Phenomenology of unpolarised TMDs

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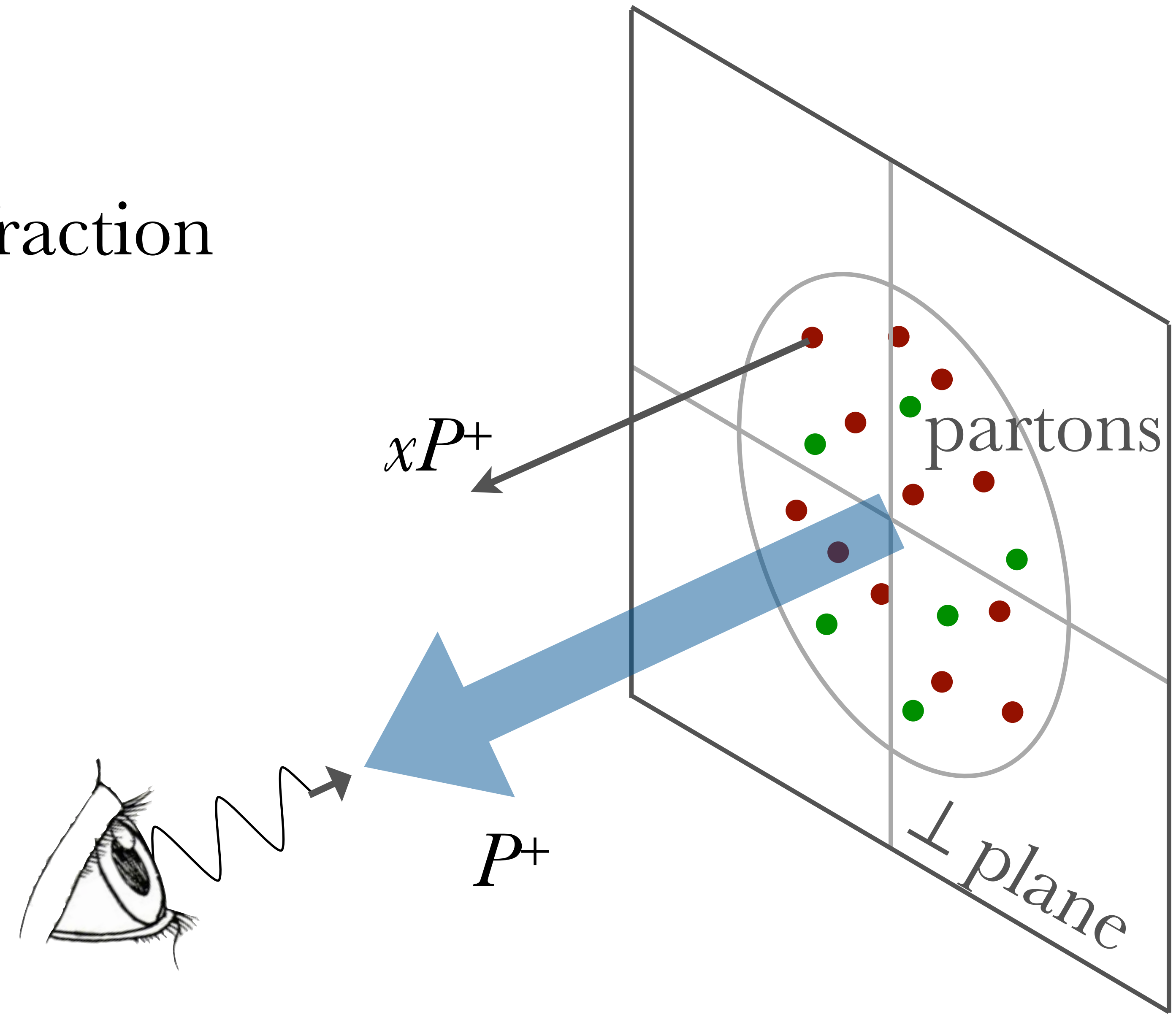
STVDIORVM UNIVERSITAS CARALITANA

Collinear PDF (FF)

Collinear PDF $f(x)$
depend on:

- x = longitudinal-momentum fraction



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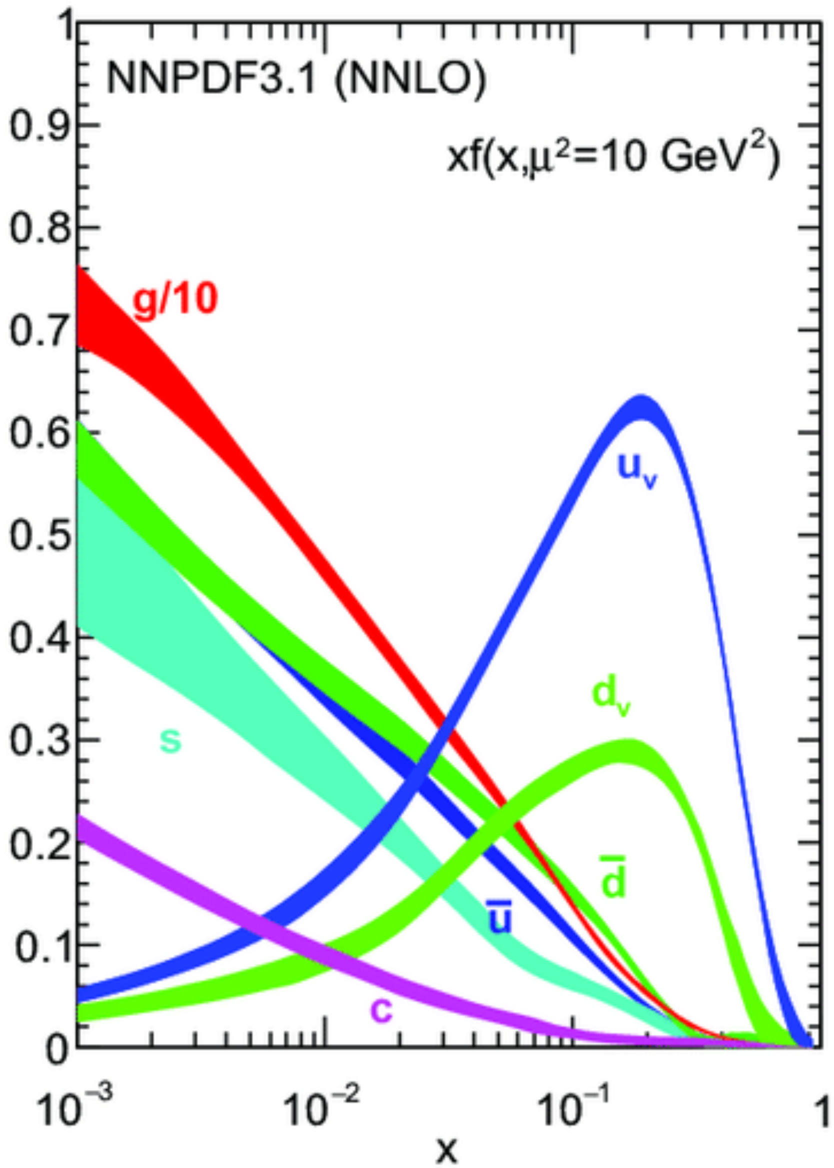
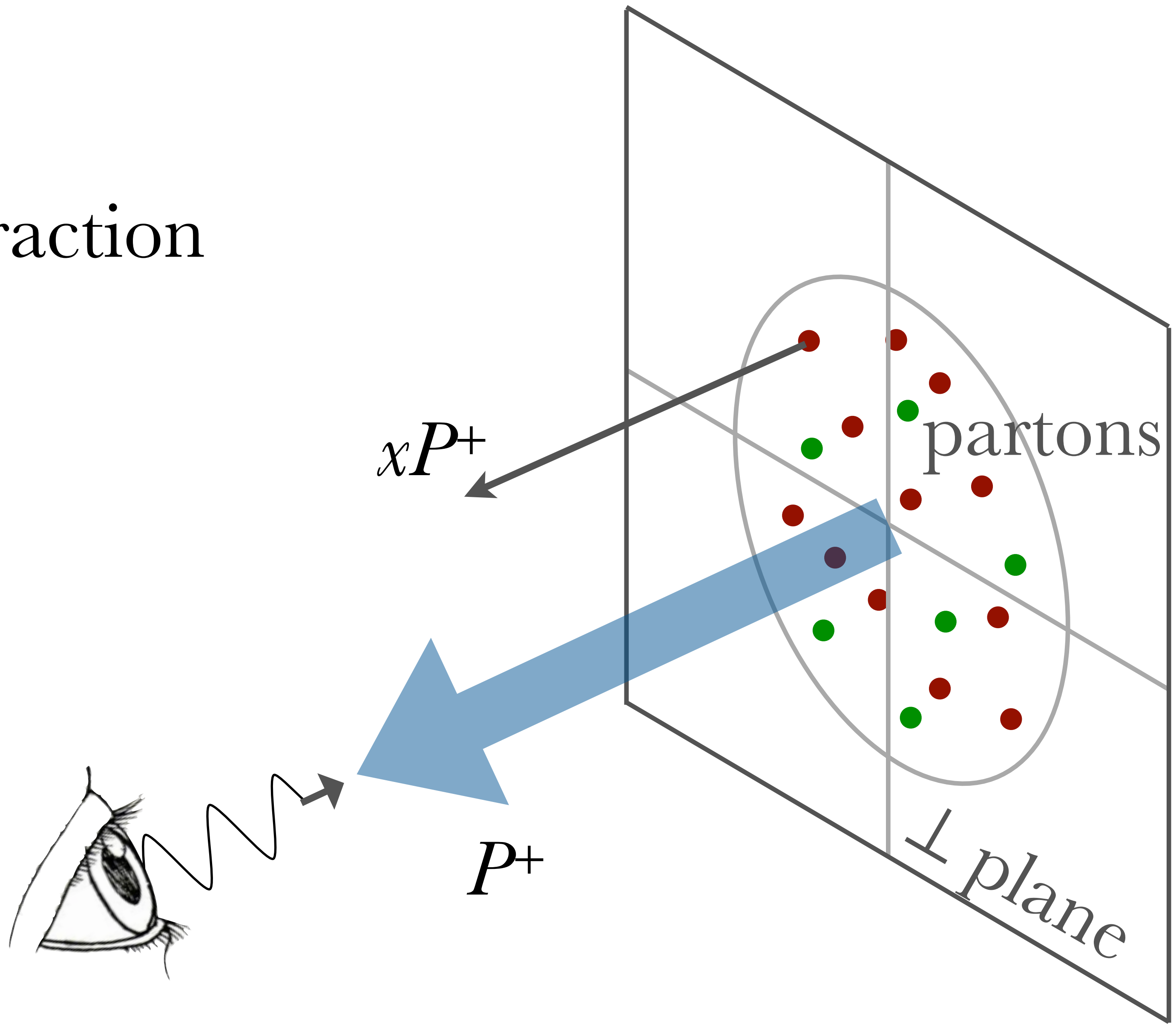
1-dim imaging



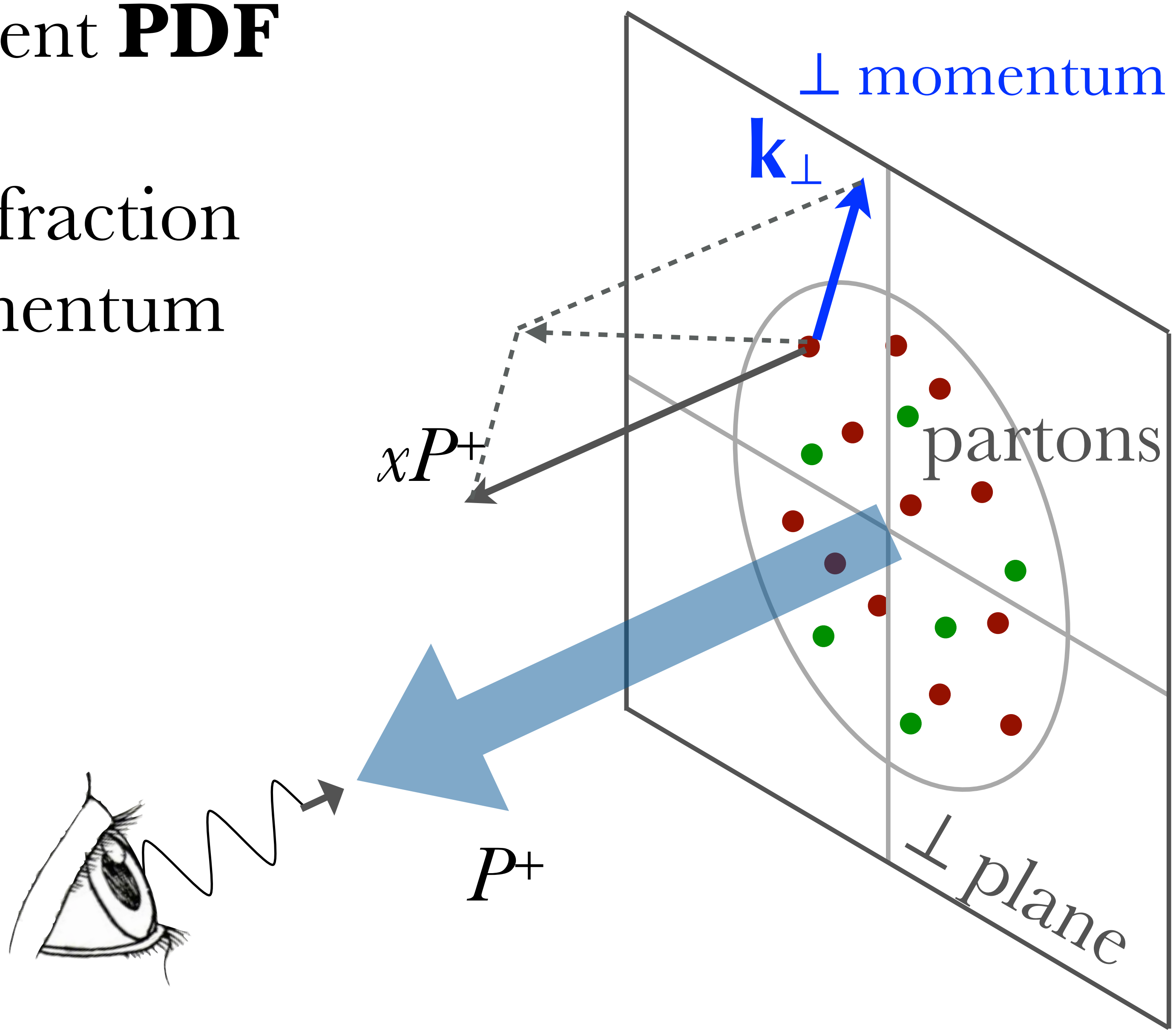


TMD PDF (FF)

Transverse **M**omentum **D**ependent **PDF**

$F(x, k_\perp)$ depend on:

- x = longitudinal-momentum fraction
- $k_\perp$ = *(intrinsic)* transverse-momentum



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Transverse **M**omentum **D**ependent **PDF**

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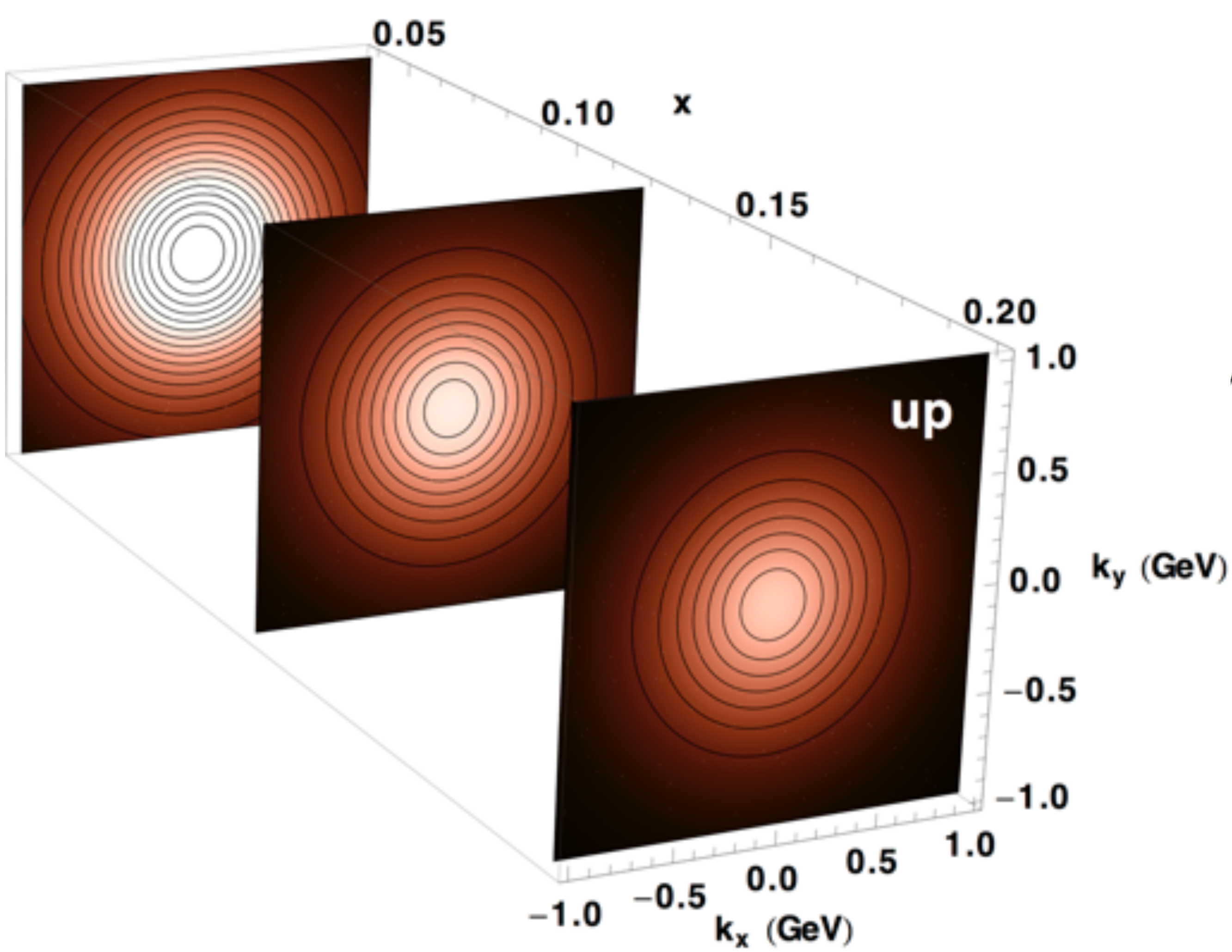
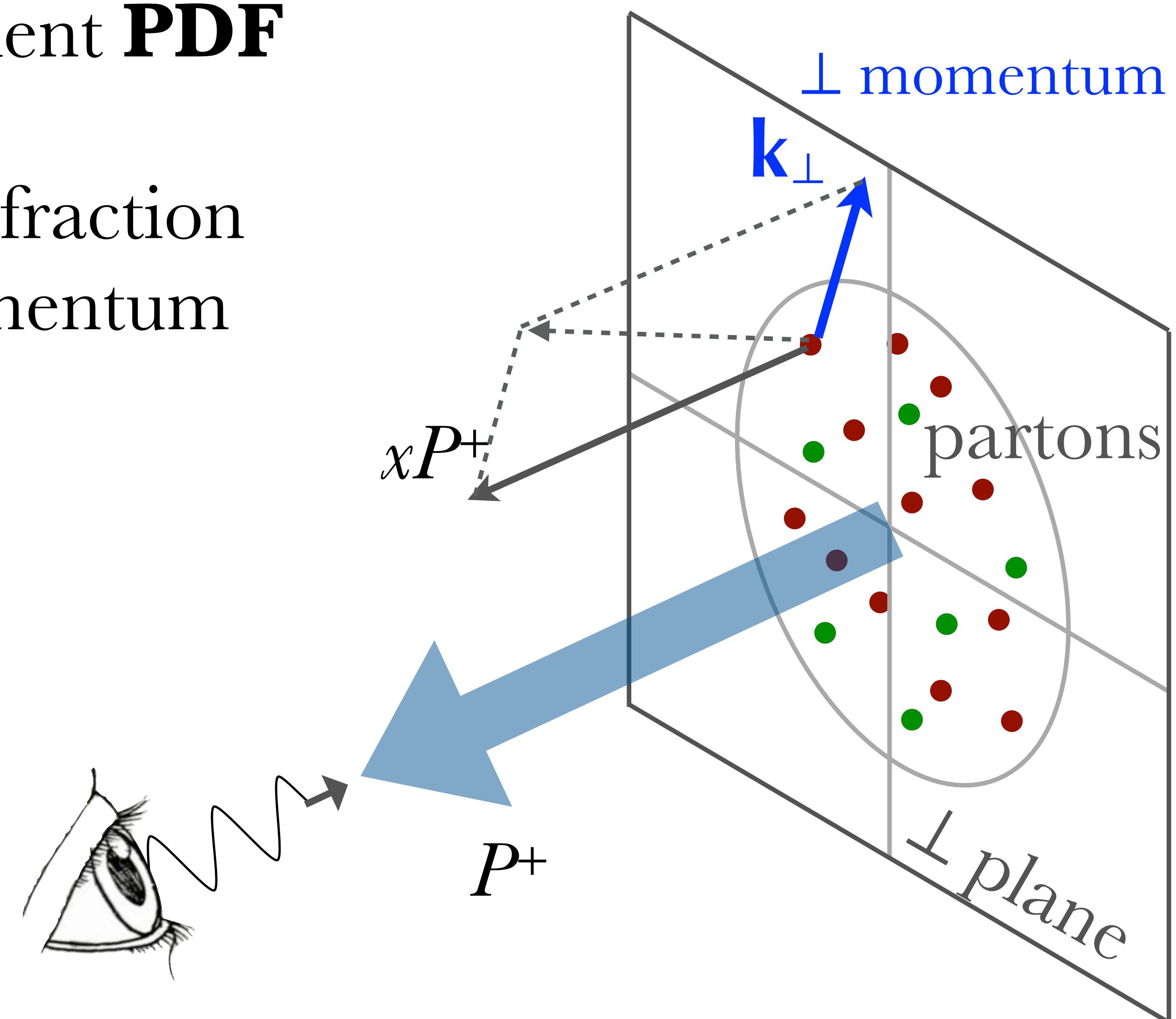
3-dim imaging





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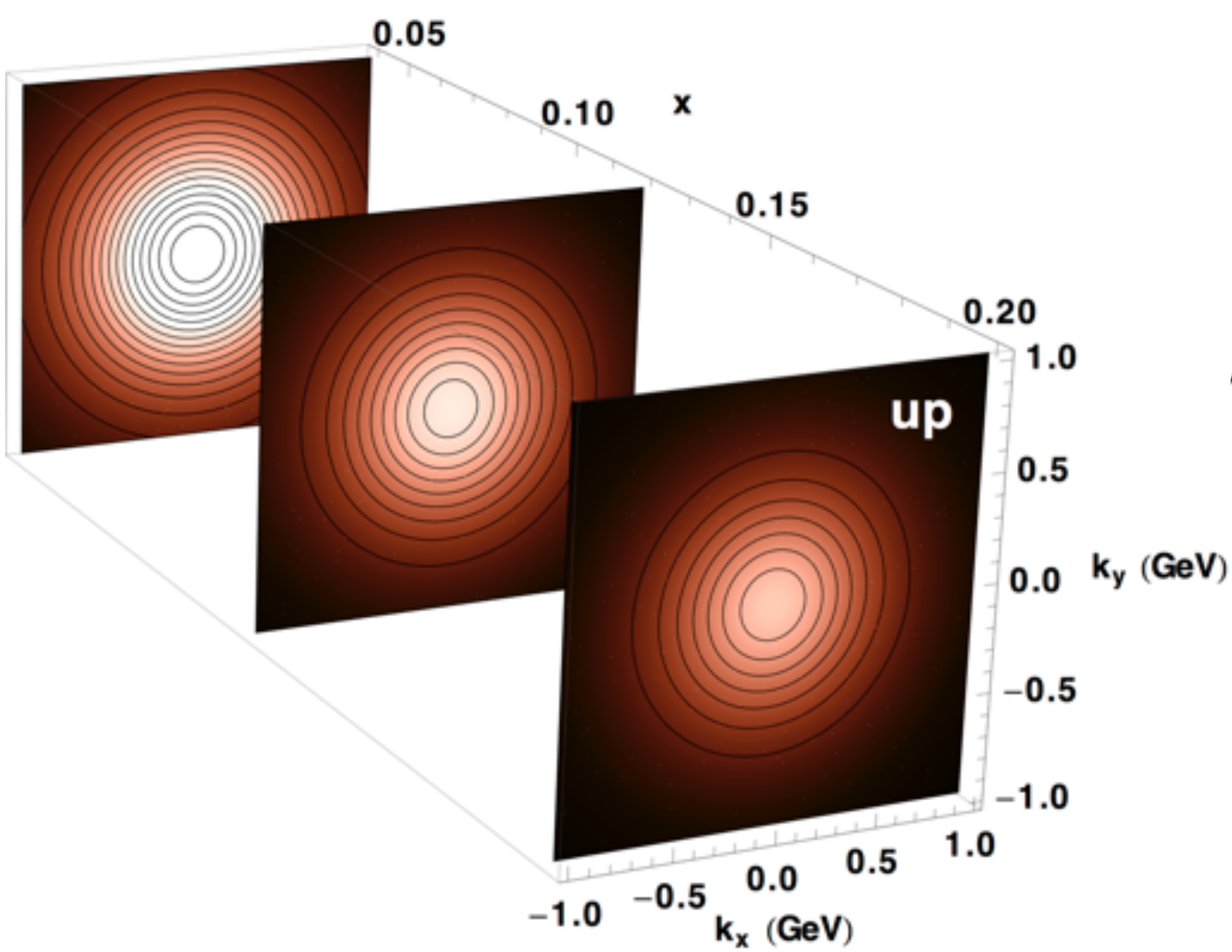

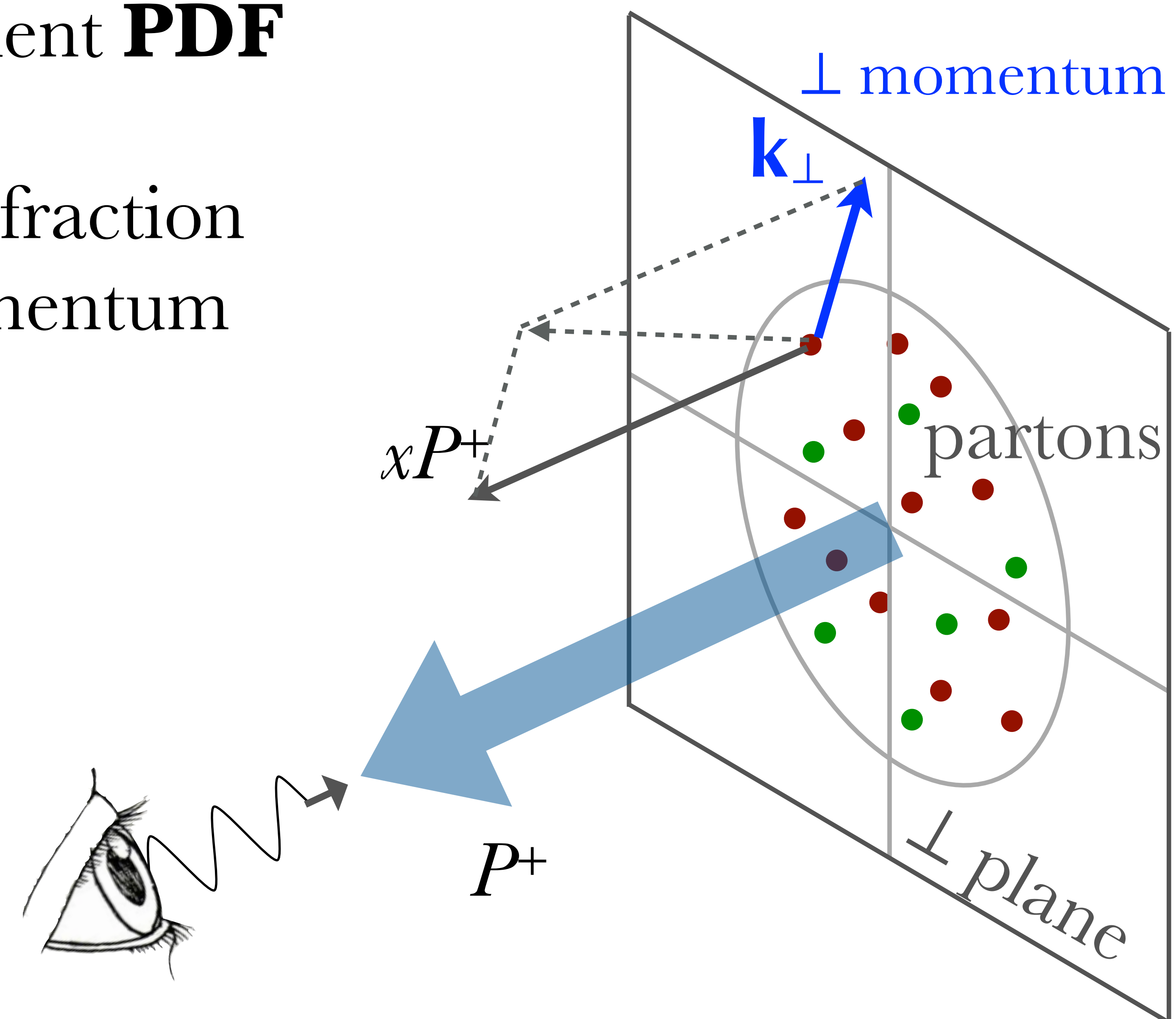

I will focus on
<u>unpolarised</u> <u>quark</u> TMDs

Factorising processes

- Processes for which TMD factorisation has been **proven**:

