in

the momentum space and

the Electron-Ion Collider

THE PLAN

- **Lecture I:**

  Structure of the nucleon

  Transverse Momentum Dependent distributions (TMDs)

  Semi Inclusive Deep Inelastic Scattering (SIDIS)

  Calculations of SIDIS structure functions in google colab

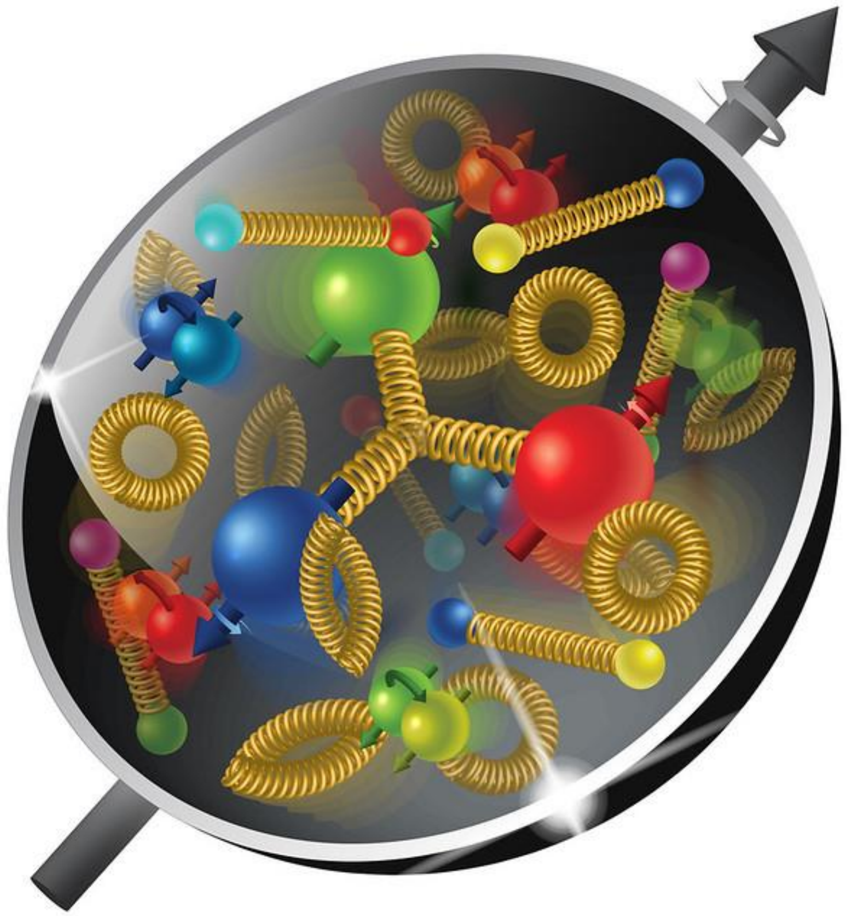
- **Lecture II:**

  Solution of TMD evolution equations

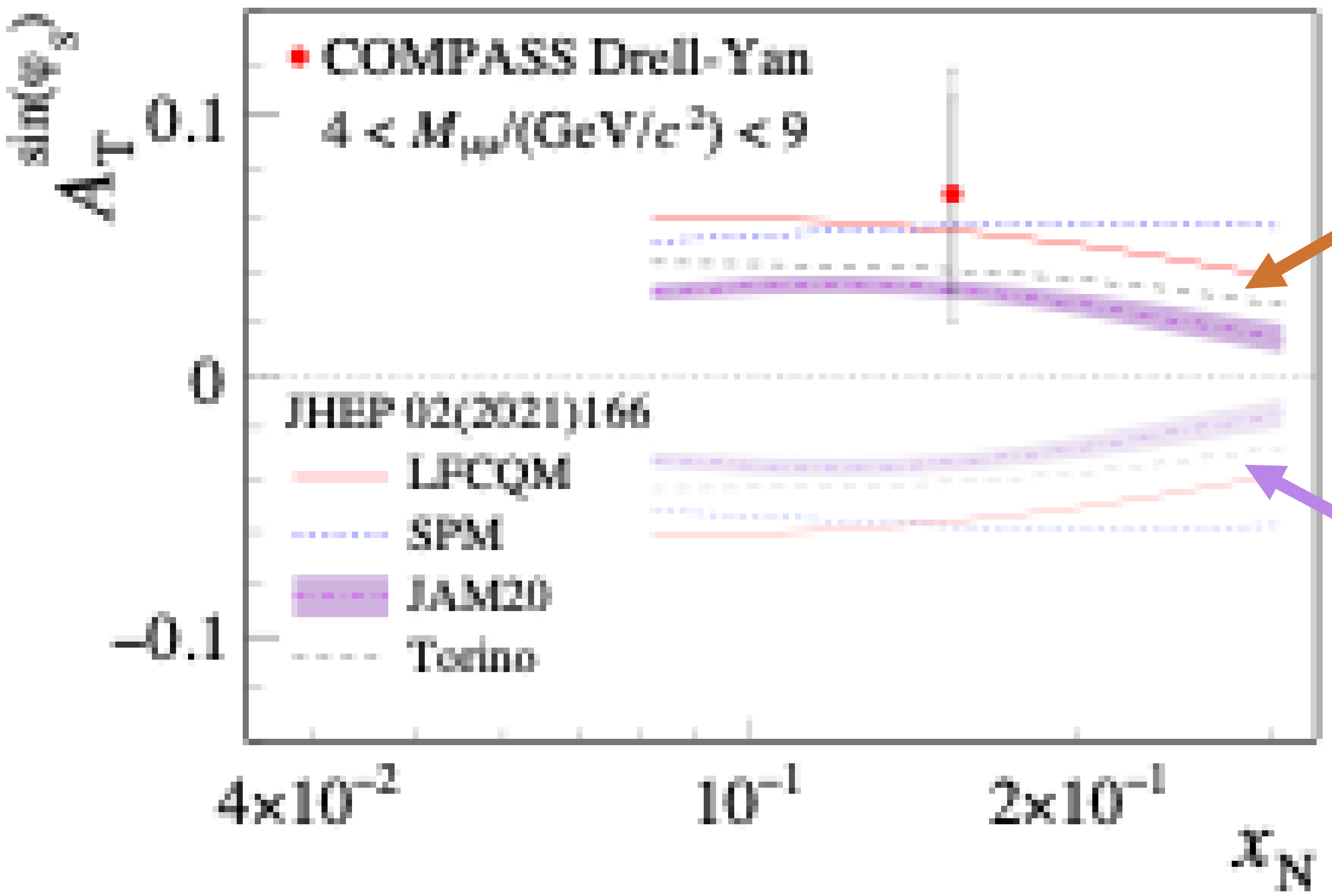
  Collins-Soper-Sterman (CSS) formalism

- **Lecture III: Giuseppe Bozzi**

  Phenomenology of unpolarized TMDs



