# in

# the momentum space and

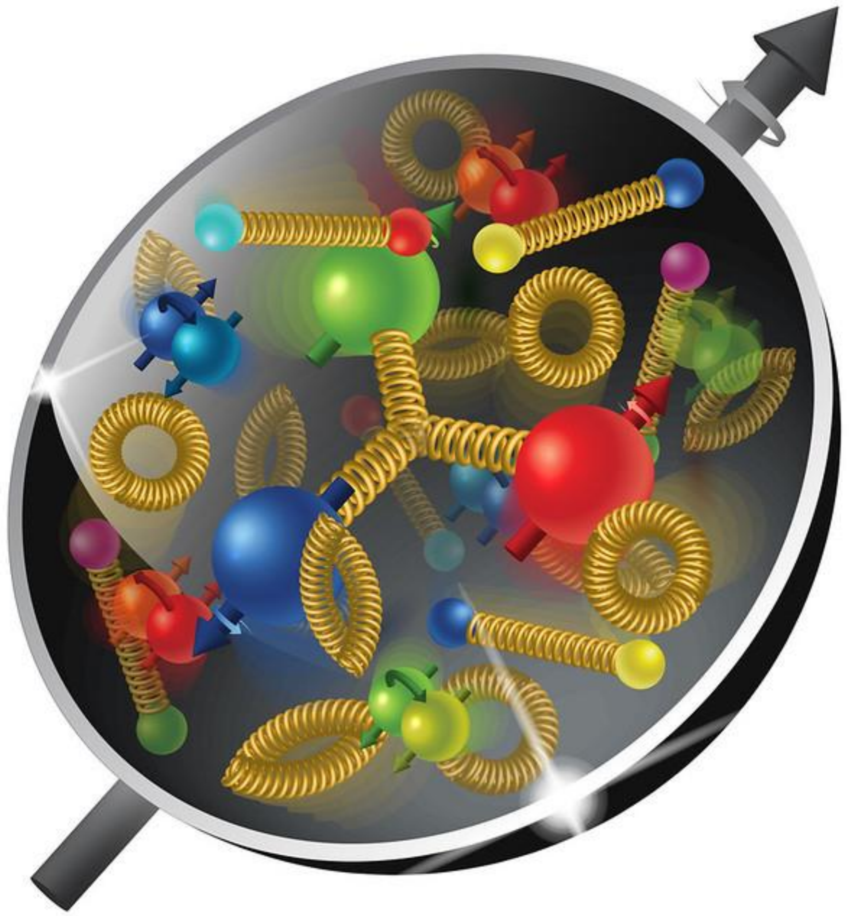
# the Electron-Ion Collider

## THE MENU

- What is 3D structure?
- Why to be excited about it?
- Why the Electron-Ion Collider?

Our group consists of theorists and experimentalists.

I will try to provide useful information for all.



# THE PLAN

- **Lecture I:**

  Structure of the nucleon

  Transverse Momentum Dependent distributions (TMDs)

  Semi Inclusive Deep Inelastic Scattering (SIDIS)

