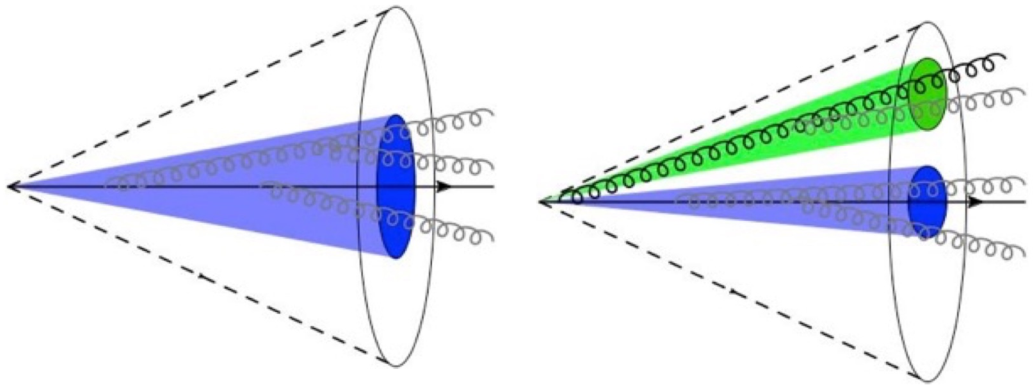
$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma^{\mathrm{vac}}_{qg}}{\mathrm{d}\theta\mathrm{d}z} = \frac{\alpha_{\mathrm{s}}(\theta Q)}{\pi}\ C_{\mathrm{F}}\frac{1+(1-z)^2}{z\ \theta} + \mathcal{O}(\alpha_{\mathrm{s}}^2,\theta^0)$$

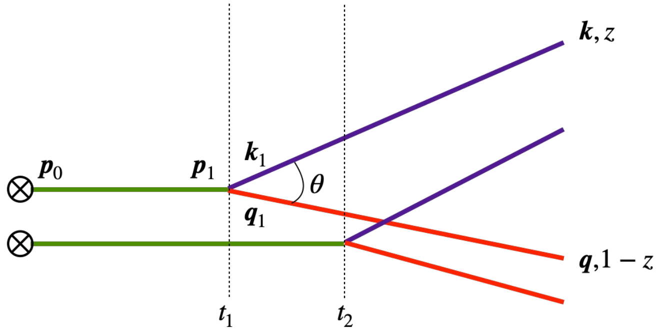
$$g^{(1)} = \left(\left[\left(\frac{\alpha_{\mathrm{s}}(Q)}{\alpha_{\mathrm{s}}(\theta Q)}\right)^{\frac{\hat{\gamma}(3)}{\beta_0}}\right]_{qq} + \frac{2n_f(\gamma_{qg}(2)-\gamma_{qg}(3))+\gamma_{gg}(2)-\gamma_{gg}(3)}{\gamma_{qq}(2)-\gamma_{qq}(3)+\gamma_{gq}(2)-\gamma_{gq}(3)}\left[\left(\frac{\alpha_{\mathrm{s}}(Q)}{\alpha_{\mathrm{s}}(\theta Q)}\right)^{\frac{\hat{\gamma}(3)}{\beta_0}}\right]_{gq}\right)$$
$$+\,\mathcal{O}\left(\alpha_{\mathrm{s}}(Q)^n \ln(\theta)^{n-1}\big|_{n\geq 1}\right) + \mathcal{O}(\theta), \qquad \text{(A.23)}$$

where $\hat{\gamma}(J) = \begin{pmatrix} \gamma_{qq}(J), & 2n_f\gamma_{qg}(J), & 0 \\ \gamma_{gq}(J), & \gamma_{gg}(J), & 0 \\ 0, & 0, & \gamma_{g\tilde{g}}(J) \end{pmatrix}$ is the spin-J twist-2 QCD anomalous dimension matrix.

Computing F_{med}

Leading structure to study is a (quark) jet fragmenting into a jet and a subjet in the presence of a medium.



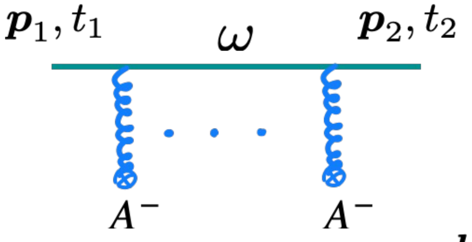


$$\mathcal{M}^{\alpha\beta} = \frac{1}{2E}\int_{\boldsymbol{p}_0\boldsymbol{p}_1\boldsymbol{k}_1\boldsymbol{q}_1}\int_{t_0}^{\infty} dt_1\,(2\pi)^2\delta^{(2)}(\boldsymbol{p}_1-\boldsymbol{k}_1-\boldsymbol{q}_1)\mathcal{G}^{\alpha\alpha_1}_{R_b}(\boldsymbol{k},L;\boldsymbol{k}_1,t_1;zE)$$
$$\times\,\mathcal{G}^{\beta\beta_1}_{R_c}(\boldsymbol{q},L;\boldsymbol{q}_1,t_1;(1-z)E)\,V(\boldsymbol{k}_1-z\boldsymbol{p}_1,z)T^{\alpha_1\beta_1\gamma_1}\,\mathcal{G}^{\gamma_1\gamma}_{R_a}(\boldsymbol{p}_1,t_1;\boldsymbol{p}_0,t_0;E)\,\mathcal{M}_0^{\gamma}(E,\boldsymbol{p}_0)$$

Computing F_{med}

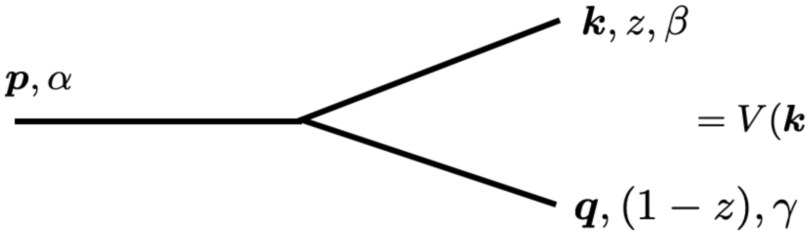
The formalism we use, based in BDMPS-Z:

- All particles have a large longitudinal momentum compared to their transverse momenta and therefore there is a decoupling between transverse and longitudinal dynamics
- We work in a mixed representation with momentum coordinates in the transverse direction and “time” (+ coordinate) in the longitudinal direction.
- Multiple scatterings resumed through propagators in a background field



$$= \mathcal{G}_R(\boldsymbol{p}_2, t_2; \boldsymbol{p}_1, t_1; \omega)$$

- Vacuum vertices



$$= V(\boldsymbol{k} - z\boldsymbol{p}, z) T^{\alpha\beta\gamma} (2\pi)^2 \delta^{(2)}(\boldsymbol{p} - \boldsymbol{k} - \boldsymbol{q})$$

- Background field averaged at the level of the cross section

$$\left\langle A^{a-}(\boldsymbol{q}_1, t_1) A^{b-\dagger}(\boldsymbol{q}_2, t_2) \right\rangle = \delta^{ab} \delta(t_2 - t_1) \delta^{(2)}(\boldsymbol{q}_1 - \boldsymbol{q}_2) v(\boldsymbol{q}_1)$$

Computing F_{med}

Full evaluation keeping z and θ not yet achieved.

Two available approximations:

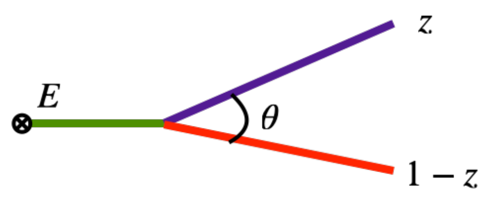
- Opacity expansion ($N = 1$) arXiv:1807.03799
    - Unitarity problems can lead to negative cross sections.
    - Recursive formulas to generate all orders (not yet implemented numerically).

- "Tilted" Wilson lines
    - Resums multiple scatterings in the eikonal approximation.
    - Assumes semi-hard splittings (z not too small).
    - We implement this using both a Yukawa and HO potential for medium scatterings and for now using the leading colour limit.

arXiv:1907.03653

arXiv:2107.02542

Computing F_{med}



For intuition, focus on HO for a bit.

- For a static medium of length within the harmonic approximation one can read off the relevant scales directly from the formulas arXiv:1907.03653
  - (Vacuum) formation time:

$$t_f = \frac{2}{z(1-z)E\theta^2} \qquad \theta_L \sim (EL)^{-1/2}$$

Below θ_L all emissions have a formation time larger than L. This emerges as near complete cancellations between dipole and quadrapole (in-out and in-in) terms in $\mathrm{F}_{\mathrm{med}}^{(ij)}$ driving it rapidly to zero below θ_L.

