Instanton properties in the chiral symmetry broken phase of QCD and implications for the EIC

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Why QCD Instantons are relevant?

- Symmetry of 2+1 flavor massless QCD is $SU_V(2) \times SU_A(2) \times U_V(1)$
- $U_A(1)$ is always broken due to quantum effects $\rightarrow$ Mass of η' is heavier than the octet mesons.

$$\partial_\mu \, j_5^\mu \; = \; N_f \frac{1}{16\pi^2} tr \left[G_{\mu\nu}(x) \tilde{G}_{\mu\nu}(x) \right] \, , \; m_{\eta'}^2 = \frac{2 N_f \chi_{top}}{F_\pi^2} \; .$$

- Spontaneous symmetry breaking of $SU_A(2) \rightarrow$ the order parameter related to density of near-zero modes through Banks-Casher relation,

$$\langle \bar{\psi}\psi \rangle \; = \; -\pi\rho(0).$$

- The challenge is to establish the connection between $\rho(0)$ and the interacting ensemble of topological objects called instantons

What are the instantons

- Solutions of the gauge fields which represent tunneling between the infinitely degenerate vacuum of QCD.
- In Euclidean time these correspond to self-dual classical solutions with a finite action which are topological

$$S = -\frac{1}{4g^2}\int d^4x\ tr\left[G(x)\tilde{G}(x)\right] = \frac{8\pi^2}{g^2}|Q|\ ,\ Q = 0, \pm 1, \pm 2, \ldots$$

- They are also the zero energy solutions of the QCD Hamiltonian.
- The solutions are characterized by 8 collective coordinates which are the radius (ρ), color orientations and its center (x_0) where this is localized

$$A^a_\mu(x)_I \ = \ \frac{2}{g}\frac{\eta_{a\mu\nu}x^\nu}{((x-x_0)^2+\rho^2)^2}$$

How do they look like

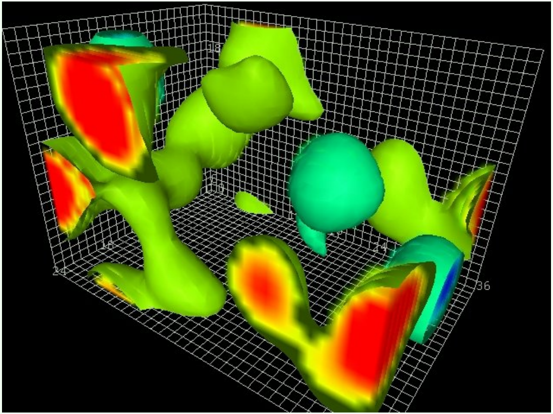


Figure 1: Vaccum structure of QCD after removing UV fluctuations
http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/QCDvacuum/

Detecting the Instantons

- Since these are non-perturbative solutions lattice field theory techniques are the most reliable to detect them.
- There are two methods used:
    1. Remove the ultra-violet fluctuations of the gauge fields and then measure $tr\left[G_{\mu\nu}(x)\tilde{G}_{\mu\nu}(x)\right]$
    2. Use the index theorem and count the number of instantons from fermion zero modes $Q = n_R - n_L$.

- Due to Nielsen-Ninomiya no-go theorem it is difficult to realize fermions with definite chirality on the lattice. Only known lattice discretization of Dirac operator which has definite chirality and an index theorem are overlap fermions.

- In our work, we use the exact zero modes of the overlap Dirac operator to detect the instantons on the gauge configurations of 2+1 falvor QCD with physical quark mass.

The zero modes of Dirac operator and instantons

- On the lattice we can calculate the eigensystem of the Dirac operator

$$i\not{D}|\psi_i\rangle \; = \; \lambda_i|\psi_i\rangle \; .$$

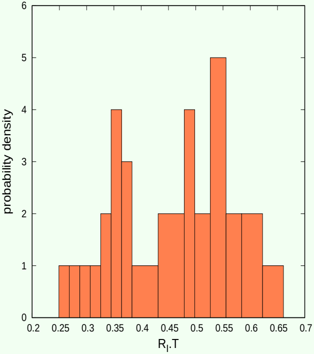
- In presence of an instanton denoted by $A^a_\mu(x)_I$, the Dirac operator has an exact zero mode which is localized at the same location as the instanton,

$$\psi_0(x) \; = \; \frac{\rho}{\pi}\frac{1}{((x-x_0)^2+\rho^2)^{3/2}}\left(\frac{1+\gamma_5}{2}\right)\phi, \qquad \phi^{\alpha\beta} = \varepsilon^{\alpha\beta}/\sqrt{2}$$

- The small eigenvalues called the near-zero modes will arise when individual instantons are interacting.

