Spectroscopic analysis of exotic hadrons using effective theories



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What are Exotic Hadrons?

Tetraquarks and Pentaquarks are the prime examples. They cannot be described by conventional structure of mesons and baryons.

• Tetraquarks

- Tetraquarks are "Exotic Mesons" consists of a pair of quarks and antiquarks.
- Tetraquarks are Bosons (Integral spin).
- In 2003, a particle temporarily called X(3872), by the Belle experiment in Japan is identified.
- Recently, in 2022, LHCb announced the discovery of two singly heavy tetraquark structures named as $T_{c\overline{s}0} (2900)^{++}$ having quark content $c\overline{s}u\overline{d}$ and $T_{c\overline{s}0} (2900)^{00}$ with quark content $c\overline{s}d\overline{u}$.
- Other experimentally detected states are :
- Z(4430), Y(4660), Y(4140), X(4274), X(4500) and X(4700), X(6900) etc.

• Pentaquarks

- Pentaquarks are "Exotic Baryons" consists of four quarks and an anti-quark.
- Total baryon number is 1.
- Pentaquarks are Fermions (half integral spin).
- The first claim of pentaquark discovery was recorded at LEPS in Japan in 2003.
- Recently, in 2022, the LHCb collaboration
- announced the discovery of the hidden-charm
- $P_{\psi s}^{\Lambda}(4338)^0$ pentaquark state with quark content
- of $uudc\overline{c}$.
- Other experimentally detected states are :
- $P_c^+(4380)$, $P_c^+(4450)$ and $P_c^-(4312)^+$

Classification scheme

To classify the exotic hadrons like tetraquarks and pentaquarks, we make use of the special unitary representations and Young tableau Techniques:

1) SU(3) Flavor representation:

Each quark is assigned by fundamental '3' representation and an anti-quark by ' $\overline{3}$ ' representation. For example, fully light pentaquarks with configuration $qqqq\overline{q}$ can be classified as:

 $[3] \otimes [3] \otimes [3] \otimes [3] \otimes [\overline{3}] = [35] \oplus 3[27] \oplus 2[\overline{10}] \oplus 4[10] \oplus 8[8] \oplus 3[1],$

Similarly, for fully light pentaquarks with configurations $q\overline{q}q\overline{q}$ can be classified as:

 $[3] \otimes [\overline{3}] \otimes [3] \otimes [\overline{3}] = [1] \oplus [8] \oplus [1] \oplus [8] \oplus [27] \oplus [8] \oplus [8] \oplus [10] \oplus [10]$

2) SU(2) Spin representation:

Each quark as well as an anti-quark is assigned by a fundamental '2' representation. For example, a tetraquark state with quark content $q\bar{q}q\bar{q}$ can be classified as:

 $[2] \otimes [2] \otimes [2] \otimes [2] = [1] \oplus [3] \oplus [1] \oplus [3] \oplus [3] \oplus [5]$

Similarly, for fully light pentaquarks,

 $[2] \otimes [2] \otimes [2] \otimes [2] \otimes [2] = [6] \oplus 4[4] \oplus 5[2]$

Young Tableau Techniques

We can define spin-flavor wave functions of exotic hadrons using Young tableau techniques. For a particular SU(n), we denotes a quark by a box representation and an anti-quark by n-1 boxes in vertical manner.



By using these classification schemes for spin and flavor of exotic hadrons, we studied the masses and magnetic moments of exotic hadrons such as tetraquarks and pentaquarks. For such purpose, we used the following methodologies:

• An extension of Gursey-Radicati mass formula

To calculate the mass spectra of exotic hadrons such as tetraquarks and pentaquarks. We have studied the mass spectra of tetraquarks states from singly heavy to fully heavy configurations. Further, pentaquarks states with configurations like singl heavy pentaquarks, hidden-charm pentaquarks, triply heavy pentaquarks are also examined using the same technique.

• Effective mass Scheme

The effective mass scheme is a theoretical framework that simplifies the calculation of magnetic moments for multi-quark systems, such as pentaquarks, by introducing effective masses for the constituent quarks. It is used to calculate the masses as well as magnetic moments of pentaquarks with configurations like singly heavy pentaquarks, hidden-charm pentaquarks, triply heavy pentaquarks.

