Small-x TMD factorization at NLO

Farid Salazar
UC Berkeley -> INT U. Washington

Advancing the understanding of NP QCD using energy flow
Nov 7th, 2023

Outline

- Color Glass Condensate in a nutshell

- Dijet production in DIS in the CGC at NLO

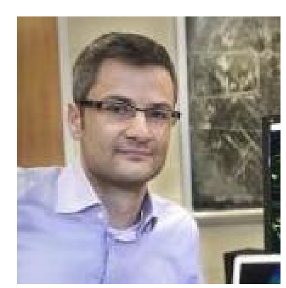
  P. Caucal, FS, R. Venugopalan. *2108.06347* *[JHEP 11 (2021) 222]*

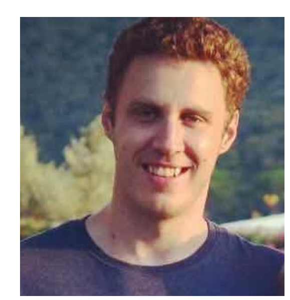
- The back-to-back limit: small-x TMD factorization at NLO

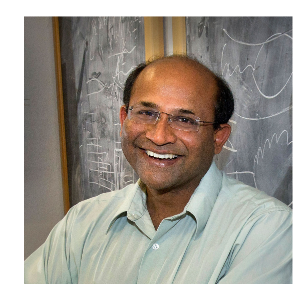
  PC, FS, B. Schenke ,RV. *2208.13872* *[JHEP 11 (2022) 169]*
  PC, FS, BS, T. Stebel, RV. *2304.03304* *[JHEP 08 (2023) 062]*
  2308.00022 *[preprint]*

- Outlook


Paul Caucal


Björn Schenke


Tomasz Stebel


Raju Venugopalan

Why gluon saturation?

- Search of gluon saturation is one of the **major goals** of the future **EIC**.
- The **Color Glass Condensate** is an EFT for this **gluon saturated regime.**
- A wide variety of observables are available to search for gluon saturation: **structure functions, diffractive processes,** and **semi-inclusive measurements**.
- Competing physical mechanisms might lead to similar signatures. Need sharper predictions (NLO era for gluon saturation).

Outstanding challenges

- Higher-order calculations for precision
- Identification of novel observables
- Modeling of initial conditions
- Spin Physics and saturation
- Event generators and global analysis
- Unification of dilute and dense QCD (beyond CGC)

recently funded SURGE Topical Collaboration supported by DOE

Discovery and characterization of gluon saturation principal goals of the future Electron-Ion Collider

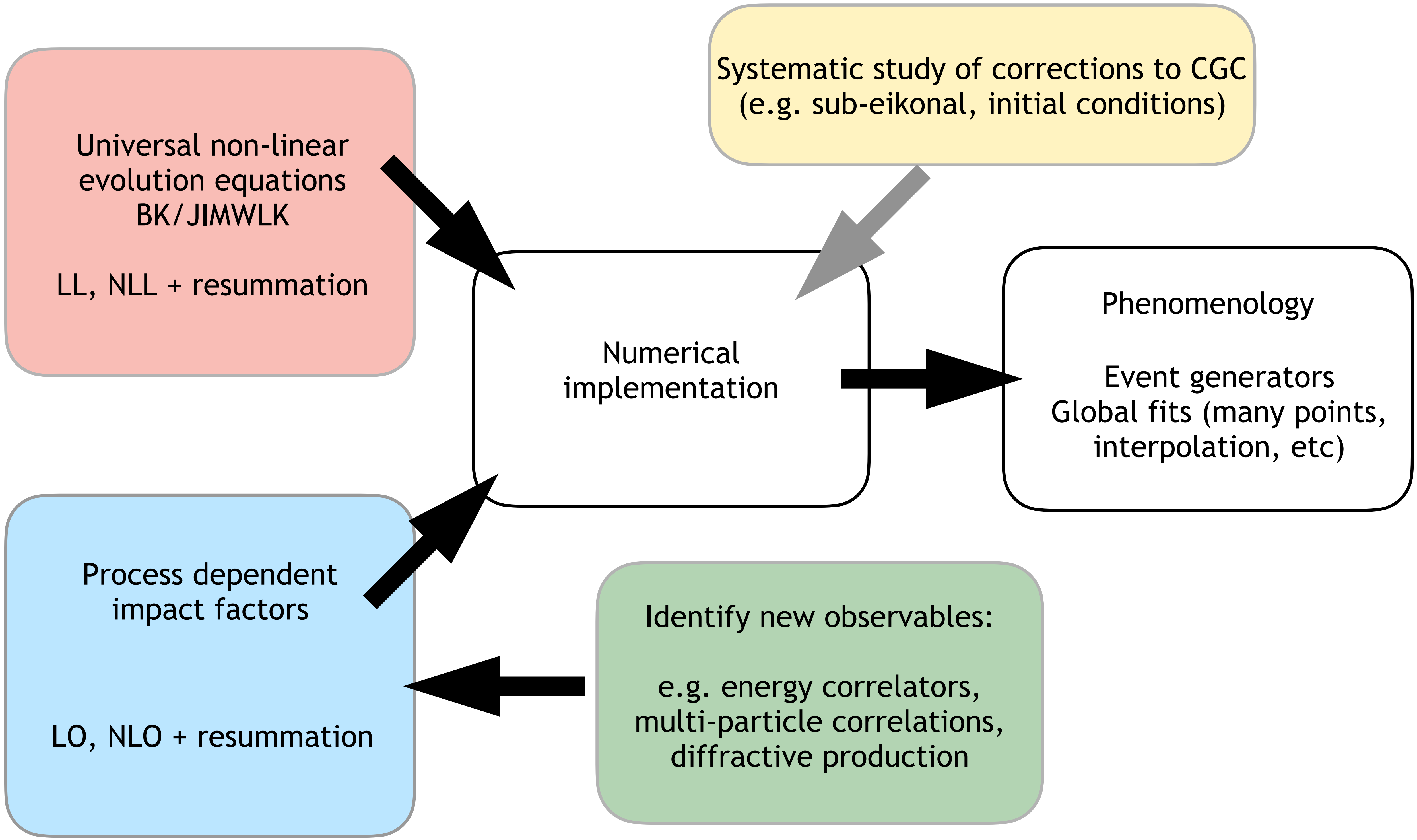
Other novel directions:

- Entanglement entropy and saturation, space-like and time-like correspondence, CGC-blackhole correspondence, color memory effect

For a short review see: *section 6 in Snowmass 2021 White Paper (arXiv: 2203.13199)*