  - Decoherence time:

$$S_{12}(\tau) = e^{-\frac{1}{12}\hat{q}(1+z^2)\theta^2\tau^3}$$

$$t_d \sim (\hat{q}\theta^2)^{-1/3} \qquad \theta_c \sim (\hat{q}L^3)^{-1/2}$$

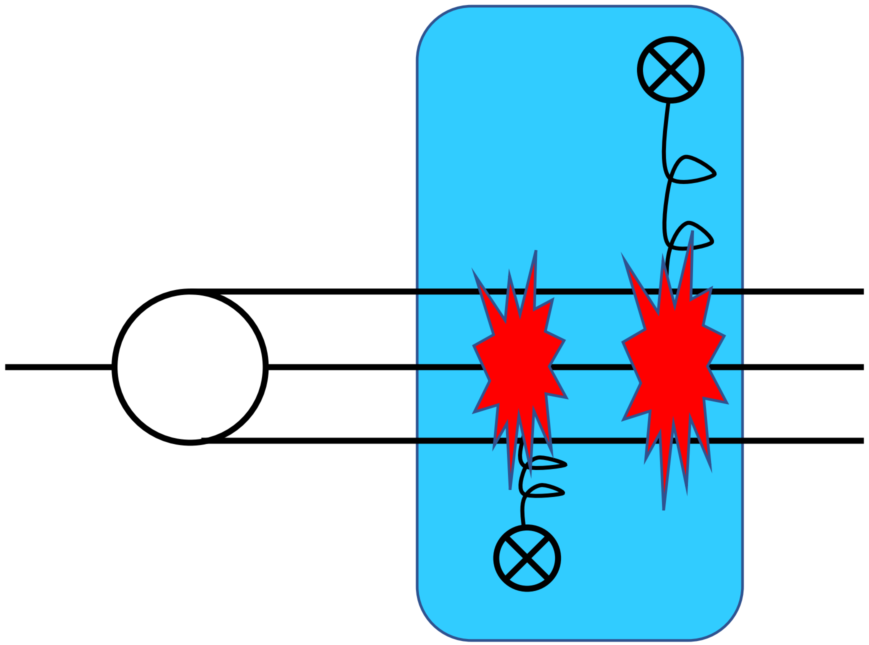
Below θ_c emissions do not colour decohere and the medium does not independently resolve them. This emerges as an exponential suppression in the factorisable dipole terms within $\mathrm{F}_{\mathrm{med}}^{(ij)}$ forcing it to become small below θ_c.

If $\theta_L > \theta_c$ then θ_c becomes irrelevant

Interpretation for F_{med}

$$\theta_{\mathrm{c}} \gg \theta_{\mathrm{L}}$$

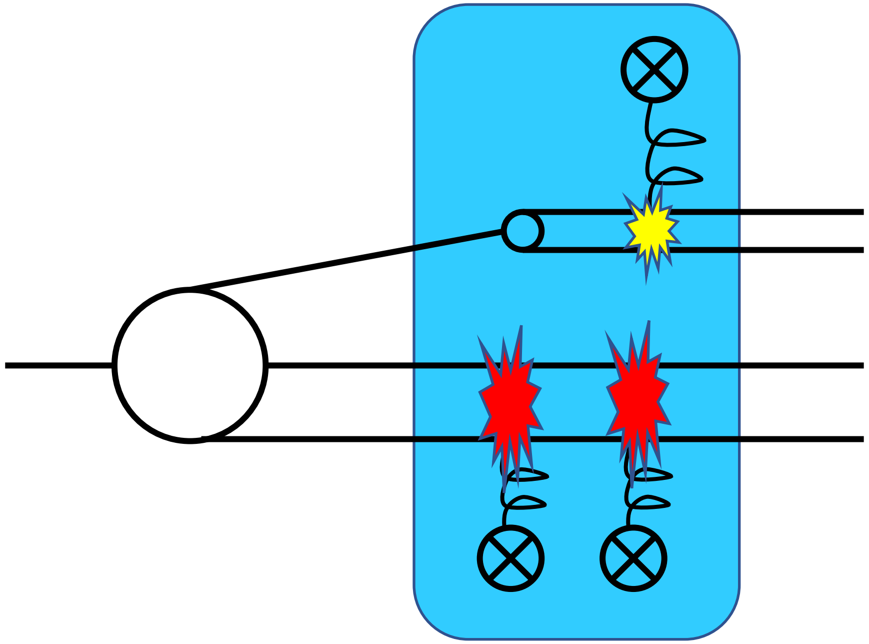
$$E \gg \hat{q}L^2$$



For angles $\theta_{\mathrm{c}} \gg \theta \gg \theta_{\mathrm{L}}$, the quark jet undergoes some minimal energy loss but the substructure is not resolved.

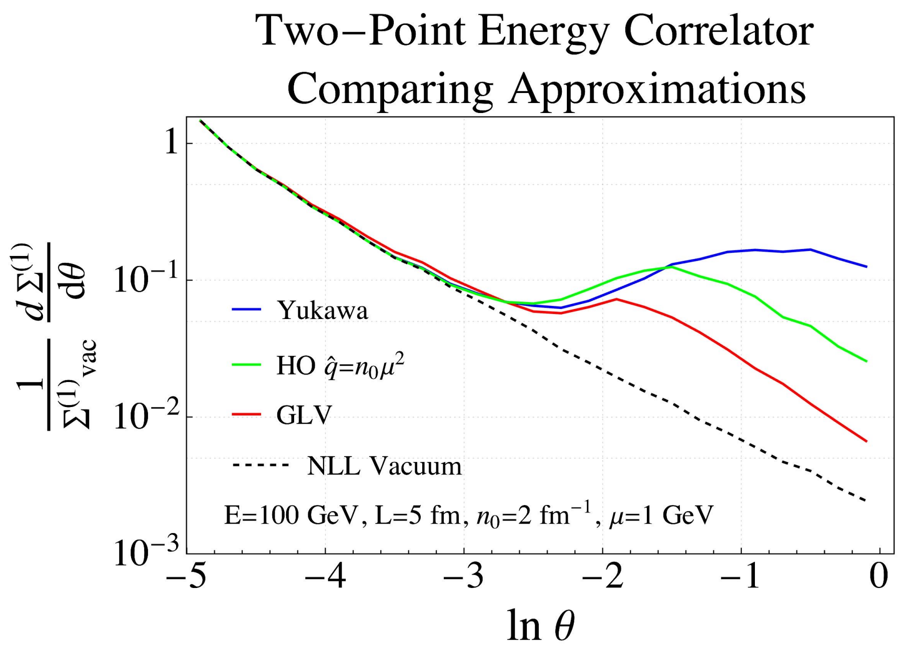
$$\theta_{\mathrm{c}} \ll \theta_{\mathrm{L}}$$

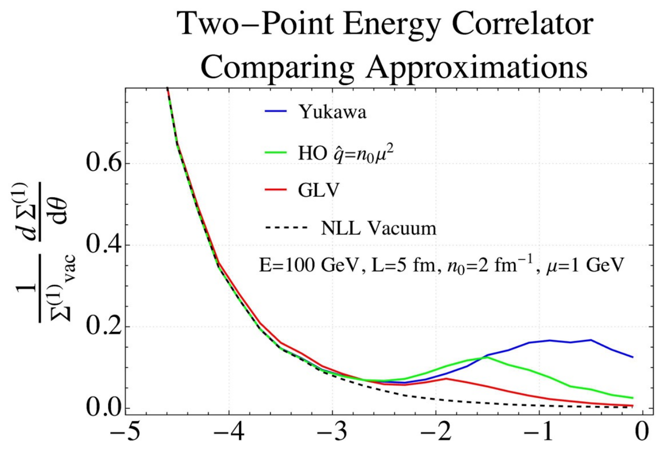
$$E \ll \hat{q}L^2$$

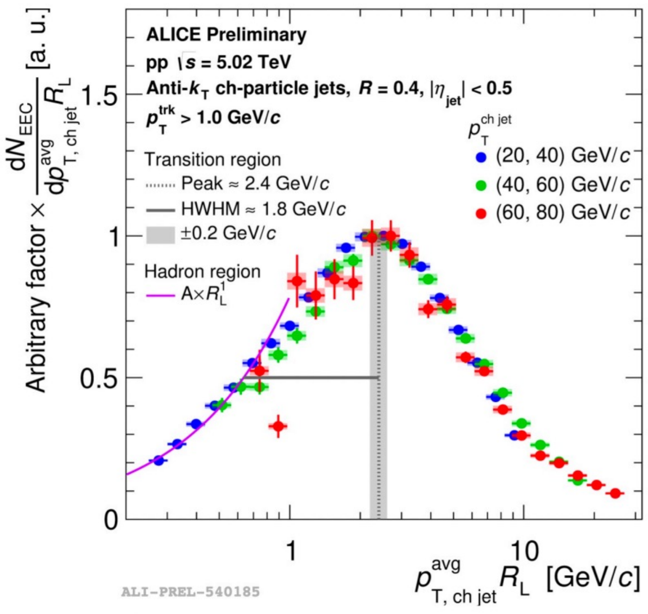


Initial splitting can be resolved by the medium when $\theta \gg \theta_{\mathrm{L}}$. Broadening and energy loss occur.