• Screened Charge scheme

The screened charge scheme is an advanced computational method used to evaluate the magnetic moments of multi-quark systems. It is used to calculate the magnetic moments of pentaquarks with configurations like singly heavy pentaquarks, hidden-charm pentaquarks, triply heavy pentaquarks.

The Gursey-Radicati mass Formula

• The original mass formula interms of Casimir operator is given by :

 $M = M_0 + C C_2[SU_S(2)] + D C_1[U_Y(1)] + E[C_2[SU_I(2)] - 1/4(C_1[U_Y(1)])^2]$

Where, $C_2[SU_S(2)]$ and $C_2[SU_I(2)]$ are the SU(2) Casimir operators for spin and isospin, respectively and $C_1[U_Y(1)]$ is the Casimir for the U(1) subgroup generated by the hypercharge Y.

In the framework of the CQM, the underlying symmetry is provided by SU(6), therefore the most general formula that can be written on the basis of a broken SU(6) symmetry :

 $M = M_0 + A C_2[SU_{SF}(6)] + B C_2[SU_F(3)] + C C_2[SU_S(2)] + D C_1[U_Y(1)] + E (C_2[SU_I(2)] - 1/4 (C_1[U_Y(1)])^2)$

By putting the eigenvalues of the Casimir operator into the mass formula,

 $<C_2[SU_1(2)]> = I(I+1), <C_1[U_Y(1)]> = Y, <C_2[SU_S(2)]> = S(S+1)$

Mass Formula is given by :

 $M_{GR} = M_0 + A s(s+1) + DY + G C_2(SU(3)) + E [I(I+1) - \frac{1}{4}Y^2]$

Initially, this was given to calculate baryon masses. M_0 is the scale parameter. A, D, E, G are constants. S, Y, I and $C_2(SU(3))$ are spin, hypercharge, isospin and Casimir operator respectively. The extended form of the mass formula parmeters are:

 M_0 is the scale parameter. A, D, E, G are constants. S, Y, I and $C_2(SU(3))$ are spin, hypercharge, isospin and Casimir operator respectively. The extended form of the mass formula parmeters are:

 $M = M_0 + A s(s+1) + DY + G C_2(SU(3)) + E [I(I+1) - \frac{1}{4}Y^2] + F N_c$

Here, N_c acts as the counter term for the number of charm quarks into the exotic hadrons. Furthermore, mass formula was further modified to include the contribution of bottom quarks and therefore modifications are done to the counter term as follows:

 $M = \xi M_0 + A s(s+1) + DY + G C_2(SU(3)) + E [I(I+1) - \frac{1}{4}Y^2] + \Sigma F_i N_i$

 ξ acts as the correction factor to the scale parameter and F_i acts as the counter term for number of charm and bottom quarks.

	M_0	A	D	E	F	G
Values[MeV]	940.0	23.0	-158.3	32.0	1354.6	52.5
Uncertainties [MeV]	1.5	1.2	1.3	1.3	18.2	1.3

Values of parameters in the GR mass formula with corresponding uncertainties

Effective mass scheme

Total mass of a pentaquark can be defined as:

$$M_P = \sum_{i=1}^{5} m_i^{\text{eff}} = \sum_{i=1}^{5} m_i + \sum_{i < j} b_{ij} s_i . s_j.$$

Here, s_i and s_j represent the spin operator for the *i*th and *j*th quarks (antiquark) and m_i^{ϵ} represents the effective mass for each of the quark (antiquark) and b_{ij} is defined as:

$$b_{ij} = \frac{16\pi\alpha_s}{9m_i m_j} \langle \Psi_0 | \ \delta^3(\vec{r}) | \Psi_0 \rangle, \tag{12}$$

where Ψ_0 is the ground state pentaquark wavefunction. For different quarks inside the pentaquark, effective masses equations are:

$$m_1^{\rm eff} = m_1 + \alpha b_{12} + \beta b_{13} + \gamma b_{14} + \eta b_{15} \tag{13}$$

$$m_2^{\text{eff}} = m_2 + \alpha b_{12} + \beta' b_{23} + \gamma' b_{24} + \eta' b_{25}$$
⁽¹⁴⁾

$$m_3^{\rm eff} = m_3 + \beta b_{13} + \beta' b_{23} + \gamma'' b_{34} + \eta'' b_{35}$$
⁽¹⁵⁾

$$m_4^{\rm eff} = m_4 + \gamma b_{14} + \gamma' b_{24} + \gamma'' b_{34} + \eta''' b_{45} \tag{16}$$

$$m_5^{\rm eff} = m_5 + \eta b_{15} + \eta' b_{24} + \eta'' b_{34} + \eta''' b_{45}. \tag{17}$$

Screened charge scheme

The charge of a quark within an exotic hadron may undergo modification analogous to the alterations observed in its mass due to the surrounding environment.

