EIC Accelerator Physics – Part 1

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CFNS School
6 June 2024

Brookhaven National Laboratory

U.S. DEPARTMENT OF ENERGY

EIC:

@BrookhavenLab

About me – Sergei Nagaitsev

EIC Technical Director, Brookhaven National Lab

Jefferson Lab Professor, ODU Department of Physics (https://fs.wp.odu.edu/snagaits/)

Recent professional experience:

2023 – 2024	EIC Lead Scientist, Jefferson Lab
2018 – 2022	Head of Accelerator Science Programs, Fermilab
2015 – 2022	Professor Part-time, The University of Chicago – Accelerator Physics
2013 – 2018	Chief Accelerator Officer, Fermilab – Accelerator Division

Community service:

2020	Chair, APS DPB
2021 – 2022	Member, HEP Snowmass Study Steering Group

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Outline (Part 1)

Accelerators – engines for discovery

EIC overview

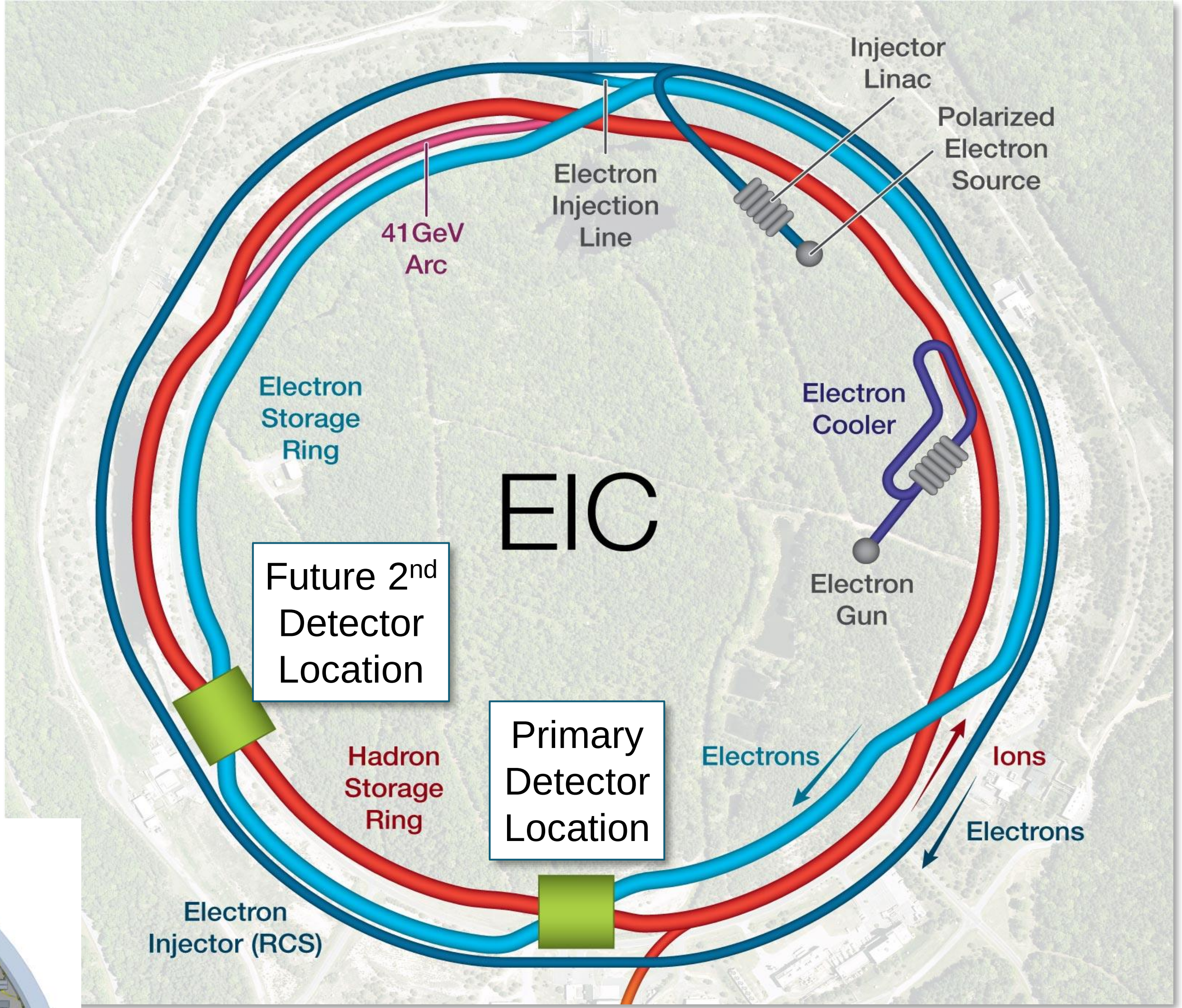
- From an accelerator physicist ☺

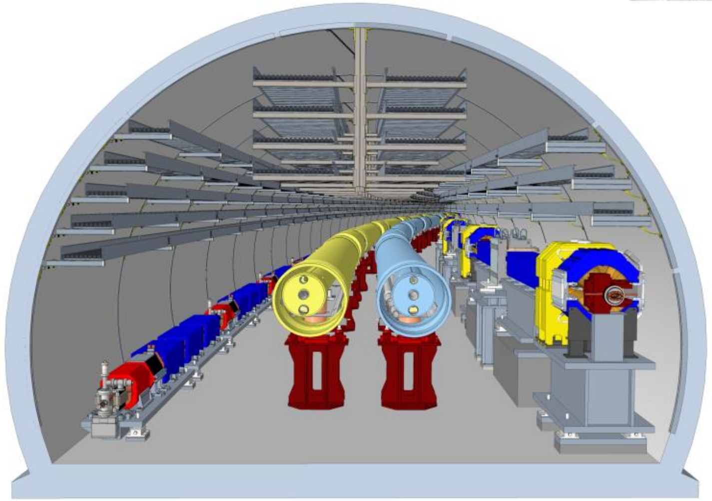
Relevant accelerator physics

- Synchrotrons
- (Brief) beam dynamics
- Phase space, emittance, “beta”

Collider interaction regions

- ”Low-beta” squeeze





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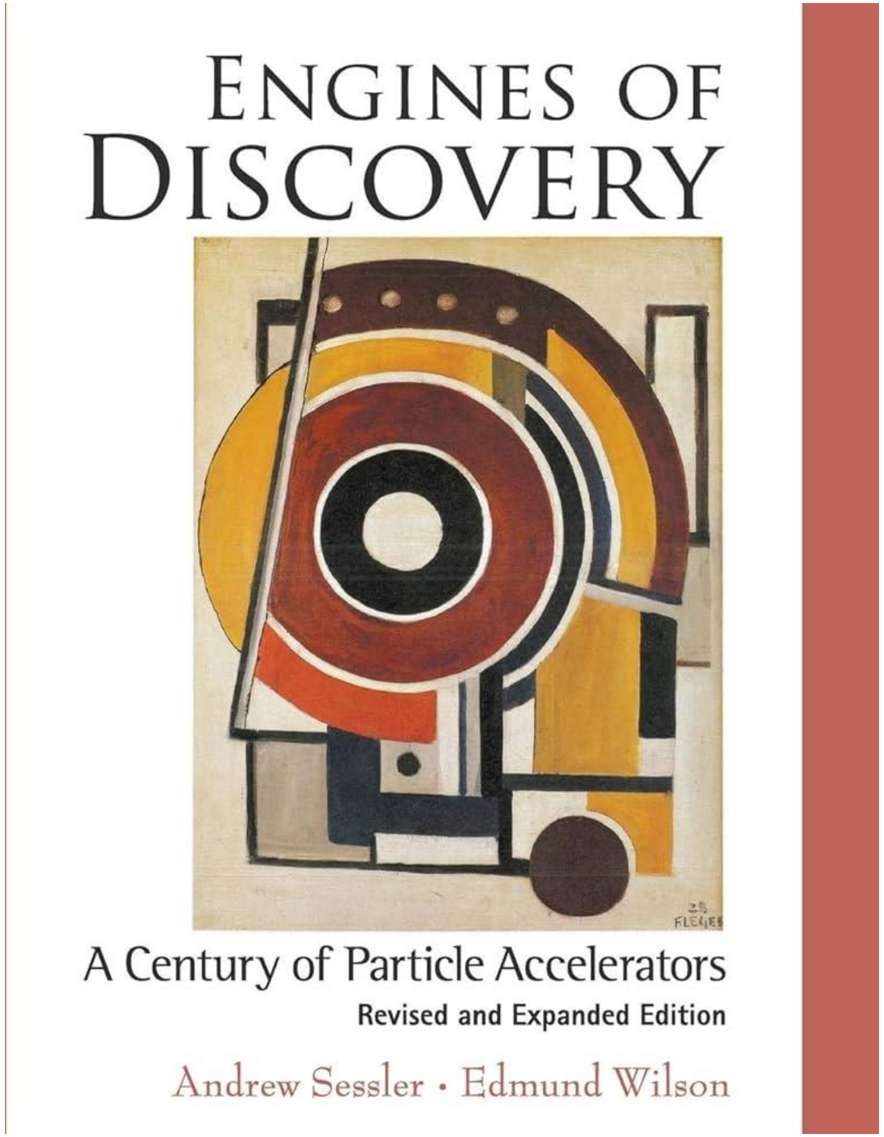
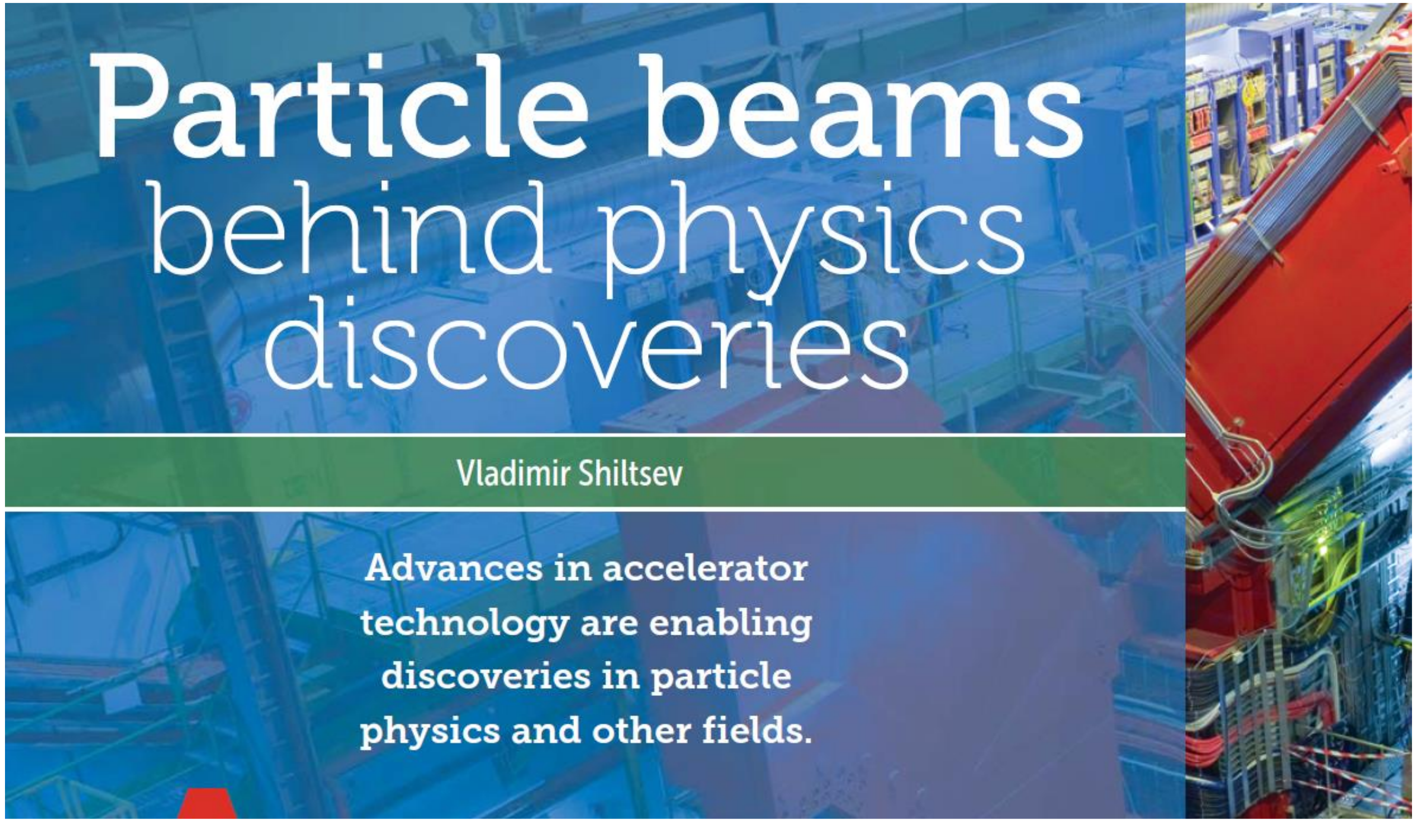
Engines of Discovery