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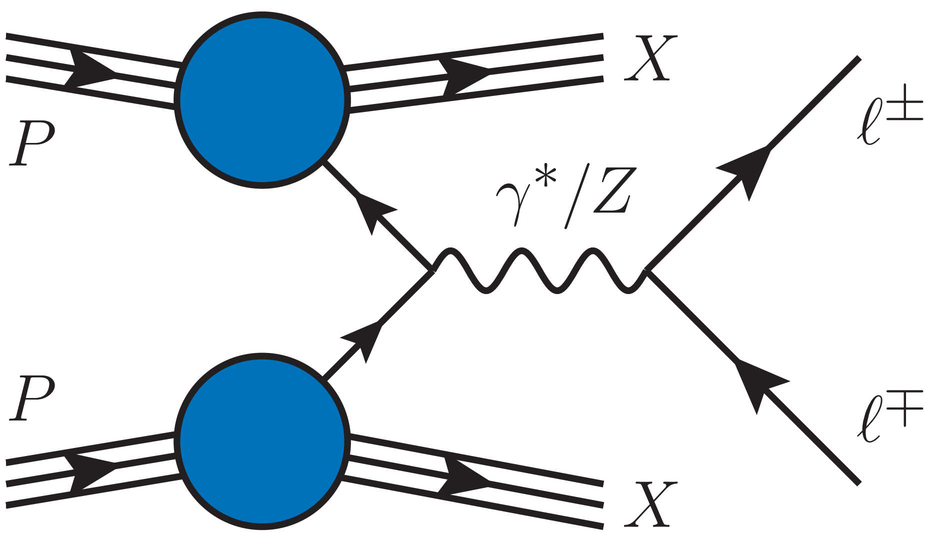
Drell-Yan



$PP \longrightarrow \ell^{\pm}\ell^{\mp}\ X$

- **Two** TMD **PDFs**
- Lots of data:
  - low-energy: FNAL
  - mid-energy: RHIC
  - high-energy: Tevatron, LHC

Factorising processes

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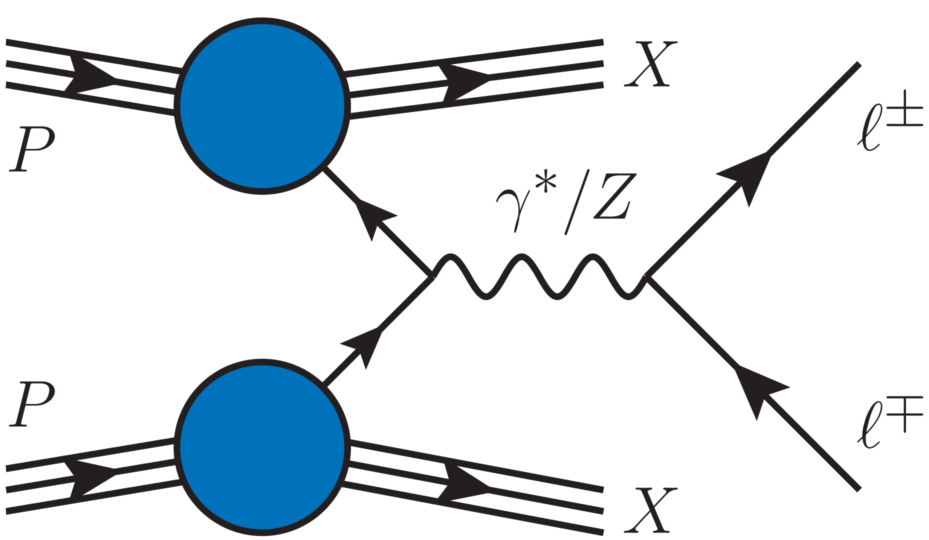
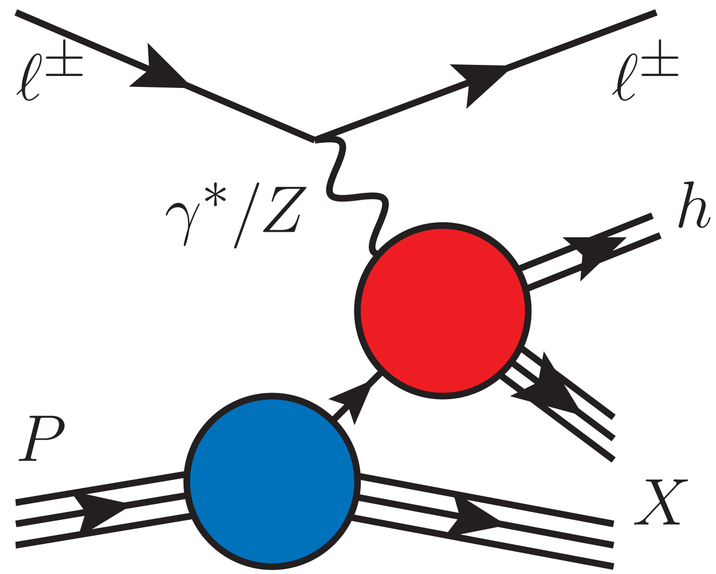
Drell-Yan



$PP \longrightarrow \ell^{\pm}\ell^{\mp}\, X$

Semi-inclusive DIS



$P\ell^{\pm} \longrightarrow \ell^{\pm} h\, X$

- **Two** TMD **PDFs**
- Lots of data:
  - low-energy: FNAL
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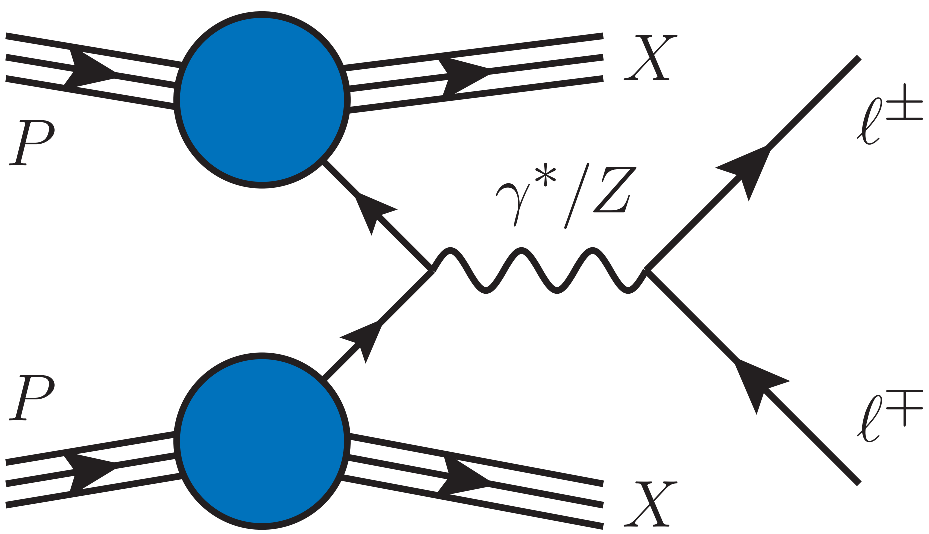
- One TMD **PDF** one **FF**
- many precise data points:
  - HERMES at DESY
  - COMPASS at CERN

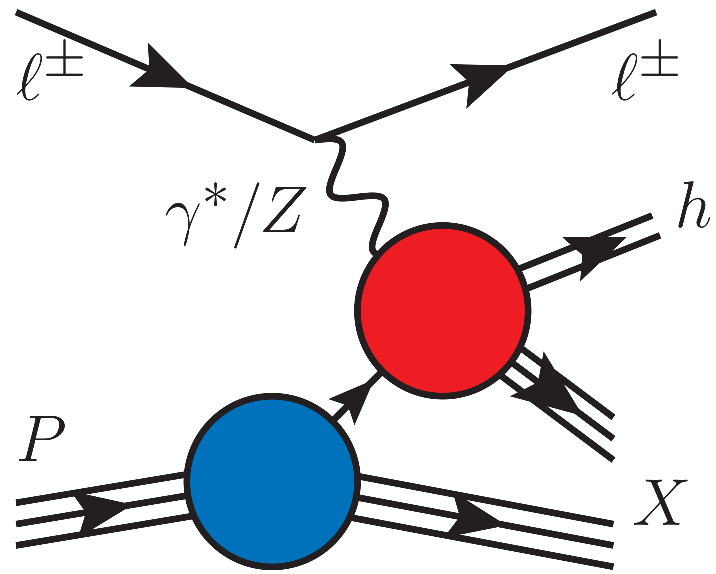
Factorising processes

- Processes for which TMD factorisation has been **proven**:

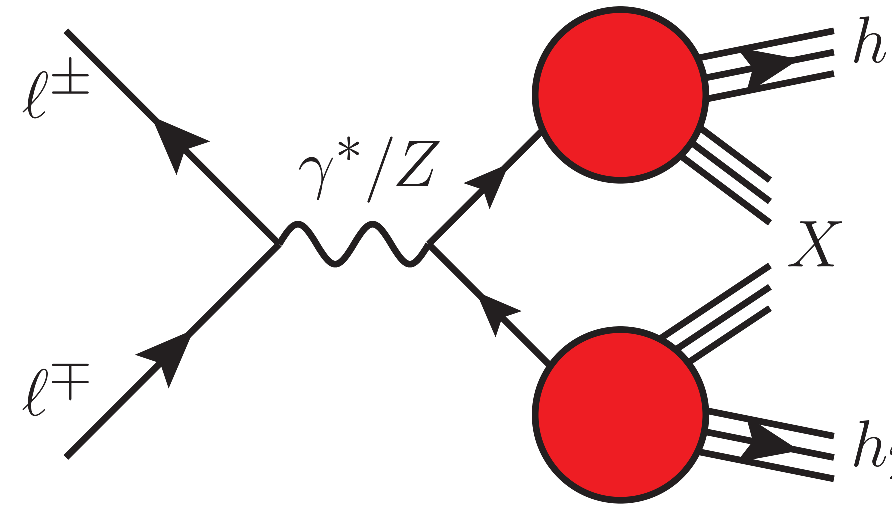
Drell-Yan	Semi-inclusive DIS	e^+e^- annihilation



$PP \longrightarrow \ell^\pm \ell^\mp\ X$



$P\ell^\pm \longrightarrow \ell^\pm h\ X$



$\ell^\pm \ell^\mp \to h_1 h_2\ X$

Drell-Yan

- **Two** TMD **PDFs**
- Lots of data:
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  - mid-energy: RHIC
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Semi-inclusive DIS

- One TMD **PDF** one **FF**
- many precise data points:
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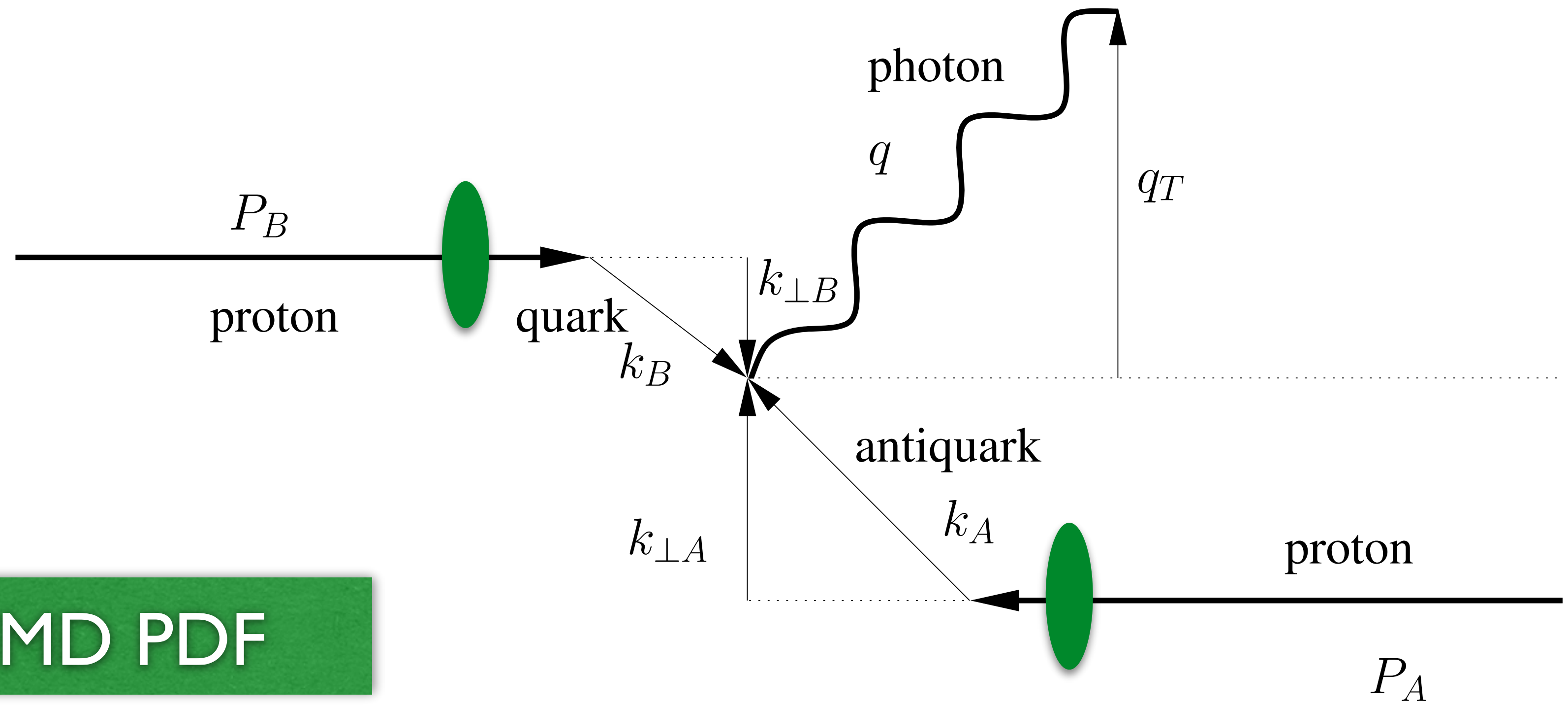
e^+e^- annihilation

- **Two** TMD **FFs**
- DIA process from:
  - BELLE at KEK
  - BABAR at SLAC

missing!

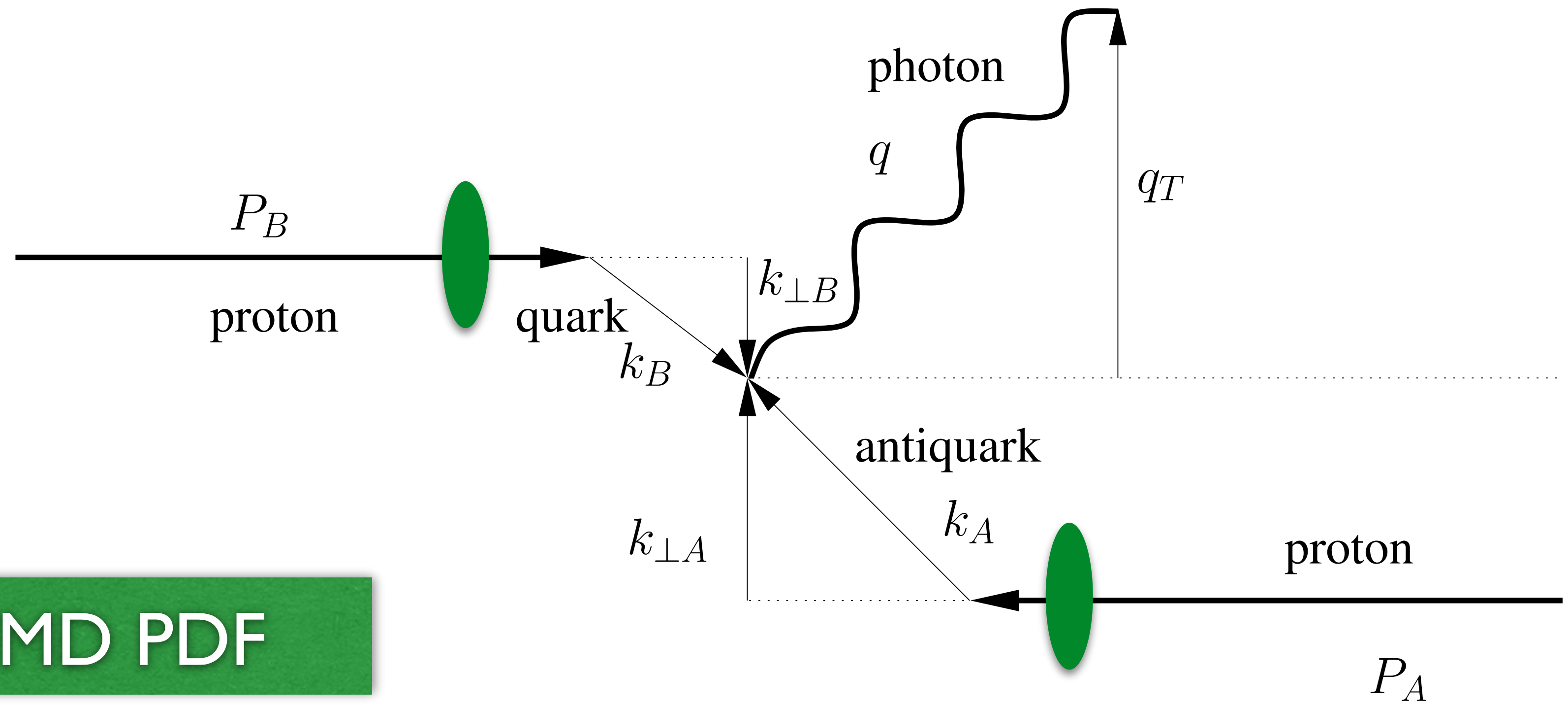
TMD factorisation for DY



quark TMD PDF

$$\frac{d\sigma}{dq_T\,dy\,dQ} \propto x_A x_B H^{DY}(Q,\mu) \sum_q c_q(Q^2) \int d^2\mathbf{k}_{\perp A}\, d^2\mathbf{k}_{\perp B}\, \boxed{F^{\bar{q}}(x_A, \mathbf{k}^2_{\perp A}; \mu, \zeta_A)}\, \boxed{F^{q}(x_B, \mathbf{k}^2_{\perp B}; \mu, \zeta_B)}\, \delta^{(2)}(\mathbf{k}_{\perp A} + \mathbf{k}_{\perp B} - \mathbf{q}_T)$$

TMD factorisation for DY



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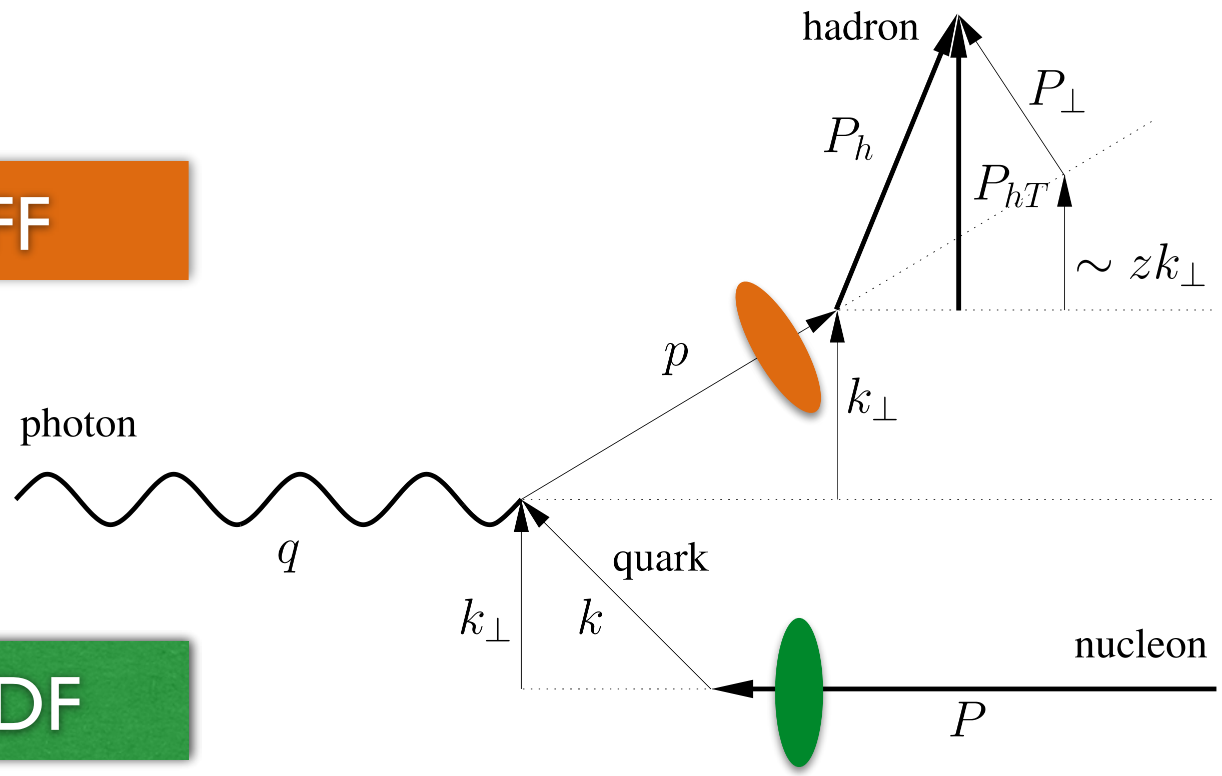
$$= x_A x_B H^{DY}(Q,\mu) \sum_q c_q(Q^2) \int \frac{db_T}{2\pi}\, b_T\, J_0(b_T q_T)\, \boxed{\hat{F}^{\bar{q}}(x_A, b_T^2; \mu, \zeta_A)}\, \boxed{\hat{F}^{q}(x_B, b_T^2; \mu, \zeta_B)}$$

TMD factorisation for SIDIS

quark TMD FF

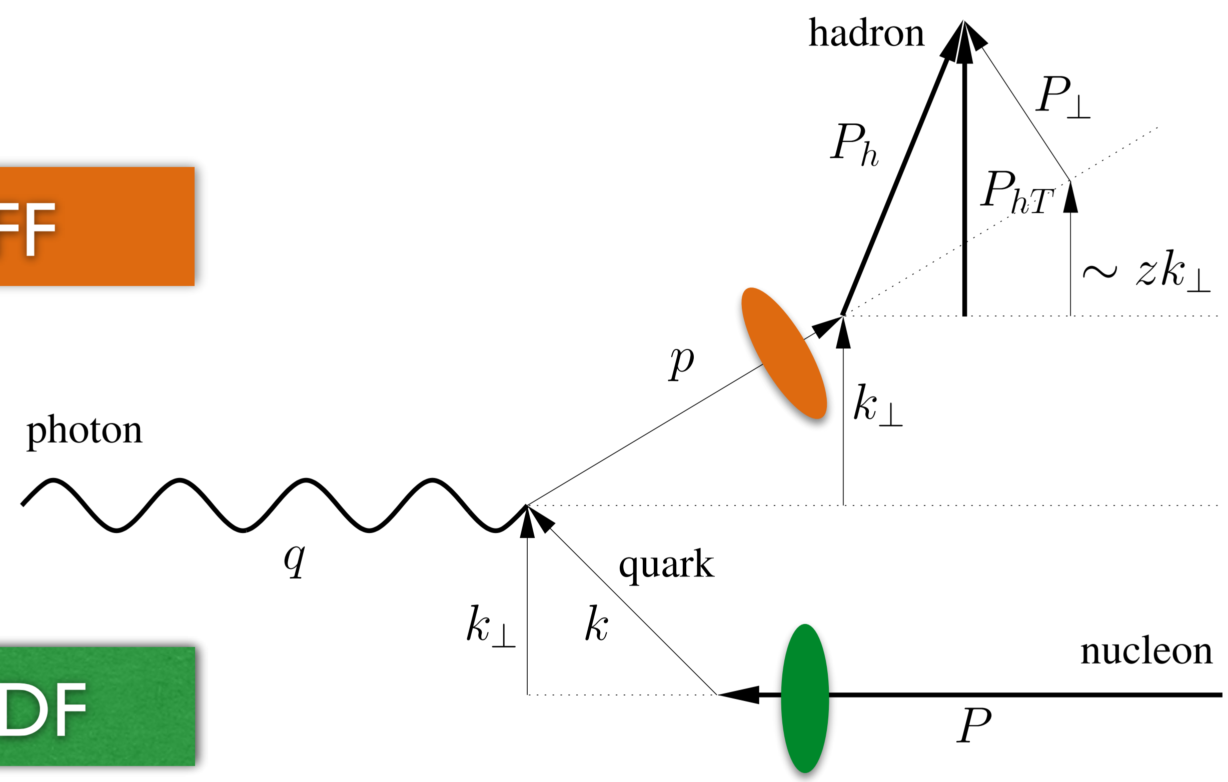
quark TMD PDF



$$\frac{d\sigma}{dx\,dz\,dq_T\,dQ} \propto xH^{SIDIS}(Q,\mu)\sum_q c_q(Q^2)\int d^2\mathbf{k}_\perp \int \frac{d^2\mathbf{P}_\perp}{z^2} \boxed{F^q(x,\mathbf{k}_\perp^2;\mu,\zeta_A)}\ \boxed{D^{q\to h}(z,\mathbf{P}_\perp^2;\mu,\zeta_B)}\ \delta^{(2)}(\mathbf{k}_\perp + \mathbf{P}_\perp/z + \mathbf{q}_T)$$

TMD factorisation for SIDIS

quark TMD FF



quark TMD PDF

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TMD factorisation for SIDIS

quark TMD FF

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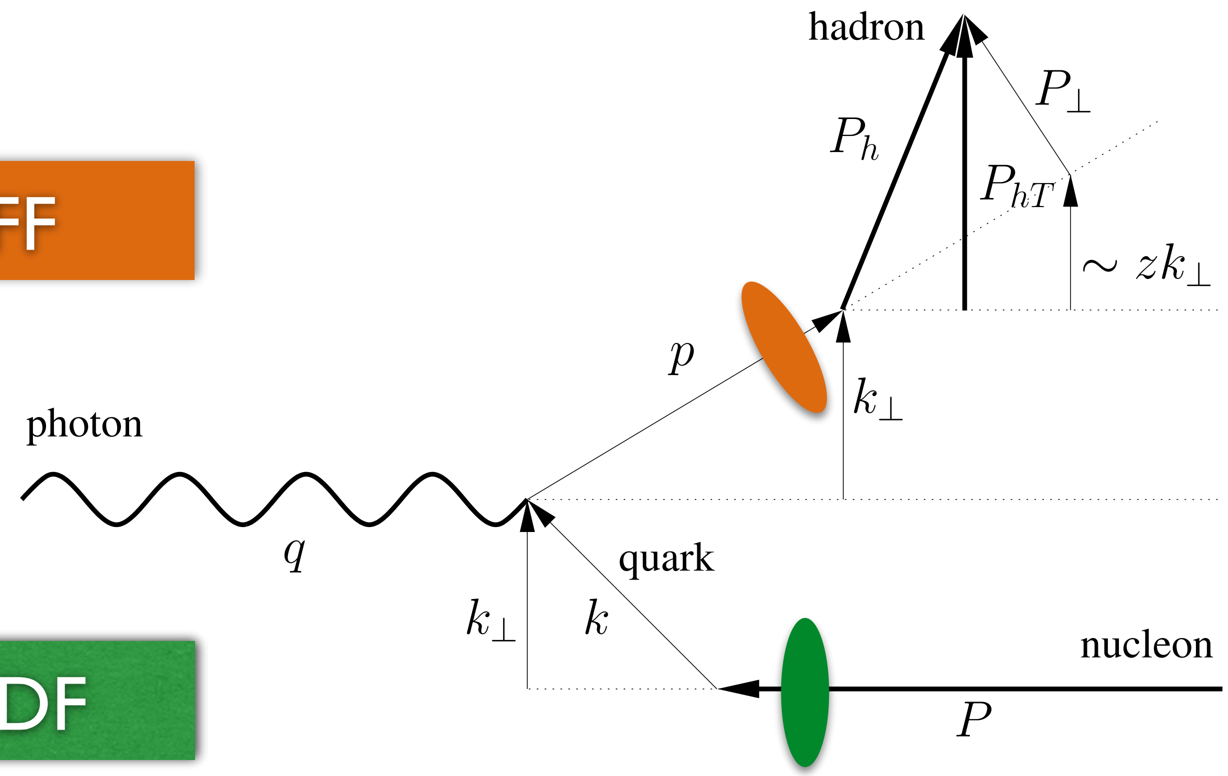

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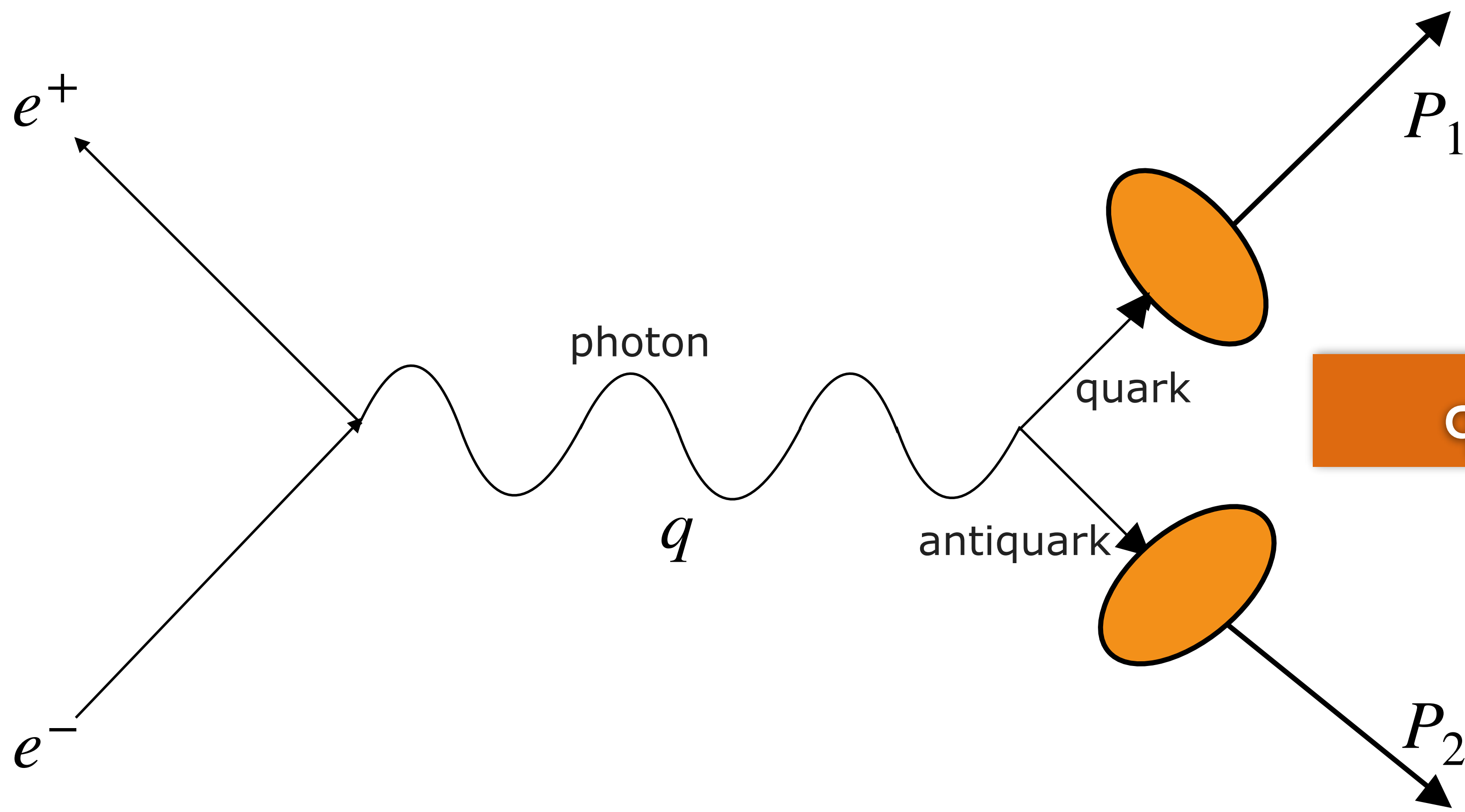
Kinematics

$$q_T^\mu = -\frac{P_{hT}^\mu}{z} - 2x\frac{q_T^2}{Q^2}P^\mu \approx -\frac{P_{hT}^\mu}{z} \longleftarrow \text{(if } q_T^2, P_{hT}^2 \ll Q^2\text{)} \longrightarrow P_{hT}^\mu \approx zk_\perp + P_\perp$$

