Stony Brook University

Center for Frontiers in Nuclear Science

Generalized Parton Distributions of low-lying octet baryons

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under the supervision of

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Outline

1 Introduction

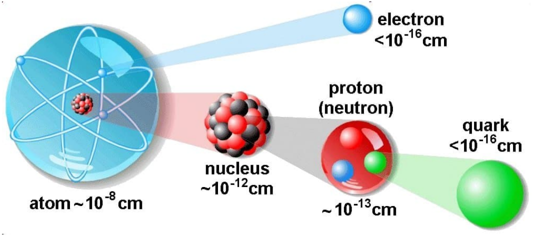
2 Generalized Parton Distributions

3 Outcomes

Introduction

Quantum chromodynamics (QCD)

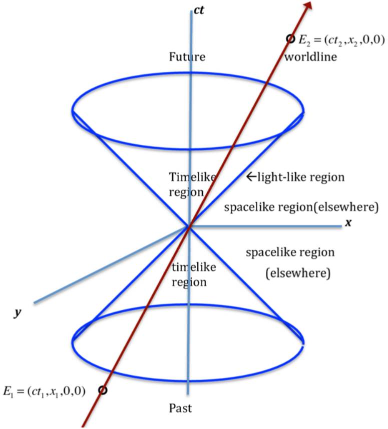
- an underlying theory for the strong interactions.
- describe the internal hadronic structure in terms of their constituent quarks and gluons degree of freedom.

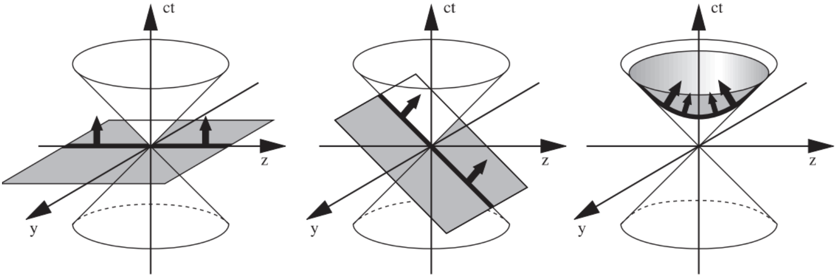


Highly preferable,
Lorentz-invariant form.

- four-vectors.
- fundamental concept in special theory of relativity.

So, **Special theory of relativity becomes an important concept that must be follow**.





The instant form	The front form	The point form
$\tilde{x}^0 = ct$	$\tilde{x}^0 = ct+z$	$\tilde{x}^0 = \tau \quad , \quad ct = \tau \cosh\omega$
$\tilde{x}^1 = x$	$\tilde{x}^1 = x$	$\tilde{x}^1 = \omega \quad , \quad x = \tau \sinh\omega \sin\theta \cos\phi$
$\tilde{x}^2 = y$	$\tilde{x}^2 = y$	$\tilde{x}^2 = \theta \quad , \quad y = \tau \sinh\omega \sin\theta \sin\phi$
$\tilde{x}^3 = z$	$\tilde{x}^3 = ct-z$	$\tilde{x}^3 = \phi \quad , \quad x = \tau \sinh\omega \cos\theta$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\tau^2 & 0 & 0 \\ 0 & 0 & -\tau^2 \sinh^2\omega & 0 \\ 0 & 0 & 0 & -\tau^2 \sinh^2\omega \sin^2\theta \end{pmatrix}$$

Light-Front Coordinates

- A generic four Vector x^μ in light-cone coordinates is describe as $x^\mu = (x^-, x^+, x_\perp)$.
- $x^+ = x^0 + x^3$ is called as light-front time.
- $x^- = x^0 - x^3$ is called as light-front longitudinal space variable.
- $x^\perp = (x^1, x^2)$ is the transverse variable.
- Similarly, we can define the longitudinal momentum $k^+ = k^0 + k^3$ and light-front energy $k^- = k^0 - k^3$.

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What makes Front form dynamics a good choice ?