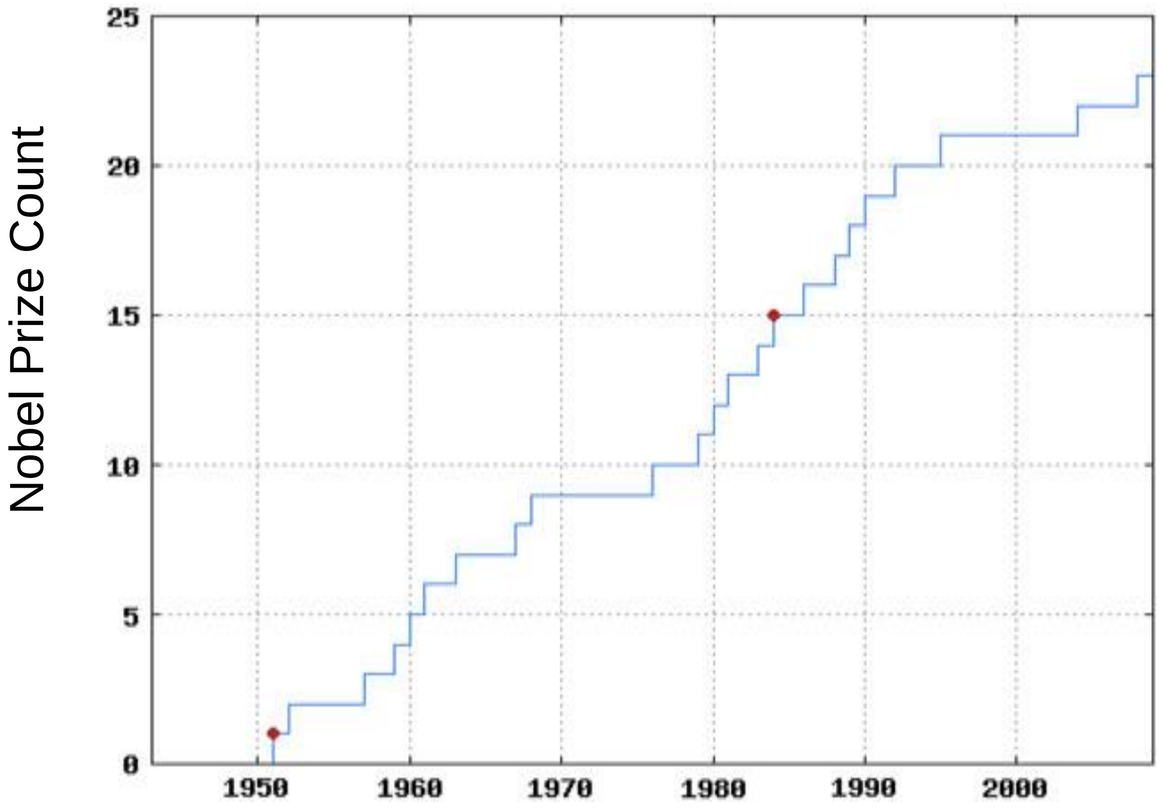
Physics Today **73**, 4, 32 (2020);

Nobel Prize Accounting: A high-level view on the influence of accelerators on physics

Reproduced from "The Influence of Accelerator Science on Physics Research," Haussecker and Chao

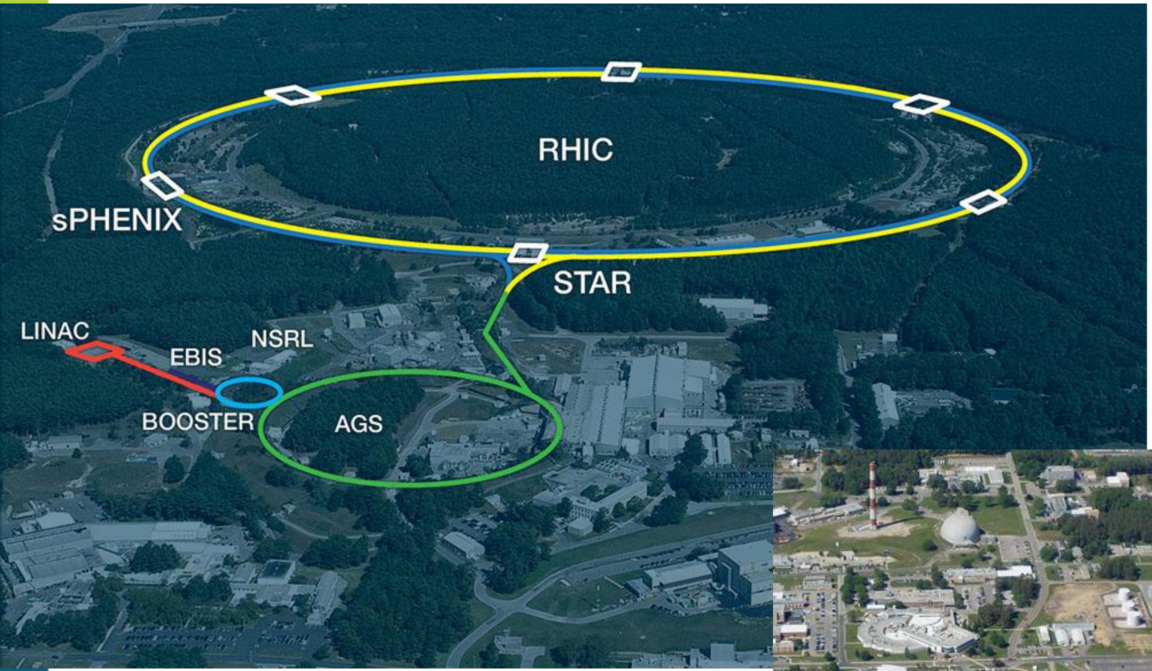
Impact of accelerators on physics was measured through Nobel Prizes:

- Criterion: Nobel awardee must have authored document citing accelerator contribution to work
- Nobel's directly influenced by Accelerators by 2009: 24
- Average frequency of contribution of accelerators to Nobel in Physics: ~3 years



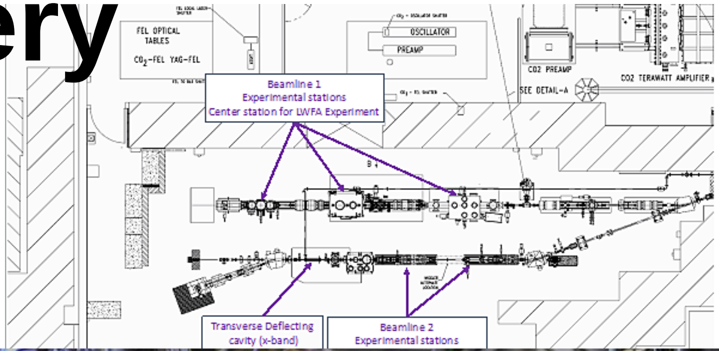
Count for Nobel Prize-winning researches or discoveries in physics from 1943 to 2009. (Haussecker and Chao, Phys Perspect. 13 (2011), p146)

Accelerators: Engines of Discovery



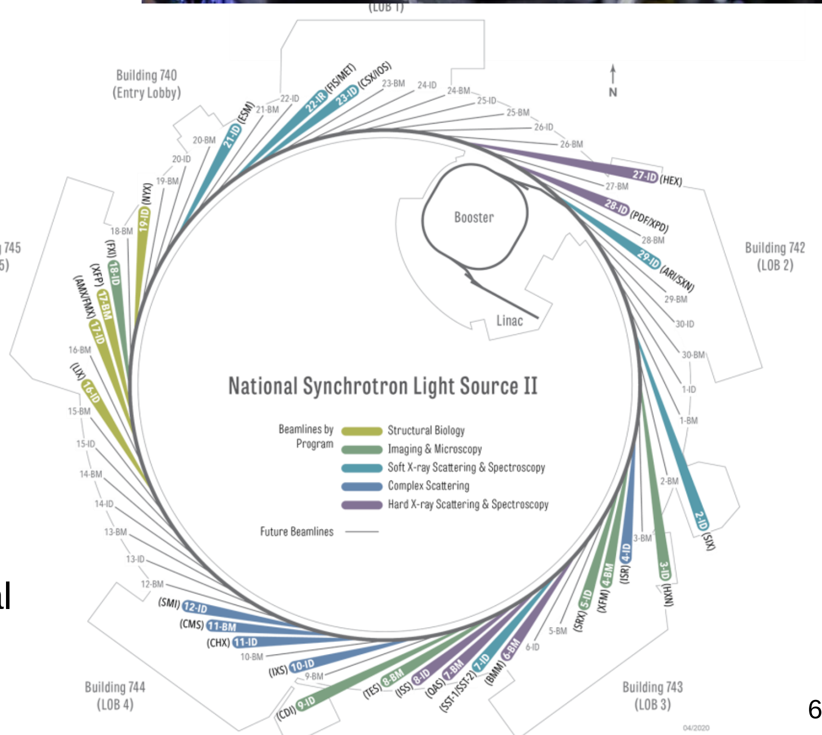
RHIC

A DOE User Facility to probe the fundamental properties of nuclear matter and to study what the universe may have looked like in the first few moments after its creation.



A DOE User Facility for Advanced Accelerator & Laser Research

A DOE User Facility to probe the fundamental properties of matter, paving the way to new scientific discoveries and innovations.



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Accelerators are instruments for providing beams of charged particles – a highly-directed form of energy – to be used as probes and tools

Beams: e-,e+, p, pbar, ions, muons

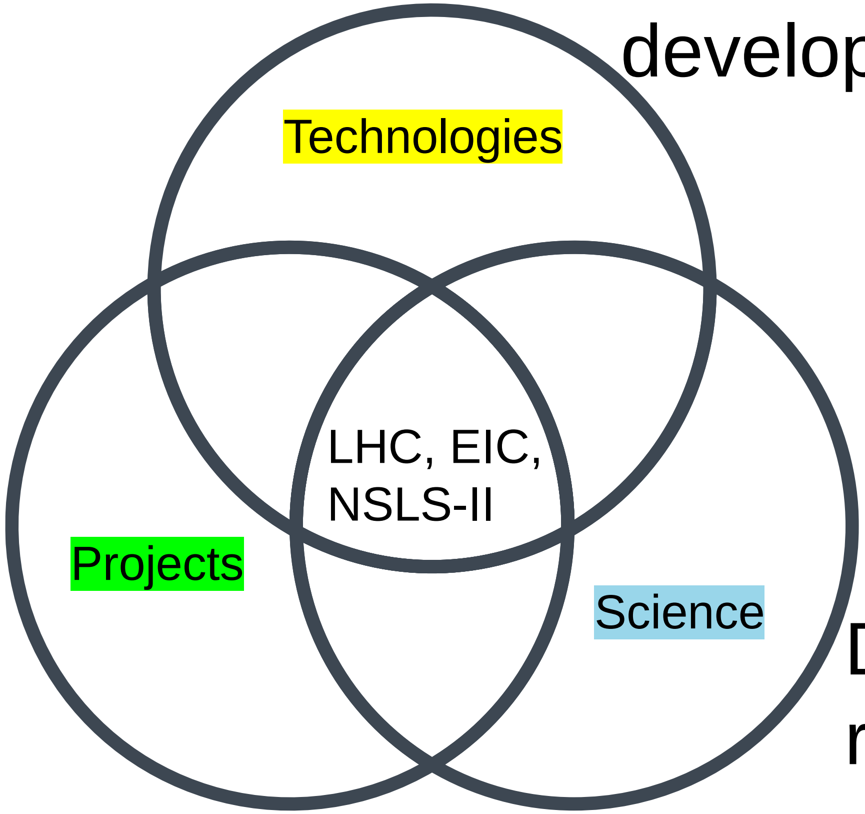
N ~ 1 to 10^{14}

$$\sqrt{\langle p_{\perp}^{2} \rangle} \sim 10^{-4} \times \langle p_{\parallel} \rangle$$

$$\Delta p_{\parallel} \sim 10^{-4} \times \langle p_{\parallel} \rangle$$

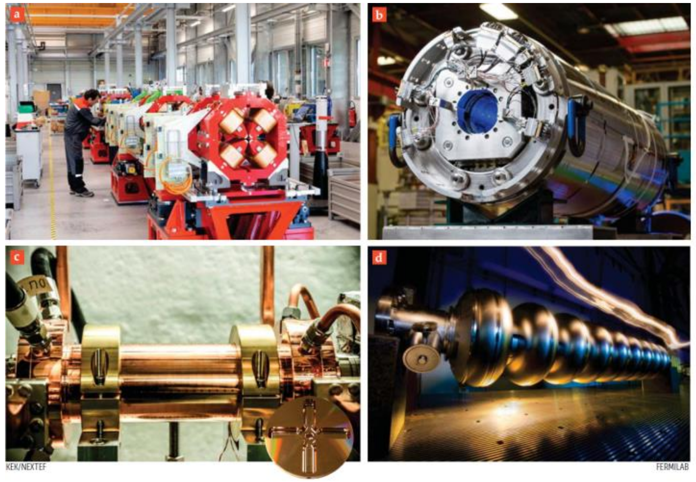
Inventions, development

Vision, concept, integration, optimization, construction



Discoveries, research

Core Accelerator Technologies



Particle beams behind physics discoveries
Physics Today 73, 4, 32 (2020)

