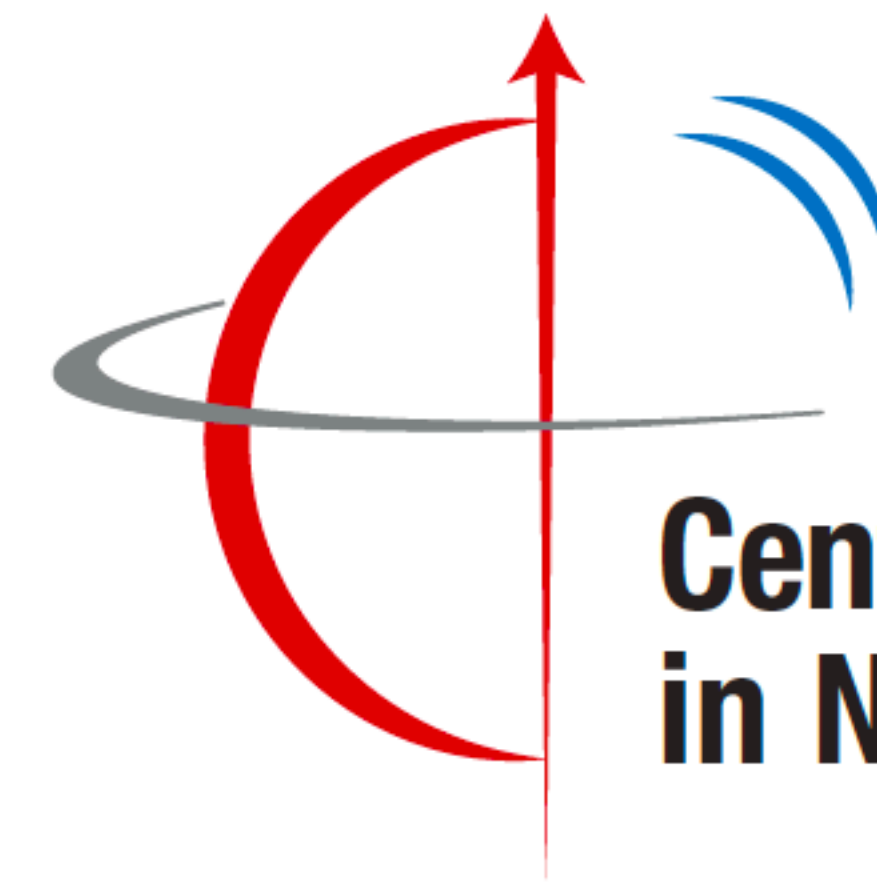
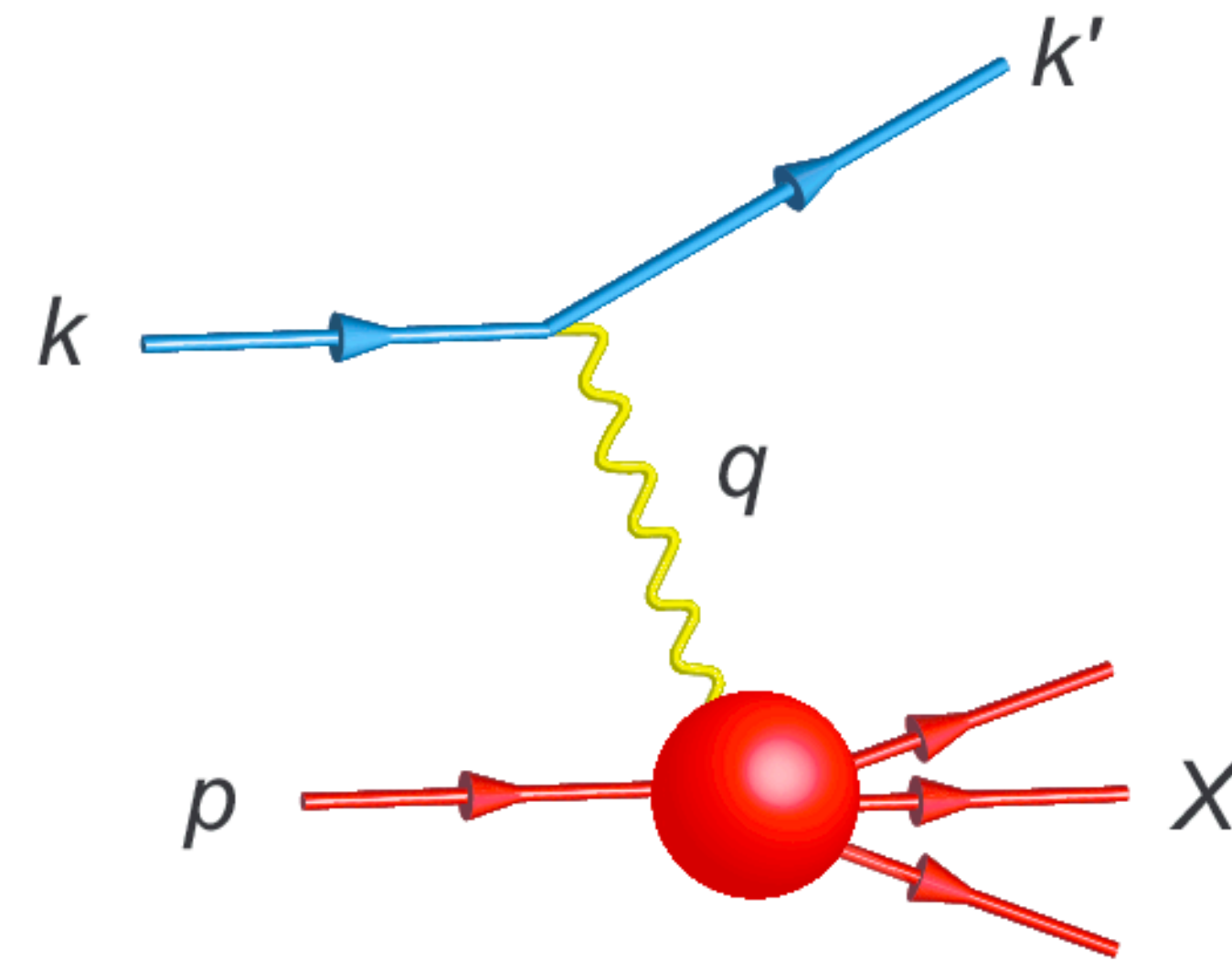
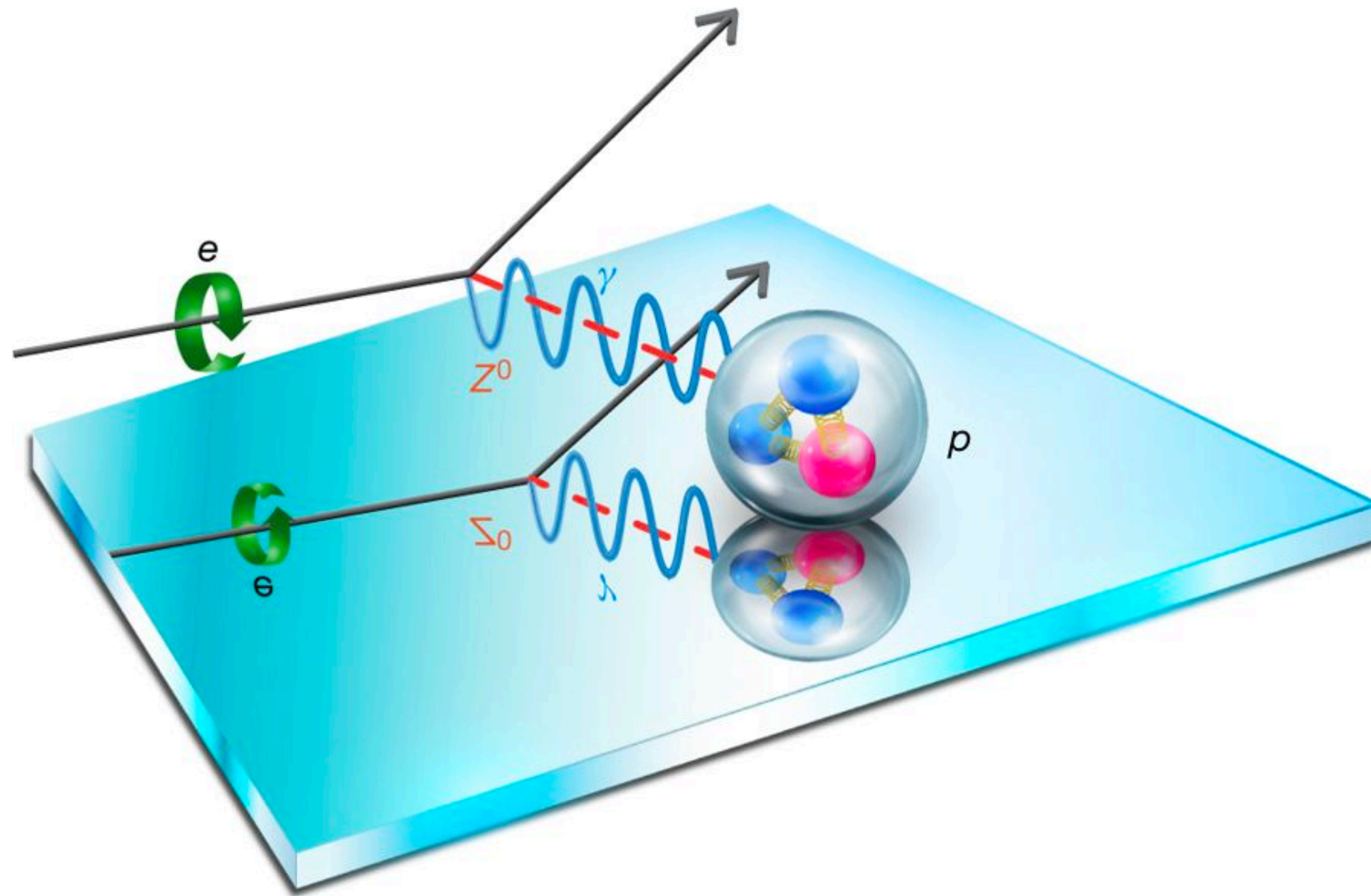


