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Spatial distributions of the energy-momentum tensor inside the nucleon

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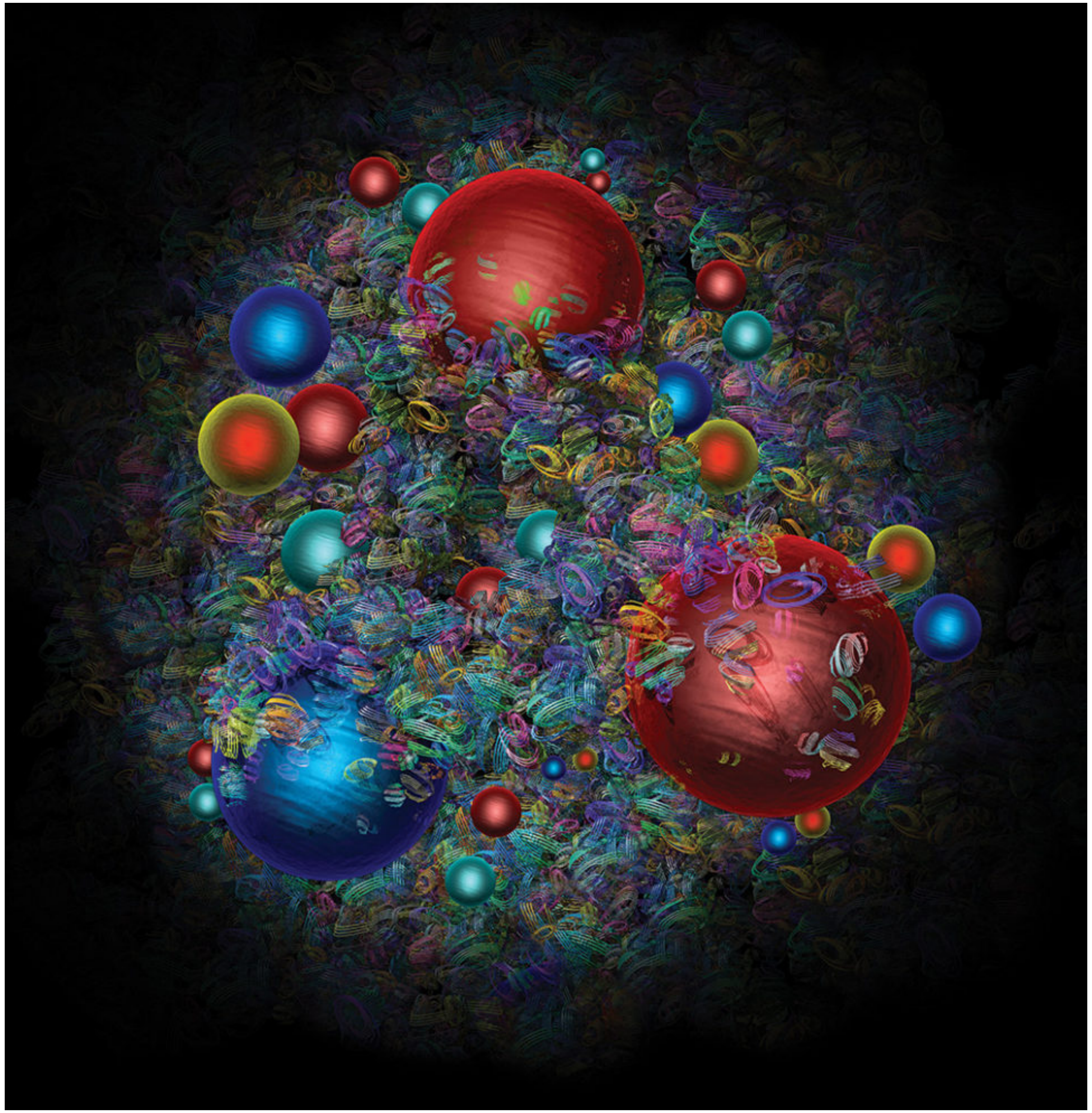
Introduction

Nucleon?

- Main building block in the visible universe
- A complex object composed of quarks and gluons via interactions such as strong, weak, etc.