The effective charge of quark 'a' in the pentaquark (a, f, x, y, z) can be expressed as:

$$e_a^P = e_a + \alpha_{af}e_f + \alpha_{ax}e_x + \alpha_{ay}e_y + \alpha_{az}e_z \tag{40}$$

Similarly, effective charge equations for other quarks (antiquarks) are defined as:

$$e_f^P = e_f + \alpha_{fa}e_a + \alpha_{fx}e_x + \alpha_{fy}e_y + \alpha_{fz}e_z, \tag{41}$$

$$e_x^P = e_x + \alpha_{xa}e_a + \alpha_{xf}e_f + \alpha_{xy}e_y + \alpha_{xz}e_z, \tag{42}$$

$$e_y^P = e_y + \alpha_{ya}e_a + \alpha_{yf}e_f + \alpha_{yx}e_x + \alpha_{yz}e_z, \tag{43}$$

$$e_z^P = e_z + \alpha_{za}e_a + \alpha_{zf}e_f + \alpha_{zx}e_x + \alpha_{zy}e_y \tag{44}$$

where the quark charges are denoted by e_a , e_f , e_x , e_y , and e_z . When we take the isospin symmetry into account, we obtain several relations:

$$\alpha_{af} = \alpha_{fa}, \quad \alpha_{ax} = \alpha_{xa}, \quad \alpha_{ay} = \alpha_{ya}, \quad \alpha_{az} = \alpha_{za} \tag{45}$$

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Thus,

$$\alpha_{uu} = \alpha_{ud} = \alpha_{dd} = \alpha_1 \tag{46}$$
$$\alpha_{us} = \alpha_{ds} = \beta_1$$
$$\alpha_{ss} = \beta_2$$

For the bottom sector,

$$\alpha_{ub} = \alpha_{db} = \beta_3, \quad \alpha_{sb} = \alpha_2 \tag{47}$$
$$\alpha_{cb} = \beta_4, \quad \alpha_{bb} = \alpha_3$$

Further, the reduction of these parameters can be achieved by applying SU(3) symmetry.

$$\alpha_1 = \beta_1 = \beta_2 \tag{48}$$

Using the Ansatz formalism, we can calculate the screening parameter α_{ij} as:

$$\alpha_{ij} = \left| \frac{m_i - m_j}{m_i + m_j} \right| \times \delta \tag{49}$$

Results

Decuplet states	Effective mass scheme
$\overline{P_c^{++(0)}}$	$3 \ \mu_u^{\text{eff}} + \mu_c^{\text{eff}} + \mu_{\bar{c}}^{\text{eff}}$
$P_c^{+(0)}$	$2\mu_u^{\text{eff}} + \mu_d^{\text{eff}} + \mu_c^{\text{eff}} + \mu_{\bar{c}}^{\text{eff}}$
$P_{c}^{0(0)}$	$\mu_u^{\mathrm{eff}} + 2\mu_d^{\mathrm{eff}} + \mu_c^{\mathrm{eff}} + \mu_{ar{c}}^{\mathrm{eff}}$
$P_{c}^{-(0)}$	$3\mu_d^{\text{eff}} + \mu_c^{\text{eff}} + \mu_{\bar{c}}^{\text{eff}}$
$P_c^{+(1)}$	$2\mu_u^{\mathrm{eff}} + \mu_s^{\mathrm{eff}} + \mu_c^{\mathrm{eff}} + \mu_{ar{c}}^{\mathrm{eff}}$
$P_{c}^{0(1)}$	$\mu_u^{\text{eff}} + \mu_d^{\text{eff}} + \mu_s^{\text{eff}} + \mu_c^{\text{eff}} + \mu_{\bar{c}}^{\text{eff}}$
$P_{c}^{-(1)}$	$2\mu_d^{ ext{eff}} + \mu_s^{ ext{eff}} + \mu_c^{ ext{eff}} + \mu_{ar{c}}^{ ext{eff}}$
$P_{c}^{0(2)}$	$\mu_u^{\mathrm{eff}} + 2\mu_s^{\mathrm{eff}} + \mu_c^{\mathrm{eff}} + \mu_{ar{c}}^{\mathrm{eff}}$
$P_{c}^{-(2)}$	$\mu_d^{ ext{eff}} + 2\mu_s^{ ext{eff}} + \mu_c^{ ext{eff}} + \mu_{ar{c}}^{ ext{eff}}$
$P_{c}^{-(3)}$	$3\mu_s^{\mathrm{eff}} + \mu_c^{\mathrm{eff}} + \mu_{\bar{c}}^{\mathrm{eff}}$

