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Quarks and glue inside hadrons in the instanton vacuum

The hadronic structure dominated by QCD vacuum

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Nonperturbative perspective in QCD

- Spontaneous chiral symmetry breaking
  $\rightarrow$ light mesons, quark condensate $\langle \bar{q}q \rangle$

- Trace anomaly

$$T^{\mu}{}_{\mu} = -\frac{b}{32\pi^2} G^2 + m\bar{q}q \tag{1}$$

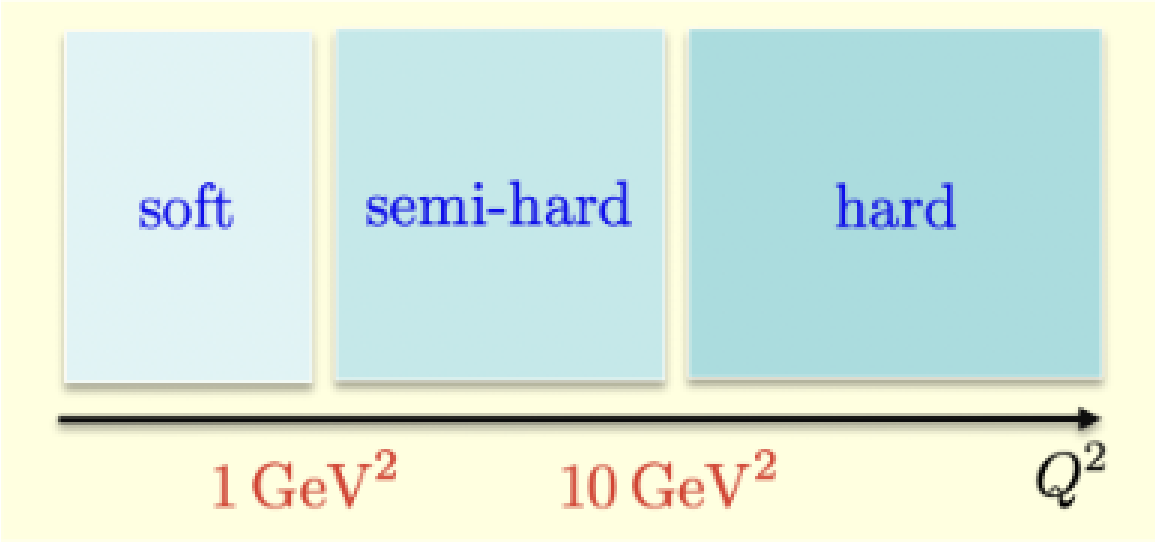
- Chiral anomaly

$$\partial_\mu J_5^\mu = \frac{N_f}{32\pi^2} G\tilde{G} + m\bar{q}i\gamma^5 q \tag{2}$$

How to describe nonperturbative phenomena by QCD?

W.-Y. Liu, E. Shuryak, I. Zahed, arXiv:2404.03047 [hep-ph] (2023)

QCD scales

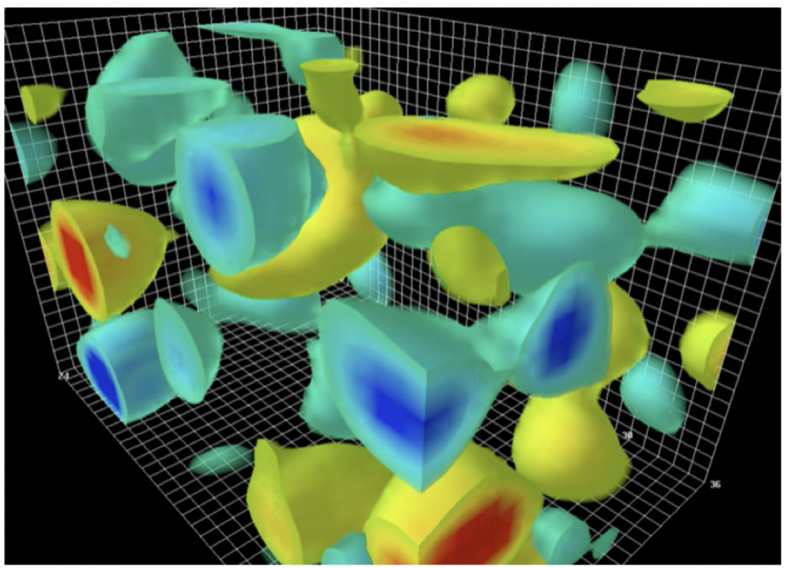


- Hard: perturbative QCD
- Semi-hard: quark-instanton, gluon-instanton coupling
- Soft: quark-meson dynamics (instanton)
- Ultrasoft: χPT, meson dynamics (instanton)

