Hadron Physics in the EIC Era

A Continuum QCD Approach

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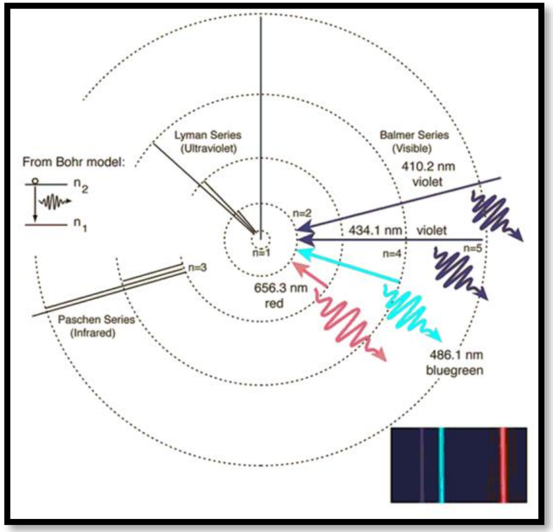
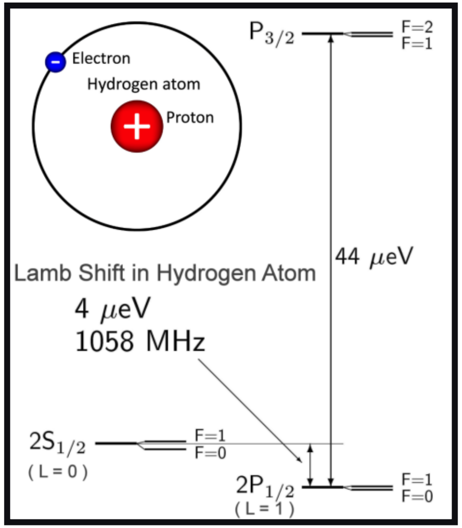
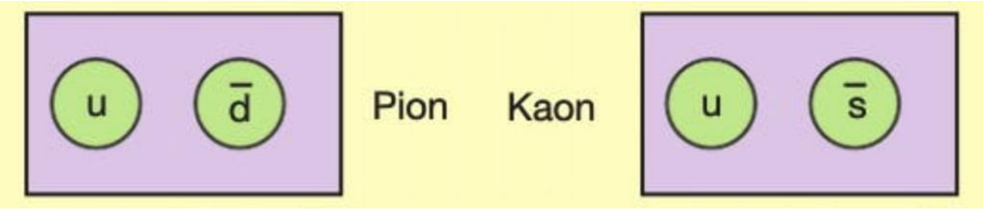
Summer School on the Physics of the Electron-Ion Collider

Center for Frontiers in Nuclear Science

Two particle bound states



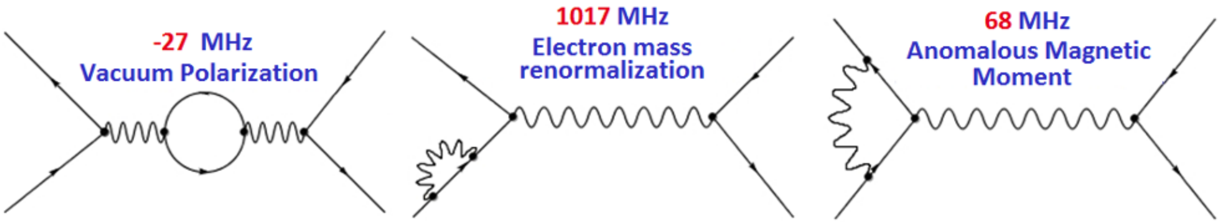




π and the **K** are the simplest two-body **bound states** in **QCD**. Unraveling their internal structure is a bigger challenge.

Nambu-Goldstone bosons associated with dynamical chiral symmetry breaking

Dyson: If you don't understand the **Hydrogen atom** (in **QED**) you don't understand anything.



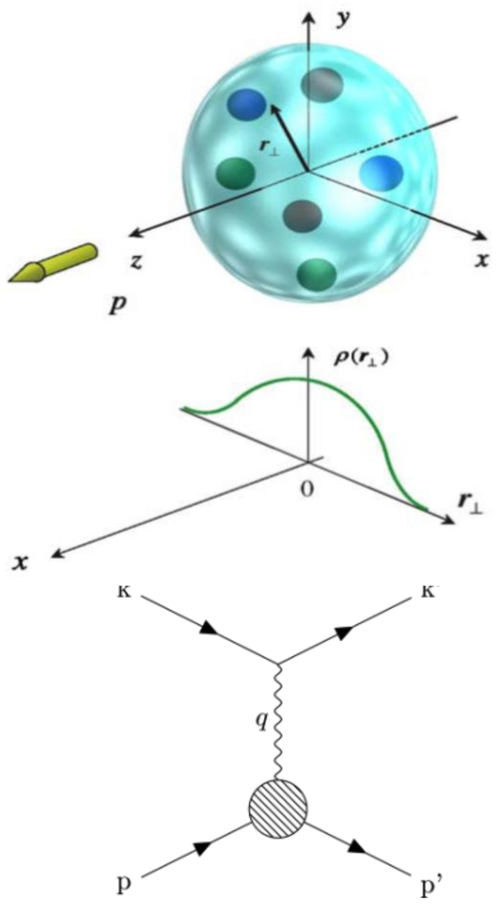
Renormalized QED

1934-1949

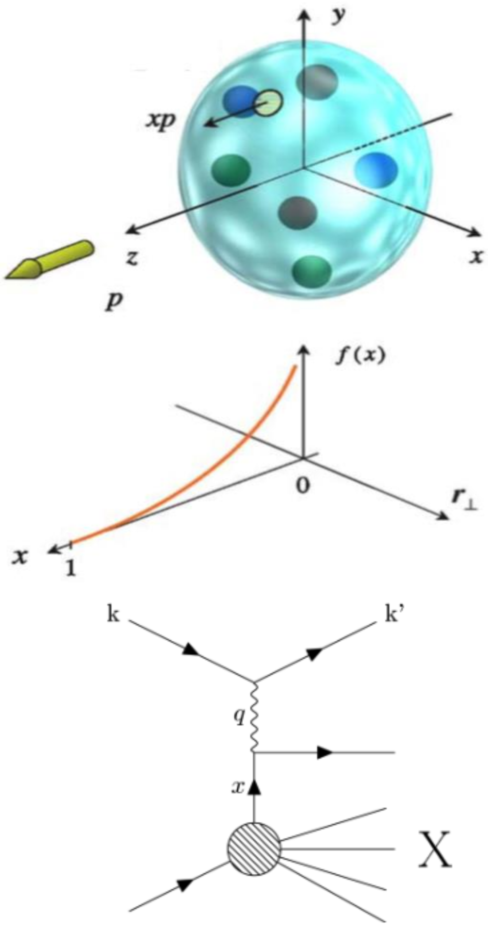
1947-1950

1960-2008

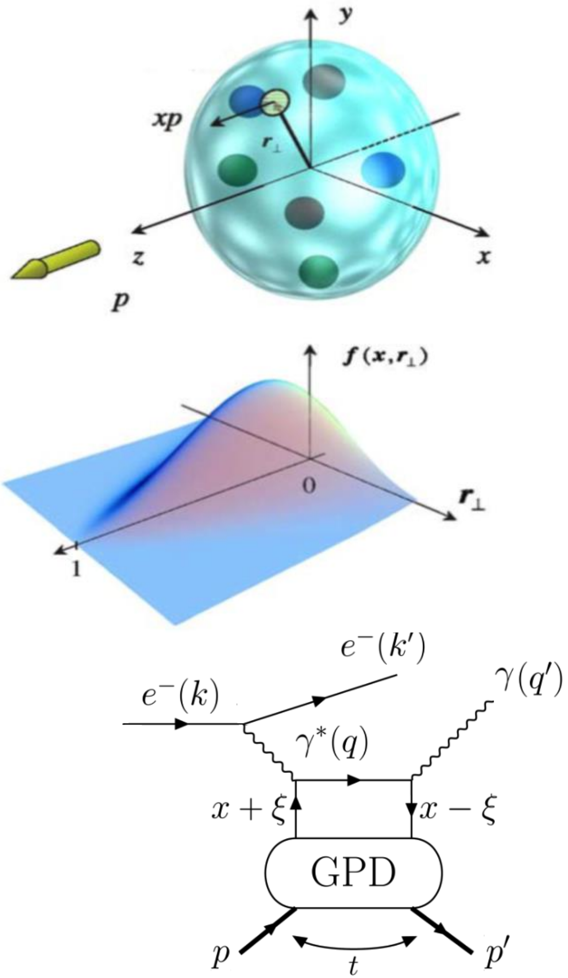
Hadrons Structure – QCD



Hofstadter, et. al., Phys. Rev. 91, 422 (1953)
Hofstadter, Rev. Mod. Phys. 28, 214 (1956)



Feynman, Phys. Rev. Lett. 23, 1415 (1969)



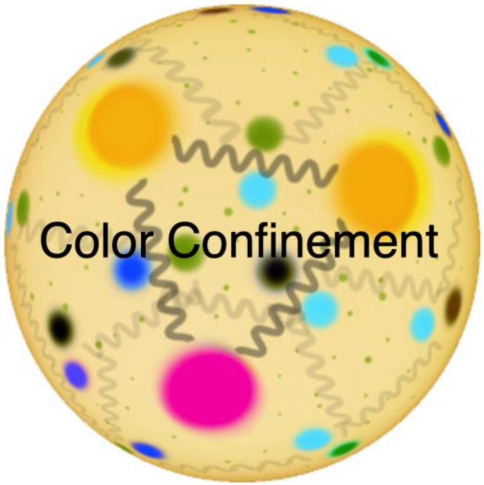
QCD: Emergent Phenomena and Challenges

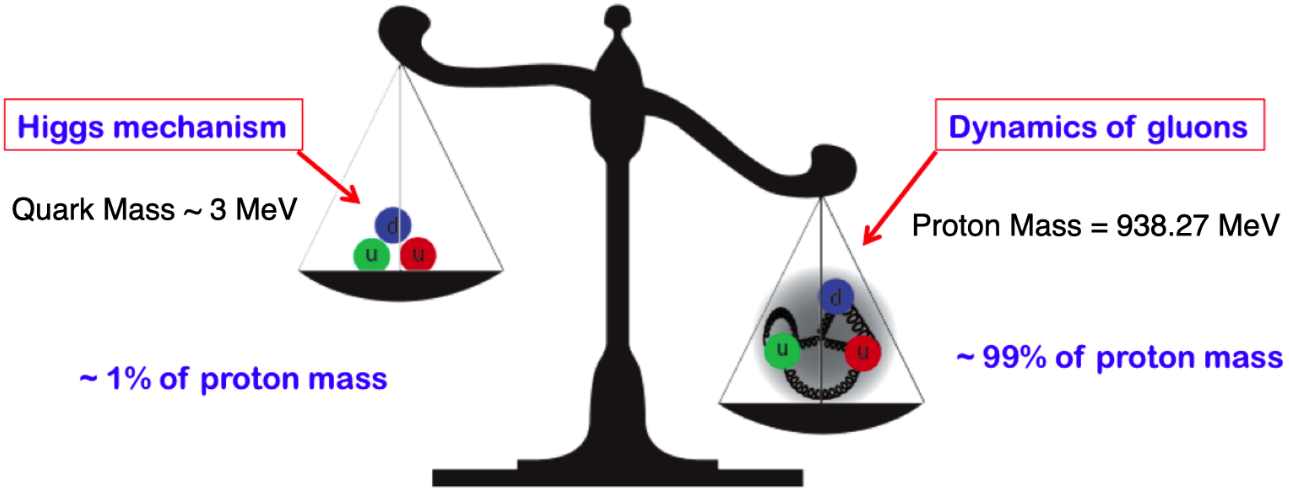
QCD is characterized by two **emergent** phenomena: **confinement** and dynamical generation of mass (**DGM**).

$$\mathcal{L}_{\text{QCD}} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$
$$D_\mu = \partial_\mu + ig\tfrac{1}{2}\lambda^a A^a_\mu\,,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu},$$

- Quarks and gluons do not reach detectors.
- Formation of color-singlet bound states: “**Hadrons**” mesons, baryons, tetraquarks, molecules

- Emergence of hadron masses (EHM) from QCD **dynamics**



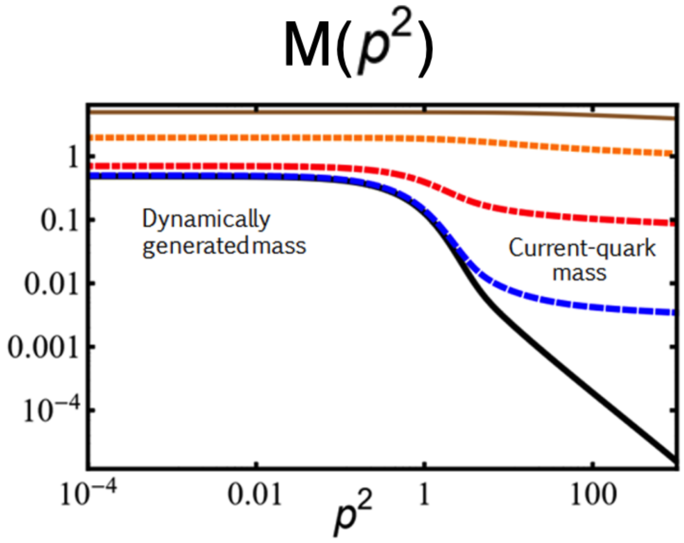


QCD: Emergent Phenomena and Challenges

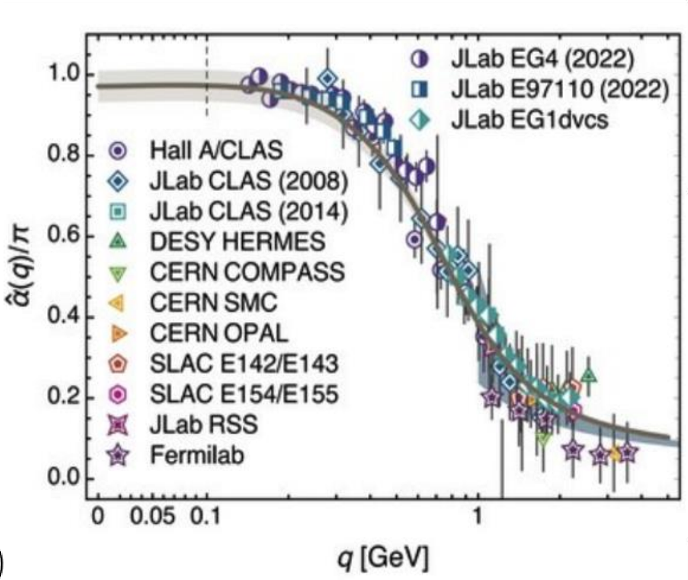
Origins of **confinement** and **dynamical mass generation** can perhaps be traced back to the Green functions of **quarks** and **gluons**.