Prime interest



Internal structure and form factor

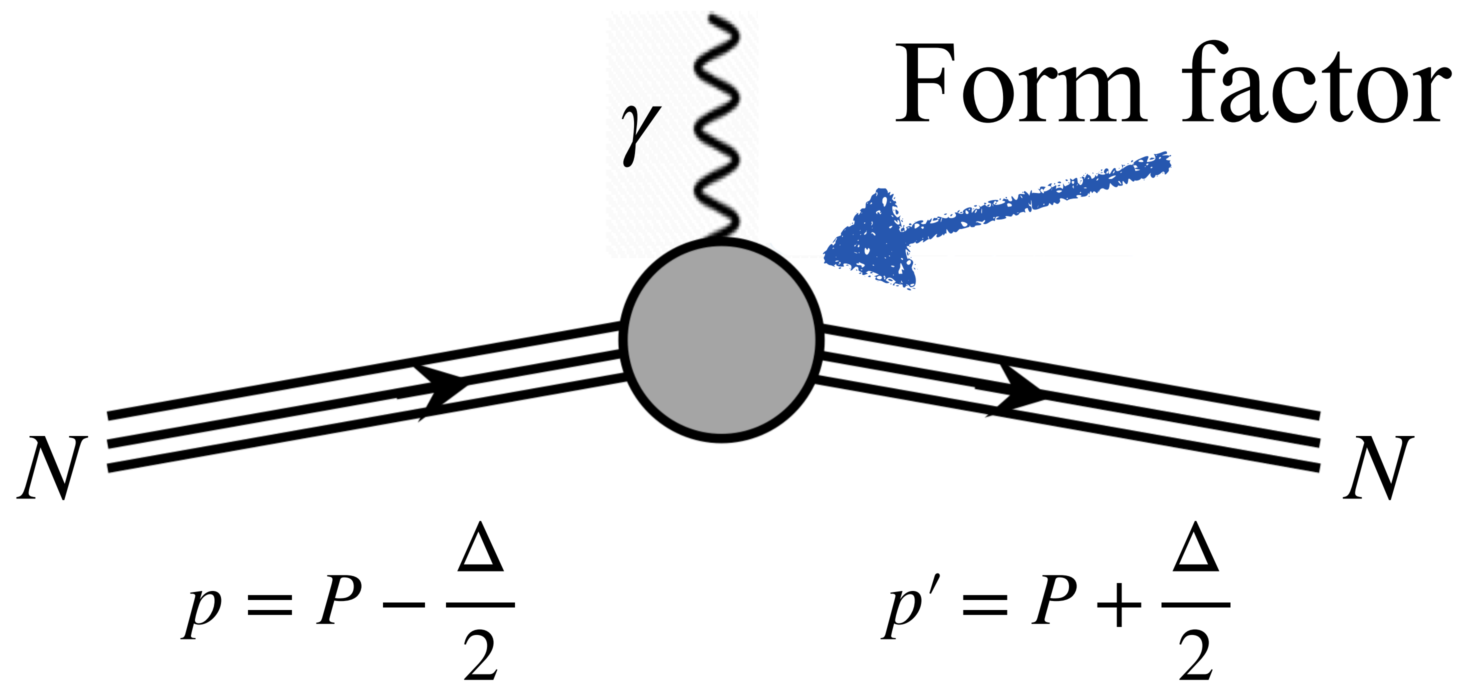
Internal structure of the nucleon?

Form factor → Spatial distribution

→ Radius e.g. proton's 'electric' radius

Form factor

A kind of structure function due to the compositeness of the particle



Classical definition of distribution $\rho(\vec{r}) = \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} F(-\Delta^2)$

However, "relativistic challenge"

Reduced Compton wavelength vs. Proton charge radius

$\bar{\lambda} = \frac{\hbar}{M_p c} \sim 0.21$ fm vs. $\sqrt{\langle r^2 \rangle^Q_{\text{exp}}} = 0.84$ fm [1]

$\frac{\bar{\lambda}}{\sqrt{\langle r^2 \rangle^Q_{\text{exp}}}} \sim 25\ \%$

Large relativistic effect

The internal dynamics of the nucleon is fully relativistic.