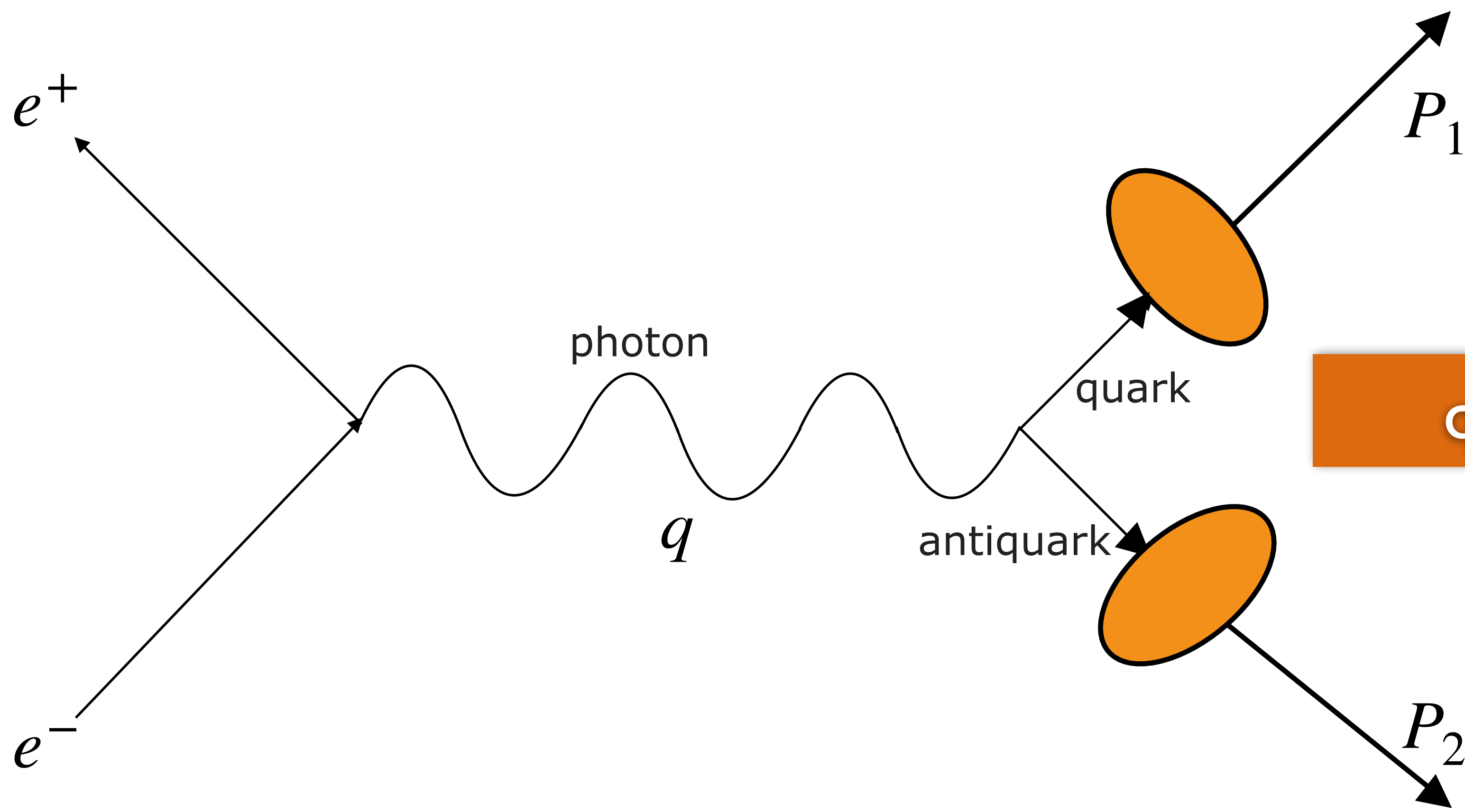
TMD factorisation for DIA



quark TMD FF

$$\frac{d\sigma}{dq_T\, dy\, dQ} \propto H^{DIA}(Q,\mu) \sum_q c_q(Q^2) \int \frac{d^2\mathbf{P}_{\perp 1}}{z_1^2} \frac{d^2\mathbf{P}_{\perp 2}}{z_2^2} \boxed{D^{\bar{q}}(z_A, \mathbf{P}^2_{\perp 1}; \mu, \zeta_1)} \boxed{D^{q}(z_B, \mathbf{P}^2_{\perp 2}; \mu, \zeta_2)} \delta^{(2)}(\mathbf{P}_{\perp 1}/z_1 + \mathbf{P}_{\perp 2}/z_2 - \mathbf{q}_T)$$

TMD factorisation for DIA



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TMD structure

$$F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) = \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) \qquad : A$$

$$\times \exp\left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} \qquad : B$$

$$\times \exp\left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} \qquad : C$$

TMD structure

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- matching to collinear PDF at $b_T \ll 1/\Lambda_{QCD}$
- **perturbative**

TMD structure

$$
\begin{aligned}
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\end{aligned}
$$

- matching to collinear PDF at $b_\mathrm{T} \ll 1/\Lambda_\mathrm{QCD}$
- **perturbative**

- CS and RGE evolution to large b_T
- **perturbative**

Perturbative accuracy

Accuracy	H and C	K and γ_F	γ_K	PDF and α_s evolution
LL	0	-	1	-
NLL	0	1	2	LO
NNLL	1	2	3	NLO
N^{3}LL	2	3	4	NNLO

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NLL $\quad C^0 \quad$ $\alpha_S^n \ln^{2n}\left(\frac{Q^2}{\mu_b^2}\right), \quad \alpha_S^n \ln^{2n-1}\left(\frac{Q^2}{\mu_b^2}\right)$

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NLL' $\quad \left(C^0 + \alpha_S C^1\right) \quad \alpha_S^n \ln^{2n}\left(\frac{Q^2}{\mu_b^2}\right), \quad \alpha_S^n \ln^{2n-1}\left(\frac{Q^2}{\mu_b^2}\right)$

Perturbative accuracy

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$$\text{NLL} \qquad C^0 \qquad \alpha_S^n \ln^{2n}\left(\frac{Q^2}{\mu_b^2}\right), \quad \alpha_S^n \ln^{2n-1}\left(\frac{Q^2}{\mu_b^2}\right)$$

$$\text{NLL'} \qquad \left(C^0 + \alpha_S C^1\right) \qquad \alpha_S^n \ln^{2n}\left(\frac{Q^2}{\mu_b^2}\right), \quad \alpha_S^n \ln^{2n-1}\left(\frac{Q^2}{\mu_b^2}\right)$$

same logarithmic accuracy (difference = NNLL)

TMD structure

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$$\times \boxed{\exp\left\{ K(b_{\boxed{*}}; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\}} \qquad : B$$

$$\times \exp\left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} \qquad : C$$

- matching to collinear PDF at $b_T \ll 1/\Lambda_{QCD}$
- **perturbative**

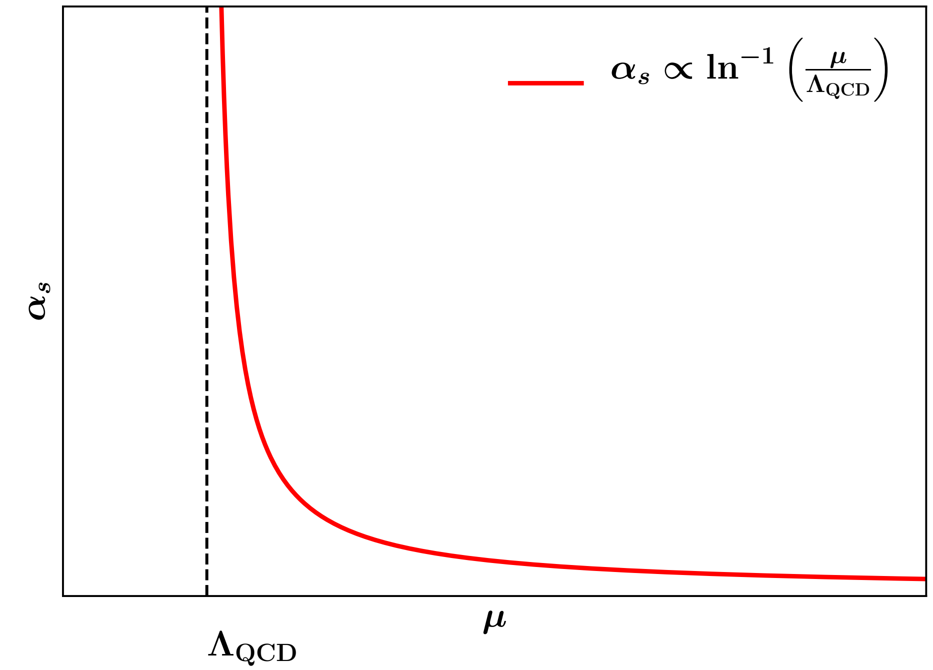
$$\left(\mu_b = 2e^{-\gamma_E}/b_* \right)$$

- CS and RGE evolution to large b_T
- **perturbative**

- b_* prescription to avoid Landau pole

Non-perturbative: b* and f_{NP}

$$\alpha_s(\mu_b) = \alpha\left(\frac{2e^{-\gamma_E}}{b}\right) \gg 1 \quad \text{for large } b \text{ values}$$



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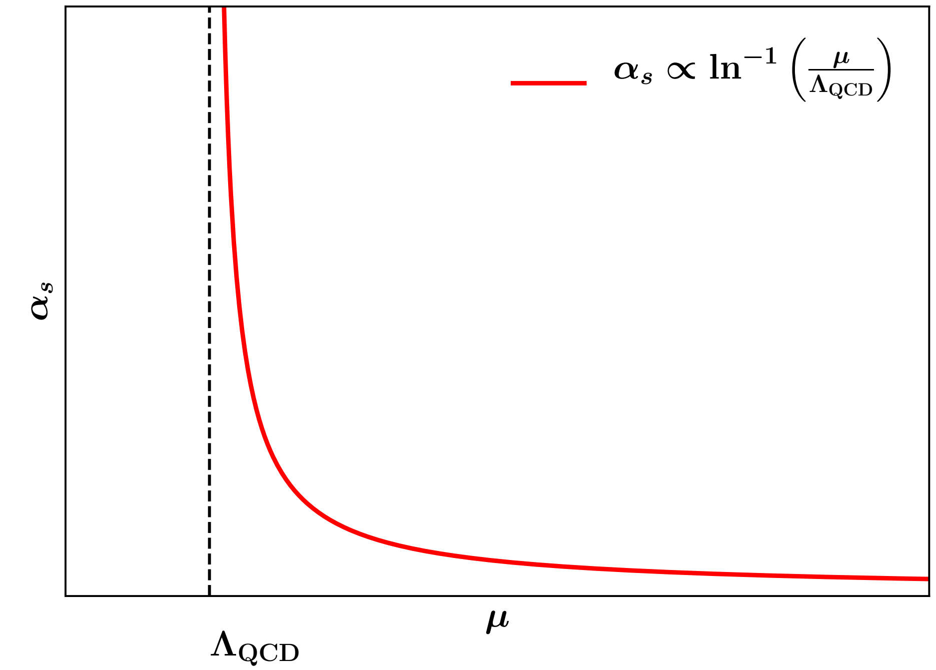
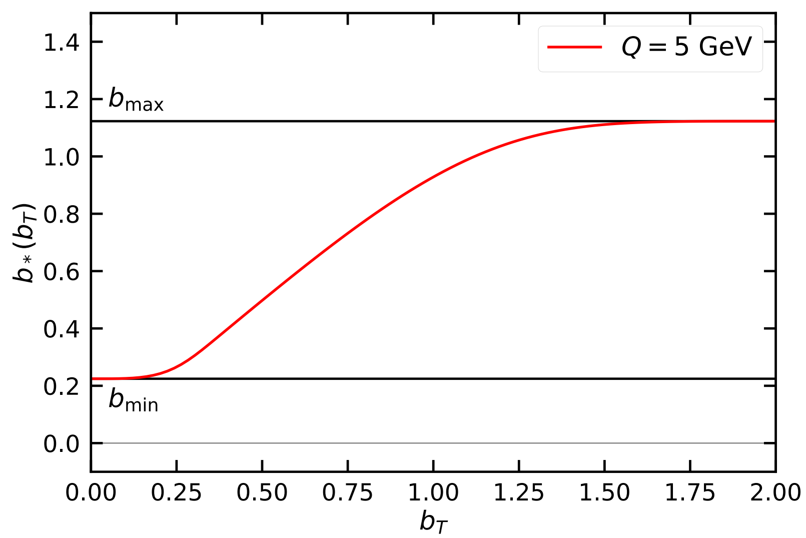


$$b_*(b) = b_{\max}\left(\frac{1-\exp\left(-\frac{b^4}{b_{\max}^4}\right)}{1-\exp\left(-\frac{b^4}{b_{\min}^4}\right)}\right)^{\frac{1}{4}}$$



$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = 2e^{-\gamma_E}/Q$$

TMD structure

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$$\times \boxed{\exp\left\{K(b_{\boxed{*}}; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'}\right]\right\}} \quad : B$$

$$\times \boxed{\exp\left\{g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}}\right\}} \quad f_{NP} \quad : C$$

- matching to collinear PDF at $b_T \ll 1/\Lambda_{QCD}$
- **perturbative**

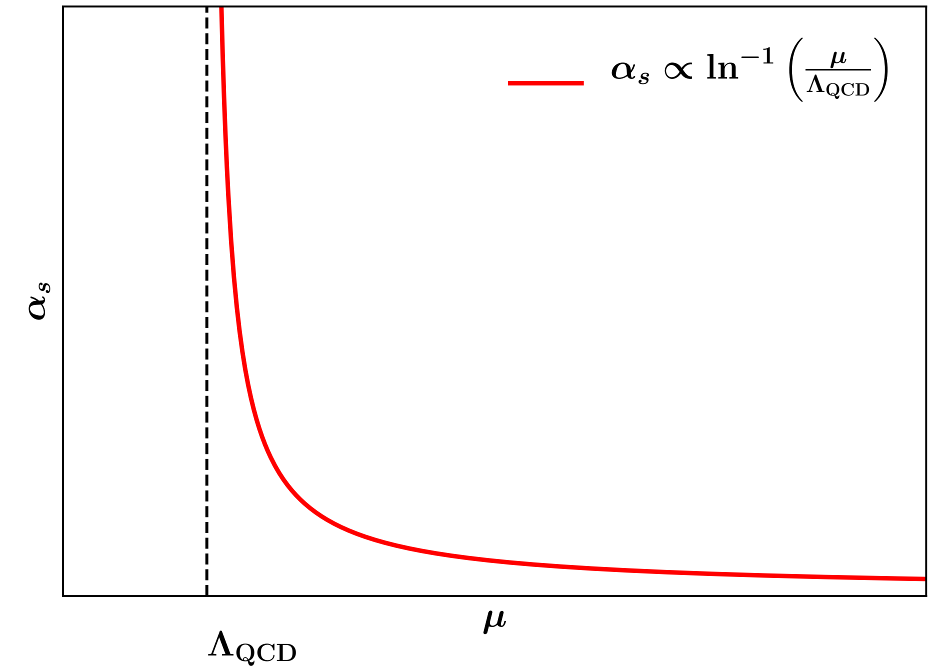
$$(\mu_b = 2e^{-\gamma_E}/b_*)$$

- CS and RGE evolution to large b_T
- **perturbative**

- b_* prescription to avoid Landau pole
- f_{NP} "parametrises" the **non-perturbative** transverse modes
- **fit** f_{NP} to data

Non-perturbative: b* and f_{NP}

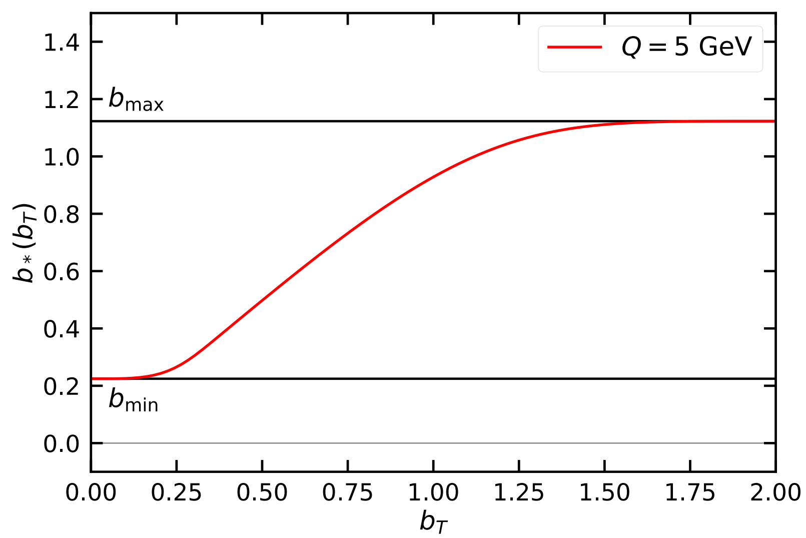
$$\alpha_s(\mu_b) = \alpha\left(\frac{2e^{-\gamma_E}}{b}\right) \gg 1 \quad \text{for large } b \text{ values}$$



$$b_*(b) = b_{\max}\left(\frac{1-\exp\left(-\frac{b^4}{b_{\max}^4}\right)}{1-\exp\left(-\frac{b^4}{b_{\min}^4}\right)}\right)^{\frac{1}{4}}$$

$$b_{\max} = 2e^{-\gamma_E}$$

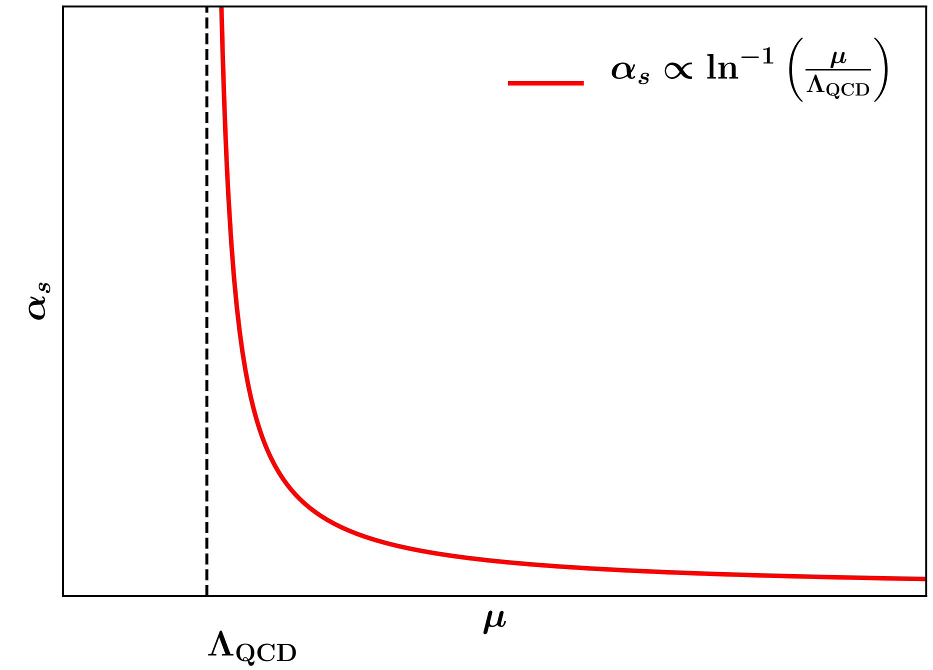
$$b_{\min} = 2e^{-\gamma_E}/Q$$



$$F(x,b;\mu,\zeta) = \left[\frac{F(x,b;\mu,\zeta)}{F(x,b_*(b);\mu,\zeta)}\right] F(x,b_*(b);\mu,\zeta)$$

Non-perturbative: b* and f_{NP}

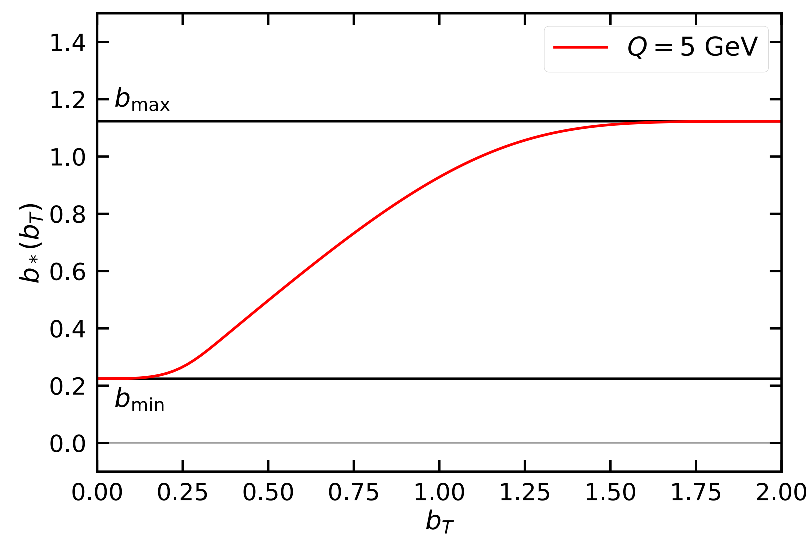
$$\alpha_s(\mu_b) = \alpha\left(\frac{2e^{-\gamma_E}}{b}\right) \gg 1 \quad \text{for large } b \text{ values}$$



$$b_*(b) = b_{\max}\left(\frac{1-\exp\left(-\frac{b^4}{b_{\max}^4}\right)}{1-\exp\left(-\frac{b^4}{b_{\min}^4}\right)}\right)^{\frac{1}{4}}$$

$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = 2e^{-\gamma_E}/Q$$



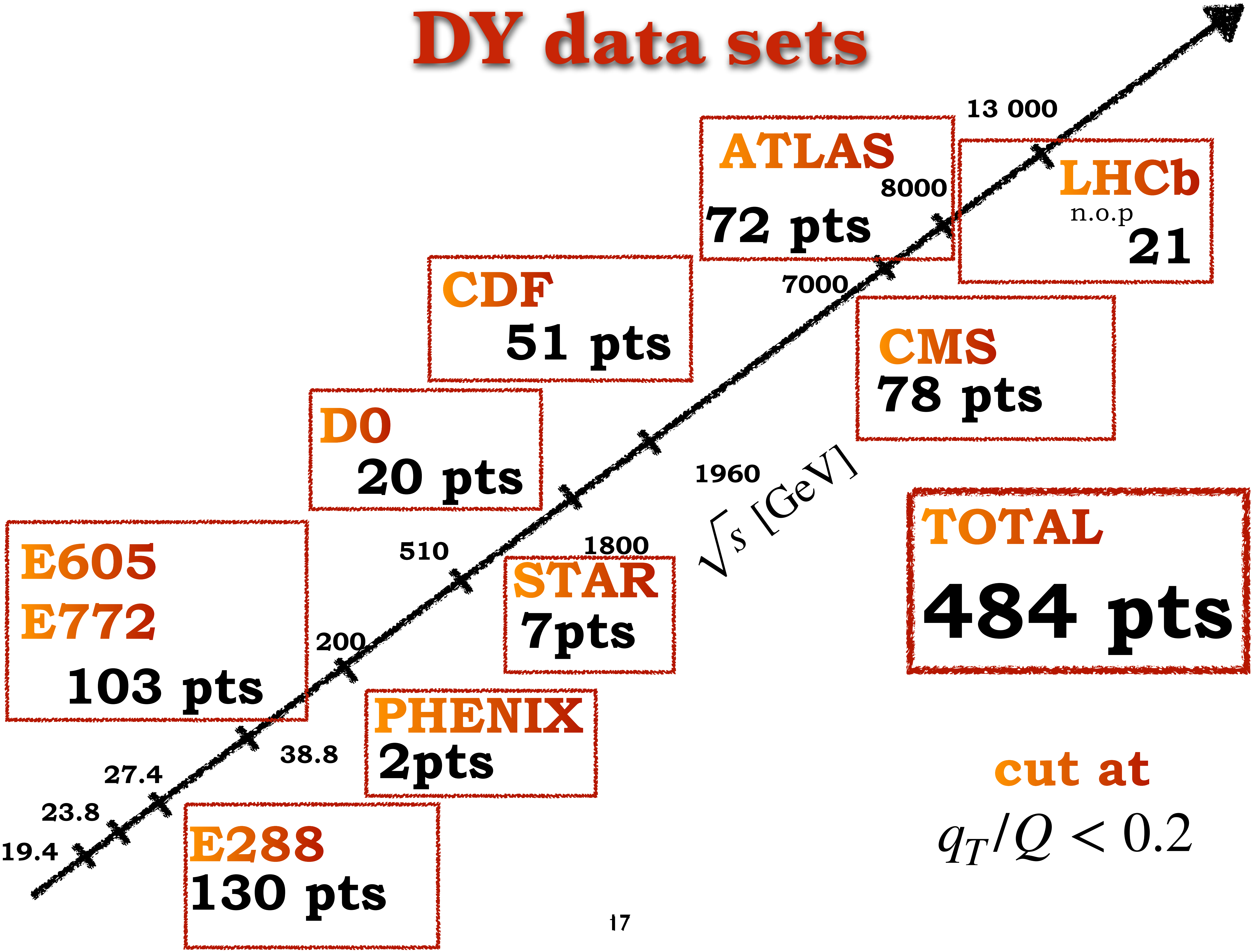
f_{NP}

$$F(x,b;\mu,\zeta) = \left[\frac{F(x,b;\mu,\zeta)}{F(x,b_*(b);\mu,\zeta)}\right] F(x,b_*(b);\mu,\zeta)$$

- NP is unavoidable: intrinsically tied to regularisation procedure
- There is not a universal form factor:
  - depends on details of b* and collinear PDFs
  - requires definition of a functional form
  - determined through a fit to experimental data

The extraction of TMD PDFs and FFs from low-p_T data

DY data sets



ATLAS
8000
72 pts
13 000
LHCb
n.o.p
21
CDF
51 pts
7000
CMS
78 pts
D0
20 pts
1960
$\sqrt{s}$ [GeV]
TOTAL
484 pts
E605
E772
103 pts
510
1800
STAR
7pts
200
PHENIX
2pts
38.8
27.4
23.8
19.4
E288
130 pts
cut at
$q_T/Q < 0.2$