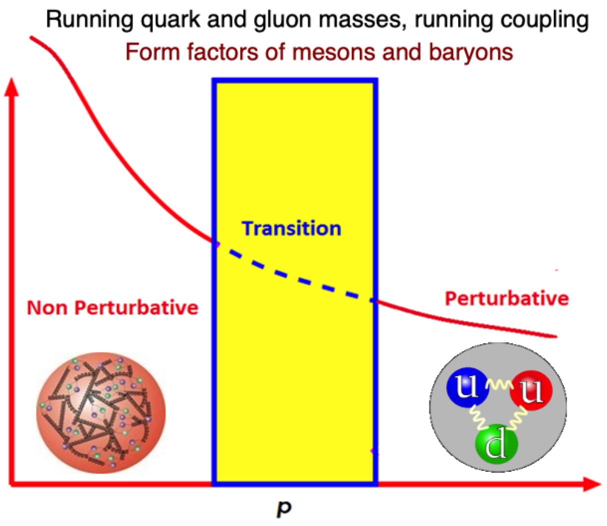
These emergent phenomena of **QCD**, non-existent in perturbation theory are naturally linked to the infrared enhancement of the **strong running coupling**.

The effects of the pattern of **dynamical mass generation** are traceable in the **Q^2 evolution** of the π and **K form factors** explored and planned in the **JLab** and the **EIC**.



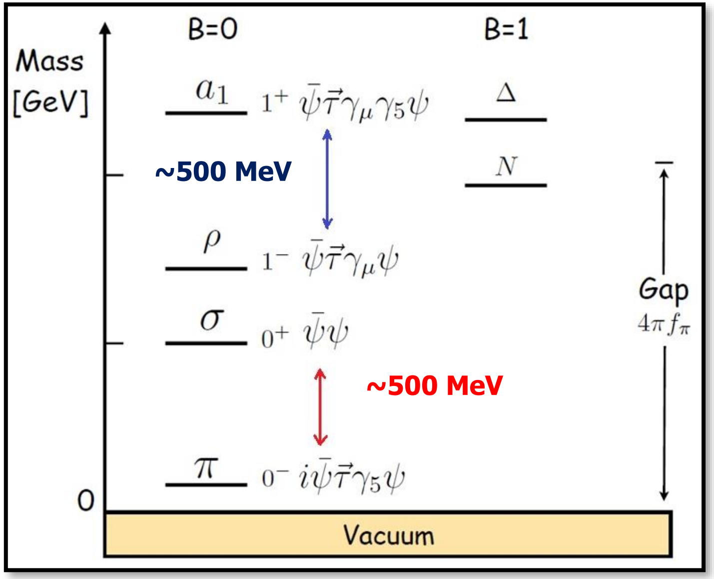
$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma \cdot p + \mathbf{M_f(p^2)})$$





DCSB: Mass Spectrum of Mesons and Baryons

Experimental signature of **DCSB** is observed in meson **masses**.



$$\psi \longrightarrow e^{-i\gamma_5\frac{\vec\tau}{2}\cdot\vec\Theta}\psi \simeq \left(1 - i\gamma_5\frac{\vec\tau}{2}\cdot\vec\Theta\right)\psi$$

$$\bar\psi \longrightarrow \bar\psi e^{-i\gamma_5\frac{\vec\tau}{2}\cdot\vec\Theta} \simeq \bar\psi\left(1 - i\gamma_5\frac{\vec\tau}{2}\cdot\vec\Theta\right)$$

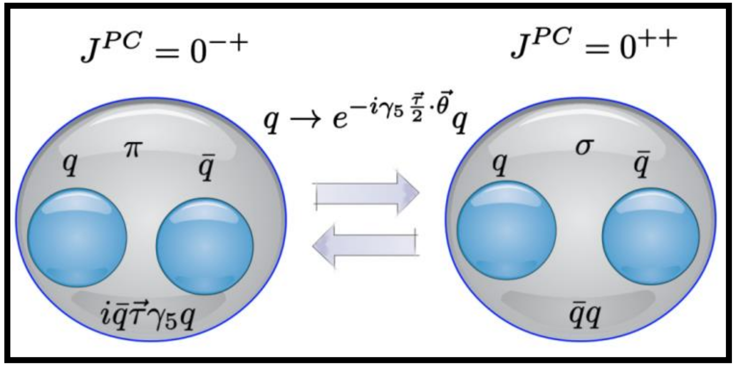
$$\pi_i\colon\ i\bar\psi\tau_i\gamma_5\psi \rightarrow i\bar\psi\tau_i\gamma_5\psi + \Theta_j\left(\bar\psi\tau_i\gamma_5\gamma_5\frac{\tau_j}{2}\psi + \bar\psi\gamma_5\frac{\tau_j}{2}\tau_i\gamma_5\psi\right)$$
$$= i\bar\psi\tau_i\gamma_5\psi + \Theta_i\bar\psi\psi$$

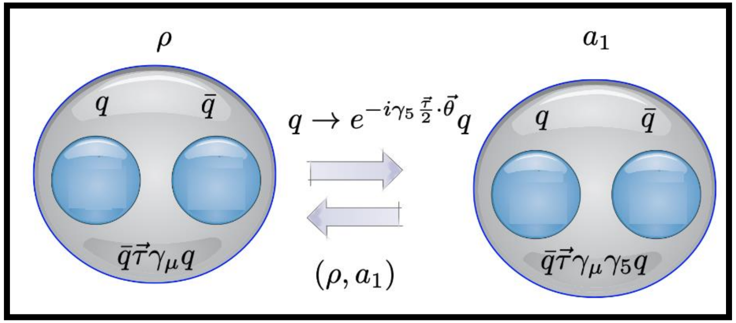
$$\vec\pi \longrightarrow \vec\pi + \vec\Theta\sigma$$
$$\sigma \longrightarrow \sigma - \vec\Theta.\vec\pi$$

Axial, Chiral Transformations

$$\vec\rho_\mu \longrightarrow \vec\rho_\mu + \vec\Theta \times \vec a_{1\mu}$$

$$\vec a_{1\mu} \longrightarrow \vec a_{1\mu} - \vec\Theta \times \vec\rho_\mu$$



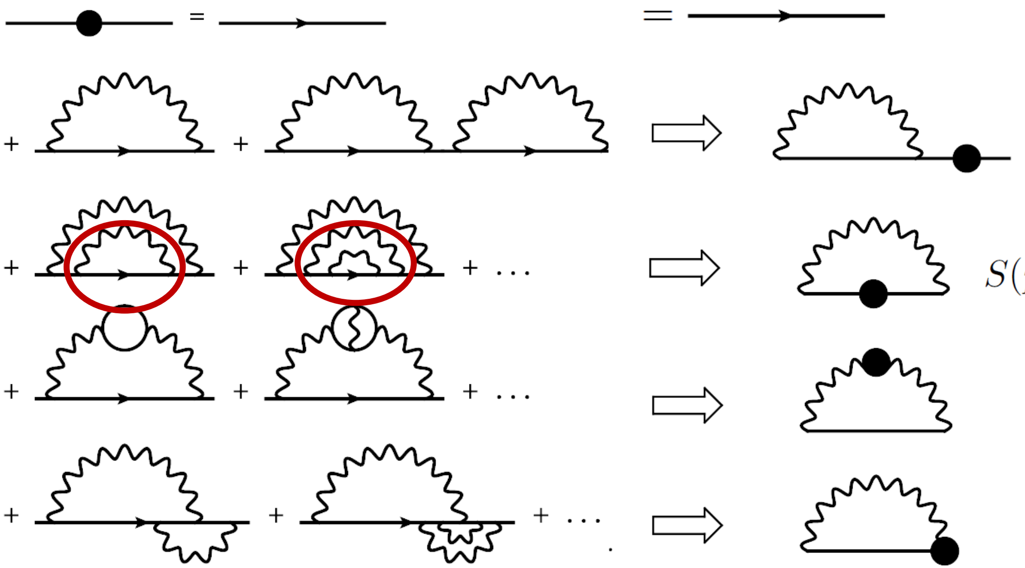


Schwinger-Dyson Equations - QED

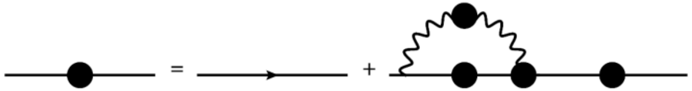
The Electron Propagator

Bare

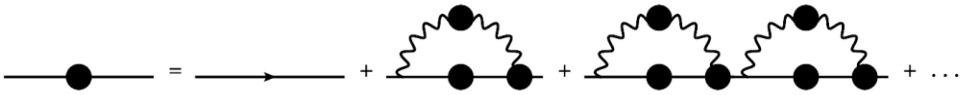
$$S^0(p) = \frac{1}{\not{p} - m}$$



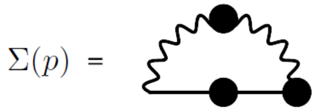
The perturbative series can be summed up:



The last blob can again be expanded:



Define self-energy as:



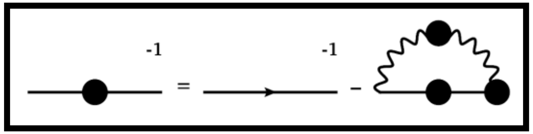
$$S(p) = S^0(p) + S^0(p)\Sigma(p)S^0(p) + S^0(p)\Sigma(p)S^0(p)\Sigma(p)S^0(p) + \ldots$$

It can be written in a compact form:

$$S(p) = S^0(p) + S^0(p)\Sigma(p)\underbrace{\left[S^0(p) + S^0(p)\Sigma(p)S^0(p) + \ldots\right]}_{S(p)}$$

$$= S^0(p) + S^0(p)\Sigma(p)S(p)$$

$$S^{-1}(p) = S^{0^{-1}}(p) - \Sigma(p)$$



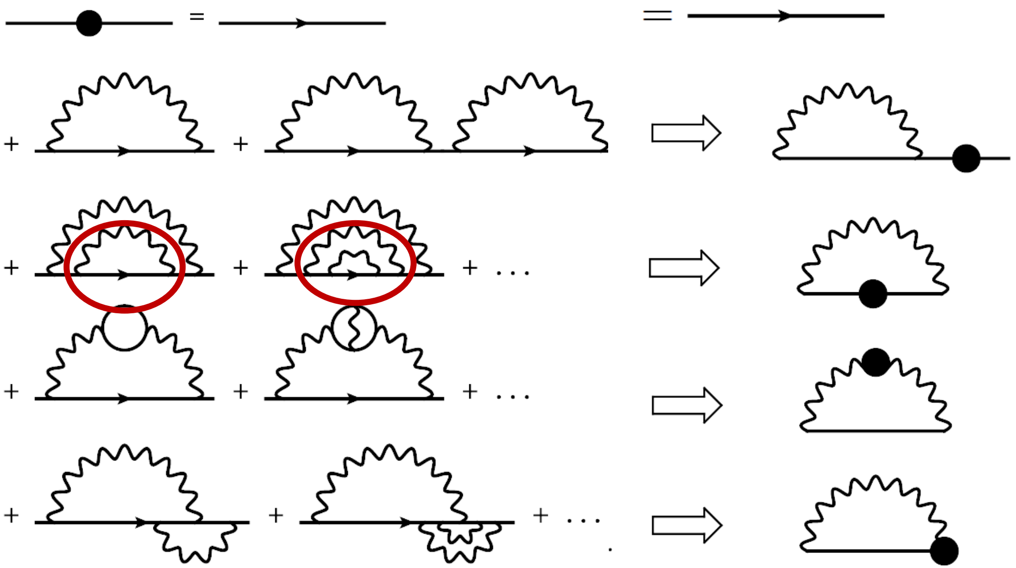
The gap equation

Schwinger-Dyson Equations - QED

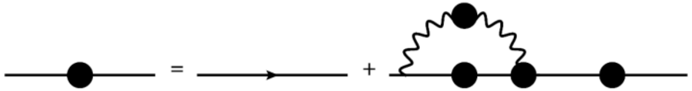
The Quark Propagator

Bare

$$S^0(p) = \frac{1}{\not{p} - m}$$



The perturbative series can be summed up:



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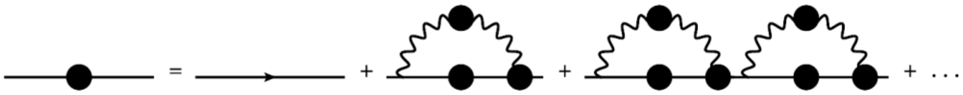
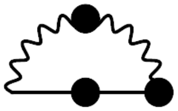
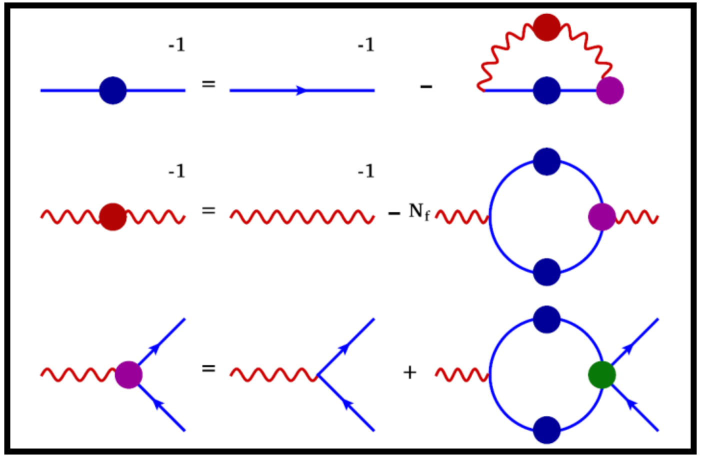


Define self-energy as: $\Sigma(p) =$





Schwinger-Dyson equations

QCD – Schwinger-Dyson Equations

Gauge Technique – Non Perturbative Solutios

- A. Salam, R. Delbourgo, Phys. Rev. 135 (1964) 6, B1398-B1427.

DCSB - Non-perturbative QED

- P.I. Fomin, V.A. Miransky, Phys. Lett. B64 (1976) 166-168.

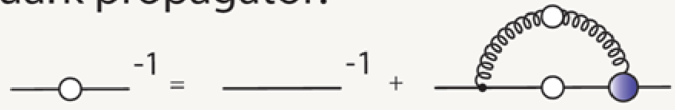
DCSB – Non-abelian Gauge Theories

- V. Miransky, V. Gusynin, Y. Sitenko, Phys. Lett. B100 (1981) 157-162

DCSB – MT Model - Vector Mesons

- P. Maris, P. Tandy, Phys. Rev. C60 (1999)

Quark propagator:



$$S(p^2,\mu^2) = \frac{Z(p^2,\mu^2)}{i\,\gamma\cdot p + M(p^2)}$$

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Renormalizable Electrodynamics of Scalar and Vector Mesons. II

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The "gauge technique" for solving field theories introduced in an earlier paper is applied to scalar and vector electrodynamics.

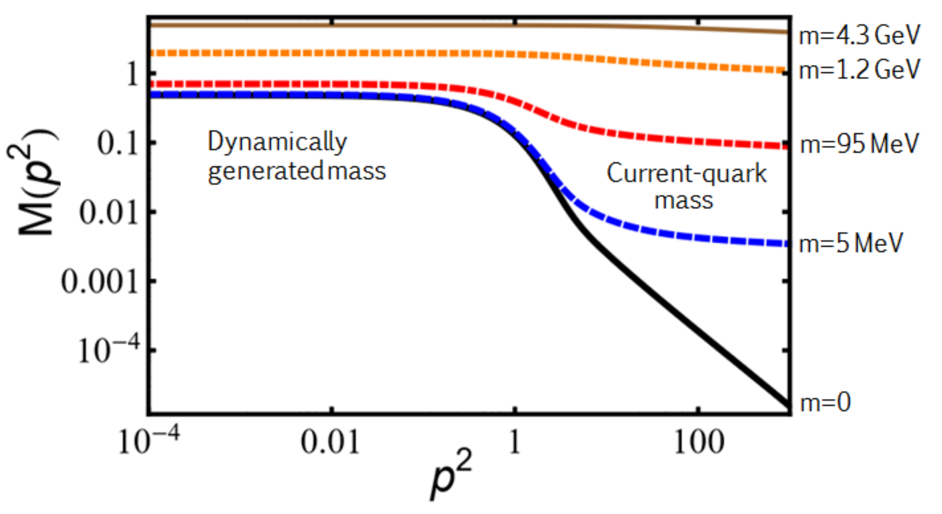
A. Dyson-Schwinger Set;

For a typical 3-field (e.g., electron-photon) interaction the well-known Dyson equations

$$S^{-1} = Z_2 S_0^{-1} + Z_1 e^2 \int \Gamma S \Gamma_0 D \quad \text{(I.1)}$$

← **SDE: electron propagator**

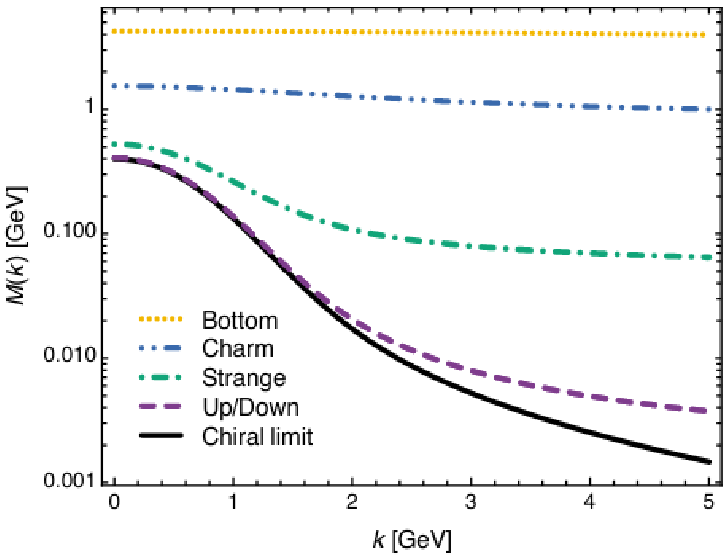
$$D^{-1} = Z_3 D_0^{-1} + Z_1 e^2 \int \Gamma S \Gamma_0 S \quad \text{(I.2)}$$

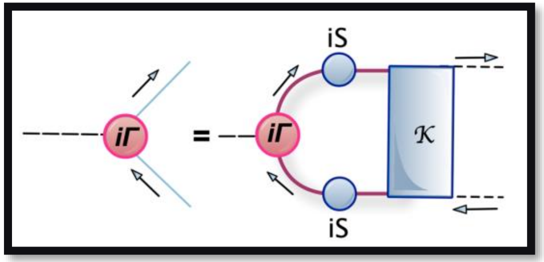


π and K: Bound States and Goldstone Bosons

The pattern of **dynamical chiral symmetry breaking** and the **Bethe-Salpeter amplitude** to be computed by solving the **Bethe-Salpeter equation**.

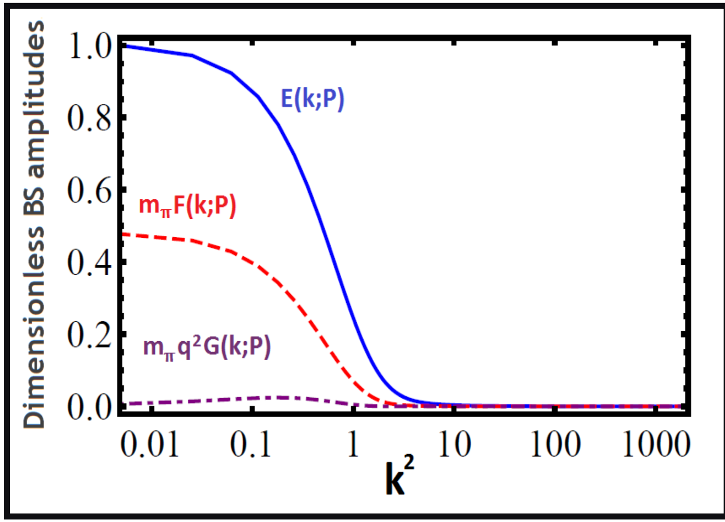
$$\Gamma_\pi(k,P) = \gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P G_\pi(k;P) + \sigma_{\mu\nu}k_\mu P_\nu H_\pi(k;P)\right]$$





$$S(p) = \frac{1}{i\gamma\cdot p\,A(p^2) + B(p^2)}$$

$$f_\pi E_\pi(k;P=0) = B(p^2)$$
$$F_R(k;0) + 2\,f_\pi F_\pi(k;0) = A(k^2)$$
$$G_R(k;0) + 2\,f_\pi G_\pi(k;0) = 2A'(k^2)$$
$$H_R(k;0) + 2\,f_\pi H_\pi(k;0) = 0$$



π and K: Probing Quarks with Photons

In studying the **elastic form factors**, it is the **photon** which probes the **dressed quarks** inside the **bound states**, highlighting the importance of the **quark-photon vertex**.

Gauge covariance:

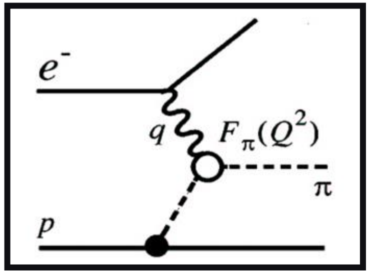
Ward Identities
Transverse Takahashi Identities
Landau-Khalatnikov-Fradkin Transformations

$$\Gamma^L_\mu(p,k,q) = \sum_{i=1}^{4} \lambda_i\, L^i_\mu(p,k)$$

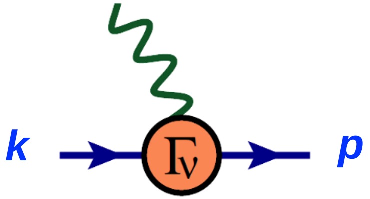
$$L^1_\mu = \gamma_\mu$$
$$L^2_\mu(p,k) = (\not p + \not k)(p+k)_\mu$$
$$L^3_\mu(p,k) = -(p+k)_\mu$$
$$L^4_\mu(p,k) = -\sigma_{\mu\nu}\,(p+k)^\nu$$

$$\Gamma^T_\mu(p,k,q) = \sum_{i=1}^{8} \tau_i(p^2,k^2,q^2) T^i_\mu(p,k)$$

$$T^1_\mu = p_\mu(k\cdot q) - k_\mu(p\cdot q)\,,$$
$$T^2_\mu = [p_\mu(k\cdot q) - k_\mu(p\cdot q)]\,(\not p + \not k)\,,$$
$$T^3_\mu = q^2\gamma_\mu - q_\mu\,\not q\,,$$
$$T^4_\mu = q^2\left[\gamma^\mu\,(\not k + \not p) - (k+p)^\mu\right] + 2(k-p)^\mu \sigma_{\nu\lambda}\, p^\nu k^\lambda\,,$$
$$T^5_\mu = -\sigma_{\mu\nu}\, q^\nu\,,$$
$$T^6_\mu = \gamma_\mu(p^2-k^2) + (p+k)_\mu\,\not q\,,$$
$$T^7_\mu = \tfrac{1}{2}(p^2-k^2)\left[\gamma_\mu(\not p + \not k) - (p+k)_\mu\right] - (p+k)_\mu \sigma_{\nu\lambda}\, p^\nu k^\lambda\,,$$
$$T^8_\mu = \gamma_\mu \sigma_{\nu\lambda}\, p^\nu k^\lambda - p_\mu\,\not k + k_\mu\,\not p\,.$$



Pion Form Factor



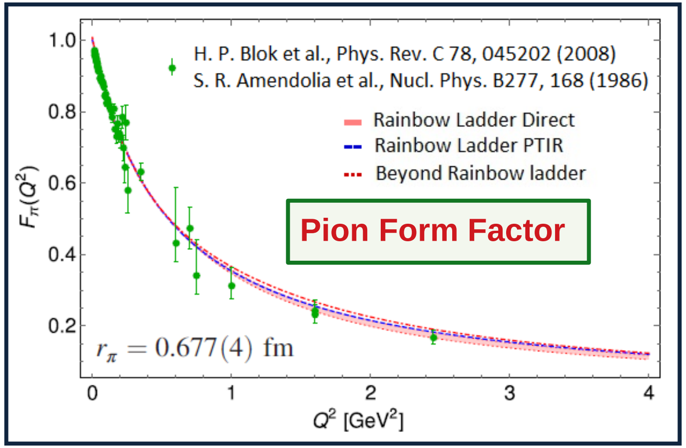
γ^μ k^μ p^μ

1 $\not k$ $\not p$ $\not k \not p$

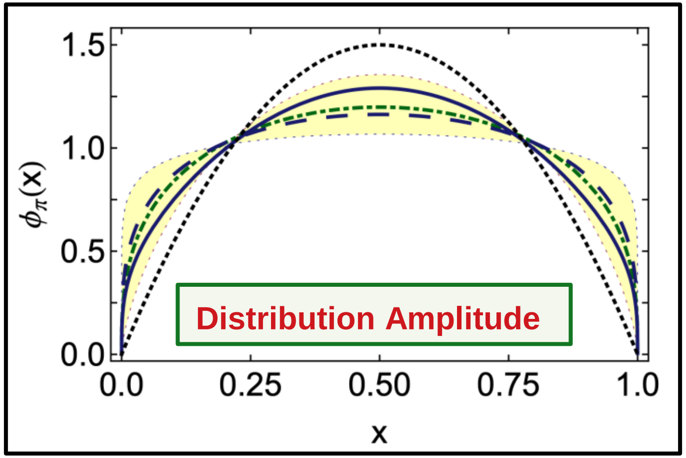
$$\Gamma^\mu(k,p) = \sum_{i=1}^{12} v_i(k,p) V^\mu_i\,,$$

$$V^\mu_1 = \gamma^\mu,\quad V^\mu_2 = k^\mu,\quad V^\mu_3 = p^\mu,$$
$$V^\mu_4 = \not k\gamma^\mu,\quad V^\mu_5 = \not k k^\mu,\quad V^\mu_6 = \not k p^\mu,$$
$$V^\mu_7 = \not p\gamma^\mu,\quad V^\mu_8 = \not p k^\mu,\quad V^\mu_9 = \not p p^\mu,$$
$$V^\mu_{10} = \not k\not p\gamma^\mu,\quad V^\mu_{11} = \not k\not p k^\mu,\quad V^\mu_{12} = \not k\not p p^\mu.$$

