Hadronic Physics via lattice QCD

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2023 lattice conference

No way I can do justice to the field in 2hrs!

12:00 - 13:30

14:00

15:00

Algorithms and Artificial Intelligence	Hadronic and Nuclear Spectrum and Interactions	Particle Physics Beyond the Standard Model	QCD at Non-zero Temperature	Quantum Computing and Quantum Information	Quark and Lepton Flavor Physics	Structure of Hadrons and Nuclei	Tests of Fundamental Symmetries
Chulwoo Jung	*Anthony Francis*	*Daniel Nogradi*	*Alexei Bazavov*	*Henry Lamm*	*Christopher Bouchard*	*Huey-Wen Lin*	*Rajan Gupta*
Ramsey Auditorium	*Curia II, WH2SW*	*Conjectorium, WH3NE*	*Sunrise, WH11NE*	*Comitium, WH2SE*	*Hornets' Nest, WH8X*	*One West, WH1W*	*Theory, WH3NW*
13:30 - 15:30	13:30 - 15:30	13:30 - 15:30	13:30 - 15:30	13:30 - 15:30	13:30 - 15:30	13:30 - 15:30	13:30 - 15:30

Coffee break

15:30 - 16:00

16:00

17:00

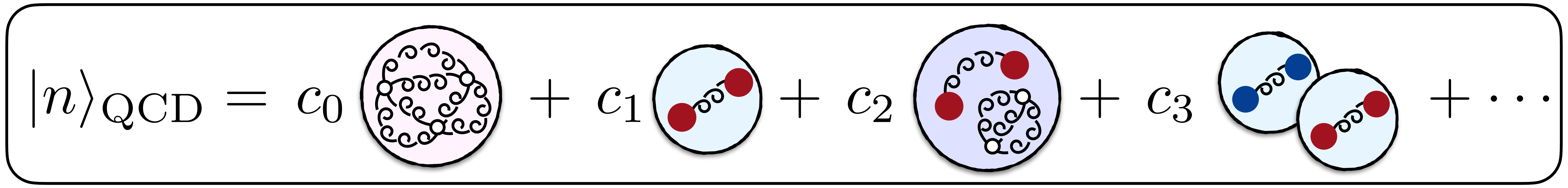
Algorithms and Artificial Intelligence	Hadronic and Nuclear Spectrum and Interactions	QCD at Non-zero Density	Quark and Lepton Flavor Physics	Software Development and Machines	Theoretical Developments	Vacuum Structure and Confinement
Sam Foreman	*Antonin Portelli*	*Christian Schmidt*	*Masaaki Tomii*	*Frank Winter*	*Evan Berkowitz*	*Akihiro Shibata*
Ramsey Auditorium	*Curia II, WH2SW*	*Sunrise, WH11NE*	*Hornets' Nest, WH8X*	*Comitium, WH2SE*	*One West, WH1W*	*Theory, WH3NW*
16:00 - 17:40	16:00 - 17:40	16:00 - 17:40	16:00 - 17:40	16:00 - 17:40	16:00 - 17:40	16:00 - 17:40

Break

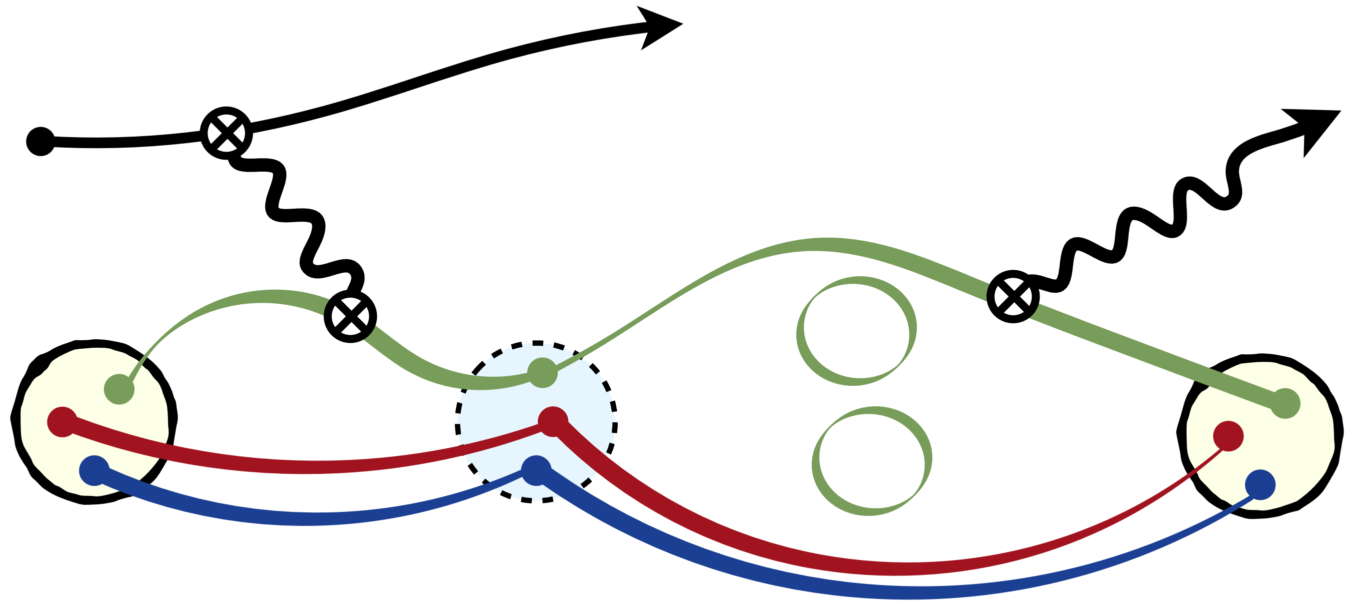
17:40 - 18:00

Hadronic/nuclear physics

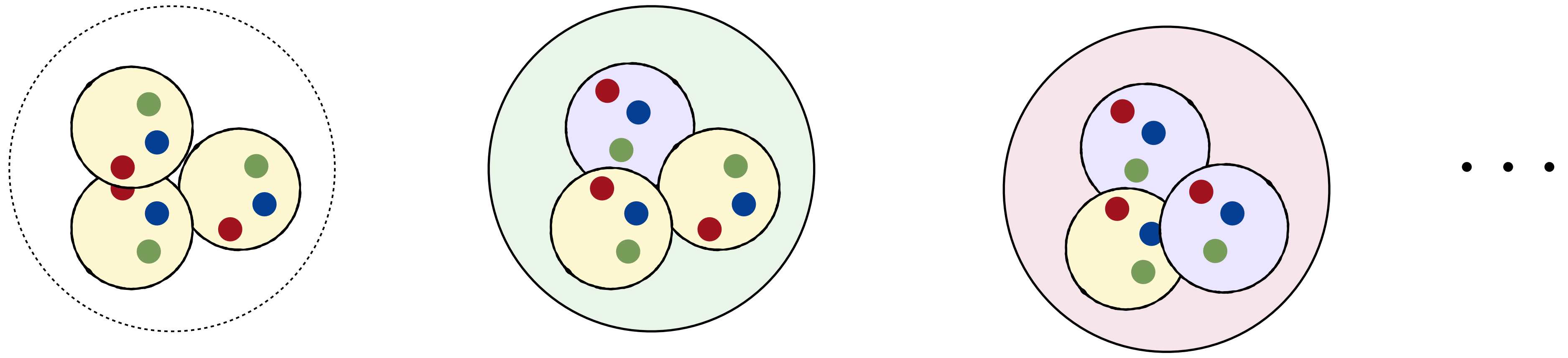
- QCD Spectroscopy



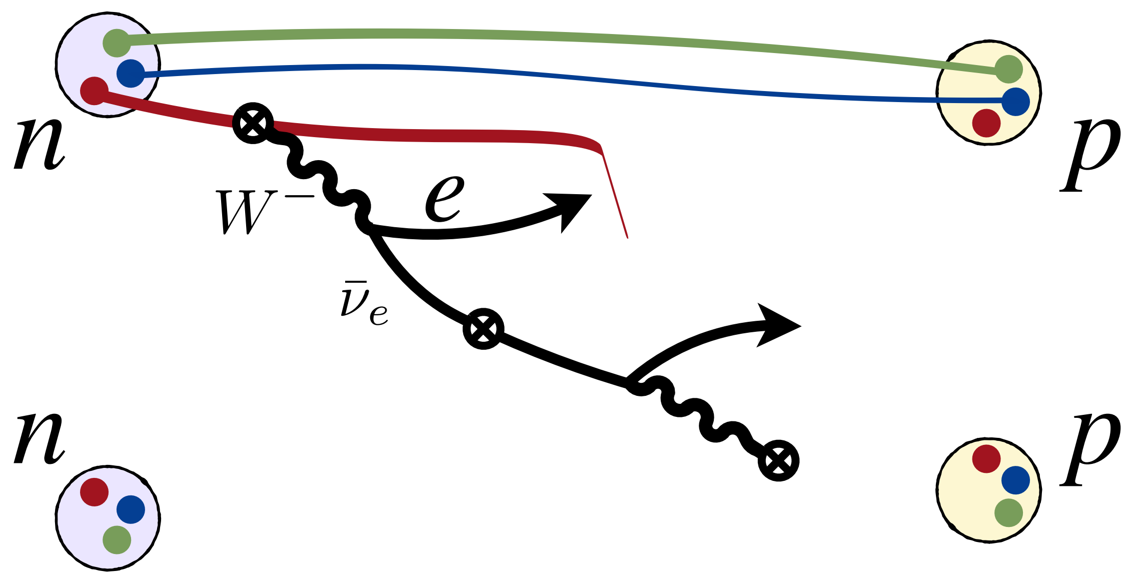
- Hadron structure



- Nuclear Structure

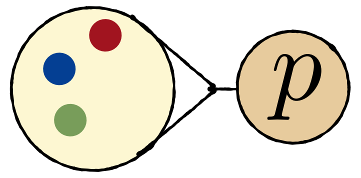


- Fundamental symmetries,



The nature of hadrons

Stable states



proton

& nuclei, and that's it!

Electroweak unstable

but QCD-stable states

QCD unstable states

The nature of hadrons

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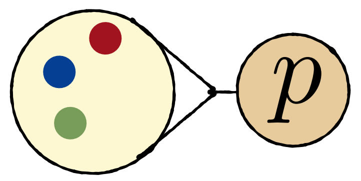

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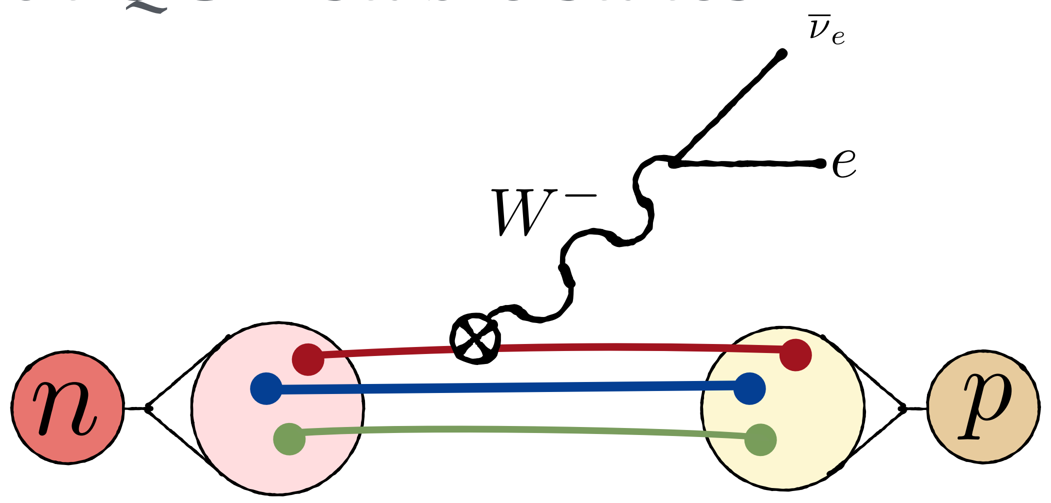
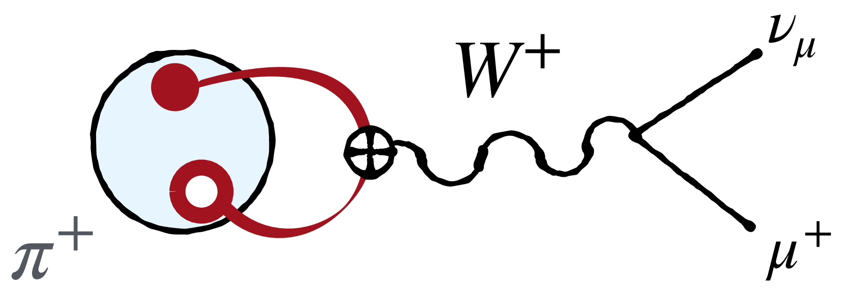
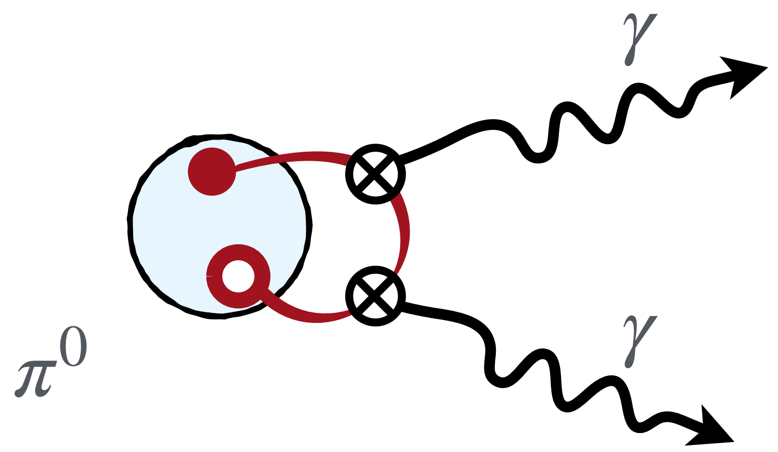