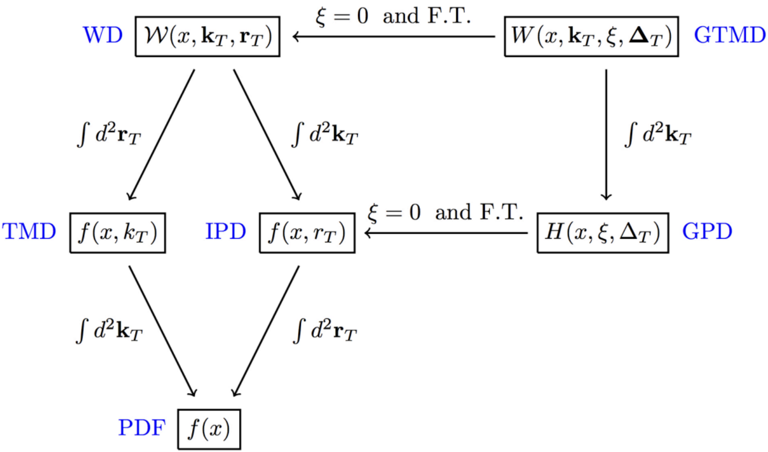
- NO SQUARE ROOT FACTOR in energy-momentum dispersion relation

$$k^- = \frac{k_\perp^2 + m^2}{k^+},$$

where

  - k^-: energy
  - k^+: longitudinal momentum
  - $k_\perp$: transverse momentum
- Vacuum expectation value is ZERO.
- Seven out of ten fundamental quantities are KINEMATICAL.

Distribution functions



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Generalized Parton Distributions

$$F^{[\Gamma]}_{\lambda\lambda'} = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \left\langle p', \lambda' \middle| \bar{\psi}\left(\frac{-z}{2}\right) \Gamma \mathcal{W}\left(\frac{-z}{2}, \frac{z}{2}\right) \middle| \mathcal{N} \right) \psi\left(\frac{z}{2}\right) \left| p, \lambda \right\rangle \Big|_{z^+=0, \mathbf{z}_\perp=0}$$

JHEP 08, 056 (2009)

with

- $\lambda(\lambda')$: Initial (final) state helicity.
- $P = \frac{p+p'}{2}$, average momentum.
- $\Delta = p' - p$, momentum transfer.
- Impact parameter space, $\mathbf{\Delta}_\perp \leftrightarrow b_\perp$ via FT.
- Γ basis $[1, \gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, \sigma^{\mu\nu}]$.

$\Gamma = \gamma^+$: H and E
unpolarized

$\Gamma = \gamma^+\gamma^5$: $\tilde{H}$ and $\tilde{E}$
longitudinal polarized

$\Gamma = i\sigma^{+i}\gamma^5$:
$H_T, E_T, \tilde{H}_T$ and $\tilde{E}_T$
transversely polarized

Generalized Parton Distributions

$$\frac{1}{2\bar{P}^{+}}\bar{u}(P')\Big[H_X^q(x,0,\mathbf{\Delta}_\perp)\,\gamma^+ + E_X^q(x,0,\mathbf{\Delta}_\perp)\,\frac{i\sigma^{+\alpha}(-\Delta_\alpha)}{2M}\Big]u(P)$$

Eur. Phys. J. A 52(2016)163.

quark pol. / nucleon pol.

nucleon pol.	U	L	T	
U	$\boldsymbol{H}$		$E_T + 2\tilde{H}_T$	
L		$\boldsymbol{\tilde{H}}$	$\tilde{E}_T$	
T	E	$\tilde{E}$	$\boldsymbol{H_T}$	$\tilde{H}_T$

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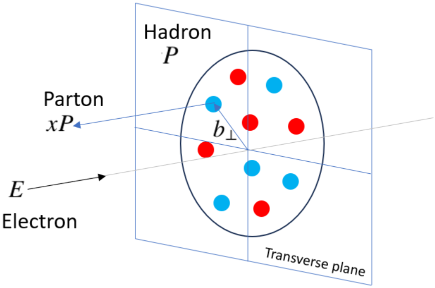
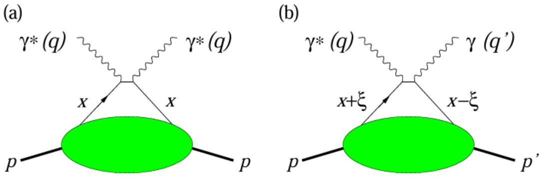


Momentum fraction carried by an active quark, $x = \frac{k^+}{P^+}$

Skewness: Fraction of longitudinal momentum transfer, $\zeta = -\frac{\Delta^+}{2P^+}$

Momentum transfer squared, $t = (p' - p)^2 = \Delta^2$

How to access GPDs ?