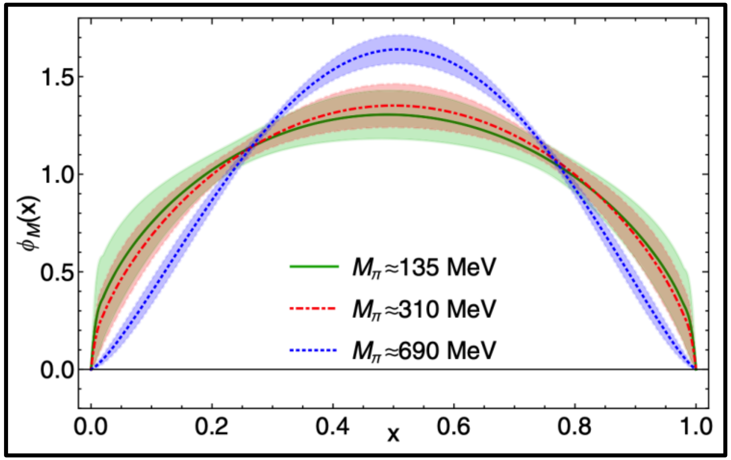
π Electromagnetic Form Factor



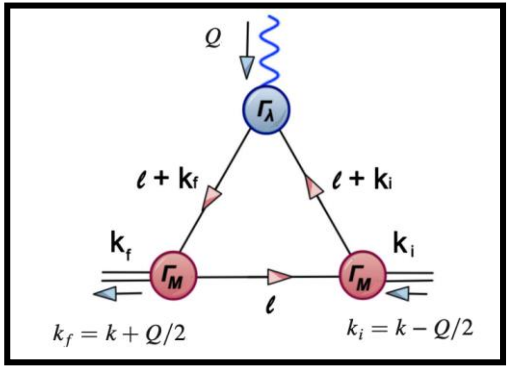
A. Miramontes et. al., Phys. Rev. D 105 (2022) 7, 074013

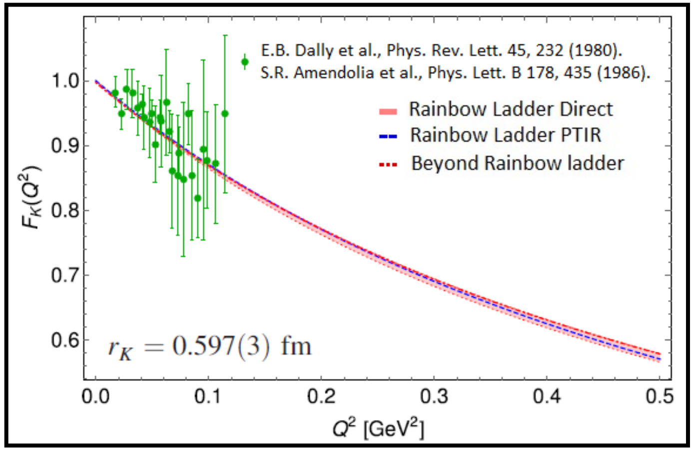


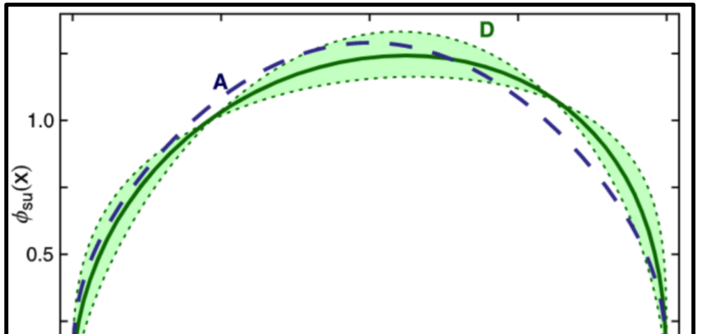
Phys. Rev. Lett. 111 (2013) 092001



Phys. Rev. D 102 (2020) 9, 094519







Phys. Lett. B 731 (2014) 13-18

π and K Form Factors: Probing the Standard Model

A muon with spin s has a **magnetic moment**: $\mu = g\frac{e}{2m}s$

The factor **g** is called the gyro-magnetic factor. The **Dirac equation** for a charged elementary fermion with spin 1/2 implies **g = 2**.

The **anomalous magnetic moment** is the deviation from g = 2, parameterized by $a_\mu = (g-2)/2$.

It appears due to radiative corrections. **Renormalization** of **QED** was established in 1943 and 1947-1948 by Tomonaga, Schwinger and Feynman.

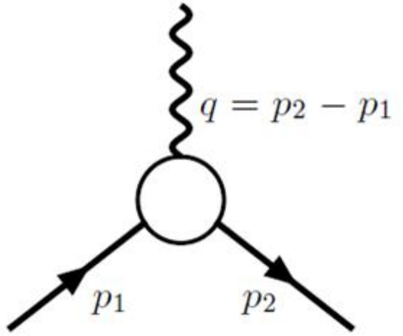
The leading contribution to a_μ, calculated by Schwinger in 1949, is: $a_\mu = \frac{\alpha}{2\pi}$

The **amplitude** of a muon scattering off an external electromagnetic field A is: ($q=p_2-p_1$):

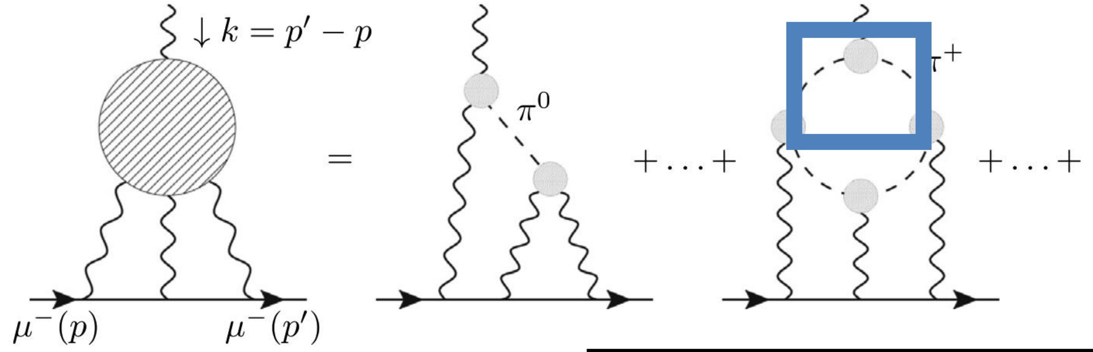
$$\mathcal{M} = -ie\langle\mu_{p2}|J^\mu(0)|\mu_{p1}\rangle A_\mu(q)$$

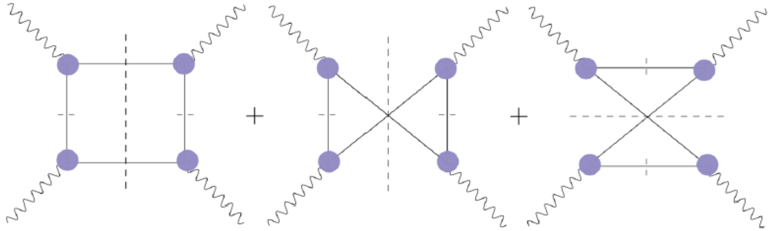
$$\langle\mu_{p2}|J^\mu(0)|\mu_{p1}\rangle = \bar{u}_{p2}\Gamma^\mu(p_2,p_1)u_{p1}$$

$$\Gamma^\mu(p_2,p_1) = \left[F_D(q^2)\gamma^\mu + F_P(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m}\right]$$

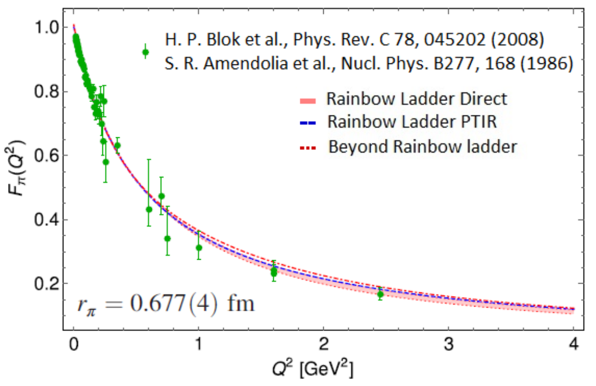


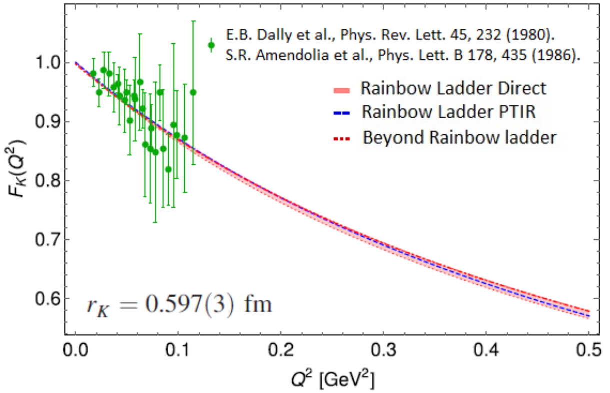
π and K Form Factors: Probing the Standard Model





Leading order **P**-box contributions to a_μ^{HLbL}, where the corresponding **P** meson EFFs are highlighted by the purple filled circles.





A. Miramontes, AB, K. Raya, P. Roig, Phys. Rev. D 105 (2022) 7, 074013

Radial excitations of π and **K**:

A. Miramontes, submitted to PRD

$$a_\mu^{\pi_1-\mathrm{box}} = (-3.2 \pm 0.6) \times 10^{-13}$$

$$a_\mu^{K_1-\mathrm{box}} = -6.8 \times 10^{-14}$$

$$a_\mu^{\pi^\pm-\mathrm{box}} = -(15.6 \pm 0.2) \times 10^{-11}$$

$$a_\mu^{K^\pm-\mathrm{box}} = -(0.48 \pm 0.02) \times 10^{-11}$$

Dispersive methods:

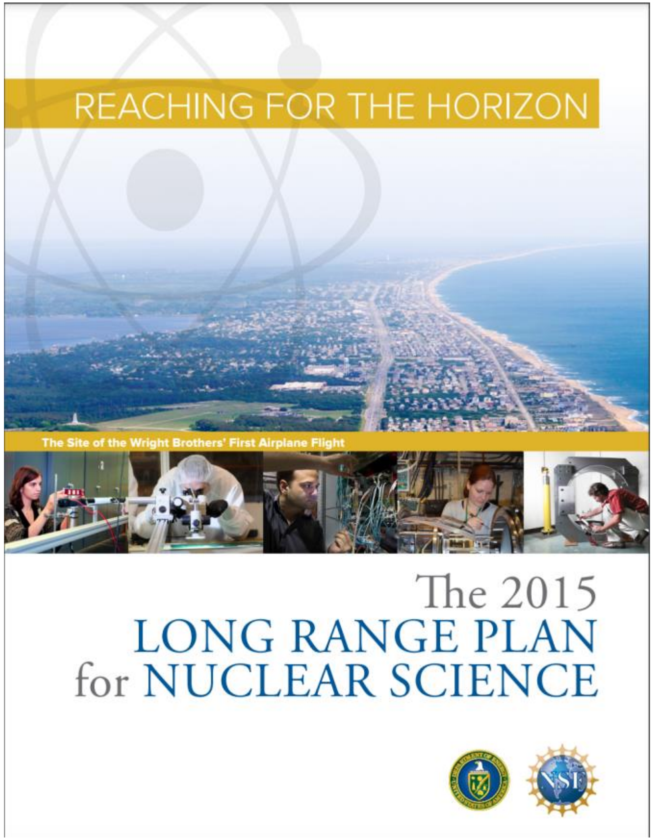
$$a_\mu^{\pi\text{-box}} = -15.9(2) \times 10^{-11}$$

$$a_\mu^{K^+\text{-box,VMD}} = -0.50 \times 10^{-11}$$

$$a_\mu^{K^+\text{-box,DSE}} = -0.48(2) \times 10^{-11}$$

Eichmann, et. al. Phys.Rev.D 101 (2020) 5, 054015

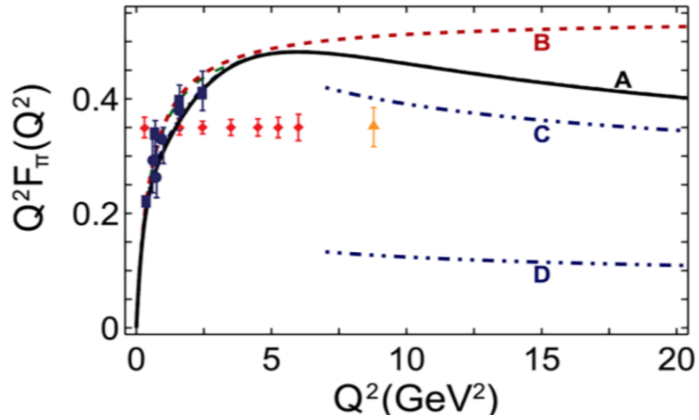
π and K Form Factor - JLab 12 GeV Upgrade