DY data sets

Experiment	N_{dat}	Observable	$\sqrt{s}$ [GeV]	Q [GeV]	y or x_F	Lepton cuts
E605	50	$Ed^3\sigma/d^3\boldsymbol{q}$	38.8	7 - 18	$x_F = 0.1$	-
E772	53	$Ed^3\sigma/d^3\boldsymbol{q}$	38.8	5 - 15	$0.1 < x_F < 0.3$	-
E288 200 GeV	30	$Ed^3\sigma/d^3\boldsymbol{q}$	19.4	4 - 9	$y = 0.40$	-
E288 300 GeV	39	$Ed^3\sigma/d^3\boldsymbol{q}$	23.8	4 - 12	$y = 0.21$	-
E288 400 GeV	61	$Ed^3\sigma/d^3\boldsymbol{q}$	27.4	5 - 14	$y = 0.03$	-
STAR 510	7	$d\sigma/d\lvert\boldsymbol{q}_T\rvert$	510	73 - 114	$\lvert y\rvert < 1$	$p_{T\ell} > 25$ GeV $\lvert\eta_\ell\rvert < 1$
PHENIX200	2	$d\sigma/d\lvert\boldsymbol{q}_T\rvert$	200	4.8 - 8.2	$1.2 < y < 2.2$	-
CDF Run I	25	$d\sigma/d\lvert\boldsymbol{q}_T\rvert$	1800	66 - 116	Inclusive	-
CDF Run II	26	$d\sigma/d\lvert\boldsymbol{q}_T\rvert$	1960	66 - 116	Inclusive	-
D0 Run I	12	$d\sigma/d\lvert\boldsymbol{q}_T\rvert$	1800	75 - 105	Inclusive	-
D0 Run II	5	$(1/\sigma)d\sigma/d\lvert\boldsymbol{q}_T\rvert$	1960	70 - 110	Inclusive	-
D0 Run II (μ)	3	$(1/\sigma)d\sigma/d\lvert\boldsymbol{q}_T\rvert$	1960	65 - 115	$\lvert y\rvert < 1.7$	$p_{T\ell} > 15$ GeV $\lvert\eta_\ell\rvert < 1.7$
LHCb 7 TeV	7	$d\sigma/d\lvert\boldsymbol{q}_T\rvert$	7000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$
LHCb 8 TeV	7	$d\sigma/d\lvert\boldsymbol{q}_T\rvert$	8000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$
LHCb 13 TeV	7	$d\sigma/d\lvert\boldsymbol{q}_T\rvert$	13000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$
CMS 7 TeV	4	$(1/\sigma)d\sigma/d\lvert\boldsymbol{q}_T\rvert$	7000	60 - 120	$\lvert y\rvert < 2.1$	$p_{T\ell} > 20$ GeV $\lvert\eta_\ell\rvert < 2.1$
CMS 8 TeV	4	$(1/\sigma)d\sigma/d\lvert\boldsymbol{q}_T\rvert$	8000	60 - 120	$\lvert y\rvert < 2.1$	$p_{T\ell} > 15$ GeV $\lvert\eta_\ell\rvert < 2.1$
CMS 13 TeV	70	$d\sigma/d\lvert\boldsymbol{q}_T\rvert$	13000	76 - 106	$\lvert y\rvert < 0.4$ $0.4 < \lvert y\rvert < 0.8$ $0.8 < \lvert y\rvert < 1.2$ $1.2 < \lvert y\rvert < 1.6$ $1.6 < \lvert y\rvert < 2.4$	$p_{T\ell} > 25$ GeV $\lvert\eta_\ell\rvert < 2.4$
ATLAS 7 TeV	6 6 6	$(1/\sigma)d\sigma/d\lvert\boldsymbol{q}_T\rvert$	7000	66 - 116	$\lvert y\rvert < 1$ $1 < \lvert y\rvert < 2$ $2 < \lvert y\rvert < 2.4$	$p_{T\ell} > 20$ GeV $\lvert\eta_\ell\rvert < 2.4$
ATLAS 8 TeV on-peak	6 6 6 6 6 6	$(1/\sigma)d\sigma/d\lvert\boldsymbol{q}_T\rvert$	8000	66 - 116	$\lvert y\rvert < 0.4$ $0.4 < \lvert y\rvert < 0.8$ $0.8 < \lvert y\rvert < 1.2$ $1.2 < \lvert y\rvert < 1.6$ $1.6 < \lvert y\rvert < 2$ $2 < \lvert y\rvert < 2.4$	$p_{T\ell} > 20$ GeV $\lvert\eta_\ell\rvert < 2.4$
ATLAS 8 TeV off-peak	4 8	$(1/\sigma)d\sigma/d\lvert\boldsymbol{q}_T\rvert$	8000	46 - 66 116 - 150	$\lvert y\rvert < 2.4$	$p_{T\ell} > 20$ GeV $\lvert\eta_\ell\rvert < 2.4$
ATLAS 13 TeV	6	$(1/\sigma)d\sigma/d\lvert\boldsymbol{q}_T\rvert$	13000	66 - 113	$\lvert y\rvert < 2.5$	$p_{T\ell} > 27$ GeV $\lvert\eta_\ell\rvert < 2.5$
Total	484					

SIDIS data sets

Hermes

344 pts

Compass

1203 pts

TOTAL

1547 pts

cut at

$Q > 1.4$ GeV *(collinear factorisation)*

$0.2 < z < 0.7$ *(no exclusive processes)*

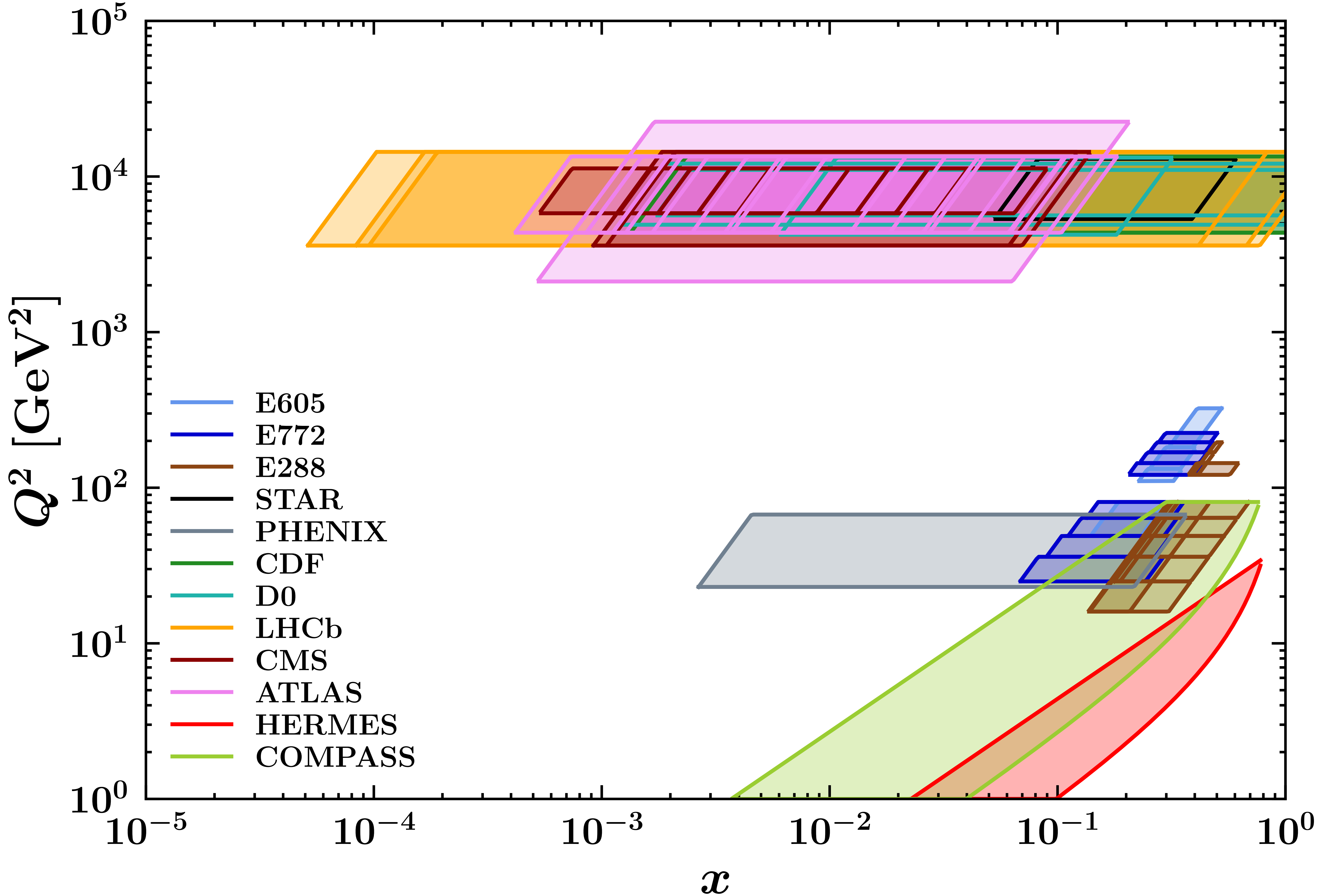
$$P_{hT}|_{max} = min[min[\,0.2Q,\, 0.5zQ\,] + 0.3 \text{ GeV},\, zQ\,]$$

$(q_T/Q < 0.2)$

SIDIS data sets

Experiment	N_{dat}	Observable	Channels	Q [GeV]	x	z	Phase space cuts
HERMES	344	$M(x, z, \lvert \boldsymbol{P}_{hT} \rvert, Q)$	$p \to \pi^+$ $p \to \pi^-$ $p \to K^+$ $p \to K^-$ $d \to \pi^+$ $d \to \pi^-$ $d \to K^+$ $d \to K^-$	1 - $\sqrt{15}$	$0.023 < x < 0.6$ (6 bins)	$0.1 < z < 1.1$ (8 bins)	$W^2 > 10$ GeV^2 $0.1 < y < 0.85$
COMPASS	1203	$M(x, z, \boldsymbol{P}_{hT}^2, Q)$	$d \to h^+$ $d \to h^-$	1 - 9 (5 bins)	$0.003 < x < 0.4$ (8 bins)	$0.2 < z < 0.8$ (4 bins)	$W^2 > 25$ GeV^2 $0.1 < y < 0.9$
Total	1547						

(x, Q^2) coverage



Experimental uncertainties

$$m_i \pm \sigma_{i,\mathrm{stat}} \pm \sigma_{i,\mathrm{unc}} \pm \sigma^{(1)}_{i,\mathrm{corr}} \pm \cdots \pm \sigma^{(k)}_{i,\mathrm{corr}}$$

uncorrelated

correlated

additive

multiplicative

$$\chi^2 = \sum_{i,j=1}^{n} (m_i - t_i)\, V^{-1}_{ij}\, (m_j - t_j)$$

$$\sigma^{(l)}_{i,\mathrm{corr}} \equiv \delta^{(l)}_{i,\mathrm{corr}} m_i$$

covariance matrix

$$V_{ij} = s_i^2 \delta_{ij} + \left(\sum_{l=1}^{k_a} \delta^{(l)}_{i,\mathrm{add}} \delta^{(l)}_{j,\mathrm{add}} + \sum_{l=1}^{k_m} \delta^{(l)}_{i,\mathrm{mult}} \delta^{(l)}_{j,\mathrm{mult}} \right) m_i m_j$$

Shifted predictions

systematic shift

$$d_i = \sum_{\alpha=1}^{k} \lambda_\alpha \boxed{\sigma_{i,\mathrm{corr}}^{(\alpha)}}$$

$$\frac{\partial \chi^2}{\partial \lambda_\alpha} = 0$$

nuisance
parameters

$$\bar{t}_i = t_i + d_i$$

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{m_i - \bar{t}_i}{s_i} \right)^2 + \sum_{\alpha=1}^{k} \lambda_\alpha^2$$

recover the form of
the uncorrelated definition

penalty term

TMD global fits

	Accuracy	HERMES	COMPASS	DY fixed target	DY collider	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL^-	✔	✔	✔	✔	1039	1.06
MAP22 arXiv:2206.07598	N^3LL^-	✔	✔	✔	✔	2031	1.06

+ brand new MAP24, N3LL, flavour-dependent, arXiv: 2405.13833

Global extractions: quick facts

PV17 (NLL)	SV19 (N3LL)	MAP22 (N3LL)	MAP24 (N3LL)
11 parameters	11 parameters	21 parameters	20(96) parameters
JHEP 06 (2017) 081	**JHEP 06 (2020) 137**	**JHEP 10 (2022) 127**	**arXiv: 2405.13833**

Functional forms

SV

$$\boxed{f_{NP}(x,b) = \exp\left(-\frac{\lambda_1(1-x)+\lambda_2 x + x(1-x)\lambda_5}{\sqrt{1+\lambda_3 x^{\lambda_4}\boldsymbol{b}^2}}\boldsymbol{b}^2\right)}$$

MAP

$$\boxed{f_{1\mathrm{NP}}(x,b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}}\right)}$$

$$\boxed{g_1(x) = N_1 \frac{(1-x)^\alpha\, x^\sigma}{(1-\hat{x})^\alpha\, \hat{x}^\sigma}}$$

Collins-Soper kernel

$$\boxed{\mathcal{D}(\mu,b) = \mathcal{D}_{\mathrm{resum}}(\mu, b^*(b)) + c_0 b b^*(b),}$$

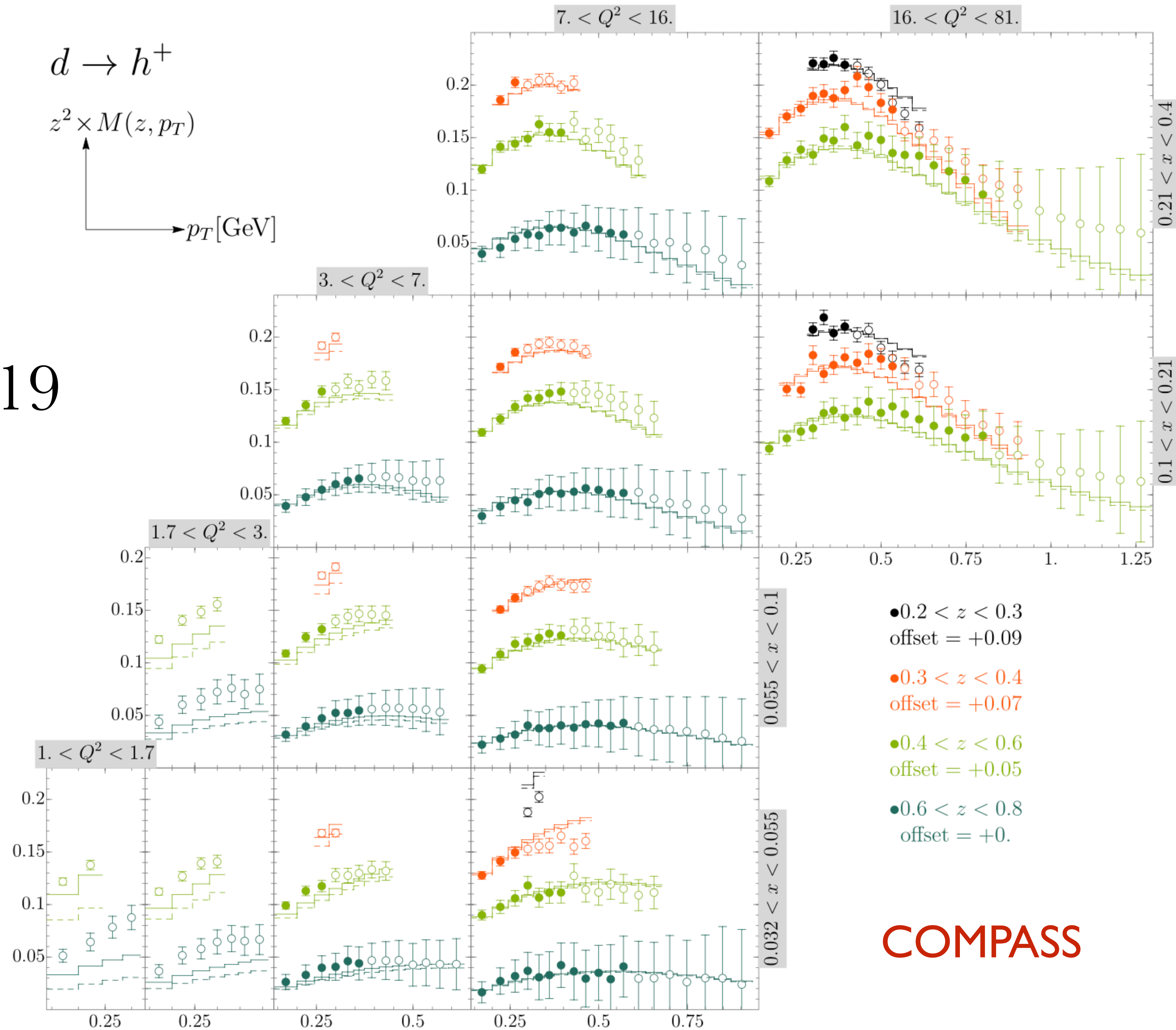
$$\boxed{b^*(b) = \frac{b}{\sqrt{1+\boldsymbol{b}^2/B_{\mathrm{NP}}^2}}.}$$

$$\boxed{f_{NP}(x,b_T^2)\exp\left\{g_K(b_T^2)\ln\frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\right\}}$$

$$\boxed{g_K(b_T^2) = -g_2^2\frac{b_T^2}{4}}$$

Fit quality: SIDIS

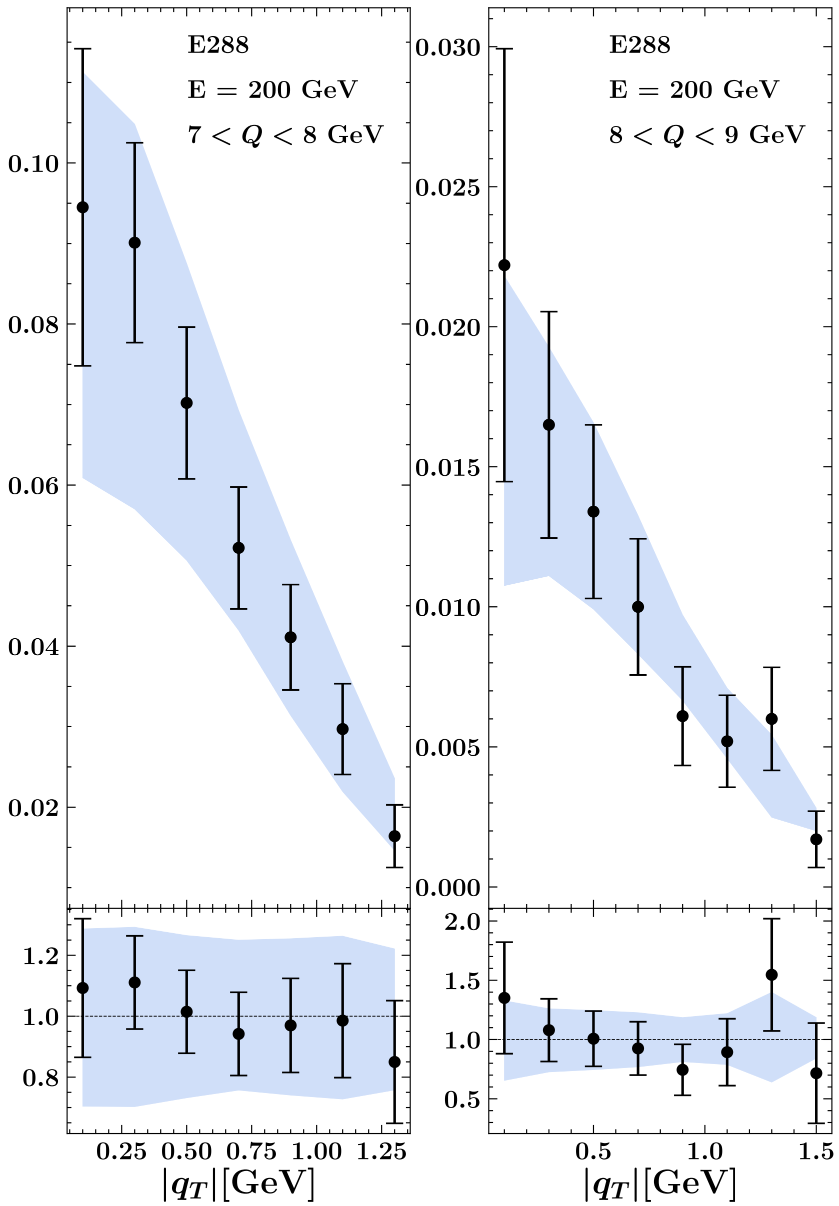
SV19



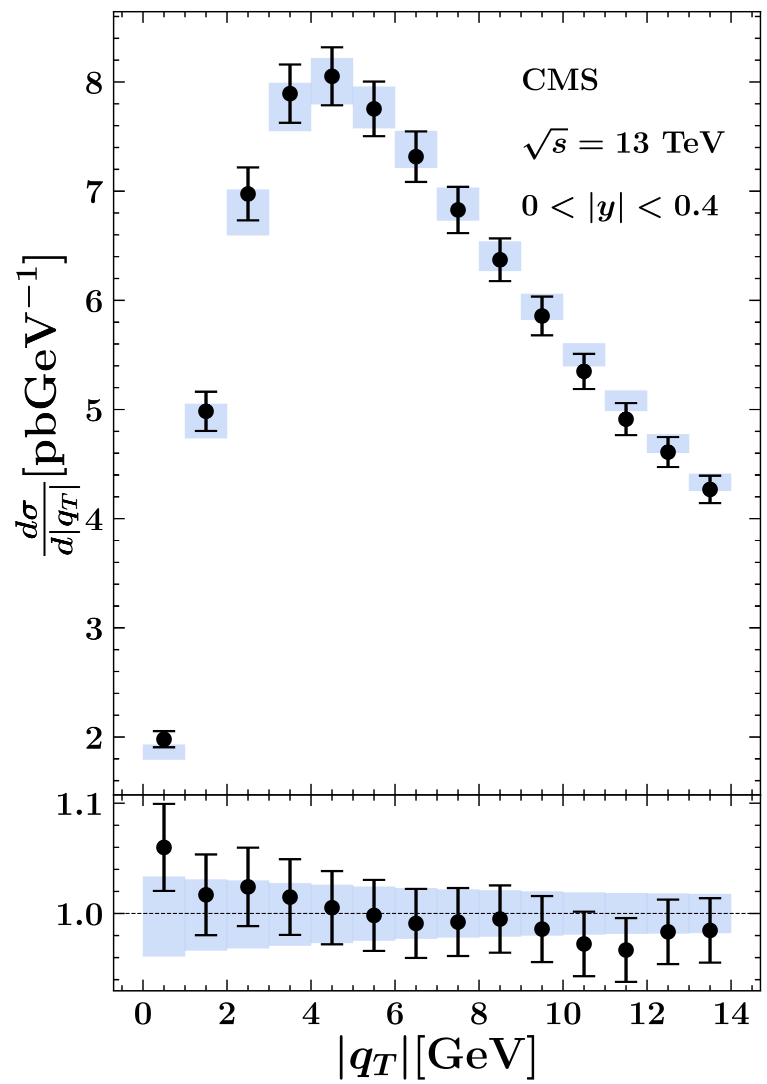
COMPASS

Fit quality: Drell-Yan

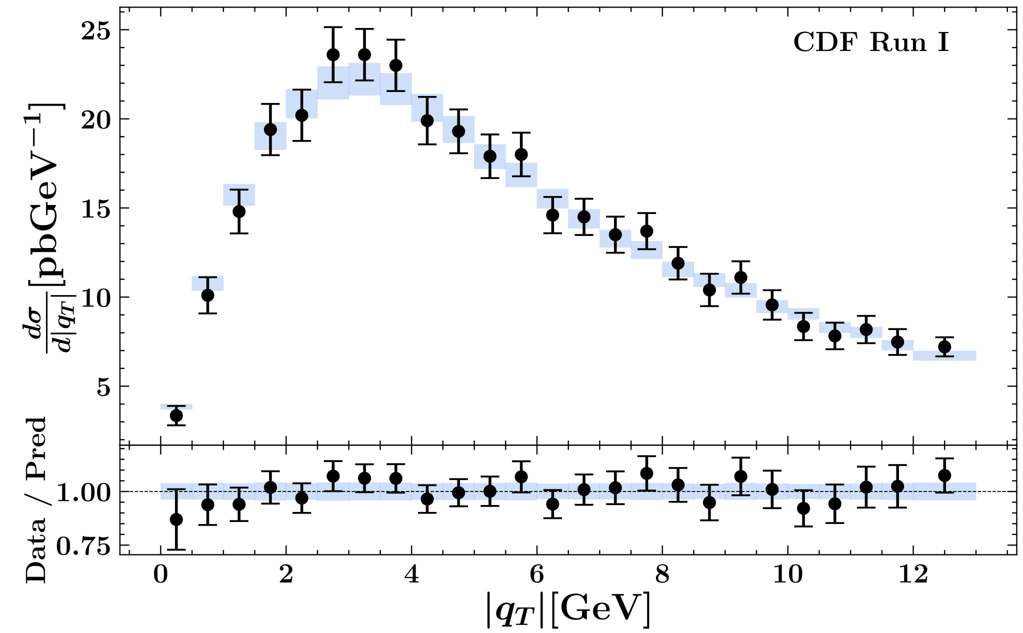
E288



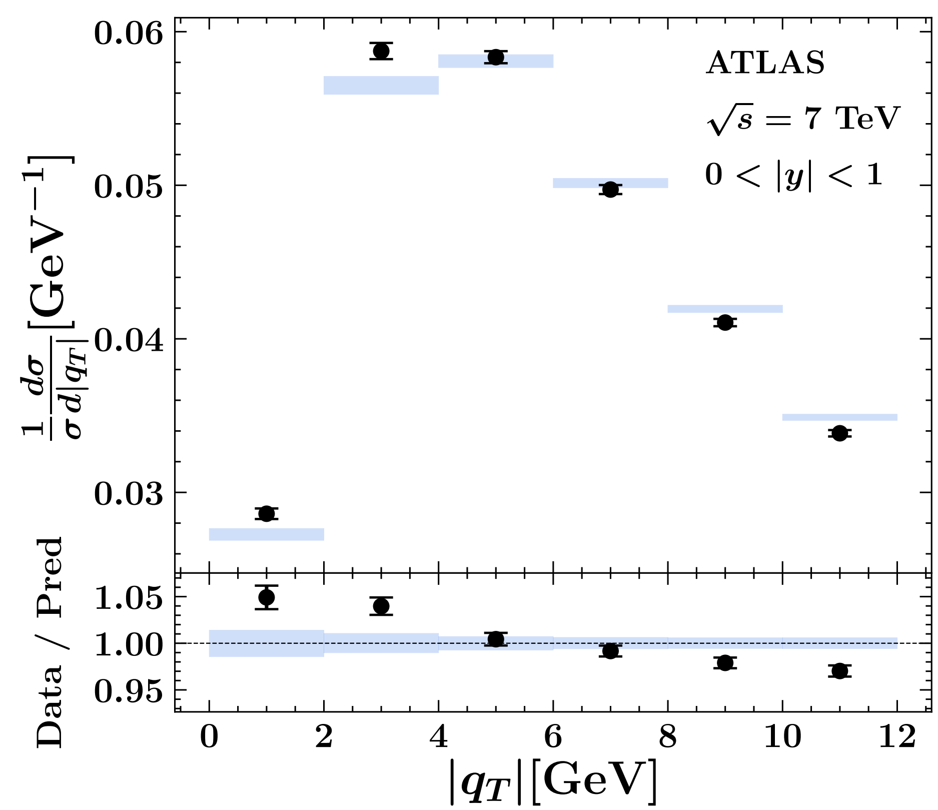
CMS



CDF

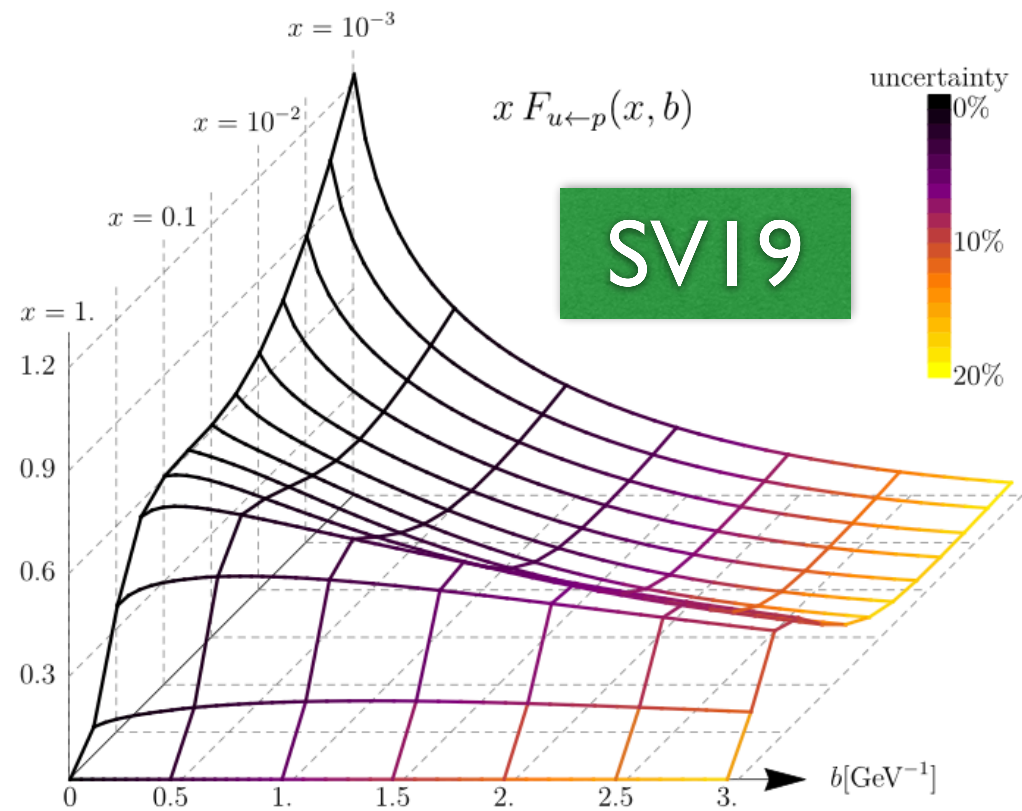


ATLAS

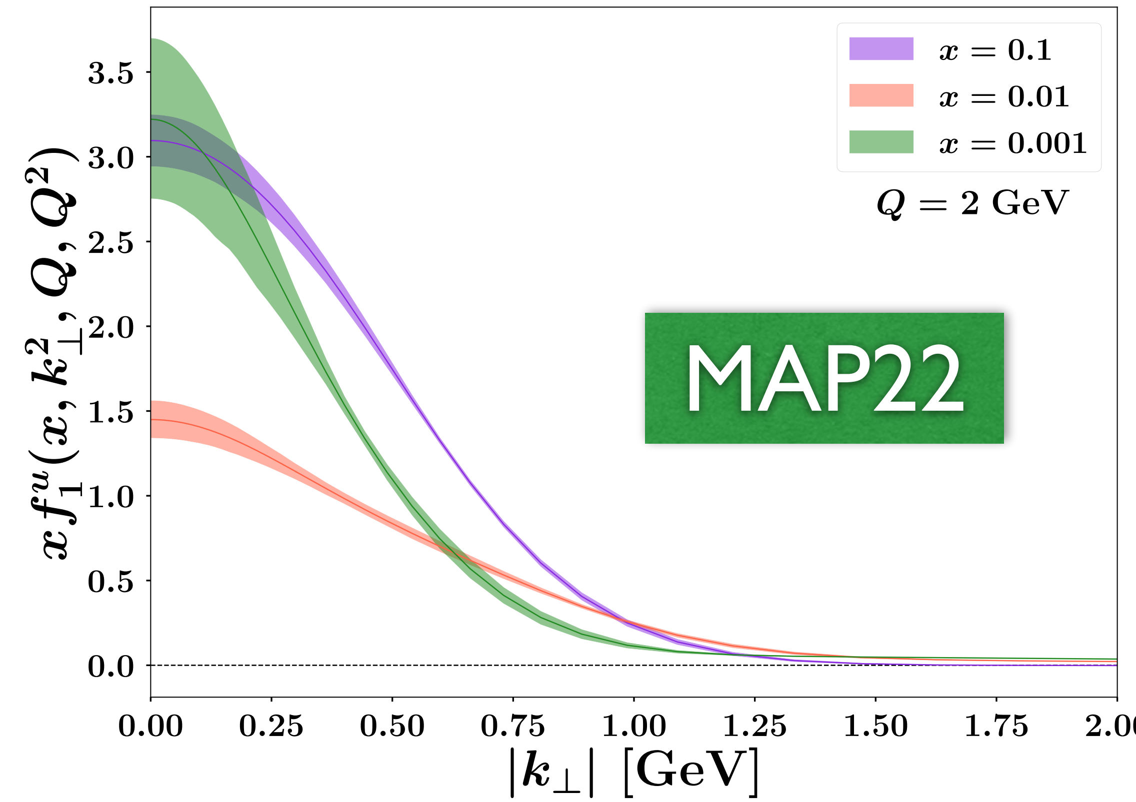


MAP

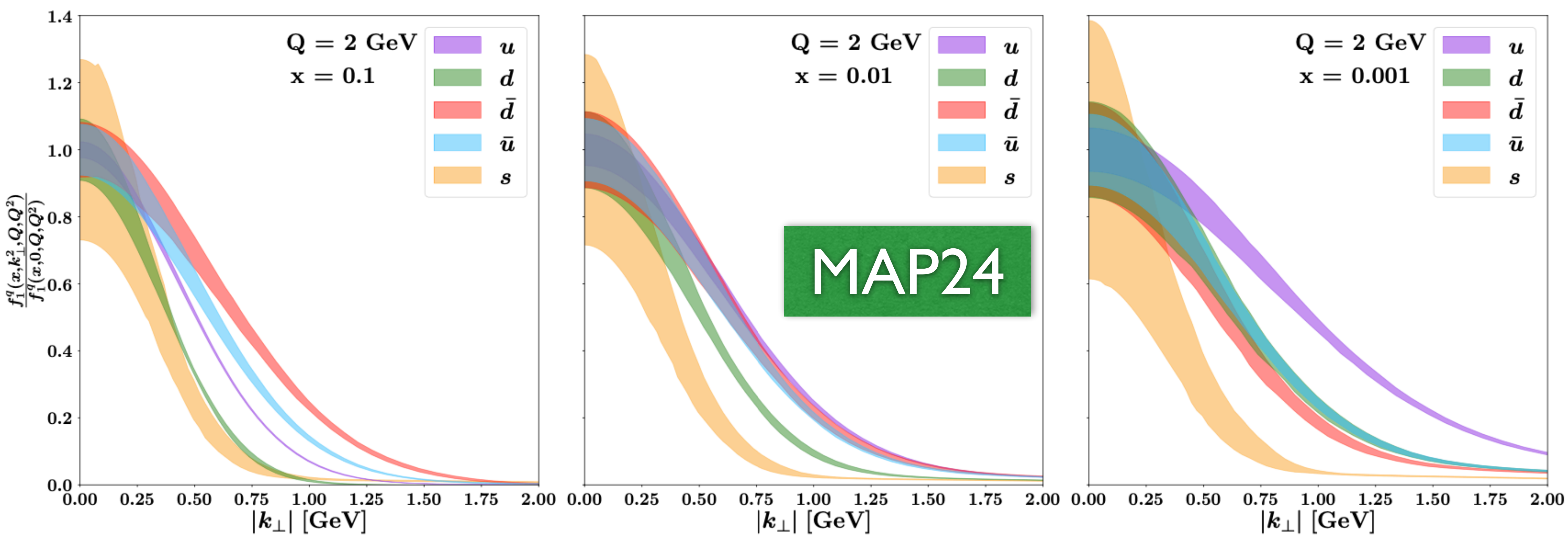
TMD PDFs



$x = 10^{-3}$
$x = 10^{-2}$
$x = 0.1$
$x = 1.$
$x\, F_{u\leftarrow p}(x,b)$
SV19
uncertainty
0%
10%
20%
1.2
0.9
0.6
0.3
0
0.5
1.
1.5
2.
2.5
3.
$b[\mathrm{GeV}^{-1}]$