  Calculations of SIDIS structure functions in google colab

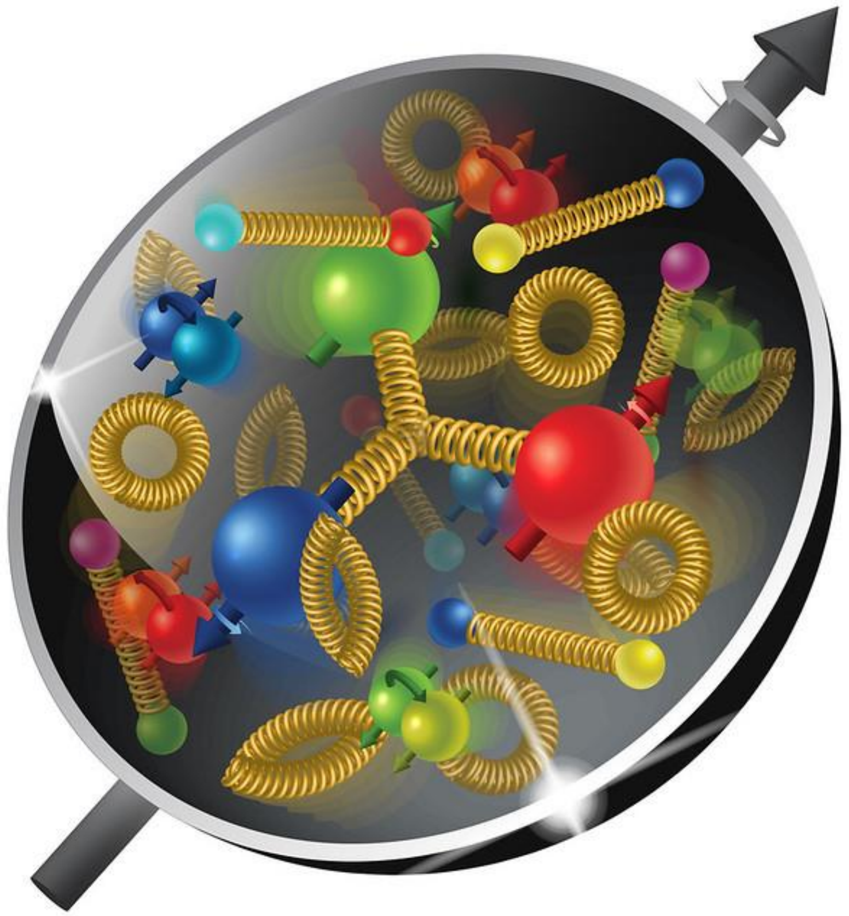
- **Lecture II:**

  Solution of TMD evolution equations

  Collins-Soper-Sterman (CSS) formalism

- **Lecture III: Giuseppe Bozzi**

  Phenomenology of unpolarized TMDs