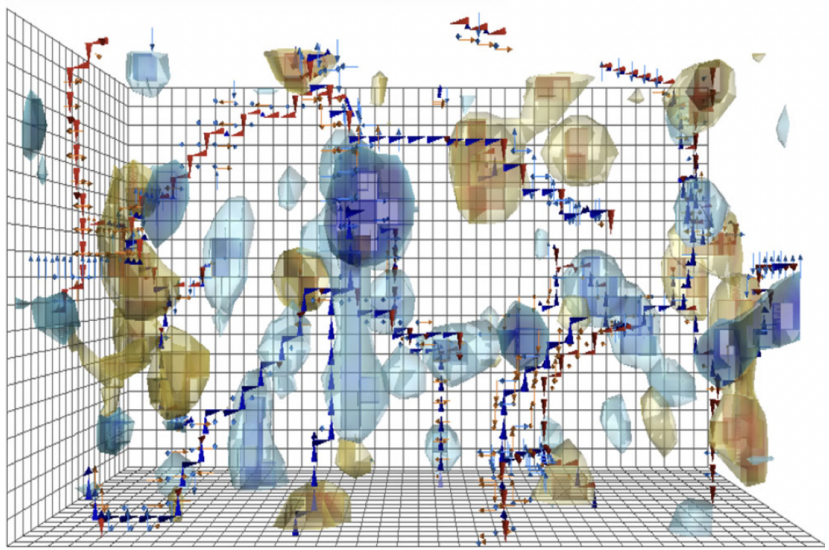
Instanton Vacuum

- QCD vacuum is topologically active at $0.6 - 0.7$ GeV
- Instanton vacuum stabilized at ($n_I = 1$ fm^{-4} and $\rho = 1/3$ fm)
- Zero energy tunnelling event between topological vacua $\Delta n_{CS} = \pm 1$

- Gluon plane waves are almost negligible



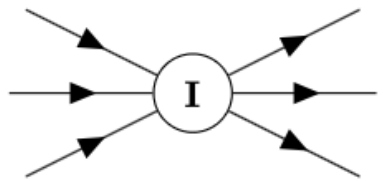
P. J. Moran, D. B. Leinweber (2008)



J. C. Biddle, W. Kamleh, D. B. Leinweber (2020)

Instanton Liquid Model

$$Z_{N_\pm} = \overbrace{\frac{1}{N_+!N_-!}\prod_{I,A}\int d\rho_I d^4 z_I dU_I n(\rho_I)}^{\text{glue}} \overbrace{\int D\psi D\bar{\psi} e^{-\int d^4x \bar{\psi}\partial\!\!\!/\psi} \prod_{I,A}\Theta_I\Theta_A}^{\text{quarks-instanton}}$$

$\Theta_I =$ 

- Gluon dynamics encoded in the instanton density $n_\pm$

$$\begin{aligned}\langle \mathcal{O}_{QCD}\rangle_{N_\pm} =& n_+\langle \mathcal{O}_+[q,\bar{q}]\rangle + n_-\langle \mathcal{O}_-[q,\bar{q}]\rangle \\ &+ \frac{n_+^2}{2}\langle \mathcal{O}_{++}[q,\bar{q}]\rangle + n_+n_-\langle \mathcal{O}_{+-}[q,\bar{q}]\rangle + \frac{n_-^2}{2}\langle \mathcal{O}_{--}[q,\bar{q}]\rangle \\ &+ \cdots\end{aligned}$$

Grand Canonical Ensemble: Fluctuations

- $G^2 \to \bar{N} = N_+ + N_-$ and $G\tilde{G} \to \Delta = N_+ - N_-$
- Fluctuations

$$\mathcal{P}(N_+, N_-) \propto \left(\frac{\bar{N}^N}{N!}\right)^{b/4} \frac{1}{\sqrt{2\pi\chi_t}} \exp\left(-\frac{\Delta^2}{2\chi_t}\right) \tag{3}$$