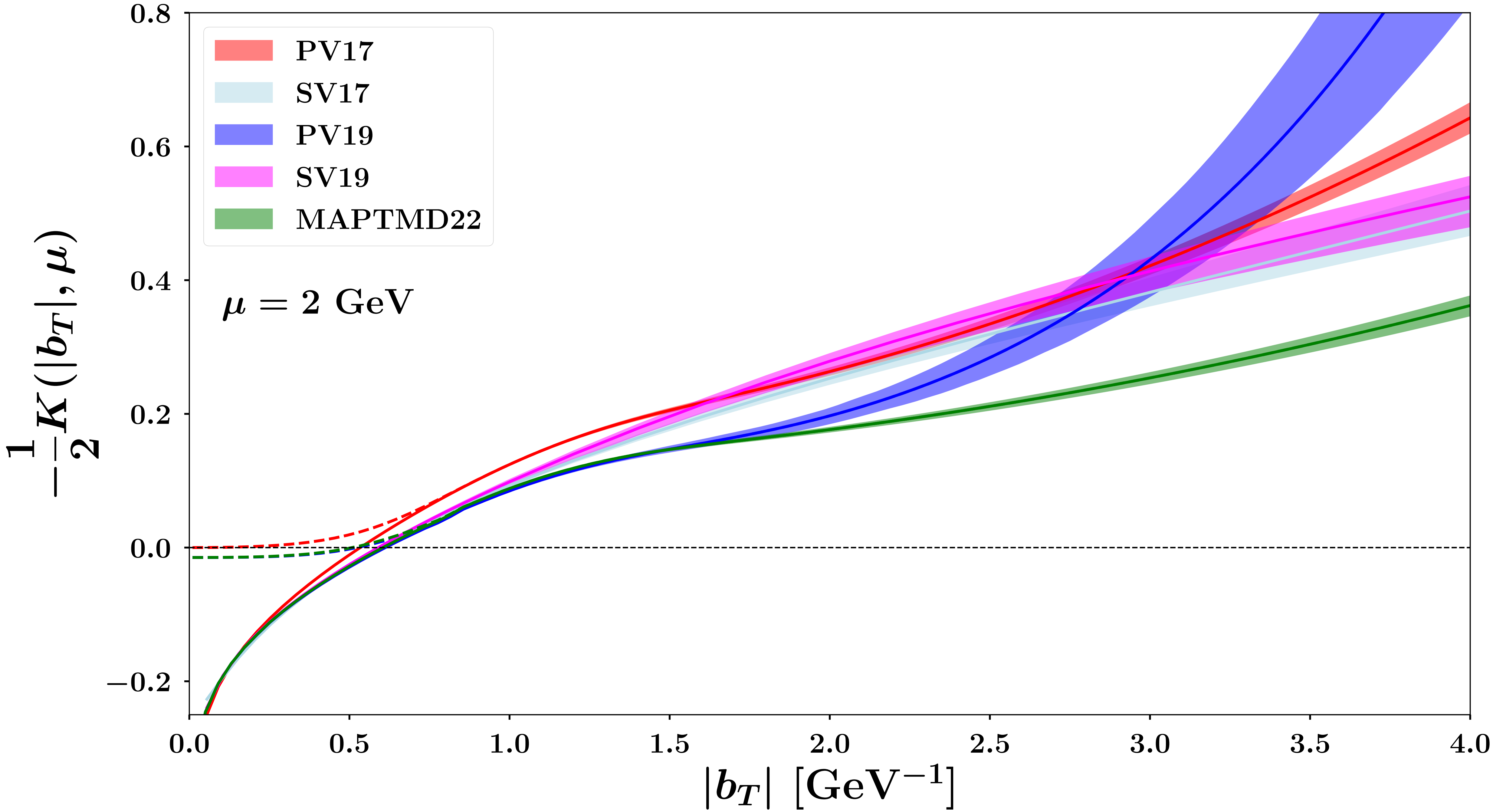
$x f_1^u(x, k_\perp^2, Q, Q^2)$
$x = 0.1$
$x = 0.01$
$x = 0.001$
$Q = 2$ GeV
MAP22
$|k_\perp|$ [GeV]



$\frac{f_1^q(x,k_\perp^2,Q,Q^2)}{f_1^q(x,0,Q,Q^2)}$
Q = 2 GeV
x = 0.1
x = 0.01
x = 0.001
u
d
$\bar{d}$
$\bar{u}$
s
MAP24
$|k_\perp|$ [GeV]

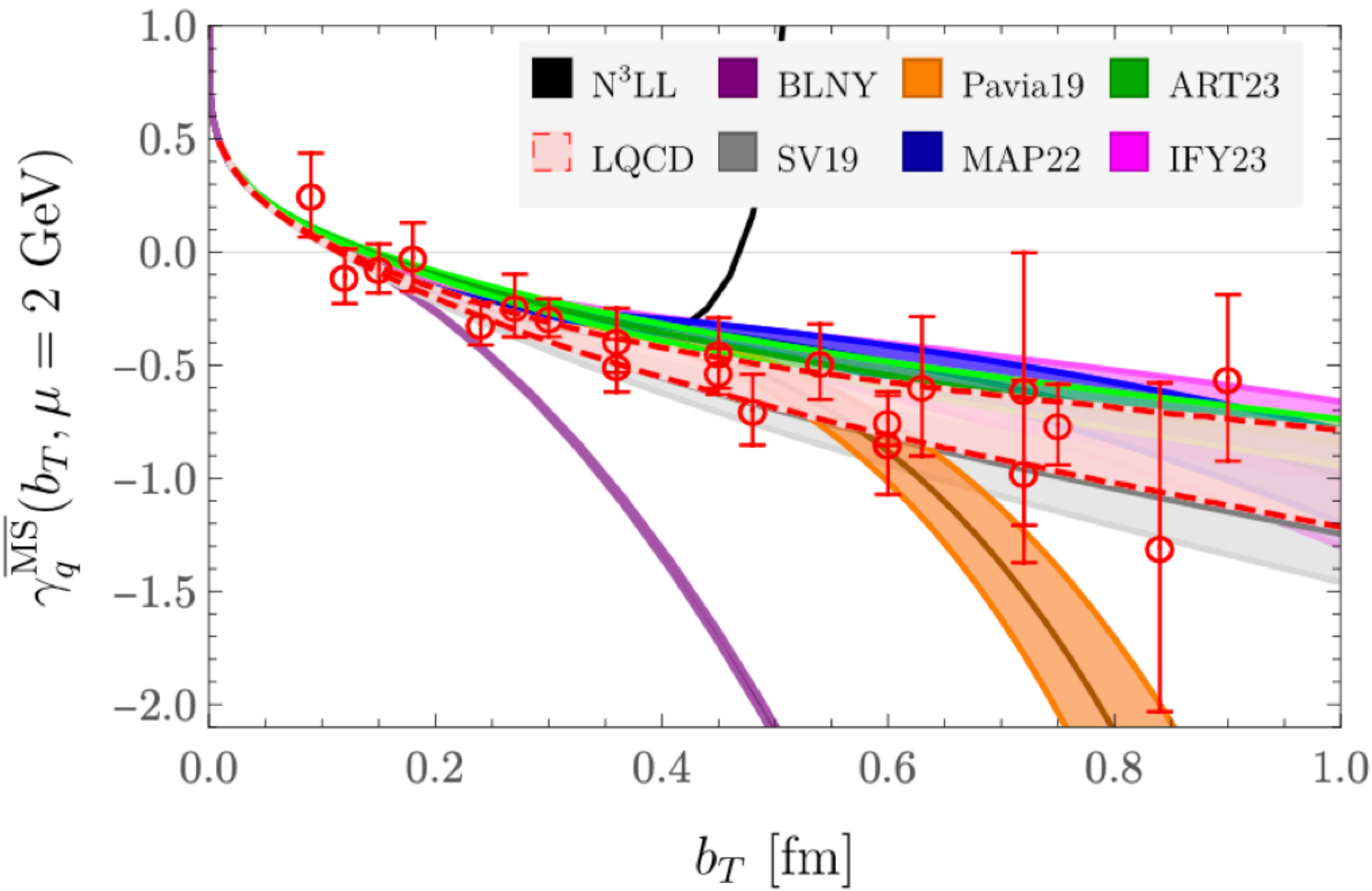
Collins-Soper kernel

$$K(|\boldsymbol{b}_T|, \mu_{b_*}) = K(b_*, \mu_{b_*}) + g_K(|\boldsymbol{b}_T|)$$



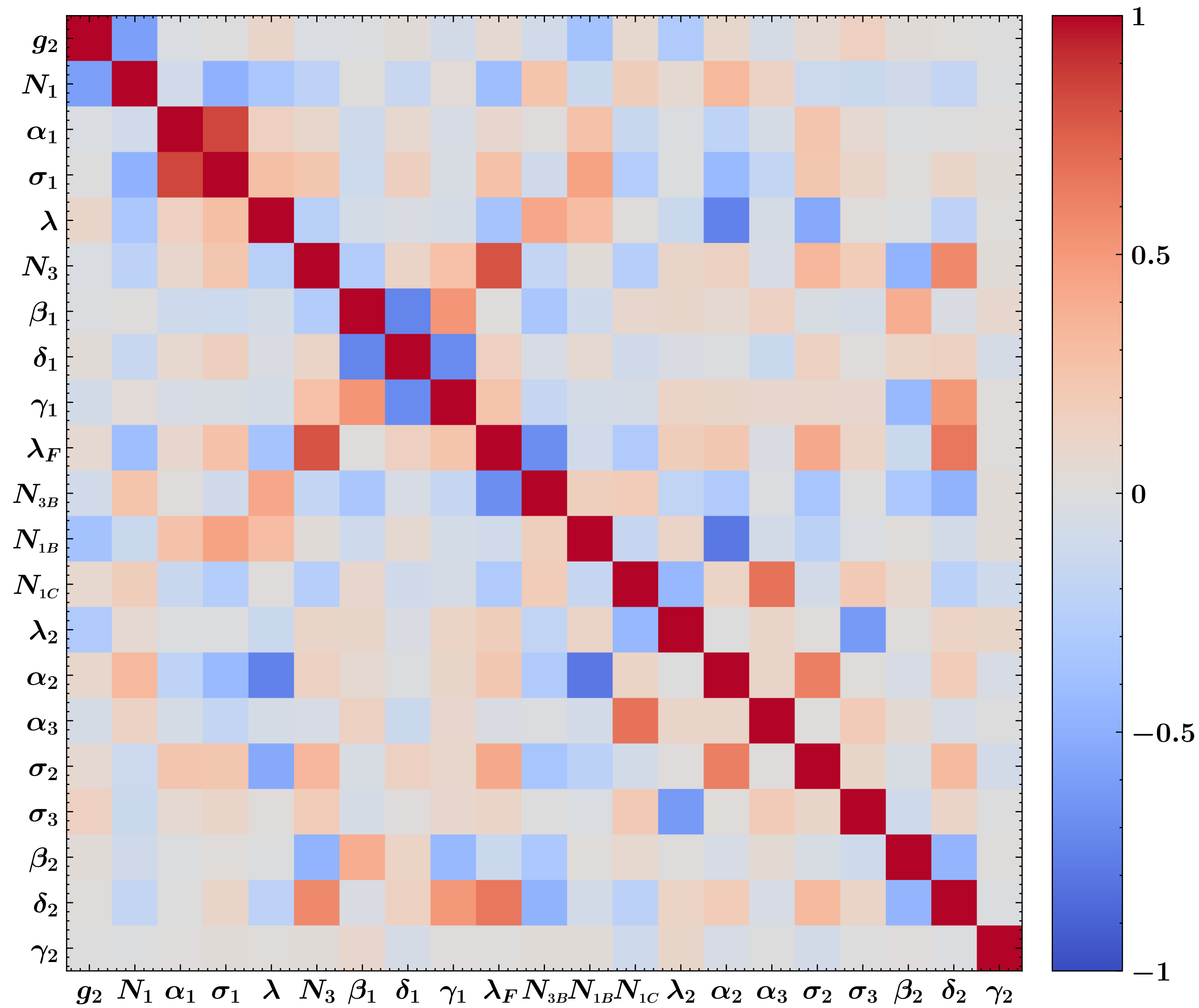
Collins-Soper kernel

$$K(|\boldsymbol{b}_T|, \mu_{b_*}) = K(b_*, \mu_{b_*}) + g_K(|\boldsymbol{b}_T|)$$



Correlation matrix

Parameter	Average over replicas
g_2 [GeV]	0.248 ± 0.008
N_1 [GeV^2]	0.316 ± 0.025
α_1	1.29 ± 0.19
σ_1	0.68 ± 0.13
λ [GeV^{-1}]	1.82 ± 0.29
N_3 [GeV^2]	0.0055 ± 0.0006
β_1	10.23 ± 0.29
δ_1	0.0094 ± 0.0012
γ_1	1.406 ± 0.084
λ_F [GeV^{-2}]	0.078 ± 0.011
N_{3B} [GeV^2]	0.2167 ± 0.0055
N_{1B} [GeV^2]	0.134 ± 0.017
N_{1C} [GeV^2]	0.0130 ± 0.0069
λ_2 [GeV^{-1}]	0.0215 ± 0.0058
α_2	4.27 ± 0.31
α_3	4.27 ± 0.13
σ_2	0.455 ± 0.050
σ_3	12.71 ± 0.21
β_2	4.17 ± 0.13
δ_2	0.167 ± 0.006
γ_2	0.0007 ± 0.0110



- $\lambda \sim 2$: weighted Gaussian important
- $\lambda_2 \neq 0$: third Gaussian non-negligible
- g_2 very small standard deviation
- correlation matrix nearly diagonal

MAP

Test: x-dependence

Test: *x*-independent fit at N³LL with Davies, Webber, Stirling (1985) NP parameterisation:

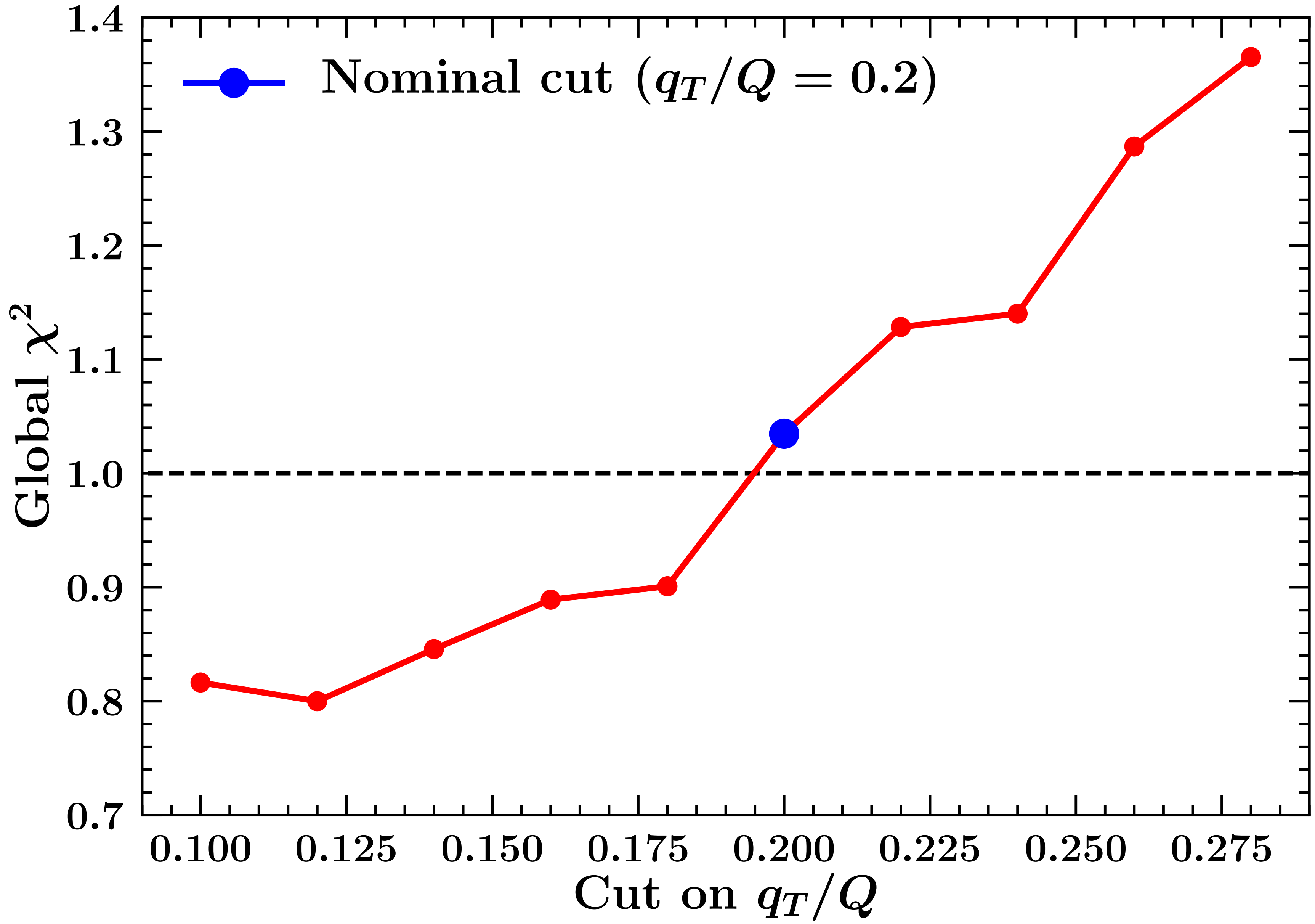
$$f_{\rm NP}^{\rm DWS}(b_T,\zeta) = \exp\left[-\frac{1}{2}\left(g_1 + g_2 \ln\left(\frac{\zeta}{2Q_0^2}\right)\right) b_T^2\right]$$

with and without ATLAS data

	Full dataset	No y-differential data
Global $\chi^2/N_{\rm dat}$	1.339	0.895
g_1	0.304	0.207
g_2	0.028	0.093

- χ^2 significantly higher for full dataset (1.339 vs. 1.020)
  - x-dependence required to describe data
- χ^2 significantly lower without ATLAS data
  - x-dependence at N³LL driven by ATLAS data

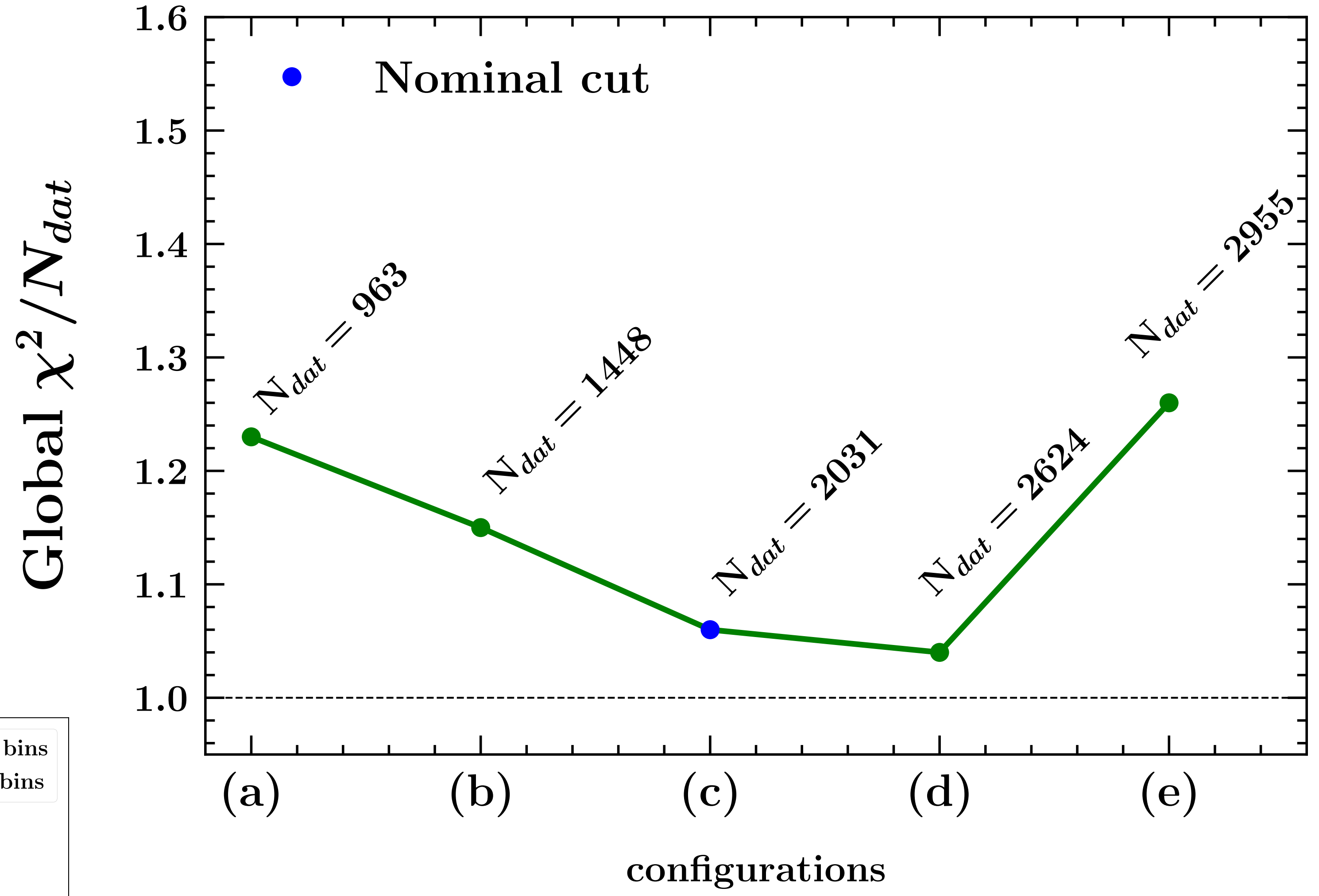
Test: dependence on q_T cut

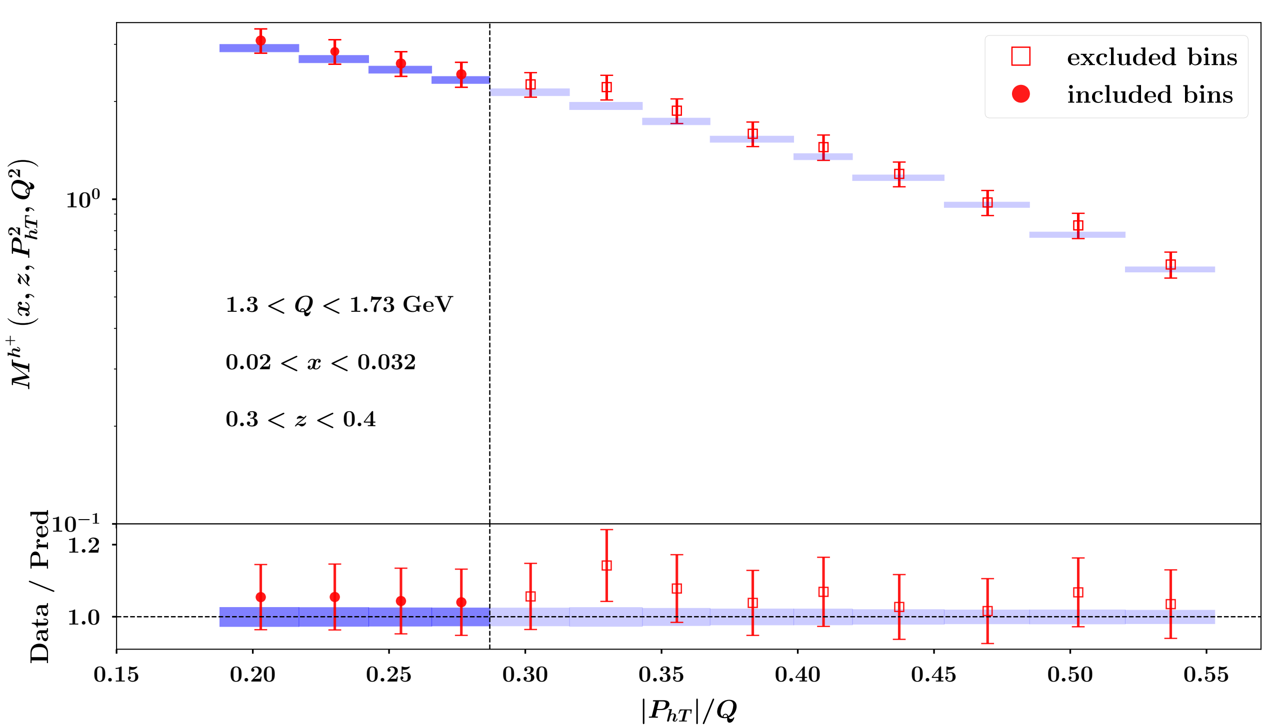


Test: different SIDIS cuts

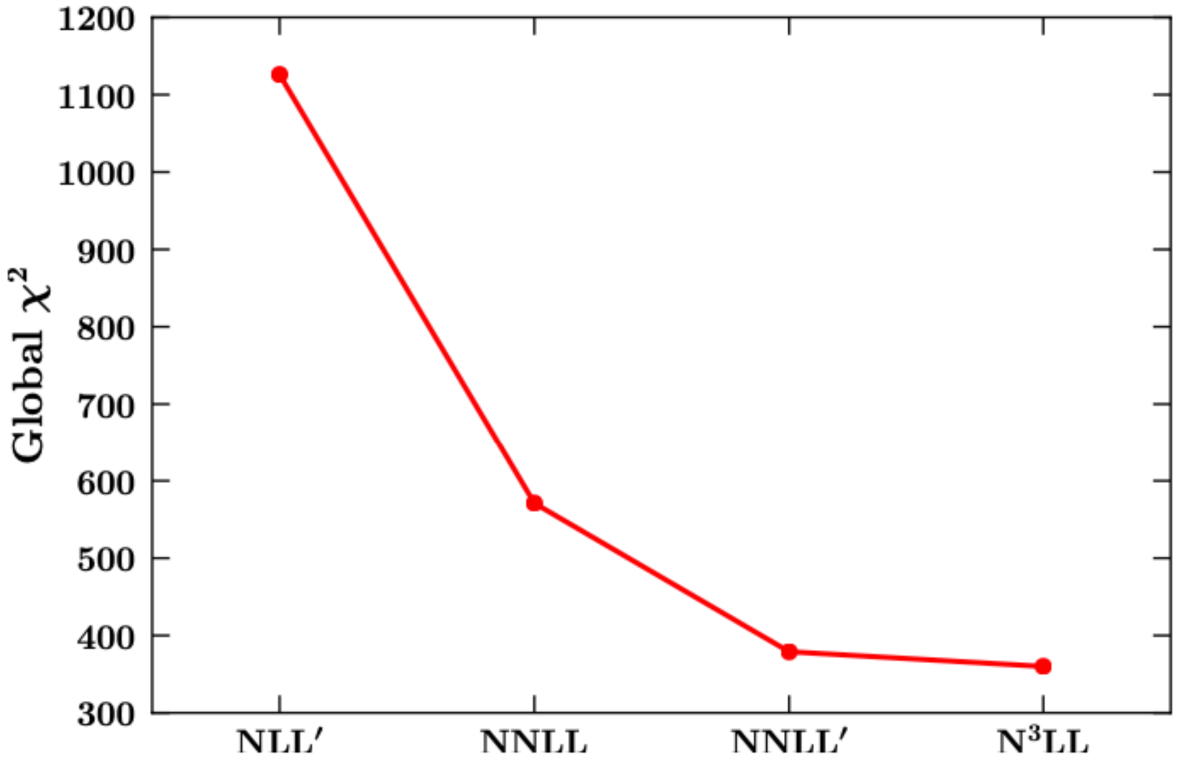
$$P_{hT}|_{max} = min[min[\, c_1 Q,\ c_2 z Q\,] + c_3\ \text{GeV},\ zQ\,]$$