# MATERIAL

https://inspirehep.net/literature/2650019

# TMD Handbook

April 6, 2023

A modern introduction to the physics of
Transverse Momentum Dependent distributions

TMD Collaboration

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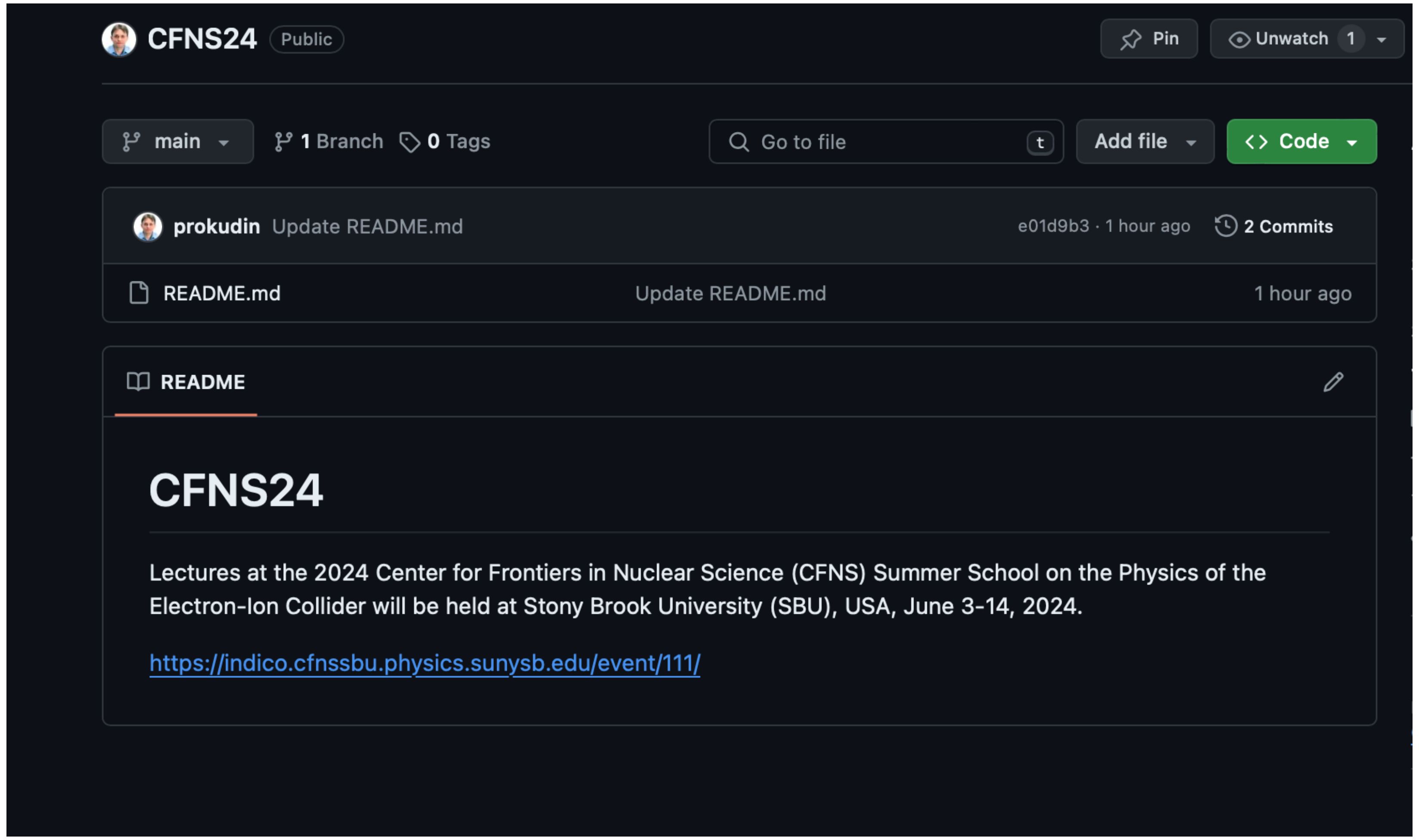
https://github.com/prokudin/CFNS24