Numerical evaluation of F_{med}



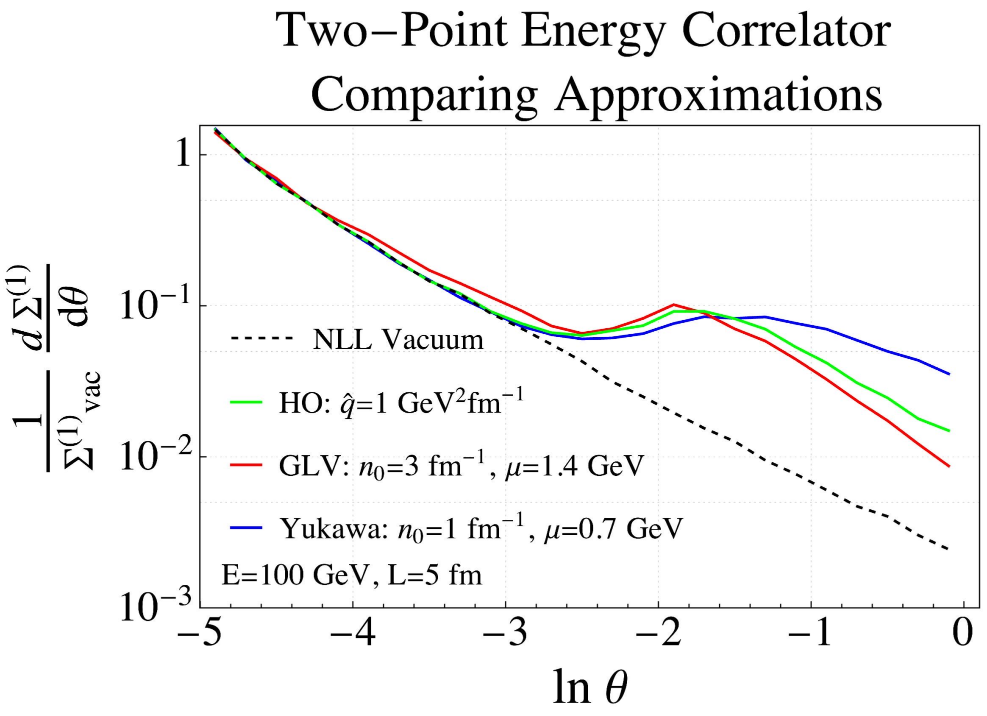




General features: an enhancement which begins above θ_L, at $\theta \gg \theta_L$ the enhancement peaks and then settles into a new medium dependent scaling law.

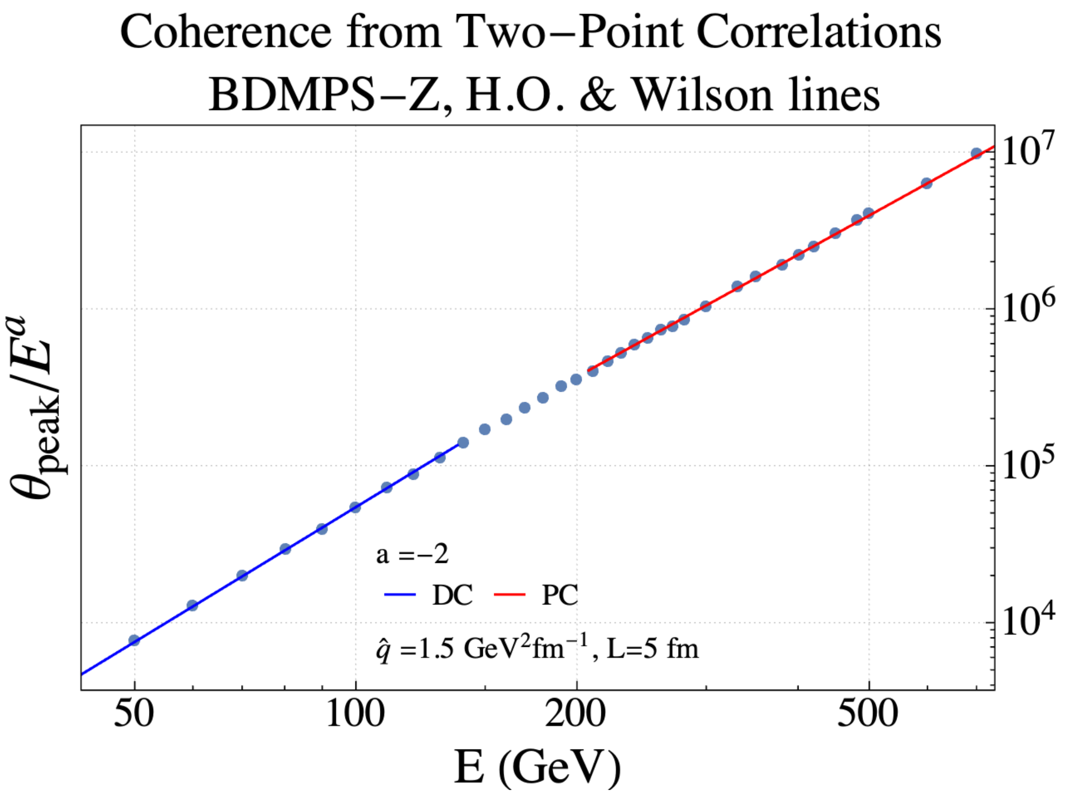
Amplitudes appear model dependent.

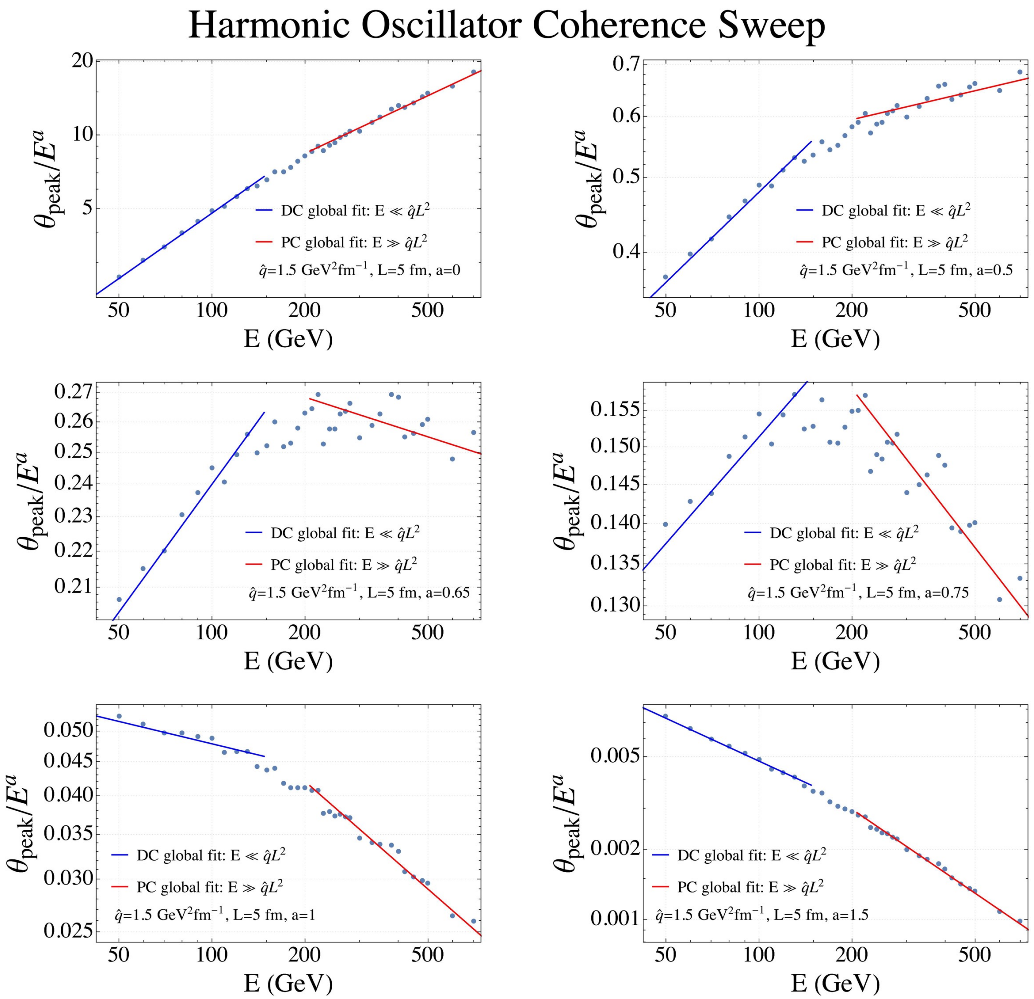
Numerical evaluation of F_{med}



Whilst amplitudes are very model dependent, the differences can be fairly well absorbed into variation of the model parameters (not so much the wide angle though).