SIVERS FUNCTION

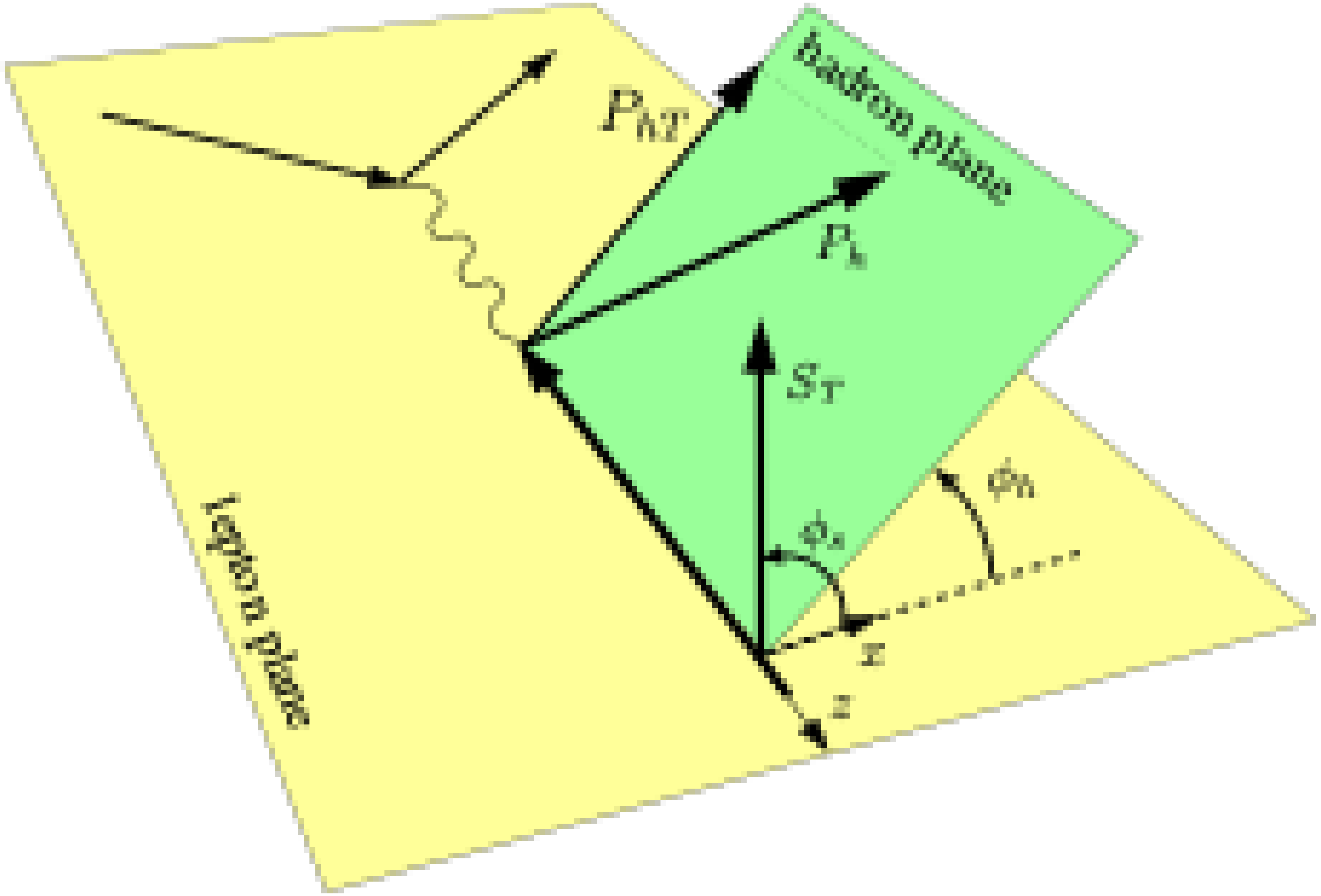


Sign change

No sign change

[2312.17379](https://arxiv.org/abs/2312.17379)

SEMI INCLUSIVE DEEP INELASTIC SCATTERING



$$\ell(l) + p(P) \longrightarrow \ell(l') + h(P_h) + X$$

SEMI INCLUSIVE DEEP INELASTIC SCATTERING

$$\begin{aligned}\frac{d^6\sigma}{dx\,dy\,dz_h\,d\phi_S\,d\phi_h\,dP_{hT}^2} = {} & \frac{\alpha_{em}^2}{x\,y\,Q^2}\left(1-y+\frac{1}{2}y^2\right)\Big[F_{UU,T} + \cos(2\phi_h)\,p_1\,F_{UU}^{\cos(2\phi_h)} \\ & + S_L \sin(2\phi_h)\,p_1\,F_{UL}^{\sin(2\phi_h)} + S_L\,\lambda p_2\,F_{LL} \\ & + S_T \sin(\phi_h-\phi_S)\,F_{UT,T}^{\sin(\phi_h-\phi_S)} \\ & + S_T \sin(\phi_h+\phi_S)\,p_1\,F_{UT}^{\sin(\phi_h+\phi_S)} + \lambda\,S_T \cos(\phi_h-\phi_S)\,p_2\,F_{LT}^{\cos(\phi_h-\phi_S)} \\ & + S_T \sin(3\phi_h-\phi_S)\,p_1\,F_{UT}^{\sin(3\phi_h-\phi_S)}\Big], \end{aligned} \tag{2.186}$$

$$p_1 = \frac{1-y}{1-y+\frac{1}{2}y^2}, \quad p_2 = \frac{y(1-\frac{1}{2}y)}{1-y+\frac{1}{2}y^2}, \quad p_3 = \frac{(2-y)\sqrt{1-y}}{1-y+\frac{1}{2}y^2}, \quad p_4 = \frac{y\sqrt{1-y}}{1-y+\frac{1}{2}y^2}$$