- We use the zero-modes to measure the widths of the instantons and the near-zero modes to quantify the distance between them.

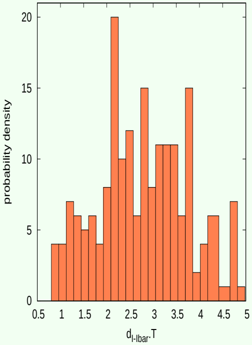
Results- Radius distribution of Instantons

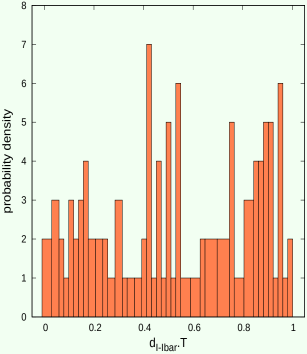


We perform lattice studies first in pure $SU(3)$ on a $32^3 \times 8$ lattice almost at critical temperature $T = 270$ MeV. The most probable size is ~ 0.4 fm.

[Ref: Sayantan Sharma & S.T., work in preparation], need careful treatment of discretization errors

Results- Distance distribution between $I\bar{I}$

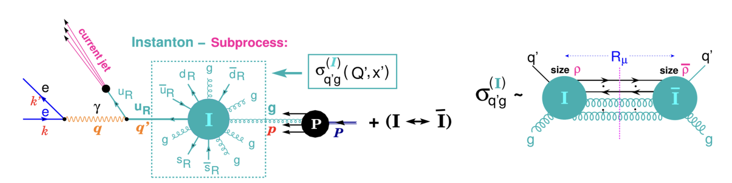




Typical separation between a pair of instantons in pure $SU(3)$ pure gauge theory (left) is larger compared to in QCD with $2+1$ flavour domain wall fermions near T_c(right).

[Ref: Sayantan Sharma & S.T., work in preparation, need higher statistics and careful treatment of systematics

Role of the instantons in DIS



[F. Schrempp, 2005]

- At very small Bjorken x, QCD dynamics is dominated by gluons and is non-perturbative (due to large density) $\rightarrow$ non-perturbative topological objects instantons become relevant
- EIC will be studying the small x region of QCD, a instanton induced DIS sub-processes is $\gamma^* + g \rightarrow \sum_{n=1}^{N_f} \bar{q}_L + q_R$, whose cross-section is

$$\sigma_I(q'p \rightarrow X) = \int d^4R \int d\rho\, d\bar{\rho}\, D(\rho) D(\bar{\rho}) \int dU e^{\frac{-4\pi}{g^2}\Sigma(U, \frac{R^2}{\rho\bar{\rho}}, \frac{\rho}{\bar{\rho}})} \times \text{parton cross.sec}$$

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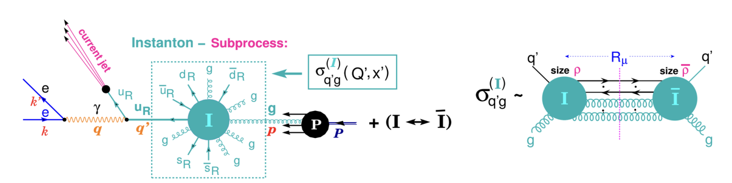

[F. Schrempp, 2005]

- We have to be in a parameter range where sphaleron processes are subdominant.
- Radius distribution $D(\rho)$ and their separations are the input that we can provide.
- Depending on the Q^2, only instantons of size $\rho < Q^{-1}$ or smaller are relevant.

Thank You

Now you know what are Instantons!!!, I guess