+

Frame dependence for the spatial distributions

[1] PDG, PTEP 2022 (2022) 083C01

Relativistic spatial distributions

I. Relativistic amplitude

Spin-0 particle

$$\langle p', s' | \hat{T}^{\mu\nu}(0) | p, s \rangle = \Lambda^{\mu}_{\alpha}\Lambda^{\nu}_{\beta}\langle p'_{\rm BF}, s'_{\rm BF} | \hat{T}^{\alpha\beta}(0) | p_{\rm BF}, s_{\rm BF}\rangle,$$

Trivial

Spin-$\frac{1}{2}$ particle

$$\langle p', s' | \hat{T}^{\mu\nu}(0) | p, s \rangle$$

Complicated structure appears

$$= \sum_{s'_{\rm BF}, s_{\rm BF}} D_{s_{\rm BF}s}(p_{\rm BF}, \Lambda)\, D^{*}_{s'_{\rm BF}s'}(p'_{\rm BF}, \Lambda)\, \Lambda^{\mu}_{\alpha}\Lambda^{\nu}_{\beta}\langle p'_{\rm BF}, s'_{\rm BF} | \hat{T}^{\alpha\beta}(0) | p_{\rm BF}, s_{\rm BF}\rangle,$$

Relativistic spin dynamics[2]

$$p'^{\mu} = \Lambda^{\mu}_{\nu} p^{\nu}$$

$$|p, s\rangle \xrightarrow{U(\Lambda)} |p', s'\rangle$$

$U(p)$: $|0,s\rangle \to |p,s\rangle$; $U^{-1}(\Lambda p)$: $|p',s'\rangle \to \sum_{s'} D_{ss'}(p,\Lambda)|0,s'\rangle$

$$|0,s\rangle \neq \sum_{s'} D_{ss'}(p, \Lambda)\, |0,s'\rangle$$

Wigner rotation

$$\rightarrow \quad [K^i, K^j] = -i\varepsilon^{ijk}J^k$$

[2] W.N. Polyzou et al., FBS 54 (2013) 1667

II. Elastic frame ($\Delta^0 = 0$)[3]

Connection between the rest (Breit) frame and the moving frame.

$$P \cdot \Delta = 0$$

$$P = \frac{p' + p}{2} = (P^0, \vec{0}_\perp, P_z) \qquad \Delta = p' - p = (0, \vec{\Delta}_\perp, 0)$$

$P_z = 0$ $\qquad$ $P_z \to \infty$

Breit frame (BF)[4]

$$P = (P^0, \vec{0}_\perp, 0)$$
$$\Delta = (0, \vec{\Delta}_\perp, 0)$$

Infinite momentum frame (IMF)[5]

$$P \approx (P_z, \vec{0}_\perp, P_z)$$
$$\Delta = (0, \vec{\Delta}_\perp, 0)$$

cf. light front frame

$$P^+ \sim P_z \qquad P^- \approx 0$$

[3] C. Lorcé, PRL 125 (2020) 232002
[4] R. G. Sachs, PR126 (1962) 2256
[5]M. Murkardt, IJMPA18 (2003) 173
G. Miller, PRL99 (2007) 112001

Energy-momentum tensor

Energy-momentum tensor (EMT)