Table showing expression for $J^p = 5/2^-$ hidden-charm pentaquarks magnetic moments using the effective mass scheme in decuplet representation.

Penta	aquark States	Masses [MeV]	Effective Mass Scheme	[MeV] Ref. [36] [MeV]
$P_c^{++(0)}, P_c^{+(0)}, P_c^{0(0)}, P_c^{-(0)}$		4772.54 ± 38.99	4615.64	4743
$P_{c}^{+(1)}$	$, P_c^{0(1)}, P_c^{-(1)}$	4881.84 ± 38.79	4772.42	4850
P_{c}	$P_{c}^{-(2)}, P_{c}^{-(2)}$	4991.14 ± 38.73	4931.88	5008
	$P_{c}^{-(3)}$	5100.44 ± 38.81	5094.02	5140
	Quark		Screened	Effective mass +
State	content	Effective mass	charge	Screened charge
$P_{c}^{++(0)}$	ииис̄с	4.78	6.34	5.93
$P_c^{+(0)}$	$uudc\bar{c}$	2.39	3.17	2.97
$P_{c}^{0(0)}$	$uddc\bar{c}$	0	0	0
$P_{c}^{-(0)}$	$dddc\bar{c}$	-2.39	-3.17	-2.96
$P_{c}^{+(1)}$	uuscē	2.66	3.59	3.37
$P_c^{0(1)}$ (1)	$udscar{c}$	0.246	0.14	0.12
$P_{c}^{-(1)}$	$ddscar{c}$	-2.17	-3.30	-3.14
$P_c^{0(2)}$ (2)	usscē	0.50	0.74	0.68
$P_c^{-(2)}$ (2)	$dsscar{c}$	-1.94	-2.98	-2.87
$P_{a}^{-(3)}$	SSSCĒ	-1.69	-2.20	-2.15

TABLE V. Magnetic moments assignments of hidden-bottom octet of pentaquarks with $J^P = 3/2^-$	utilising the effective mass
scheme, screened charge scheme, and combining the effective mass and screened charge schemes.	

State	Quark Content	Effective mass	Screened Charge	Eff. mass $+$ Screen.	Charge Ref. [23]
$P_{b}^{+(0)}$	$uudb\overline{b}$	2.00	2.28	2.04	2.78
$P_{b}^{0(0)}$	$uddbar{b}$	-0.09	-0.09	-0.09	-0.04
$P_{b}^{+(1)}$	$uusbar{b}$	2.26	2.62	2.39	2.60
$P_b^{0(1)}, P_b^{1'0}$	$udsbar{b}$	0.12	0	-0.02	0.10
$P_{b}^{-(1)}$	$ddsbar{b}$	-2.01	-2.62	-2.43	-2.41
$P_{b}^{0(2)}$	$ussbar{b}$	0.36	0.50	0.45	0.51
$P_{b}^{-(2)}$	$dssbar{b}$	-1.81	-2.36	-2.24	-2.43

TABLE VI. Table for masses of predicted hidden-bottom Pentaquark Decuplet using the extended form of the Gursey-Radicati mass formula and the effective mass scheme. The nomenclature for pentaquark states is identical to that seen in the figure.². All masses are measured in MeV.