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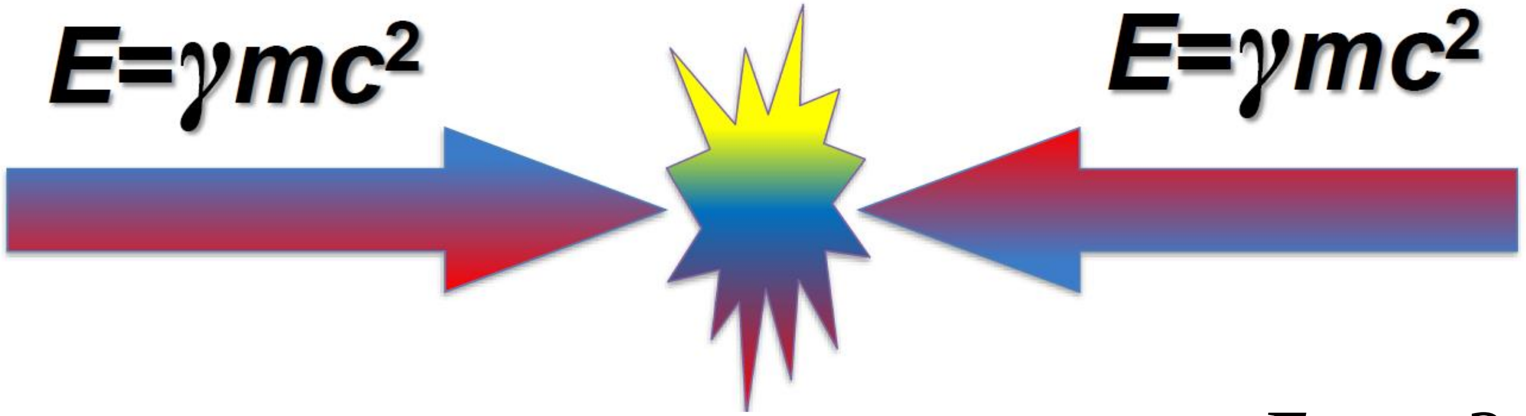
Colliders: 31 colliders built since 1959



Center-of-mass energy is E_{CM}= 2*E* – available for *'new physics'*

$$E_{CM} \approx 2\sqrt{E_1 E_2}$$

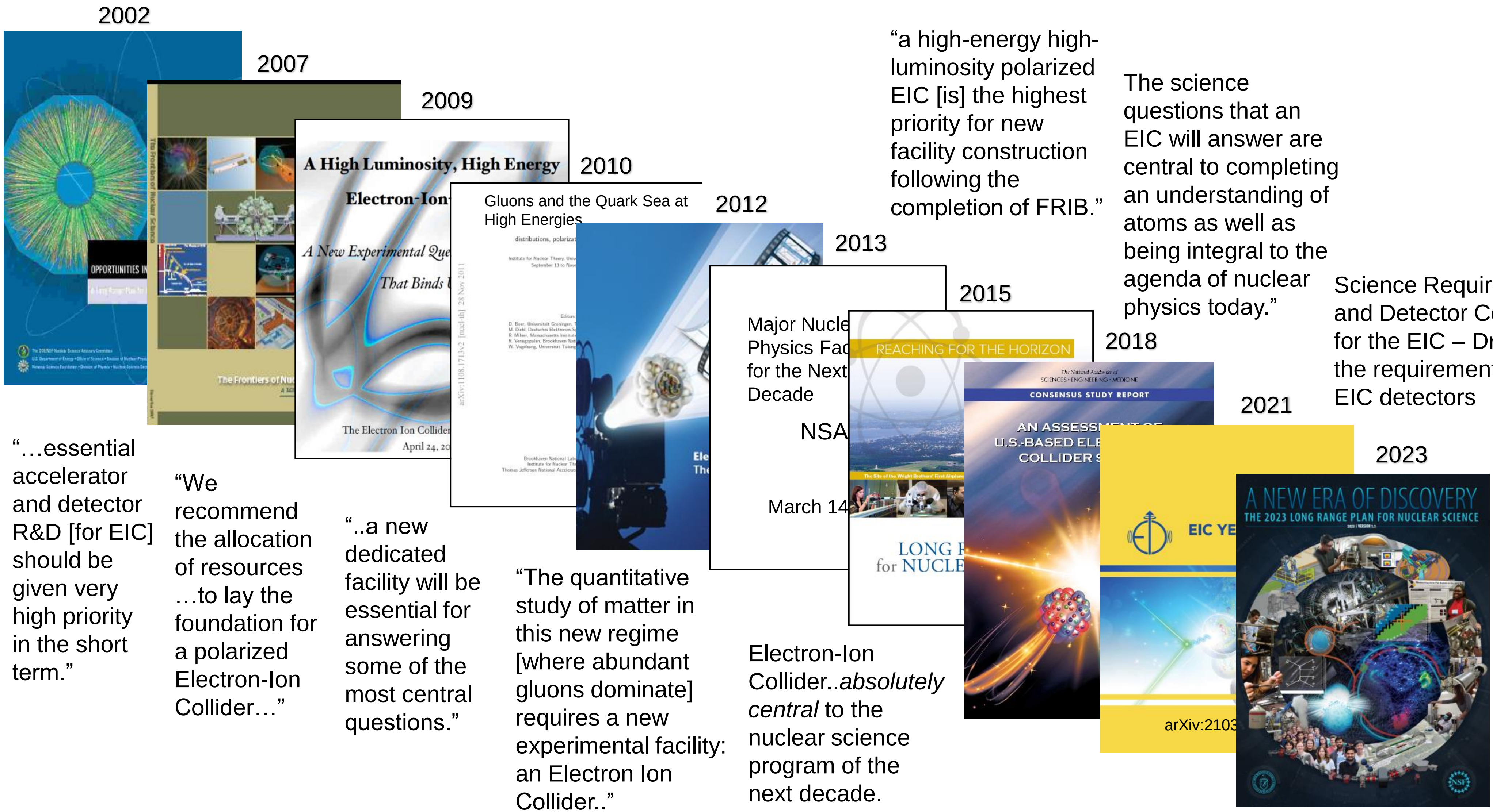
7 colliders are in operation today, 2 under construction (EIC and NICA)

- LHC, RHIC, BEPC, Super-KEKB, DAFNE, VEPP-4M, VEPP-2000

April 26, 2024: a joint "Statement of Intent between the United States of America and the European Organization for Nuclear Research (CERN) concerning Future Planning for Large Research Infrastructure Facilities, Advanced Scientific Computing, and Open Science" was signed at The White House.

The SOI expresses an intention by the United States to collaborate on a future **FCC Higgs Factory** should the CERN Member States determine the project feasible.

The Scientific Foundation for an EIC was Built Over Two Decades



"…essential accelerator and detector R&D [for EIC] should be given very high priority in the short term."

"We recommend the allocation of resources …to lay the foundation for a polarized Electron-Ion Collider…"

"..a new dedicated facility will be essential for answering some of the most central questions."

"The quantitative study of matter in this new regime [where abundant gluons dominate] requires a new experimental facility: an Electron Ion Collider.."

Electron-Ion Collider..*absolutely central* to the nuclear science program of the next decade.

"a high-energy high-luminosity polarized EIC [is] the highest priority for new facility construction following the completion of FRIB."

The science questions that an EIC will answer are central to completing an understanding of atoms as well as being integral to the agenda of nuclear physics today."

Science Requirements and Detector Concepts for the EIC – Drives the requirements of EIC detectors

We recommend the expeditious completion of the EIC as the highest priority for facility construction.

The DOE CD0 Mission-Need 2019

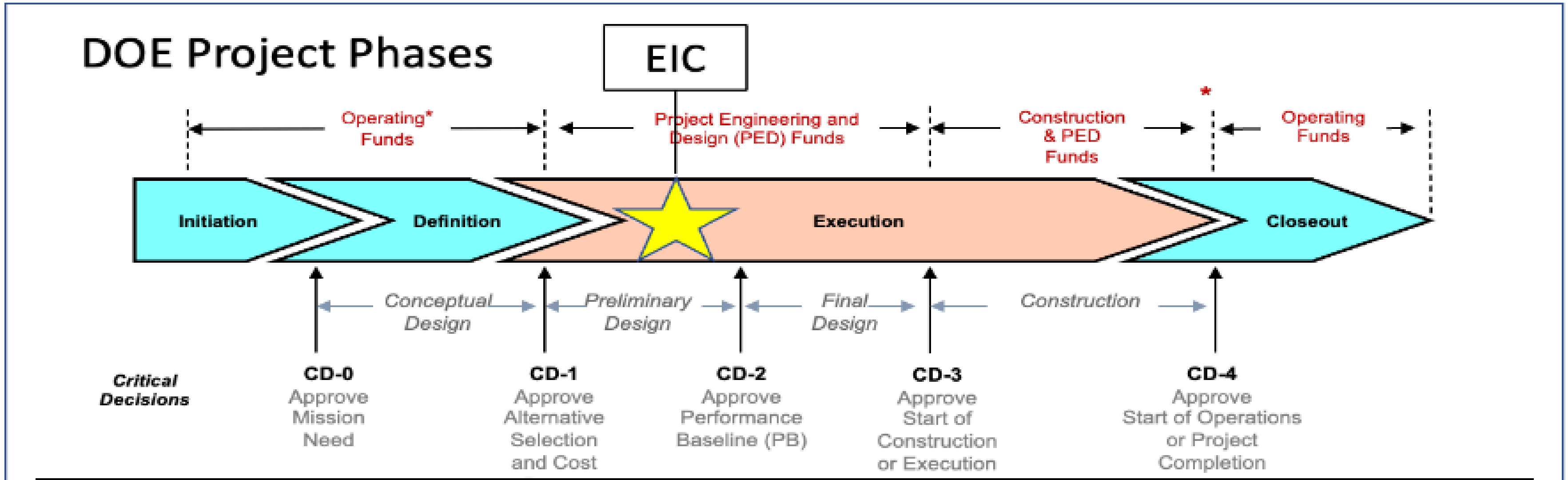
The EIC is required in order to probe the role of gluons and sea quarks and to examine nature's possible adherence to the predictions of dense, and ultimately saturated, gluon matter. An EIC capable of making a considerable leap in technical capabilities beyond previous electron scattering programs must reach collision energies far higher than are currently available world-wide. The key EIC machine parameters required to address its scientific agenda are listed in the 2015 LRP. These include a high degree of beam polarization (~70%) for electrons and light ions, availability of ion beams from deuterons to the heaviest stable nuclei, variable center of mass energies ~20–100 GeV, upgradable to ~140 GeV (e-p), high collision luminosity ~$10^{33\text{-}34}$ $\mathrm{cm}^{-2}\mathrm{s}^{-1}$, and possibly more than one interaction region.

$$E_{COM} \approx 2\sqrt{E_e E_h}$$

$E_e = 5$ GeV	$E_e = 5$ GeV	$E_e = 10$ GeV	$E_e = 18$ GeV
$E_p = 40$ GeV	$E_p = 100$ GeV	$E_p = 275$ GeV	$E_p = 275$ GeV
$E_{COM} \approx 28$ GeV	$E_{COM} \approx 45$ GeV	$E_{COM} \approx 100$ GeV	$E_{COM} \approx 140$ GeV

EIC Project Critical Decisions and Plans

CD-0, Mission Need Approved	December 2019
DOE Site Selection Announced	January 2020
CD-1, Alternative Selection and Cost Range Approved	June 2021
CD-3A, Long-Lead Procurement Approved	**March 2024**
CD-3B, Long-Lead Procurement Planned Approval	**March 2025**
CD-2/3, Performance Baseline/Construction Start Plan	**End 2025**



DOE Independent Project Review for CD-3B and Project Status on Jan 7-9, 2025.

EIC Requirements

EIC design goals

- High luminosity: **$L = (0.1\text{-}1)\cdot 10^{34}\ \mathrm{cm^{-2}\ s^{-1}}$**
  - ➔ 10-100 $\mathrm{fb^{-1}}$ "High"
- Collisions of highly **polarized (>70%)** e and p (and light ion) beams Unique
  - with flexible bunch by bunch spin patterns
- Large range of CM energies:
  - **E_{cm} = 20-140 GeV** "Low"
- Large range of ion species:
  - Protons – Uranium Diverse
- Ensure accommodation of a second IR
- Large detector acceptances; good background
  - Hadron particle loss
  - IR synchrotron radiation backgrounds

uniquely challenging

EIC Accelerator Design Overview

Hadron storage ring (HSR): 40-275 GeV (existing)

- up to 1160 bunches, 1A avg beam current (3x RHIC, polarized)
- bright vertical beam emittance (1.5 nm); new vac sleeves
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Electron storage ring (ESR): 5–18 GeV (new)

- up to 1160 polarized bunches
  - high polarization by continual reinjection from RCS
- 2.5 A avg beam current ➔ 9 MW SR power
- superconducting RF cavities

Compare to 50 MW at FCC

Current 6x NSLS-

Rapid cycling synchrotron (RCS): 0.4-18 GeV (new)

- 2 bunches at 1 Hz; spin transparent due to high periodicity

High luminosity interaction region (new; future 2nd IR)

- $L = 10^{34}\,cm^{-2}s^{-1}$ = 10 kHz-uba, superconducting magnets
- 25 mrad crossing angle with crab cavities/crab crossing
- spin rotators (produce longitudinal polarization at IP)