neutron

~15min



$\mathcal{O}(10^{-8}s)$



$\mathcal{O}(10^{-16}s)$

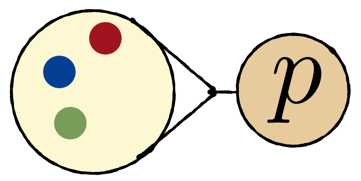
~dozens of such states

QCD unstable states

~99% of states fall under this category

The nature of hadrons

Stable states



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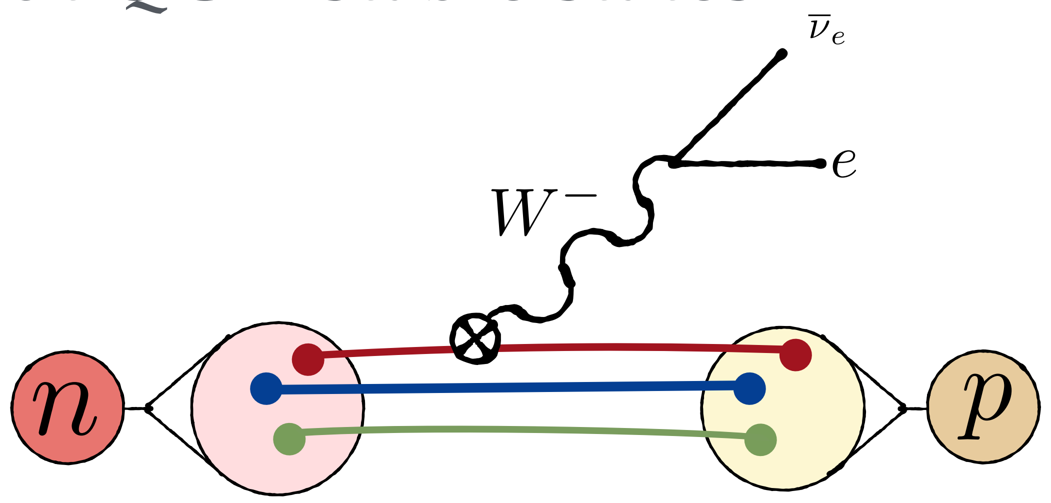

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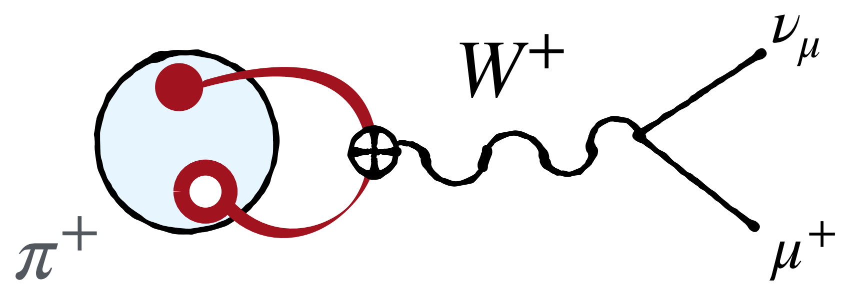
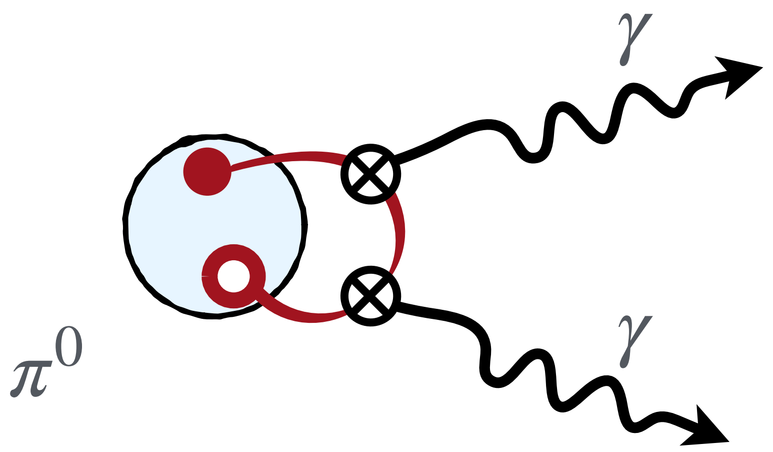


$\mathcal{O}(10^{-8}s)$



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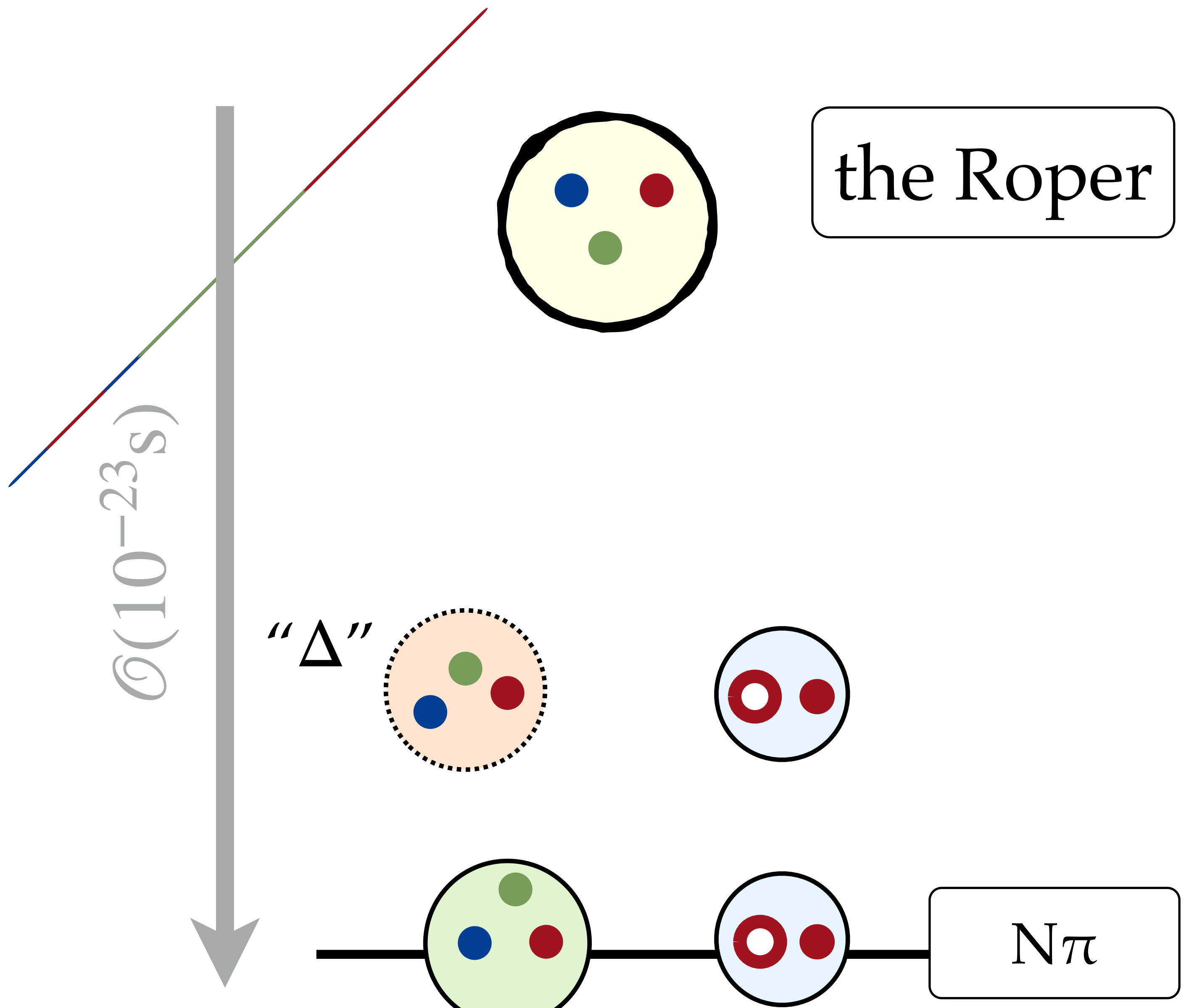

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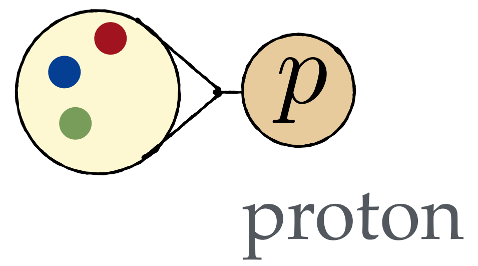
The nature of hadrons

Stable states



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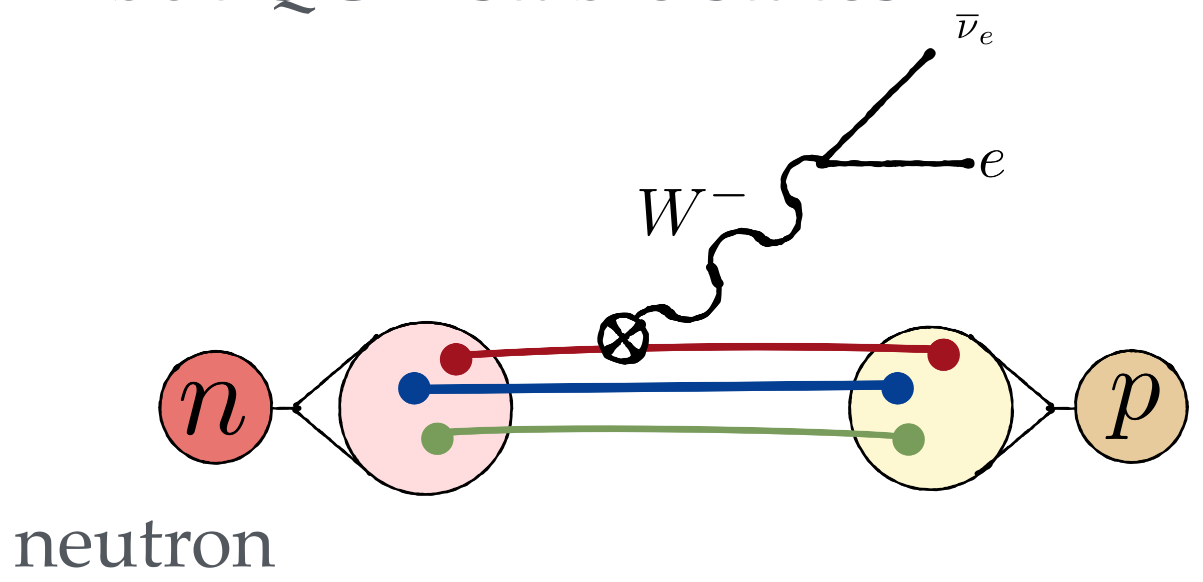

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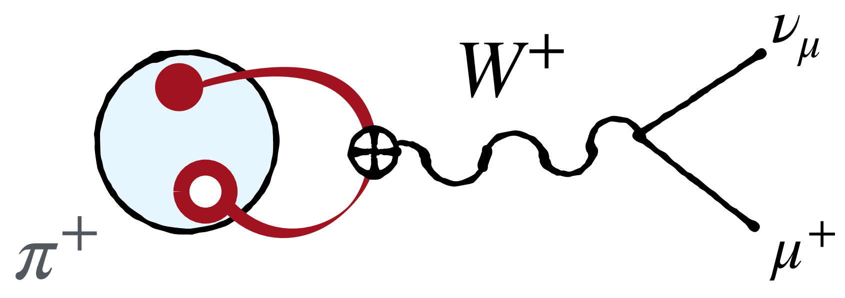

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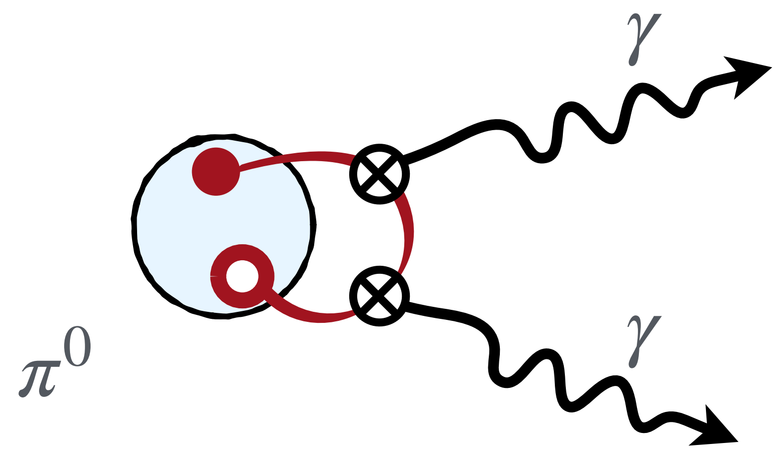