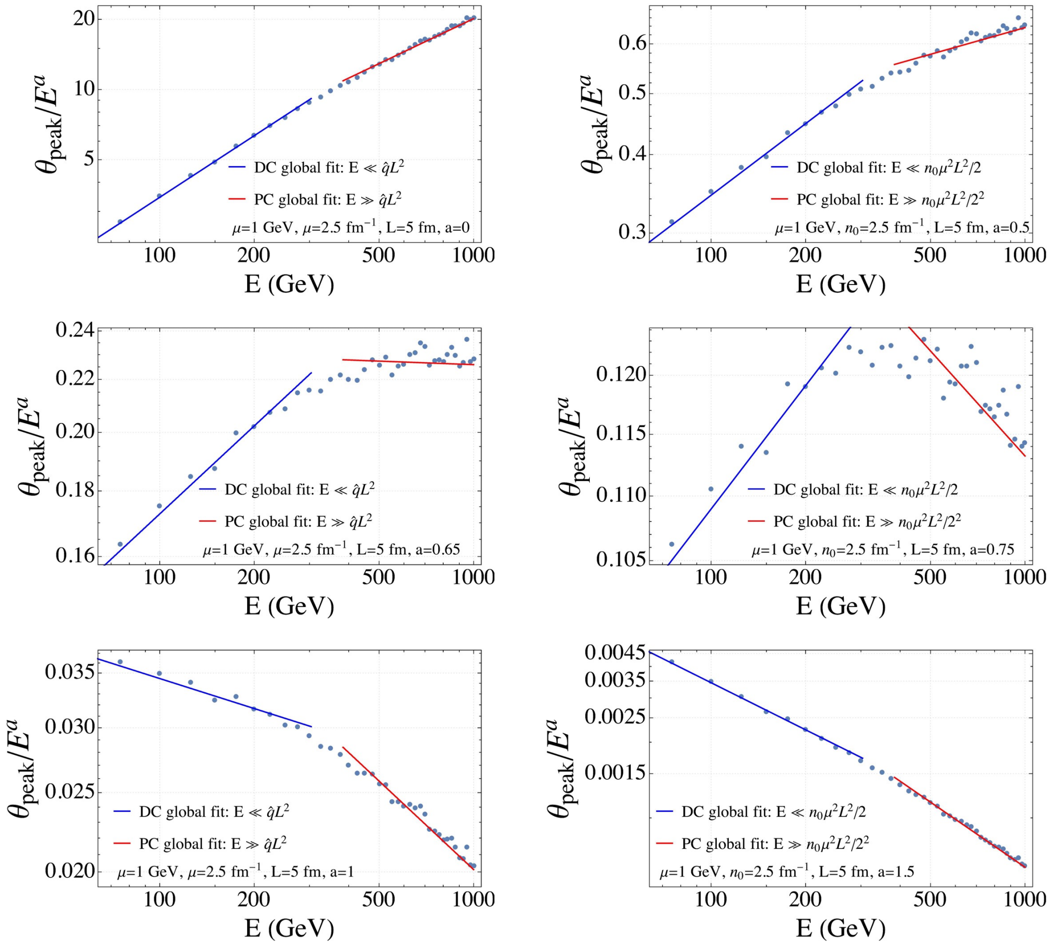
Numerical evaluation of F_{med}



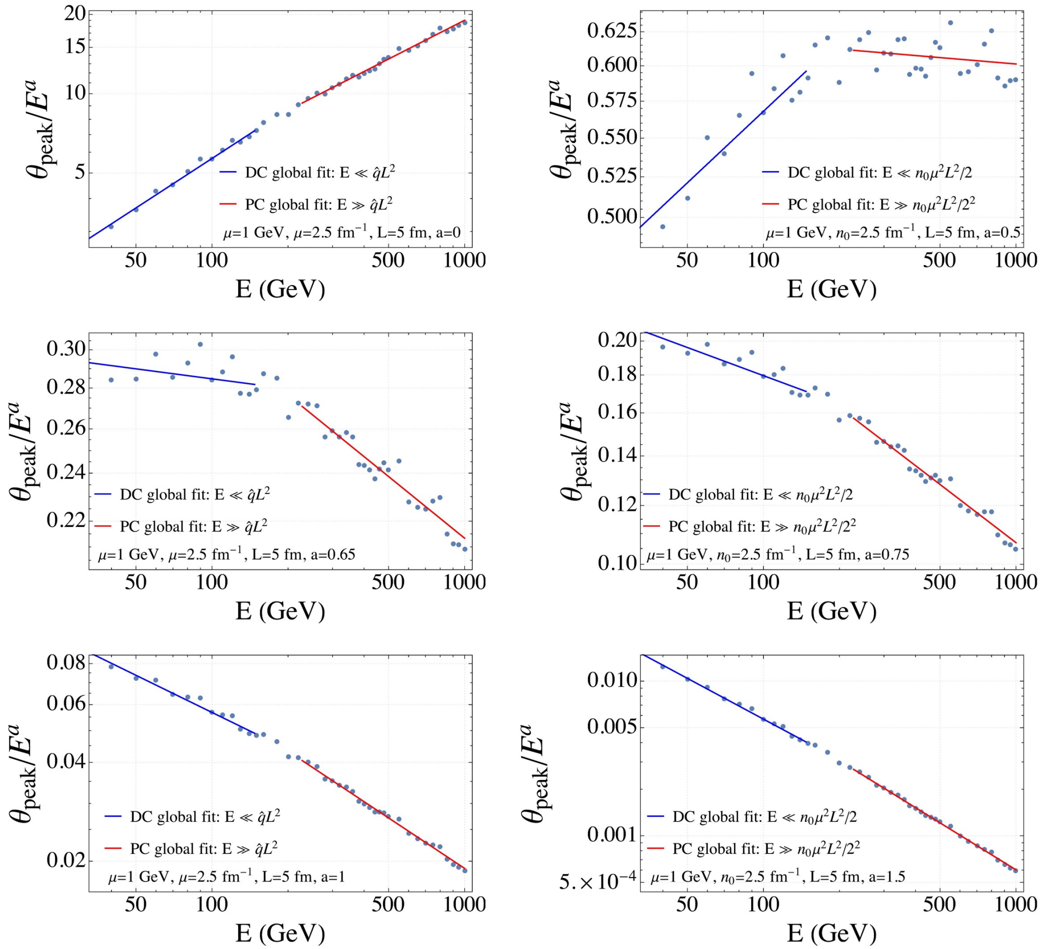


Numerical evaluation of F_{med}

Yukawa Potential Coherence Sweep

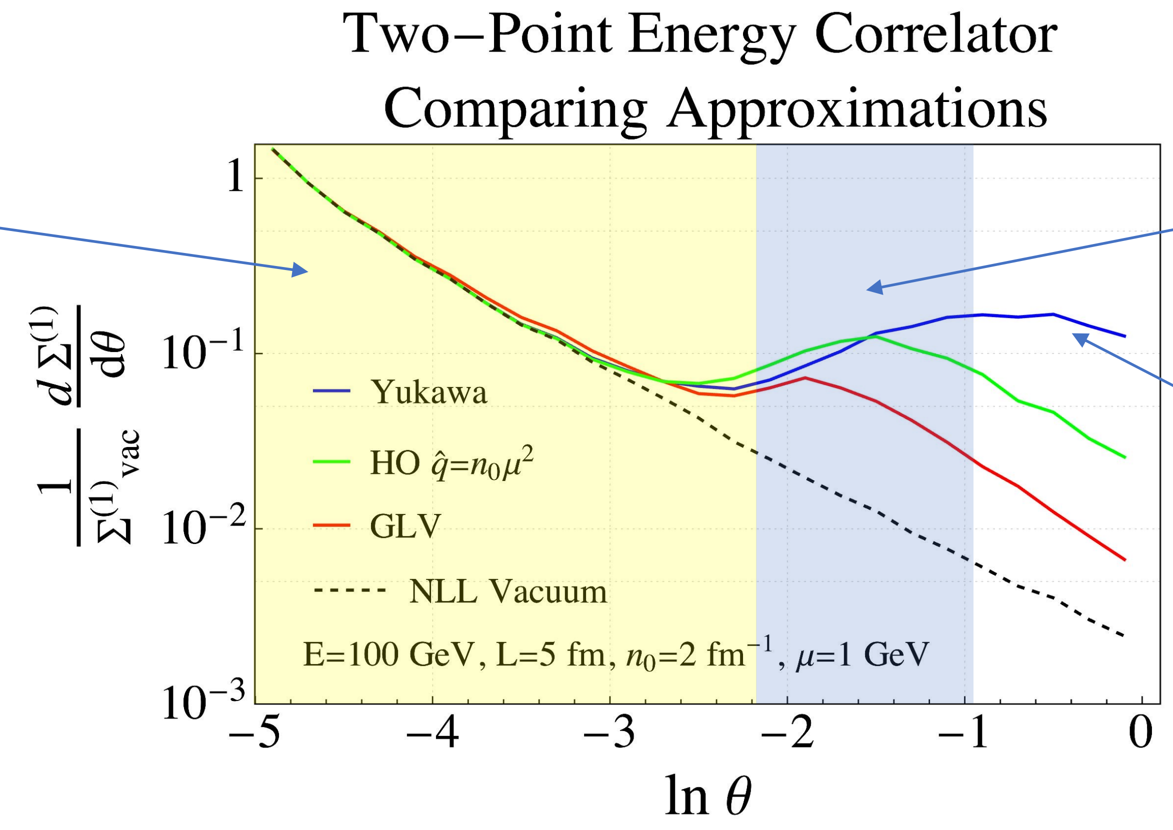


GLV Approximation Coherence Sweep



Numerical evaluation of F_{med}

Controlled by θ_L

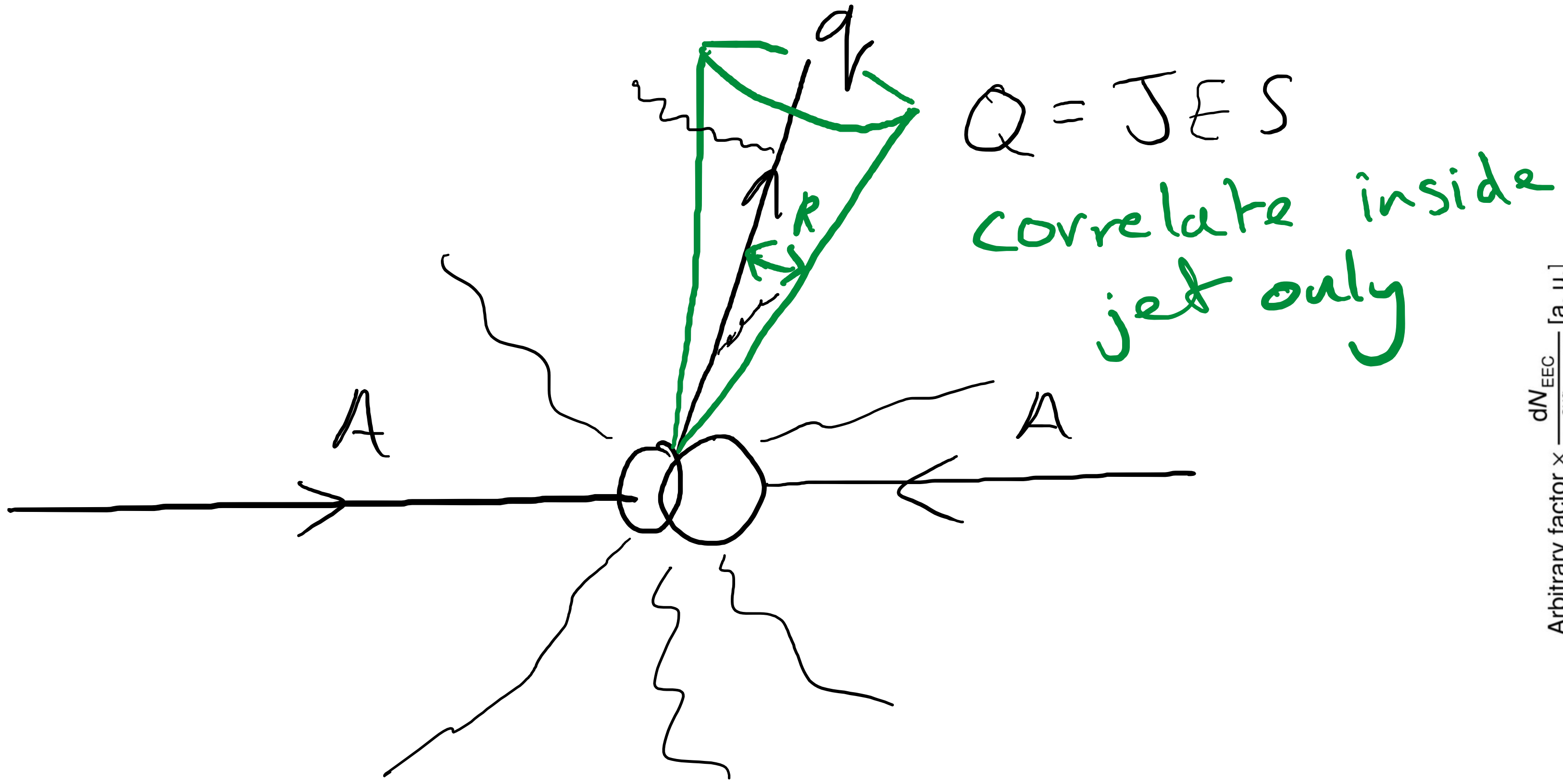


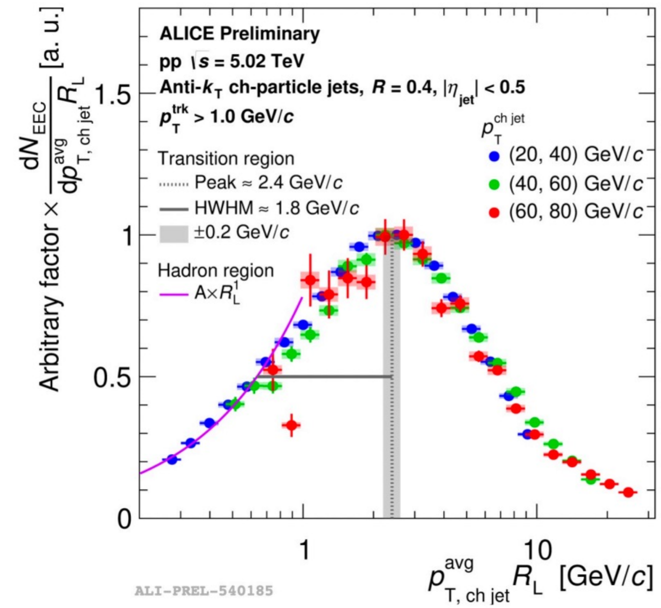
Controlled by $(\theta_c - \theta_L)$

Controlled by number of large scatters limit. Behaves as if medium is infinitely long and so always decoherent.