Pentaquark States	G-R Formula	Eff. Mass Scheme	Ref. [9]	Ref. [6]	Ref. [23]
$P_b^{++(0)}, P_b^{+(0)}, P_b^{0(0)}, P_b^{-(0)}$	11676.60 ± 70.20	11314.1	11052	11246.5	11235
$P_b^{+(1)}, P_b^{0(1)}, P_b^{-(1)}$	11786.90 ± 70.09	11462.2	-	11429.9	11524
$P_b^{0(2)}, P_b^{-(2)}$	11897.20 ± 70.06	11614.4	11141	11575.7	11669
$P_{b}^{-(3)}$	12007.50 ± 70.10	11771.2	-		11673

Quark Content	S I	Y	$SU(3)_f$	$C_2(SU(3)_f$	$) N_c$	Our prediction(MeV) Reference
uuuūc, uudūc, uddūc	1/2 0	2/3	[4]15	16/3	1	3109.42 ± 19.57
dddūc, uuddc	1/2 1	2/3	4115	16/3	1	3173.42 ± 19.72
uddde nuude	1/2 2	2/3	4115	16/3	1	3301.42 ± 21.01
uusse, udsse, ddsse	3/2 0	2/3	4 15	16/3	ĩ	3178.42 ± 20.06
,,	3/2 1	2/3	4 15	16/3	1	3242.42 ± 20.21
	3/2 2	2/3	4 15	16/3	1	3370.42 ± 21.47
	5/2 0	2/3	4 15	16/3	1	3293.42 ± 22.19
	5/2 1	2/3	4 15	16/3	1	3357.42 ± 22.32
	5/2 2	2/3	[4]15	16/3	1	3485.42 ± 23.47
unusc. undsc	1/21/	25/3	[4] 15	16/3	1	2956.45+ 19.67
uddāc, dddāc	1/2 3/	25/3	4 15	16/3	1	3052.45 ± 20.07
	1/25/	25/3	4 15	16/3	1	3212.45 ± 22.28
	3/2 1/	25/3	4 15	16/3	1	3035.45 ± 20.16
	3/2 3/	25/3	4 15	16/3	1	3121.45 ± 20.54
	3/25/	25/3	4 15	16/3	1	3281.45 ± 22.71
	5/21/	25/3	4 15	16/3	1	3140.45 ± 22.28
	5/2 3/	25/3	4 15	16/3	1	3236.45 ± 22.63
	5/2 5/	25/3	[4]15	16/3	1	3396.45 ± 24.62
uusūc, ussāc, dssāc	1/2 1/	2 - 1/3	[4]15	16/3	1	3294.39 ± 19.58
ddsūc. ddsdc	1/2 3/	2 - 1/3	4115	16/3	1	3390.39 ± 20.14
unsde udsuc	1/2 5/	2 -1/3	4 15	16/3	1	3550.39 ± 22.60
	3/2 1/	2 - 1/3	4 15	16/3	1	3363.39 ± 20.07
	3/2 3/	2 - 1/3	4 15	16/3	1	3459.39 ± 20.62
	3/25/	2 - 1/3	4 15	16/3	1	3619.39 ± 23.03
	5/21/	2 - 1/3	4 15	16/3	1	3478.39 ± 22.20
	5/23/	2 - 1/3	4 15	16/3	1	3574.39 ± 22.70
	5/25/	2 - 1/3	[4]15	16/3	1	3734.39 ± 24.91
ussue ussde	1/2 0	-4/3	[4]15	16/3	1	3415 35+ 19 63
desde	1/2 1	-4/2	4	16/3	1	347935 + 1973
LB+7+71.5L	3/2 0	-1/2	4 15	16/3	1	3484.35 ± 20.12
	3/2 1	-4/3	4 15	16/3	1	3548.35 ± 20.22
	5/2 0	-4/3	4 15	16/3	î	3599.35 ± 22.52
	5/2 1	-4/3	[4]15	16/3	1	3663.35 ± 22.33
sssuc sesde	1/21/	2 -7/3	[4]	16/3	1	3568 32+ 19 80
treater, marriet	3/2 3/	2 -7/3	4 15	16/3	i	3637.32 ± 20.28
	5/2 5/	2 7/3	111	16/3	1	3752 32+ 22 30