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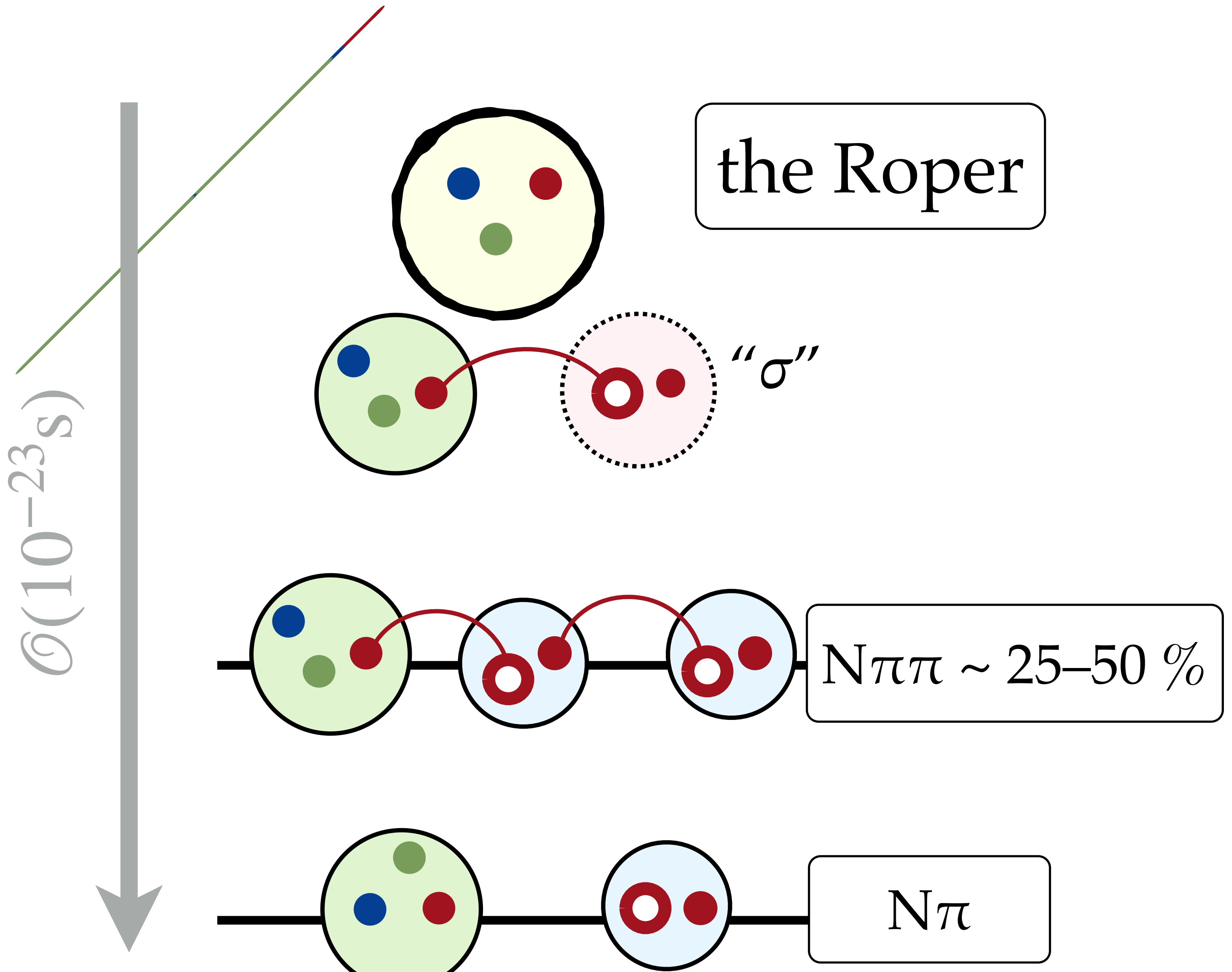

Strongly coupled system involving few-body states…!

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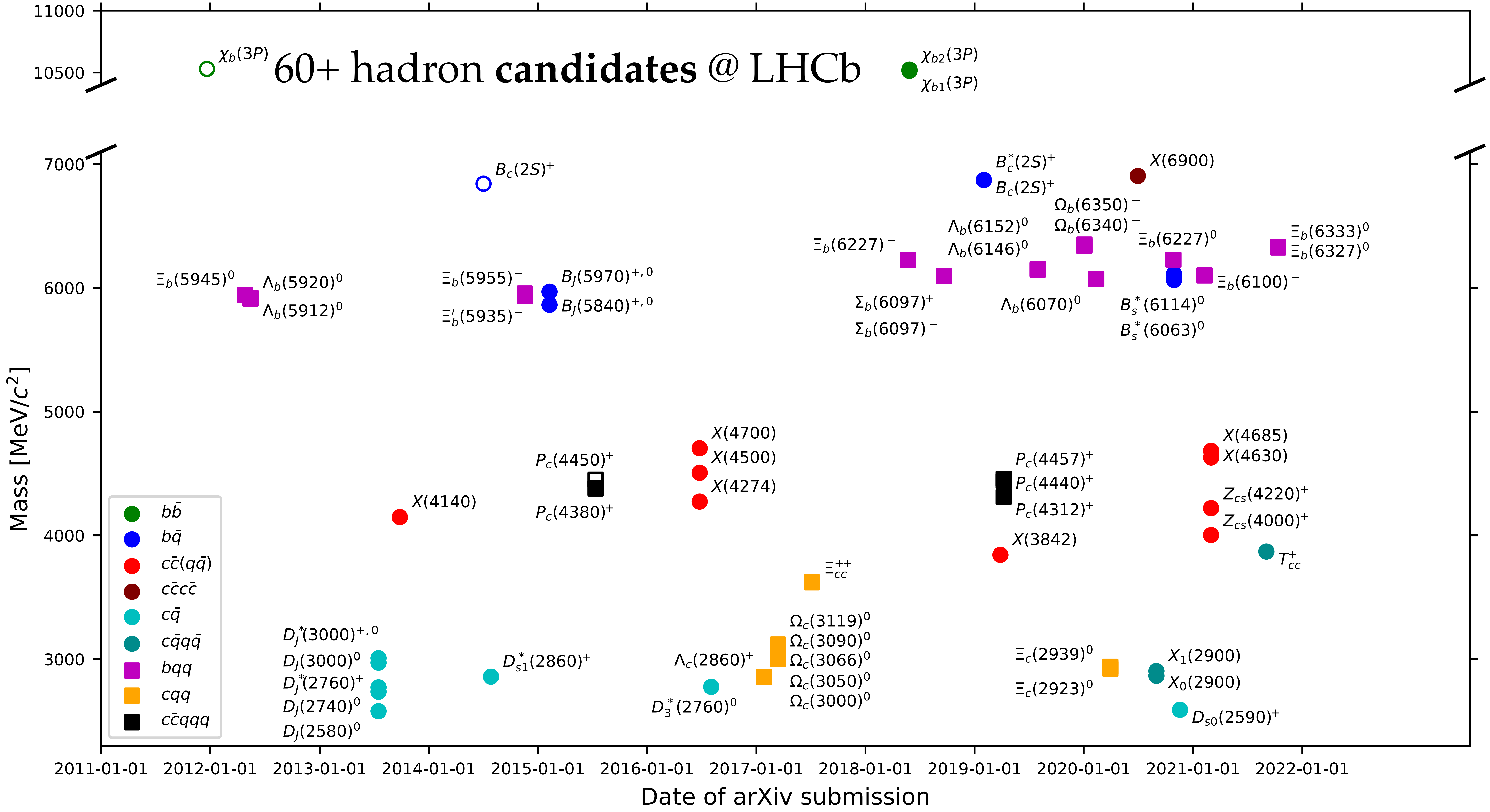
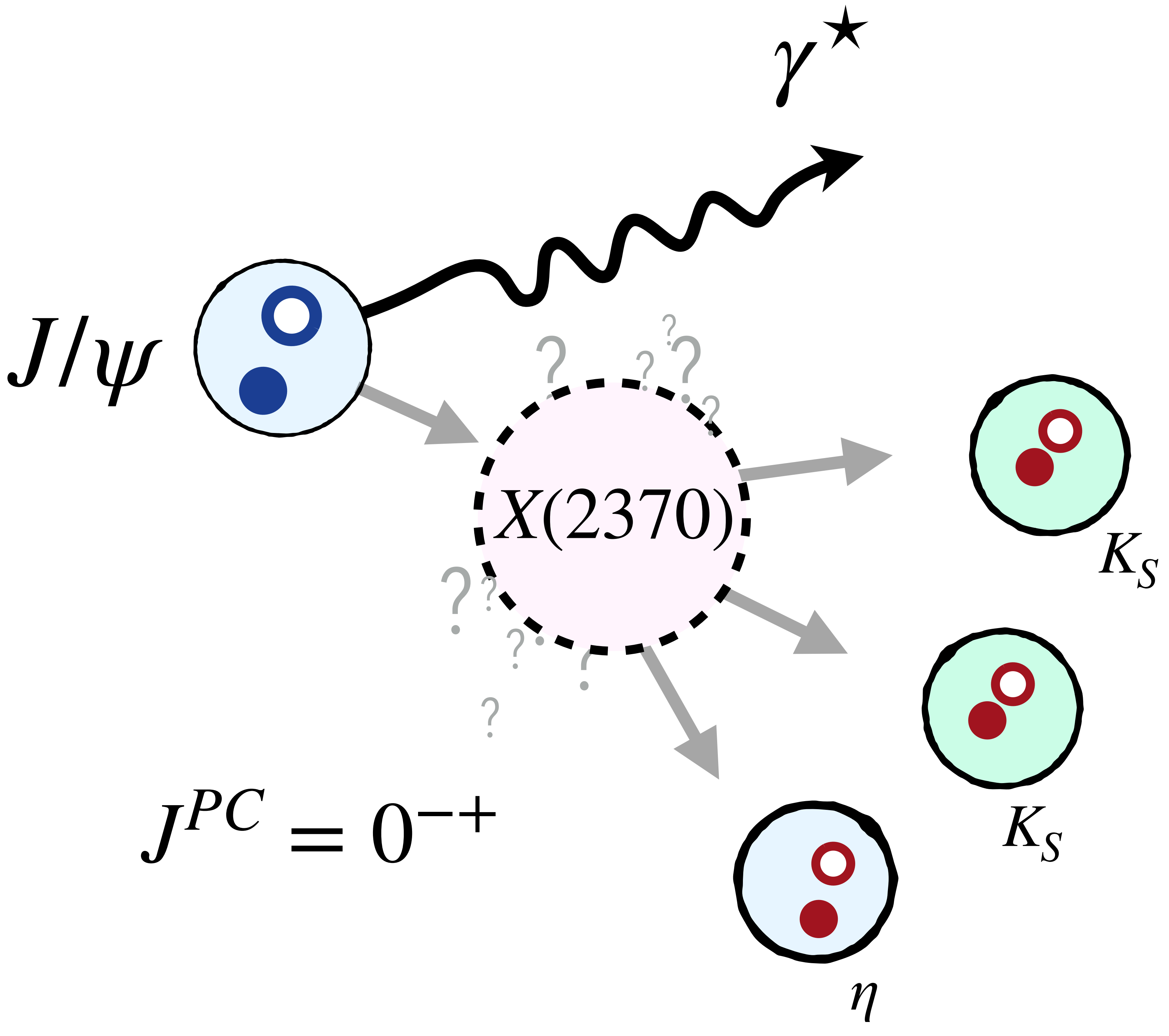
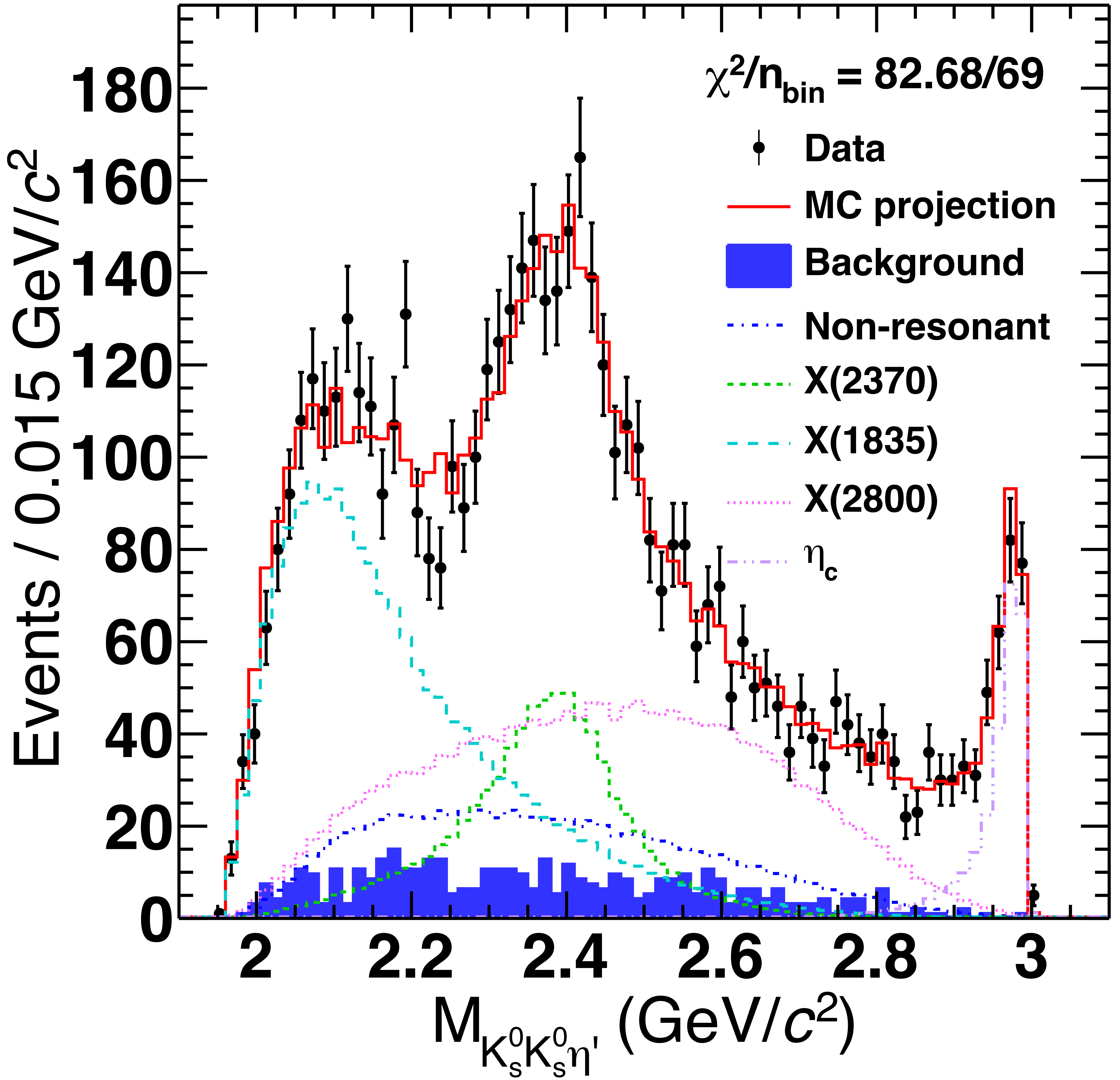
The particle zoo the remake