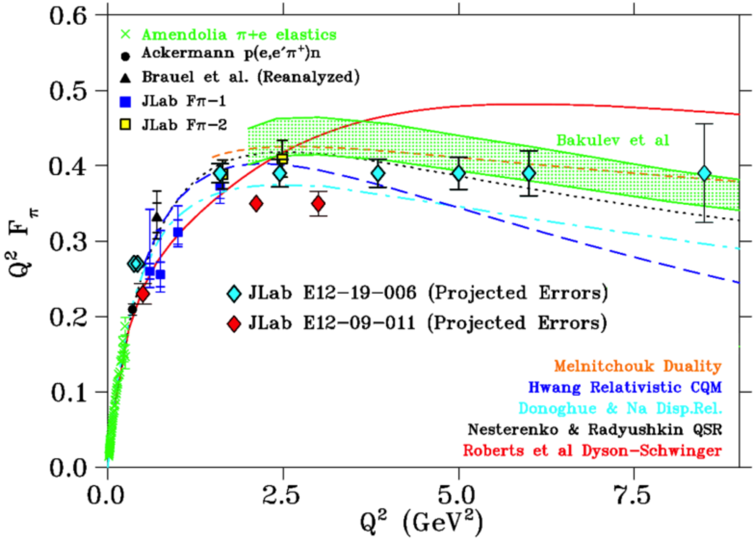


The study of the **pion form factor** is one of the **flagship goals** of the **JLab 12-GeV upgrade**… regime in which the phenomenology of **QCD** begins a **transition** from **large-** to **short-distance** scale behavior.

The **pion form factor** can potentially be measured till **Q^2 ~ 6-8** in the **12 GeV** upgrade of the **JLab**.

Courtesy Garth Huber

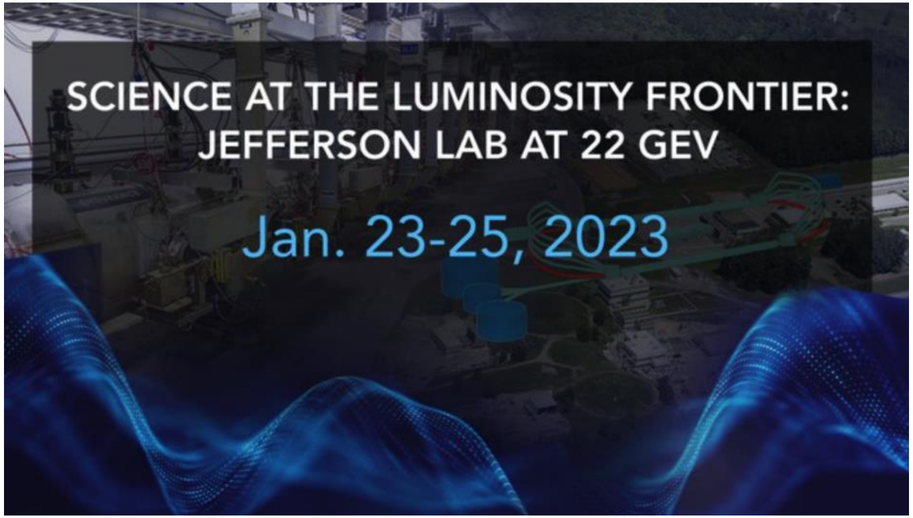


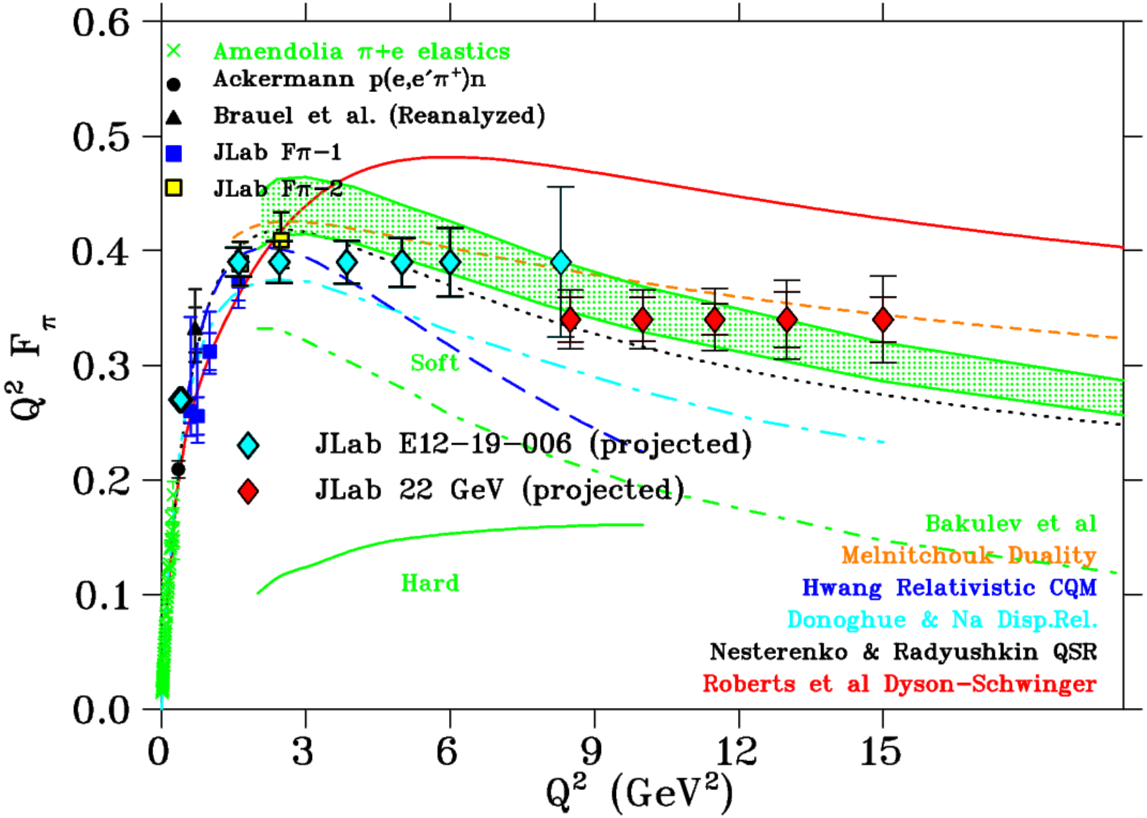


The **electromagnetic form factors** of **K** can be measured till 5 GeV^2 in the **12 GeV** upgrade of **JLab**

π and K Form Factor - JLab 22 GeV Upgrade

With a potential next **22-GeV** upgrade of the **JLab**, the **pion electromagnetic form factor** could be measured till **Q^2 ~ 15 GeV^2**.





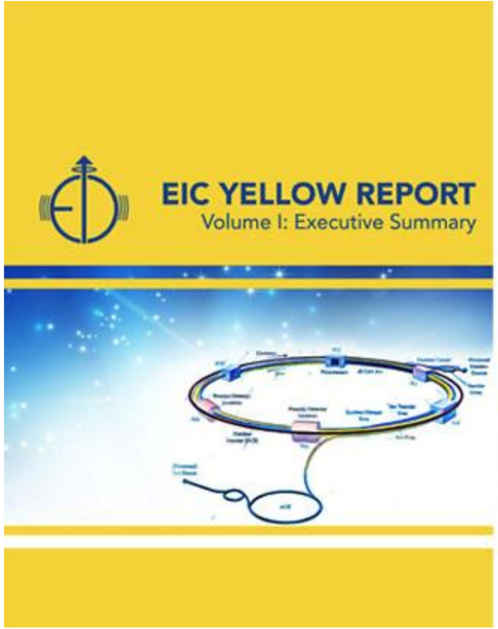
The **form factors** of **K** can be measured till 10 GeV^2 in the **22 GeV** upgrade of **JLab**

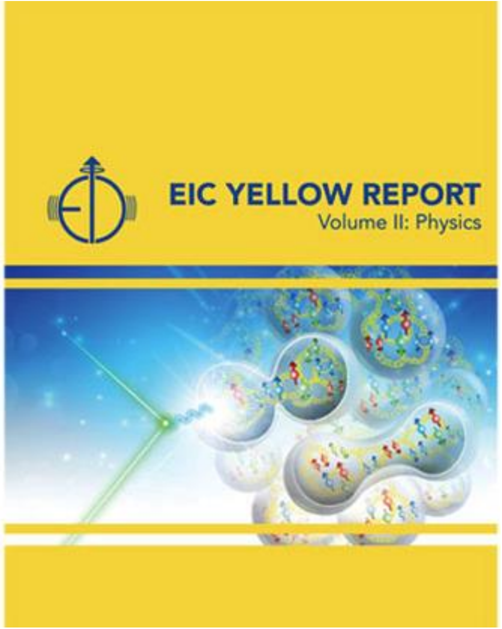
Courtesy Garth Huber

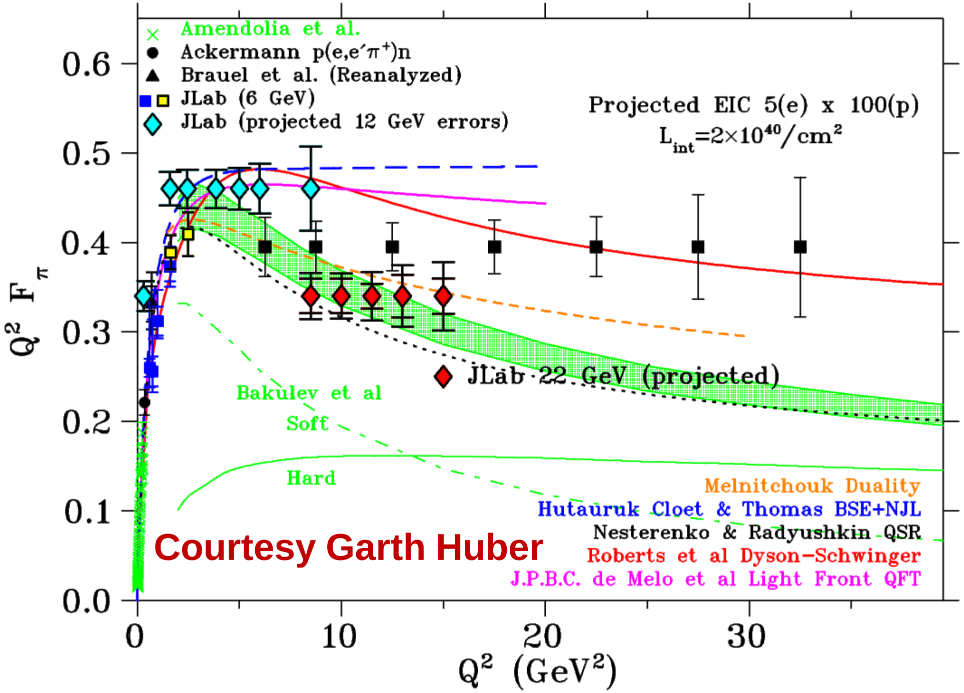
π and K Form Factor at Large Q^2 in EIC Era

Science Question: Can we get quantitative guidance on the **emergent pion mass** mechanism**?**
Key measurement: **Pion form factor** data for **Q^2: 10-40 GeV^2**.