Provided $E \gg E_c \sim \hat{q}L^2$

Computation on inclusive jets

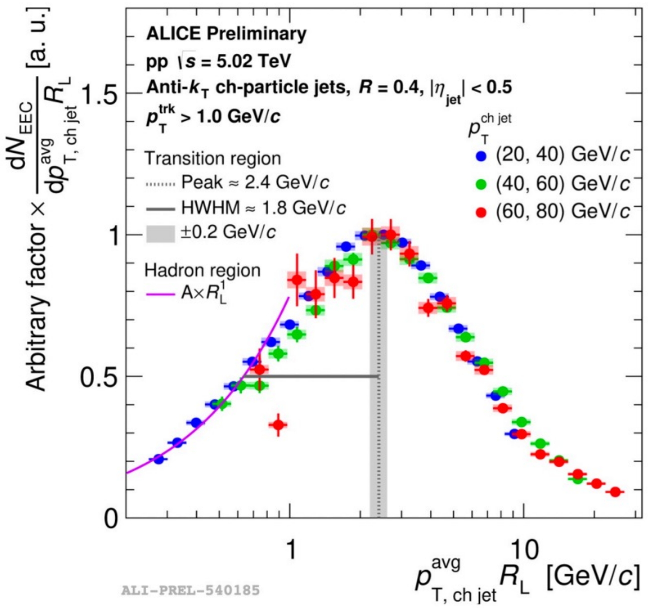




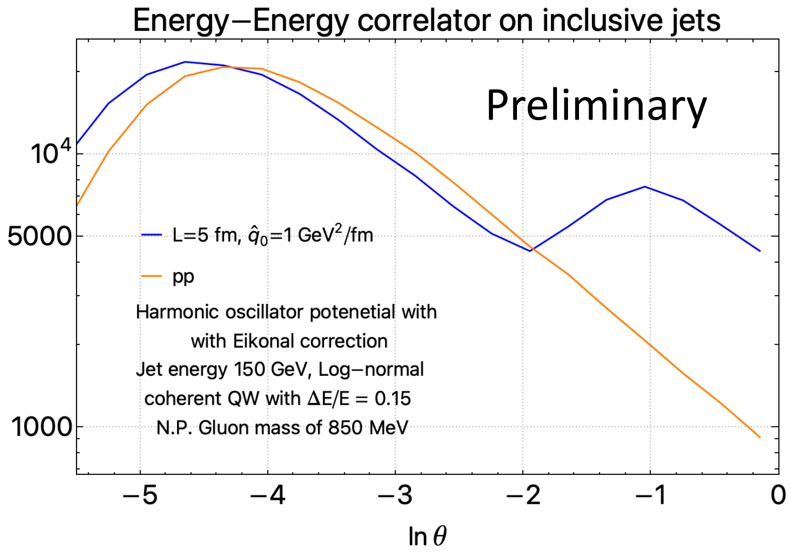
Computation on inclusive jets

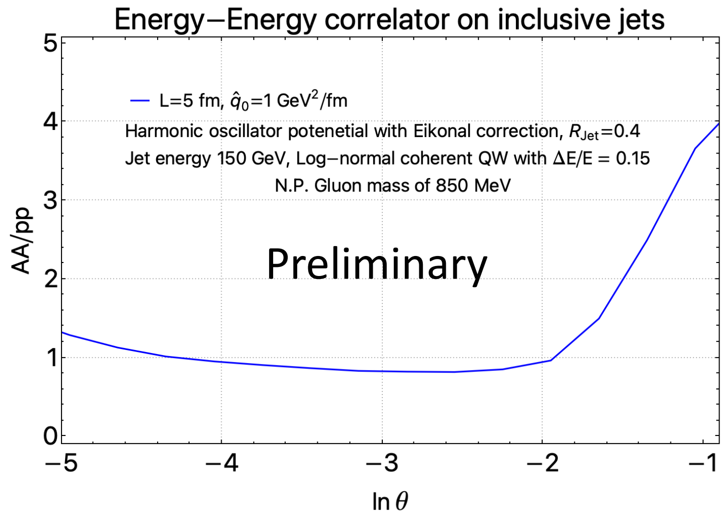
Energy loss will now be important because of the dependence on the JES and due to its large overall effect on narrower jets. Quenching weights now are an important addition to the calculation. [2307.08943](2307.08943)