Clone the repository, or download files from github

# TMD TABLE

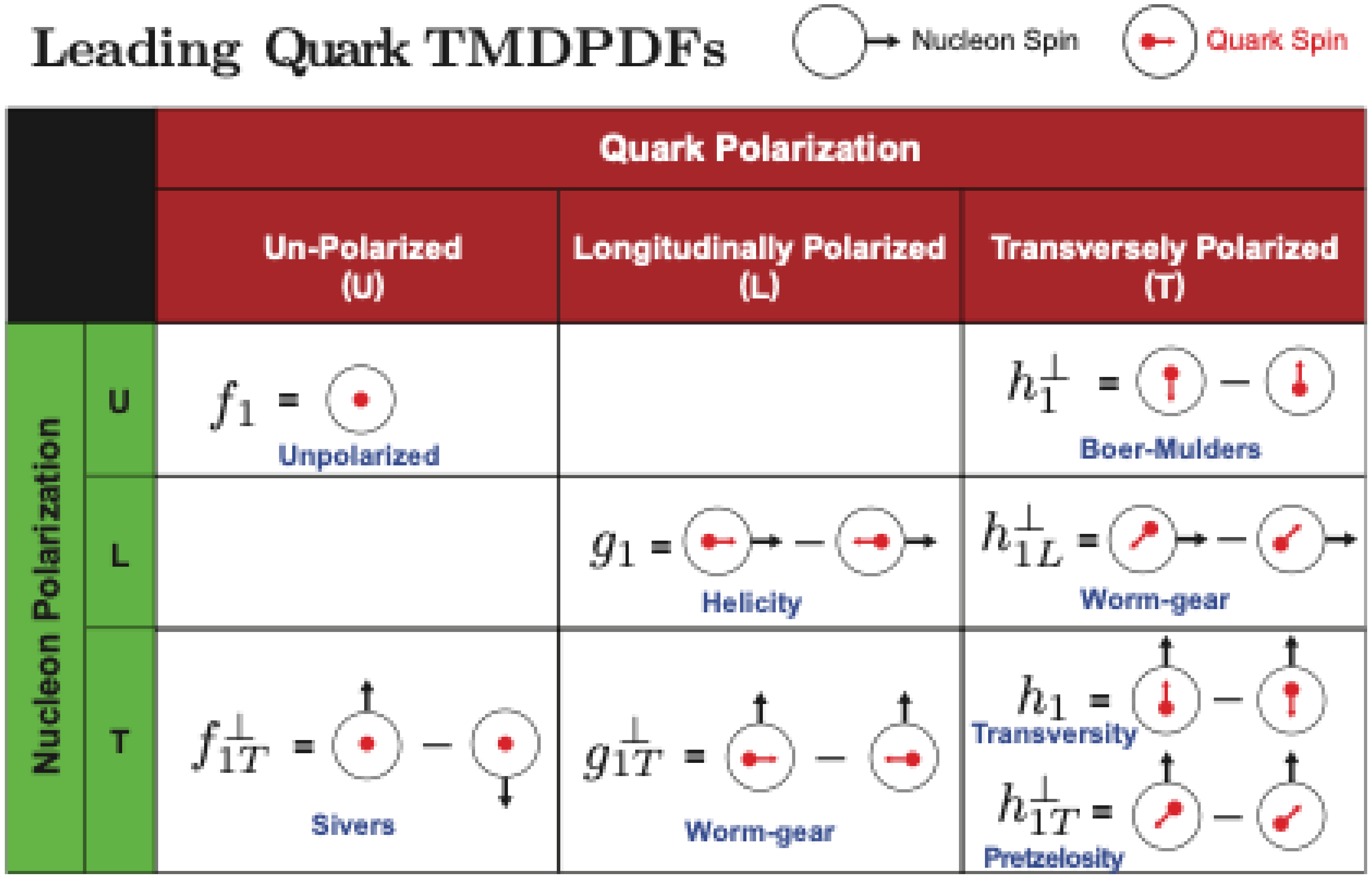



Figure 2.5: Leading power quark parton distribution functions for the proton or a spin-1/2 hadron.

# TRANSVERSE MOMENTUM DEPENDENT DISTRIBUTIONS

$$f_{i/p_S}^{[\gamma^+]}(x,\mathbf{k}_T,\mu,\zeta) = f_1(x,k_T) - \frac{\epsilon_T^{\rho\sigma} k_{T\rho} S_{T\sigma}}{M} \kappa\, f_{1T}^{\perp}(x,k_T)\,, \tag{2.123}$$

$$f_{i/p_S}^{[\gamma^+\gamma_5]}(x,\mathbf{k}_T,\mu,\zeta) = S_L\, g_1(x,k_T) - \frac{k_T\cdot S_T}{M} g_{1T}^{\perp}(x,k_T)\,,$$

$$f_{i/p_S}^{[i\sigma^{\alpha+}\gamma_5]}(x,\mathbf{k}_T,\mu,\zeta) = S_T^\alpha h_1(x,k_T) + \frac{S_L k_T^\alpha}{M} h_{1L}^{\perp}(x,k_T)$$
$$- \frac{\mathbf{k}_T^2}{M^2}\left(\frac{1}{2} g_T^{\alpha\rho} + \frac{k_T^\alpha k_T^\rho}{\mathbf{k}_T^2}\right) S_{T\rho} h_{1T}^{\perp}(x,k_T) - \frac{\epsilon_T^{\alpha\rho} k_{T\rho}}{M} \kappa\, h_1^{\perp}(x,k_T)\,.$$