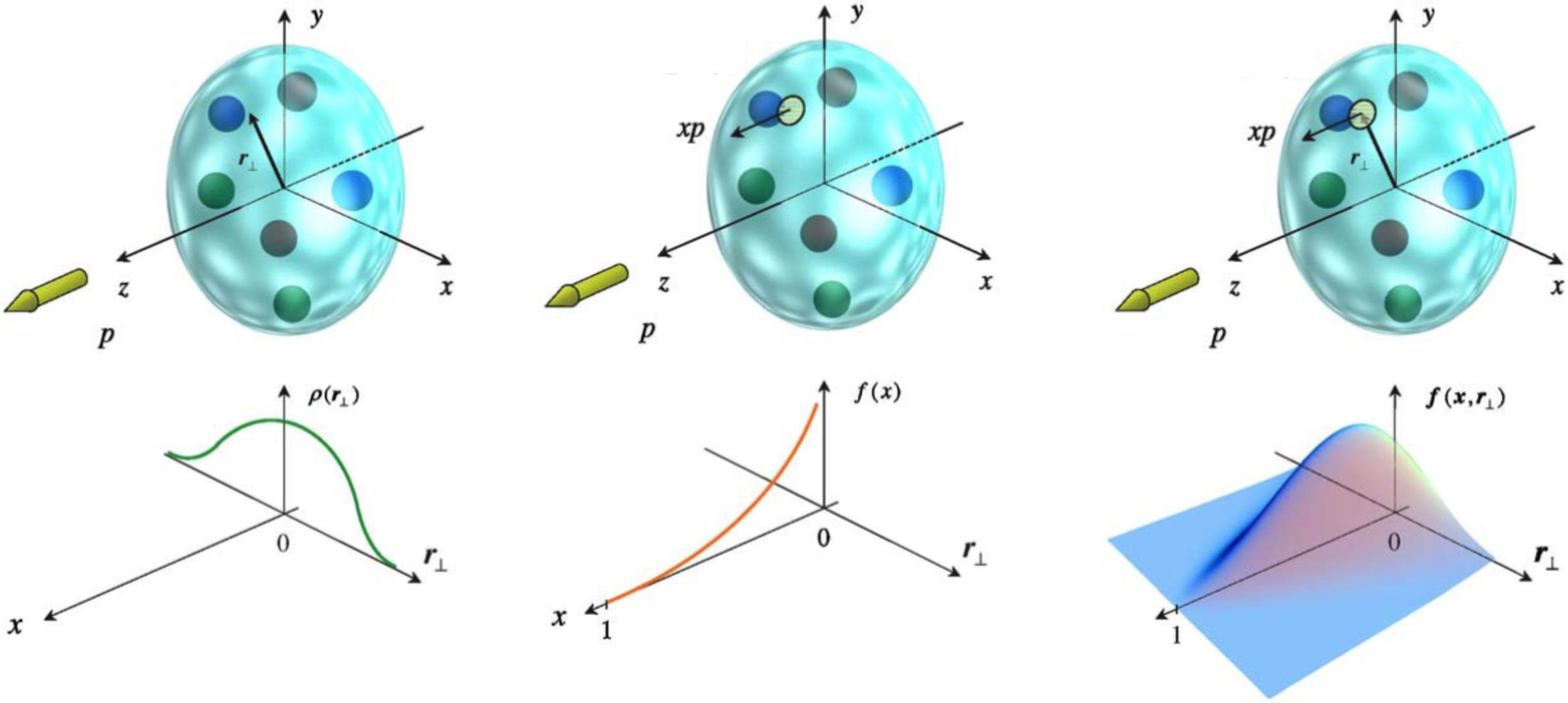
Science Question: How much interference is between emergent and Higgs mass mechanism?
Key measurement: **Kaon form factor** data for **Q^2: 10-20 GeV^2**.







π and K: Towards a 3-Dimensional Picture



Fourier transform of the elastic form factor

Parton Distribution

Skew-less Generalized Parton Distribution

Towards Algebraic Models

Sophisticated Truncations

Numerically Demanding
QCD Predictions

QCD Green Functions
Mesons – Static, FFs, PDAs, PDFs

Schwinger-Dyson Equations

Most Observables
Mesons, Baryons

Contact Interaction

Simple Implementation
Insightful - Mindful

Moderate Simplicity
QCD-like Predictions

Algebraic Models

LFWFs, GPDs, TMDs,
PDFs, FFs (spin-1)

The Algebraic Model (AM)

- It retains the **constant term** from original models, setting it to $\mathbf{M_q}$.
- There is a **term linear** in **w** with the coefficient $\mathbf{(M_h^2 - M_q^2)}$. For same **flavored quarks**, it ceases to contribute by construction.
- There is a **quadratic term w^2** with coefficient $\mathbf{m_M^2}$. The condition

$$|M_{\bar{h}} - M_q| \leq m_{\mathrm{M}} \leq M_{\bar{h}} + M_q$$

- It guarantees the **positivity** of

$$\Lambda^2(w)$$

The quark propagator:

$$S_{q(\bar{h})}(k) = [-i\gamma \cdot k + M_{q(\bar{h})}]\Delta(k^2, M^2_{q(\bar{h})})$$

$$\Delta(s,t) = (s+t)^{-1}$$

Bethe-Salpeter Amplitude:

$$n_{\mathrm{M}}\Gamma_{\mathrm{M}}(k,P) = i\gamma_5 \int_{-1}^{1} dw\, \rho_{\mathrm{M}}(w)[\hat{\Delta}(k_w^2, \Lambda_w^2)]^{\nu}$$

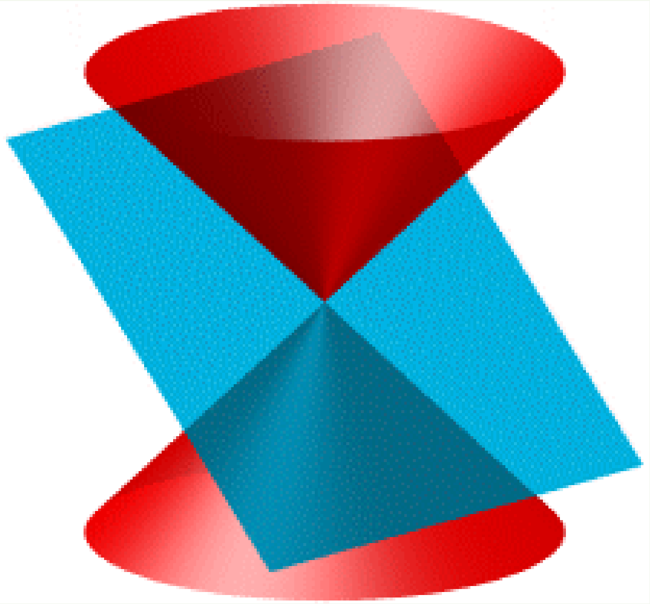
$$\hat{\Delta}(s,t) = t\Delta(s,t), \quad k_w = k + (w/2)P$$

$M_{q(\bar{h})}$ is constituent quark mass for a given flavor
n_{M} is a normalization constant
$\rho_{\mathrm{M}}(w)$ is a spectral density

$$\Lambda^2(w) = M_q^2 - \frac{1}{4}(1-w^2)m_{\mathrm{M}}^2 + \frac{1}{2}(1-w)(M_{\bar{h}}^2 - M_q^2)$$

The Light Front Wavefunction

For a quark in pseudo-scalar meson **M**, the **leading twist** (2-particle) **light front wave function**, ψ_M , can be obtained via the light front projection of the meson's **BSWF**.



Bethe-Salpeter Wavefunction:

$$\chi_M(k,P) = S_q(k+P/2)\Gamma_M(k,P)S_{\bar{h}}(k-P/2)$$

Light Front Wavefunction:

$$\psi_M^q(x,k_\perp^2) = \mathrm{tr}\int_{dk_\parallel} \delta(n\cdot k - x n\cdot P)\gamma_5\gamma\cdot n\chi_M(k-P/2,P)$$

$$n \text{ lightlike}, n^2=0 \text{ and } n\cdot P = -m_M$$

BSA:

$$f_M\phi_M^q(x) = \frac{1}{16\pi^3}\int d^2k_\perp \psi_M^q(x,k_\perp^2)$$

The Algebraic Model:

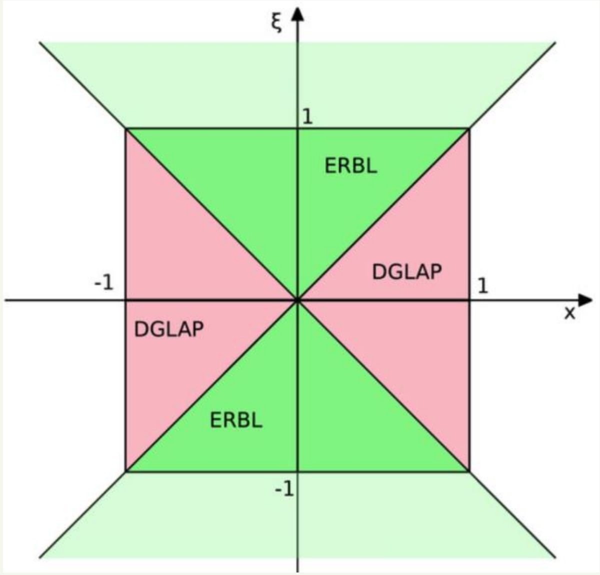
$$\psi_M^q(x,k_\perp^2) = 16\pi^2 f_M \frac{\nu\Lambda_{1-2x}^{2\nu}}{(k_\perp^2+\Lambda_{1-2x}^2)^{\nu+1}}\phi_M^q(x)$$

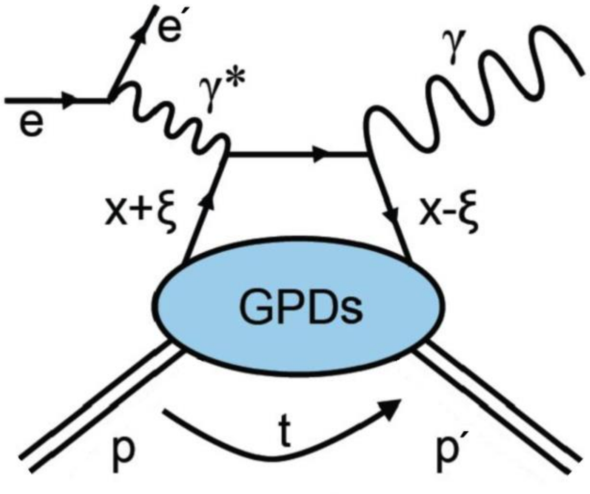
The GPDs from the overlap representation

Valence quark **GPD** from the overlap representation of the **LFWF**.

Both x and ξ have support on [−1,1].

The overlap representation is only valid in the **DGLAP** region, **|x|>ξ**.





$$P = (p + p')/2$$

$$-t = \Delta^2 = (p - p')^2$$

$$x = \frac{n \cdot k}{n \cdot P}$$

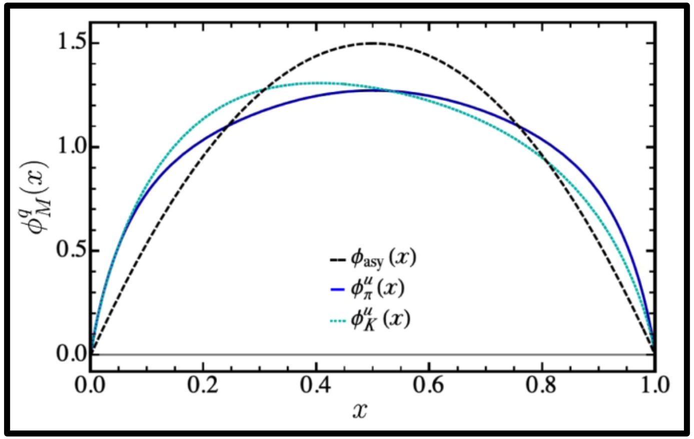
$$\xi = -\frac{n \cdot \Delta}{2n \cdot P}$$

$$H^q_{\mathrm{M}}(x, \xi, t) = \int \frac{d^2 k_\perp}{16\pi^3} \psi^{q*}_{\mathrm{M}}(x^-, (\mathbf{k}^-_\perp)^2) \psi^q_{\mathrm{M}}(x^+, (\mathbf{k}^+_\perp)^2)$$

$$x^\pm = \frac{x \pm \xi}{1 \pm \xi} \qquad \mathbf{k}^\pm_\perp = k_\perp \mp \frac{\Delta_\perp}{2} \frac{1 - x}{1 \pm \xi}$$

$$\Delta^2_\perp = \Delta^2(1 - \xi^2) - 4\xi^2 m^2_{\mathrm{M}}$$

From the PDAs to the LFWFs



π and K PDAs

Z.-F. Cui, et. al.,
Eur. Phys. J. C 80, 1064 (2020).

$$(\bar{x} = 1 - x)$$

$$\phi_\pi^u(x) = 20.226\, x\bar{x}\,[1 - 2.509\sqrt{x\bar{x}} + 2.025 x\bar{x}]$$

$$\phi_K^u(x) = 18.04\, x\bar{x}\,[1 + 5x^{0.032}\bar{x}^{0.024} - 5.97x^{0.064}\bar{x}^{0.048}]$$

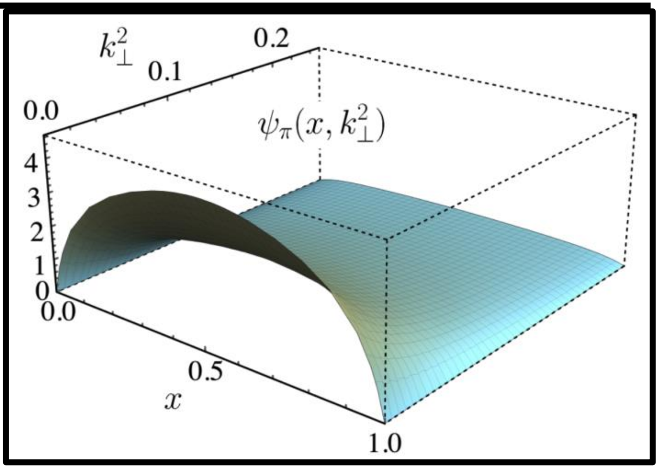
Drawing upon accumulated information on the **PDAs** of **π** and **K**, we parameterize them as corrections to the asymptotic PDA form on the hadronic scale.