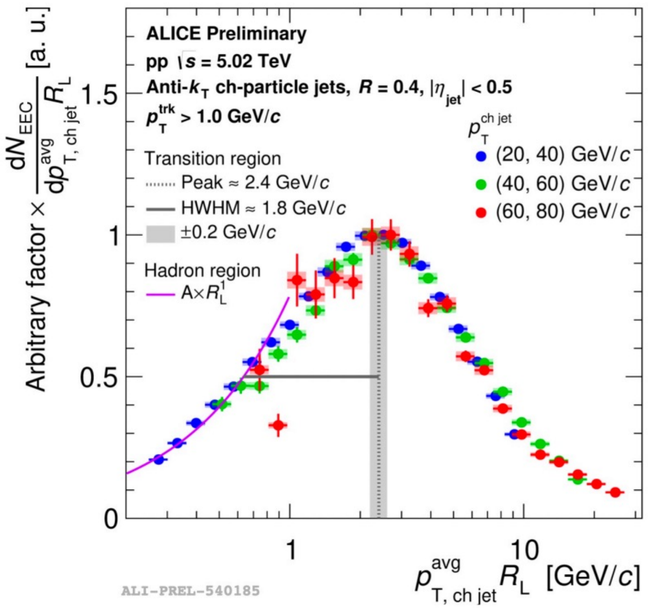
Additionally, the NP transition is a large JES dependent feature. It will be subject to energy loss with small knock-on effects throughout the spectrum. A model is needed, we use a finite NP gluon mass. [0802.1870](0802.1870) [hep-ph/9808392](hep-ph/9808392)



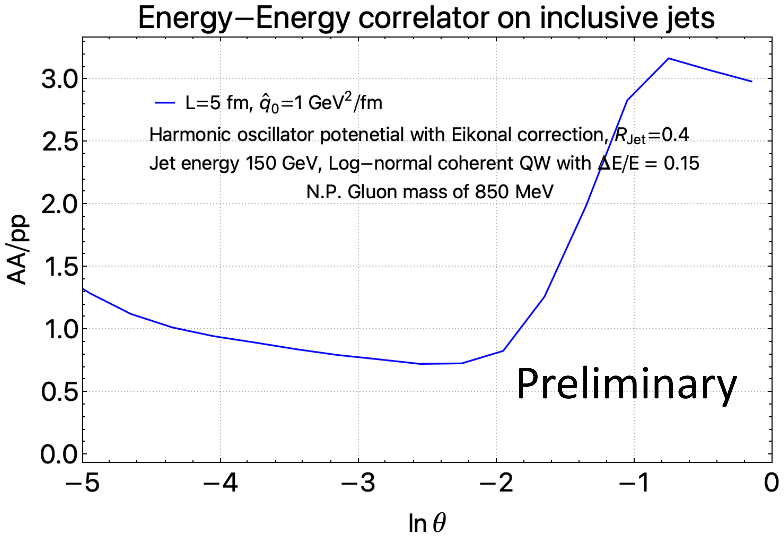
Computation on inclusive jets

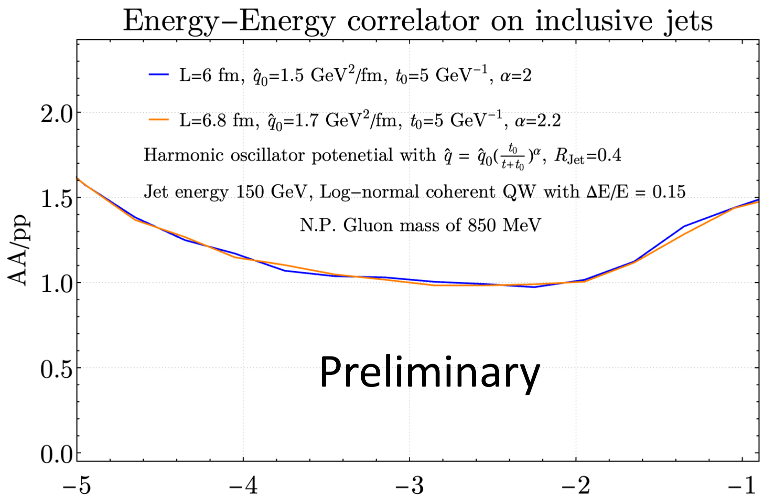


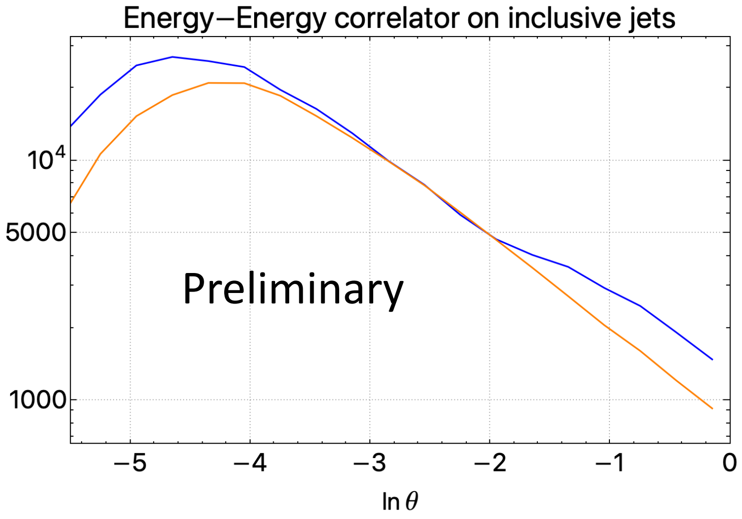




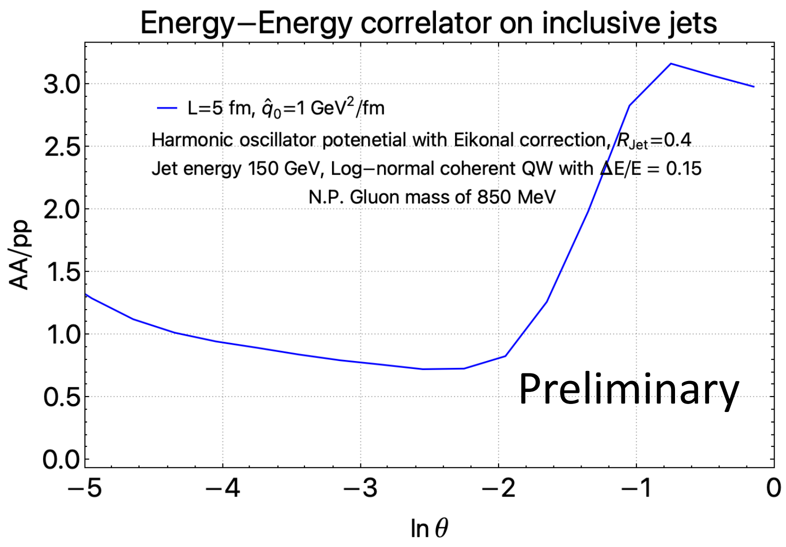
Improving computations on inclusive jets