Numerous other experimental searches…

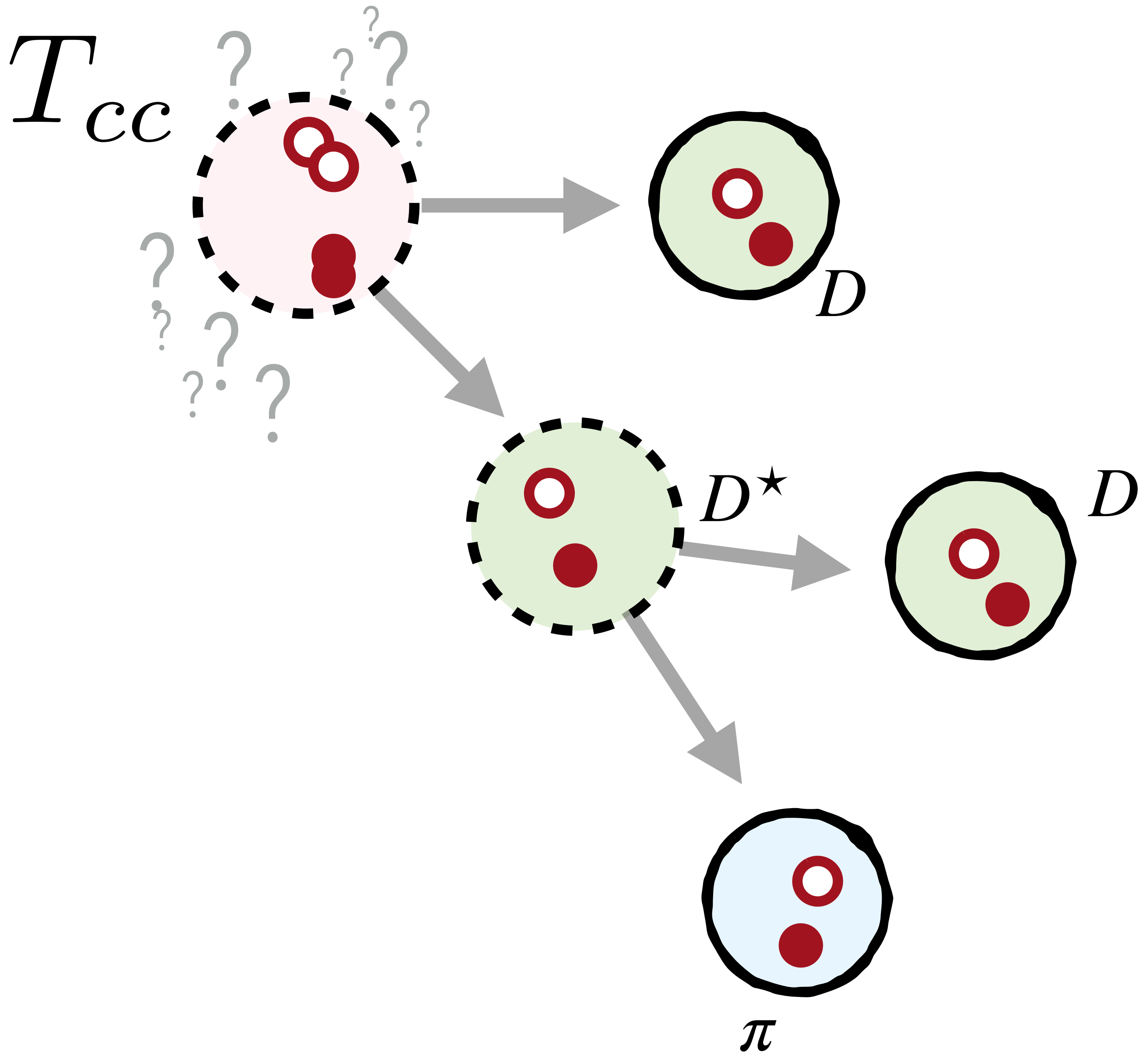
Glueballs?

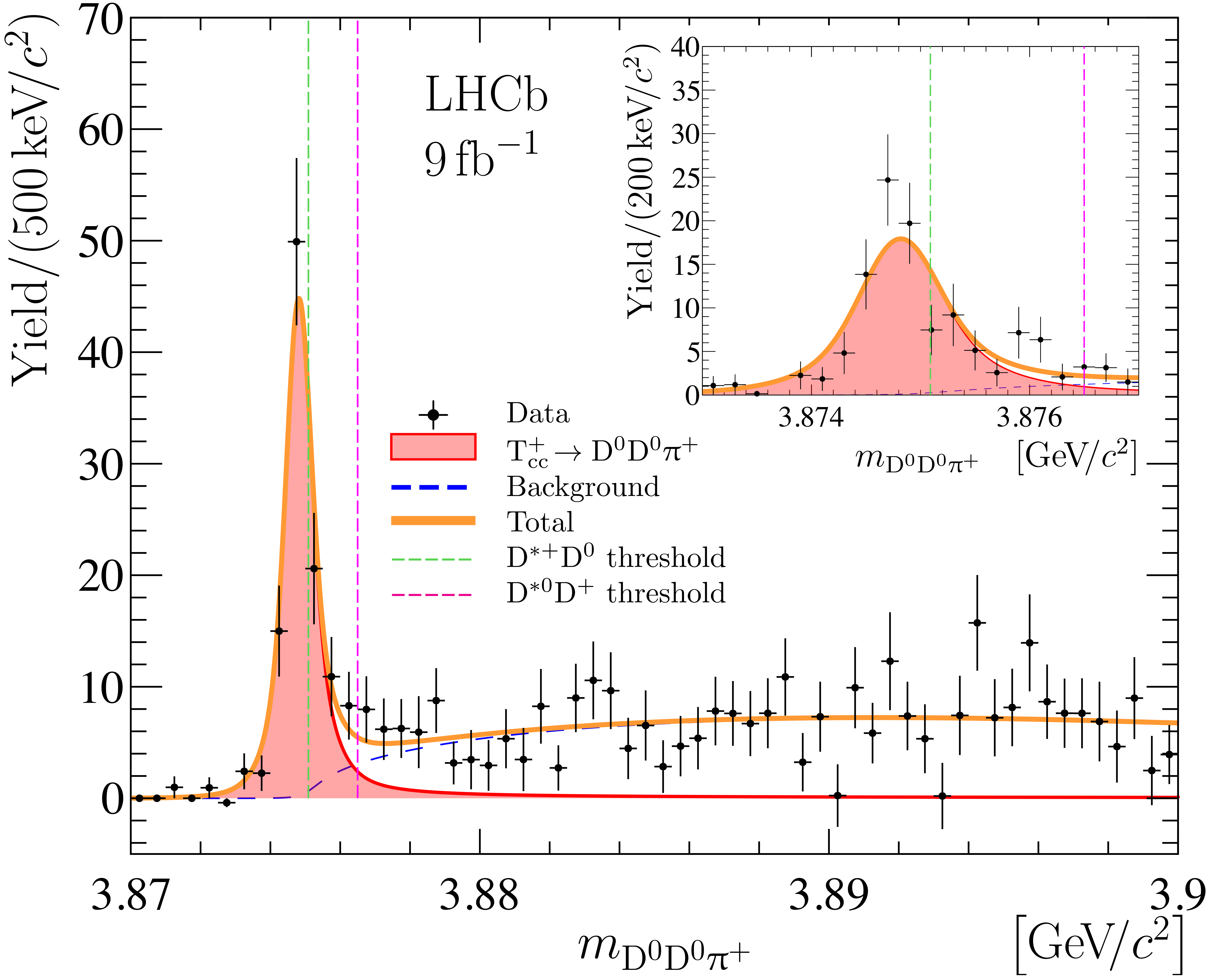




BESIII

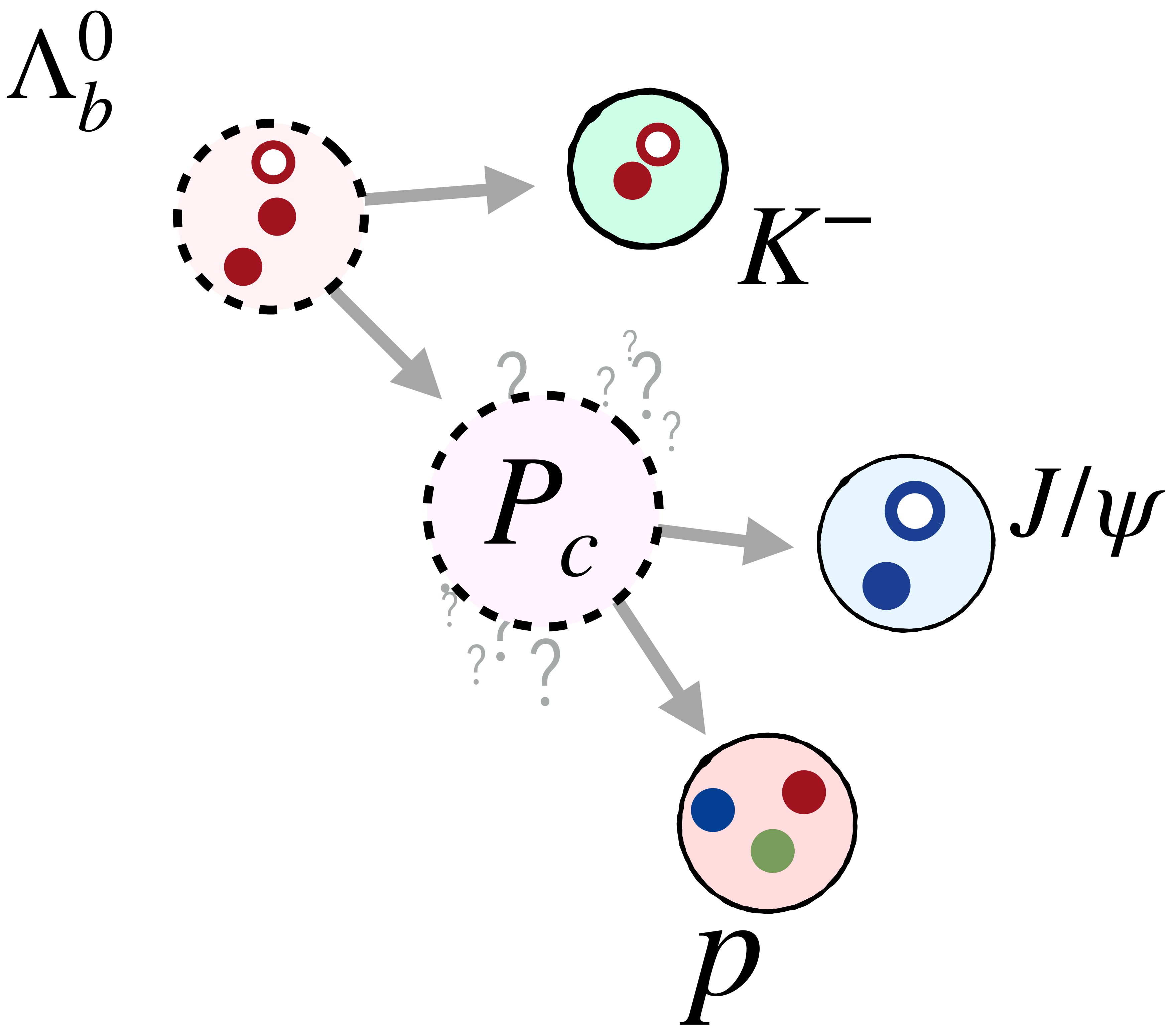
Tetraquarks?

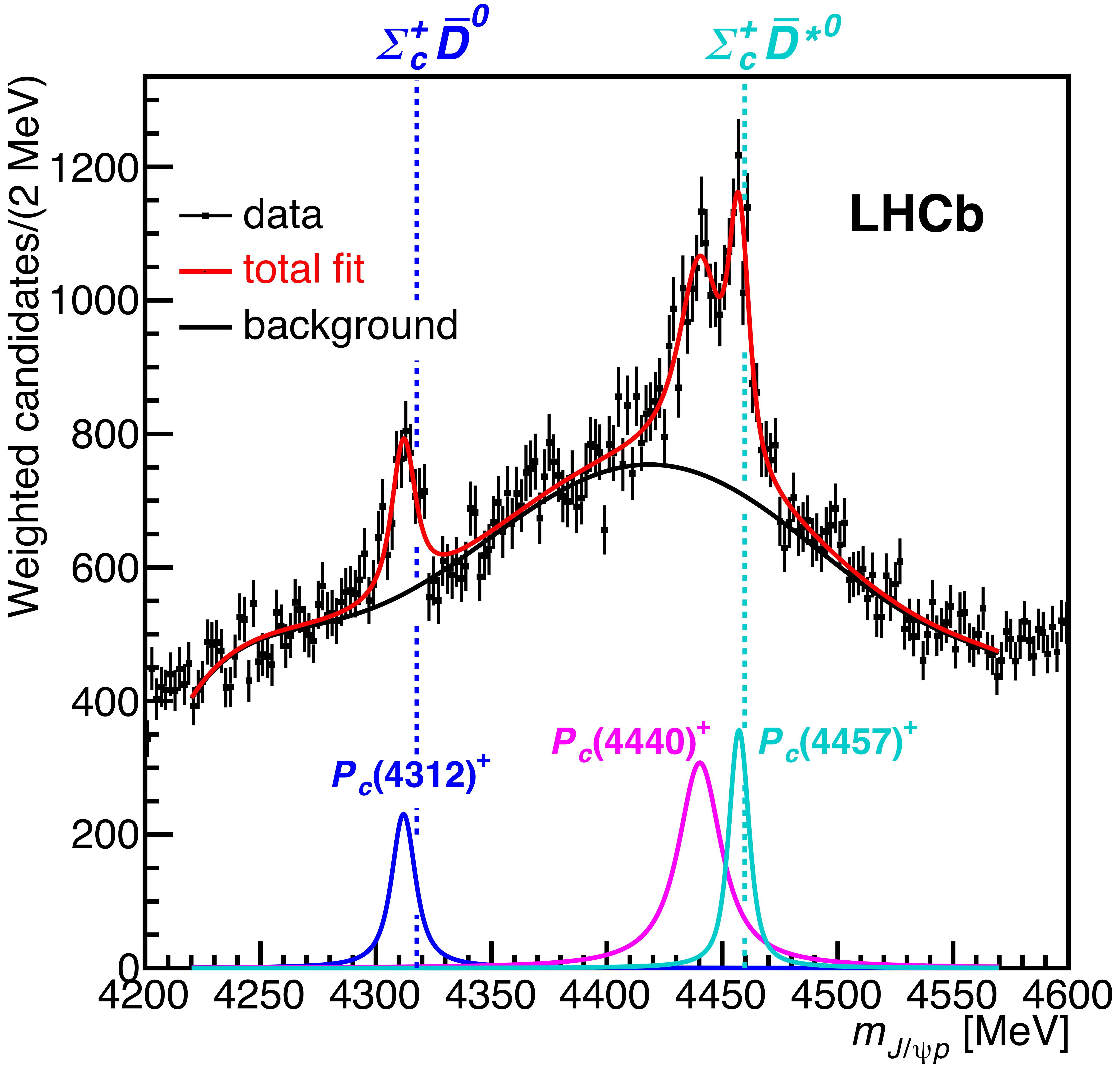




(2021)

Pentaquarks?