Phys. Rep. 388, 41–277 (2003)

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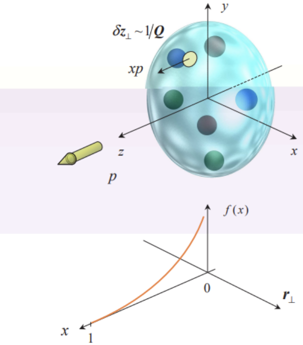
Generalized parton distributions (GPDs)
For zero skewness

- Reduced to PDFs,
  $\int f^q(x,0,t)\,dt \Rightarrow$ PDFs
- Pathway to FFs
  $\int f^q(x,0,t)\,dx \Rightarrow$ FFs.

$$f^q(x,0,t) \xrightarrow{FT} f^q(x,0,\mathbf{r}_\perp)$$

$$\int f^q(x,0,\mathbf{r}_\perp)\,dx \Rightarrow \text{FT FFs.}$$

- 3D hybrid distribution
  - 1D momentum
  - 2D coordinate space



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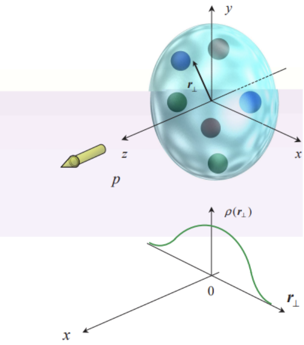

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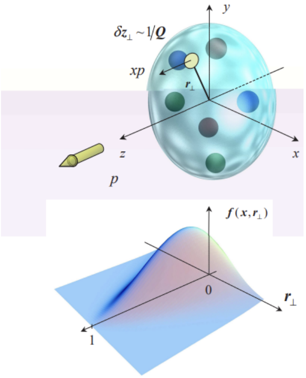

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Charge and Magnetization distributions

$$\rho_X^q(x, b_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2}\, e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} H_X^q(x, 0, \mathbf{\Delta}_\perp)$$

$$\tilde{\rho}_M^{X_q}(\mathbf{b}_\perp) = \mathbf{b}_\perp Sin^2(\phi) \int \frac{\mathbf{\Delta}_\perp^2 d\mathbf{\Delta}_\perp}{2\pi} J_1(\mathbf{\Delta}_\perp \mathbf{b}_\perp) \int dx\, E_X^q(x, 0, \mathbf{\Delta}_\perp)\,.$$

Phys. Lett. B **669**, 345-351 (2008)
Nucl. Phys. B 969, 115440 (2021)
Eur. Phys. J. A **60**, 42 (2024)

Hadron state; Quark-Scalar diquark

on complete basis of Fock-states

$$|\psi_A(P^+, \mathbf{P}_\perp)\rangle = \sum_{\mathcal{N}} \prod_{i=1}^{\mathcal{N}} \frac{dx\, d^2\mathbf{k}_{\perp i}}{2(2\pi)^3 \sqrt{x_i}}\, 16\pi^3\, \delta\Big(1 - \sum_{i=1}^{\mathcal{N}} x_i\Big)\, \delta^{(2)}\Big(\sum_{i=1}^{\mathcal{N}} \mathbf{k}_{\perp i}\Big) \times \psi_{\mathcal{N}}(x_i, \mathbf{k}_{\perp i}, \lambda_i)|\mathcal{N}; x_i P^+, x_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, \lambda_i\rangle\,,$$

Nucl. Phys. B 596, 99–124 (2001)

- $\mathbf{k}_{\perp i}$- intrinsic transverse momentum
- λ_i- helicity

$J^z = +\frac{1}{2}$,

$$\psi^{\uparrow X}_{+\frac{1}{2}}(x, \mathbf{k}_\perp) = \Big(M_X + \frac{m_q}{x}\Big)\varphi_X\,,$$
$$\psi^{\uparrow X}_{-\frac{1}{2}}(x, \mathbf{k}_\perp) = -\frac{(k^1 + \imath k^2)}{x}\varphi_X\,. \quad (1)$$