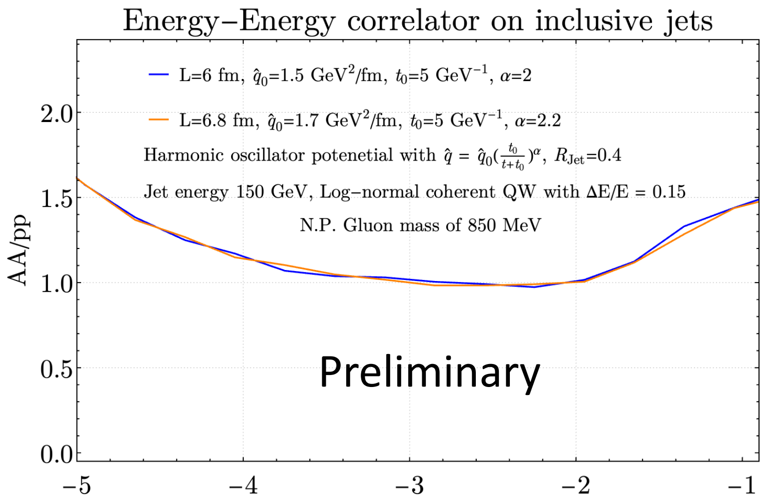


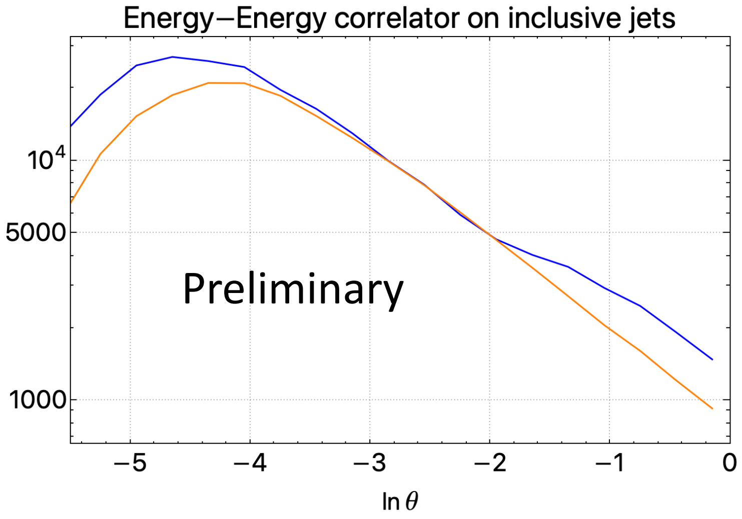




Improving computations on inclusive jets







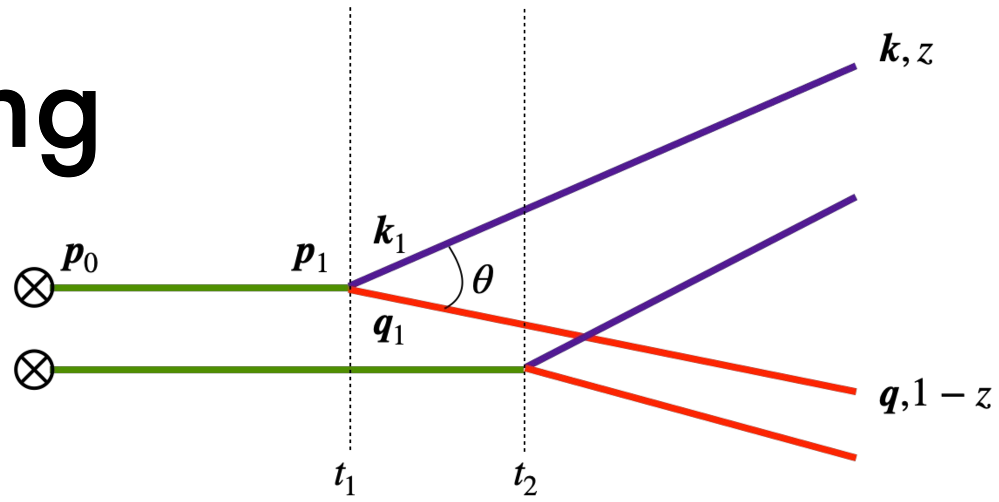
These are early results which still use relatively simple modelling.

We are working on more sophisticated calculations and hope to release first predictions in the near future!

Thanks!

Part N/A: Supplemental Material

Jet Quenching



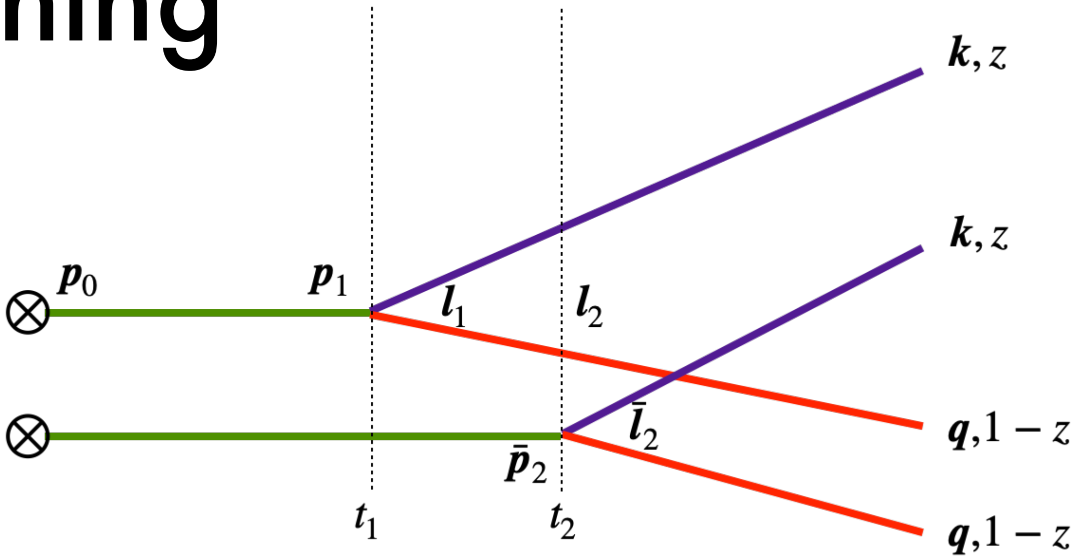
$$\mathcal{M}^{\alpha\beta} = \frac{1}{2E}\int_{\boldsymbol{p}_0\boldsymbol{p}_1\boldsymbol{k}_1\boldsymbol{q}_1}\int_{t_0}^{\infty} dt_1\,(2\pi)^2\delta^{(2)}(\boldsymbol{p}_1-\boldsymbol{k}_1-\boldsymbol{q}_1)\mathcal{G}_{R_b}^{\alpha\alpha_1}(\boldsymbol{k},L;\boldsymbol{k}_1,t_1;zE)$$
$$\times\,\mathcal{G}_{R_c}^{\beta\beta_1}(\boldsymbol{q},L;\boldsymbol{q}_1,t_1;(1-z)E)\,V(\boldsymbol{k}_1-z\boldsymbol{p}_1,z)T^{\alpha_1\beta_1\gamma_1}\,\mathcal{G}_{R_a}^{\gamma_1\gamma}(\boldsymbol{p}_1,t_1;\boldsymbol{p}_0,t_0;E)\,\mathcal{M}_0^{\gamma}(E,\boldsymbol{p}_0)$$

Where each of the in-medium propagators is of the form:

$$\mathcal{G}_R(t_2,\boldsymbol{x}_2;t_1,\boldsymbol{x}_1;\omega) = \int_{\boldsymbol{x}_1}^{\boldsymbol{x}_2}\mathcal{D}\boldsymbol{r}\,\exp\left\{\frac{i\omega}{2}\int_{t_1}^{t_2}d\xi\,\dot{\boldsymbol{r}}^2(\xi)\right\}\underbrace{\mathrm{P}\exp\left\{ig\int_{t_1}^{t_2}d\xi\,A_R^-(\xi,\boldsymbol{r}(\xi))\right\}}_{V_R(t_2,t_1;[\boldsymbol{r}])}$$

$$\left\langle|\mathcal{M}|^2\right\rangle \propto \Big\langle\mathcal{G}_{R_b}^{\alpha\alpha_1}(\boldsymbol{k},L;\boldsymbol{k}_1,t_1;zE)\mathcal{G}_{R_c}^{\beta\beta_1}(\boldsymbol{q},L;\boldsymbol{q}_1,t_1;(1-z)E)\mathcal{G}_{R_b}^{\dagger\bar{\alpha}_2\alpha}(\bar{\boldsymbol{k}}_2,t_2;\boldsymbol{k},L;zE)$$
$$\times\mathcal{G}_{R_c}^{\dagger\bar{\beta}_2\beta}(\bar{\boldsymbol{q}}_2,t_2;\boldsymbol{q},L;(1-z)E)\mathcal{G}_{R_a}^{\gamma_1\gamma}(\boldsymbol{p}_1,t_1;\boldsymbol{p}_0,t_0;E)\mathcal{G}_{R_a}^{\dagger\bar{\gamma}\bar{\gamma}_2}(\bar{\boldsymbol{p}}_0,t_0;\bar{\boldsymbol{p}}_2,t_2;E)\Big\rangle$$