# QUADRATIC AND REDUCIBLE TWO LOOP LEVEL FULL ELECTROWEAK $e$ -P SCATTERING WITH COVARIANT APPROACH



**Center for Frontiers  
in Nuclear Science**



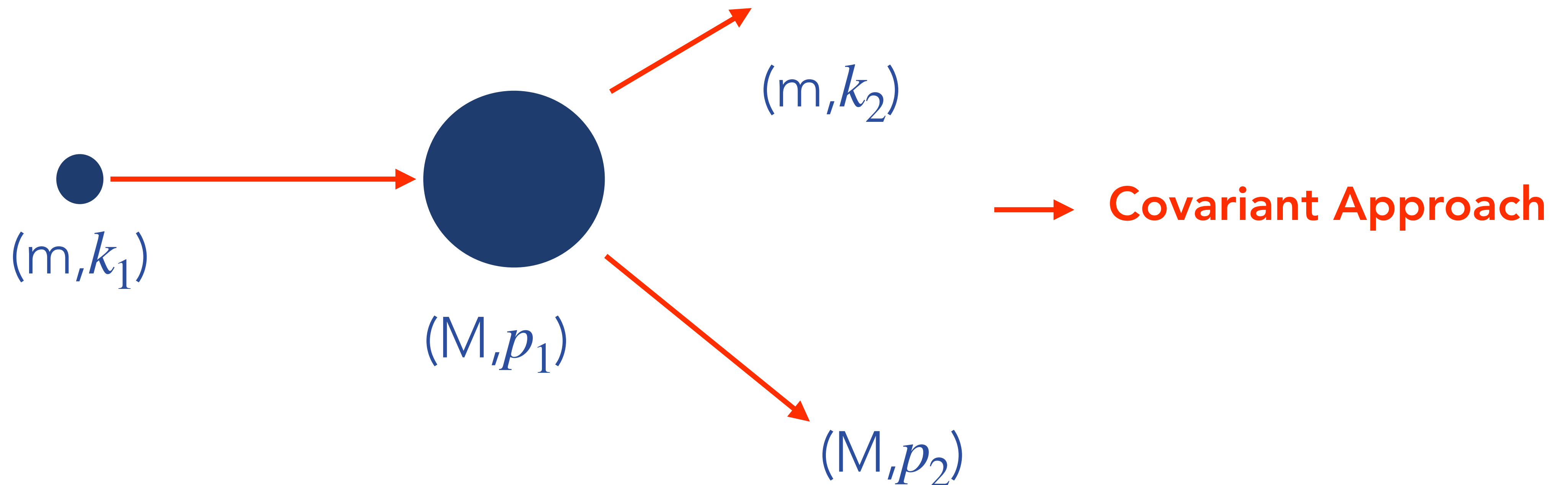
MAHUMM GHAFAR  
PHD CANDIDATE MEMORIAL UNIVERSITY OF NEWFOUNDLAND  
SUPERVISORS: DR. ALEKSANDRS ALEKSEJEVS AND DR. SVETLANA BARKANOVA