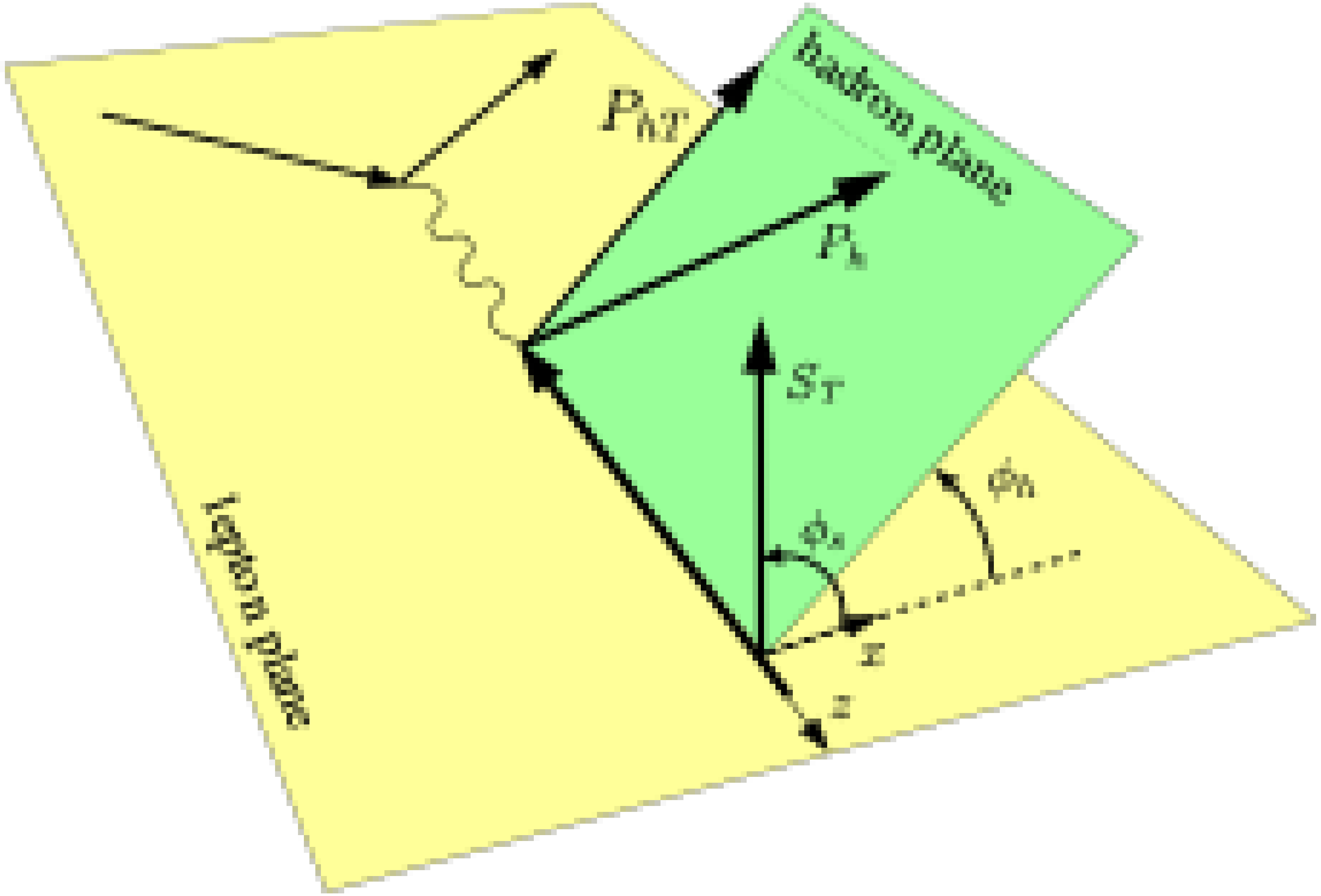
$$\kappa = \begin{cases} +1 & \text{(Drell-Yan)} \\ -1 & \text{(SIDIS)} \end{cases}$$

- $f_1(x,k_T)$ describes an unpolarized quark inside an unpolarized hadron, similar to the unpolarized collinear distribution $f_1(x)$.
- $g_1(x,k_T)$ is the helicity distribution which describes a longitudinally polarized quark inside a longitudinally polarized hadron, similar to the collinear helicity distribution $g_1(x)$.
- $h_1(x,k_T)$ is the transversity distribution which describes a transversely polarized quark inside a transversely polarized hadron, similar to the collinear transversity distribution $h_1(x)$.
- $f_{1T}^{\perp}(x,k_T)$ is the Sivers function [135] which describes an unpolarized quark inside a transversely polarized hadron. Since it is $T$-odd, it was originally believed to vanish due to symmetry arguments [61]. It was later clarified that it is non-vanishing when correctly taking the Wilson lines in the definition of the unsubtracted TMD PDF and soft function into account [38, 62, 136].
- The function $g_{1T}^{\perp}(x,k_T)$ describes longitudinally polarized quarks in a transversely polarized hadron, and vice versa $h_{1L}^{\perp}(x,k_T)$ describes transversely polarized quarks in a longitudinally polarized hadron [137]. They are referred in the literature as "worm-gear" T and L functions or Kotzinian-Mulders [64, 138] functions.
- $h_1^{\perp}(x,k_T)$ is the Boer-Mulders function [63] which describes a transversely polarized quark in an unpolarized hadron. Like the Sivers function $f_{1T}^{\perp}$, it is time-reversal odd.
- $h_{1T}^{\perp}(x,k_T)$ is the pretzelosity function, which contributes to the distribution of a transversely polarized quark in a transversely polarized hadron [132], in addition to the transversity $h_1(x,k_T)$. Curiously, the name of this function stems from its expected shape [139] published by G. Miller, which was also highlighted in the New York Times [140], exhibiting the unusual shape of the proton due to the presence of this function.