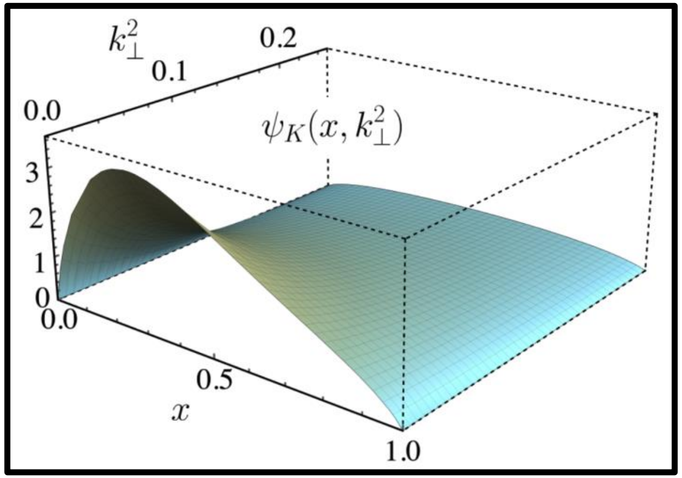
LFWF $\psi_{\rm M}^q(x, k_\perp^2)$

↓

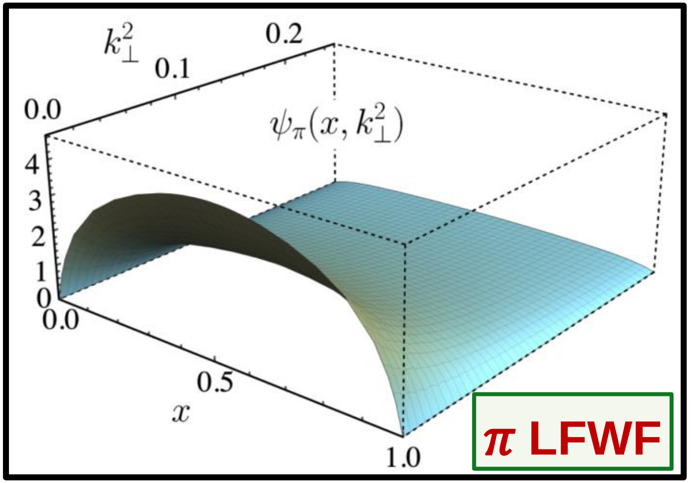
PDA

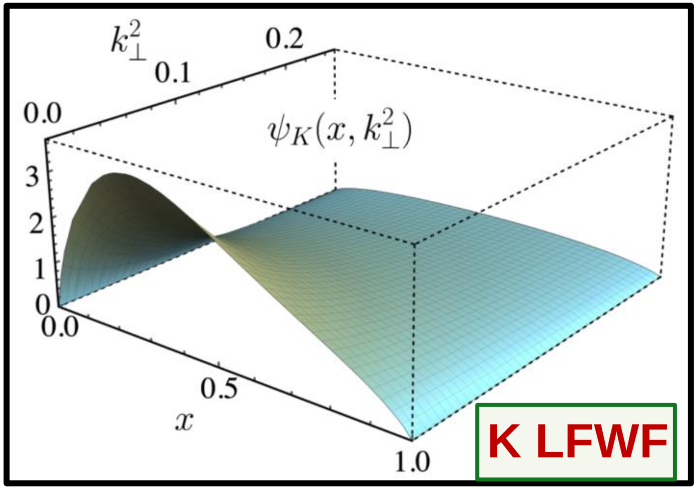
$$\frac{16\pi^2 f_{\rm M}\, \nu \Lambda_{1-2x}^{2\nu}}{(k_\perp^2 + \Lambda_{1-2x}^2)^{\nu+1}}\, \phi_{\rm M}^q(x)$$





From the LFWFs to the GPDs



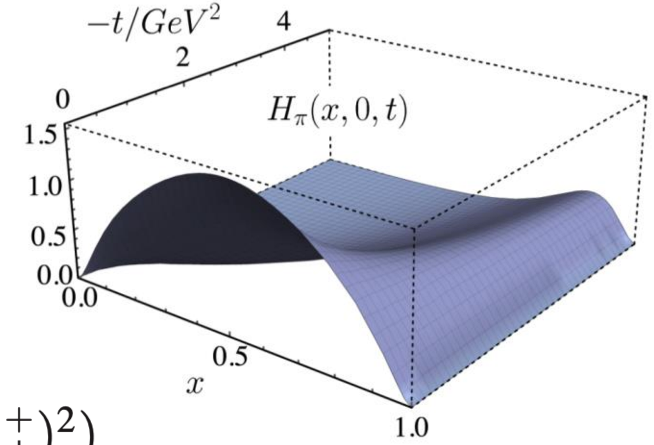


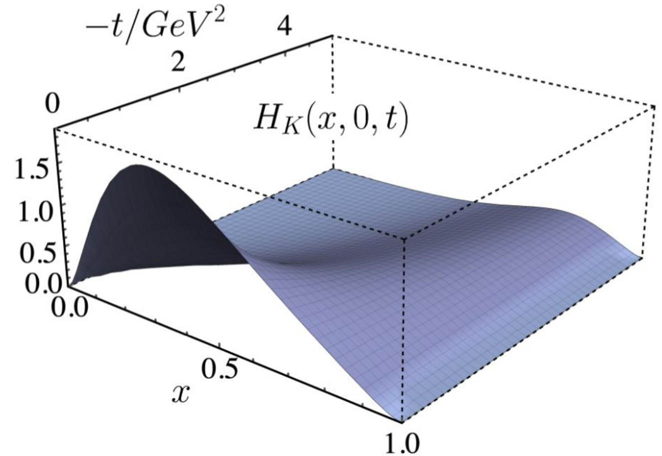
$$H_{\mathrm{M}}^{q}(x, \xi, t)$$

$$\int \frac{d^2 k_\perp}{16\pi^3} \psi_{\mathrm{M}}^{q*}(x^-, (\mathbf{k}_\perp^-)^2) \psi_{\mathrm{M}}^{q}(x^+, (\mathbf{k}_\perp^+)^2)$$

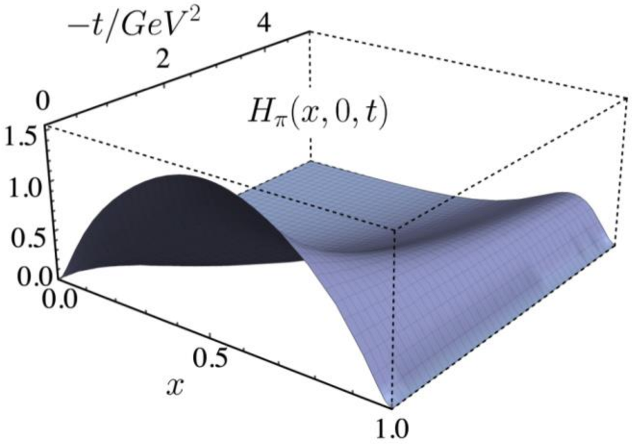
Overlap Representation of the GPDS

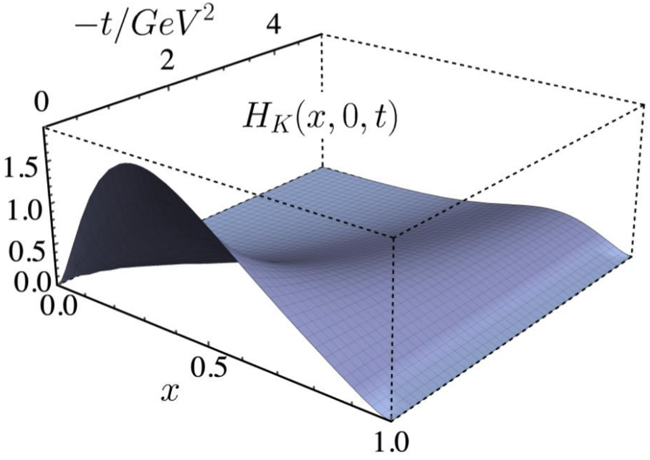
L. Albino, M. Higuera, K. Raya, AB
Phys. Rev. D 106 (2022) 3, 034003





From the GPDs to the PDFs

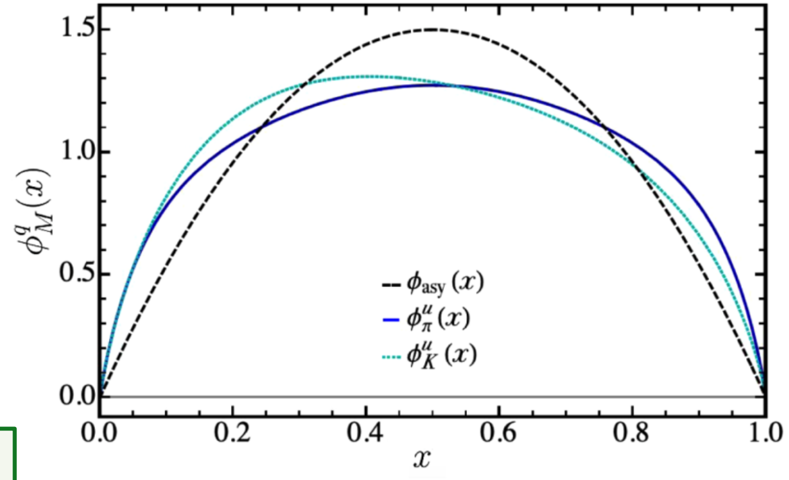


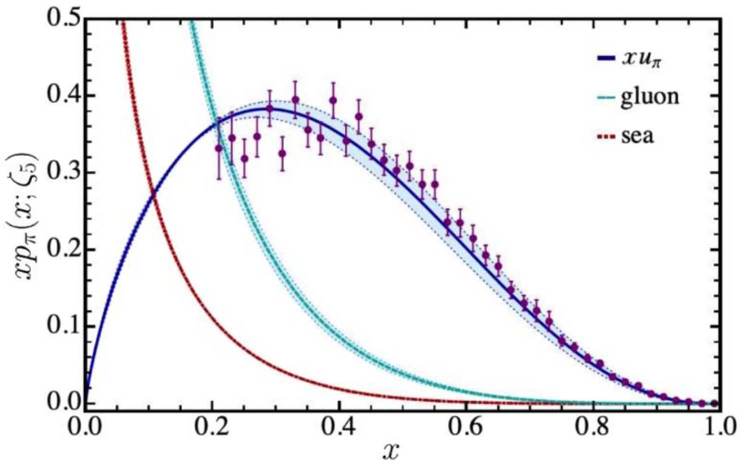


$$q_{\mathrm{M}}(x) \equiv H^q_{\mathrm{M}}(x,0,0)$$

DGLAP Evolution Equations

L. Albino, M. Higuera, K. Raya, AB
Phys. Rev. D 106 (2022) 3, 034003

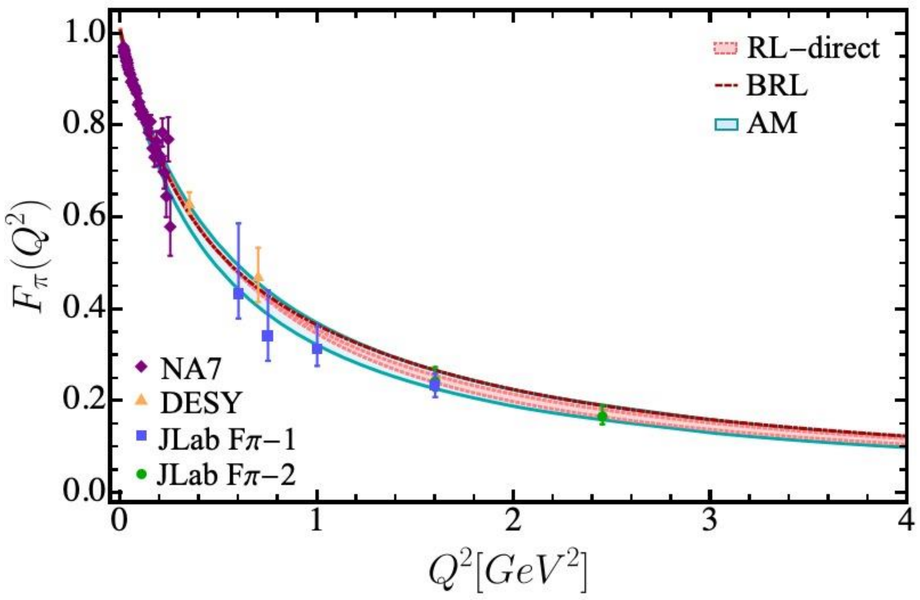


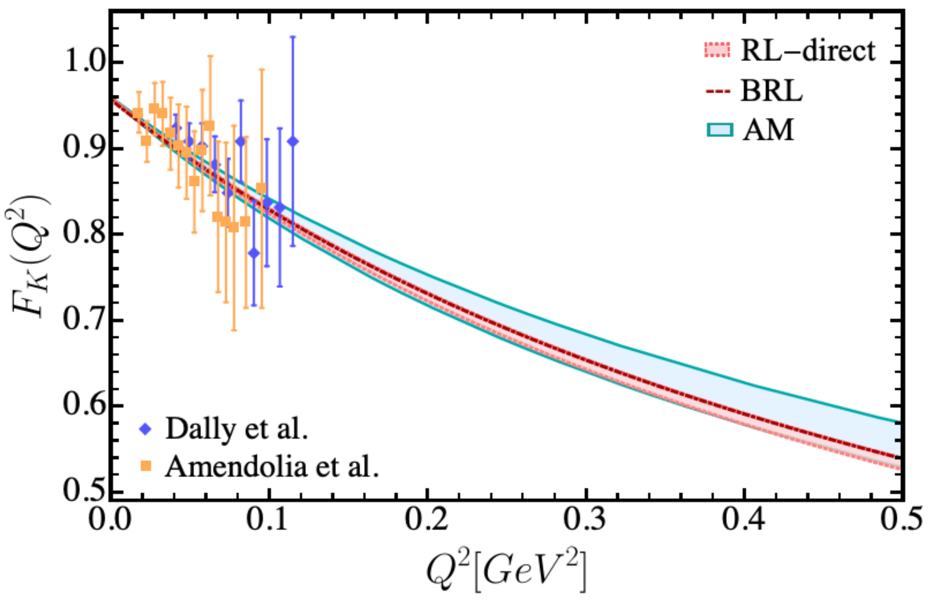


Completing the Cycle – Back to Form Factors

The **electromagnetic form factors** using our **algebraic model** can be obtained either through the knowledge of the **GPDs** or the direct evaluation of the **triangle diagram**.