# MOTIVATION

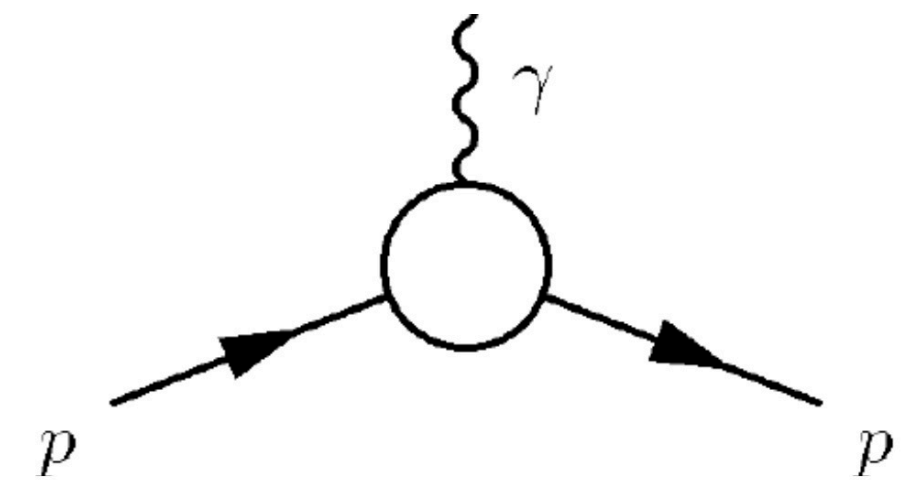
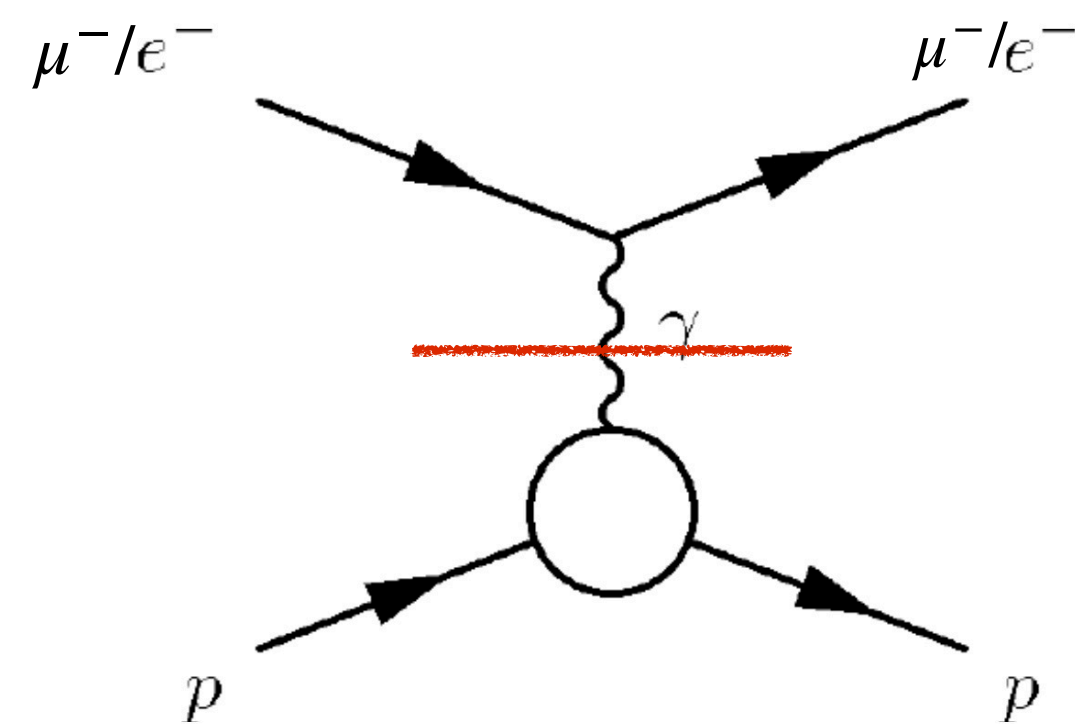
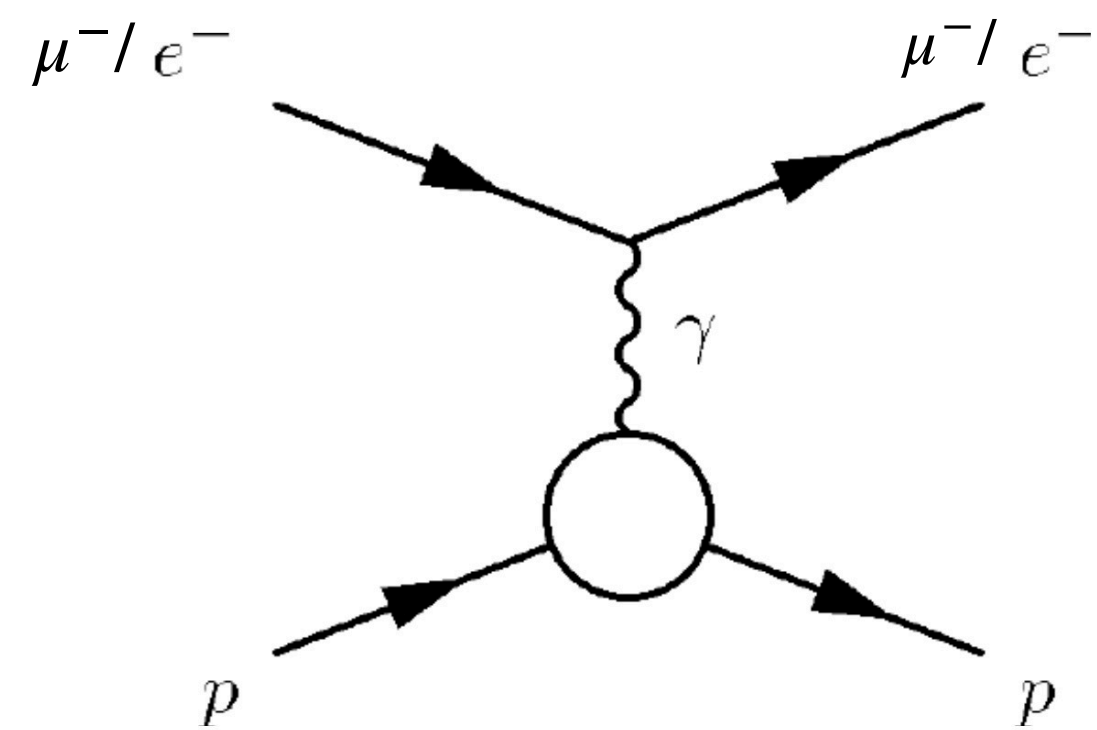
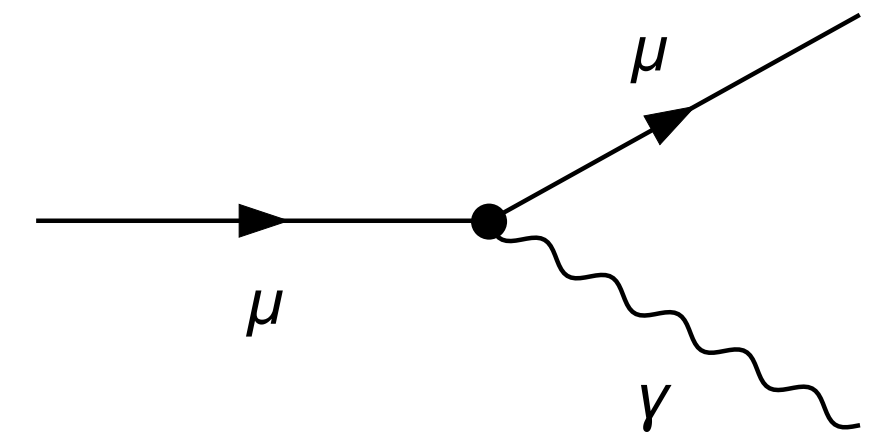
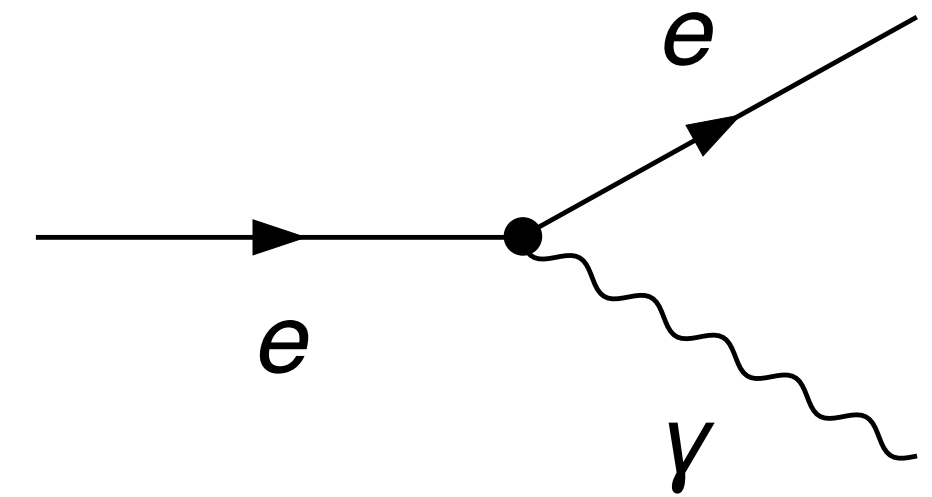
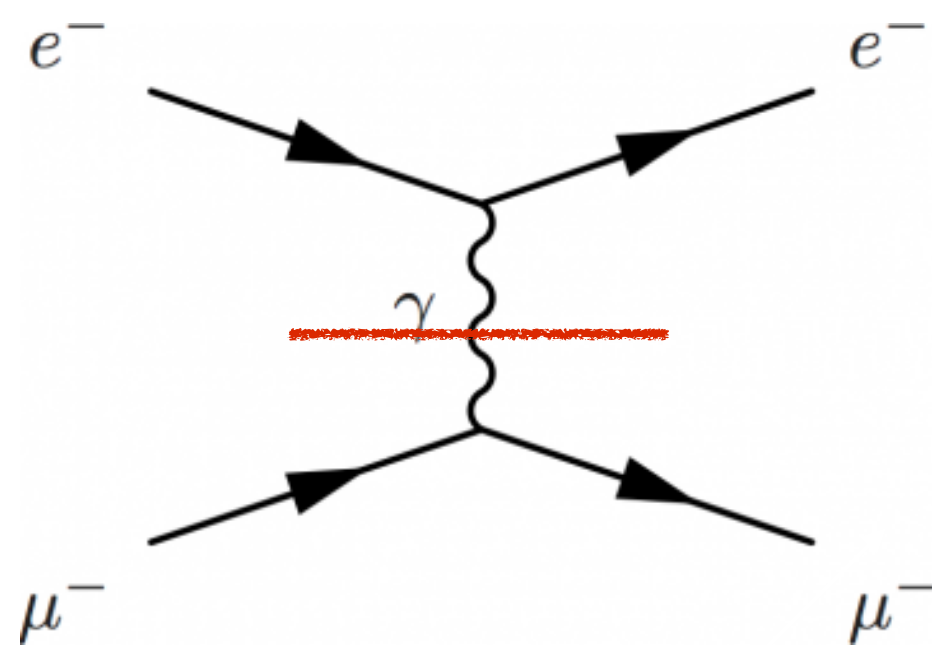
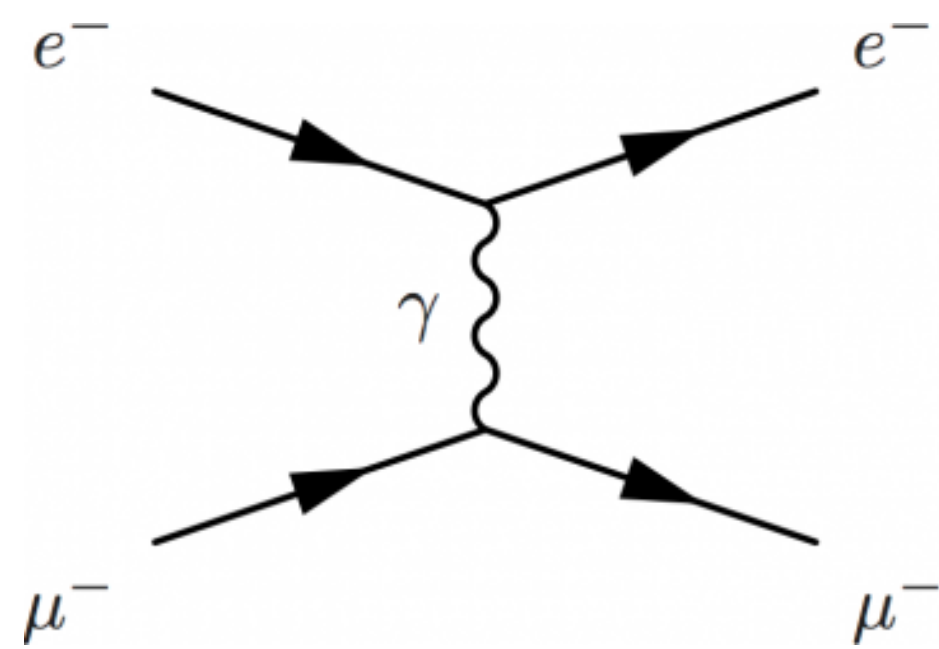
- The theory of Standard Model (SM) → unifies Electromagnetic, Weak and Strong interactions → can make predictions that match experiments to one part in ten billion.
- SM limitations → don't include gravity, dark matter/dark energy existence, hierarchies of scale related to Higgs boson etc.
- Theoretical door open for Beyond the SM (BSM) physics to be observed at TeV scale → **but** till date no concrete evidence of BSM at 13 TeV centre-of-mass energy at LHC.
- **Low energy precision physics** becomes important → provides a way to reach mass scales not directly accessible at existing high-energy colliders.
- We are doing precision physics with full electroweak Parity Violating Asymmetry ( $A_{PV}$ ) → achieve by calculating the higher order corrections up to NNLO ( $\alpha^4$ ) using **Covariant/ leptonic tensor approach**.