- Topological compressibility: trace anomaly

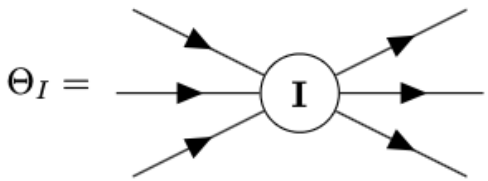
$$\sigma = \frac{4}{b}\bar{N} \tag{4}$$

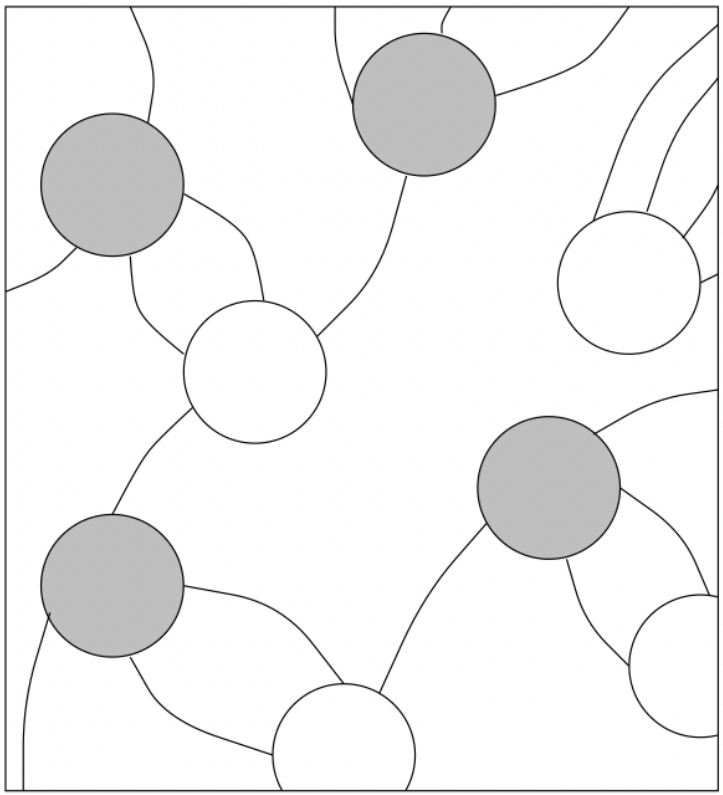
- Topological susceptibility: chiral anomaly

$$\chi_t = \bar{N}\left(1 + N_f\frac{m^*}{m}\right)^{-1} \tag{5}$$

W.-Y. Liu, E. Shuryak, I. Zahed, arXiv:2404.03047 [hep-ph] (2023)

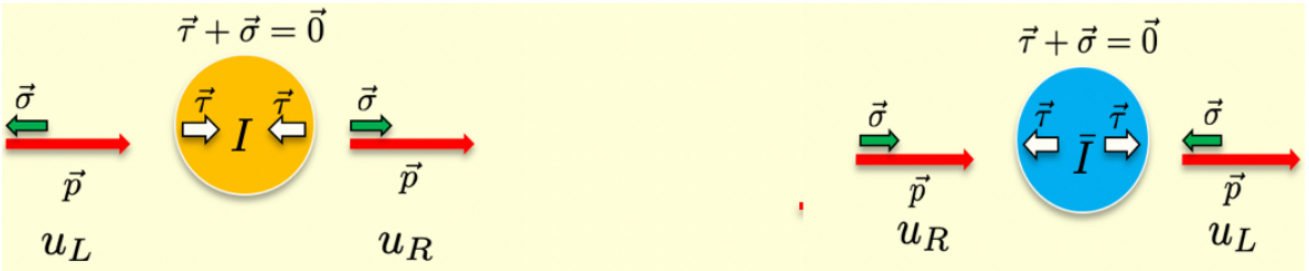
Spontaneous chiral symmetry breaking





Hadrons in Instanton Vacuum

- Constituent quarks acquire mass from delocalization



- 't Hooft effective Lagrangian

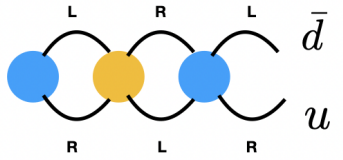
$$\mathcal{L}_I = \frac{G_I}{8N_c^2}\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\tau^a\psi)^2 - (\bar{\psi}i\gamma^5\psi)^2 + (\bar{\psi}i\gamma^5\tau^a\psi)^2\right]$$

- Constituent mass $M \sim 350 - 390$ MeV
- Instanton size $\rho \rightarrow$ natural cut-off