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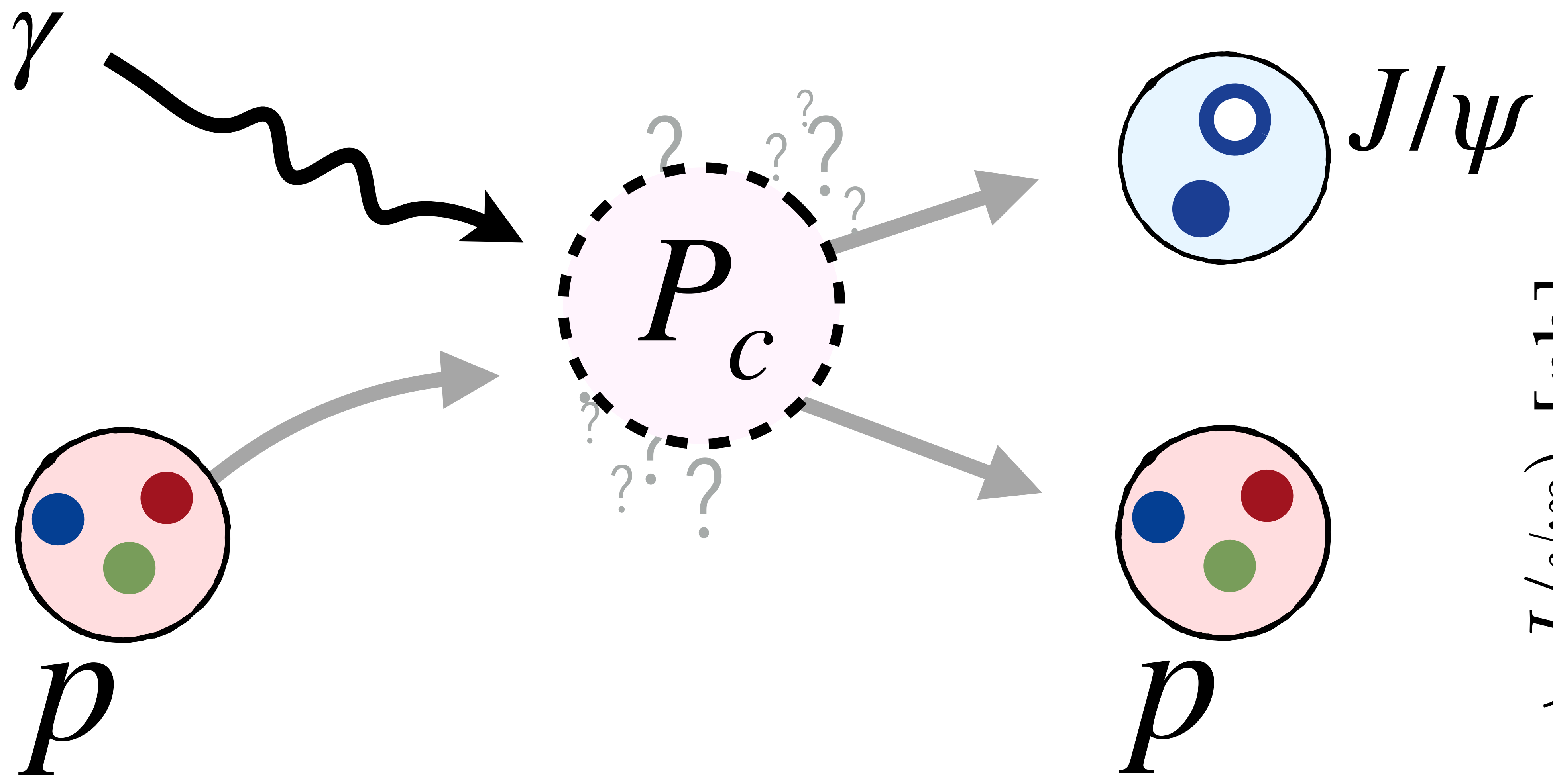
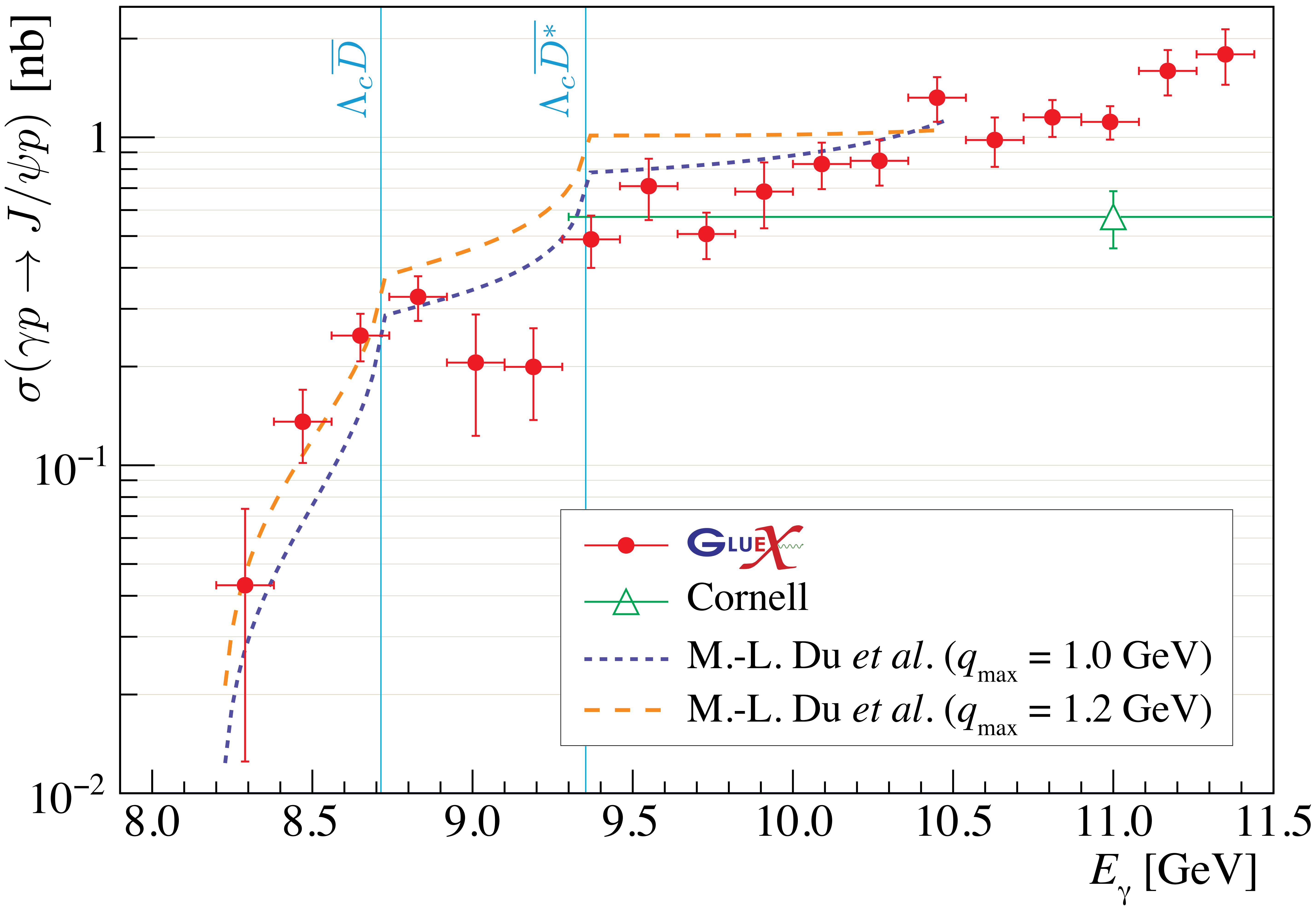
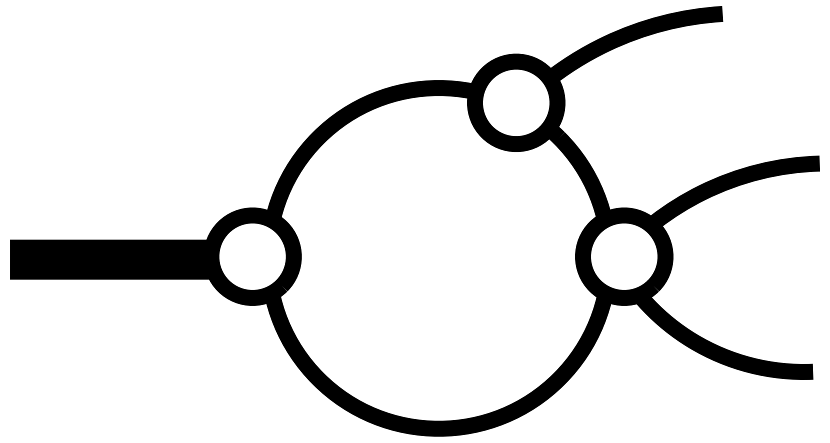


No smoking gun of pentaquarks at JLab



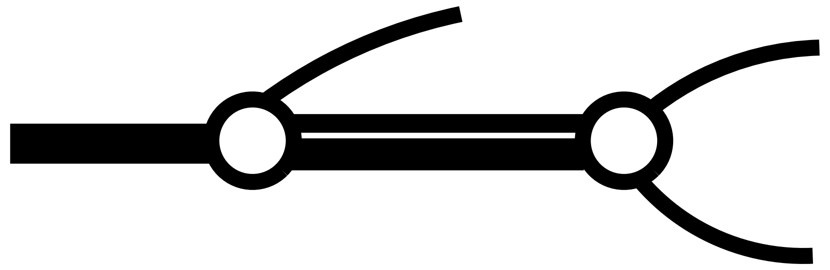
Key questions to answer

- [ ] Which enhancements in cross sections are actual QCD state?



$\sim \log(s - s_t)$

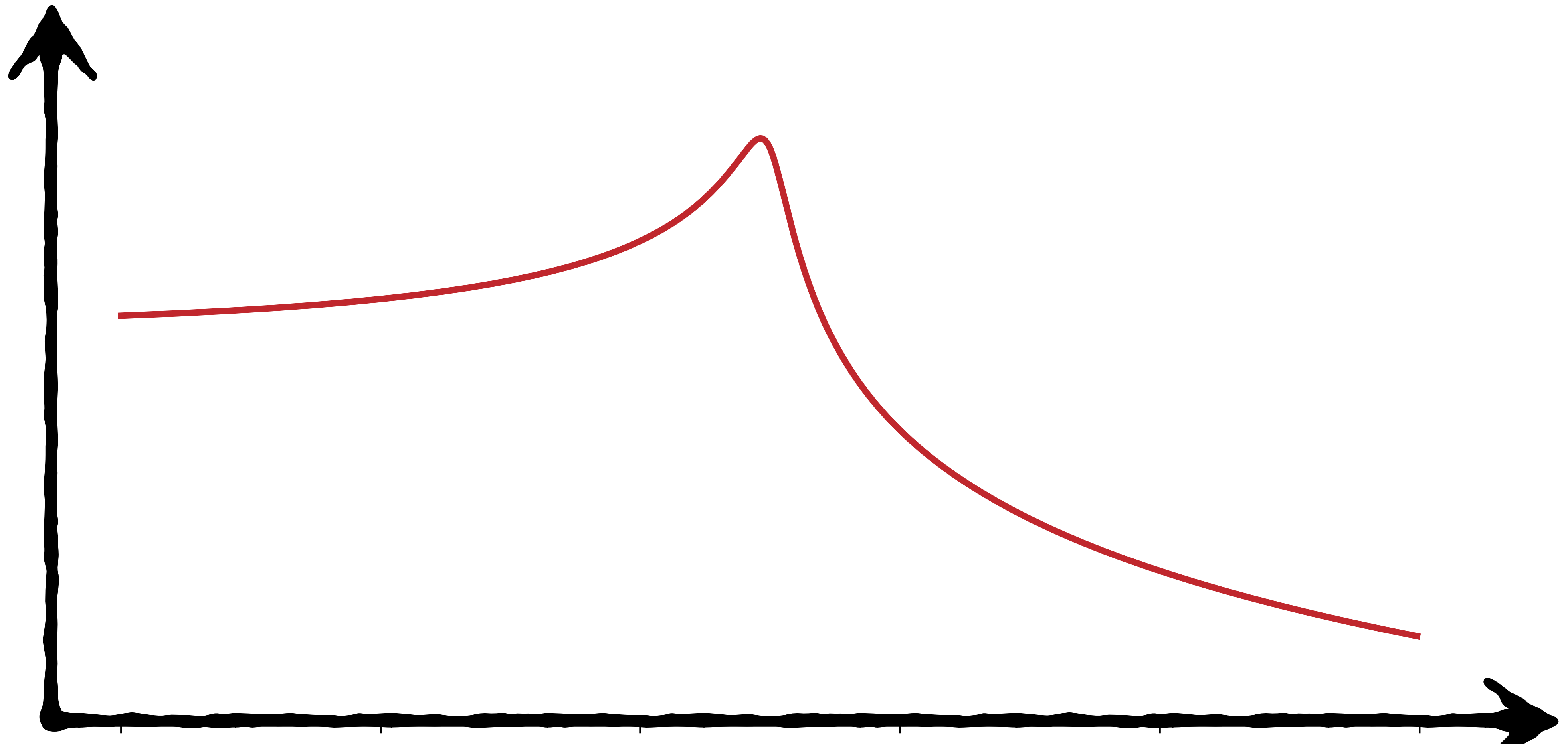
vs.