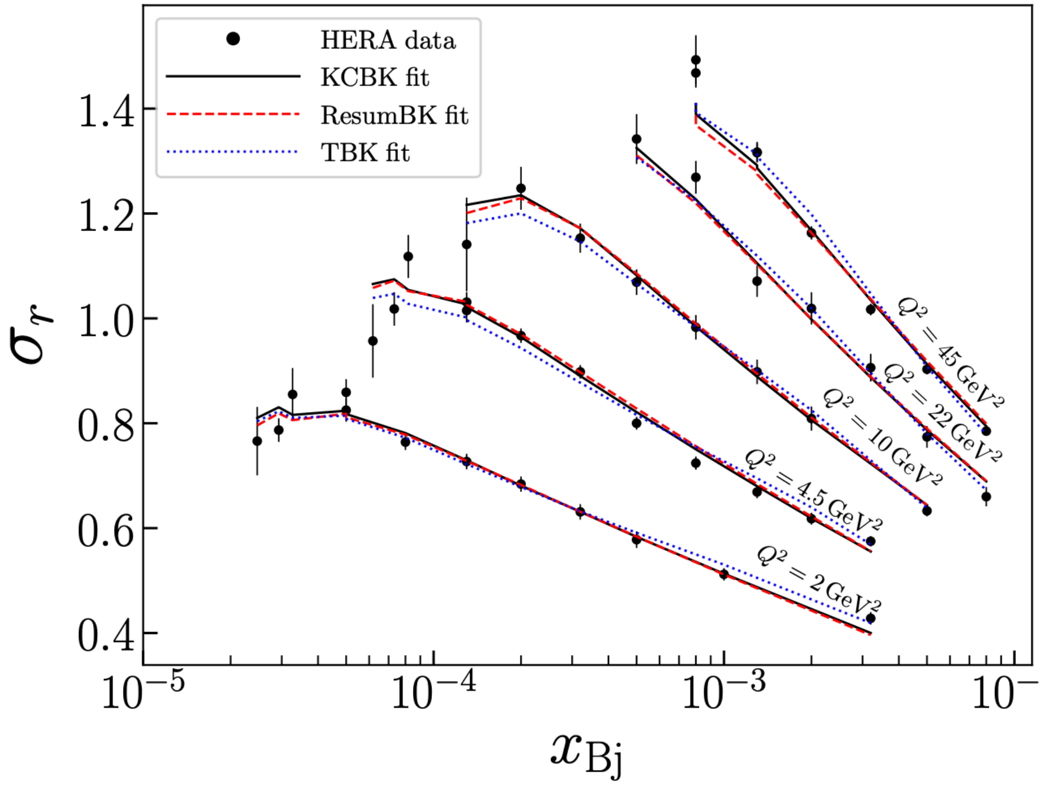
Precision frontier for gluon saturation

End-to-end precision analysis for saturation physics

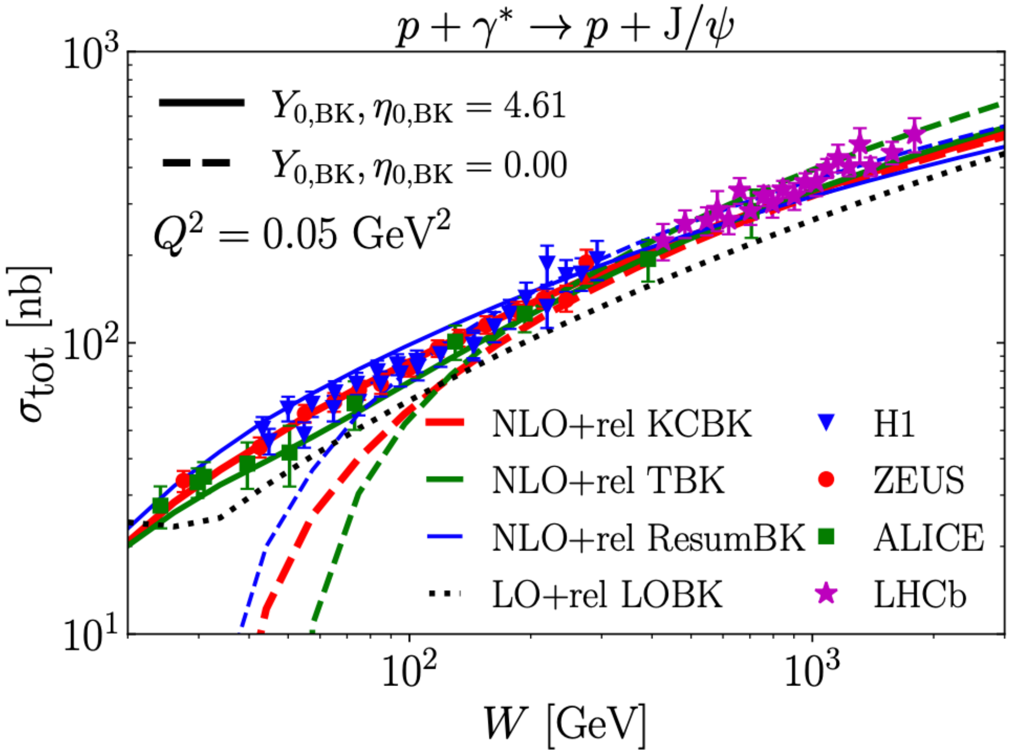


Precision frontier for gluon saturation

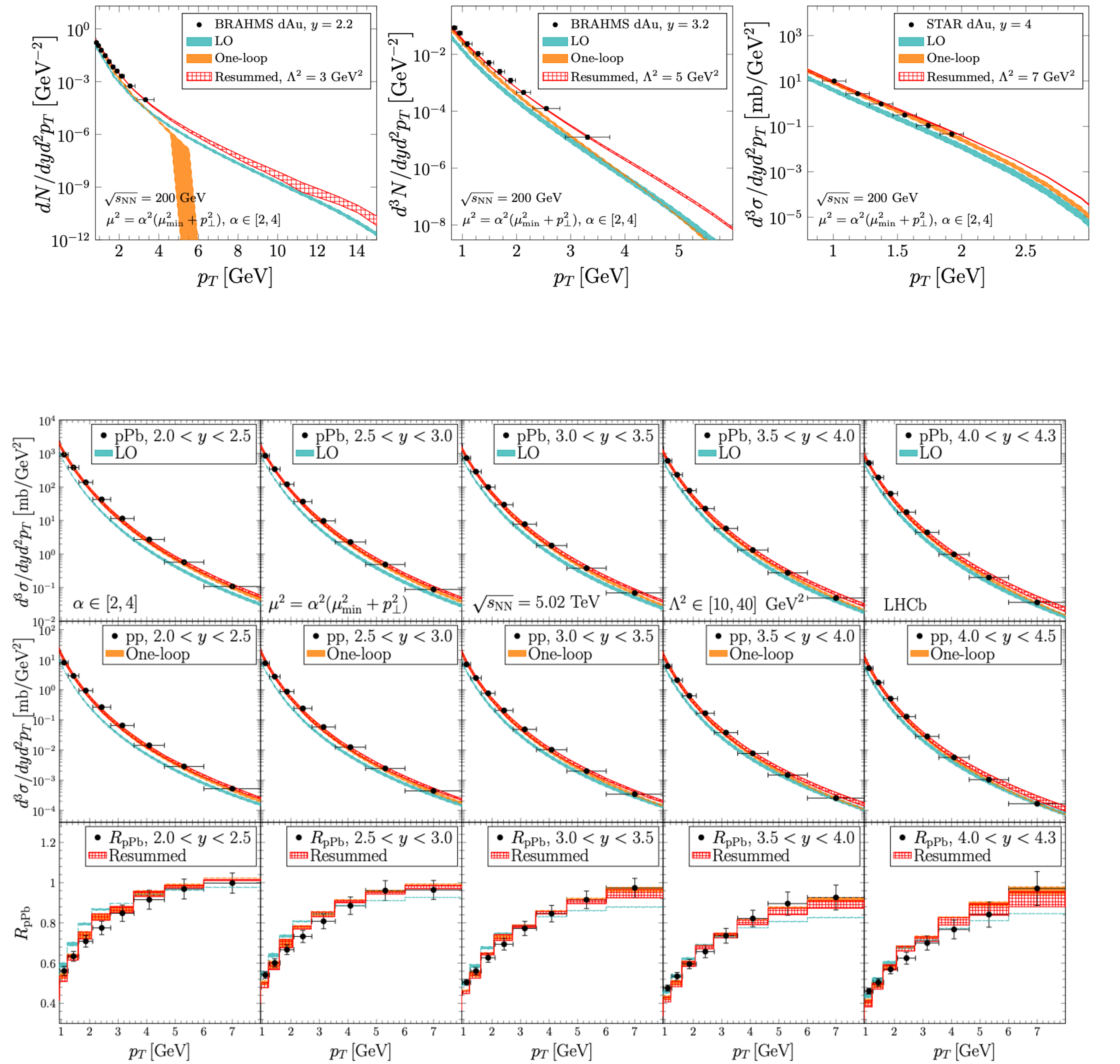
Saturation physics at NLO



Beuf, Lappi, Hänninen, Mäntysaari (2020)



Mäntysaari, Penttala (2022)



Shi, Wang, Wei, Xiao (2021)

Precision frontier for gluon saturation

Lots of **recent progress** in understanding **saturation physics** in the **precision era** (NLO) with focus on fully inclusive process or one-particle production

The **more differential** the process (e.g. two-particle correlations) the **harder** the calculations (both **analytically** and **numerically**)

Our goal:

Promote **two-particle observables** in **saturation** to **NLO**

Observable:

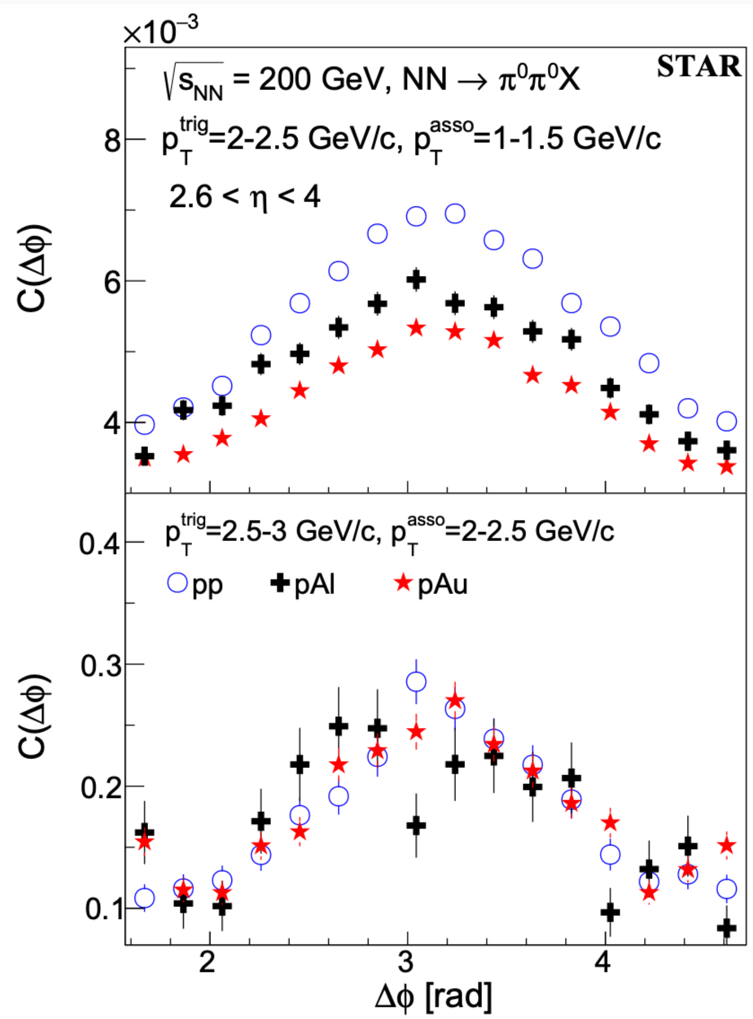
Inclusive Dijet production in **DIS**

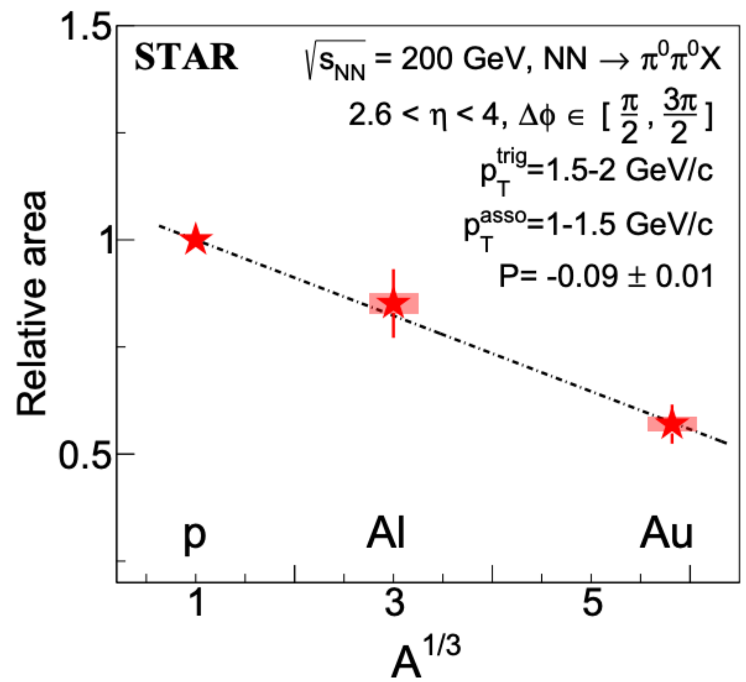
Why?