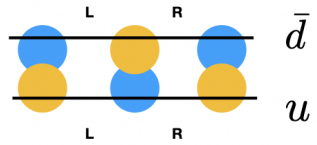
G. 't Hooft, Phys. Rev. D 14, 3432 (1976);
W.-Y. Liu, I. Zahed, Phys. Rev. D 107 (2023) 9, 094024

Instanton Molecules

- Instanton molecules are important
  $\rightarrow$ vector meson (ρ), tensor glueball (T^{++}), odd glueball ($dGGG$)
- Molecule-indueced interations penalized by instanton density



π



ρ

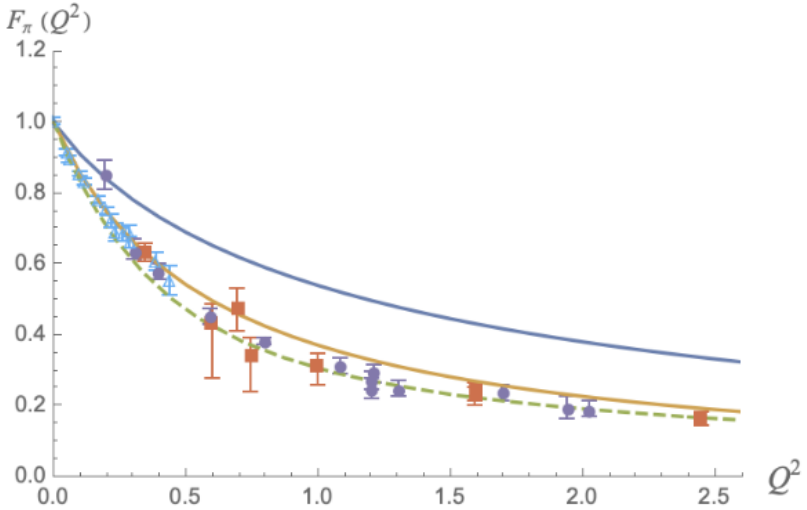
$$\mathcal{L}_{I\bar{I}} = \frac{G_{I\bar{I}}}{2N_c^2}\Big\{4\left[(\bar{\psi}\psi)^2 + (\bar{\psi}\tau^a\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2 + (\bar{\psi}i\gamma^5\tau^a\psi)^2\right]$$
$$- \left[(\bar{\psi}\gamma^\mu\psi)^2 + (\bar{\psi}\tau^a\gamma^\mu\psi)^2 - 3(\bar{\psi}\gamma^\mu\gamma^5\psi)^2 + (\bar{\psi}\tau^a\gamma^\mu\gamma^5\psi)^2\right]\Big\}$$

T. Schäfer, E. Shuryak, and J. Verbaarschot, Phys. Rev. D 51, 1267 (1995);
Edward Shuryak, Ismail Zahed, Phys.Rev.D 107 (2023) 3, 034023

Remarks

- QCD vacuum is topologically active at $0.6 - 0.7$ GeV (instanton)
- Instanton dominates light meson dynamics: m_M and g_{Mqq} $\rightarrow \sigma$, a_0, π, K, η, η', ω, ρ, f_1, a_1, etc.
- Diquarks $\rightarrow$ light baryons: p, n, Σ, Λ, Ξ
- Hadronic light front wave functions
- gluon dynamics is encoded in the instanton density fluctuation
- The instanton liquid model provides solid framework for both quark and gluonic content (glueballs, mesons) in any light hadrons.

W.-Y. Liu, I. Zahed, Phys. Rev. D 107 (2023) 9, 094024
W.-Y. Liu, I. Zahed, Phys. Rev. D 109 (2024) 7, 074029