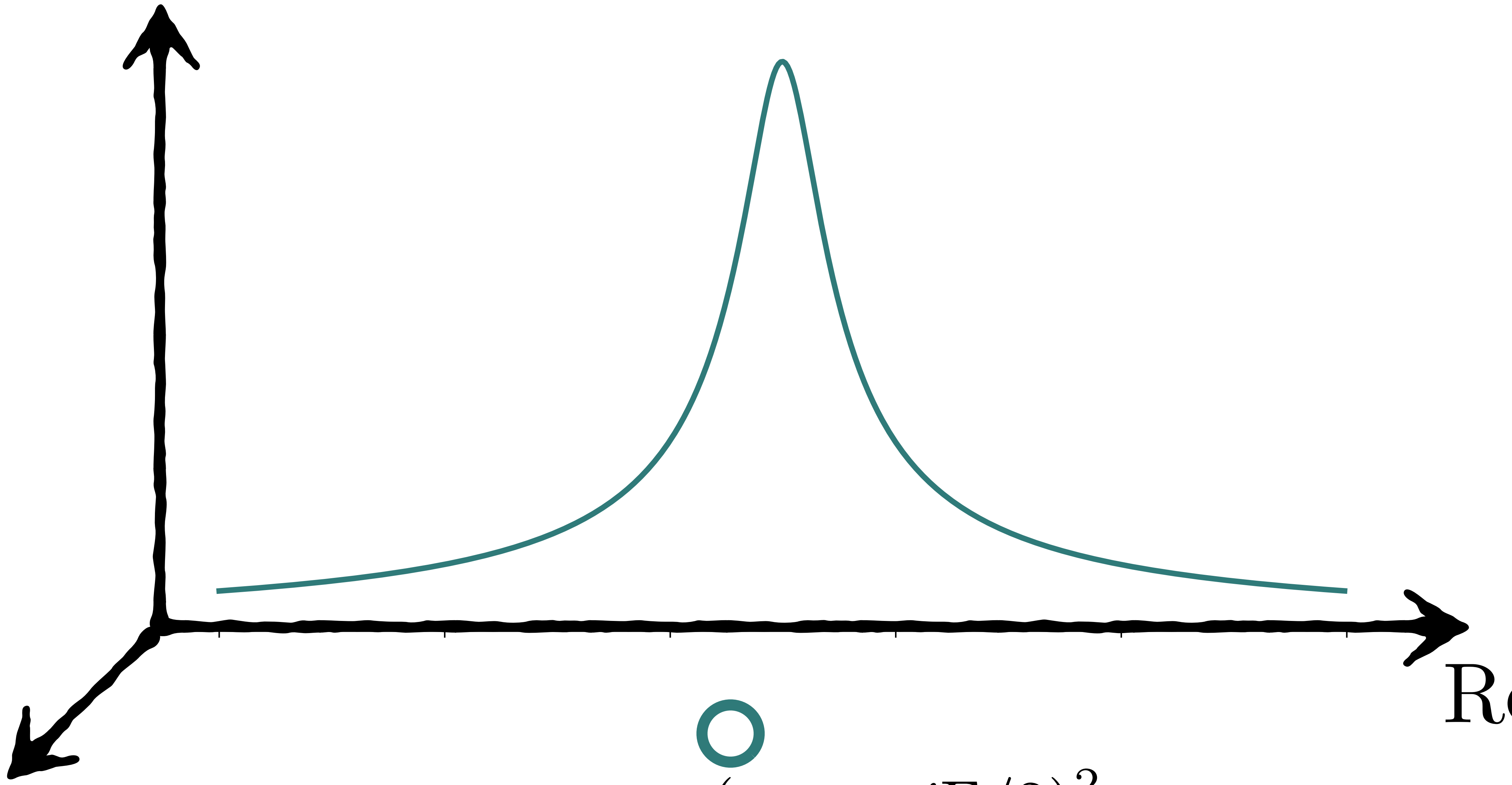


$$\sim \frac{1}{s - (m_R - \frac{i}{2}\Gamma)^2}$$

“just a poser”

“the real deal”





$s = E_{\rm cm}^2$

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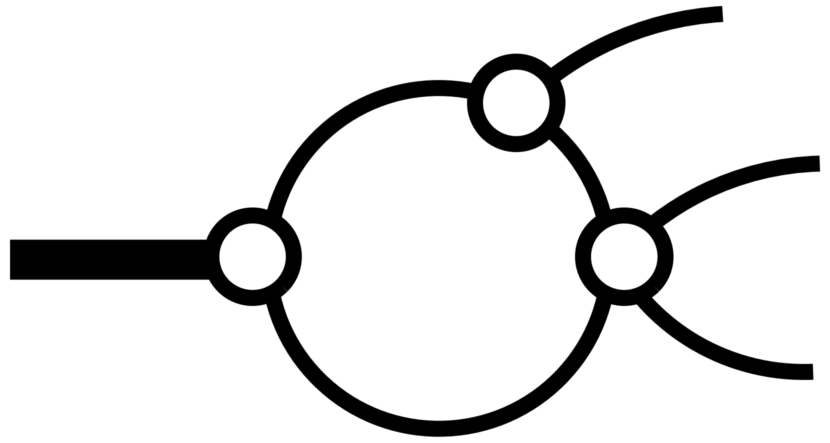

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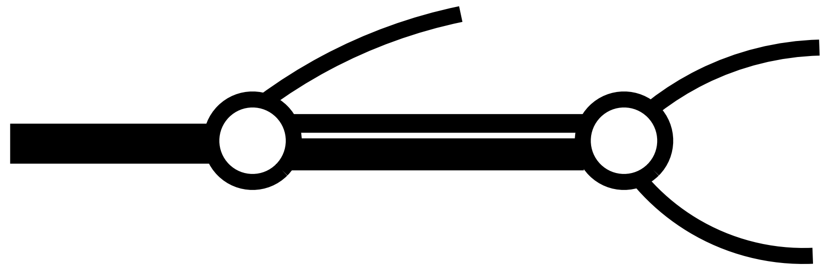

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"just a poser"

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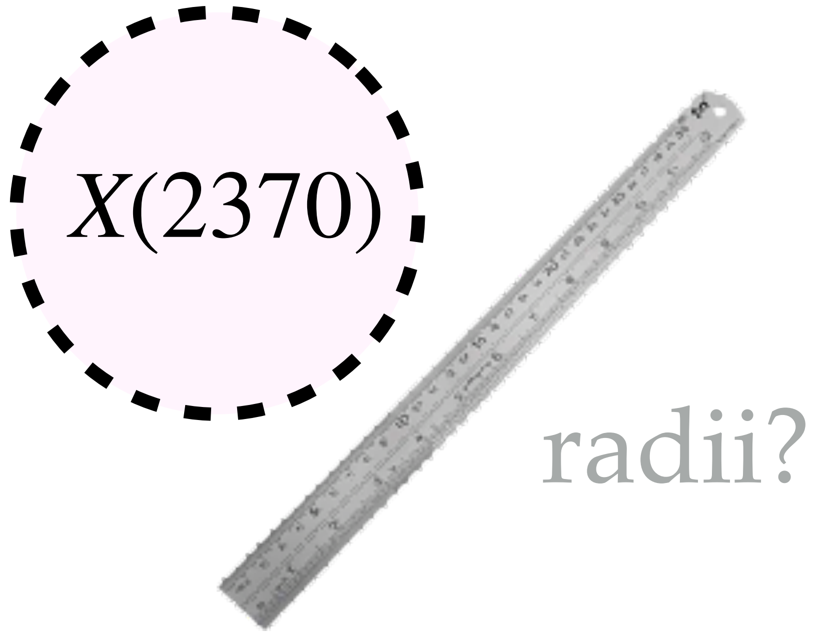
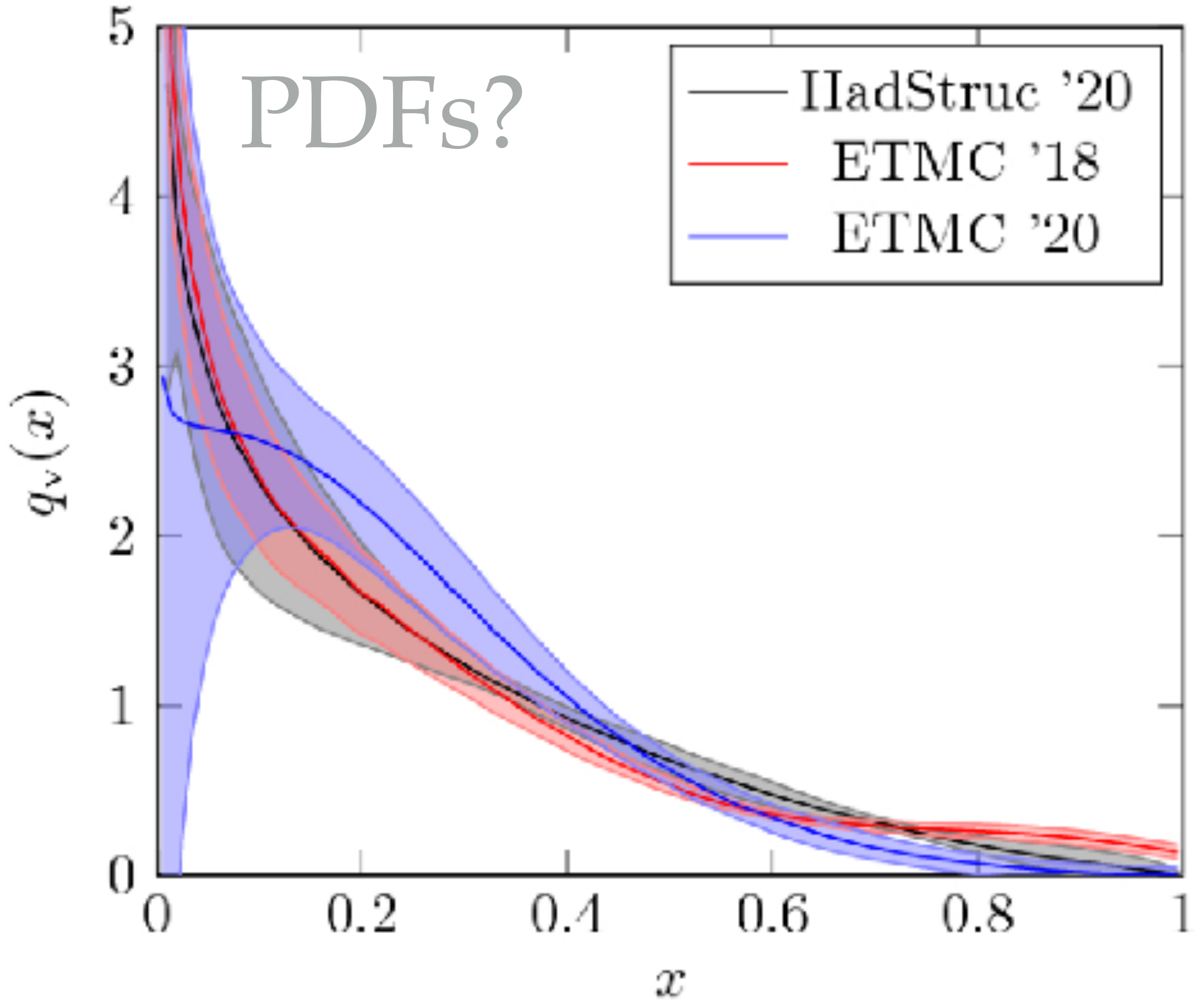
- If a real state, what is its inner structure?