Will be **measured** at **EIC** and **theoretically clean**

Azimuthal correlations a window to gluon saturation

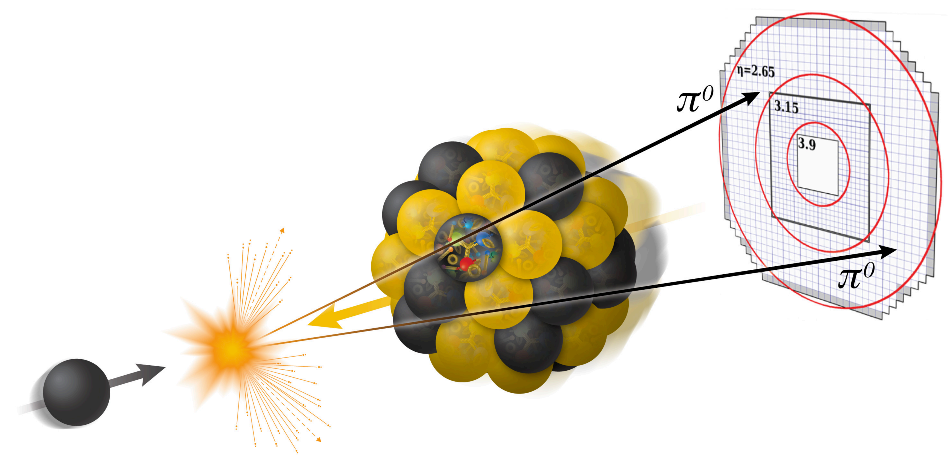
Evidence for Nonlinear Gluon Effects in QCD and Their Mass Number Dependence at STAR





Suppression characteristic of saturation

$$Q_s^2 \propto A^{1/3}$$



Xiaoxuan Chu and Elke Aschenauer

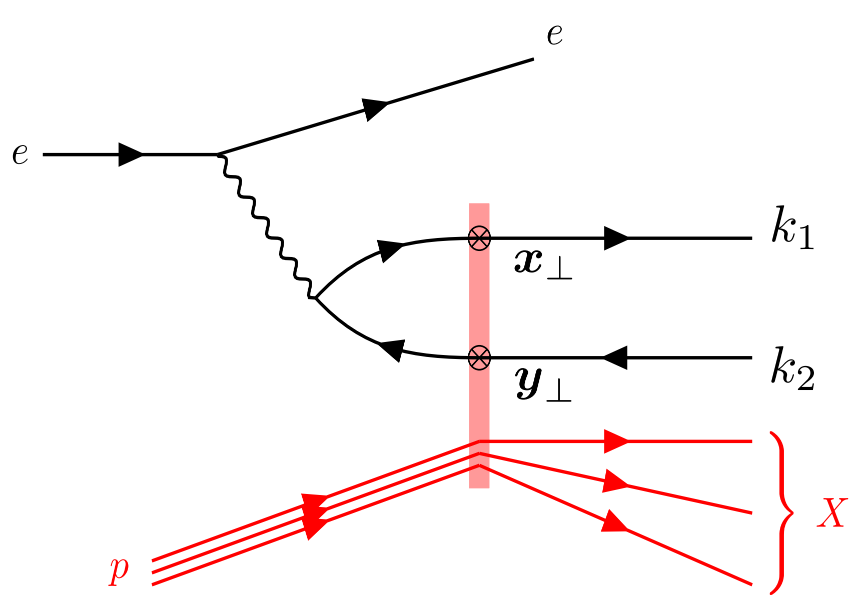
STAR Collaboration
Phys. Rev. Lett. 129, 092501 (2022)

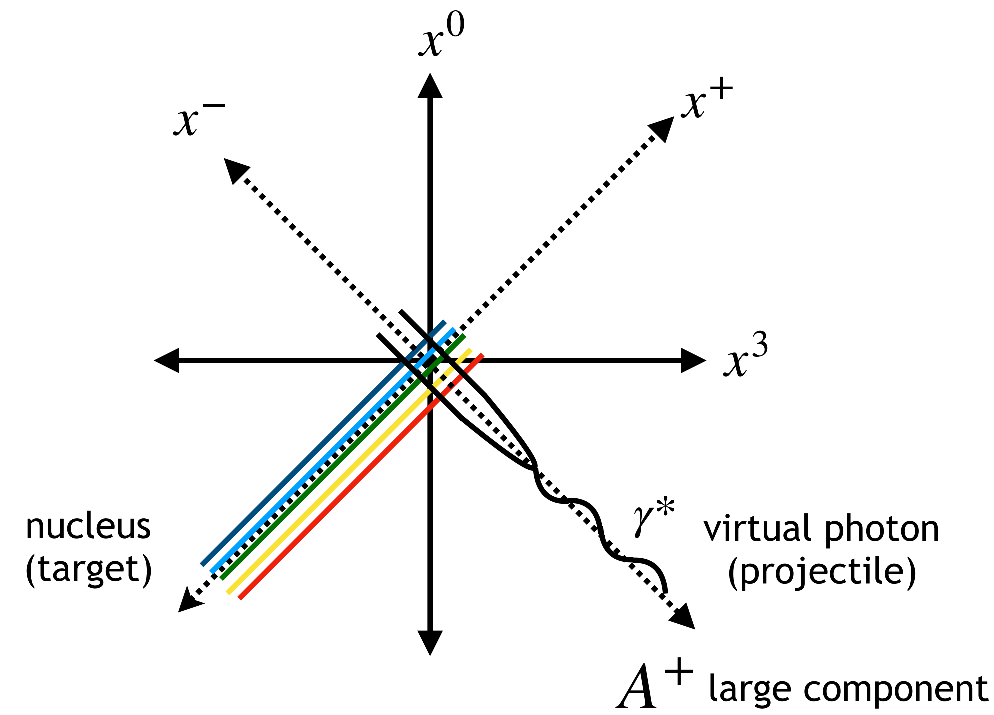
What about dihadron production at the EIC?

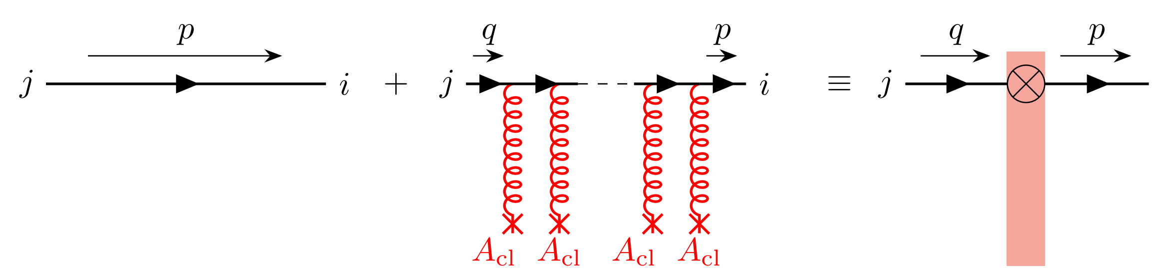
Dijet production in DIS at LO

Dijet azimuthal correlations in DIS

Jalilian-Marian, Gelis (2003)







Unpolarized differential cross-section:

$$\mathrm{d}\sigma_{\mathrm{LO}} = \mathcal{H}(Q, \boldsymbol{P}_\perp; \boldsymbol{l}_\perp, \boldsymbol{l}'_\perp) \otimes_{\boldsymbol{l}_\perp, \boldsymbol{l}'_\perp} \mathcal{G}(\boldsymbol{q}_\perp; \boldsymbol{l}_\perp, \boldsymbol{l}'_\perp)$$

Perturbatively calculable

$q\bar{q}$ interaction with nucleus

First numerical evaluation of dijet production in DIS within CGC:

PHYSICAL REVIEW LETTERS **124**, 112301 (2020)

Multigluon Correlations and Evidence of Saturation from Dijet Measurements at an Electron-Ion Collider