ILM

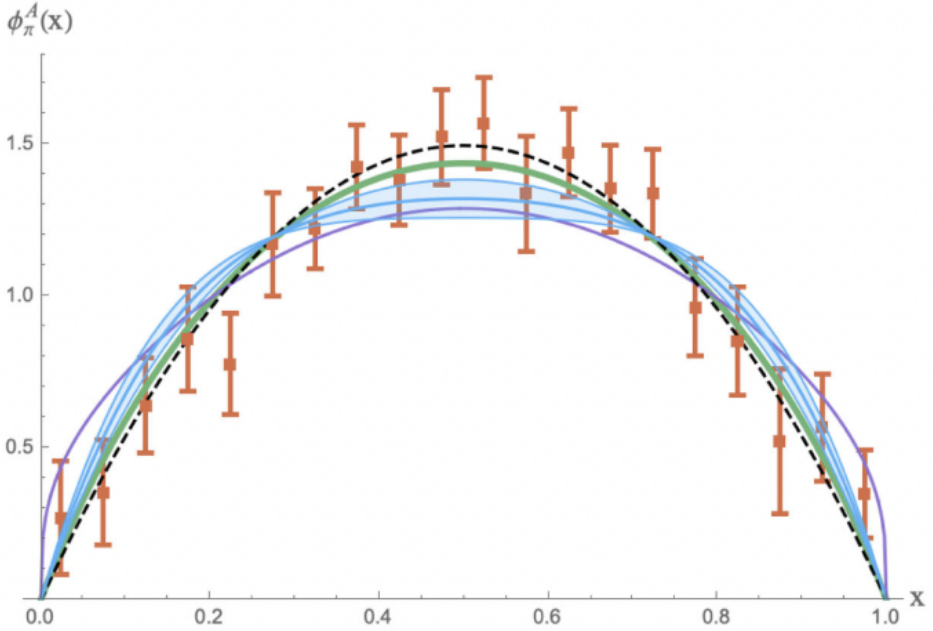
ILM + VDM

DSE

JLab Fπ Collaboration (2008)

Wilson Syncrotron Lab, Cornell (1977)