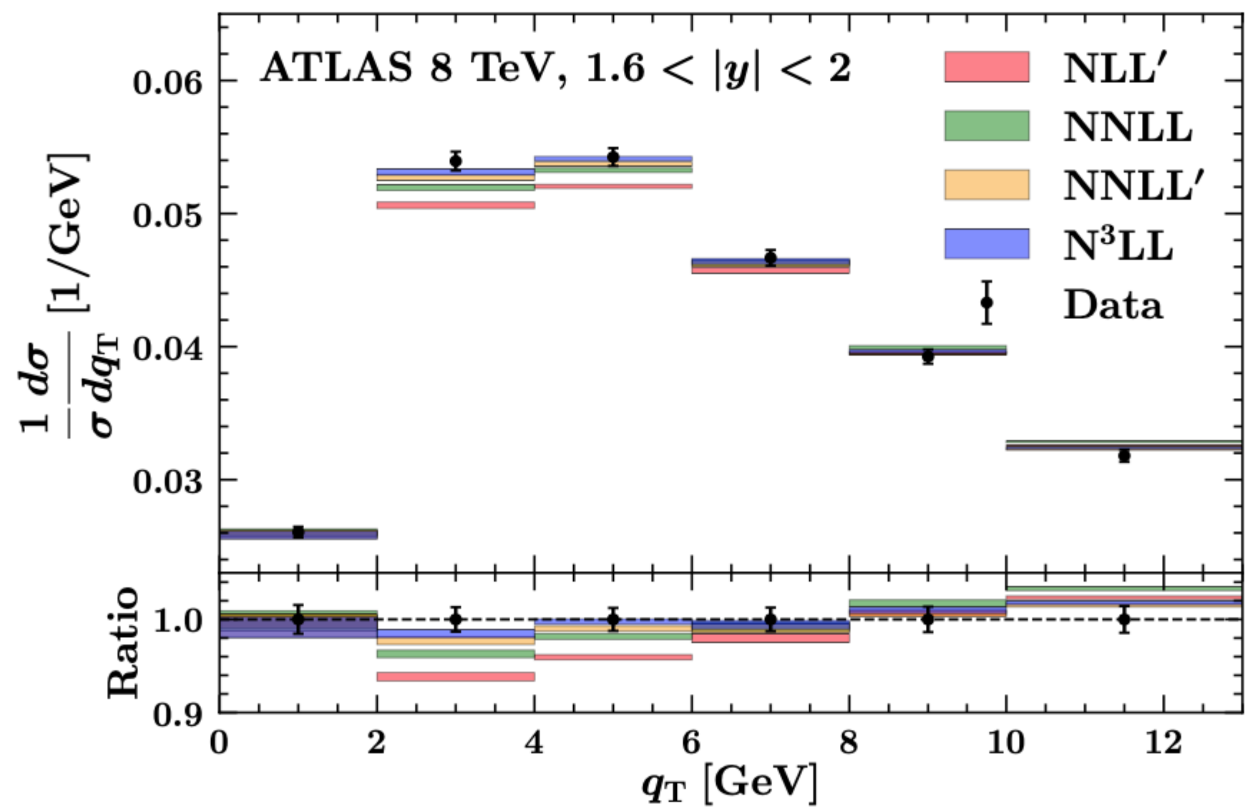
- (a) $c_1 = 0.4, c_2 = 0.4, c_3 = 0$
- (b) $c_1 = 0.15, c_2 = 0.4, c_3 = 0.2$
- (c) $c_1 = 0.2, c_2 = 0.5, c_3 = 0.3$ (baseline)
- (d) $c_1 = 0.2, c_2 = 0.6, c_3 = 0.4$
- (e) $c_1 = 0.2, c_2 = 0.7, c_3 = 0.5$