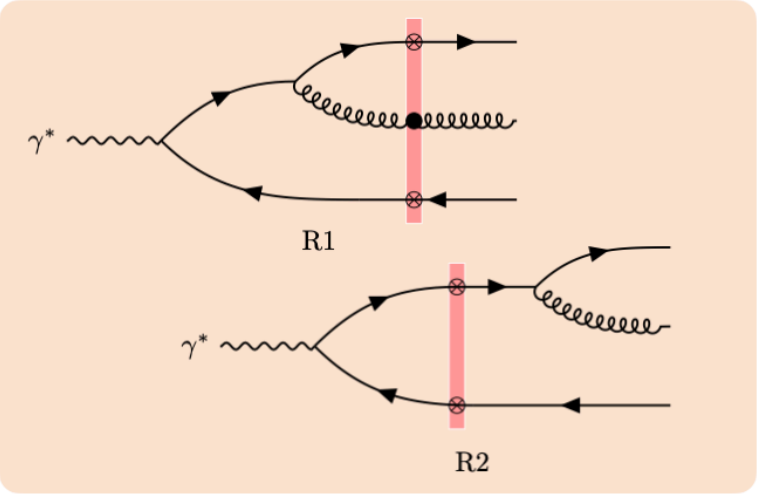
Heikki Mäntysaari,[1,2,*] Niklas Mueller,[3,†] Farid Salazar,[3,4,‡] and Björn Schenke[3,§]

Dijet production in DIS at NLO

JHEP Published for SISSA by Springer

Dijet impact factor in DIS at next-to-leading order in the Color Glass Condensate

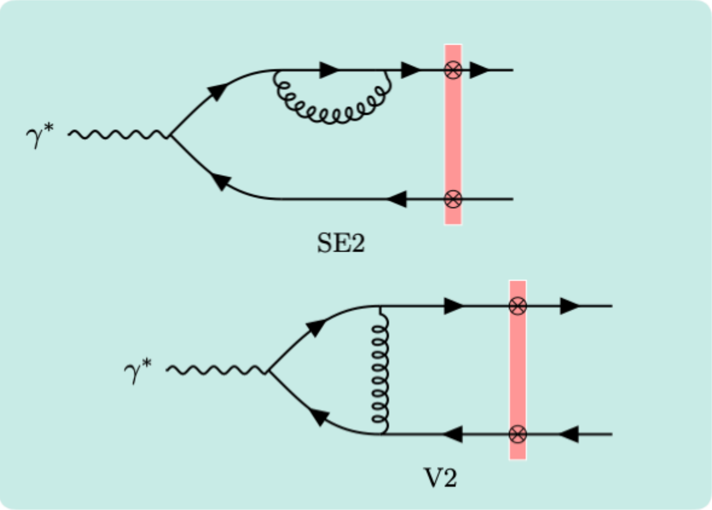
Paul Caucal,[a] Farid Salazar[a,b,c] and Raju Venugopalan[a]



- Divergences: UV, soft and collinear

  Dimensional regularization + longitudinal momentum cut-off + small-R cone algorithm

$$\int_{\Lambda^-} \frac{\mathrm{d}k_g^-}{k_g^-}\mu^\varepsilon \int \frac{\mathrm{d}^{2-\varepsilon}\boldsymbol{k_{g\perp}}}{(2\pi)^{2-\varepsilon}} f_{\Lambda^-}(k_g^-, \boldsymbol{k_{g\perp}})$$

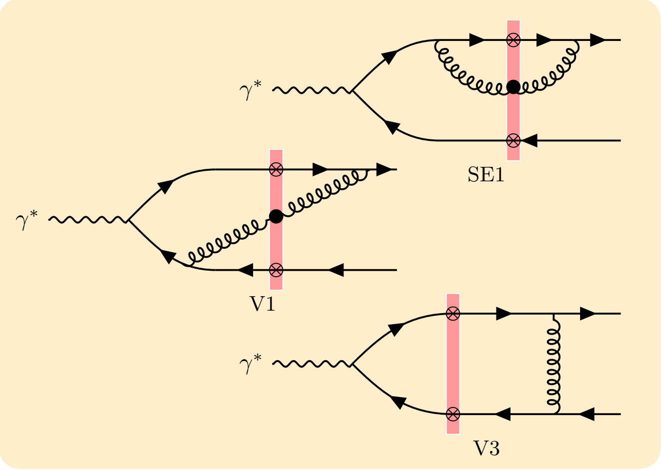


- Large rapidity (high-energy) logs

  Resummed via JIMWLK renormalization

- Impact factor

  Finite piece (free of large rapidity logs), but might contain other (potentially) large logs!



- We showed cancellation of UV, soft and collinear divergences
- Absorbed large energy/rapidity logs into JIMWLK resummation
- Isolated genuine $\mathcal{O}(\alpha_s)$ contributions (aka impact factor)

Dijet production in DIS at NLO

Evolution and impact factor

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \int_{z_0} \frac{\mathrm{d}z_g}{z_g} \mathrm{d}\widetilde{\sigma}_{\mathrm{NLO}}(z_g)$$

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \boxed{\int_{z_0} \frac{\mathrm{d}z_g}{z_g} \mathrm{d}\widetilde{\sigma}_{\mathrm{NLO}}(0)} + \boxed{\int_{z_0} \frac{\mathrm{d}z_g}{z_g} \left[\mathrm{d}\widetilde{\sigma}_{\mathrm{NLO}}(z_g) - \mathrm{d}\widetilde{\sigma}_{\mathrm{NLO}}(0)\right]}$$

JIMWLK factorization

Impact factor

$$\mathrm{d}\widetilde{\sigma}_{\mathrm{NLO}}(0) = H_{\mathrm{LL}} \mathrm{d}\sigma_{\mathrm{LO}} = \mathcal{H} \otimes H_{\mathrm{LL}} \mathcal{G}$$

$$\mathrm{d}\sigma_{\mathrm{LO+LL}} = \mathcal{H}(Q, \boldsymbol{P}_\perp; \boldsymbol{l}_\perp, \boldsymbol{l}'_\perp) \otimes_{\boldsymbol{l}_\perp, \boldsymbol{l}'_\perp} \mathcal{G}_Y(\boldsymbol{q}_\perp; \boldsymbol{l}_\perp, \boldsymbol{l}'_\perp)$$

$$Y = \ln(z_g) \qquad \frac{\partial \mathcal{G}}{\partial Y} = H_{\mathrm{LL}} \mathcal{G}$$

JIMWLK RG evolution

Explicit expression for impact factor provided in Caucal, Salazar, Venugopalan (2021)

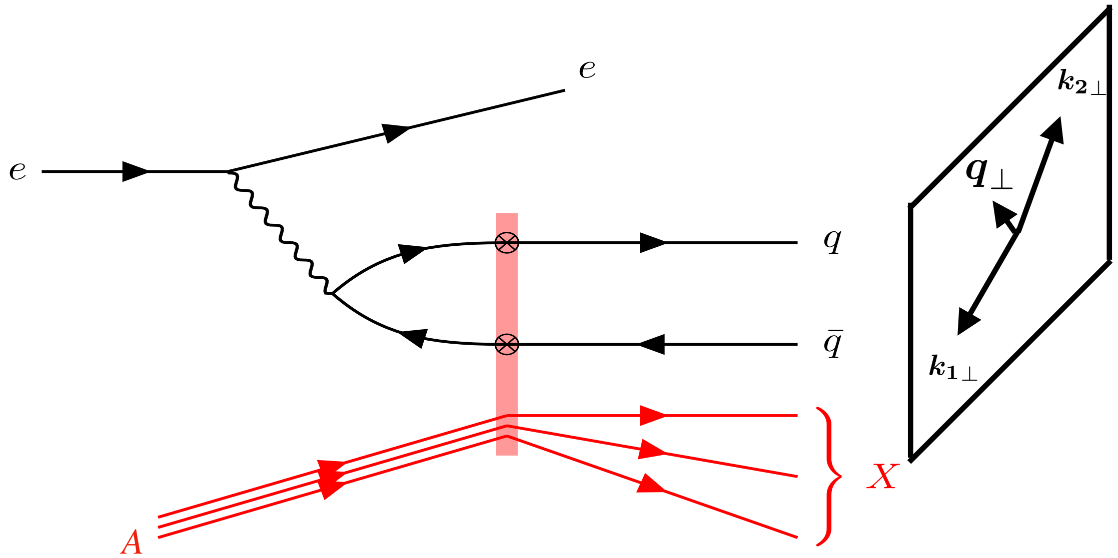
Back-to-back dijets at LO

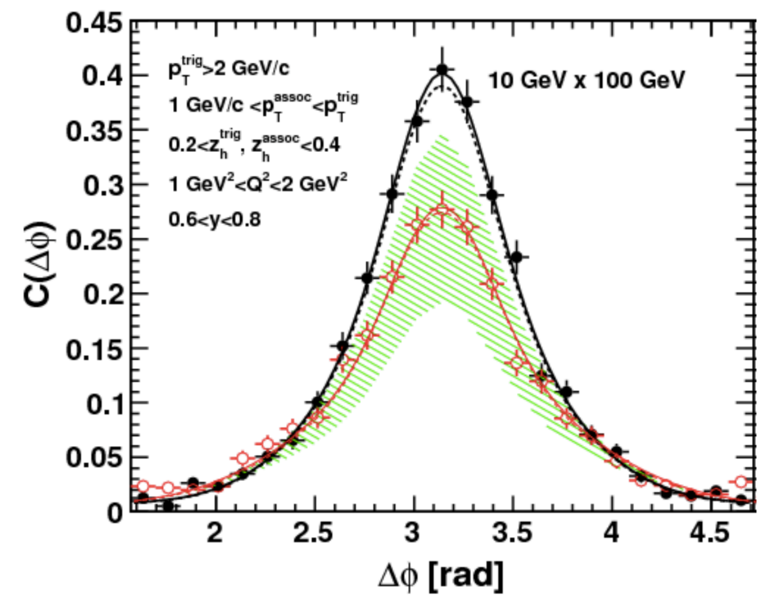
Small-x TMD factorization from CGC

F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan (2011)

In the “correlation limit” $q_\perp, Q_s \ll P_\perp$ and high-energy limit $P_\perp \ll W$