# FULL ELECTROWEAK $e^-p$ SCATTERING

- Elastic  $e^-p$  scattering is studied up to the NNLO level considering all SM particles in the loop.
- A longitudinally polarized  $e^-$  scatters off an unpolarized proton target



# WHAT IS A COVARIANT APPROACH?



# COVARIANT APPROACH WITH LEPTONIC-HADRONIC TENSORS

- The differential cross section of general lepton-lepton/hadron scattering can be obtained by:

$$d\sigma \sim L^{\mu\nu} L_{\mu\nu} \text{ or } d\sigma \sim L^{\mu\nu} W_{\mu\nu}$$

- where  $W_{\mu\nu}$  is the **hadronic tensor** which in case of elastic  $e^-p$  scattering:

$$W_{\mu\nu} = H_1 g_{\mu\nu} + H_2 p_{1\mu} p_{1\nu} + H_3 p_{2\mu} p_{2\nu} + H_4 p_{1\mu} p_{2\nu} + H_5 p_{2\mu} p_{1\nu} + H_6 \epsilon_{\mu,\nu,p_1,p_2}$$

where  $p_1$  and  $p_2$  are incoming and outgoing protons momenta.  $H_1, H_2, H_3, H_4, H_5$  and  $H_6$  are the hadronic structure functions which can be extracted from experimental data.