Spin I Masses Quark Y content $\begin{array}{ccc} P_{c0}^{++}, & P_{c}^{-} \\ P_{c0}^{0}, P_{c0}^{-}, P_{c}^{-} \end{array}$ $P_{c0}^{+},$ 4/3 3195.64 \pm 20.91 2 4/3 3264.64 \pm 21.37 4/3 3379.64 \pm 23.38 2 $P_{c1}^+, P_{c1}^0, P_{c1}^-,$ 1/3 3295.22 \pm 20.14 3/21/2 P_{c1}^{--} 3/2 3/2 1/3 3364.22 ± 20.62 $5/2 \ 3/2 \ 1/3 \ 3479.22 \pm 22.70$ $P_{c2}^0, P_{c2}^-, P_{c2}^{--}$ 1/2-2/3 3393.32 \pm 19.72 -2/3 3462.32 \pm 20.12 1 -2/3 3577.32 \pm 22.32 P_{c3}^{-}, P_{c3}^{--} $1/2 - 5/3 \ 3493.06 \pm 19.67$ 3/2 1/2 -5/3 3562.06 ± 20.15 5/2 1/2 -5/3 3677.06 ± 22.28 P_{c4}^{--} 0 -8/3 3592.80 \pm 19.98 1/20 -8/3 3661.80 \pm 20.46 3/20 -8/3 3776.80 \pm 22.56 5/2

TABLE I: Masses of singly charm pentaquark states arranged in 15-plet of SU(3) representation. Each state is labeled as P_{cs}^q , where s stands for the number of strange quarks in the system and q stands for the electric charge.

TABLE II. Pentaquark masses for $N_c = 1$ for all possible configurations. Masses are in the units of MeV.

SCI Publications:

1) Ankush Sharma, Alka Upadhyay, An analysis of Gursey-Radicati mass formula for Tetraquark systems, Phys. Ser. 08 005308 (2022) doi: (10 1088/1402 4806/acf0f2). Impact Easter 2.0

Phys. Scr. 98 095308 (2023) doi: (10.1088/1402-4896/acf0f3), Impact Factor - 2.9.

2) Ankush Sharma, Alka Upadhyay, Spectroscopy of Hidden-charm decuplet of Pentaquarks,
J. Phys. G: Nucl. Part. Phys. 51 035003 (2024) doi: (10.1088/1361-6471/ad1eb8), Impact Factor - 3.5.

3) Ankush Sharma, Alka Upadhyay, Masses and Magnetic moments of singly heavy Pentaquarks, arXiv:2401.02146 (Communicated).

4) Ankush Sharma, Alka Upadhyay, Hidden-Bottom Pentaquarks: masses, magnetic moments and partial widths, arXiv:2402.14885 (Communicated).

5) Ankush Sharma, Alka Upadhyay, Phenomenological Analysis of Triply Heavy Pentaquarks with configurations $q\bar{q}QQQ$ and $qqQQ\bar{Q}$. arXiv:2405.11479 (Communicated).

Conference Publications:

1) **Ankush Sharma**, Rashmi, Alka Upadhyay, Hidden-Charm Pentaquark masses using the extension of the Gursey-Radicati mass formula (DAE Symposia 2022 on Nuclear Physics, Guwahati, Assam, India (2022)).

2) Ankush Sharma, Alka Upadhyay, Bottom Tetraquark masses using the Extension of Gursey-Radicati mass formula (Accepted in QNP 2022 The 9th International Conference on Quark and Nuclear Physics, Tallahassee, United States of America).

3) Rashmi, **Ankush Sharma**, Alka Upadhyay, Spectroscopic analysis of Heavy Pentaquarks (Accepted for Publication in Springer Journal in DAE-HEP 2022 Symposium, IISER Mohali, India).

4) **Ankush Sharma**, Rashmi, Alka Upadhyay, Singly heavy Pentaquark masses using the extension of the Gursey-Radicati mass formula (DAE Symposia 2023 on Nuclear Physics, IIT Indore, Madhya Pradesh, India (2023)).

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1) Elena Santopinto and Alessandro Giachino, Compact Pentaquark Structures, Phys.Rev. D.96.014014.

2) M.M. Giannini, E. Santopinto, A.Vassallo,Spectroscopy of Pentaquark States, arXiv:nuclth/0506032v1.

3) Pontus Holma, Tommy Ohlsson, Phenomenological predictions for pentaquark masses from fits to baryon masses, arXiv:1906.08499 [hep-ph].

4) P. Colangelo, F. De Fazio and S. Nicotri, Phys. Lett. B 642, 48(2006).

5) Hidden charm octet tetraquarks from a diquark-antidiquark model Ruilin Zhu, Phys. Rev. D 94, 054009.

6) Implications of Heavy Quark-Diquark Symmetry for Excited Doubly Heavy Baryons and Tetraquarks, Thomas Mehen, Phys. Rev. D 96, 094028.

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