↳ Forward-jets (photon direction) but close to back-to-back in the transverse plane





Zheng, Aschenauer, Lee, Xiao (2014)

$$\boldsymbol{q}_\perp = \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}$$
$$\boldsymbol{P}_\perp = z_2 \boldsymbol{k}_{1\perp} - z_1 \boldsymbol{k}_{2\perp}$$

Perturbatively calculable

$$\mathrm{d}\sigma_{\mathrm{LO}} = \mathcal{H}^{ij}(Q, \boldsymbol{P}_\perp)\, G_Y^{ij}(\boldsymbol{q}_\perp)$$

$$G_Y^{ij}(\boldsymbol{q}_\perp) = \frac{-2}{\alpha_s} \int \frac{\mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)} \left\langle \mathrm{Tr}\left[V(\boldsymbol{b}_\perp)\partial_\perp^i V^\dagger(\boldsymbol{b}_\perp) V(\boldsymbol{b}'_\perp)\partial_\perp^j V^\dagger(\boldsymbol{b}'_\perp)\right]\right\rangle_Y$$

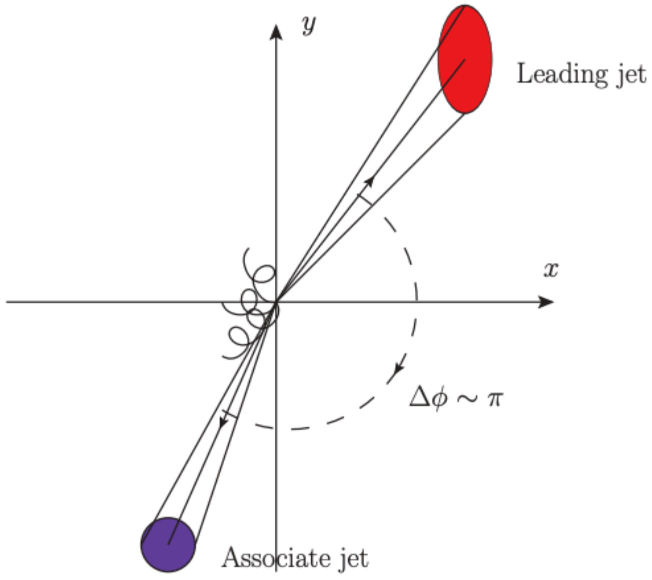
“Bare” TMD built from Wilson lines correlators, saturation scale Q_s implicitly built-in

Nuclear dependence $Q_s^2 \propto A^{1/3}$

Back-to-back dijets at NLO

Interplay of small-x & soft gluon resummation

A.H. Mueller, B-W. Xiao, F. Yuan (2013)



$$q_\perp^2 \ll P_\perp^2 \ll s \qquad \ln(s/P_\perp^2) \qquad \ln^2(P_\perp^2/q_\perp^2)$$

Joint small-x + Sudakov resummation

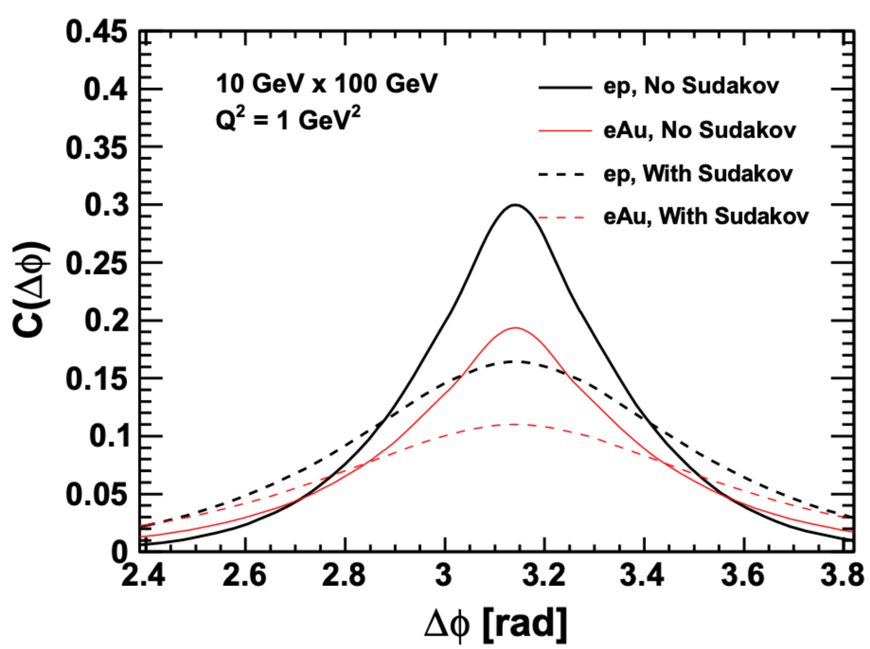
$$d\sigma = \mathcal{H}(Q, \boldsymbol{P}_\perp) \int \frac{d^2\mathbf{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp \cdot \mathbf{r}_{bb'}} \widetilde{G}_Y^0(\mathbf{r}_{bb'}) e^{-S_{\mathrm{Sud}}(\mathbf{r}_{bb'}, \boldsymbol{P}_\perp)}$$

Sudakov factor:

$$S_{\mathrm{Sud}}(\mathbf{r}_{bb'}, P_\perp) = \frac{\alpha_s N_c}{\pi} \int_{c_0^2/\mathbf{r}_{bb'}^2}^{P_\perp^2} \frac{1}{2} \ln\left(\frac{P_\perp^2}{\mu^2}\right)$$

$\widetilde{G}_Y$ obeys JIMWLK equation (non-linear)
(see also Dominguez, Mueller, Munier, Xiao 2013)

Soft gluon emissions change profile of azimuthal correlations
Zheng, Aschenauer, Lee, Xiao 2013



Our goal:

Does the CGC/TMD correspondence hold at NLO?

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \mathcal{H}^{ij}_{\mathrm{LO+NLO}}(Q, \boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2 \boldsymbol{b}_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp \cdot \mathbf{r}_{bb'}} \widetilde{G}^{ij}_{Y}(\mathbf{r}_{bb'}) e^{-S_{\mathrm{Sud}}(\mathbf{r}_{bb'}, \boldsymbol{P}_\perp)}$$

i) Is this evolution equation related to non-local RG equation known in small-x literature?

ii) Is it possible to pin down the Sudakov double and single logs?

iii) Is it possible to express the finite pieces in terms of a factorizable NLO coefficient function and the WW gluon TMD? Or do we expect factorization-breaking contributions?

iv) Can we account both for unpolarized and linearly polarized contributions?

Sudakov and small-x resummation I

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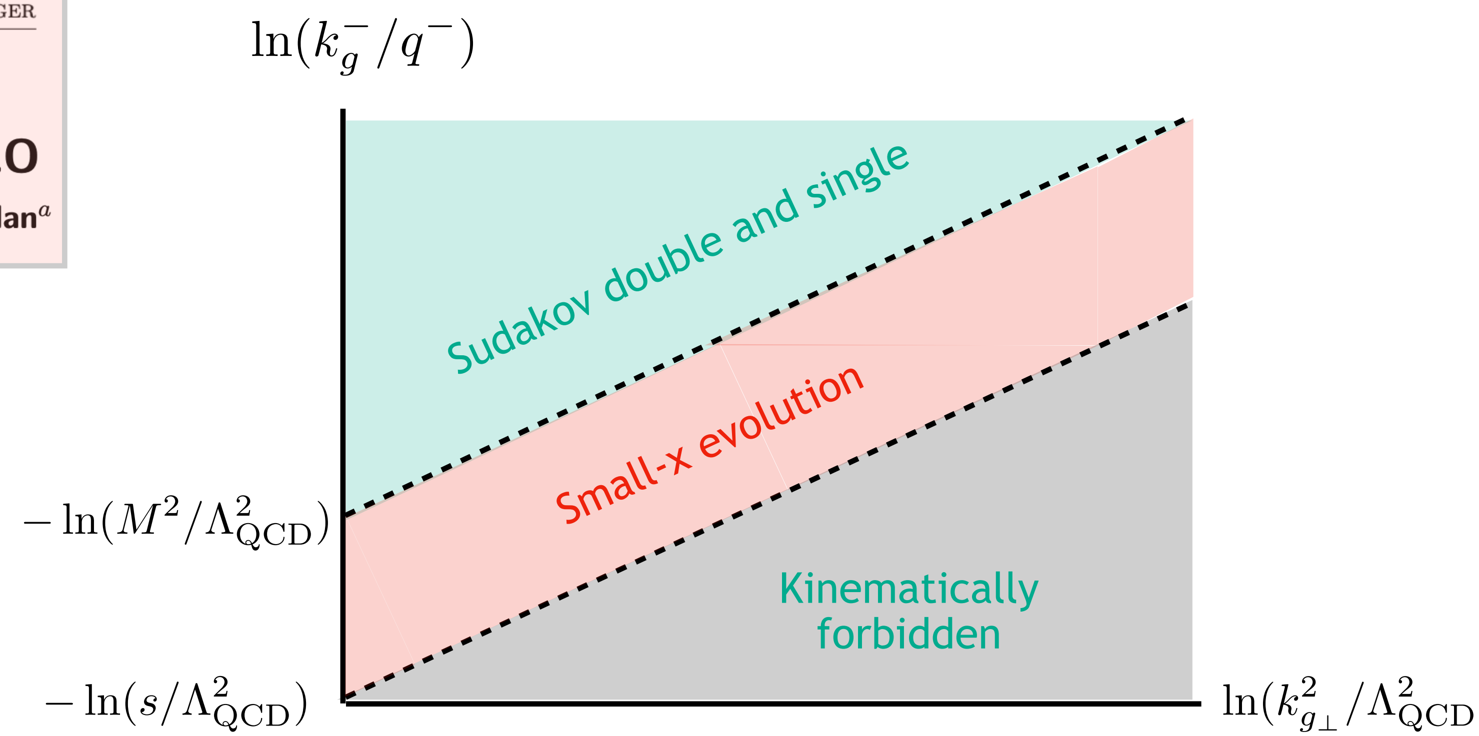
Back-to-back inclusive dijets in DIS at small x: Sudakov suppression and gluon saturation at NLO