# QED AND ELECTROWEAK HADRONIC COUPLINGS WITH FORM FACTORS

$$\Gamma_{\gamma-p}^{\mu}(q^2) = ieCnp2 \left( f2p\gamma^{\mu} + gp\gamma_L\gamma^{\mu}\omega_{-} + gp\gamma_R\gamma^{\mu}\omega_{+} - \frac{f2p(p_1^{\mu} + p_2^{\mu})}{2m_p} \right)$$

$$\Gamma_{Z-p}^{\mu}(q^2) = -ieCnp2 \left( F2W\gamma^{\mu} + gpz_L\gamma^{\mu}\omega_{-} + gpz_R\gamma^{\mu}\omega_{+} - \frac{F2W(p_1^{\mu} + p_2^{\mu})}{2m_p} \right)$$

$$F2W = \frac{F2Vp - 4 \sin^2 \theta_W f2p}{4 \cos \theta_W \sin \theta_W} \rightarrow \text{EW form factor}$$

$$gpz_{(L,R)} = \frac{F1Vp - 4 \sin^2 \theta_W f1p \pm G1p}{4 \sin \theta_W \cos \theta_W}$$

$$gp\gamma_L = gp\gamma_R = f1p(0) \rightarrow \text{Electric form factor}$$

$$G1p = 1.267 \rightarrow \text{Axial form factor}$$

$$Cnp2 = \left( \frac{\Lambda^2}{\Lambda^2 - t} \right)^2, \quad \Lambda = \sqrt{0.83 \, m_p^2}$$

$$F(1,2)Vp = f(1,2)p - f(1,2)n$$

# TREE-LEVEL LEPTONIC TENSOR ( $\alpha$ -ORDER)

$$|M|^2 \propto \left| \begin{array}{c} \text{Diagram 1: } e^- \text{ scattering} \\ \text{Diagram 2: } \mu^- \text{ scattering} \end{array} \right|^2$$

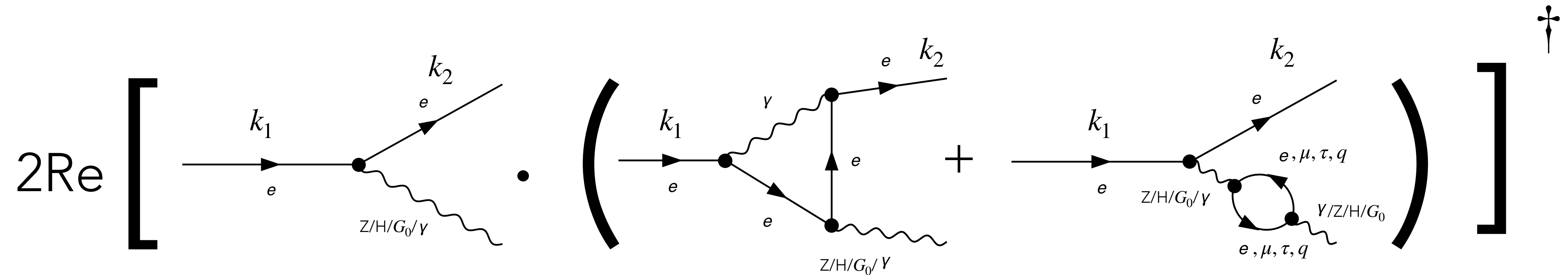
The diagram shows two Feynman diagrams for tree-level leptonic scattering. The first diagram (left) shows an incoming electron with momentum  $k_1$  and an outgoing electron with momentum  $k_2$ , interacting via a wavy line (representing a  $Z/H/G_0/\gamma$  boson) with a muon neutrino. The second diagram (right) shows an incoming muon with momentum  $k_3$  and an outgoing muon with momentum  $k_4$ , interacting via a wavy line (representing a  $Z/H/G_0/\gamma$  boson) with a muon neutrino. The diagrams are enclosed in vertical bars, and the entire expression is squared, indicating the squared magnitude of the matrix element.

- For **tree-level** upper part of the diagram (say  $e^-p$  scattering), one can calculate leptonic tensor which is:

$$L_{\mu\nu}^0 \propto 4\pi\alpha((l_1)g_{\mu\nu} + (l_2)k_{2\mu}k_{1\nu} + (l_3)k_{1\mu}k_{2\nu} + \dots)$$

where  $k_1, k_2$  are incoming and outgoing  $e^-$  momenta and  $l_{1,2..}$  are tree level leptonic tensor structure functions.

# NEXT TO THE LEADING ORDER (NLO) LEPTONIC TENSOR ( $\alpha^2$ -ORDER)



- The **NLO** leptonic tensor can be obtained by multiplying tree-level upper diagram with the sum of one-loop level self energy (SE) and triangular diagrams.

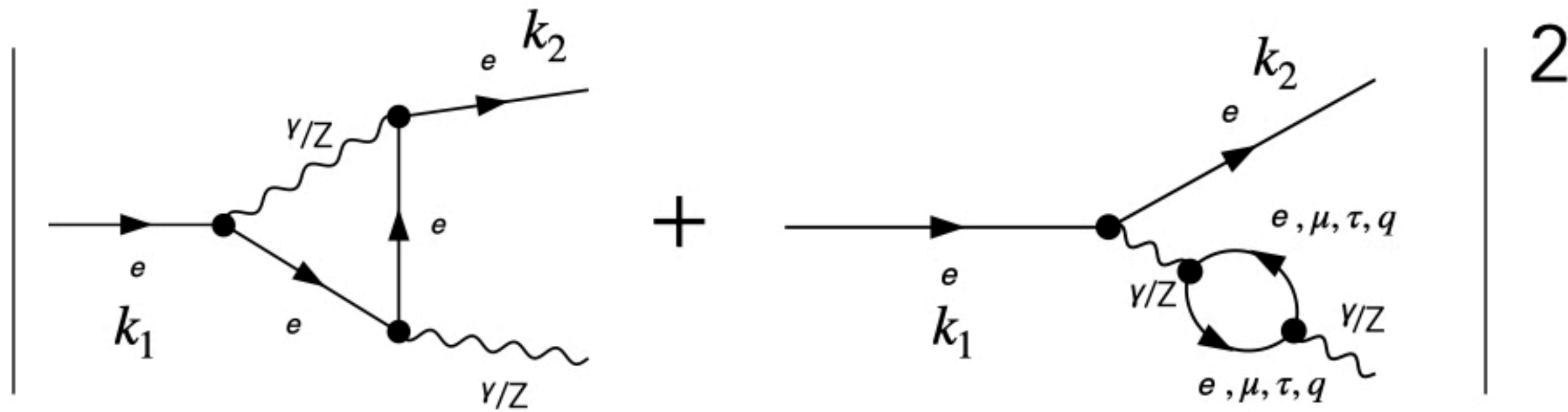
$$L_{\mu\nu}^{NLO} = (m_1)g_{\mu\nu} + (m_2)k_{1\nu}k_{2\mu} + (m_3)k_{1\mu}k_{2\nu} + (m_4)k_{1\mu}k_{1\nu} + (m_5)k_{2\mu}k_{2\nu} + \dots$$

Where  $m_{1,2,3\dots}$  are leptonic structure functions which depend on the momentum transfer " $t$ " and written in terms of Passarino-Veltman integral functions. We used LoopTools Mathematica package to calculate them.

- In total **307** graphs SE and triangular graphs.