Conserved current under space-time translations

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

: Energy
: Momentum
: Pressure
: Shear forces

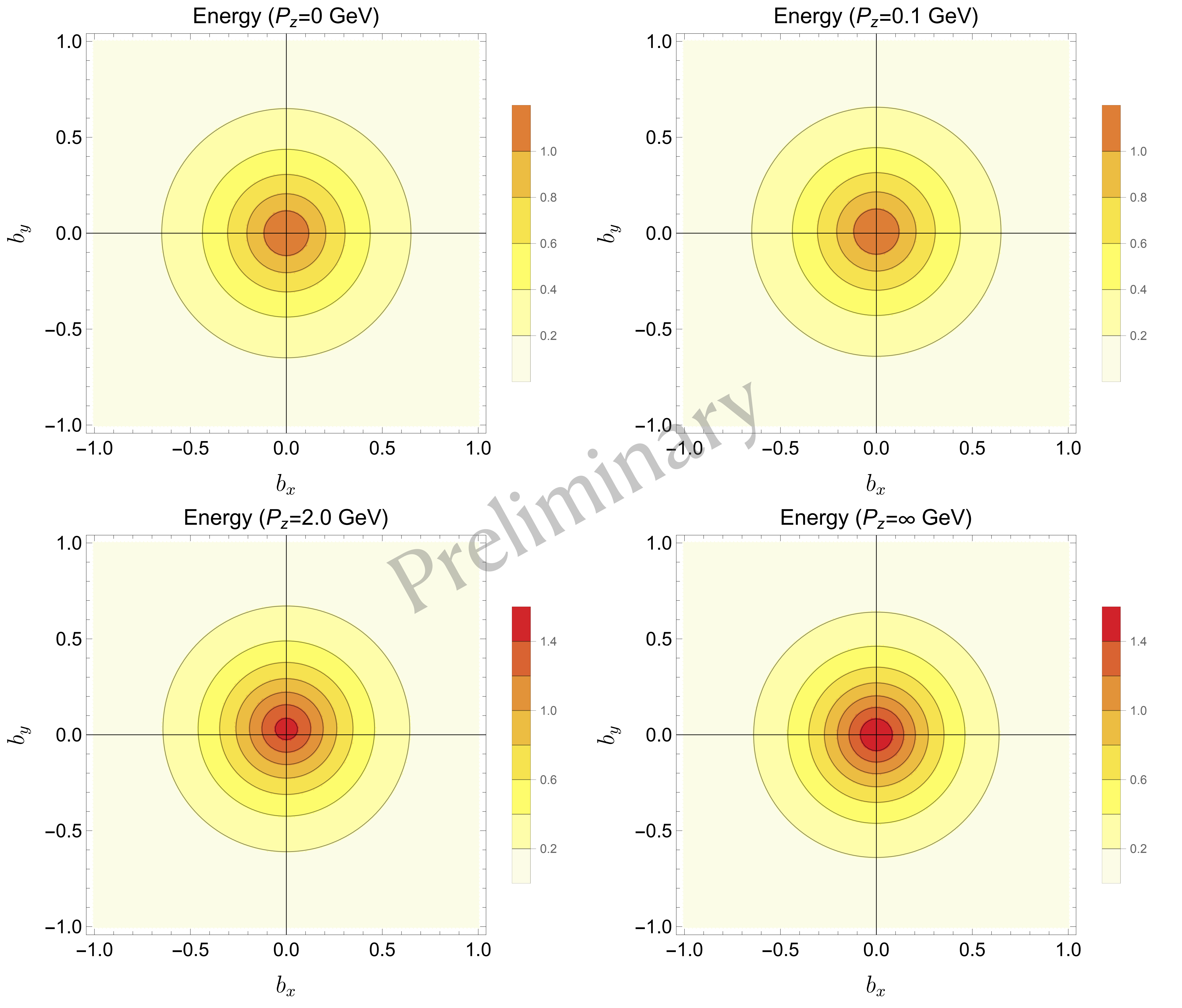
For future works…

Applying for higher spin hadrons

Transverse TAM, OAM, and intrinsic spin distributions

etc…

Polarized energy distribution



Thank you for listening

Don’t hesitate to ask questions or discuss something!

Back up

Physical spin states in the relativistic sense

$$|p, s\rangle = \sum_{s_{\mathrm{BF}}} U(\Lambda)\,|p_{\mathrm{BF}}, s_{\mathrm{BF}}\rangle D_{s_{\mathrm{BF}}s}\left(p_{\mathrm{BF}}, \Lambda\right)$$