Paul Caucal,[a] Farid Salazar,[b,c,d,e] Björn Schenke[a] and Raju Venugopalan[a]



Small-x evolution for WW follows well-known BK-JIMWLK equations **amended with a kinematic constrain** to separate small-x and soft gluons

$$\mathrm{d}\sigma_{\rm NLO} = \mathcal{H}(Q,\boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2\mathbf{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\mathbf{r}_{bb'}} \widetilde{G}^0_Y(\mathbf{r}_{bb'}) \left[1 - S_{\rm Sud}(\mathbf{r}_{bb'},\boldsymbol{P}_\perp)\right] + \mathcal{O}(\alpha_s^2)$$

$$\frac{\partial G}{\partial Y} = H_{\rm LL}\Theta\left(1/(z_g Q_f^2) - r_<^2\right) G$$

$$S_{\rm Sud}(\mathbf{r}_{bb'},P_\perp) = \frac{\alpha_s N_c}{\pi}\int_{c_0^2/\mathbf{r}^2_{bb'}}^{P_\perp^2} \frac{\mathrm{d}\mu^2}{\mu^2}\left[\frac{1}{2}\ln\left(\frac{P_\perp^2}{\mu^2}\right) + B_0\right]$$

$$B_0 = \frac{C_F}{N_c}\ln\left(\frac{1}{z_1 z_2 R^2}\right) + \ln\left(\frac{P_\perp^2 + z1 z_2 Q^2}{P_\perp^2}\right) - 1$$

Finite results *apparently* involve complicated convolution including operators beyond WW, but **needed for precision!**

Discrepancy with expected single log from CSS

Factorization of finite pieces

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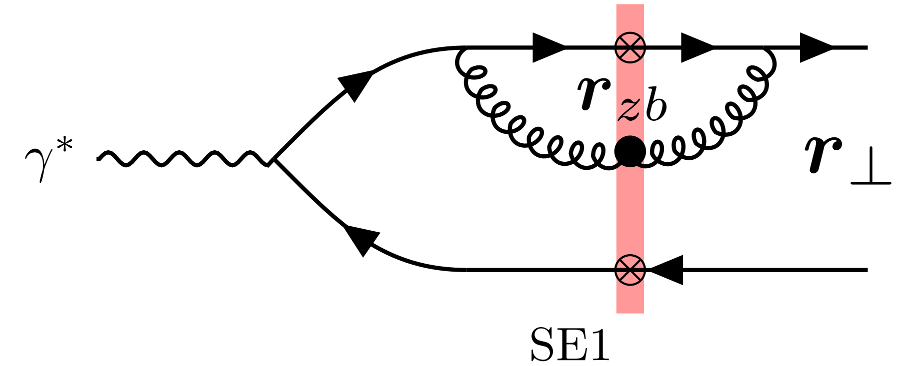
Back-to-back inclusive dijets in DIS at small x: gluon Weizsäcker-Williams distribution at NLO

Paul Caucal,[a] Farid Salazar,[b,c,d,e] Björn Schenke,[f] Tomasz Stebel[g] and Raju Venugopalan[f]



Leading power Expansion of all finite pieces can be cast in terms of the WW gluon TMD

Finite contributions can be absorbed into NLO impact factor

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \mathcal{H}_{\mathrm{NLO}}(Q, \boldsymbol{P}_\perp; Y, R) \int \frac{\mathrm{d}^2\boldsymbol{b}_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp} \widetilde{G}^0_Y(\boldsymbol{b}_\perp) \left[1 - S_{\mathrm{Sud}}(\boldsymbol{b}_\perp, P_\perp)\right] + \mathcal{O}(\alpha_s^3)$$

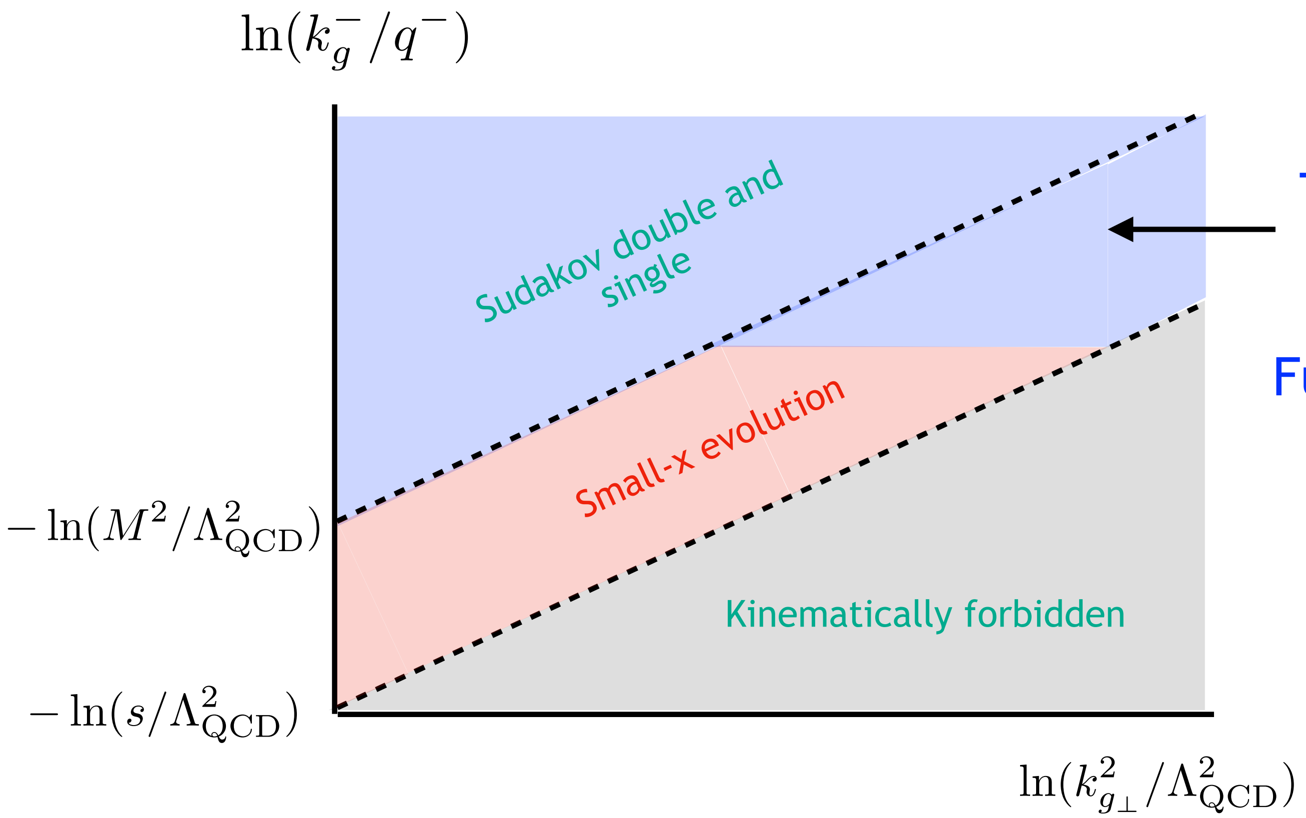
fully analytic result

- TMD-like factorization at NLO in small-x kinematics

Paves the road for

- Efficient numerical implementation and phenomenology

Sudakov and small-x resummation II



This region contains a Sudakov single log, which exactly cancels the term that caused discrepancy with CSS

Furthermore, it is convenient to formulate the evolution in terms of "target momentum fraction" $\eta = \ln(k_g^+/P^+)$

The resulting small-x equation (ordered in η) is the **JIMWLK analog** of the non-local BK equation found by Ducloué, Ian, Mueller, Soyez, Triantafyllopoulos (2019)

$$d\sigma_{\rm NLO} = \mathcal{H}_{\rm NLO}(Q, \boldsymbol{P}_\perp; R) \int \frac{d^2\boldsymbol{b}_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp} \widetilde{G}^0_\eta(\boldsymbol{b}_\perp) \left[1 - S_{\rm Sud}(\boldsymbol{b}_\perp, P_\perp)\right]$$

$$S_{\rm Sud}(\mathbf{r}_{bb'}, P_\perp) = \frac{\alpha_s N_c}{\pi} \int_{c_0^2/\mathbf{r}_{bb'}^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2} \left[\frac{1}{2}\ln\left(\frac{P_\perp^2}{\mu^2}\right) + B_0\right]$$