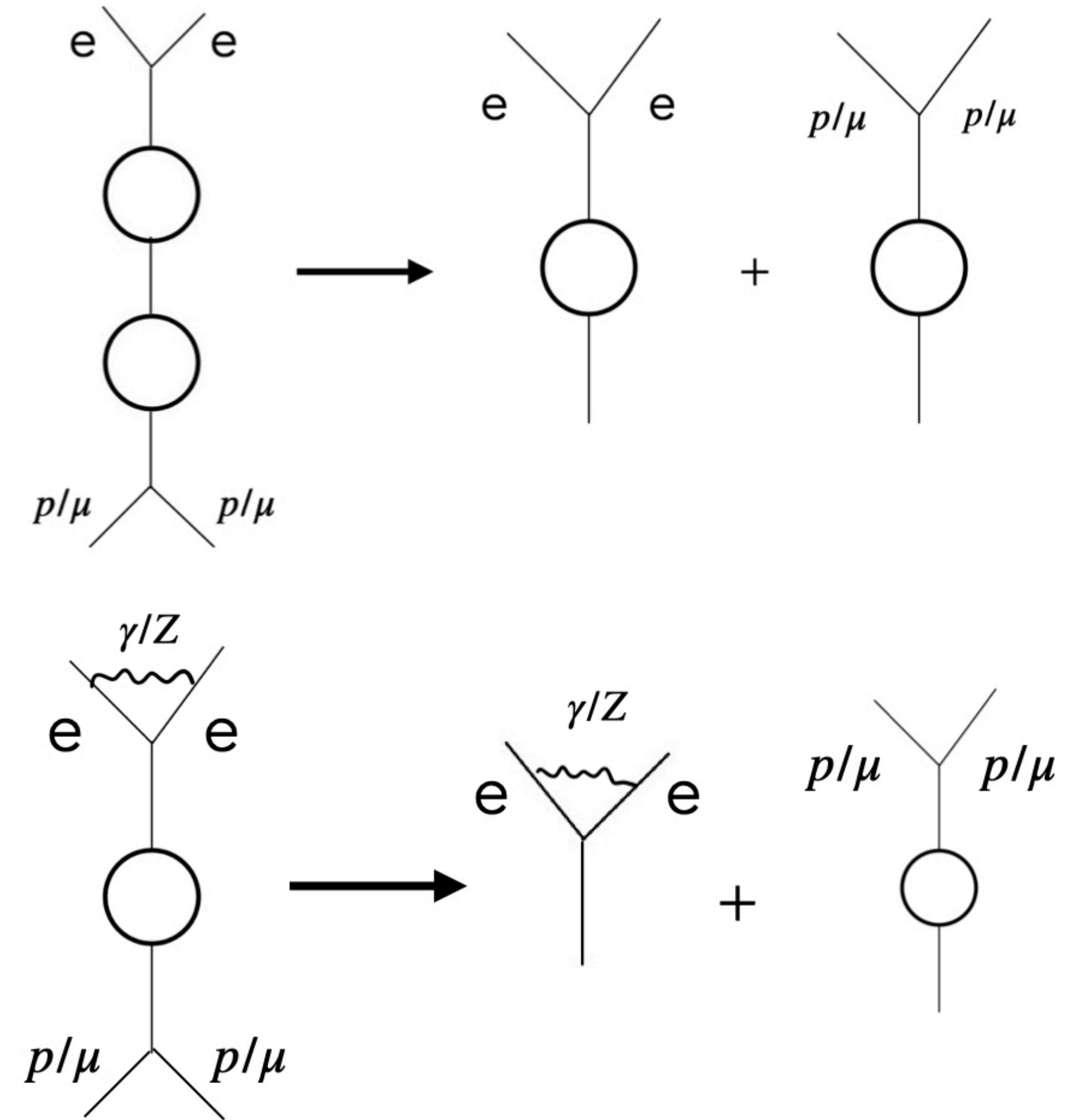


# NEW RESULTS: QED AND ELECTROWEAK NNLO LEVEL LEPTONIC TENSOR ( $\alpha^3$ -ORDER)

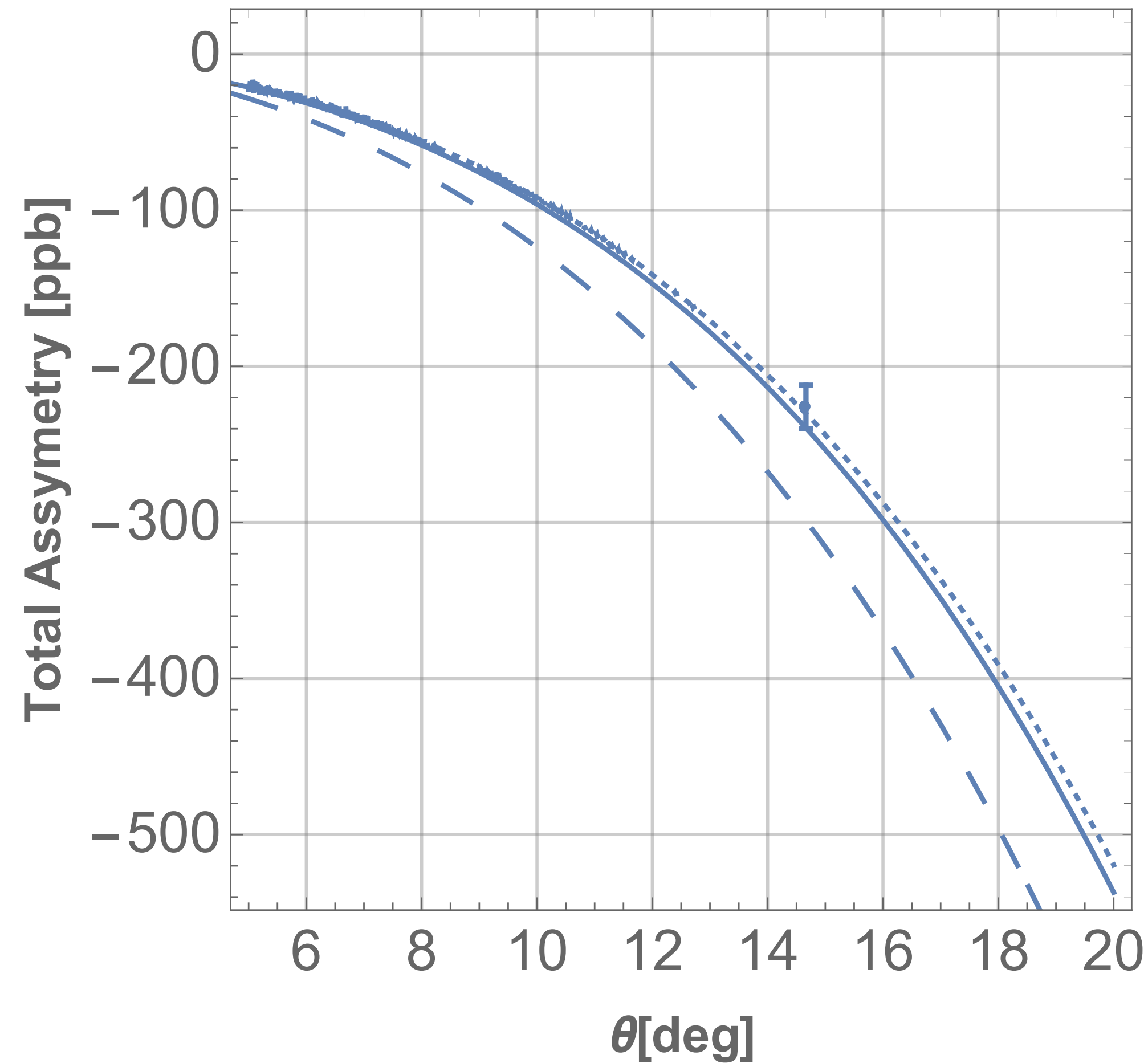


$$L_{\mu\nu}^{NNLO} = (n_1)g_{\mu\nu} + (n_2)k_{1\nu}k_{2\mu} + (n_3)k_{1\mu}k_{2\nu} + (n_4)k_{1\mu}k_{1\nu} + (n_5)k_{2\mu}k_{2\nu} + \dots$$

- ▶ FEYNARTS and FORMCALC as base languages to calculate leptonic tensor structure functions.
- ▶ Kept the mass of electron.