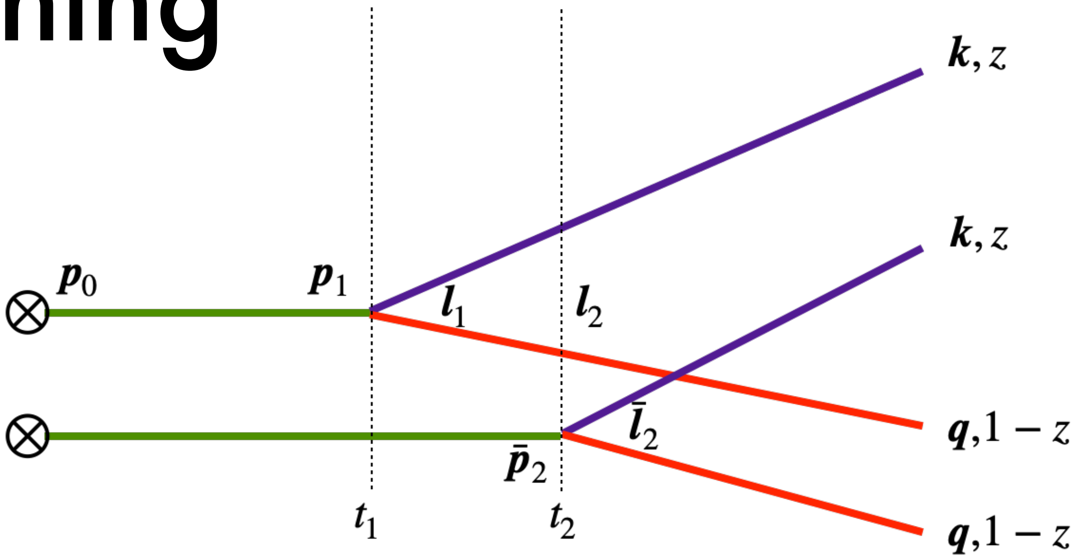
Jet Quenching



$$\frac{d\sigma}{d\Omega_k d\Omega_q} = \frac{g^2}{z(1-z)E^2} P_{a\to bc}(z)\, 2\mathrm{Re} \int_{\boldsymbol{p}_0 \boldsymbol{p}_1 \bar{\boldsymbol{p}}_2 \boldsymbol{l}_1 \boldsymbol{l}_2 \bar{\boldsymbol{l}}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\boldsymbol{l}_1 \cdot \bar{\boldsymbol{l}}_2)$$
$$\times\, \mathcal{S}^{(4)}((1-z)\boldsymbol{k} - z\boldsymbol{q}, L; \boldsymbol{l}_2, \bar{\boldsymbol{l}}_2, t_2; \boldsymbol{k}+\boldsymbol{q}-\bar{\boldsymbol{p}}_2, z)$$
$$\times\, \mathcal{K}^{(3)}(\boldsymbol{l}_2, t_2; \boldsymbol{l}_1, t_1; \bar{\boldsymbol{p}}_2 - \boldsymbol{p}_1, z)\, \mathcal{P}_{R_a}(\boldsymbol{p}_1 - \boldsymbol{p}_0; t_1, t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}}$$

Jet Quenching

BDMPS-Z



$$\frac{d\sigma}{d\Omega_k d\Omega_q} = \frac{g^2}{z(1-z)E^2} P_{a\to bc}(z)\, 2\mathrm{Re}\int_{\boldsymbol{p}_0\boldsymbol{p}_1\bar{\boldsymbol{p}}_2\boldsymbol{l}_1\boldsymbol{l}_2\bar{\boldsymbol{l}}_2}\int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\boldsymbol{l}_1\cdot\bar{\boldsymbol{l}}_2)$$
$$\times\, \mathcal{S}^{(4)}((1-z)\boldsymbol{k}-z\boldsymbol{q}, L; \boldsymbol{l}_2, \bar{\boldsymbol{l}}_2, t_2; \boldsymbol{k}+\boldsymbol{q}-\bar{\boldsymbol{p}}_2, z)$$
$$\times\, \mathcal{K}^{(3)}(\boldsymbol{l}_2, t_2; \boldsymbol{l}_1, t_1; \bar{\boldsymbol{p}}_2-\boldsymbol{p}_1, z)\, \mathcal{P}_{R_a}(\boldsymbol{p}_1-\boldsymbol{p}_0; t_1, t_0)\frac{d\sigma_{hard}}{d\Omega_{p_0}}$$

$\langle \mathcal{G}\mathcal{G}\mathcal{G}^\dagger\mathcal{G}^\dagger \rangle$

$\langle \mathcal{G}\mathcal{G}\mathcal{G}^\dagger \rangle$

$\langle \mathcal{G}\mathcal{G}^\dagger \rangle$

Part N/A: Supplemental Material

$$\mathcal{M}_{\gamma\to q\bar{q}} = \frac{e}{E}\mathrm{e}^{i\frac{\boldsymbol{p}_1^2}{2zE}L+i\frac{\boldsymbol{p}_2^2}{2(1-z)E}L}\int_0^\infty \mathrm{d}t\,\int_{\boldsymbol{k}_1,\boldsymbol{k}_2}\left[\mathcal{G}(\boldsymbol{p}_1,L;\boldsymbol{k}_1,t|zE)\,\bar{\mathcal{G}}(\boldsymbol{p}_2,L;\boldsymbol{k}_2,t|(1-z)E)\right]_{ij}$$
$$\times\,\gamma_{\lambda,s,s'}(z)\boldsymbol{k}\cdot\boldsymbol{\epsilon}_\lambda^*\,\mathcal{G}_0(\boldsymbol{k}_1+\boldsymbol{k}_2,t|E)$$

$$\mathcal{G}(\boldsymbol{p}_1,t_1;\boldsymbol{p}_0,t_0) = \int_{\boldsymbol{x}_1,\boldsymbol{x}_2}\mathrm{e}^{-i\boldsymbol{p}_1\cdot\boldsymbol{x}_1+i\boldsymbol{p}_0\cdot\boldsymbol{x}_0}\mathcal{G}(\vec{x}_1,\vec{x}_0)$$

$$\mathcal{G}(\vec{x}_1,\vec{x}_0) = \int_{\boldsymbol{r}(t_0)=\boldsymbol{x}_0}^{\boldsymbol{r}(t_1)=\boldsymbol{x}_1}\mathcal{D}\boldsymbol{r}\,\exp\left[i\frac{E}{2}\int_{t_0}^{t_1}\mathrm{d}s\,\dot{\boldsymbol{r}}^2\right]V(t_1,t_0;[\boldsymbol{r}])$$

$$V(t_1,t_0;[\boldsymbol{r}]) = \mathcal{P}\exp\left[ig\int_{t_0}^{t_1}\mathrm{d}t\,\mathbf{t}^a A^{-,a}(t,\boldsymbol{r}(t))\right]$$

$$\frac{\mathrm{d}N^{\mathrm{med}}}{\mathrm{d}z\mathrm{d}\boldsymbol{p}^2} = \frac{1}{4(2\pi)^2\,z(1-z)}\langle|\mathcal{M}_{\gamma\to q\bar{q}}|^2\rangle = \frac{1}{4(2\pi)^2\,z(1-z)}\langle\left|\mathcal{M}^{\mathrm{in}}_{\gamma\to q\bar{q}}+\mathcal{M}^{\mathrm{out}}_{\gamma\to q\bar{q}}\right|^2\rangle$$

Part N/A: Supplemental Material

$$\frac{\mathrm{d}\sigma_{qg}}{\mathrm{d}\theta\mathrm{d}z} = \frac{\mathrm{d}\sigma_{qg}^{\mathrm{vac}}}{\mathrm{d}\theta\mathrm{d}z}(1 + F_{\mathrm{med}}(z,\theta,\hat{q},L))$$

$$F_{\mathrm{med}} = 2\int_0^L \frac{\mathrm{d}t_1}{t_{\mathrm{f}}}\left[\int_{t_1}^L \frac{\mathrm{d}t_2}{t_{\mathrm{f}}}\cos\left(\frac{t_2-t_1}{t_{\mathrm{f}}}\right)\mathcal{C}^{(4)}(L,t_2)\mathcal{C}^{(3)}(t_2,t_1) - \sin\left(\frac{L-t_1}{t_{\mathrm{f}}}\right)\mathcal{C}^{(3)}(L,t_1)\right]$$

$$\mathcal{C}_{gq}^{(3)}(t_2,t_1) = \frac{1}{N_c^2-1}\left\langle \mathrm{tr}[V_2^\dagger V_1]\,\mathrm{tr}[V_0^\dagger V_2] - \frac{1}{N_c}\mathrm{tr}[V_0^\dagger V_1]\right\rangle .$$

$$\begin{aligned}\mathcal{C}_{gq}^{(3)}(t_2,t_1) &= \mathrm{e}^{-\frac{1}{2}\int_{t_1}^{t_2}\mathrm{d}s\,n(s)[N_c(\sigma_{02}+\sigma_{12})-\frac{1}{N_c}\sigma_{01}]}\\ &= \mathrm{e}^{-\frac{1}{12}\hat{q}(t_2-t_1)^3\theta^2\left(1+z^2+\frac{2z}{N_c^2-1}\right)} .\end{aligned}$$

$$\mathcal{C}_{gq}^{(4)}(L,t_2) = \frac{1}{N_c^2-1}\left\langle \mathrm{tr}[V_{\bar{1}}^\dagger V_1 V_2^\dagger V_{\bar{2}}]\,\mathrm{tr}[V_{\bar{2}}^\dagger V_2] - \frac{1}{N_c}\mathrm{tr}[V_{\bar{1}}^\dagger V_1]\right\rangle ,$$

$$\begin{aligned}&\frac{1}{N_c^2}\langle \mathrm{tr}[V_1 V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger]\,\mathrm{tr}[V_2 V_{\bar{2}}^\dagger]\rangle \simeq \mathrm{e}^{-\frac{1}{4}\hat{q}\theta^2(t-t_2)(t_2-t_1)^2(1-2z+3z^2)}\\ &\times\left(1-\frac{1}{2}\hat{q}\theta^2 z(1-z)(t_2-t_1)^2\int_{t_2}^{t}\mathrm{d}s\,\mathrm{e}^{-\frac{1}{12}\hat{q}\theta^2\left[(s-t_2)^2(2s-3t_1+t_2)+6z(1-z)(s-t_2)(t_2-t_1)^2\right]}\right)\end{aligned}$$