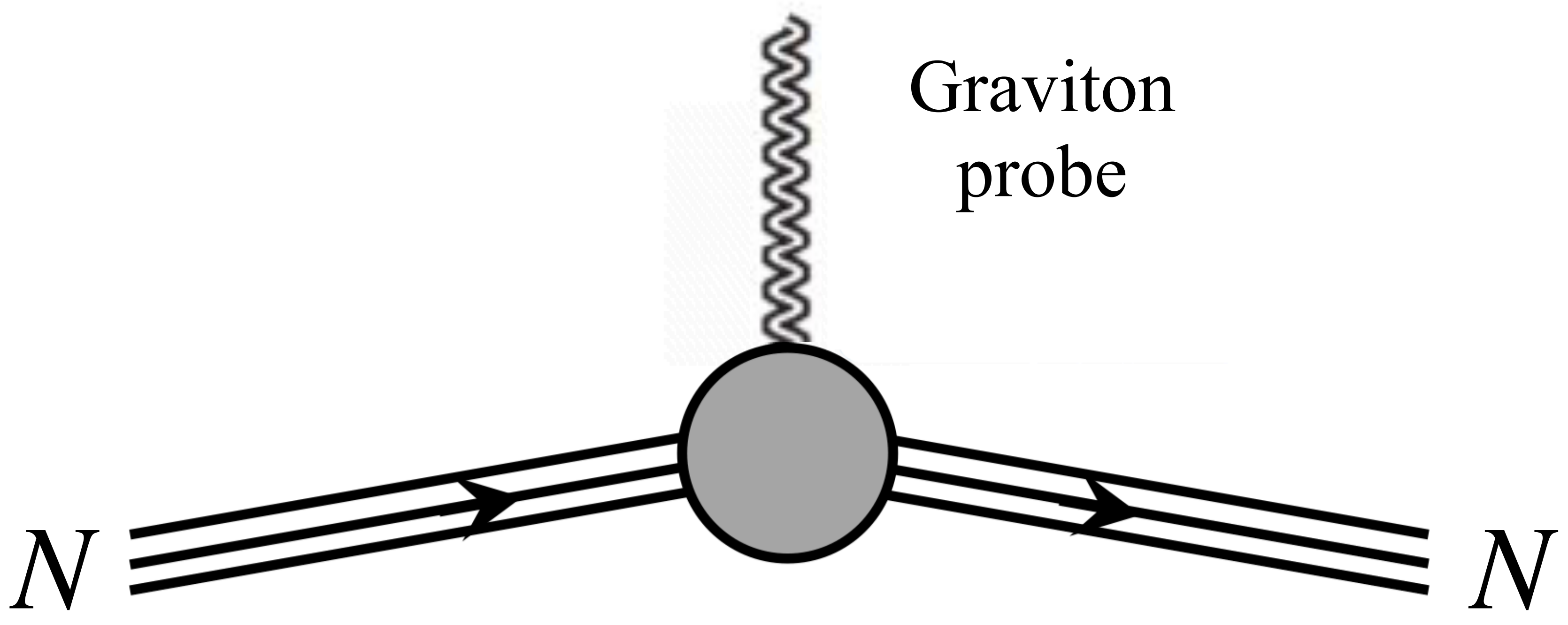
$$\rightarrow D_{s_{\mathrm{BF}}s}\left(p_{\mathrm{BF}}, \Lambda\right) = \langle 0,s\,|\,U^{-1}\left(\Lambda p_{\mathrm{BF}}\right) U(\Lambda)\, U\left(p_{\mathrm{BF}}\right)|\,0,s_{\mathrm{BF}}\rangle$$

Matrix elements of asymmetric EMT operator

$$< p', s'|\,\widehat{T}^{\mu\nu}_a(0)\,|p, s> = \bar{u}\left(p', s'\right)\left[\frac{P^\mu P^\nu}{M}A_a\left(Q^2\right) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M}D_a\left(Q^2\right) + Mg^{\mu\nu}\bar{C}_a\left(Q^2\right)\right.$$
$$\left. + \frac{iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M}J_a\left(Q^2\right) - \frac{iP^{[\mu}\sigma^{\nu]\rho}\Delta_\rho}{2M}S_a\left(Q^2\right)\right] u\left(p, s\right)$$