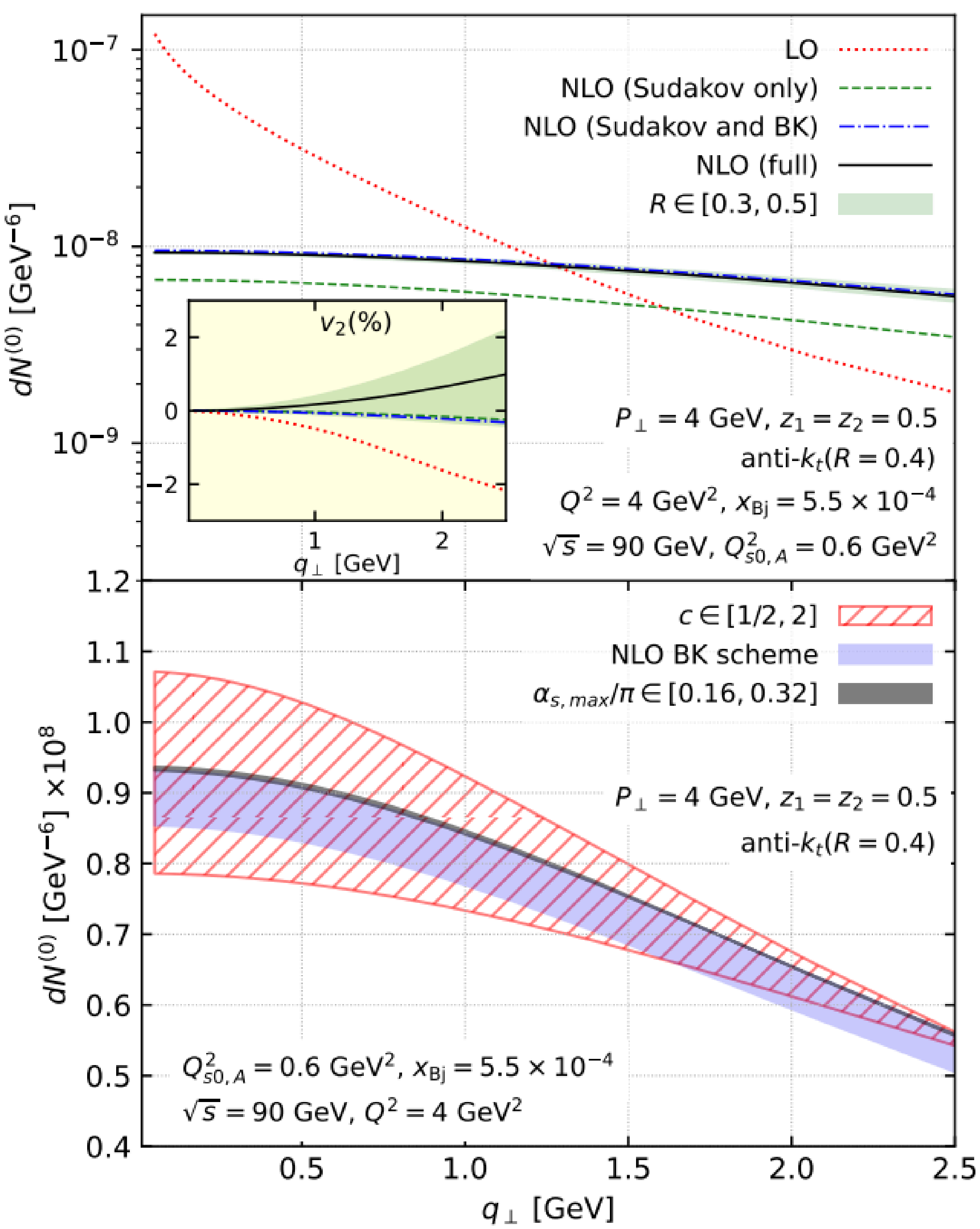
$$B_0 = \frac{C_F}{N_c}\ln\left(\frac{1}{z_1 z_2 R^2}\right) + \ln\left(\frac{P_\perp^2 + z_1 z_2 Q^2}{P_\perp^2}\right) - \frac{\pi}{N_c}\beta_0$$

For phenomenology we will exponentiate Sudakov logs ala CSS

$$[1 - S_{\rm Sud}(\boldsymbol{b}_\perp, P_\perp)] \rightarrow \exp(-S_{\rm Sud}(\boldsymbol{b}_\perp, P_\perp))$$

Caucal, FS, Schenke, Stebel, Venugopalan (preprint 2308.00022)

Numerical results for the differential cross-section



Caucal, FS, Schenke, Stebel, Venugopalan (preprint 2308.00022)

$\mathrm{d}N$ min-bias differential yield

- Sudakov significantly suppresses the yield when $q_\perp$ is small (back-to-back)
- Small-x evolution results in the growth of the yield (slower at smaller $q_\perp$ -> saturation)
- NLO corrections to hard function have a very small effect (at this kinematics)
- Uncertainties:

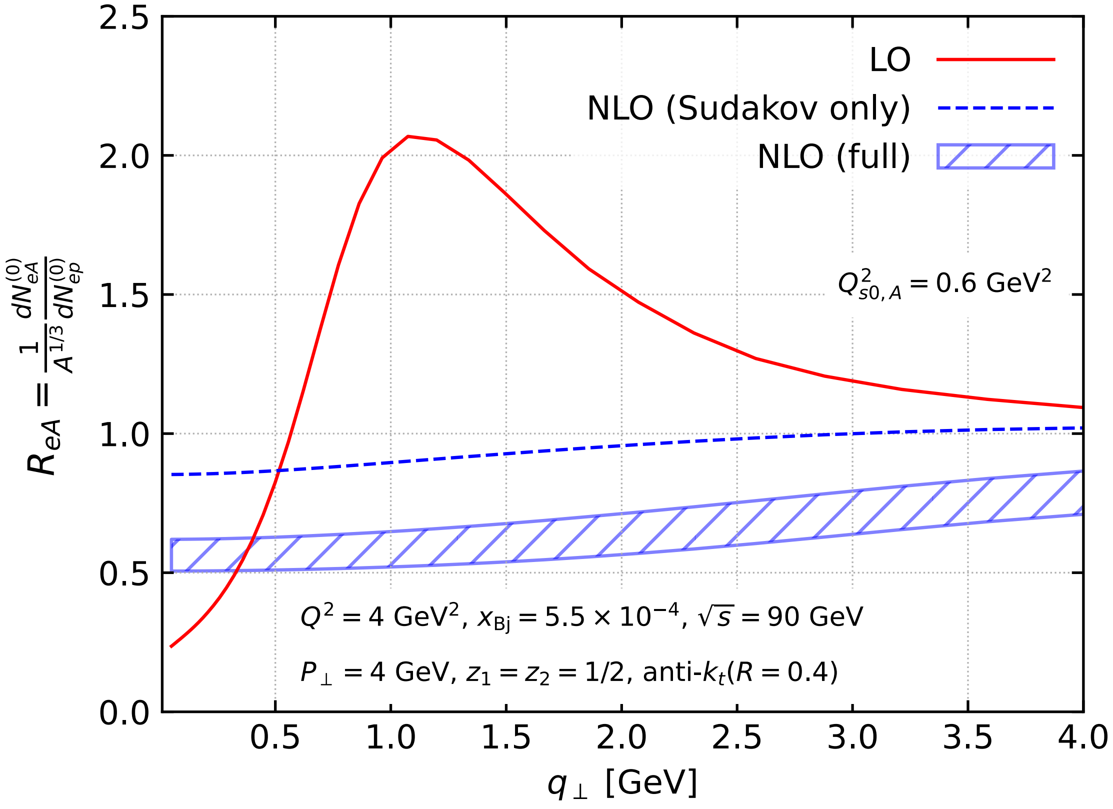
Running of the coupling in hard function and Sudakov factor

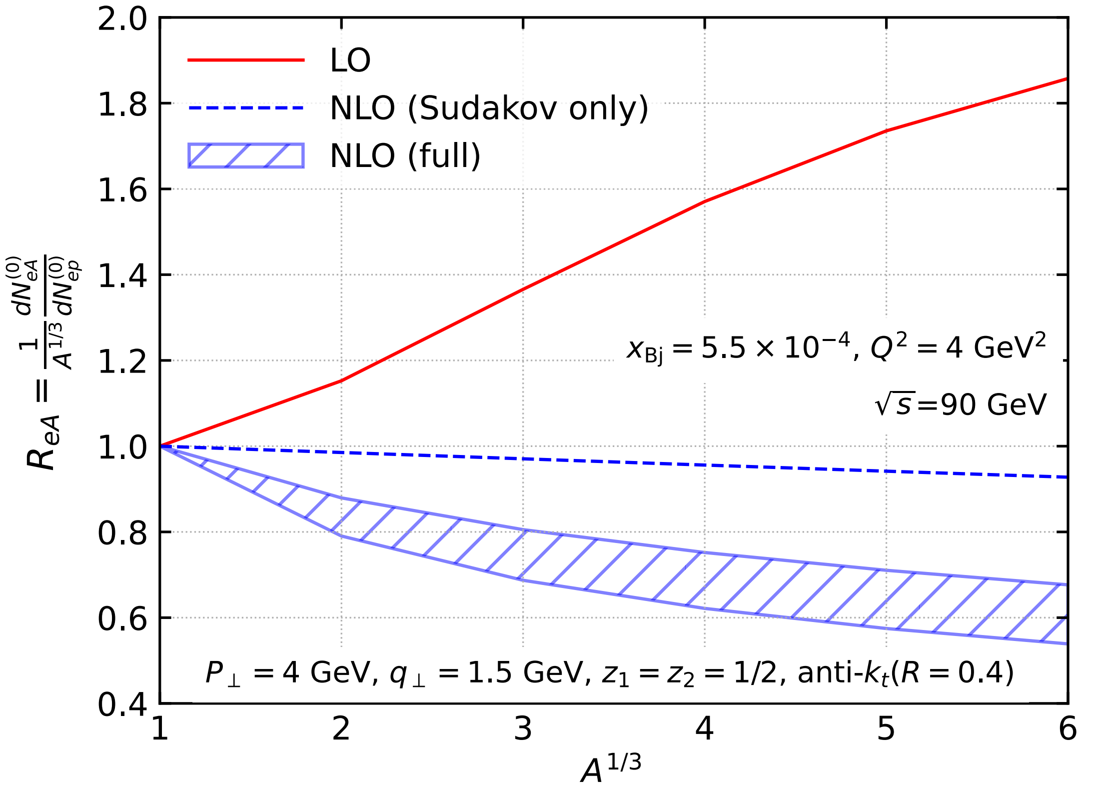
$$\alpha_s(cP_\perp) \quad \alpha_s(c\mu)$$