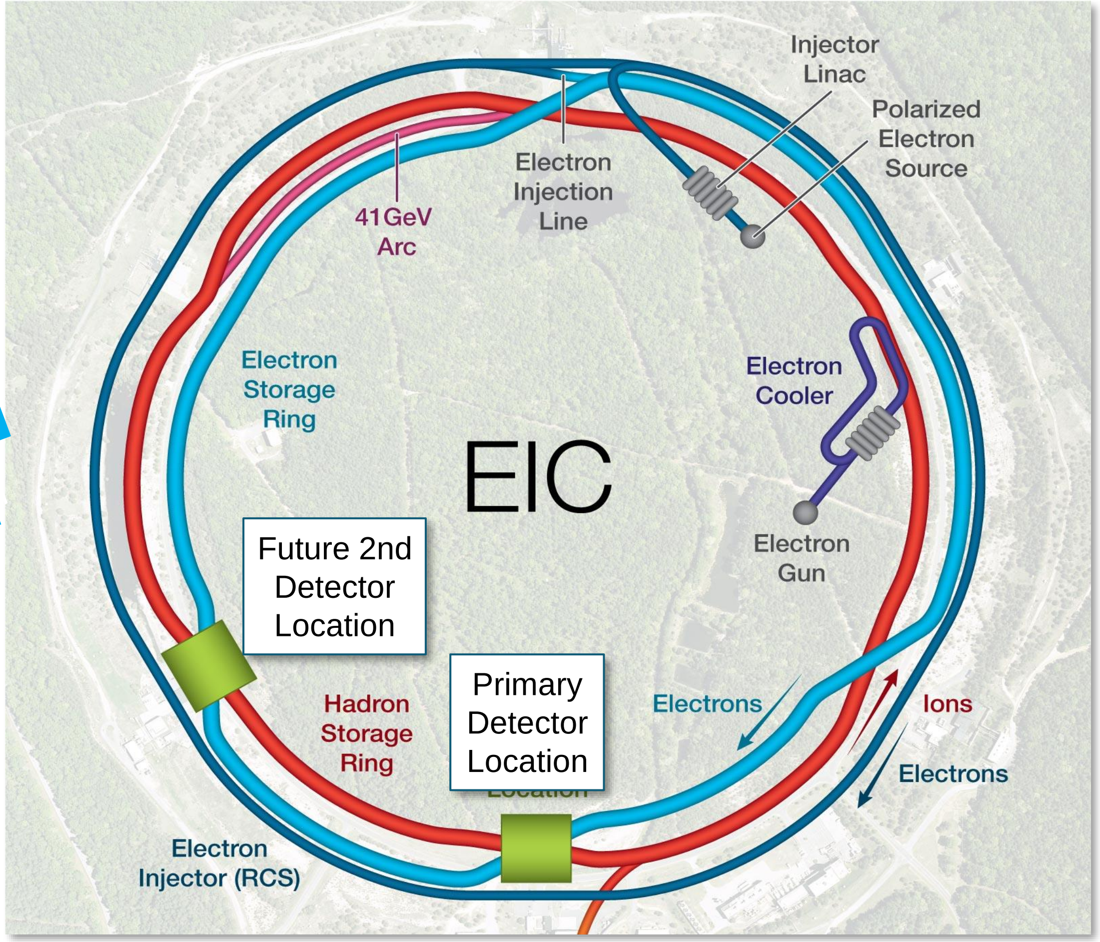

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Table 3.3: EIC beam parameters for different center-of-mass energies $\sqrt{s}$, with strong hadron cooling. High divergence configuration.

Species	proton	electron	proton	electron	proton	electron	proton	electron	proton	electron
Energy [GeV]	275	18	275	10	100	10	100	5	41	5
CM energy [GeV]	140.7		104.9		63.2		44.7		28.6	
Bunch intensity [10^{10}]	19.1	6.2	6.9	17.2	6.9	17.2	4.8	17.2	2.6	13.3
No. of bunches	290		1160		1160		1160		1160	
Beam current [A]	0.69	0.227	1	2.5	1	2.5	0.69	2.5	0.38	1.93
RMS norm. emit., h/v [μm]	5.2/0.47	845/71	3.3/0.3	391/26	3.2/0.29	391/26	2.7/0.25	196/18	1.9/0.45	196/34
RMS emittance, h/v [nm]	18/1.6	24/2.0	11.3/1.0	20/1.3	30/2.7	20/1.3	26/2.3	20/1.8	44/10	20/3.5
β^*, h/v [cm]]	80/7.1	59/5.7	80/7.2	45/5.6	63/5.7	96/12	61/5.5	78/7.1	90/7.1	196/21.0
IP RMS beam size, h/v [μm]	119/11		95/8.5		138/12		125/11		198/27	
K_x	11.1		11.1		11.1		11.1		7.3	
RMS $\Delta\theta$, h/v [μrad]	150/150	202/187	119/119	211/152	220/220	145/105	206/206	160/160	220/380	101/129
BB parameter, h/v [10^{-3}]	3/3	93/100	12/12	72/100	12/12	72/100	14/14	100/100	15/9	53/42
RMS long. emittance [10^{-3}, eV·s]	36		36		21		21		11	
RMS bunch length [cm]	6	0.9	6	0.7	7	0.7	7	0.7	7.5	0.7
RMS $\Delta p/p$ [10^{-4}]	6.8	10.9	6.8	5.8	9.7	5.8	9.7	6.8	10.3	6.8
Max. space charge	0.007	neglig.	0.004	neglig.	0.026	neglig.	0.021	neglig.	0.05	neglig.
Piwinski angle [rad]	6.3	2.1	7.9	2.4	6.3	1.8	7.0	2.0	4.2	1.1
Long. IBS time [h]	2.0		2.9		2.5		3.1		3.8	
Transv. IBS time [h]	2.0		2		2.0/4.0		2.0/4.0		3.4/2.1	
Hourglass factor H	0.91		0.94		0.90		0.88		0.93	
Luminosity [10^{33}cm^{-2}s^{-1}]	1.54		10.00		4.48		3.68		0.44	

Today's plan

- Beam sources
- Beam energy
- Luminosity
- Beam transport (bending and focusing)
- Beam size at IP
  - Emittance
  - 'beta' function
- Maybe: beam stability

How to build particle accelerators?

In three easy steps:

1. One needs particles
2. Add acceleration
3. Add focusing

For colliders: add collisions

Nuclear and Particle Physics

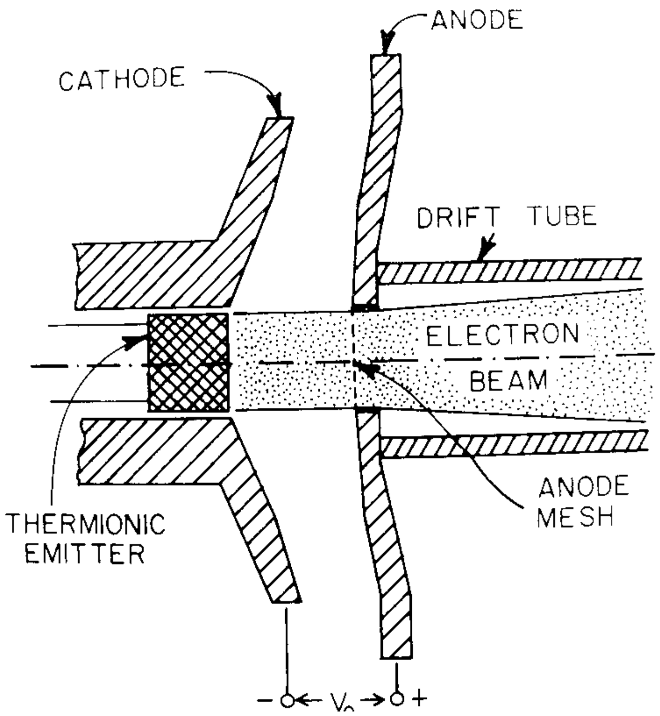
The first “high-energy” accelerator, made by Cockroft and Walton, was immediately used to understand the atomic nucleus. They made the first artificially produced nuclear reaction:

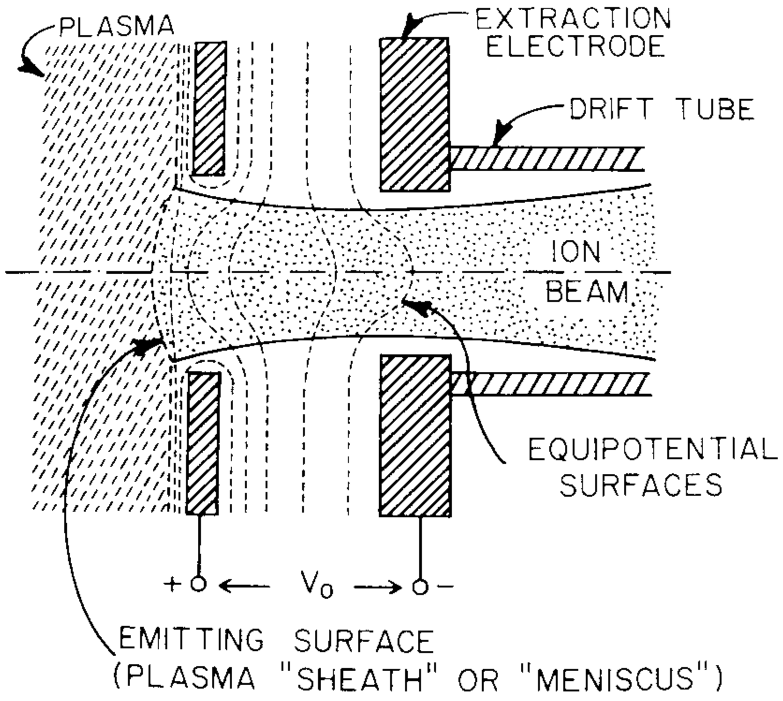
p+Li -> 2He

Early accelerator developments were driven by the quest for higher and higher particle energies, which in turn was driven by developments in nuclear physics (through the 1960s) and then elementary particle physics (1960s-onward)

The largest accelerator is at CERN (LHC). It collides two proton beams of energy 7 TeV each.

Particle sources





The simplest conceptual model of an electron beam source is a planar diode.

- Cathodes typically operate at 1400K ($kT \approx 0.12$ eV)
- An effective work function is about 1.6 eV

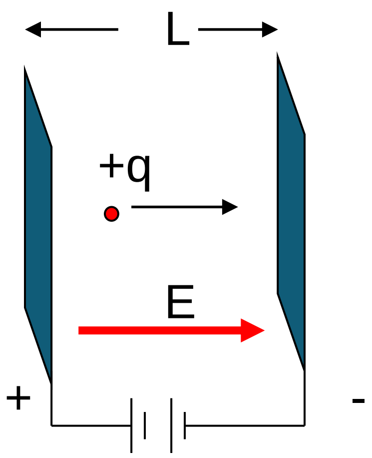
Ion sources are generally more complex. The ions are typically extracted from the plasma of a gas discharge.

Potential-Drop Accelerators

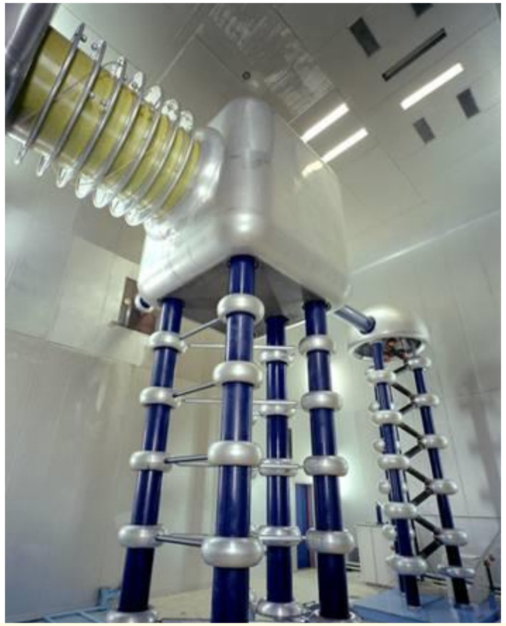
- These accelerators use at static, DC, potential difference between two conductors to impart a kinetic energy

$$\Delta W = qV_0$$

- Earliest particle accelerators were the Cockcroft-Walton generator and the Van de Graaff generator
- Highest voltage achieved is 25 MV
- It is difficult to establish and maintain a static DC field of 20+ MV