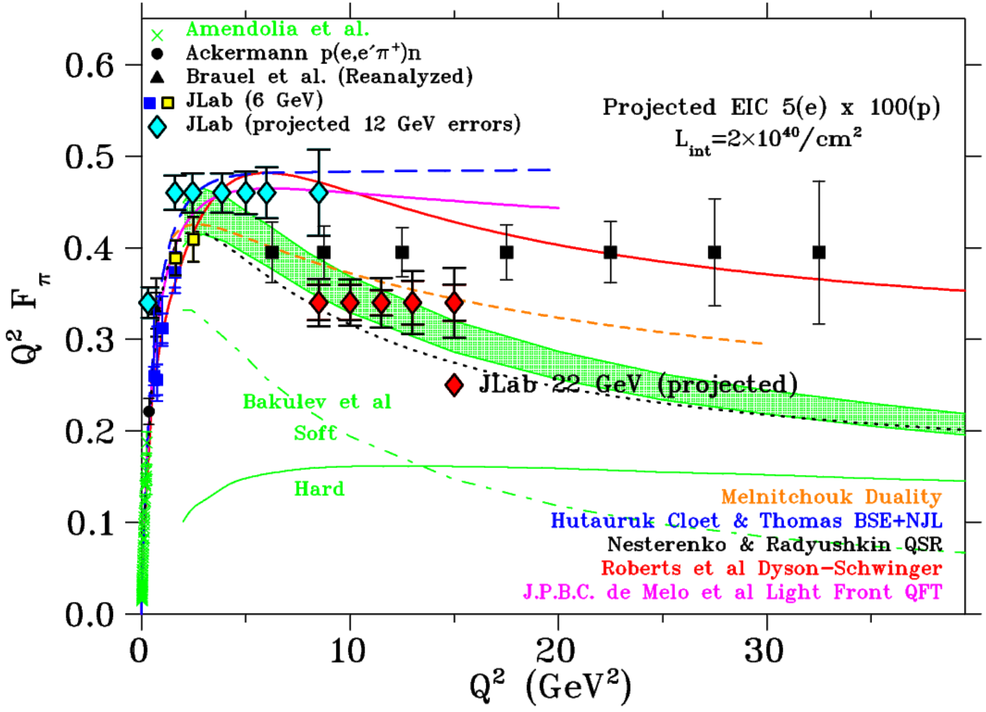
Such an exercise provides stringent constraints on the efficacy of the **algebraic model** we have constructed by direct comparison with the refined calculation of these **form factors.**

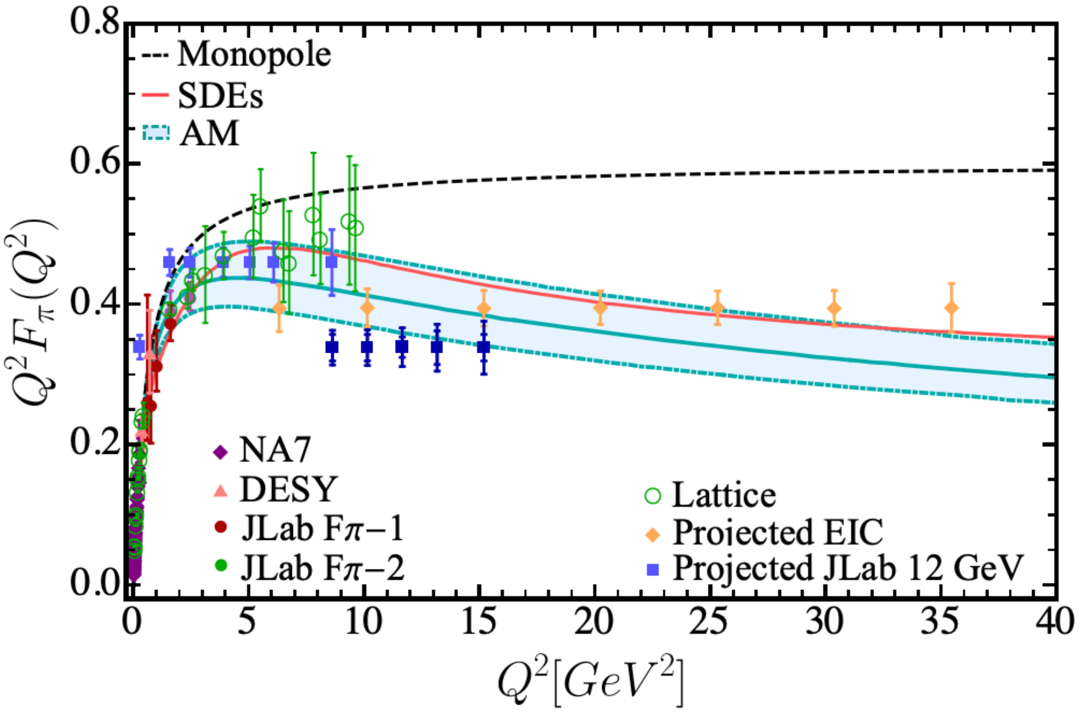




Completing the Cycle – Back to Form Factors

We can extend this analysis of the **Algebraic Model** to compute the **pion electromagnetic form factors** to larger **Q^2** range: **0-40 GeV^2** which would likely cover the photon virtualities accessible to the **JLab12**, **JLab22** and **EIC programs**:

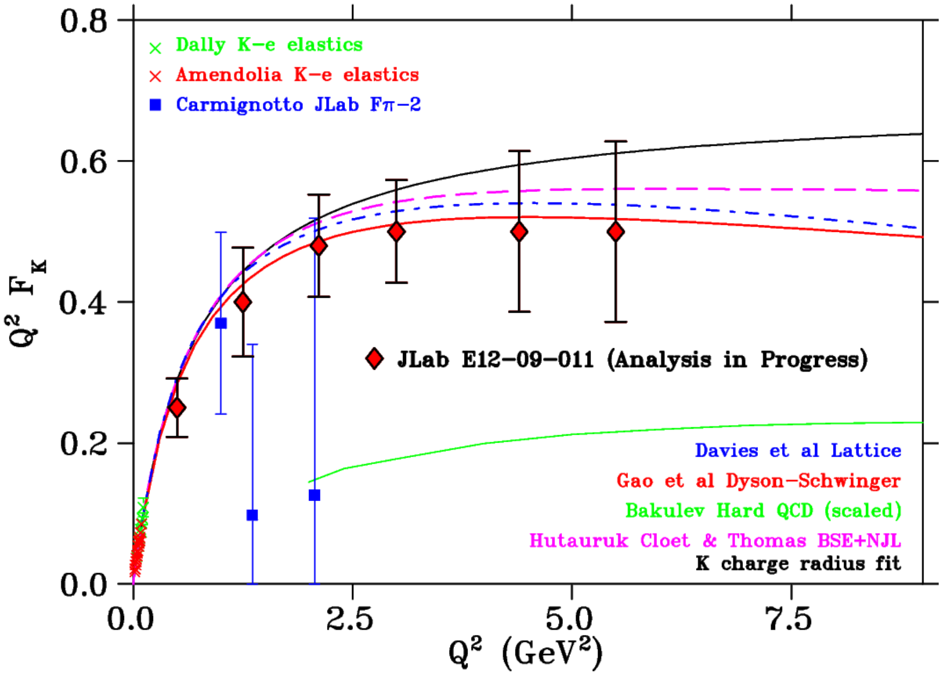




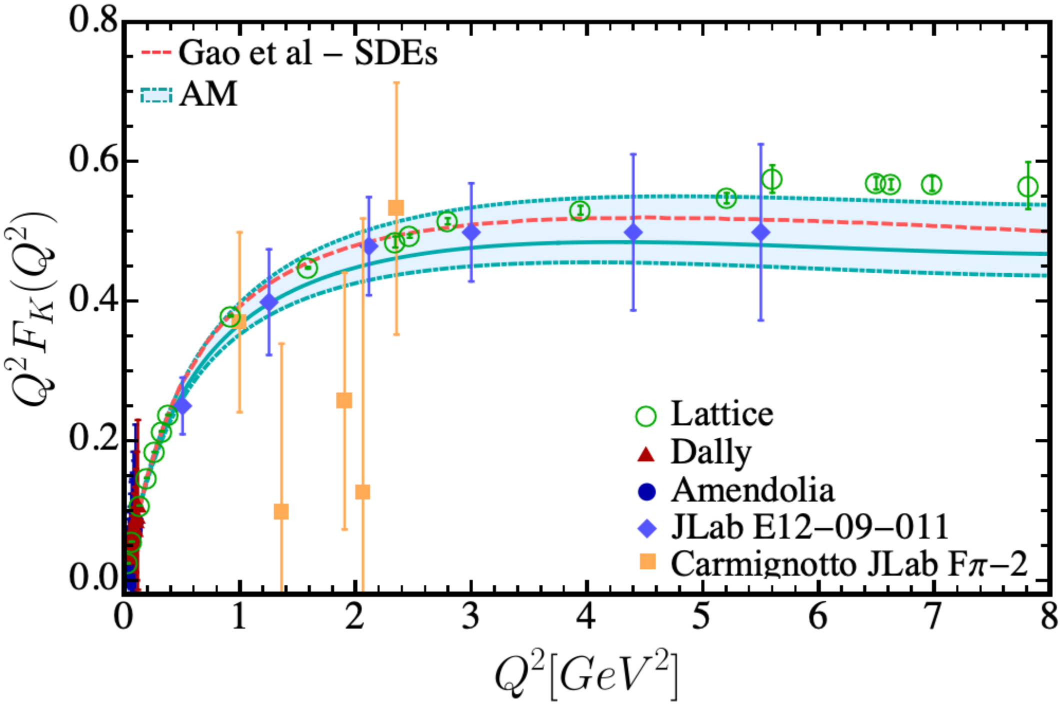
Completing the Cycle – Back to Form Factors

There is an analysis underway of the **kaon electromagnetic form factor** till **5.5 GeV²** of the data obtained in **JLab E12-09-011** experiment.

Courtesy Garth Huber



Algebraic Model results



Summary and Outlook

- The interplay of **QCD akin** truncations of **Schwinger-Dyson equations** and **algebraic model** based upon these studies shed important light on the **internal structure** of **pion** and **kaon**.

- **QCD akin** refined computation of **pion** and **kaon electromagnetic form factors** at low and intermediate virtualities of the probing photon in electroproduction processes:

A. Miramontes AB, K. Raya, P. Roig, Phys. Rev. D 105 (2022) 7, 074013
L. Chang, I.C. Cloët, C.D. Roberts, S.M. Schmidt, P.C. Tandy, Phys. Rev. Lett. 111 (2013) 14, 141802

- Results for the **pion electromagnetic form factor** at large photon virtualities accessible to the potential **22GeV upgrade** of the **JLab** and **EIC** are also available:

L. Chang, I.C. Cloët, C.D. Roberts, S.M. Schmidt, P.C. Tandy, Phys. Rev. Lett. 111 (2013) 14, 141802
J. Arrington, et al. (Feb 23, 2021, J.Phys. G 48 (2021) 7, 075106

- More recently, **pion** and **kaon form factors** have been computed in the the **time-like region**

A.S. Miramontes, H. Sanchis Alepuz, R. Alkofer, Phys. Rev. D 103 (2021) 11, 116006
A.S. Miramontes, AB, Phys. Rev. D 107 (2023) 1, 014016

Summary and Outlook

- Carefully constructed **Algebraic Models** can enable computation of the **GPDs**, **PDFs** and **EFF** with relative ease which is reminiscent of a **contact interaction** while mimicking the reliability of **QCD akin** refined truncations of **Schwinger-Dyson equations**.

L. Albino, M. Higuera, K. Raya, AB Phys. Rev. D 106 (2022) 3, 034003

- Despite these encouraging results and synergy with experimental endeavors at **JLab** and **EIC**, further improvements and extensions in the **continuum QCD approach** are desirable.

- Deeper research into the theoretical foundations of the truncations involved at the level of the **Green functions** of the fundamental degrees of freedom, i.e., **quarks**, **gluons**, as well as **quark-gluon** and **gluon-gluon** interactions continues vigorously.

- **Schwinger-Dyson equations** have also been of substantial success in the studies of **baryons** such as the **transition form factors** of **nucleon** to its **excited states** which is a hallmark of **CLAS**, **CLAS12** and **CLAS22** programs at **JLab** and hold the promise to offer a reliable tool for the future **JLab** and **EIC era** research into the heart of **hadronic matter**.

Thank you for your attention