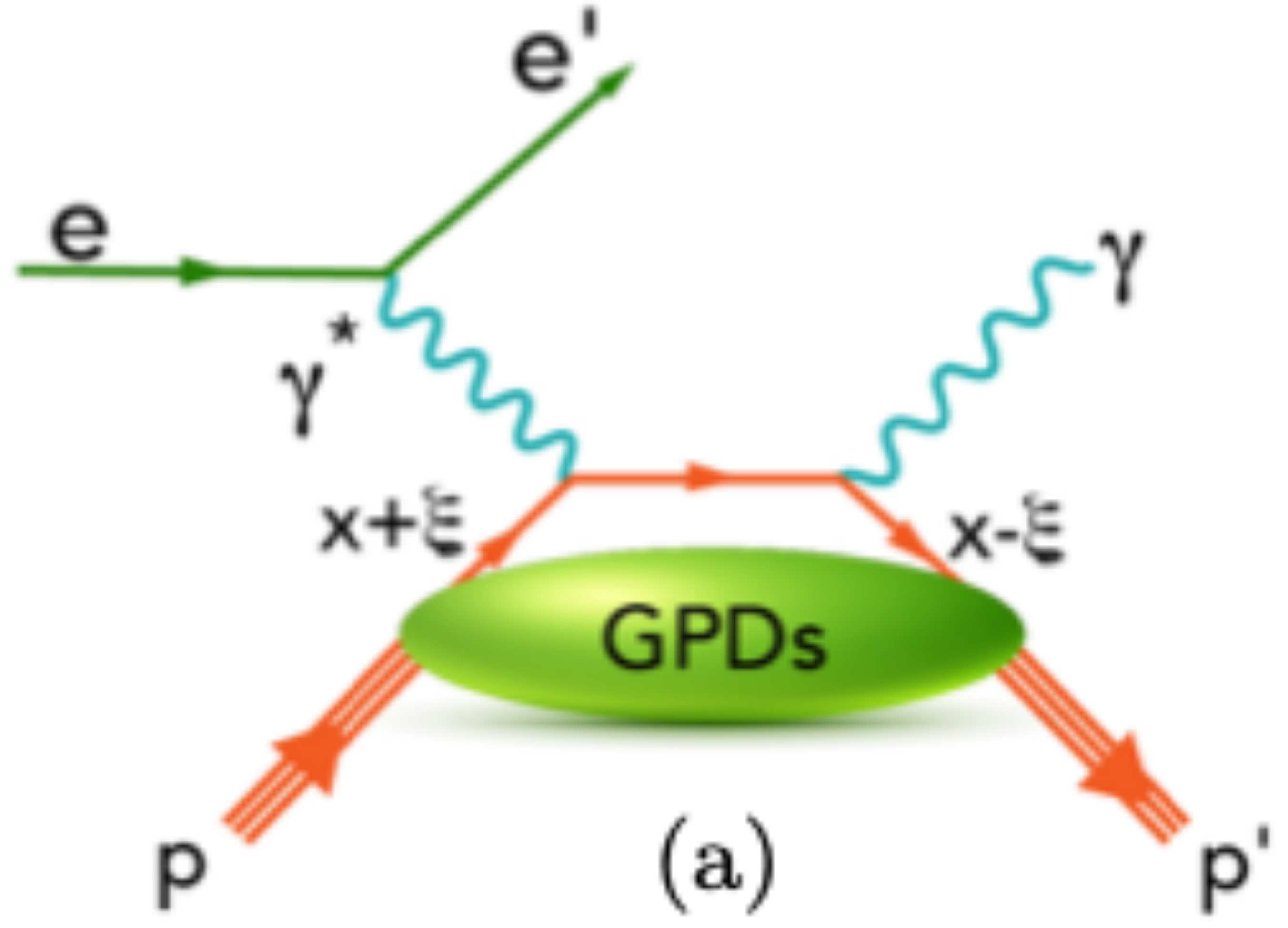
Back up

Direct measure



Too weak gravitation

Indirect measure[6]



Deeply virtual Compton scattering

[6] X.-D. Ji, PRL 78 (1997) 610

V.D. Burkert et al., Rev.Mod.Phys. 95 (2023) 4

Back up

Matrix elements of non-local quark operator; Generalized Parton Distributions (GPDs)

$$\frac{1}{2}\int\frac{d\lambda}{2\pi}e^{ix(P\cdot z)} < p',s'|\overline{\psi}_q\left(-\frac{\lambda n}{2}\right)\gamma^\mu n_\mu \psi_q\left(\frac{\lambda n}{2}\right)|p,s > \Big|_{z=\lambda n}$$

$$= \frac{1}{2\left(P\cdot n\right)}\overline{u}\left(p',s'\right)\left[H^q\left(x,\xi,t\right)\gamma^\mu n_\mu + E^q\left(x,\xi,t\right)\frac{i\sigma^{\mu\nu}n_\mu \Delta_\nu}{2M}\right]u\left(p,s\right)$$