SEMI INCLUSIVE DEEP INELASTIC SCATTERING

$$
\begin{aligned}
F_{UU,T} &= \mathcal{C}\big[f_1 D_1\big], \\
F_{UU}^{\cos 2\phi_h} &= \mathcal{C}\left[\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T) - \boldsymbol{p}_T\cdot\boldsymbol{k}_T}{z_h M_N M_h}\, h_1^{\perp} H_1^{\perp}\right], \\
F_{UL}^{\sin 2\phi_h} &= \mathcal{C}\left[\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T) - \boldsymbol{p}_T\cdot\boldsymbol{k}_T}{z_h M_N M_h}\, h_{1L}^{\perp} H_1^{\perp}\right], \\
F_{LL} &= \mathcal{C}\big[g_1 D_1\big], \\
F_{LT}^{\cos(\phi_h-\phi_S)} &= \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_N}\, g_{1T}^{\perp} D_1\right], \\
F_{UT}^{\sin(\phi_h+\phi_S)} &= \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{z_h M_h}\, h_1 H_1^{\perp}\right], \\
F_{UT}^{\sin(\phi_h-\phi_S)} &= \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_N}\, f_{1T}^{\perp} D_1\right], \\
F_{UT}^{\sin(3\phi_h-\phi_S)} &= \mathcal{C}\left[\frac{4\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)^2 - 2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\boldsymbol{k}_T\cdot\boldsymbol{p}_T) - (\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\,\boldsymbol{k}_T^2}{2 z_h M_N^2 M_h}\, h_{1T}^{\perp} H_1^{\perp}\right], \qquad (2.188)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{C}\left[\omega\, f\, D\right] = x \sum_i H_{ii}(Q^2,\mu) \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{p}_T\, \delta^{(2)}(z_h \boldsymbol{k}_T + \boldsymbol{p}_T - \boldsymbol{P}_{hT}) \\
\times\, \omega\, f_{i/p_S}(x, k_T, \mu, \zeta_1)\, D_{h/i}(z_h, p_T, \mu, \zeta_2),
\end{aligned}
$$

SEMI INCLUSIVE DEEP INELASTIC SCATTERING

$$F_{UU}(x, z_h, P_{hT}, Q^2) = \mathcal{B}\left[\tilde{f}_1^{(0)}\, \tilde{D}_1^{(0)}\right],$$

$$F_{UU}^{\cos 2\phi_h}(x, z_h, P_{hT}, Q^2) = M_N\, M_h\, \mathcal{B}\left[\tilde{h}_1^{\perp(1)}\, \tilde{H}_1^{\perp(1)}\right],$$

$$F_{UL}^{\sin 2\phi_h}(x, z_h, P_{hT}, Q^2) = M_N\, M_h\, \mathcal{B}\left[\tilde{h}_{1L}^{\perp(1)}\, \tilde{H}_1^{\perp(1)}\right],$$

$$F_{LL}(x, z_h, P_{hT}, Q^2) = \mathcal{B}\left[\tilde{g}_1^{(0)}\, \tilde{D}_1^{(0)}\right],$$

$$F_{LT}^{\cos(\phi_h-\phi_S)}(x, z_h, P_{hT}, Q^2) = M_N\, \mathcal{B}\left[\tilde{g}_{1T}^{\perp(1)}\, \tilde{D}_1^{(0)}\right],$$

$$F_{UT}^{\sin(\phi_h+\phi_S)}(x, z_h, P_{hT}, Q^2) = M_h\, \mathcal{B}\left[\tilde{h}_1^{(0)}\, \tilde{H}_1^{\perp(1)}\right],$$