# Tree level, NLO and NNLO level $A_{PV}$ for $e^-p$ scattering versus $\theta_{CM}$ using QWEAK kinematics



$(\theta = 14.6^\circ)$

- Tree  $A_{PV} \sim -298.9 \text{ ppb}$
- NLO  $A_{PV} \sim -239 \text{ ppb}$
- ..... NNLO  $A_{PV} \sim -230 \text{ ppb}$

**QWEAK Measured**

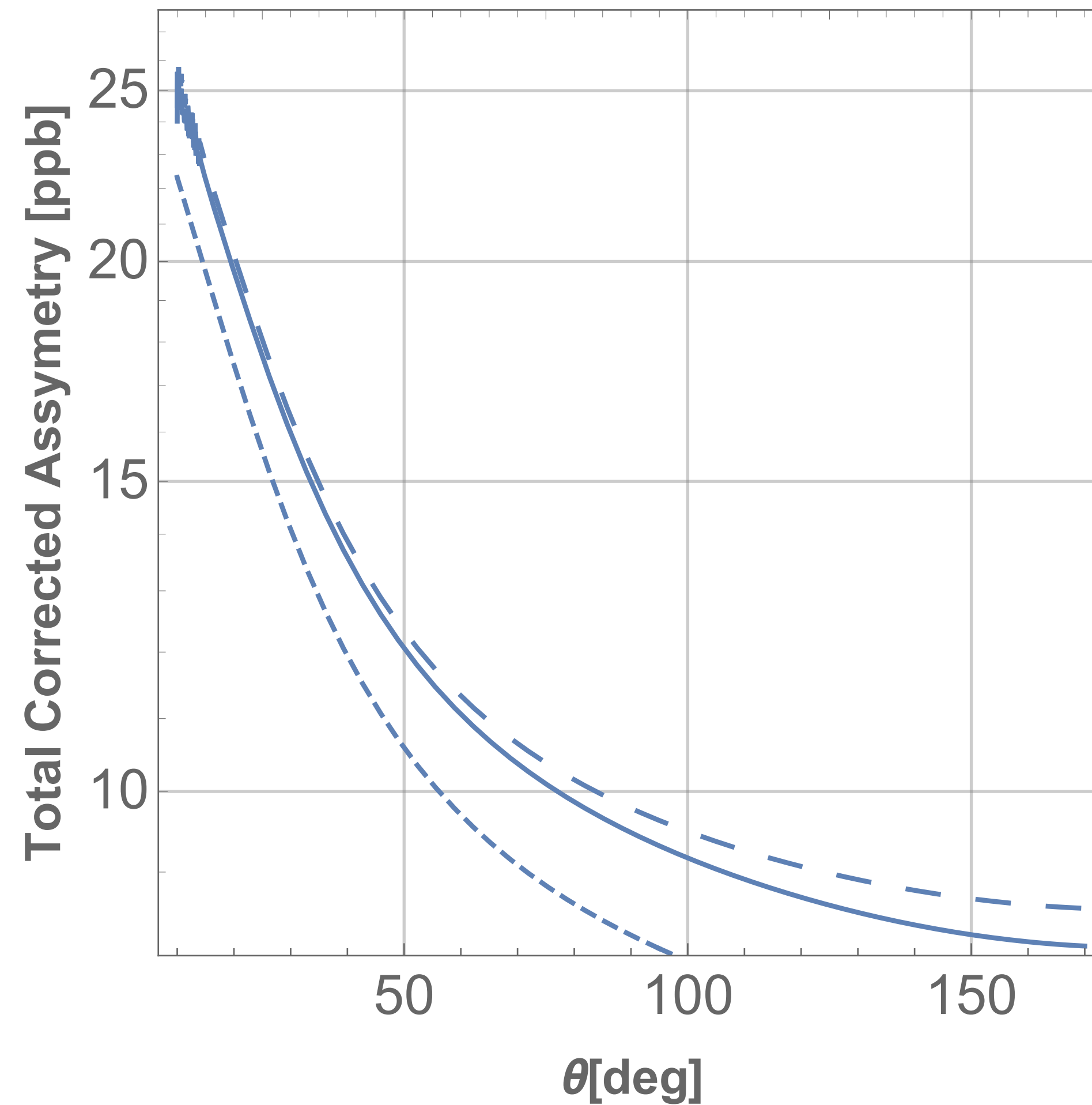
$\sim -226.5 \pm 7.3(\text{statistical}) \pm 5.8(\text{systematic}) \text{ ppb}$

Phys. Rev. C 101, 055503

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

$(e^-p)$  Tree level, NLO and NNLO level  $A_{PV}$  versus  $\theta_{CM}$

# NLO and NNLO level Corrected $A_{PV}$ Asymmetry with Correction factors in percentage for QWEAK kinematics



$(\theta = 14.6^\circ)$

..... NLO level Corrected  $A_{PV} = 19.9 \%$

—— Quadratic level Corrected  $A_{PV} = 22.5 \%$

- - - Total Corrected  $A_{PV} = 22.9 \%$

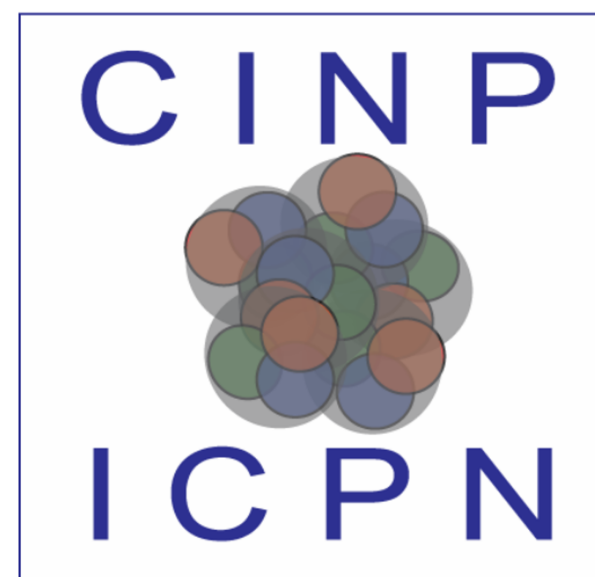
$$\text{Corrected } A_{PV} \% = \left( \frac{\text{Tree level } A_{PV} - (NLO, NNLO)A_{PV}}{\text{Tree level } A_{PV}} \right) \times 100$$

NLO and NNLO level Corrected  $A_{PV}$  versus  $\theta_{CM}$

# RESULTS:

- For completeness, work in progress to include soft and hard photon bremsstrahlung cross sections in the results.
- Next goal is to consider the polarized proton and study the  $A_{PV}$  effects in electron-proton scattering.
- We make predictions for the  $e^-p$  NNLO level radiative corrections. These theoretical predictions will be important for many experimental programs such as QWEAK, P2, EIC and MOLLER (background studies) searching for physics beyond the Standard Model at the precision frontier.