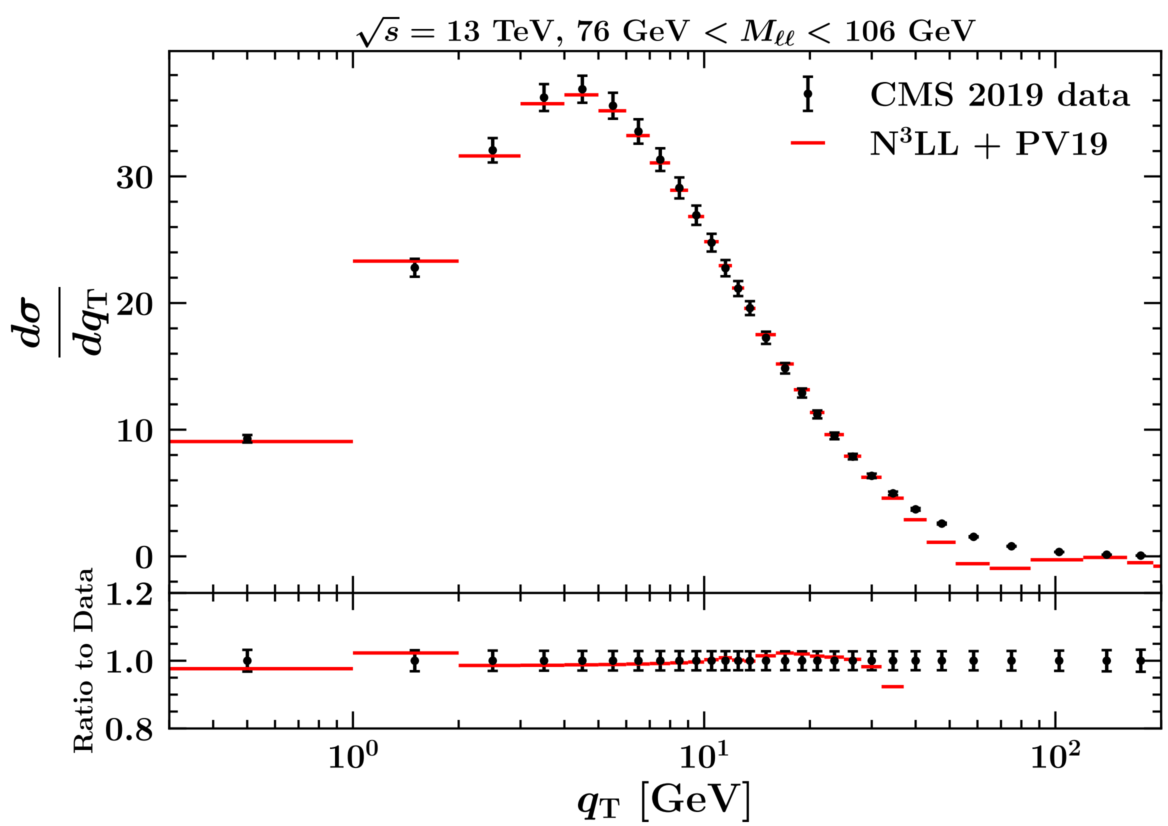
Test: convergence

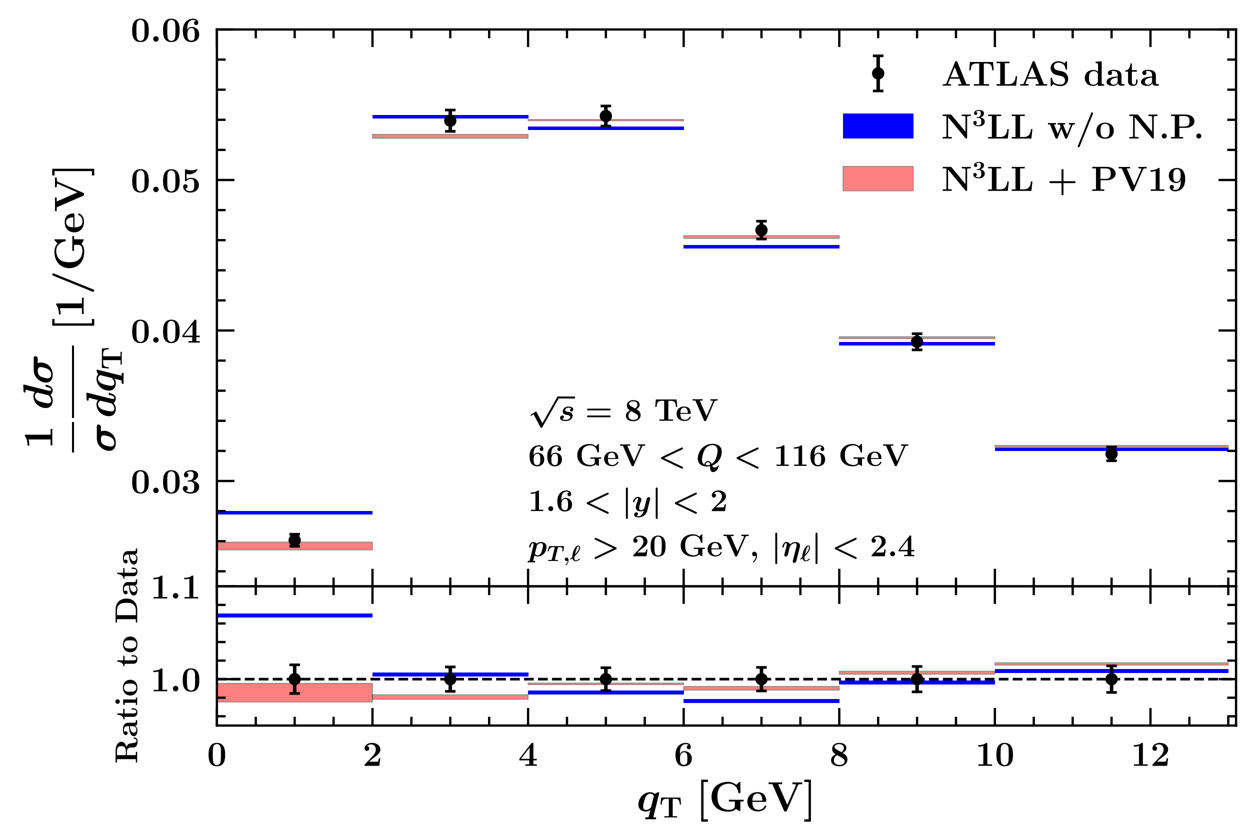




Order	NLL’	NNLL	NNLL’	N³LL
χ^2 /d.o.f.	3.19	1.62	1.07	**1.02**

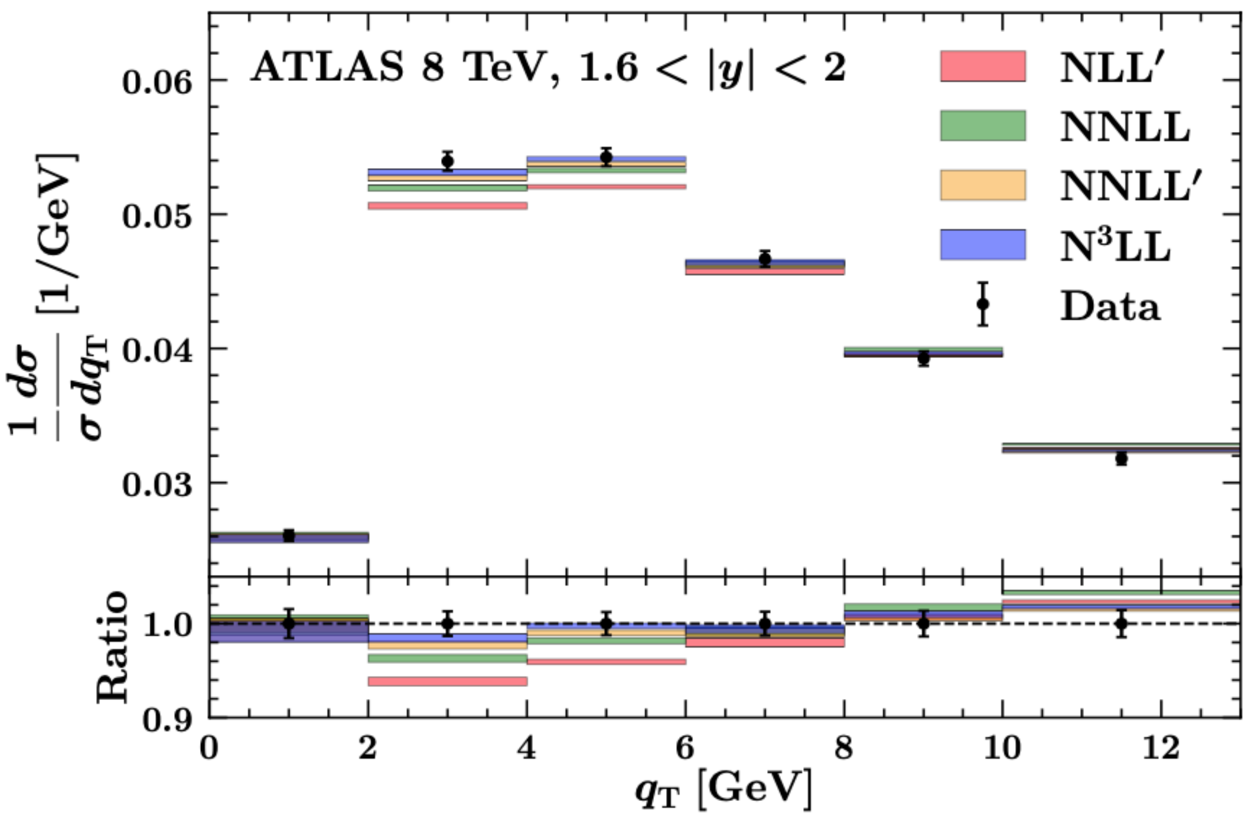
Test: impact on LHC data





Open problems: SIDIS

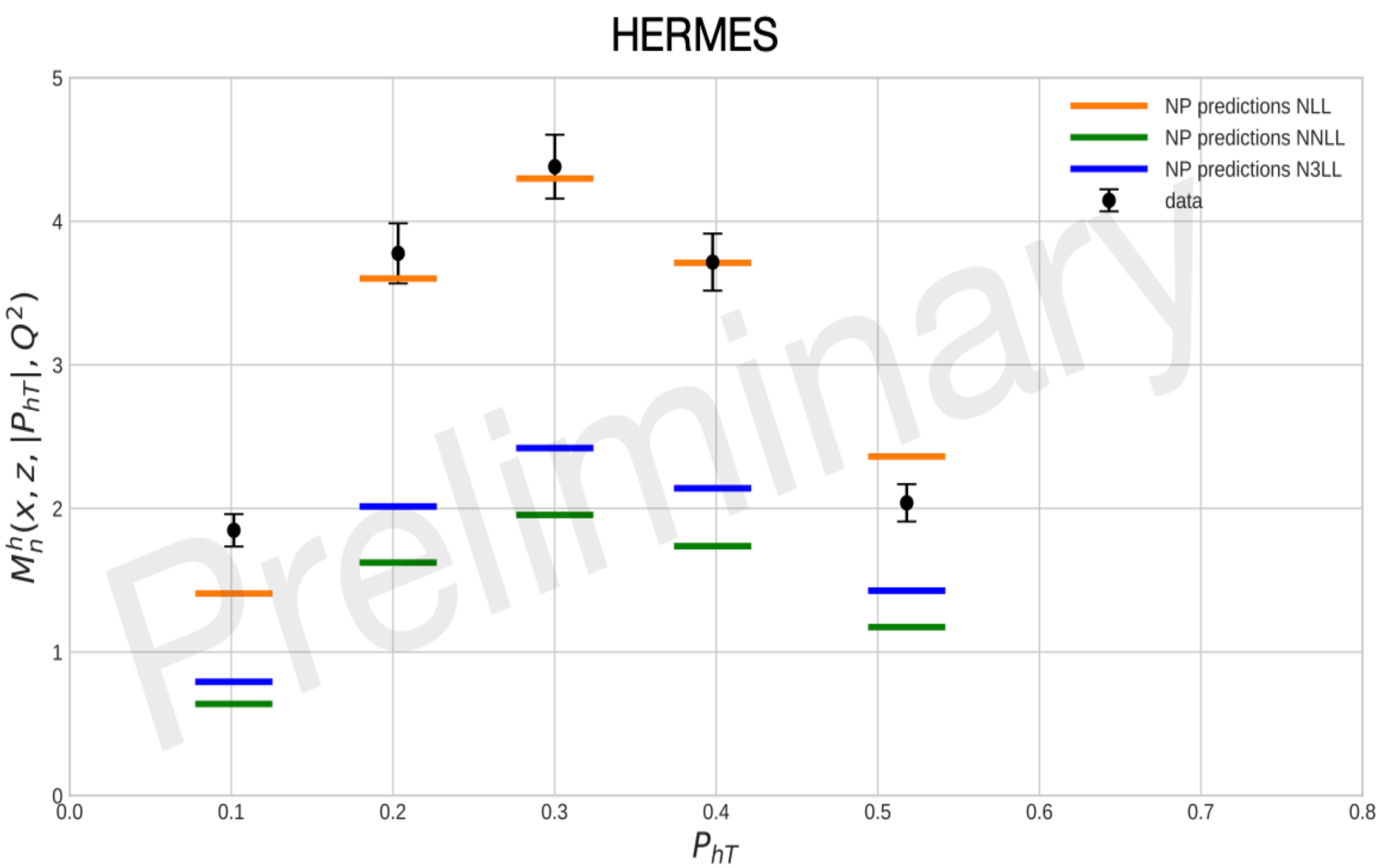
DY beyond NLL

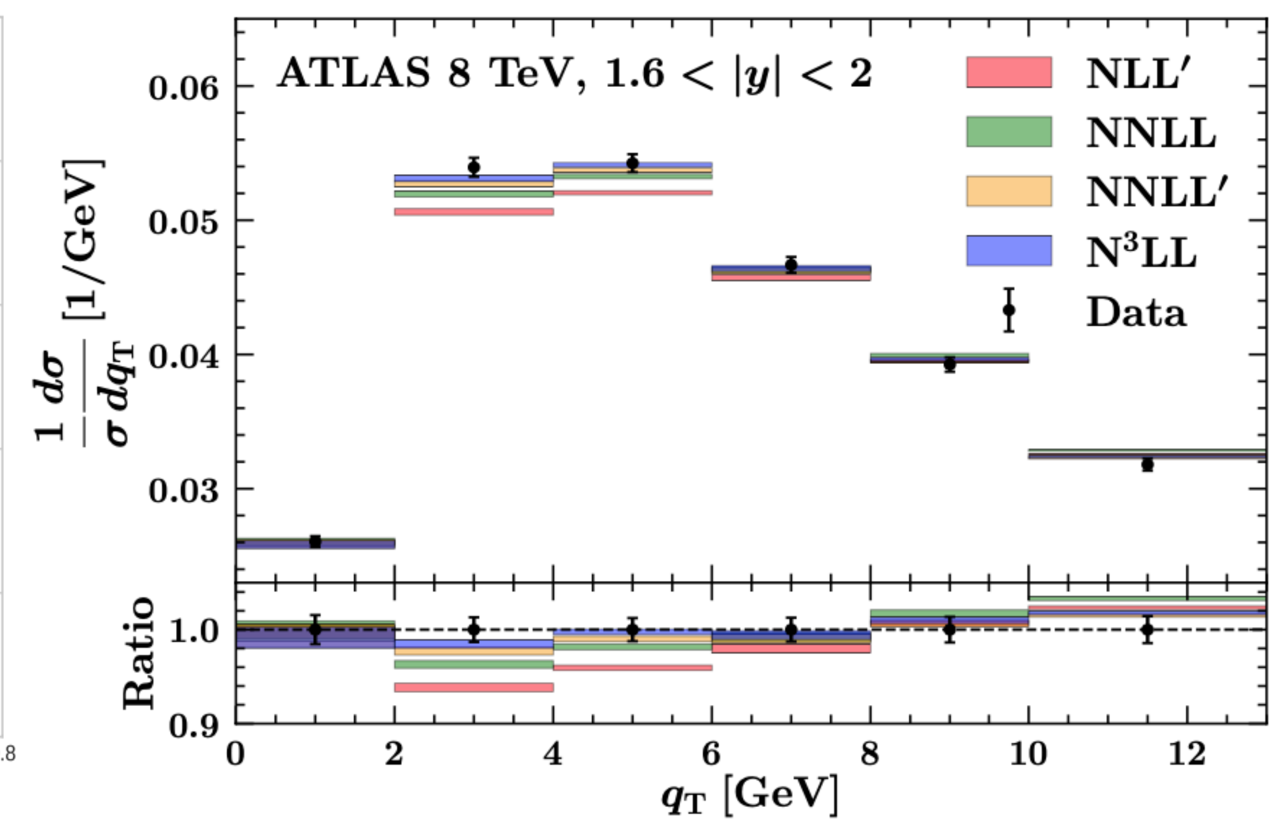


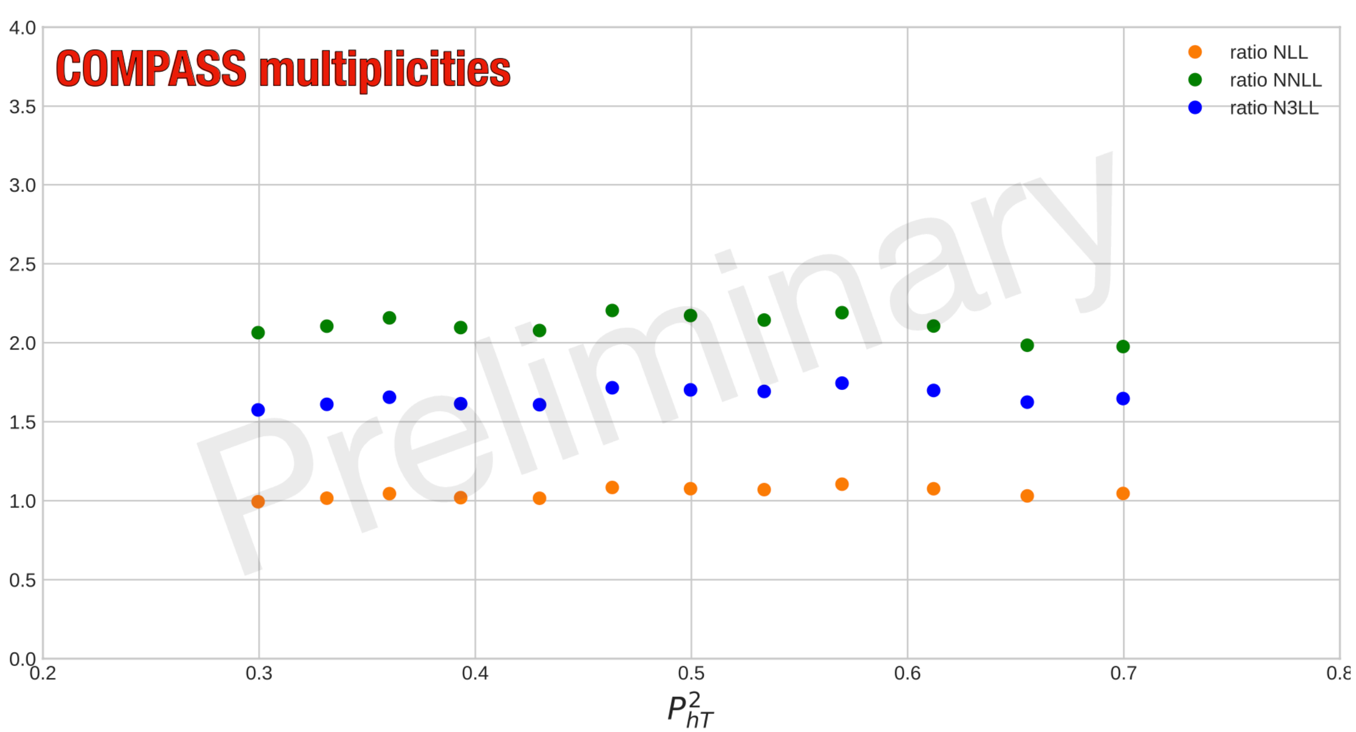
Open problems: SIDIS

SIDIS beyond NLL

DY beyond NLL

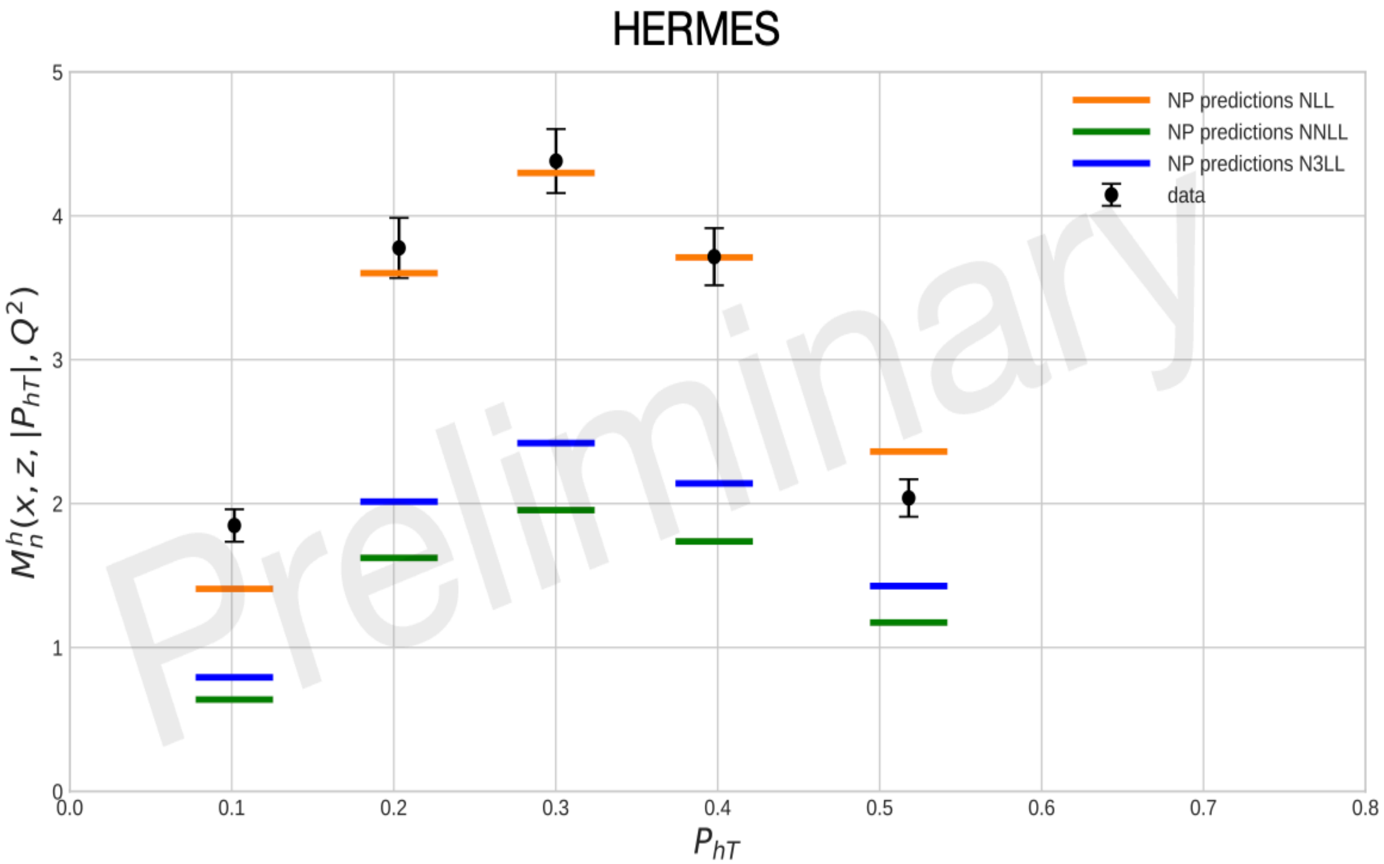




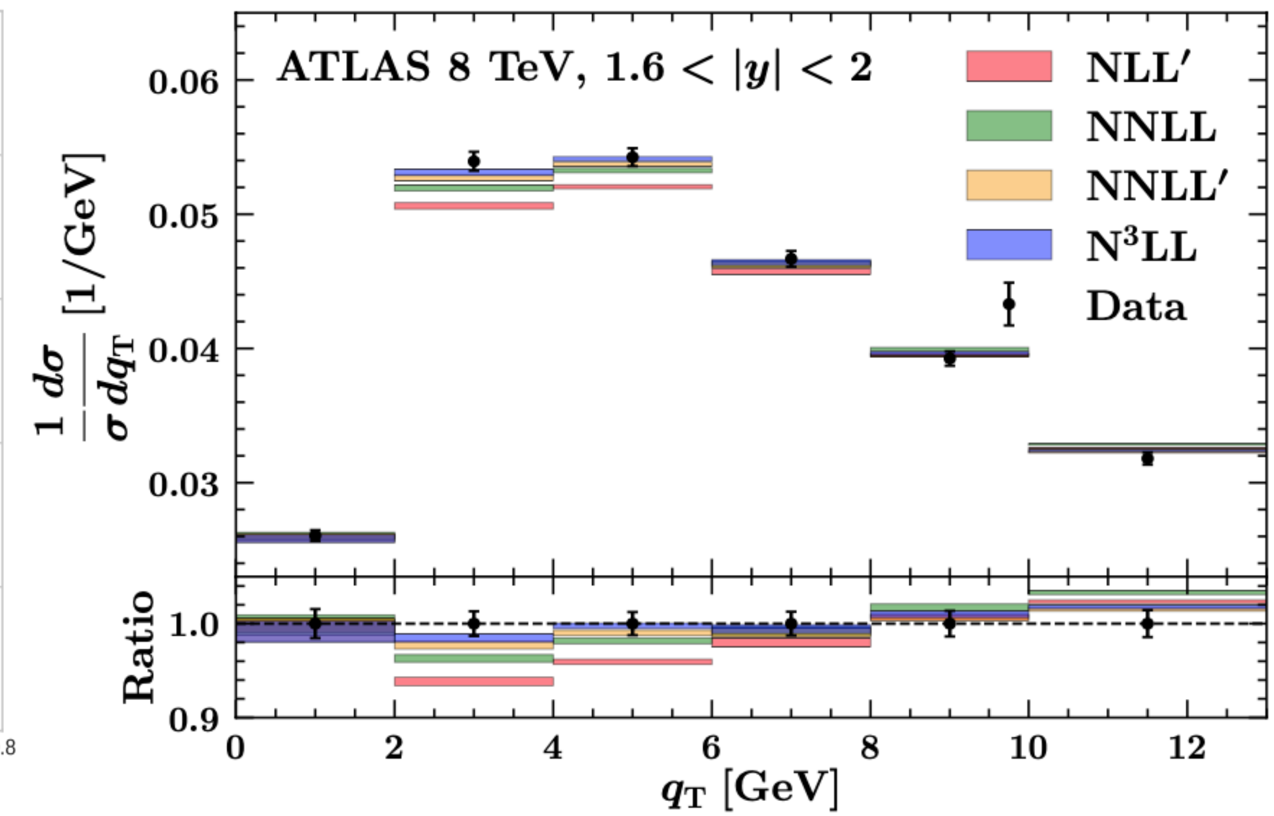


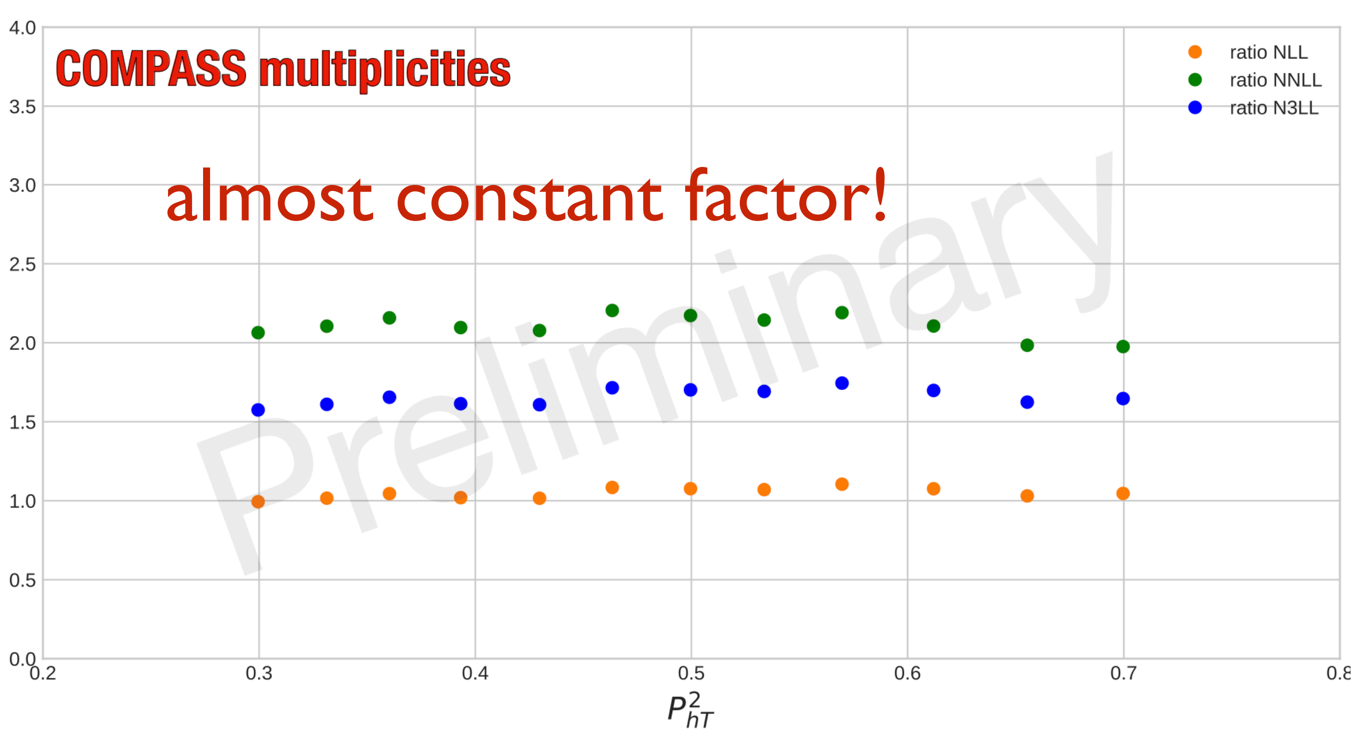
Open problems: SIDIS

SIDIS beyond NLL



DY beyond NLL





<u>normalisation factor for SIDIS</u>

- computed a priori, before the fit
- independent on the fitting parameters
- dependent on collinear PDFs
- alternatively: kinematic power corrections [Vladimirov, JHEP 12 (2023), 008

A note on TMD scales

- A sensible choice of the scales is important to allow perturbation theory to be reliable:
  - **no large unresummed logarithms** should be introduced,
  - each scale has to be set in the **vicinity of its natural (central) value**,
  - **scale variations** give an estimate of h.o. corrections
- In TMD factorisation for DY the relevant scales are q_{T} and Q:
  - natural to expect $\mu_0 \sim \sqrt{\zeta_0} \sim q_T \sim b_T^{-1}$ and $\mu \sim \sqrt{\zeta} \sim Q$
- In fact, it turns out that (in the $\overline{\mathrm{MS}}$ scheme) the **central scales** are:

$$\mu_0 = \sqrt{\zeta_0} = \frac{2e^{-\gamma_E}}{b_T} \equiv \mu_b \quad \text{and} \quad \mu = \sqrt{\zeta} = Q$$

- This choice **nullifies** all unresummed logs. One should thus consider:

$$\mu_0 = C_i^{(1)} \mu_b, \quad \sqrt{\zeta_0} = C_i^{(2)} \mu_b, \quad \mu = C_f^{(1)} Q, \quad \sqrt{\zeta} = \cancel{C_f^{(2)}} Q,$$

A note on TMD scales

- To reason why variations of ζ have **no effect** is that:

$$\frac{d\sigma}{dq_T} \propto H\left(\frac{\mu}{Q}\right) F_1(\mu,\zeta_1) F_2(\mu,\zeta_2) \quad \text{with} \quad \boxed{\zeta_1\zeta_2 \stackrel{!}{=} Q^4}$$

- It is easy to see that:

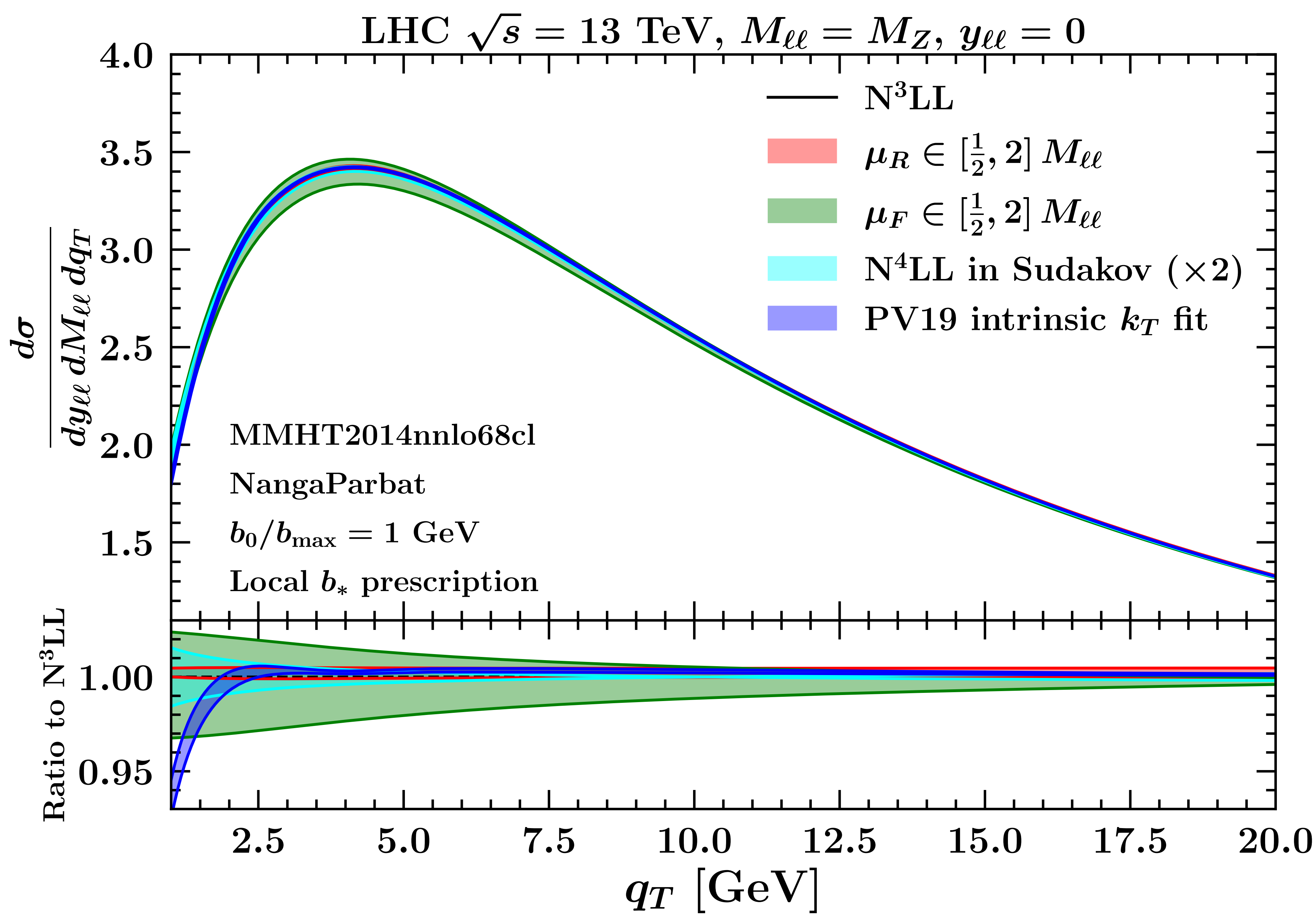
$$F_1(\mu,\zeta_1)F_2(\mu,\zeta_2) = \underbrace{R\left[(\mu,\zeta_1) \leftarrow (\mu_0,\zeta_0)\right] R\left[(\mu,\zeta_2) \leftarrow (\mu_0,\zeta_0)\right]}_{f(\zeta_1\zeta_2)\,=\,f(Q^4)} F_1(\mu_0,\zeta_0)F_2(\mu_0,\zeta_0)$$

- The single dependence on ζ_1 and ζ_2 **drops** in the combination:
  - i.e., $\zeta_1 = \zeta_2 = Q^2$ but any other choice such that $\zeta_1\zeta_2 = Q^4$ is **identical**.
- One can choose $\mu_0 = \sqrt{\zeta_0}$:
  - not strictly necessary but **probably a conservative choice**.
- At the end of the day, we have **two scales** to be varied:

$$\boxed{\mu_0 = \sqrt{\zeta_0} = C_i\mu_b \quad \text{and} \quad \mu = C_f Q}$$

Estimate of uncertainties

- Theoretical uncertainty estimate on **N³LL**:
  - estimate of subleading logarithmic corrections by including N⁴LL corrections in the Sudakov (**mimicking resummation scale variations**),
  - variations of μ_r and μ_f by a factor 2 up and down w.r.t. M_{ll},
  - inclusion of non-perturbative effects as determined in the **PV19** fit.



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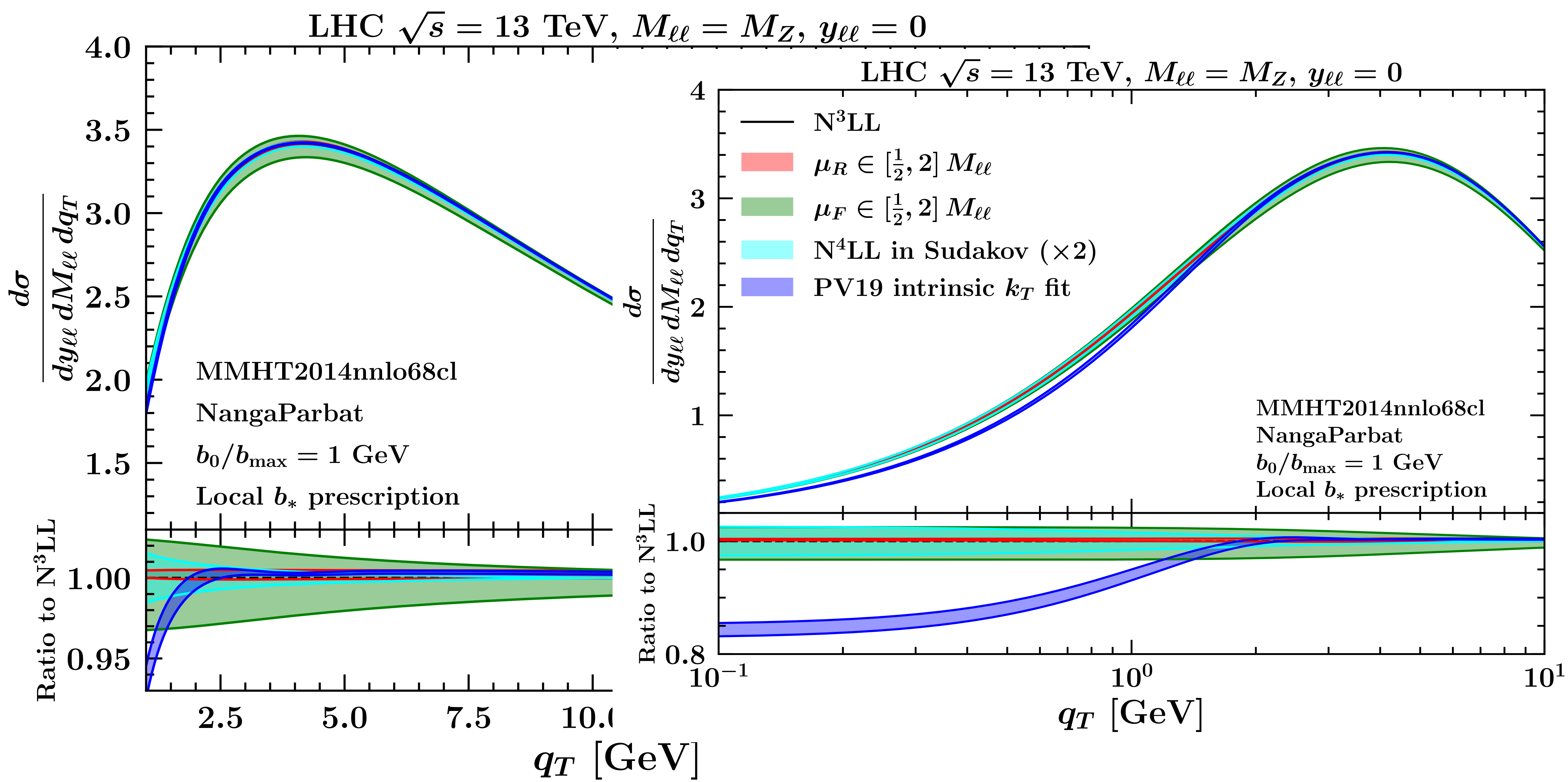
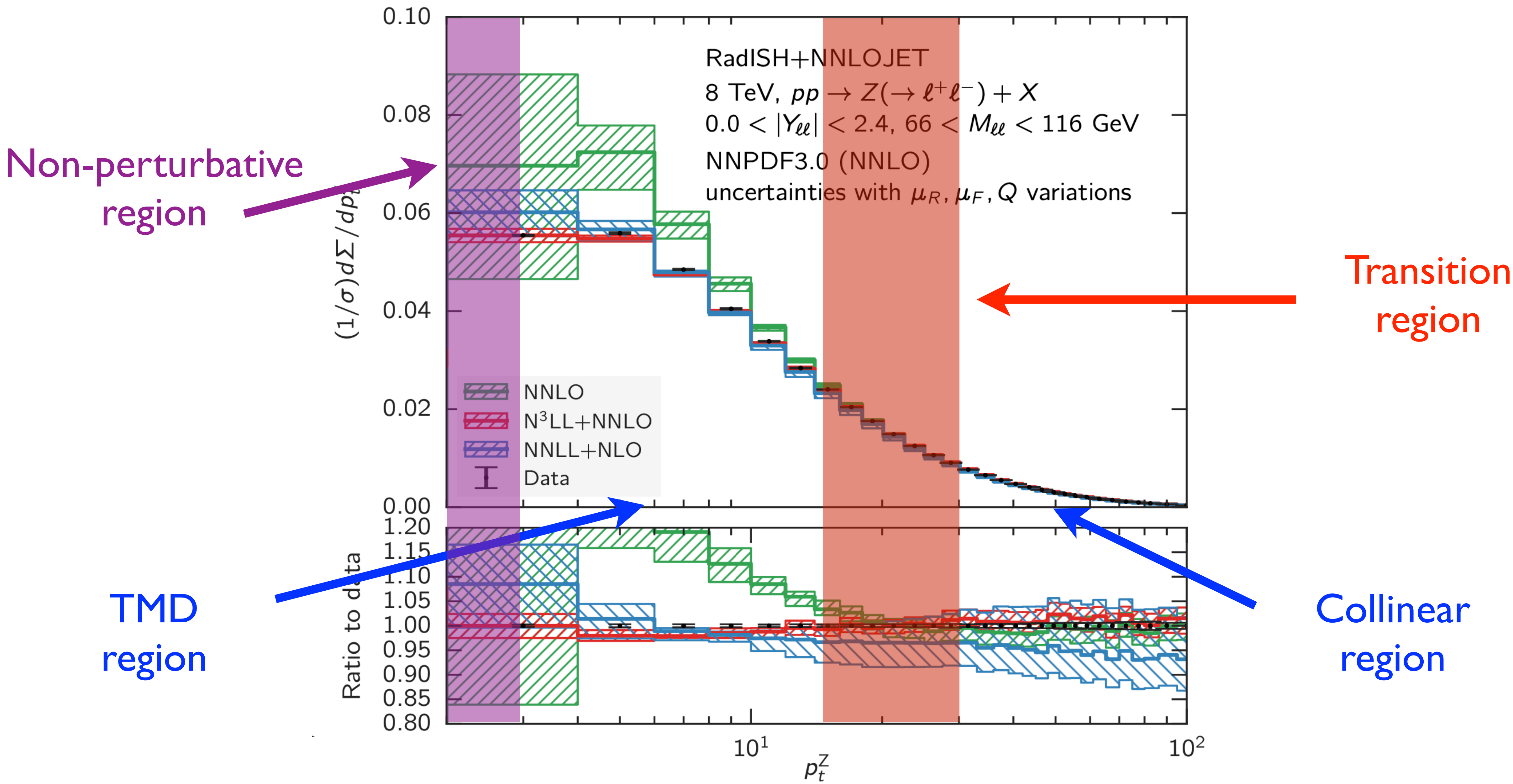


Future: matching with F.O.

- **Matching** between TMD and collinear factorisations:

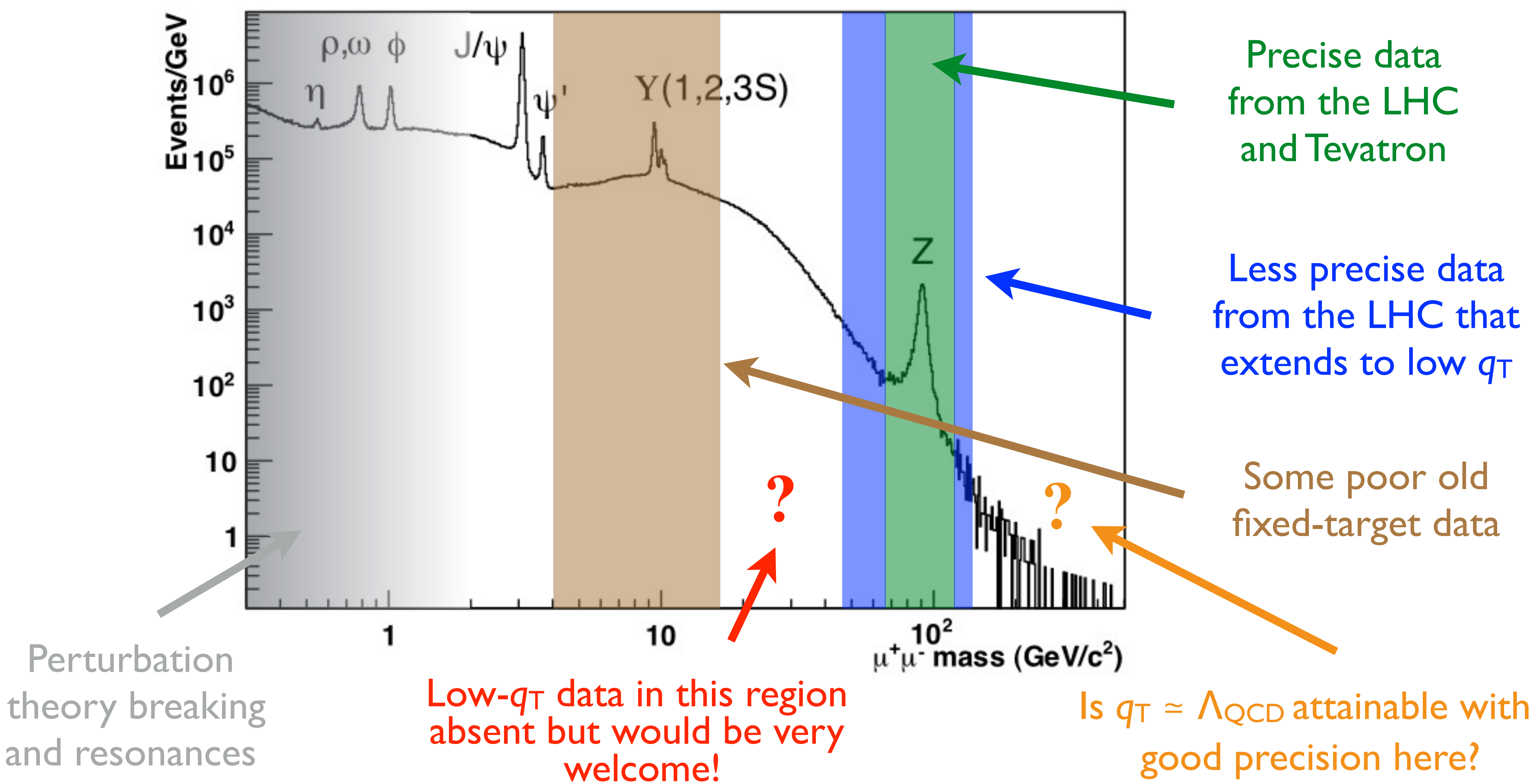


- Well-understood procedure at the LHC energies where usually $Q \gg \Lambda_{\mathrm{QCD}}$:
  - clear separation of TMD and collinear, non-perturbative confined to very low q_{T}.
- Not so much so for current (and future) SIDIS data due to smaller Q:
  - need to *identify* and *study* the transition region.

Future: Exp. Measurements

- TMD factorisation applies for $q_T \ll Q$:
  - the region $q_T \simeq \Lambda_{QCD}$ is relevant for hadron structure, no matter how large Q.
  - As Q increases the cross section drops and low q_T becomes hard to access.

Need as many (low-q_T + y-binned) data as possible!



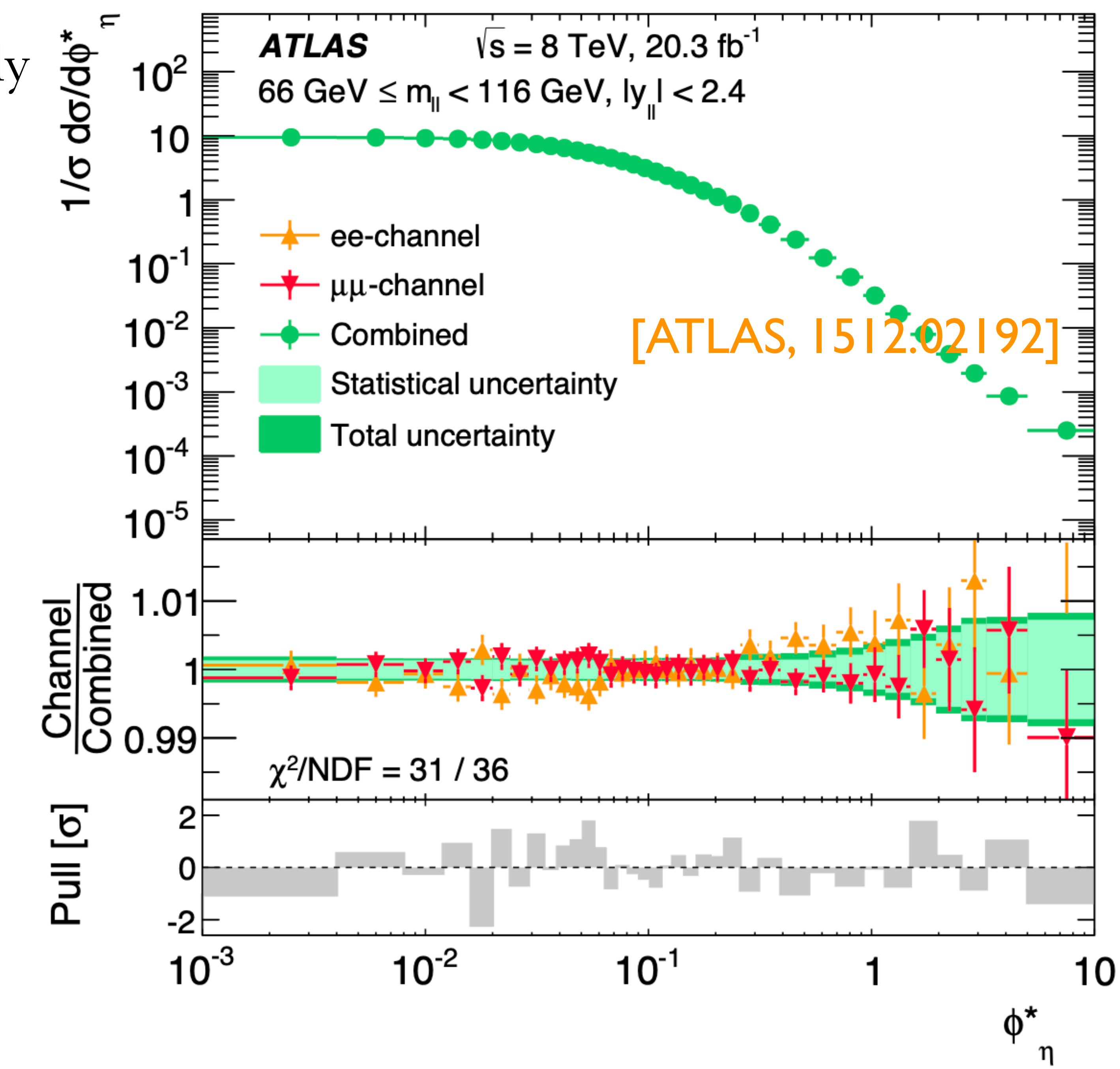
Future: Exp. Measurements

$$\phi^*_\eta = \tan\left(\frac{\pi - \Delta\phi_\ell}{2}\right)\sqrt{1 - \tanh^2\left(\frac{\Delta\eta_\ell}{2}\right)}$$

[Banfi et *al*., 1009.1580]

- Small ϕ^* is mapped onto small q_T, this observable is expected to carry important information on hadron structure.

- Experimentally very clean because it only involves angles.