Van de Graff (tandem) generator



Cockroft-Walton generator

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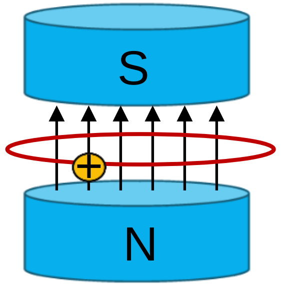
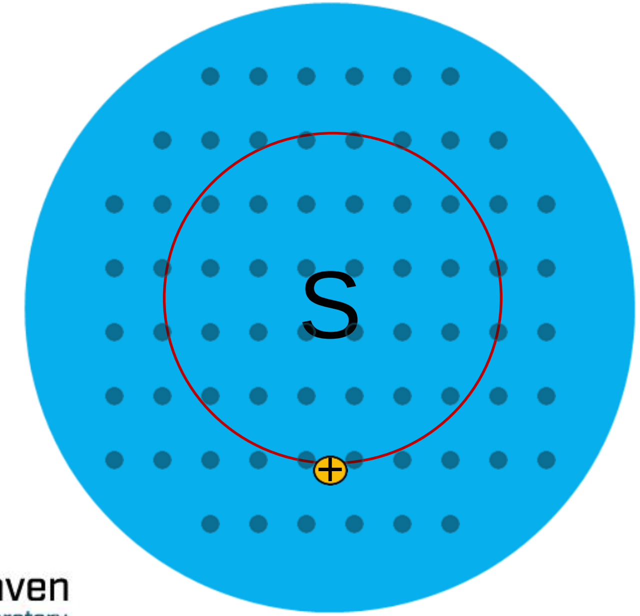
Cyclotron (Larmor) frequency

$$\omega_C = \frac{qB}{\gamma mc}$$

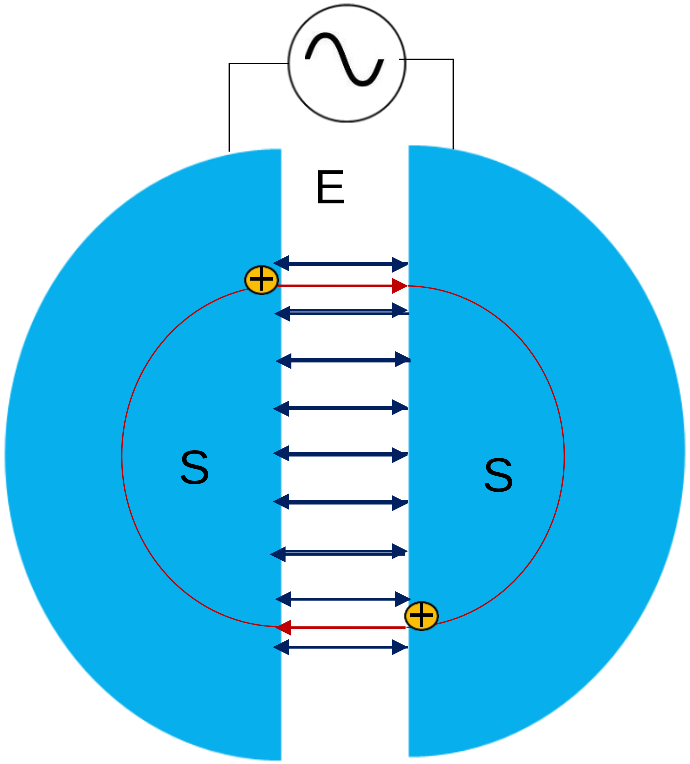
- Frequency weakly depends on particle energy (for non-relativistic particles)
- Led to invention of a cyclotron

The Cyclotron Concept





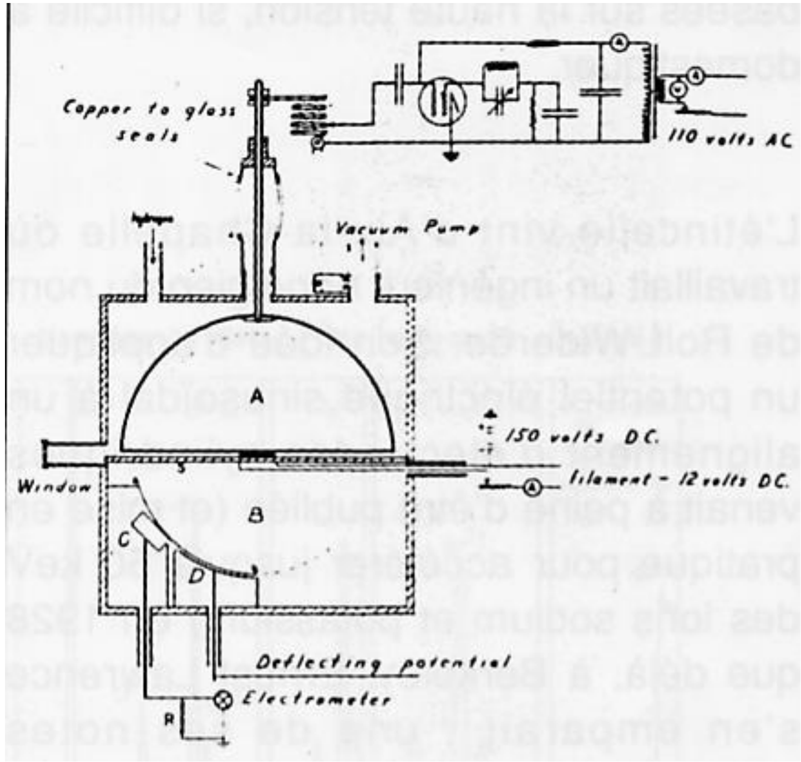
Sinusoidal voltage source

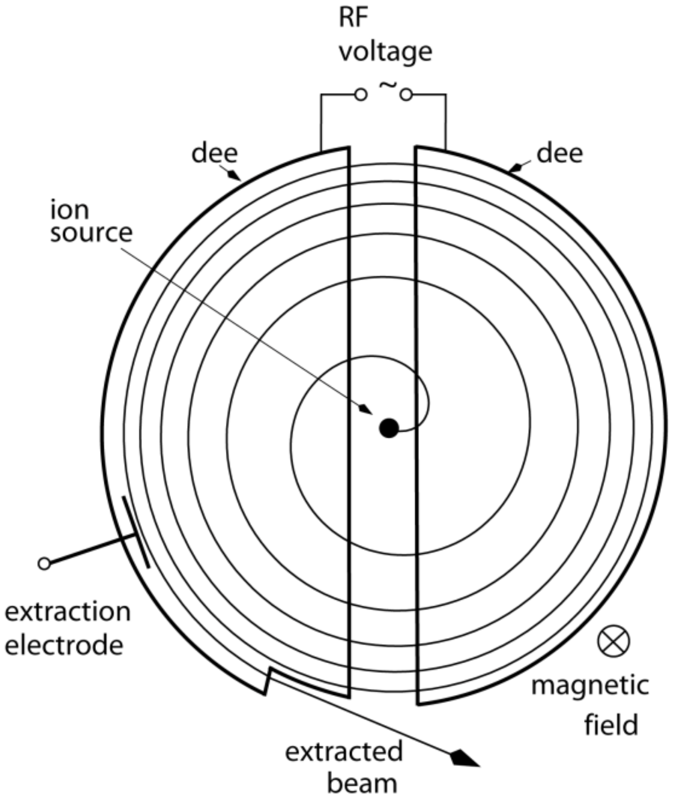


<u>The Recipe:</u>

1. A magnetic field to bend a particle around.
2. A sinusoidally varying electric field.

The First Cyclotron





First cyclotron (1931):

- 1 kV gap voltage
- 80 kV protons
- 4.5” in diameter

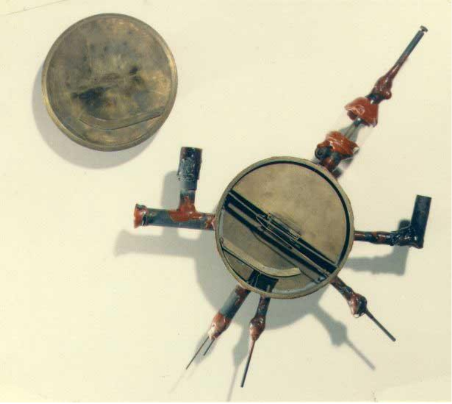
Origins of Accelerator Physics Earn Two Nobels

Two Nobels were awarded for advent of accelerator technologies

Nobel 1939

"for the invention and development of the cyclotron and for results obtained with it, especially with regard to artificial radioactive elements."

Photo from the Nobel Foundation archive.
Ernest Orlando Lawrence



https://history.aip.org/

Photo courtesy Lawrence Berkeley Lab

Nobel 1951

"for their pioneer work on the transmutation of atomic nuclei by artificially accelerated atomic particles."

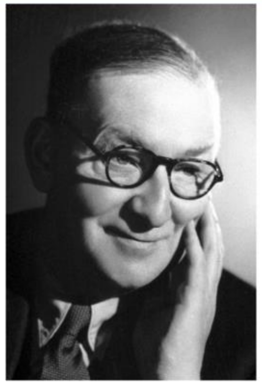
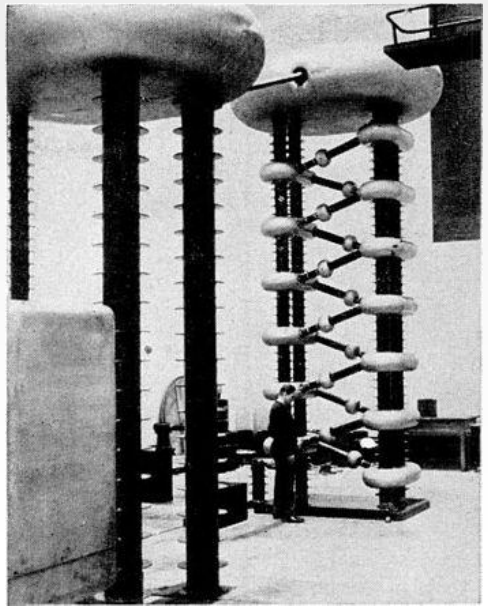


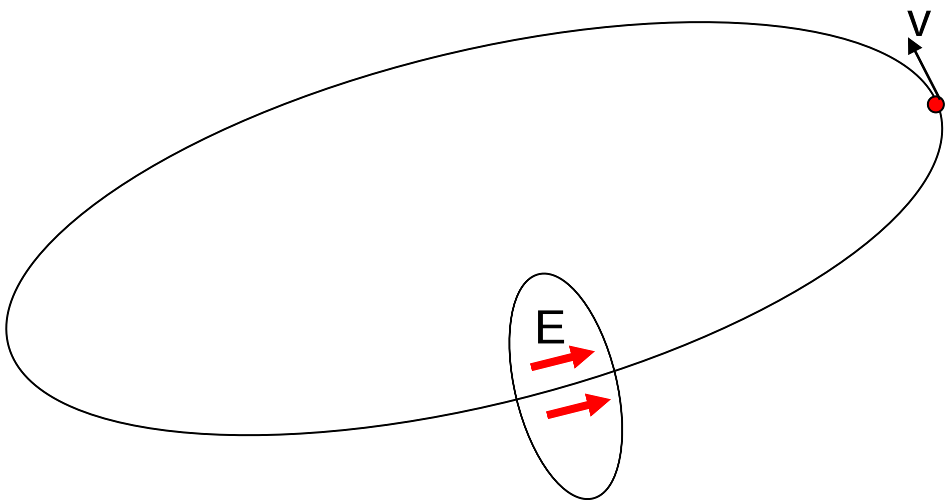
Photo from the Nobel Foundation archive.
Sir John Douglas Cockcroft

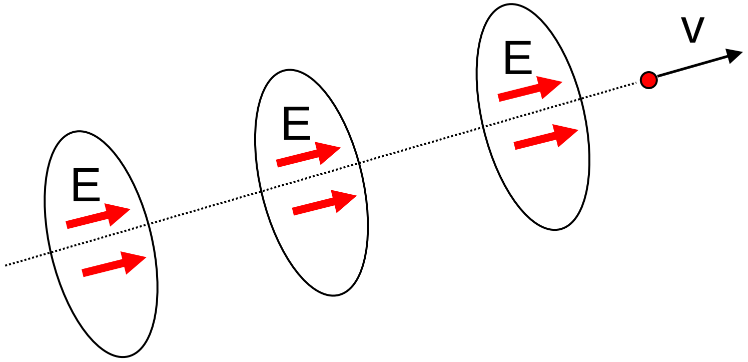
Photo from the Nobel Foundation archive.
Ernest Thomas Sinton Walton



Acceleration by repeated application of time-varying accelerating fields

Two approaches for accelerating with time-varying fields





Circular Accelerators

Use one or a small number of Radio-frequency accelerating cavities and make use of repeated passage through them. This approach is realized in circular accelerators: Cyclotrons, synchrotrons and their variants

Linear Accelerators

Use many accelerating cavities through which the particle beam passes once:

These are linear accelerators

Beam rigidity

$$B\rho \text{ [G-cm]} \approx \frac{pc \text{ [eV]}}{300}$$

Beam rigidity plays an important role in beam dynamics. For example, the LHC beam is *pc* = 7 TeV.