- We acknowledge the support of the Canadian Institute of Nuclear Physics (CINP) for the travel grant and the Natural Sciences and Engineering Research Council of Canada (NSERC) for this project.



**Canadian Institute of  
Nuclear Physics**

**Institut canadien de  
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# Backup Slides

# PARITY VIOLATING ASYMMETRY

- Formula:  $A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$ , where:  $\sigma_R \propto |M_R|^2$  and  $\sigma_L \propto |M_L|^2$
- For QED,  $|M_{\gamma R}| = |M_{\gamma L}|$ , numerator contains just weak+electroweak cross terms.
- Denominator contains just QED terms as  $m_Z$  (90 GeV)  $>$   $m_e$  (0.5 MeV)

- $$A_{PV} = \frac{|M_{ZZ}|_R^2 - |M_{ZZ}|_L^2 + |M_{\gamma Z}|_R^2 - |M_{\gamma Z}|_L^2}{|M_{\gamma\gamma}|_R^2 + |M_{\gamma\gamma}|_L^2}$$

- Using covariant approach → calculated tree level, one-loop level and quadratic level scattering amplitudes squared along with the parity violating asymmetry.

## QED

$$i\mathcal{M} = \bar{u}(k_2)(-ie\gamma_\mu)u(k_1)\frac{-i}{t}\bar{u}(p_2)(-ie\Gamma_{\gamma-p}^\mu)u(p_1)$$

$$\Gamma_{\gamma-p}^\mu = F_1^p(t)\gamma^\mu + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2^p(t)$$

$F_1(t)$  and  $F_2(t) \rightarrow$  Dirac and Pauli form factors depending on momentum transfer "t".

## WEAK

$$i\mathcal{M} = \bar{u}(k_2)(-ie(a_v\gamma_\mu + a_p\gamma_\mu\gamma_5))u(k_1)\frac{-i}{t - M_Z^2}\bar{u}(p_2)(-ie\Gamma_{Z-p}^\mu)u(p_1)$$

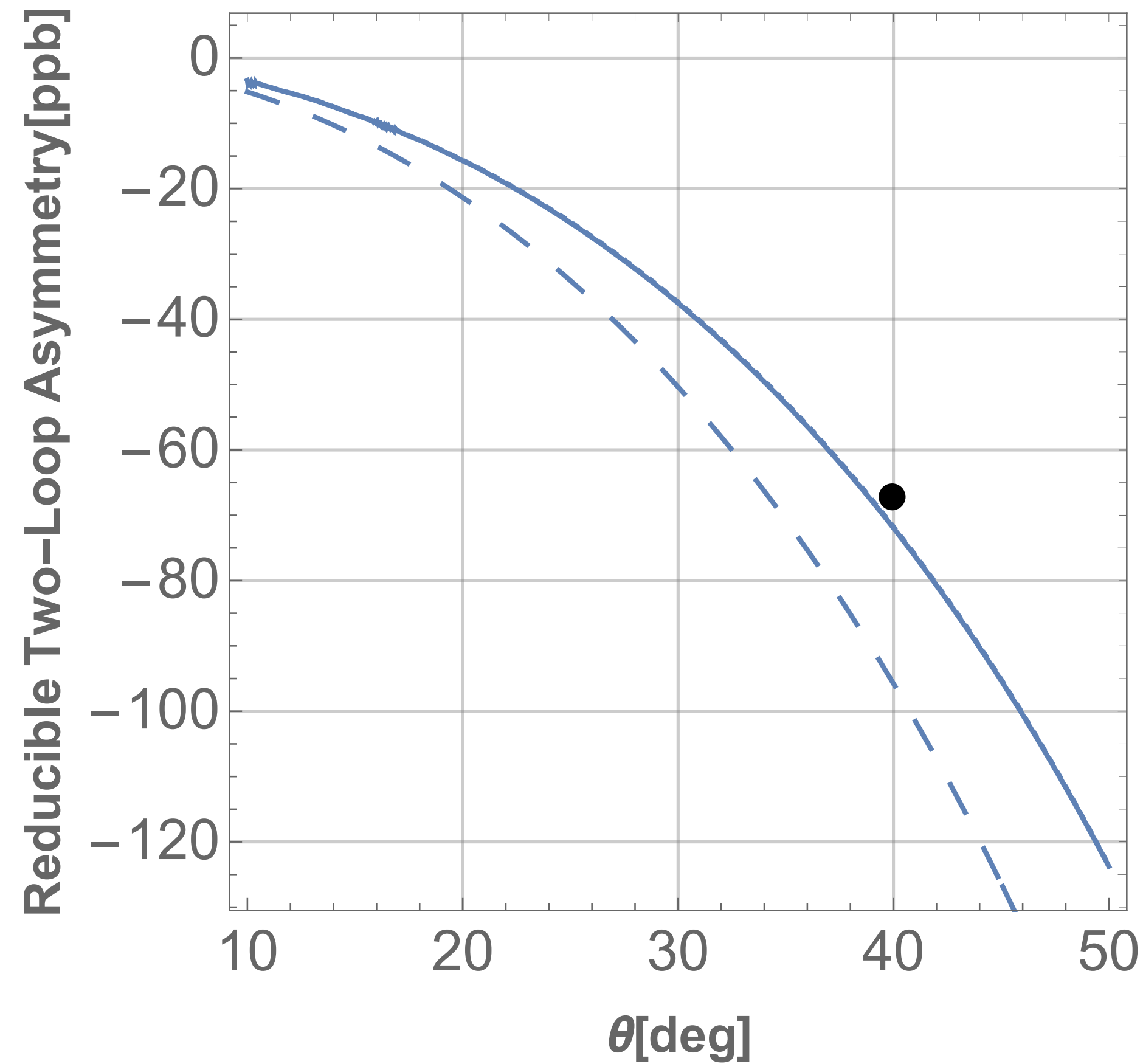
$$\Gamma_{Z-p}^\mu = f_1^p(t)\gamma^\mu + \frac{i\sigma^{\mu\nu}q_\nu}{2M}f_2^p(t) + g_1^p(t)\gamma^\mu\gamma_5$$

$f_1^p(t)$ ,  $f_2^p(t)$  and  $g_1^p(t) \rightarrow$  weak electric, magnetic and axial vector form factors.

$$a_v = \frac{I^3 - 2\sin^2\theta_W Q_f}{2\sin\theta_W \cos\theta_W}, \quad a_p = \frac{I^3}{2\sin\theta_W \cos\theta_W}, \quad \text{where}$$

$$Q_f = -1(e^-) \text{ and } I_3 = -\frac{1}{2}.$$

# Tree level, NLO and NNLO level $A_{PV}$ for $e^-p$ scattering versus $\theta_{CM}$ using P2 kinematics



$(\theta = 39.97^\circ)$

--- Tree  $A_{PV} \sim -95.6 \text{ ppb}$

— NLO  $A_{PV} \sim -71.81 \text{ ppb}$

..... NNLO (Reducible Two loop)  $A_{PV} \sim -71.6 \text{ ppb}$

**P2 Proposed  $A_{PV} \sim -67.34 \text{ ppb}$**

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

$(e^-p)$  Tree level, NLO and NNLO level  $A_{PV}$  versus  $\theta_{CM}$

# REFERENCES FOR BOX DIAGRAMS

- [1] M. Gorchtein, Phys. Rev. C **73**, 055201 (2006)
- [2] Peter G. Blunden et al., Physical Review Letters 91(14)