# TRANSVERSE MOMENTUM DEPENDENT DISTRIBUTIONS

$$\tilde{f}_{i/p_S}^{[\gamma^+]}(x, \mathbf{b}_T, \mu, \zeta) = \tilde{f}_1(x, b_T) + i\epsilon_{\rho\sigma} b_T^\rho S_T^\sigma M \tilde{f}_{1T}^\perp(x, b_T)\,,$$
$$\tilde{f}_{i/p_S}^{[\gamma^+\gamma_5]}(x, \mathbf{b}_T, \mu, \zeta) = S_L \tilde{g}_1(x, b_T) + i b_T \cdot S_T M \tilde{g}_{1T}^\perp(x, b_T)\,,$$
$$\tilde{f}_{i/p_S}^{[i\sigma^{\alpha+}\gamma_5]}(x, \mathbf{b}_T, \mu, \zeta) = S_T^\alpha \tilde{h}_1(x, b_T) - i S_L b_T^\alpha M \tilde{h}_{1L}^\perp(x, b_T) + i\epsilon^{\alpha\rho} b_{\perp\rho} M \tilde{h}_1^\perp(x, b_T)$$
$$+ \frac{1}{2}\mathbf{b}_T^2 M^2 \left(\frac{1}{2} g_T^{\alpha\rho} + \frac{b_T^\alpha b_T^\rho}{\mathbf{b}_T^2}\right) S_{\perp\rho} \tilde{h}_{1T}^\perp(x, b_T)\,. \qquad (2.126)$$

# SEMI INCLUSIVE DEEP INELASTIC SCATTERING




$$\ell(l) + p(P) \longrightarrow \ell(l') + h(P_h) + X$$

# SEMI INCLUSIVE DEEP INELASTIC SCATTERING

$$\frac{d^6\sigma}{dx\,dy\,dz_h\,d\phi_S\,d\phi_h\,dP_{hT}^2} = \frac{\alpha_{em}^2}{x\,y\,Q^2}\left(1-y+\frac{1}{2}y^2\right)\Big[F_{UU,T} + \cos(2\phi_h)\,p_1\,F_{UU}^{\cos(2\phi_h)}$$
$$+ S_L\sin(2\phi_h)\,p_1\,F_{UL}^{\sin(2\phi_h)} + S_L\,\lambda p_2\,F_{LL}$$
$$+ S_T\sin(\phi_h-\phi_S)\,F_{UT,T}^{\sin(\phi_h-\phi_S)}$$
$$+ S_T\sin(\phi_h+\phi_S)\,p_1\,F_{UT}^{\sin(\phi_h+\phi_S)} + \lambda\,S_T\cos(\phi_h-\phi_S)\,p_2\,F_{LT}^{\cos(\phi_h-\phi_S)}$$
$$+ S_T\sin(3\phi_h-\phi_S)\,p_1\,F_{UT}^{\sin(3\phi_h-\phi_S)}\Big], \qquad (2.186)$$

$$p_1 = \frac{1-y}{1-y+\frac{1}{2}y^2}, \quad p_2 = \frac{y(1-\frac{1}{2}y)}{1-y+\frac{1}{2}y^2}, \quad p_3 = \frac{(2-y)\sqrt{1-y}}{1-y+\frac{1}{2}y^2}, \quad p_4 = \frac{y\sqrt{1-y}}{1-y+\frac{1}{2}y^2}$$