$$B\rho \approx 2.3\times 10^{10} \text{ G-cm} = 2.3\times 10^{4} \text{ T-m}$$

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SC dipoles

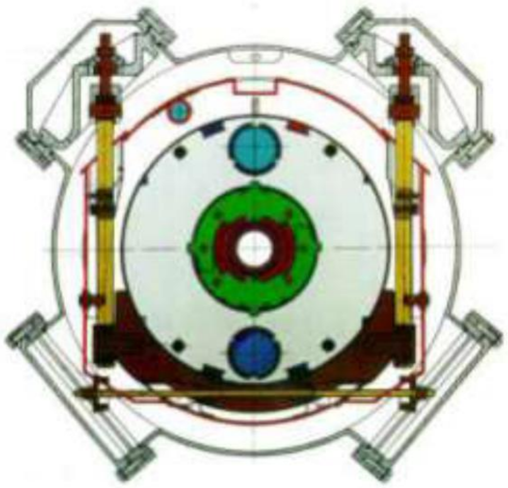
4.5T

Tevatron,
6 m, 76 mm
774 dipoles

4.5 K He, NbTi
+ warm iron
small He-plant

5.3T

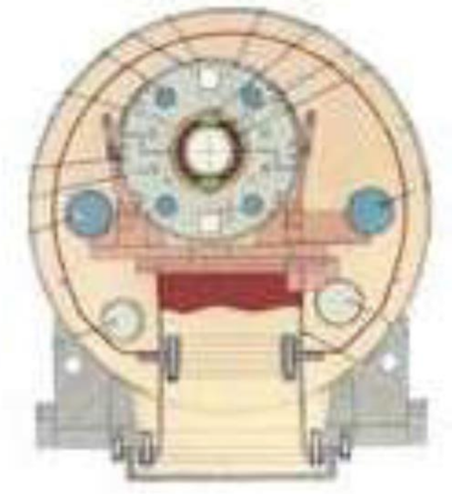
HERA,
9 m, 75 mm
416 dipoles



NbTi cable
cold iron
Al collar

3.5T

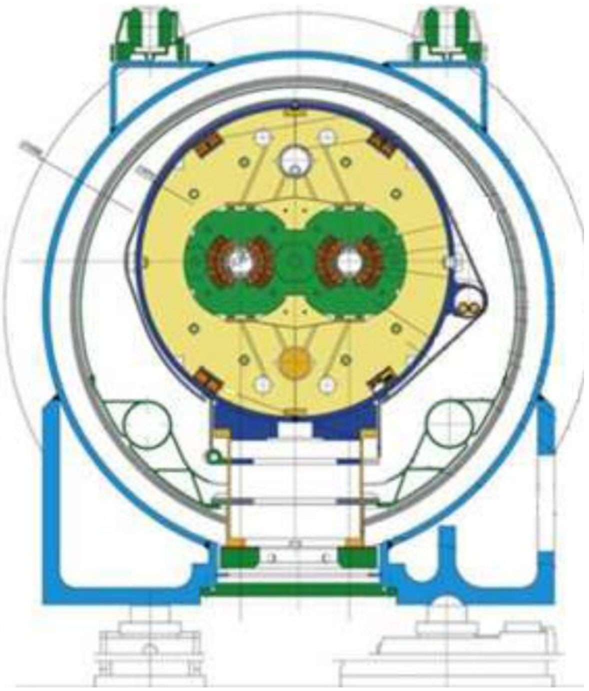
RHIC,
9 m, 80 mm
264 dipoles



NbTi cable
simple & cheap

8.3T

LHC,
15 m, 56 mm
1276 dipoles



NbTi cable
2K He
two bores

EIC Accelerator Design Overview

Hadron storage ring (HSR): 40-275 GeV (existing)

- up to 1160 bunches, 1A avg beam current (3x RHIC, polarized)
- bright vertical beam emittance (1.5 nm); new vac sleeves
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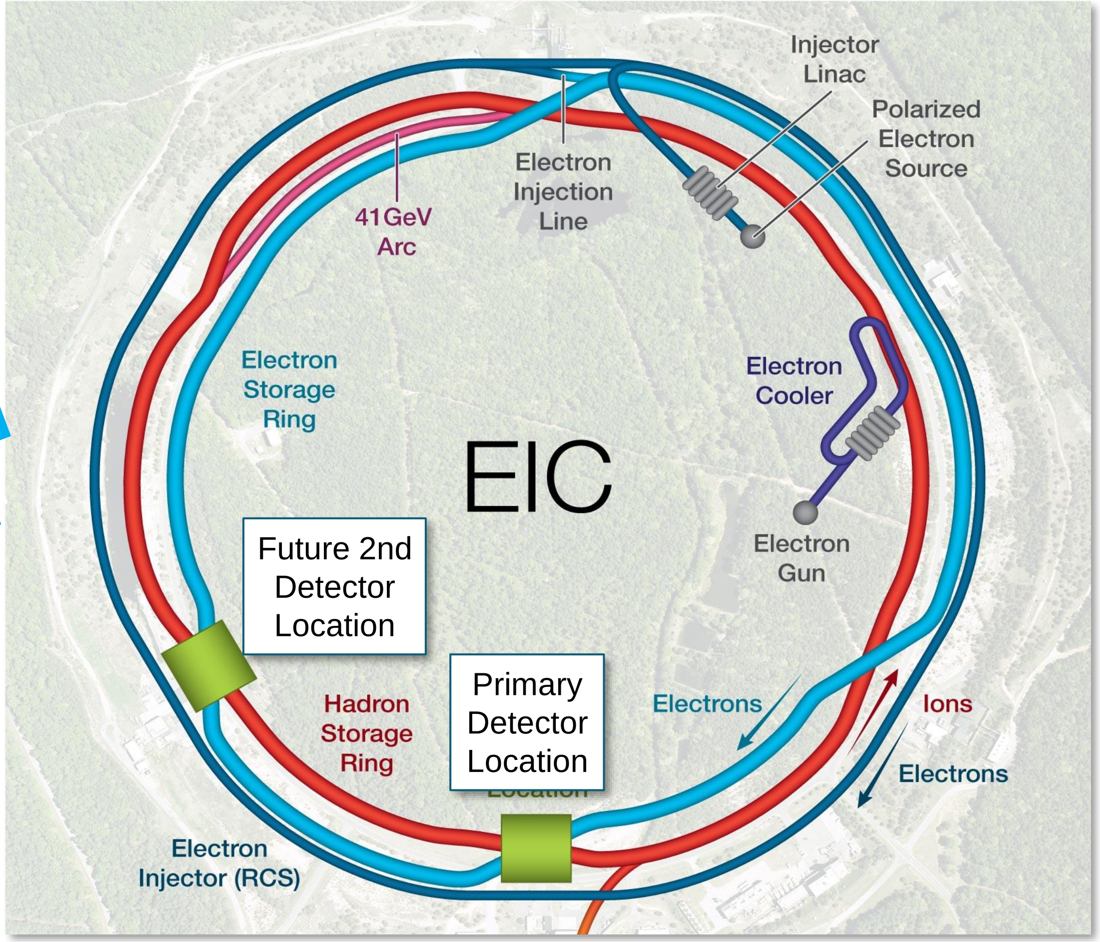

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Beam current [A]	0.69	0.227	1	2.5	1	2.5	0.69	2.5	0.38	1.93
RMS norm. emit., h/v [μm]	5.2/0.47	845/71	3.3/0.3	391/26	3.2/0.29	391/26	2.7/0.25	196/18	1.9/0.45	196/34
RMS emittance, h/v [nm]	18/1.6	24/2.0	11.3/1.0	20/1.3	30/2.7	20/1.3	26/2.3	20/1.8	44/10	20/3.5
β^*, h/v [cm]]	80/7.1	59/5.7	80/7.2	45/5.6	63/5.7	96/12	61/5.5	78/7.1	90/7.1	196/21.0
IP RMS beam size, h/v [μm]	119/11		95/8.5		138/12		125/11		198/27	
K_x	11.1		11.1		11.1		11.1		7.3	
RMS $\Delta\theta$, h/v [μrad]	150/150	202/187	119/119	211/152	220/220	145/105	206/206	160/160	220/380	101/129
BB parameter, h/v [10^{-3}]	3/3	93/100	12/12	72/100	12/12	72/100	14/14	100/100	15/9	53/42
RMS long. emittance [10^{-3}, eV·s]	36		36		21		21		11	
RMS bunch length [cm]	6	0.9	6	0.7	7	0.7	7	0.7	7.5	0.7
RMS $\Delta p/p$ [10^{-4}]	6.8	10.9	6.8	5.8	9.7	5.8	9.7	6.8	10.3	6.8
Max. space charge	0.007	neglig.	0.004	neglig.	0.026	neglig.	0.021	neglig.	0.05	neglig.
Piwinski angle [rad]	6.3	2.1	7.9	2.4	6.3	1.8	7.0	2.0	4.2	1.1
Long. IBS time [h]	2.0		2.9		2.5		3.1		3.8	
Transv. IBS time [h]	2.0		2		2.0/4.0		2.0/4.0		3.4/2.1	
Hourglass factor H	0.91		0.94		0.90		0.88		0.93	
Luminosity [10^{33}cm^{-2}s^{-1}]	1.54		10.00		4.48		3.68		0.44	

Luminosity

Interaction rate per unit cross-section

$$L = f\,\frac{N_1 N_2}{A} \quad [\mathrm{cm}^{-2}\mathrm{s}^{-1}]$$

For Gaussian cross-sections

$$L \approx f\,\frac{N_1 N_2}{4\pi\sigma_x\sigma_y} F(H,\sigma_L,..)$$

Paraxial approximation in beams

$$\theta \approx \frac{p_\perp}{p_z} \approx \frac{p_\perp}{p}$$

$$\sin\theta \approx \theta, \quad \tan\theta \approx \theta \quad \text{and} \quad \cos\theta \approx 1.$$

Linear approximation for all transverse forces.

Ray transfer matrix analysis for beam optics

- Similar to light optics.

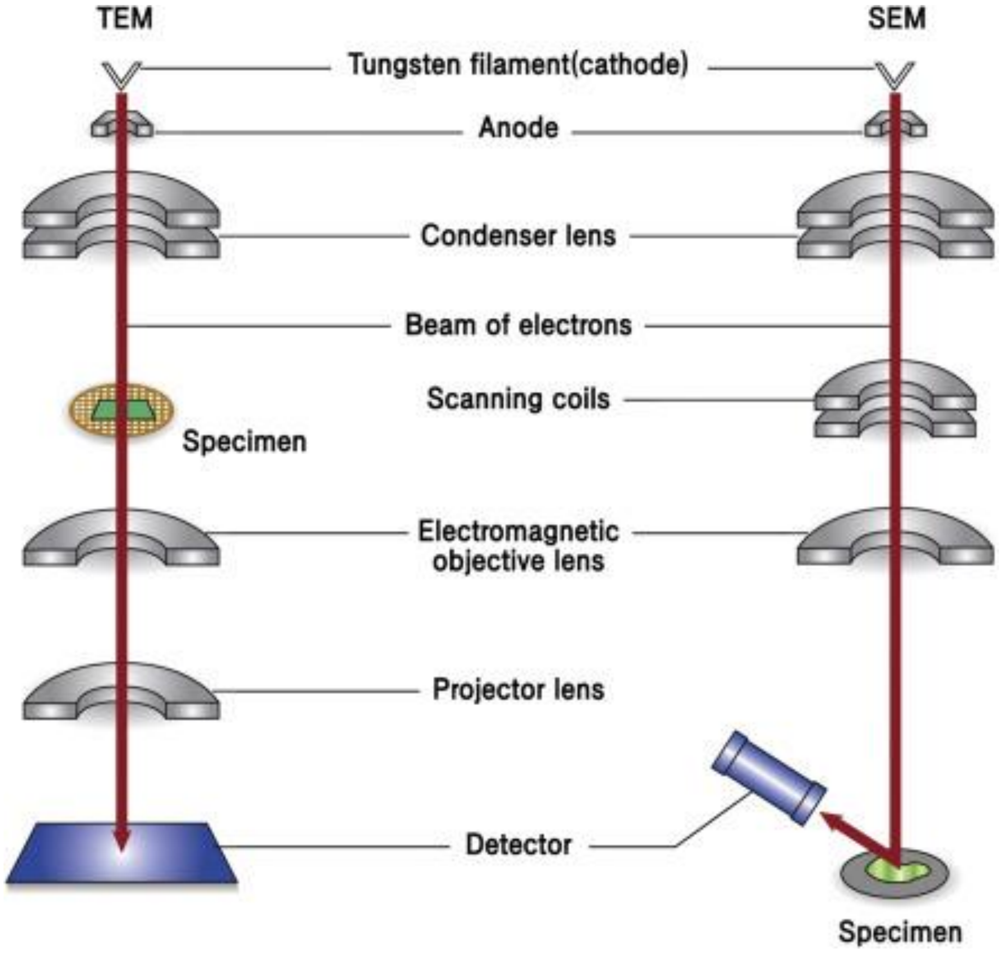
Drift space

$$\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

Thin lens

$$\begin{pmatrix} 1 & 0 \\ \pm\frac{1}{F} & 1 \end{pmatrix}$$

Example: electron microscopes

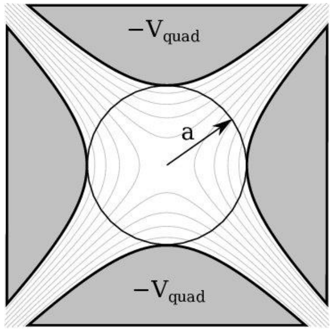


Various axially-symmetric lenses are used in EM’s

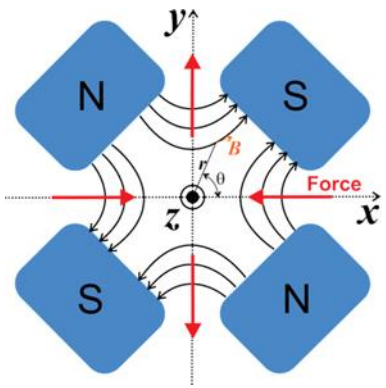
Quadrupoles

Consider examples of lenses:

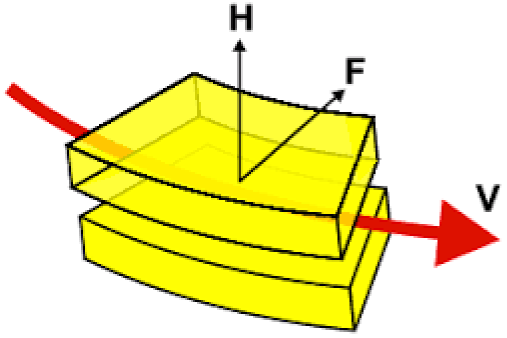
- Electrostatic quadrupole

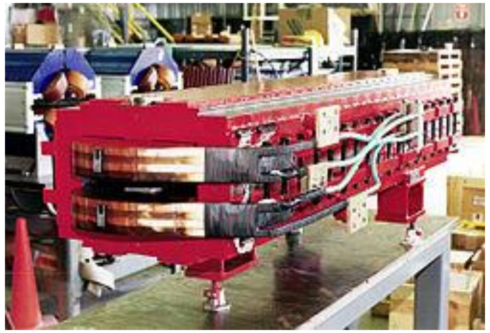


- Magnetostatic quadrupole



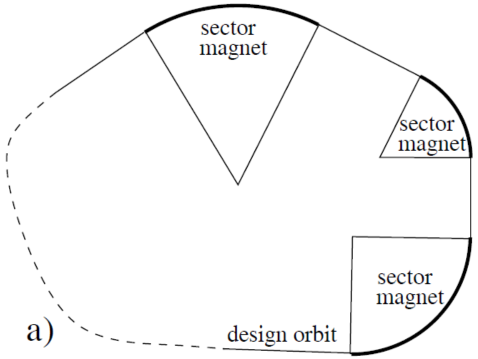
Bends (dipole magnets)





We consider the design orbit in a circular accelerator as a closed planar curve made up of arcs, each with a constant radius of curvature r_0. Each bend in the orbit is caused by a sector magnet, see Fig. 1a. The cylindrical coordinates r, θ, z are applied in each sector magnet (Fig. 1b), and its magnetic field $\mathbf{B} = (B_r, B_\theta, B_z)$ is two-dimensional:

$$B_\theta = 0, \quad \frac{\partial B_{r,z}}{\partial \theta} = 0.$$



Dipole magnets

We start with the relativistic equation of motion in the horizontal plane

$$\gamma m \ddot{r} = \gamma m \frac{v_\theta^2}{r} - \frac{e}{c} B_z v_\theta \qquad (1)$$

with the centrifugal and Lorentz forces on the right side. Here e and m are the particle's charge and mass, $\mathbf{v} = (v_r, v_\theta, v_z)$ is its velocity, c is the speed of light, $\beta = v/c$ and $\gamma = (1 - v^2/c^2)^{-1/2}$ are the relativistic factors. On the design orbit the vertical field B_0 is related to the nominal momentum p_0,

$$p_0 c = \gamma_0 \beta_0 m c^2 = \cdot e B_0 r_0, \qquad (2)$$

where the subscript zero indicates nominal particle parameters. Consider a particle's trajectory in a neighborhood close to the design orbit and use the development around the design orbit

$$v_{z,r} \ll v_\theta \approx \beta_0 c, \quad x = r - r_0 \ll r_0 \, .$$

Dipole magnets

Instead of equations of motion it is convenient to use equations of trajectories, changing the independent variable in Eq. (1) from time t to the path s along the design orbit, $ds = v_\theta dt \approx v_0 dt$,

$$r'' = \frac{1}{r} - \frac{eB_z}{\gamma\beta mc^2}, \tag{3}$$

where the prime indicates the derivative over s. Development to first order on the right side of Eq. (3), taking account of the particle's momentum offset $p = p_0 + \Delta p$, and relation (2) yields

$$\frac{1}{r_0 + x} - \frac{e\left(B_0 + \partial B_z/\partial r\, x + \ldots\right)}{\gamma_0\beta_0 mc^2(1 + \Delta p/p_0)} \approx -\frac{x}{r_0^2} - \frac{e}{p_0 c}\frac{\partial B_z}{\partial r}x + \frac{1}{r_0}\frac{\Delta p}{p}.$$

Thus, for *x* we obtain

$$x'' + K_x x = \frac{1}{r_0}\frac{\Delta p}{p}, \qquad K_x = \frac{1}{r_0^2} + \frac{e}{p_0 c}\frac{\partial B_z}{\partial r}.$$

Dipole magnets

For *z* we have similar

$$\gamma m\ddot{z} = \frac{e}{c} B_r v_\theta \,,$$

$$B_r = 0 + \frac{\partial B_r}{\partial z} z + \ldots \approx \frac{\partial B_z}{\partial r} z, \qquad \frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} = 0,$$

$$z'' + K_z z = 0,$$

where K_z is the vertical focusing function,

$$K_z = -\frac{e}{p_0 c} \frac{\partial B_z}{\partial r}.$$

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Matrix elements

$$\frac{d^2x}{ds^2} = 0 \qquad \begin{pmatrix} x \\ x' \end{pmatrix}_s = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

Drift space:

$$\frac{d^2x}{ds^2} + Kx = 0,\ K \ge 0$$

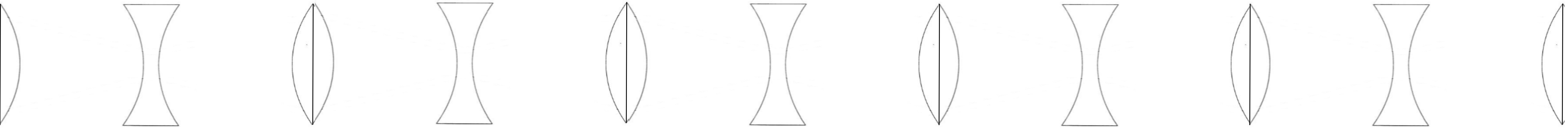
Harmonic oscillator:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = \begin{pmatrix} \cos\sqrt{K}s & \dfrac{\sin\sqrt{K}s}{\sqrt{K}} \\ -\sqrt{K}\sin\sqrt{K}s & \cos\sqrt{K}s \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = \begin{pmatrix} \cosh\sqrt{|K|}s & \dfrac{\sinh\sqrt{|K|}s}{\sqrt{|K|}} \\ \sqrt{|K|}\sinh\sqrt{|K|}s & \cosh\sqrt{|K|}s \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0, \quad K < 0$$

FODO

By convention horizontal focusing/defocusing is shown; vertical is opposite



Most accelerator lattices (including EIC) are designed in modular ways

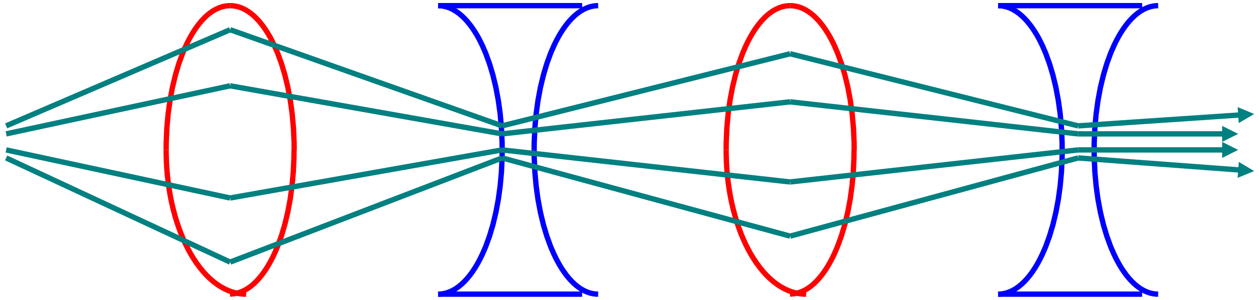
- Design and operational clarity, separation of functions

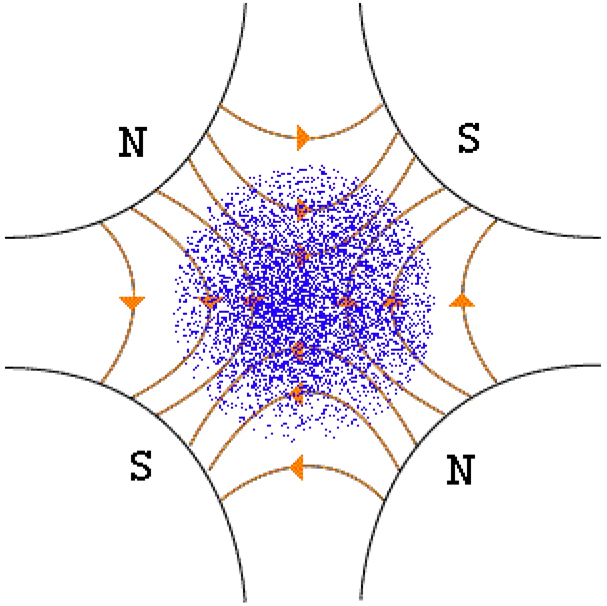
One of the most common modules is a FODO module

- Alternating focusing and defocusing “strong” quadrupoles
- Spaces between are combinations of drifts and dipoles
- Strong quadrupoles dominate the focusing
- **Periodicity** is **one FODO “cell”**
- **Horizontal beam size largest at centers of focusing quads**
- **Vertical beam size largest at centers of defocusing quads**

How To Get Net-Focusing from Quadrupoles???

The key is to alternate focusing and defocusing quadrupoles. This is called a FODO lattice (Focus-Drift-Defocus-Drift). :





Synchrotrons: Periodic Focusing Systems

Periodic focusing systems with periodic boundary conditions

- A long academic history!
- Solid state, quantum mechanics...

Periodic **1D linear** focusing system

- Hill's Equation (homogeneous)

$$x'' + K(s)x = 0 \qquad K(s) \equiv \frac{1}{(B\rho)} \left(\frac{\partial B_y}{\partial x} \right)(s) \qquad K(s+C) = K(s)$$

- Overall synchrotron is periodic, as are repeated FODO "cells"

Symmetry is locally broken for "insertions"

- Match boundary conditions between sections with different K values (focusing)

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Phase Space and Emittance

Equations of motion are **second-order differential equations**

Like classical mechanics, particle trajectories are determined by their **equation of motion** (Hill's equation) and **two initial conditions**

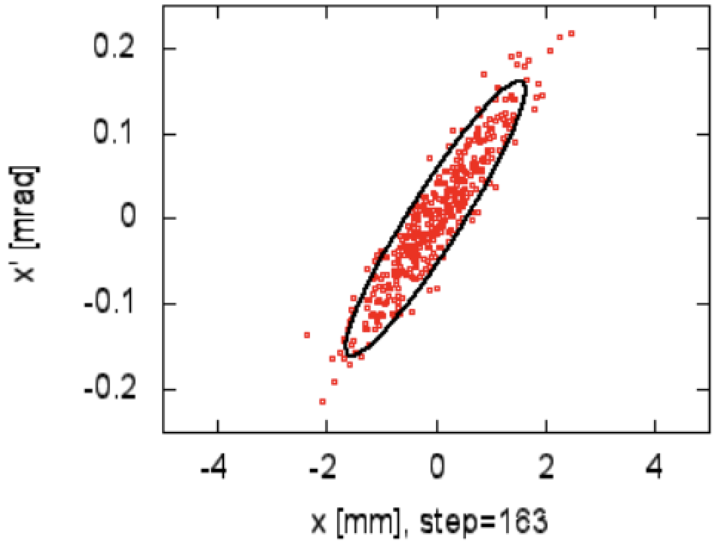
- **Horizontal** $(x, x' \equiv dx/ds)$ x',y' are angles
- **Vertical** $(y, y' \equiv dy/ds)$

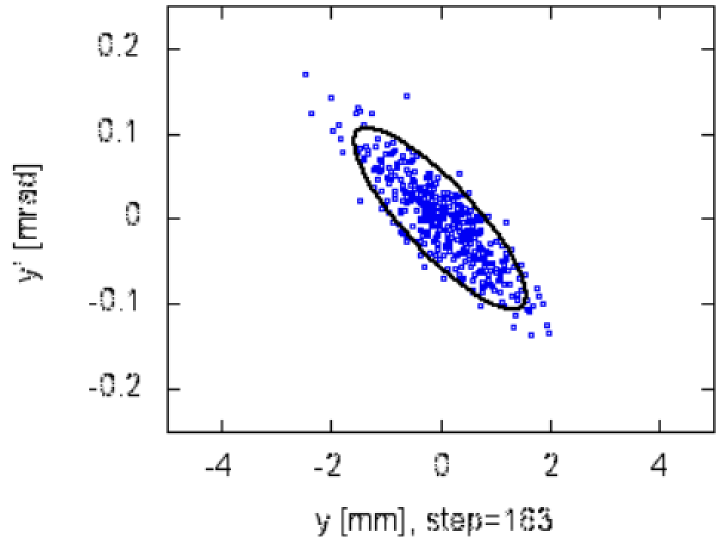
x',y' are small for high-energy accelerators

- Paraxial optics approximations apply

Transverse distributions are **usually Gaussian**

- RMS distribution ellipse area: **emittance** [mm-mrad]





Longitudinal similar but different

Table 3.3: EIC beam parameters for different center-of-mass energies $\sqrt{s}$, with strong hadron cooling. High divergence configuration.