Lattice (2022), m_π = 140 MeV



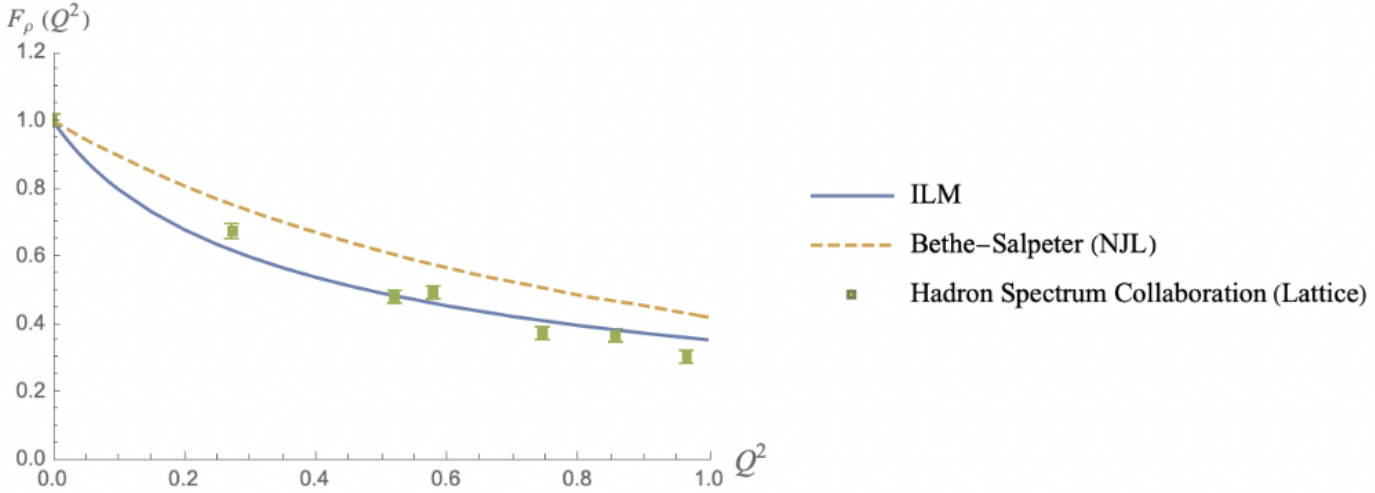
ILM, μ = 2 GeV

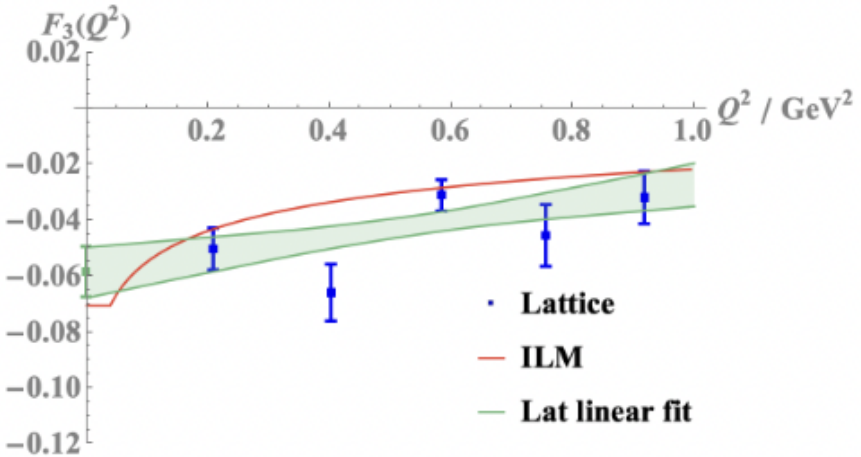
E791 dijet

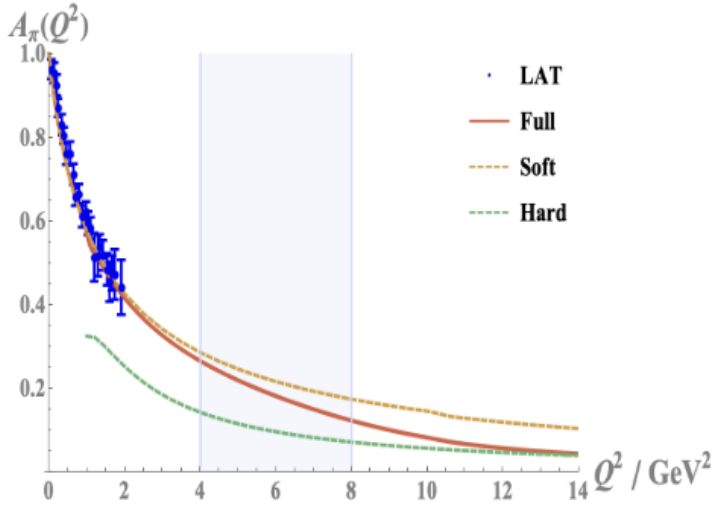
DSE

Asymptotic

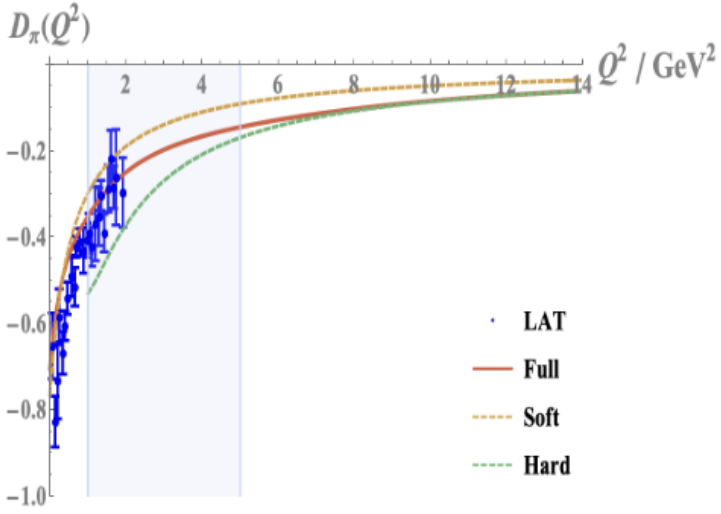
Lattice, μ = 2 GeV


$F_\rho\,(Q^2)$
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Q^2
ILM
Bethe–Salpeter (NJL)
Hadron Spectrum Collaboration (Lattice)

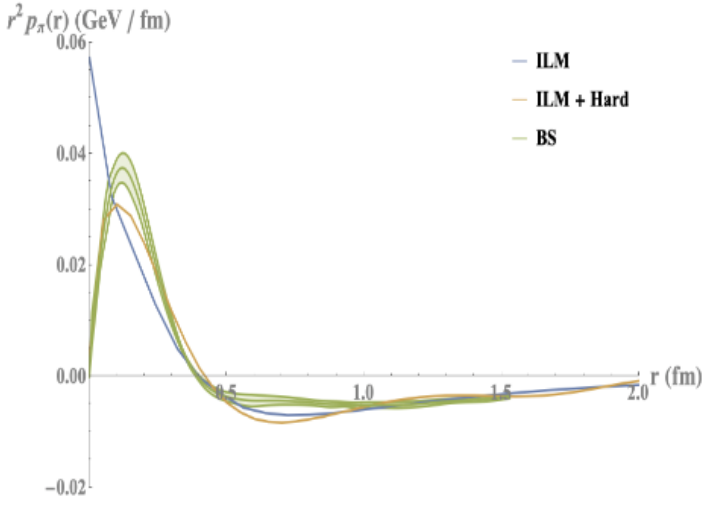

$F_3(Q^2)$
0.02
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
0.2
0.4
0.6
0.8
1.0
Q^2 / GeV^2
Lattice
ILM
Lat linear fit