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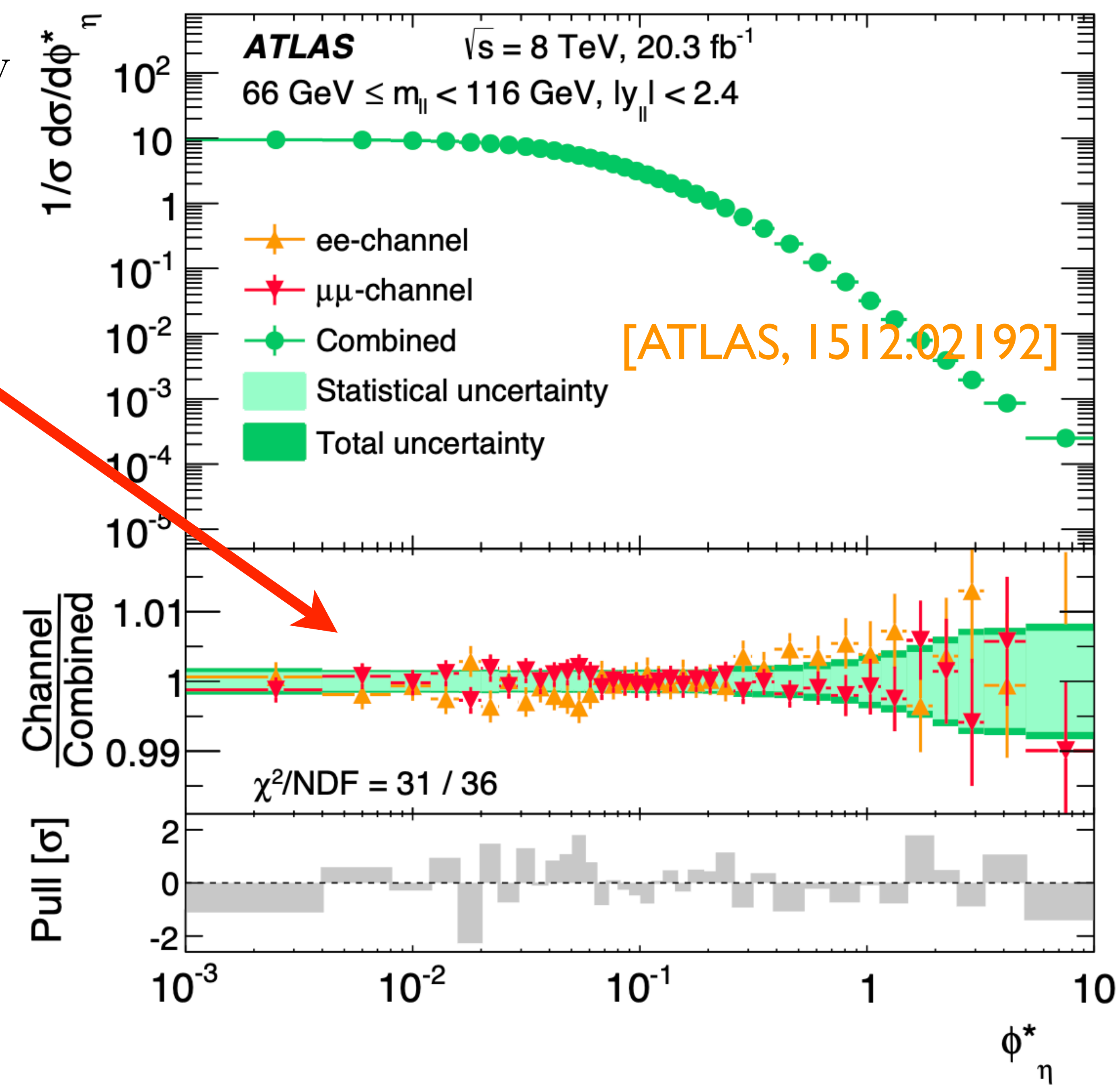

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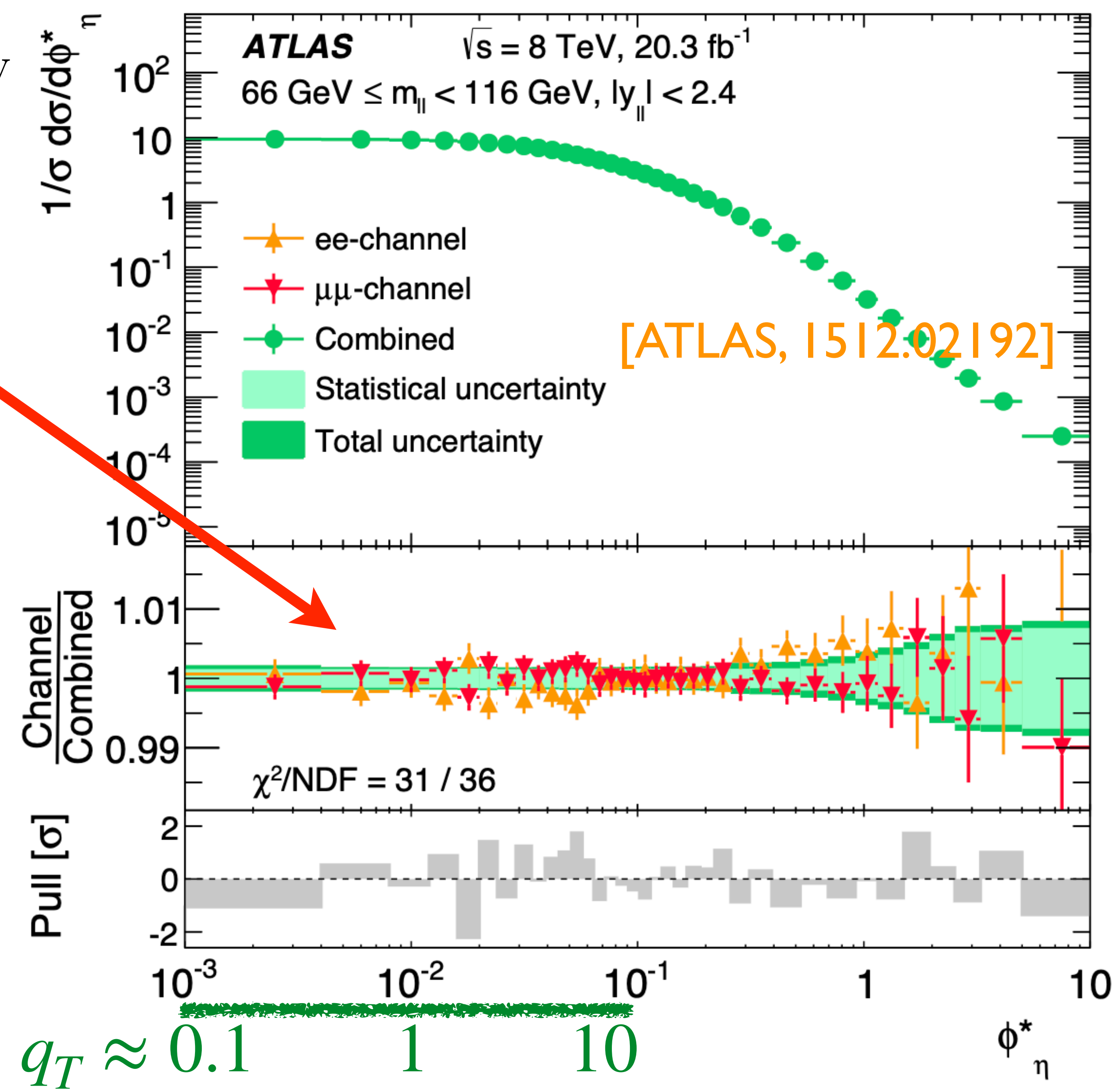

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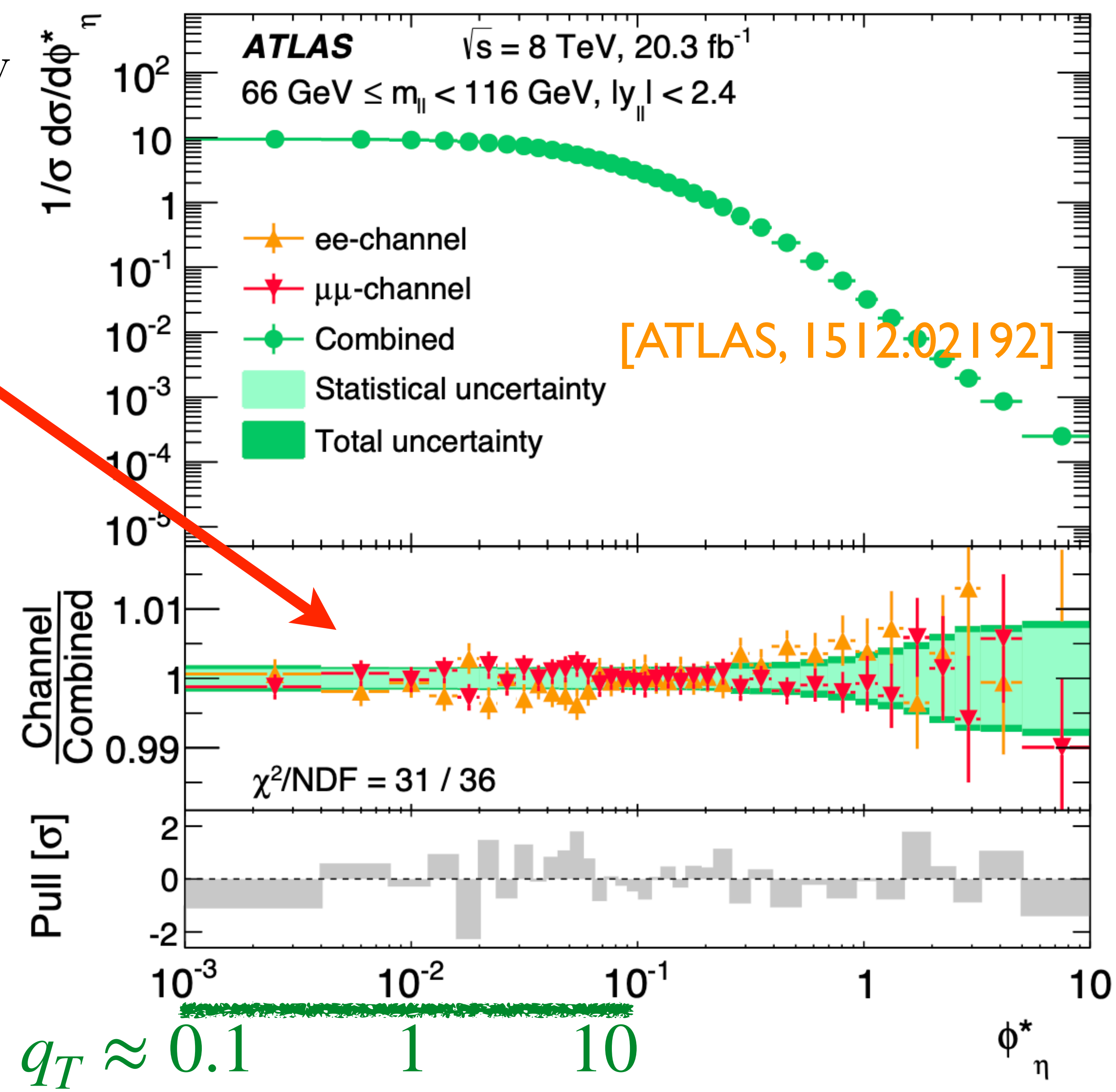

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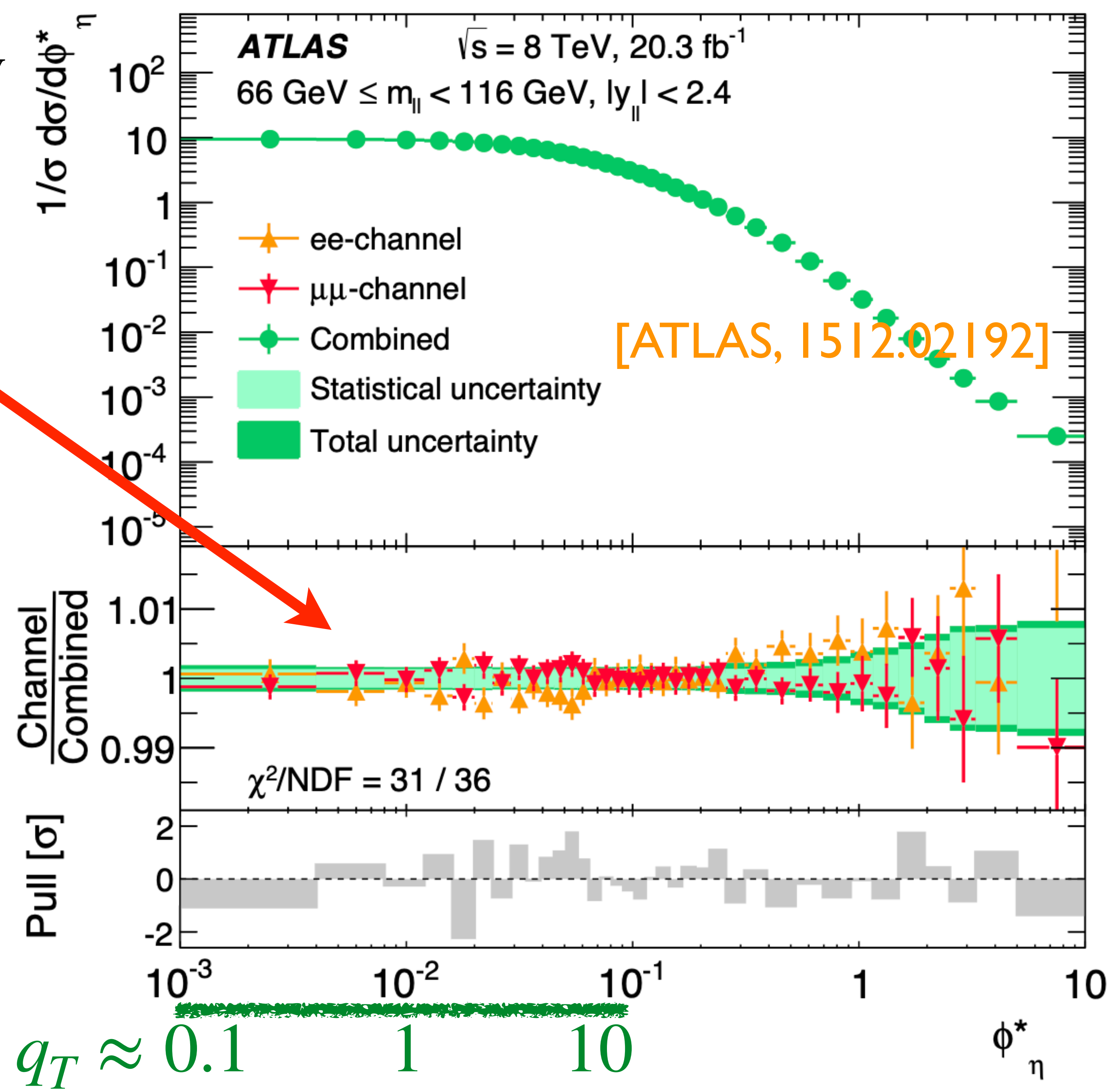
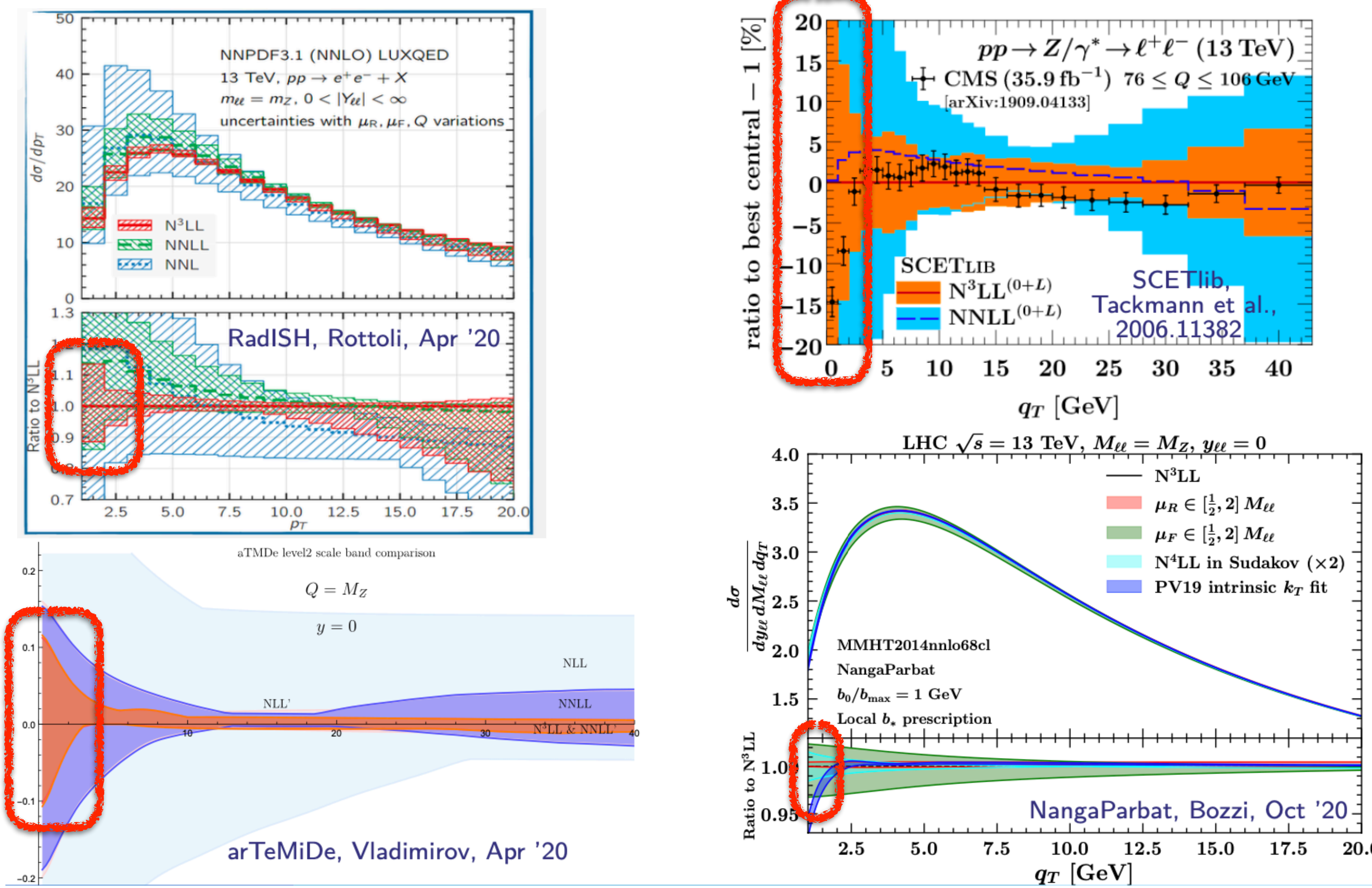
  - it might be interesting to check shape variation with rapidity and m_{ll} at low ϕ^*_η(TMD (x, Q^2)-dependence)



Future: Interplay of P and NP

- Understanding of **theoretical uncertainties** is crucial to achieve a reliable extraction of the non-perturbative components from data.



[T. Cridge, last EWWG general meeting]

Future: W mass measurements

- $p_{Tl} \leftarrow q_{TW} \leftarrow$ resummation + intrinsic-k_T
- <u>All analyses assume flavour-independence</u>
- <u>impact of flavour-dependent intrinsic-k_T comparable to PDF variations</u>

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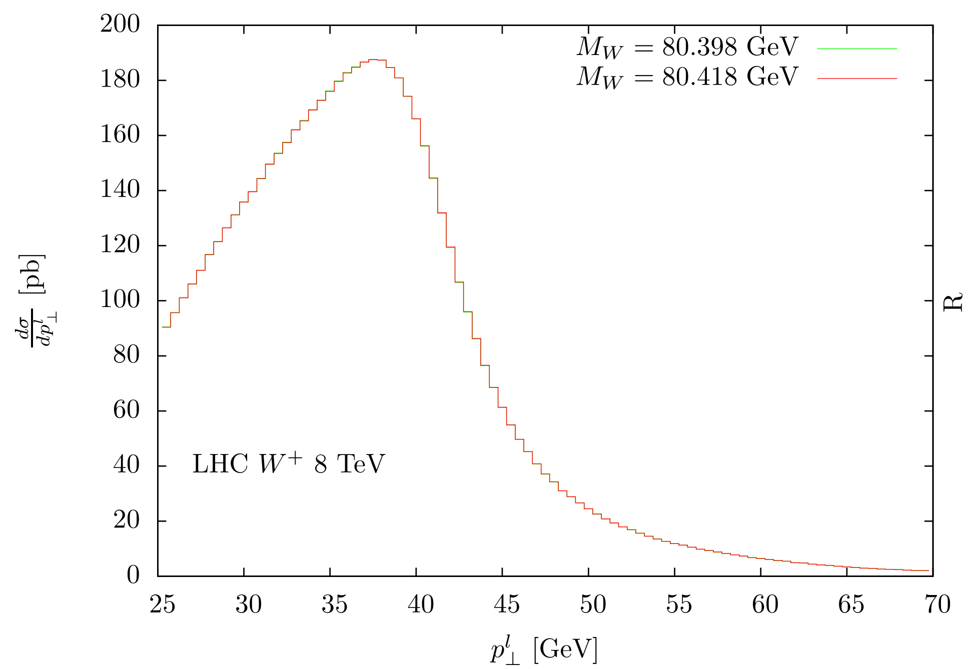

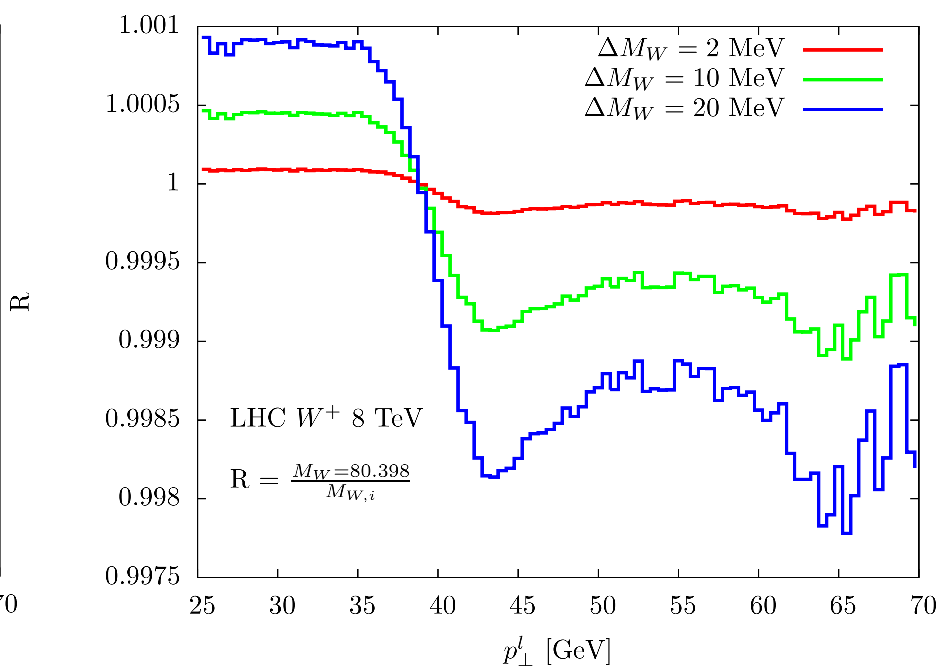

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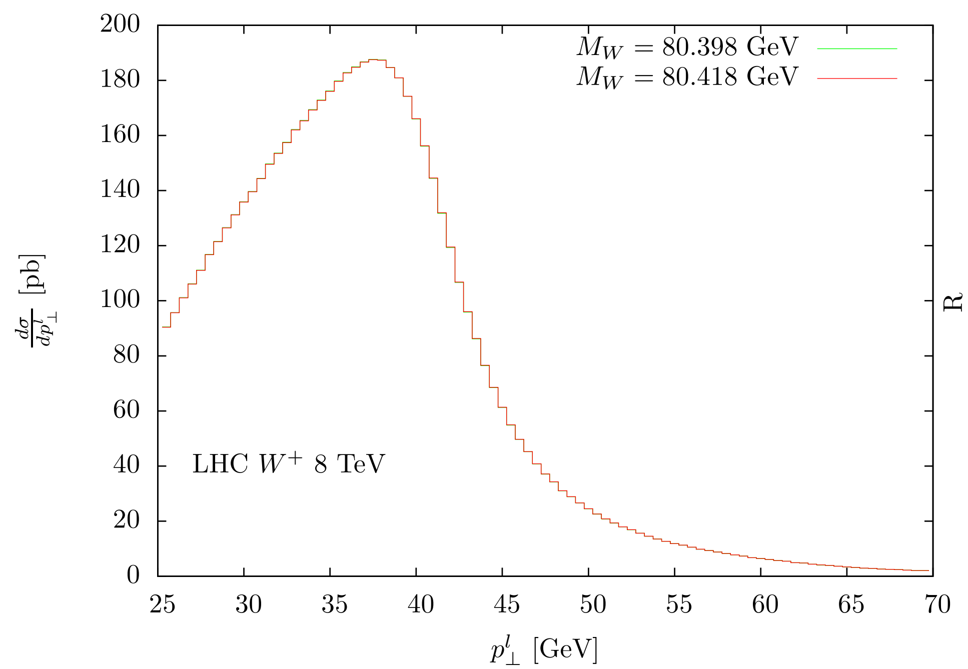

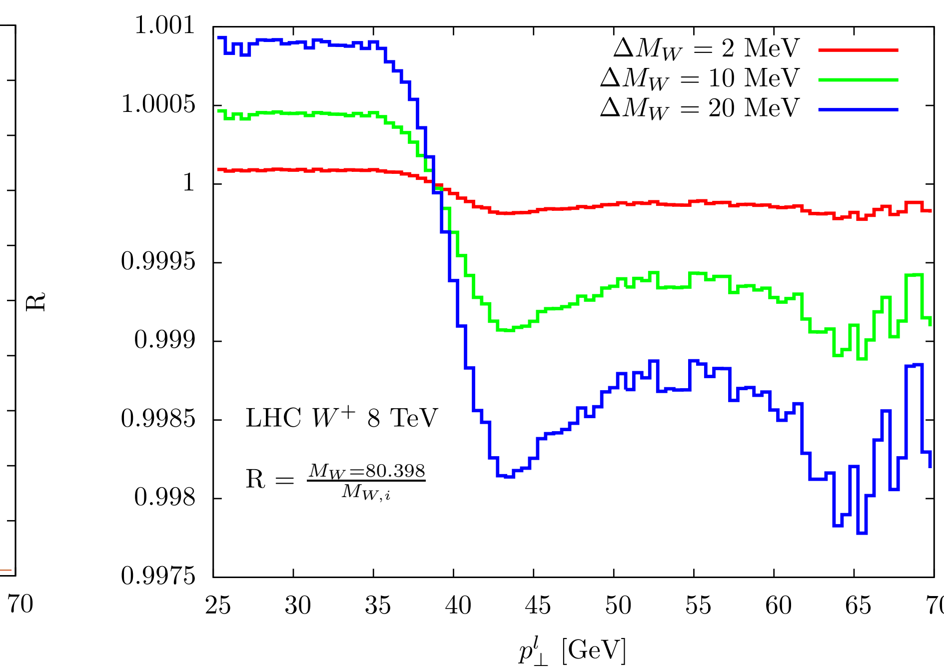

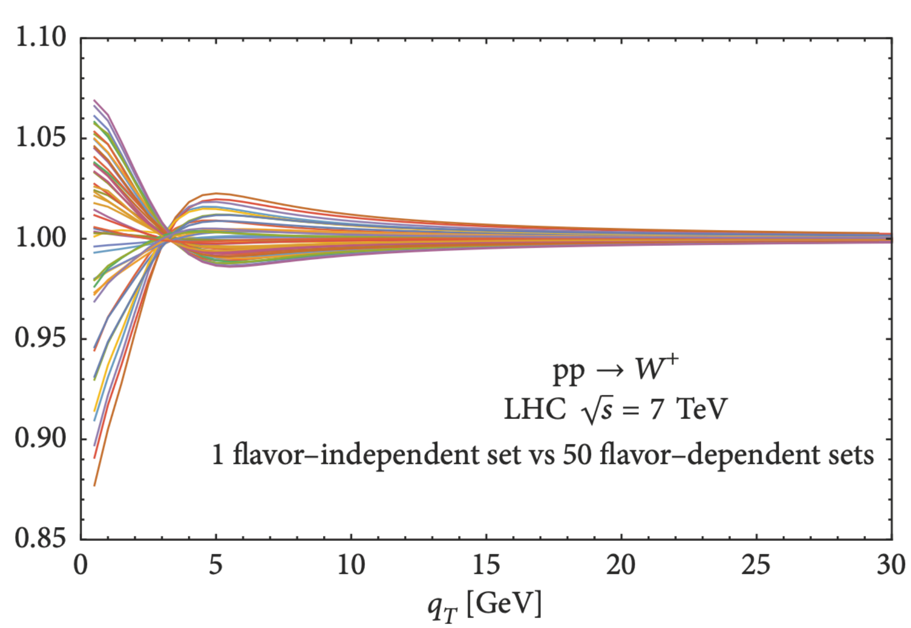

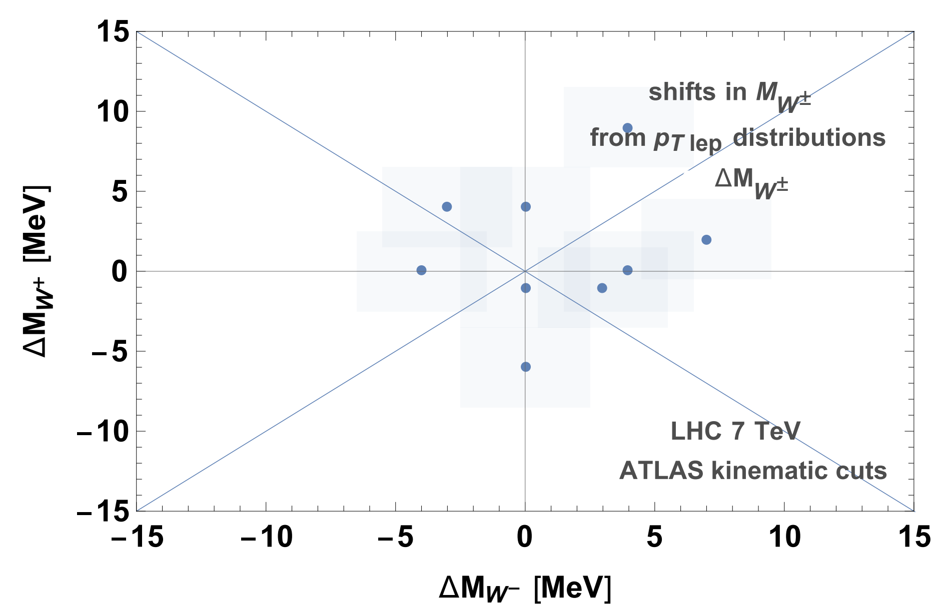

Thank you!