Species	proton	electron	proton	electron	proton	electron	proton	electron	proton	electron
Energy [GeV]	275	18	275	10	100	10	100	5	41	5
CM energy [GeV]	140.7		104.9		63.2		44.7		28.6	
Bunch intensity [10^{10}]	19.1	6.2	6.9	17.2	6.9	17.2	4.8	17.2	2.6	13.3
No. of bunches	290		1160		1160		1160		1160	
Beam current [A]	0.69	0.227	1	2.5	1	2.5	0.69	2.5	0.38	1.93
RMS norm. emit., h/v [μm]	5.2/0.47	845/71	3.3/0.3	391/26	3.2/0.29	391/26	2.7/0.25	196/18	1.9/0.45	196/34
RMS emittance, h/v [nm]	18/1.6	24/2.0	11.3/1.0	20/1.3	30/2.7	20/1.3	26/2.3	20/1.8	44/10	20/3.5
β^*, h/v [cm]]	80/7.1	59/5.7	80/7.2	45/5.6	63/5.7	96/12	61/5.5	78/7.1	90/7.1	196/21.0
IP RMS beam size, h/v [μm]	119/11		95/8.5		138/12		125/11		198/27	
K_x	11.1		11.1		11.1		11.1		7.3	
RMS $\Delta\theta$, h/v [μrad]	150/150	202/187	119/119	211/152	220/220	145/105	206/206	160/160	220/380	101/129
BB parameter, h/v [10^{-3}]	3/3	93/100	12/12	72/100	12/12	72/100	14/14	100/100	15/9	53/42
RMS long. emittance [10^{-3}, eV·s]	36		36		21		21		11	
RMS bunch length [cm]	6	0.9	6	0.7	7	0.7	7	0.7	7.5	0.7
RMS $\Delta p/p$ [10^{-4}]	6.8	10.9	6.8	5.8	9.7	5.8	9.7	6.8	10.3	6.8
Max. space charge	0.007	neglig.	0.004	neglig.	0.026	neglig.	0.021	neglig.	0.05	neglig.
Piwinski angle [rad]	6.3	2.1	7.9	2.4	6.3	1.8	7.0	2.0	4.2	1.1
Long. IBS time [h]	2.0		2.9		2.5		3.1		3.8	
Transv. IBS time [h]	2.0		2		2.0/4.0		2.0/4.0		3.4/2.1	
Hourglass factor H	0.91		0.94		0.90		0.88		0.93	
Luminosity [$10^{33}\mathrm{cm}^{-2}\mathrm{s}^{-1}$]	1.54		10.00		4.48		3.68		0.44	

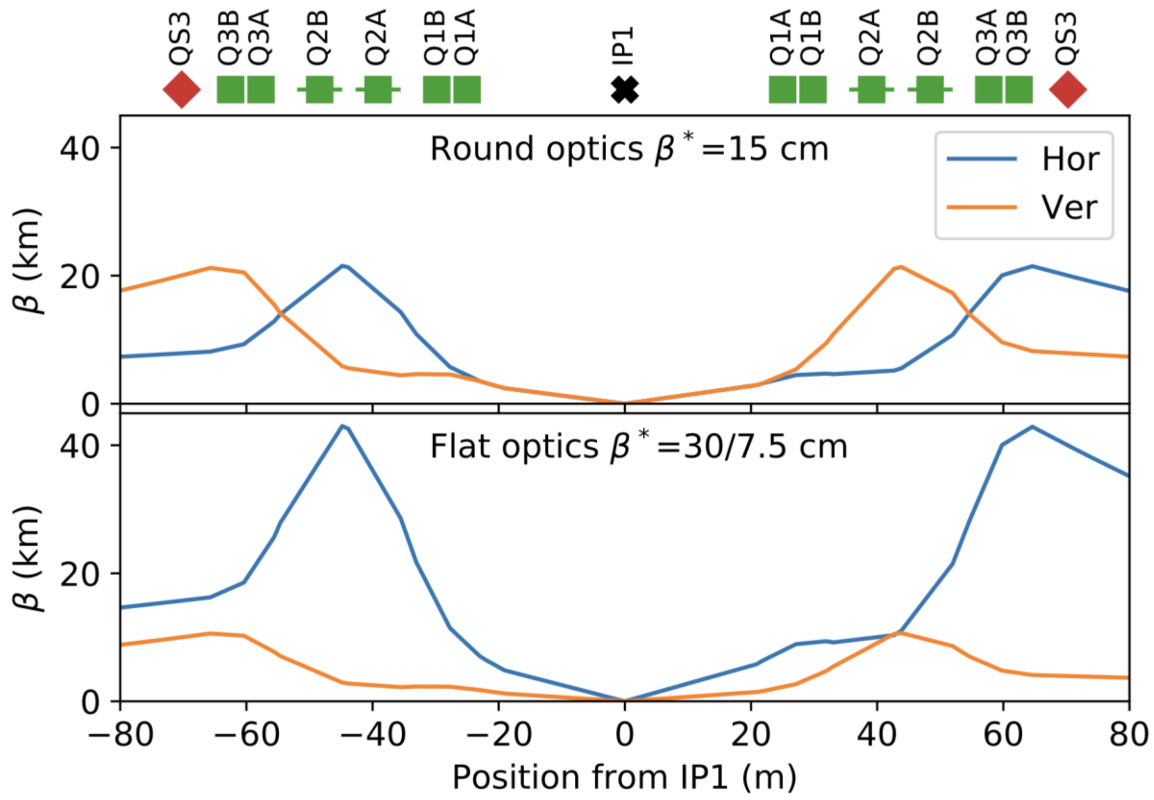
Beta star

magnets

beam

Beam size depends on emittance and optics “beta”: $\beta(s) = \sigma(s)^2/\epsilon$

- Look for kinks in beta function plots



PHYS. REV. ACCEL. BEAMS 23, 041001 (2020)

$$\beta^\star : \quad \beta \text{ at Interaction Point}$$

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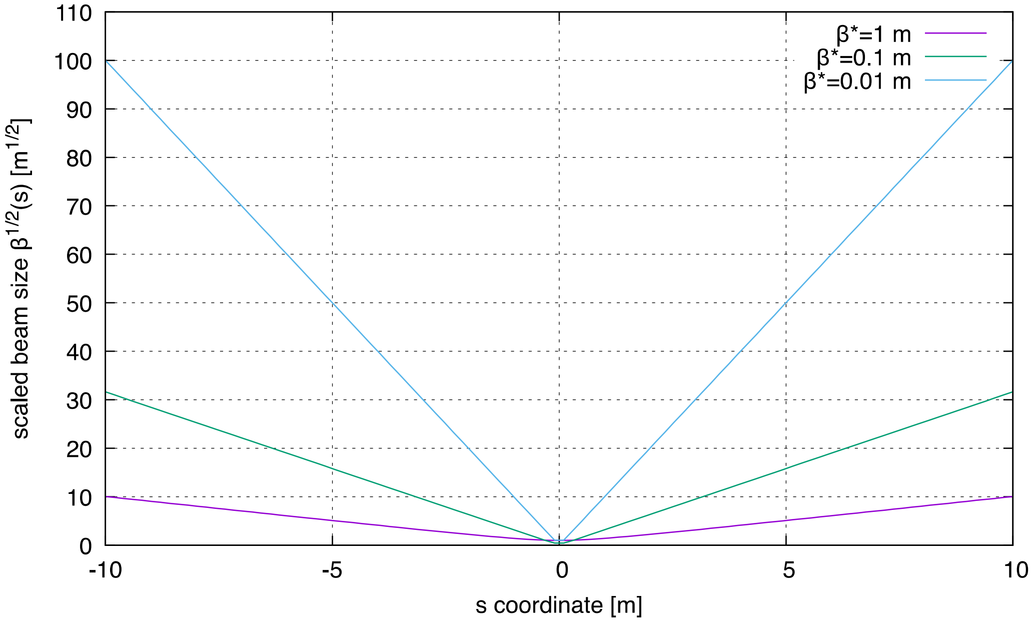
Beta in Free Space

Can minimum beam size be achieved in ”free space”

- Yes! We can demonstrate in free space

$$\beta(s) = \beta^\star + s^2/\beta^\star$$

- **Design tensions**:
  - Low beta* for high luminosity
  - Proximity of quadrupoles to IP
  - Large-aperture quads feasibility/quality
  - (Beam divergence near IP; ”hourglass”)
  - (Downstream secondary focus)



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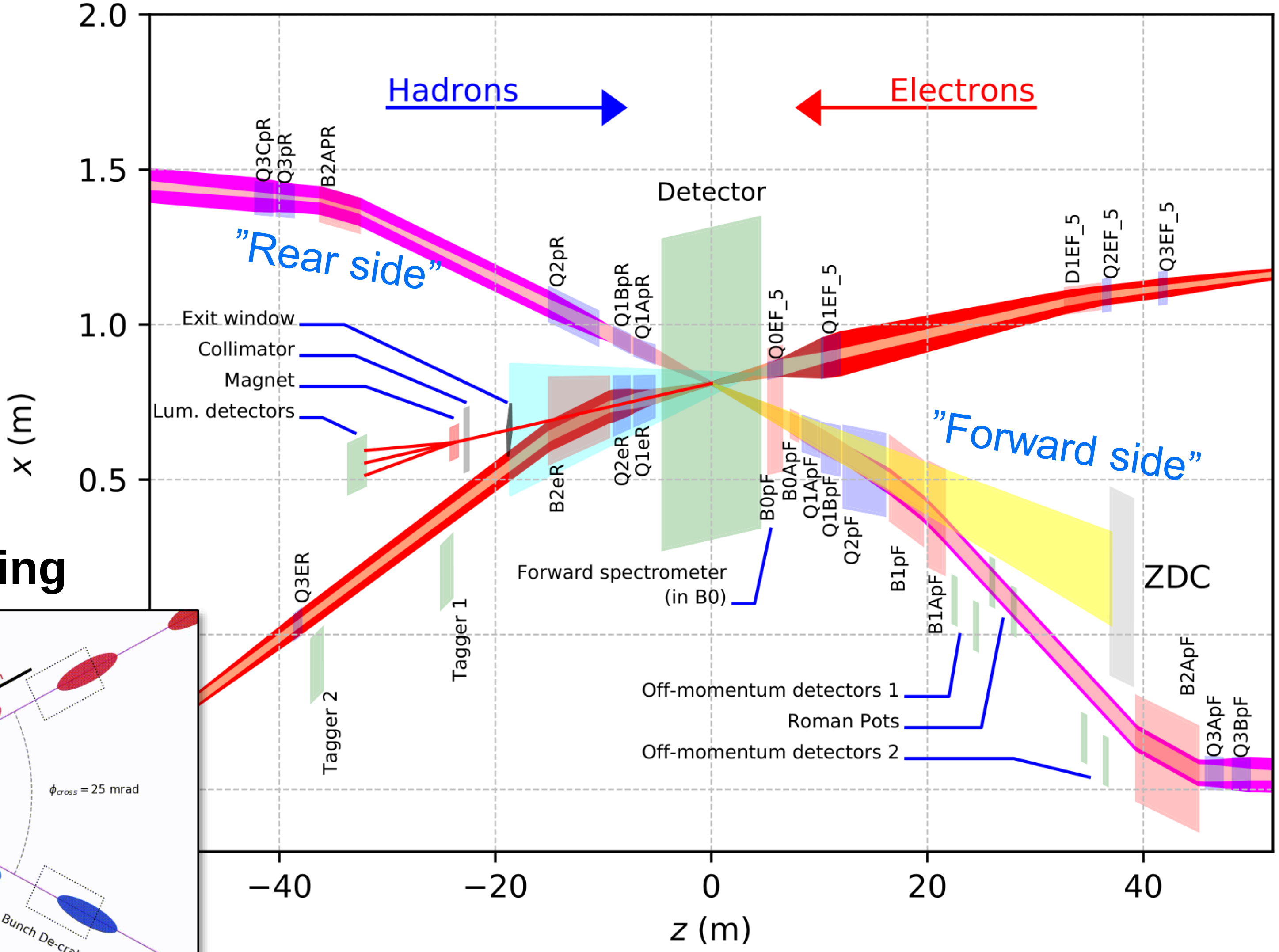
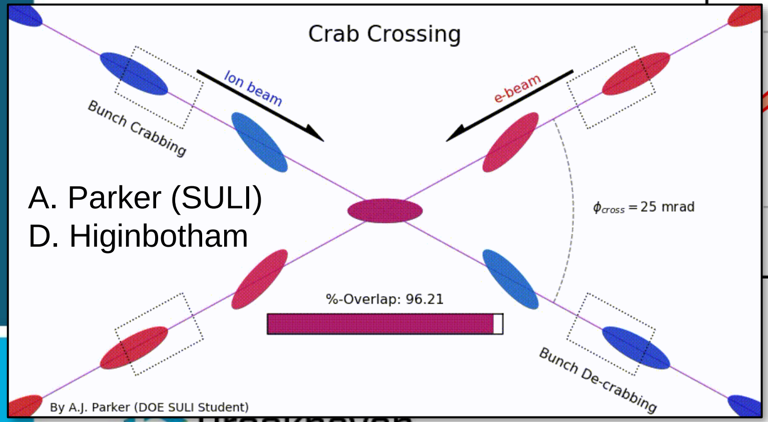
EIC Primary Interaction Region

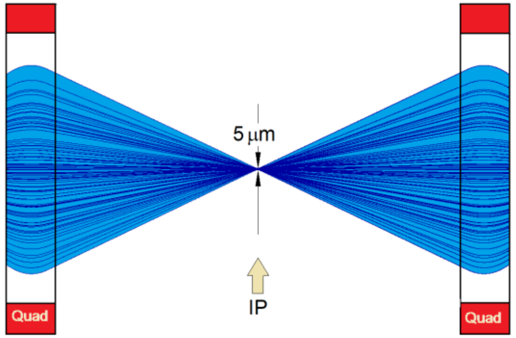
Existing tunnel and experiment halls

Different axis scales!

25 mrad crab crossing







Focusing (quads) as close to IP as possible (~5m!)

Tensions in magnet requirements:

- high field
- large apertures
- e/p magnet proximity near IP

Chromaticity: focusing dependence on particle energy