$J^z = -\frac{1}{2}$,

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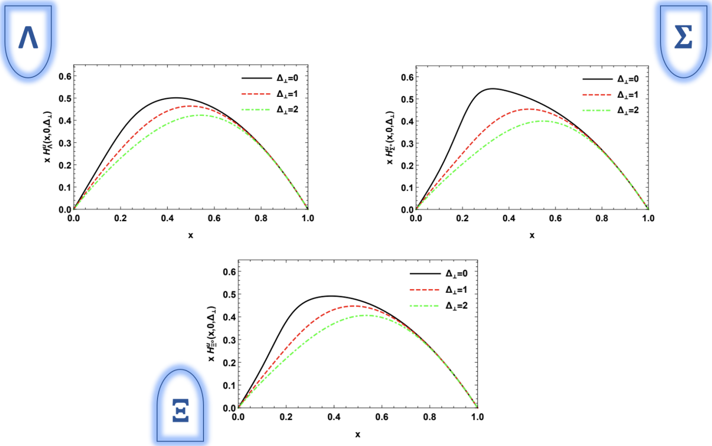
Outcome

GPD $H(x, 0, \Delta_\perp)$; $u - flavor$

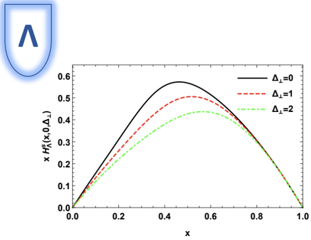
Momentum space

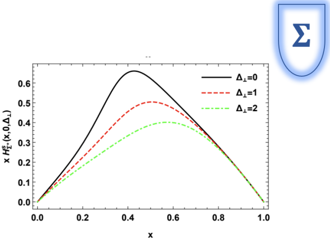


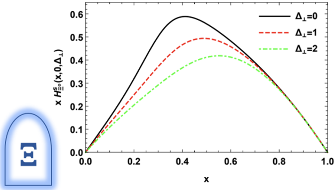
Outcome

GPD $H(x, 0, \Delta_\perp)$; $s - flavor$

Momentum space



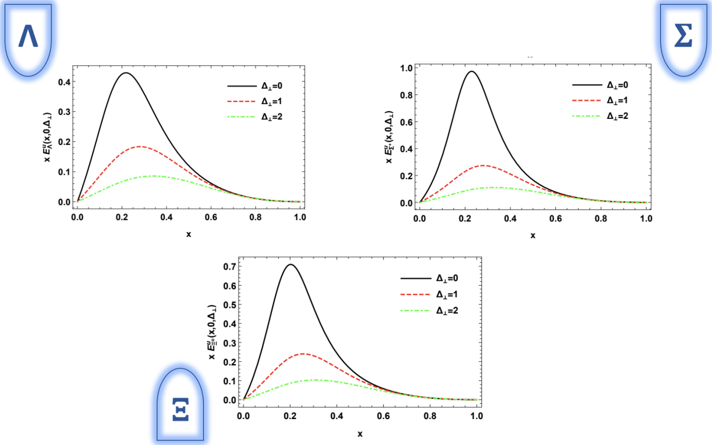




Outcome

GPD $E(x, 0, \Delta_\perp)$; $u - flavor$

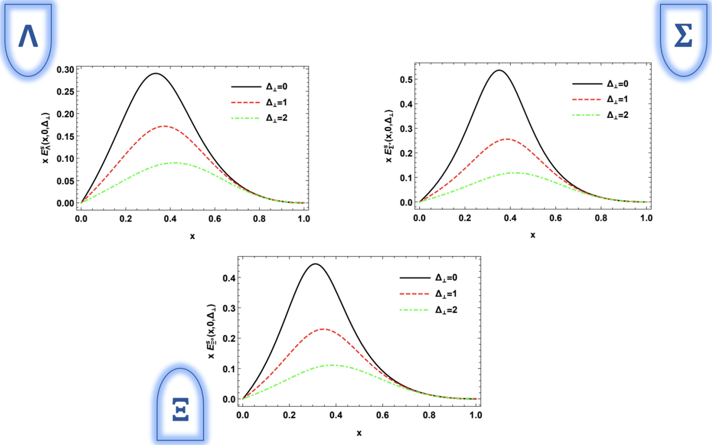
Momentum space



Outcome

GPD $E(x, 0, \Delta_\perp)$; $s - flavor$

Momentum space



"Curiosity is the beginning of knowledge.
Action is the beginning of change."
-James Clear