# SEMI INCLUSIVE DEEP INELASTIC SCATTERING

$$F_{UU,T} = \mathcal{C}\left[f_1 D_1\right],$$

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T) - \boldsymbol{p}_T\cdot\boldsymbol{k}_T}{z_h M_N M_h}\, h_1^{\perp} H_1^{\perp}\right],$$

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T) - \boldsymbol{p}_T\cdot\boldsymbol{k}_T}{z_h M_N M_h}\, h_{1L}^{\perp} H_1^{\perp}\right],$$

$$F_{LL} = \mathcal{C}\left[g_1 D_1\right],$$

$$F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_N}\, g_{1T}^{\perp} D_1\right],$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{z_h M_h}\, h_1 H_1^{\perp}\right],$$

$$F_{UT}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_N}\, f_{1T}^{\perp} D_1\right],$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[\frac{4\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)^2 - 2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\boldsymbol{k}_T\cdot\boldsymbol{p}_T) - (\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\,\boldsymbol{k}_T^2}{2 z_h M_N^2 M_h}\, h_{1T}^{\perp} H_1^{\perp}\right], \quad (2.188)$$

$$\mathcal{C}\left[\omega\, f\, D\right] = x \sum_i H_{ii}(Q^2,\mu) \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{p}_T\, \delta^{(2)}(z_h \boldsymbol{k}_T + \boldsymbol{p}_T - \boldsymbol{P}_{hT}) \times \omega\, f_{i/p_S}(x, k_T, \mu, \zeta_1)\, D_{h/i}(z_h, p_T, \mu, \zeta_2),$$

# SEMI INCLUSIVE DEEP INELASTIC SCATTERING

$$F_{UU}(x, z_h, P_{hT}, Q^2) = \mathcal{B}\left[\tilde{f}_1^{(0)} \tilde{D}_1^{(0)}\right],$$
$$F_{UU}^{\cos 2\phi_h}(x, z_h, P_{hT}, Q^2) = M_N M_h \mathcal{B}\left[\tilde{h}_1^{\perp(1)} \tilde{H}_1^{\perp(1)}\right],$$
$$F_{UL}^{\sin 2\phi_h}(x, z_h, P_{hT}, Q^2) = M_N M_h \mathcal{B}\left[\tilde{h}_{1L}^{\perp(1)} \tilde{H}_1^{\perp(1)}\right],$$
$$F_{LL}(x, z_h, P_{hT}, Q^2) = \mathcal{B}\left[\tilde{g}_1^{(0)} \tilde{D}_1^{(0)}\right],$$
$$F_{LT}^{\cos(\phi_h-\phi_S)}(x, z_h, P_{hT}, Q^2) = M_N \mathcal{B}\left[\tilde{g}_{1T}^{\perp(1)} \tilde{D}_1^{(0)}\right],$$
$$F_{UT}^{\sin(\phi_h+\phi_S)}(x, z_h, P_{hT}, Q^2) = M_h \mathcal{B}\left[\tilde{h}_1^{(0)} \tilde{H}_1^{\perp(1)}\right],$$
$$F_{UT}^{\sin(\phi_h-\phi_S)}(x, z_h, P_{hT}, Q^2) = -M_N \mathcal{B}\left[\tilde{f}_{1T}^{\perp(1)} \tilde{D}_1^{(0)}\right],$$
$$F_{UT}^{\sin(3\phi_h-\phi_S)}(x, z_h, P_{hT}, Q^2) = \frac{M_N^2 M_h}{4} \mathcal{B}\left[\tilde{h}_{1T}^{\perp(2)} \tilde{H}_1^{\perp(1)}\right],$$

$$\mathcal{B}[\tilde{f}^{(m)} \tilde{D}^{(n)}] \equiv x \sum_i H_{ii}(Q^2, \mu) \int_0^\infty \frac{\mathrm{d}b_T}{2\pi} b_T b_T^{m+n} J_{m+n}(q_T b_T)$$
$$\times \tilde{f}_{i/N}^{(m)}(x, b_T, \mu, \zeta_1) \tilde{D}_{h/i}^{(n)}(z_h, b_T, \mu, \zeta_2).$$

# NOTEBOOK EXAMPLE