$$F_{UT}^{\sin(\phi_h-\phi_S)}(x, z_h, P_{hT}, Q^2) = -M_N\, \mathcal{B}\left[\tilde{f}_{1T}^{\perp(1)}\, \tilde{D}_1^{(0)}\right],$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)}(x, z_h, P_{hT}, Q^2) = \frac{M_N^2\, M_h}{4}\, \mathcal{B}\left[\tilde{h}_{1T}^{\perp(2)}\, \tilde{H}_1^{\perp(1)}\right],$$

$$\mathcal{B}[\tilde{f}^{(m)}\, \tilde{D}^{(n)}] \equiv x \sum_i H_{ii}(Q^2, \mu) \int_0^\infty \frac{db_T}{2\pi}\, b_T\, b_T^{m+n}\, J_{m+n}(q_T b_T) \times \tilde{f}_{i/N}^{(m)}(x, b_T, \mu, \zeta_1)\, \tilde{D}_{h/i}^{(n)}(z_h, b_T, \mu, \zeta_2).$$

TMD EVOLUTION EQUATIONS

Differential equations, diagonal in the flavor space. RHS can be expanded in perturbative series.

$$\frac{d\ln\tilde{F}(x,b_T,\mu,\zeta)}{d\ln\mu}=\gamma_F(\mu)$$

TMD anomalous dimension

$$\frac{\partial\ln\tilde{F}(x,b_T,\mu,\zeta)}{\partial\ln\sqrt{\zeta}}=\tilde{K}(b_T,\mu)$$

Collins-Soper kernel is specific for TMDs

$$\frac{d\tilde{K}(b_T,\mu)}{d\ln\mu}=-\gamma_K(\mu)$$

Cusp anomalous dimension

ζ = Collins-Soper parameter

μ = UV renormalization scale

CSS SOLUTION OF EVOLUTION EQUATIONS

Collins-Soper-Sterman (CSS) organization of the solution of TMD evolution equations for the Drell-Yan cross section:

$$
\begin{aligned}
\frac{d\sigma}{d^4q\, d\Omega} = & \frac{2}{s}\sum_{j,j_A,j_B}\frac{d\hat{\sigma}_{j\bar{j}}(Q,\mu_Q,\alpha_s(\mu_Q))}{d\Omega}\int\frac{d^2 b_T}{(2\pi)^2}e^{iq_T\cdot b_T} \\
& \times e^{-g_{j/A}(x_A,b_T;b_{\max})}\int_{x_A}^1\frac{d\hat{x}_A}{\hat{x}_A}f_{j_A/A}(\hat{x}_A;\mu_{b_*})\,\tilde{C}_{j/j_A}\left(\frac{x_A}{\hat{x}_A},b_*;\mu_{b_*}^2,\mu_{b_*},\alpha_s(\mu_{b_*})\right) \\
& \times e^{-g_{\bar{j}/B}(x_B,b_T;b_{\max})}\int_{x_B}^1\frac{d\hat{x}_B}{\hat{x}_B}f_{j_B/B}(\hat{x}_B;\mu_{b_*})\,\tilde{C}_{\bar{j}/j_B}\left(\frac{x_B}{\hat{x}_B},b_*;\mu_{b_*}^2,\mu_{b_*},\alpha_s(\mu_{b_*})\right) \\
& \times\left(\frac{Q^2}{Q_0^2}\right)^{-g_K(b_T;b_{\max})}\left(\frac{Q^2}{\mu_{b_*}^2}\right)^{\tilde{K}(b_*;\mu_{b_*})}\exp\left\{\int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[2\gamma_j(\alpha_s(\mu');1)-\ln\frac{Q^2}{(\mu')^2}\gamma_K(\alpha_s(\mu'))\right]\right\} \\
& + \text{polarized terms} + \text{large-}q_T\text{ correction, } Y + \text{p.s.c.}
\end{aligned}
$$

Here μ_{b_*} is chosen to allow perturbative calculations of b_*-dependent quantities without large logarithms:

$$\mu_{b_*} = C_1/b_*,$$

where C_1 is a numerical constant typically chosen to be $C_1 = 2e^{-\gamma_E}$.

From https://arxiv.org/pdf/1412.3820