$A_\pi(Q^2)$
LAT
Full
Soft
Hard
Q^2 / GeV^2

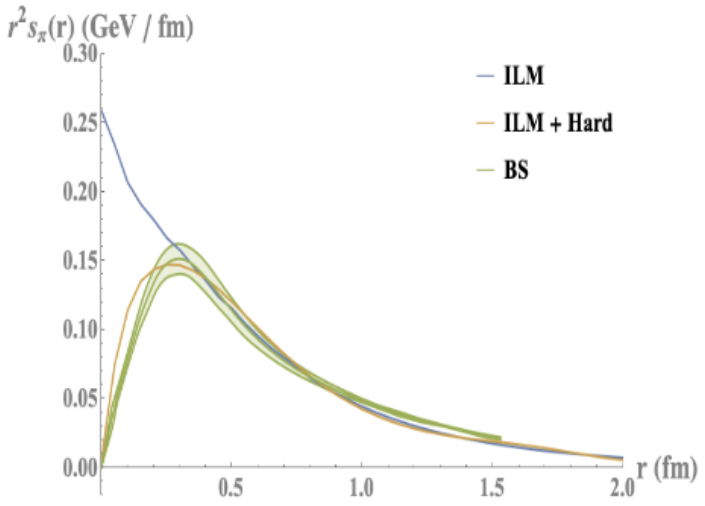
(a)


$D_\pi(Q^2)$
Q^2 / GeV^2
LAT
Full
Soft
Hard

(b)

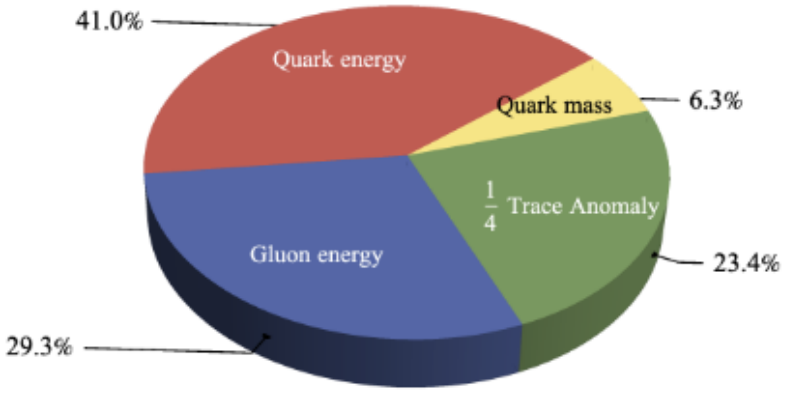

$r^2 p_\pi$(r) (GeV / fm)
ILM
ILM + Hard
BS
r (fm)

(a)


$r^2 s_\pi$(r) (GeV / fm)
ILM
ILM + Hard
BS
r (fm)

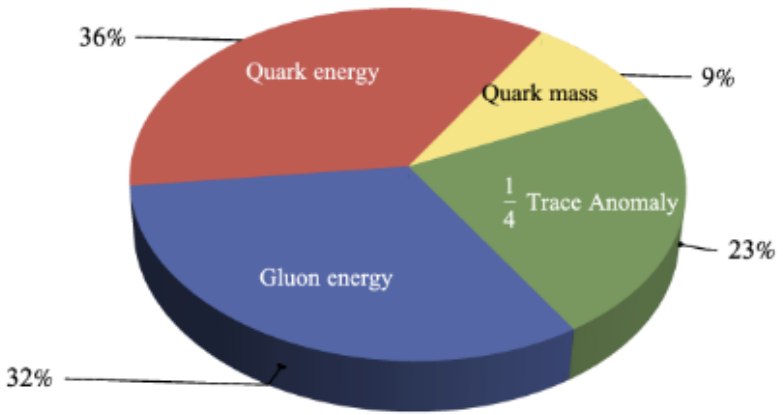
(b)

$\mu = 2$ GeV



(a)

Lattice (χQCD collaboration)



(b)