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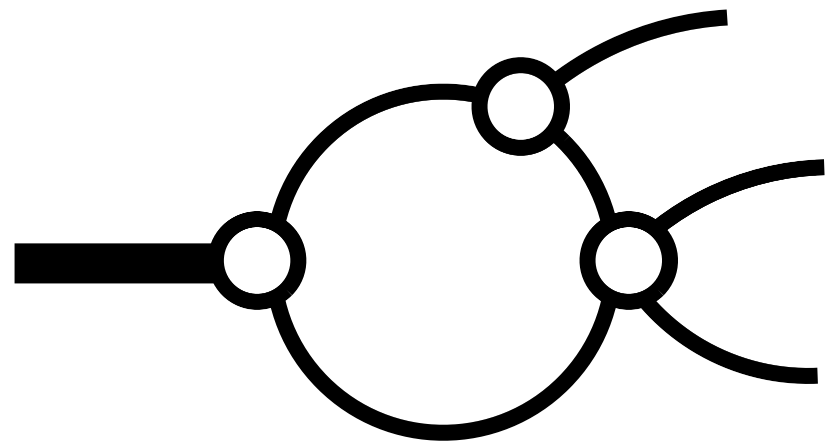

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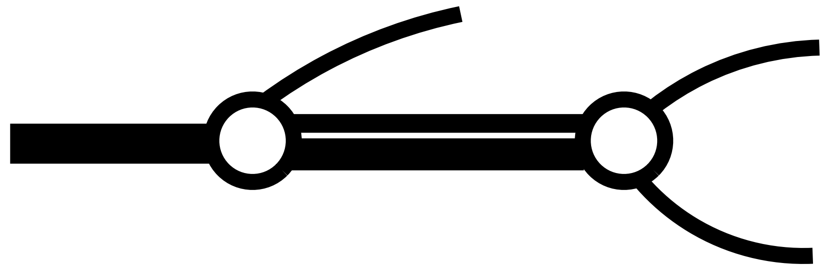

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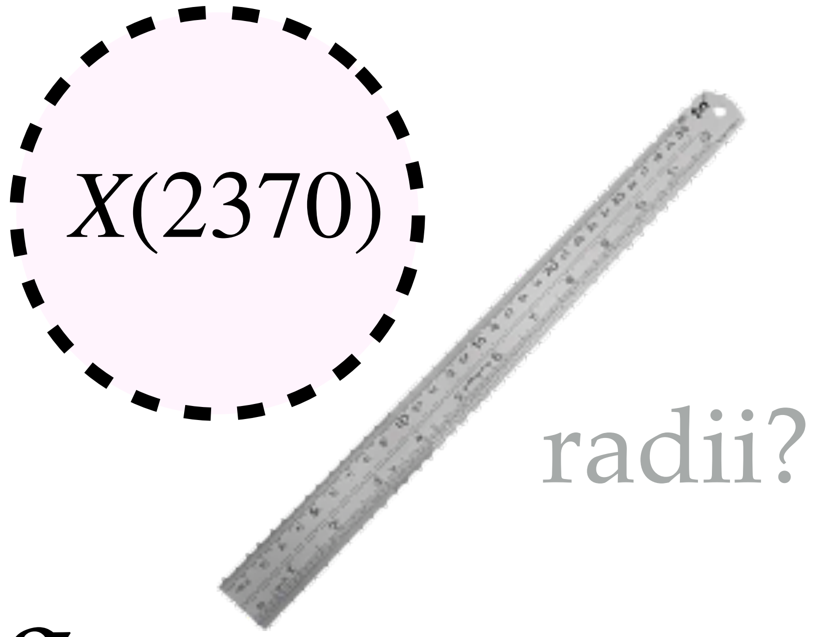
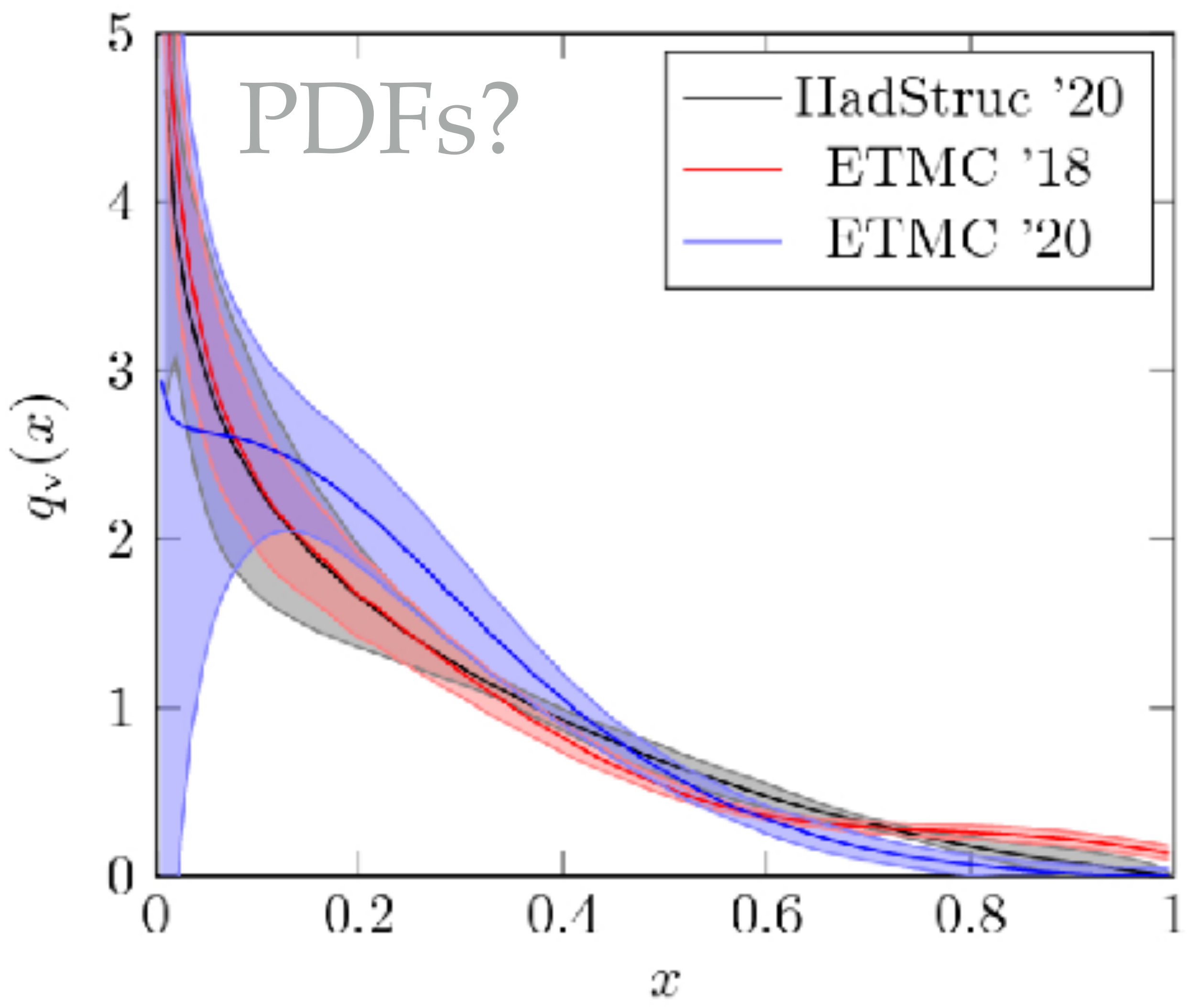
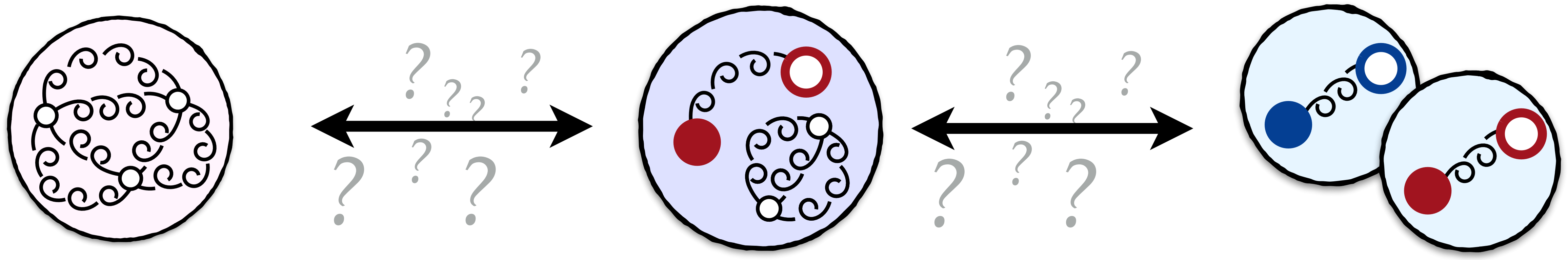




- ☐ Given structural information, can we say anything about the nature?



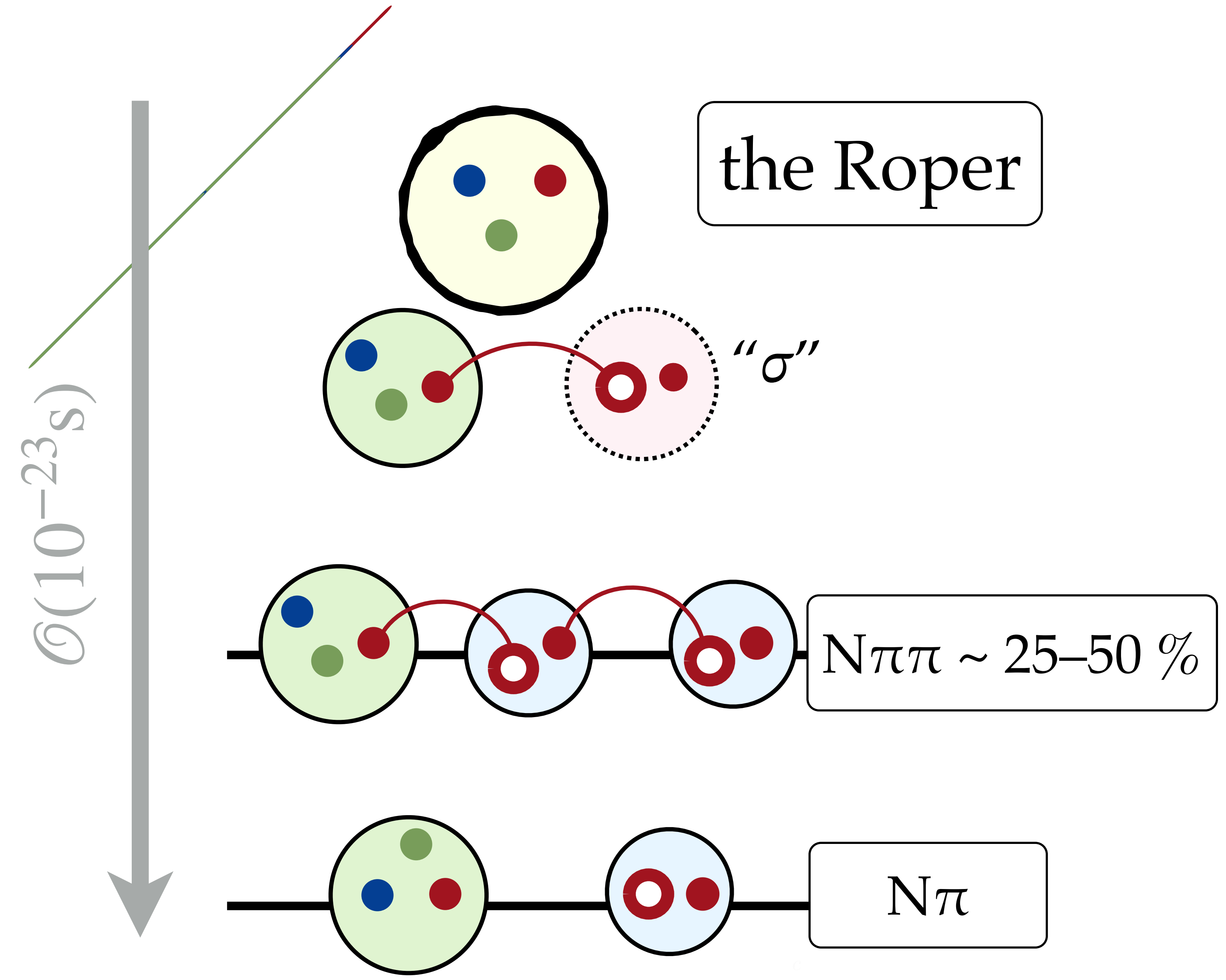
- ☐ Can we deduce general principles from the QCD spectrum?

Need for lattice QCD

{ If we want to claim understanding of a given theory, we must first understand its spectrum. }

{ requires a method that treats all non-perturbative aspects of QCD exactly… including coupling to large number of states }

QCD unstable states



Strongly coupled system involving few-body states…!

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Outline

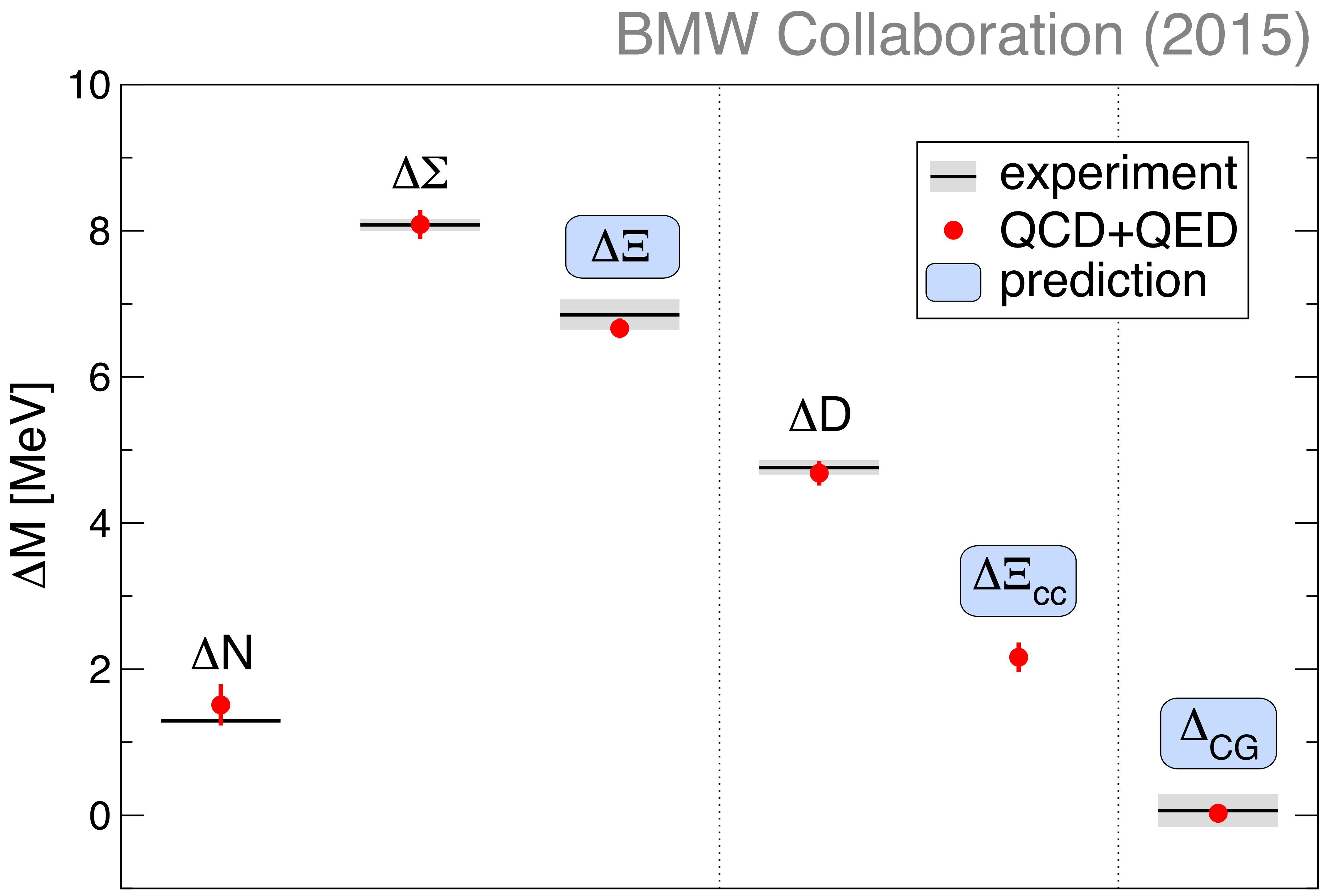
Lattice QCD in a nutshell **[today & tomorrow]**