In the DVCS, the actual observables are Compton form factors (CFFs) at leading order α_s

$$\mathrm{Re}\mathscr{H}\left(\xi,t\right) + i\mathrm{Im}\mathscr{H}\left(\xi,t\right) = \sum_q e_q^2 \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] H_q\left(x,\xi,t\right)$$

Back up

2D spatial distributions in the EF

$$T_a^{\mu\nu}(\boldsymbol{b}_\perp, P_z; s', s) = \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\, \frac{\langle p', s'|\widehat{T}_a^{\mu\nu}(0)|p, s\rangle}{2P^0}\bigg|_{\Delta_z=0}$$

Generic Lorentz tensor and EMT distribution

$$t_a^{\mu\nu} = \begin{pmatrix} \gamma_P^2\rho_a & \gamma_P\mathscr{P}_a^i & \gamma_P^2\mathscr{P}_a^z \\ \gamma_P\mathscr{J}_a^i & \mathscr{M}_a^{ij} & \gamma_P\Pi_a^{iz} \\ \gamma_P^2\mathscr{J}_a^z & \gamma_P\Pi_a^{zi} & \gamma_P^2\sigma_a^z \end{pmatrix}, \qquad t_a^{\mu\nu} = \gamma_P T_a^{\mu\nu}$$

Back up

EF energy distribution

$$\rho_a^X(\boldsymbol{b}_\perp, P_z; s', s) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[\delta_{s's}\tilde{\rho}_a^{U,X}(Q^2, P_z) + \frac{(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta}_\perp)_z}{2M}\tilde{\rho}_a^{T,X}(Q^2, P_z)\right],$$

where the spin-independent amplitudes are given by

$$\tilde{\rho}_a^{U,E}(Q^2, P_z) = \frac{M}{\sqrt{P_z^2+M^2}}\frac{P^0\left[P^0 + M(1+\tau)\right]}{M(P^0+M)(1+\tau)} ME_a(Q^2),$$

$$\tilde{\rho}_a^{U,J}(Q^2, P_z) = \frac{M}{\sqrt{P_z^2+M^2}}\frac{2\tau P_z^2}{M(P^0+M)(1+\tau)} MJ_a(Q^2),$$

$$\tilde{\rho}_a^{U,F}(Q^2, P_z) = \frac{M}{\sqrt{P_z^2+M^2}}\frac{P_z^2\left[P^0 + M(1+\tau)\right]}{MP^0(P^0+M)(1+\tau)} MF_a(Q^2),$$

and the spin-dependent amplitudes are given by

$$\tilde{\rho}_a^{T,E}(Q^2, P_z) = -\frac{M}{\sqrt{P_z^2+M^2}}\frac{P^0 P_z}{M(P^0+M)(1+\tau)} ME_a(Q^2),$$

$$\tilde{\rho}_a^{T,J}(Q^2, P_z) = \frac{M}{\sqrt{P_z^2+M^2}}\frac{2P_z\left[P^0 + M(1+\tau)\right]}{M(P^0+M)(1+\tau)} MJ_a(Q^2),$$

$$\tilde{\rho}_a^{T,F}(Q^2, P_z) = -\frac{M}{\sqrt{P_z^2+M^2}}\frac{P_z^3}{MP^0(P^0+M)(1+\tau)} MF_a(Q^2).$$

$$E_a(Q^2) = A_a(Q^2) - \tau B_a(Q^2) + \bar{C}_a(Q^2) + \tau D_a(Q^2),$$

$$J_a(Q^2) = \frac{1}{2}\left[A_a(Q^2) + B_a(Q^2)\right],$$

$$F_a(Q^2) = -\tau D_a(Q^2) - \bar{C}_a(Q^2),$$