NLO BK scheme with or without collinear single log resummation

α_{max}: freezing of the coupling

Numerical results for nuclear modification ratio





$\mathrm{d}N$ min-bias differential yield

$$R_{\rm eA} = \frac{1}{A^{1/3}} \frac{\mathrm{d}N_{eA}}{\mathrm{d}N_{ep}}$$

- LO shows suppression and a Cronin peak (broadening)
- NLO (Sudakov only) displays a slight suppression but is close to unity
- Full NLO (Sudakov + small-x evolution) shows a significant A-dependent suppression

Caucal, FS, Schenke, Stebel, Venugopalan (preprint 2308.00022)

Summary

- Search for gluon saturation is one of the major goals of the future EIC and upcoming upgrades of the LHC (e.g. Focal)
- The Color Glass Condensate is an EFT for this saturated regime that has been applied to calculate a variety of processes.
- We performed a complete NLO calculation for dijet production in DIS within the CGC
- We showed TMD factorization (1-loop) at small-x holds when jets are back-to-back in the transverse plane
- Provide numerical predictions for EIC

Phase space for small-x resummation

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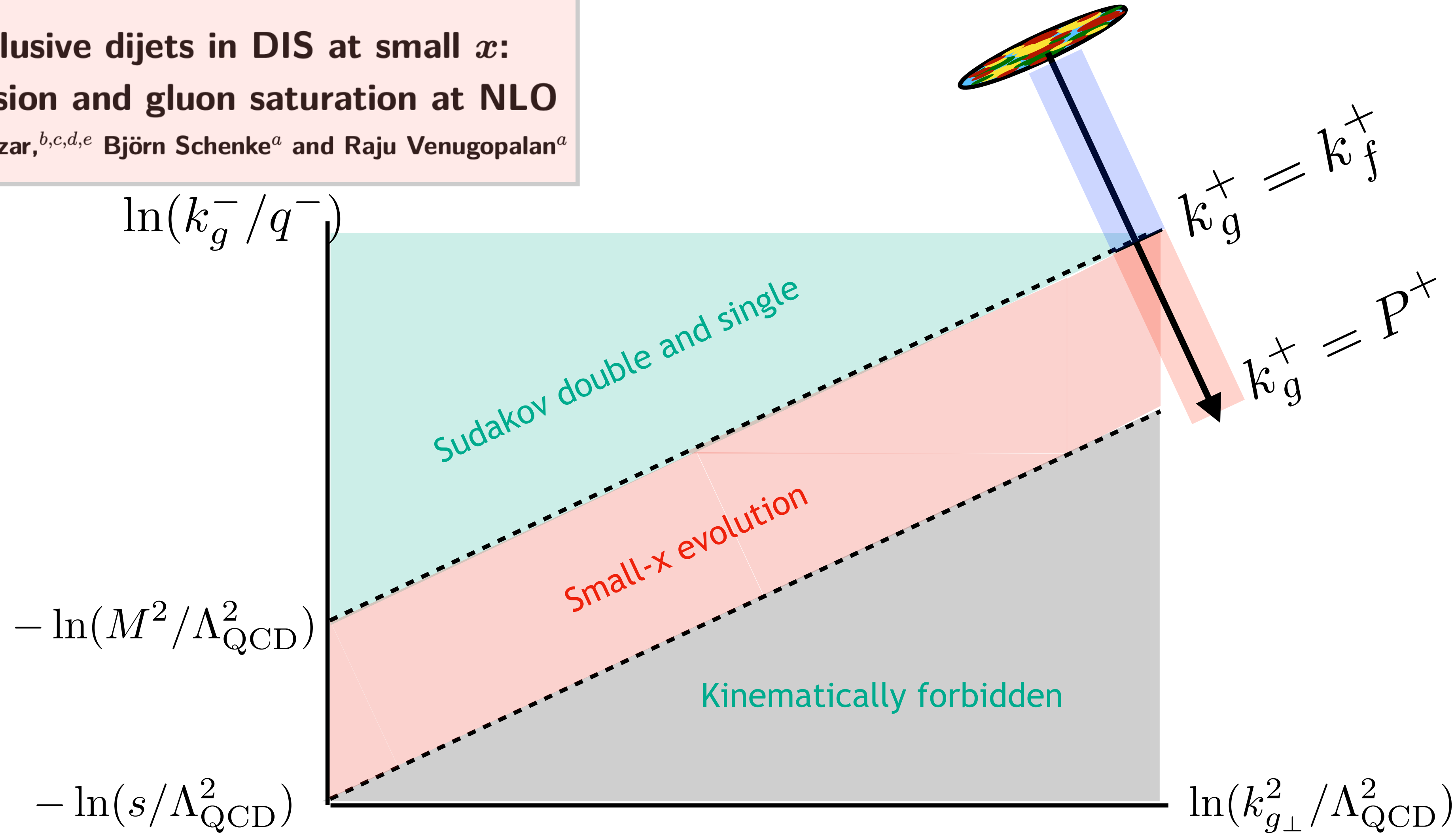
Back-to-back inclusive dijets in DIS at small x: Sudakov suppression and gluon saturation at NLO

Paul Caucal,[a] Farid Salazar,[b,c,d,e] Björn Schenke[a] and Raju Venugopalan[a]

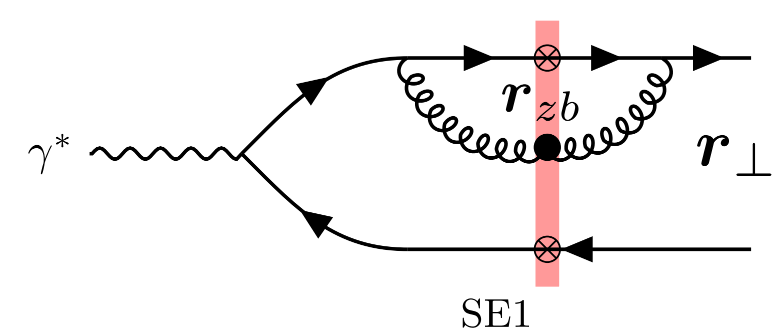


Examining “rapidity divergent term ”more closely

$$\mathrm{d}\widetilde{\sigma}_{\rm NLO}(z_g) = H_{\rm LL}\mathrm{d}\sigma_{\rm LO} \quad \text{if} \quad z_g \boldsymbol{r}^2_{zb} \lesssim z_1 z_2 \boldsymbol{r}^2_\perp \longrightarrow k_f^+ = \frac{Q_f^2}{2k_f^-} \sim \frac{M^2}{2q^-}$$

Kinematically constrained JIMWLK

$$\frac{\partial \mathcal{G}}{\partial Y} = H_{\rm LL}\Theta\left(1/(z_g Q_f^2) - r_<^2\right)\mathcal{G}$$



Back-to-back dijets at NLO

From projectile to target rapidity evolution

2307.XXXX [work in progress]

Assume Gaussian approximation, then enough to consider dipole evolution

For the BK equation, our kinematic constraint implies

$$\frac{\partial S_Y(\mathbf{r}_{bb'})}{\partial Y} = \frac{\alpha_s N_c}{\pi} \int \frac{\mathrm{d}^2 \boldsymbol{z}_\perp}{2\pi} \Theta(-Y - \ln(r_<^2 \mu_\perp^2)) \frac{\mathbf{r}_{bb'}^2}{\mathbf{r}_{zb}^2 \mathbf{r}_{zb'}^2} \left[S_Y(\mathbf{r}_{zb}) S_Y(\mathbf{r}_{zb'}) - S_Y(\mathbf{r}_{bb'}) \right]$$

Can be cast in terms of target evolution:

$$\frac{\partial \mathcal{S}_\eta(\mathbf{r}_{bb'})}{\partial \eta} = \frac{\alpha_s N_c}{\pi} \int \frac{\mathrm{d}^2 \boldsymbol{z}_\perp}{2\pi} \Theta(\eta - \delta_{bb'z}) \frac{\mathbf{r}_{bb'}^2}{\mathbf{r}_{zb}^2 \mathbf{r}_{zb'}^2} \left[\mathcal{S}_{\eta-\delta_{zb}}(\mathbf{r}_{zb}) \mathcal{S}_{\eta-\delta_{zb'}}(\mathbf{r}_{zb'}) - \mathcal{S}_\eta(\mathbf{r}_{bb'}) \right]$$

Same as non-local equation found by Iancu, Mueller, Soyez, Triantafyllopolous (2019)

$$\mathcal{S}_{\eta_f} = S_{Y_f} \qquad \eta_f = Y_f + \ln(\mathbf{r}_{bb'} Q^2) - \ln(x_{\mathrm{Bj}}/x_0)$$

$$Y_f = -\ln(\mathrm{r}_{bb'}^2 \mu_\perp^2) \longrightarrow \eta_f = \ln(x_0/x_f) \qquad x_f = \mu_\perp^2/W^2$$

This choice leads to the cancellation of -1 in the coefficient of the Sudakov single log!

Leads to a natural choice of target rapidity scale