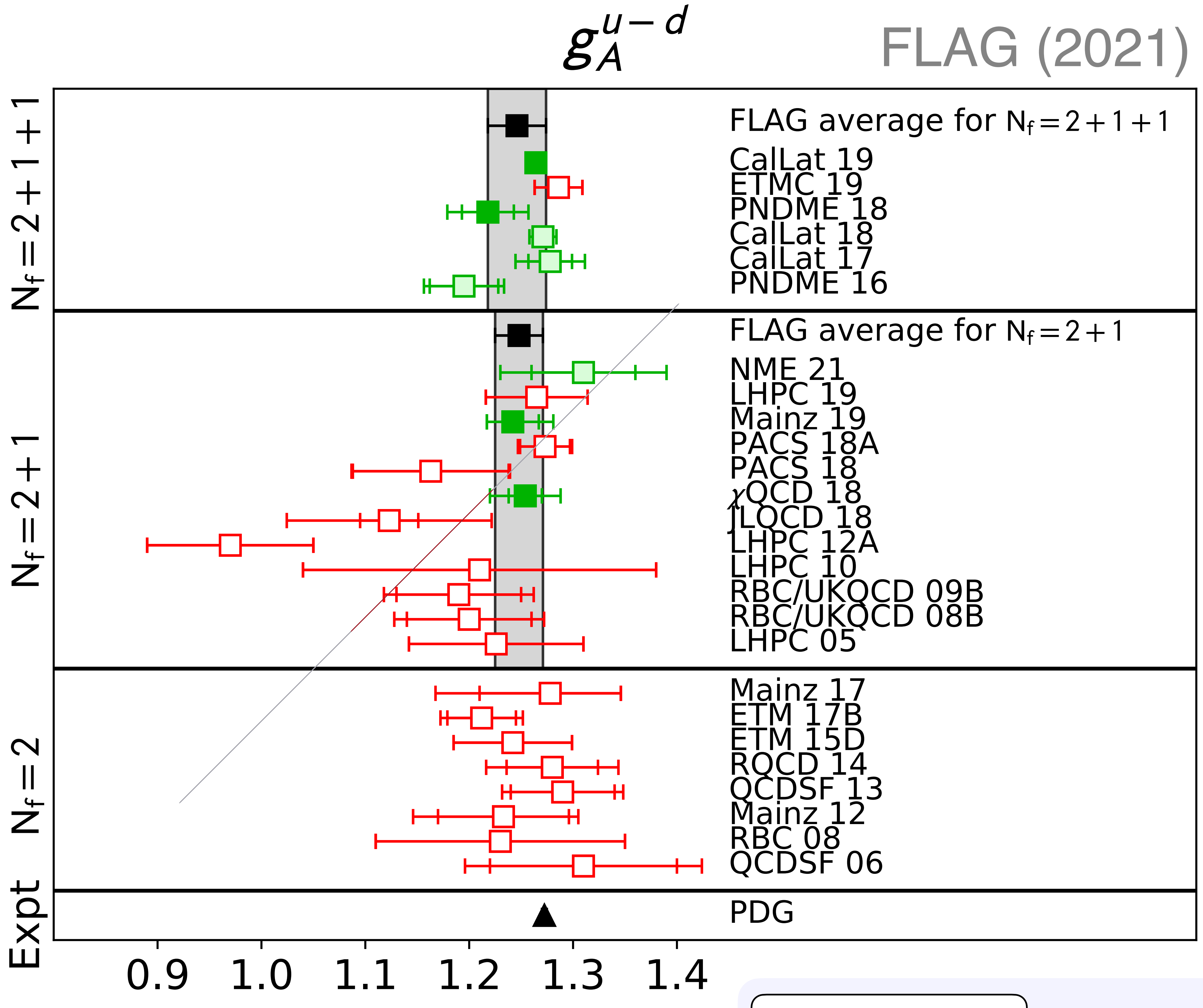
- Does lattice work?
- why does lattice QCD work?
- what can it be used for?
- what are its limitations?

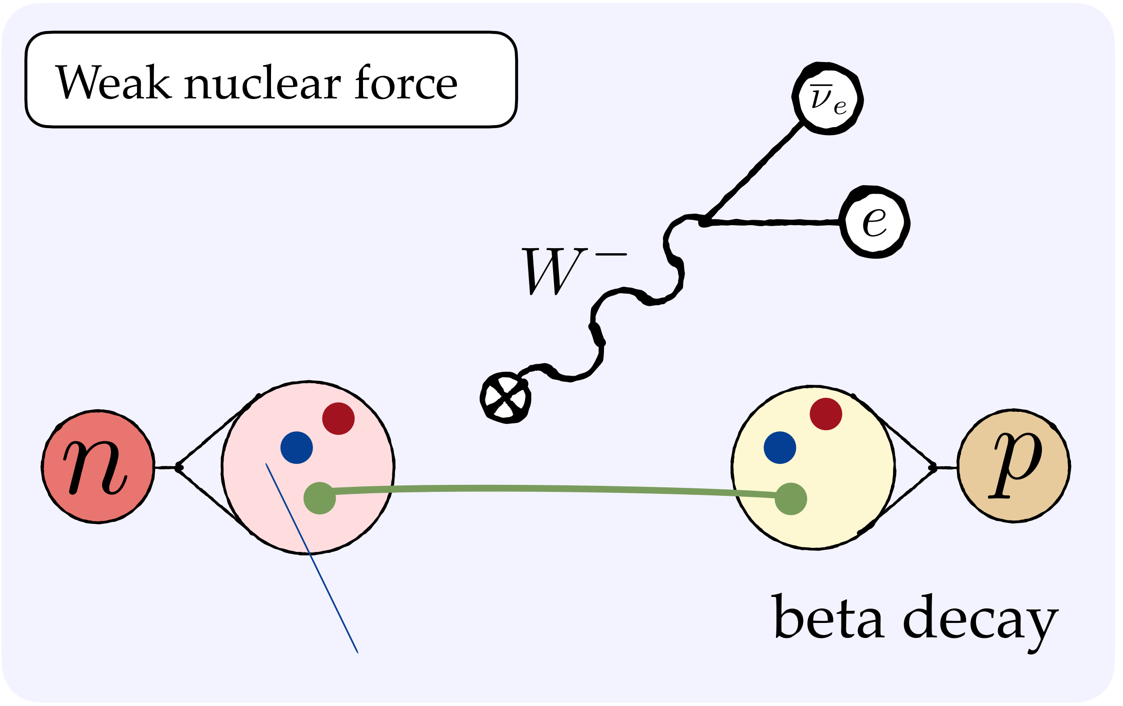
What is the cutting edge of lattice QCD? **[tomorrow]**

- hadron structure, fundamental symmetry,
- scattering processes,
- ….other stuff, I won't get to 🤓
  - finite-temperature, weak decays, BSM searches, …. ,

lattice QCD works!







Coleman-Glashow mass difference $\Delta_{CG} = \Delta M_N - \Delta M_\Sigma + \Delta M_\Xi$

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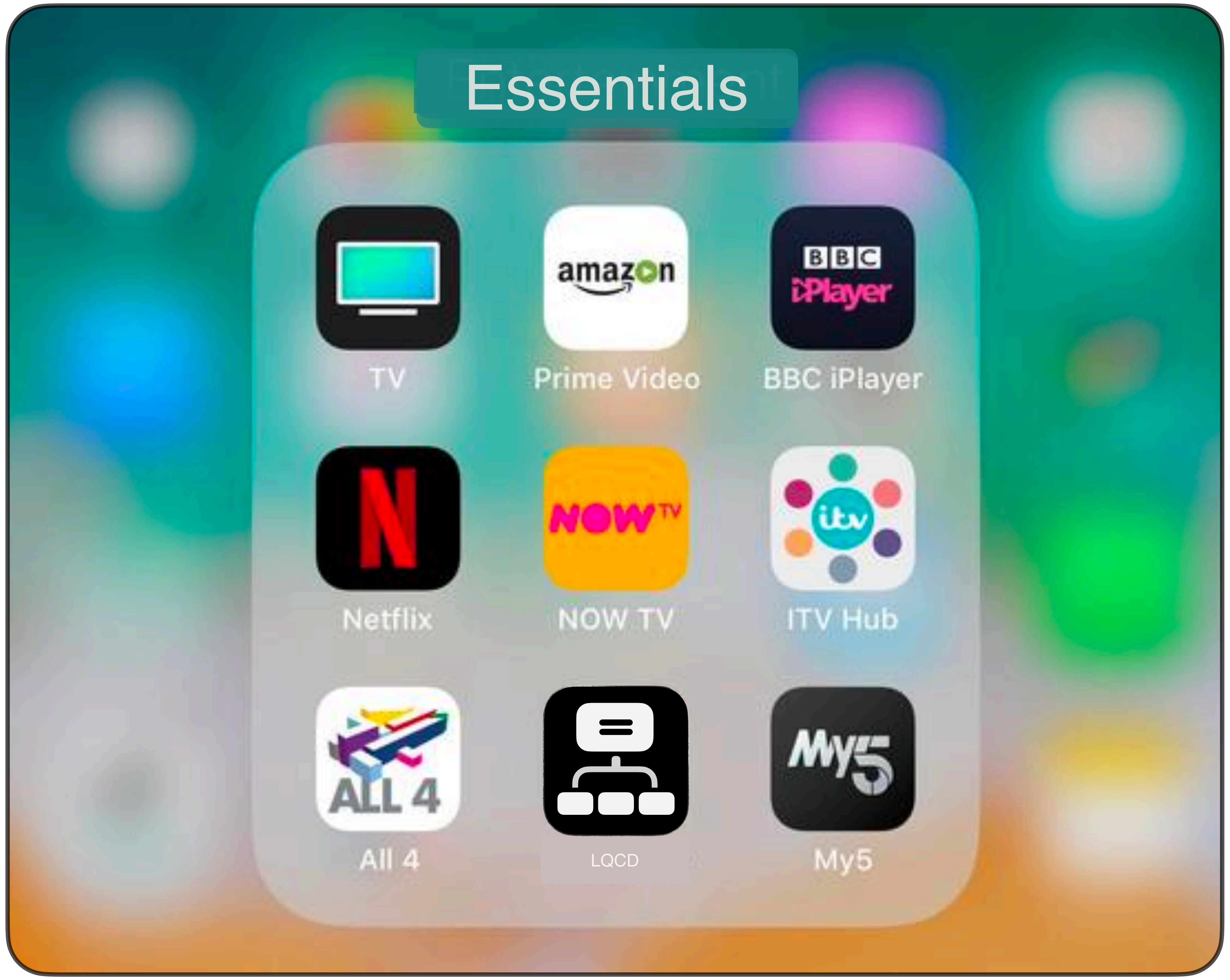
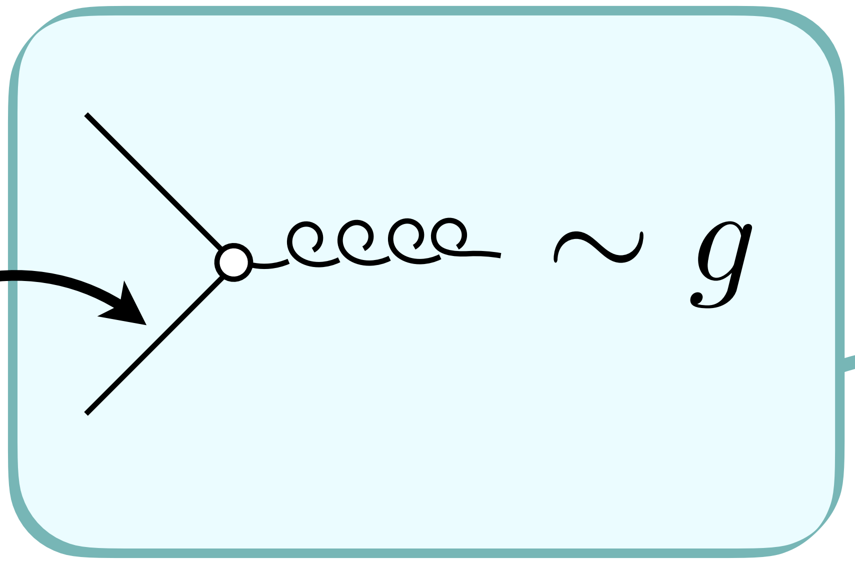
Lattice QCD in a nut shell

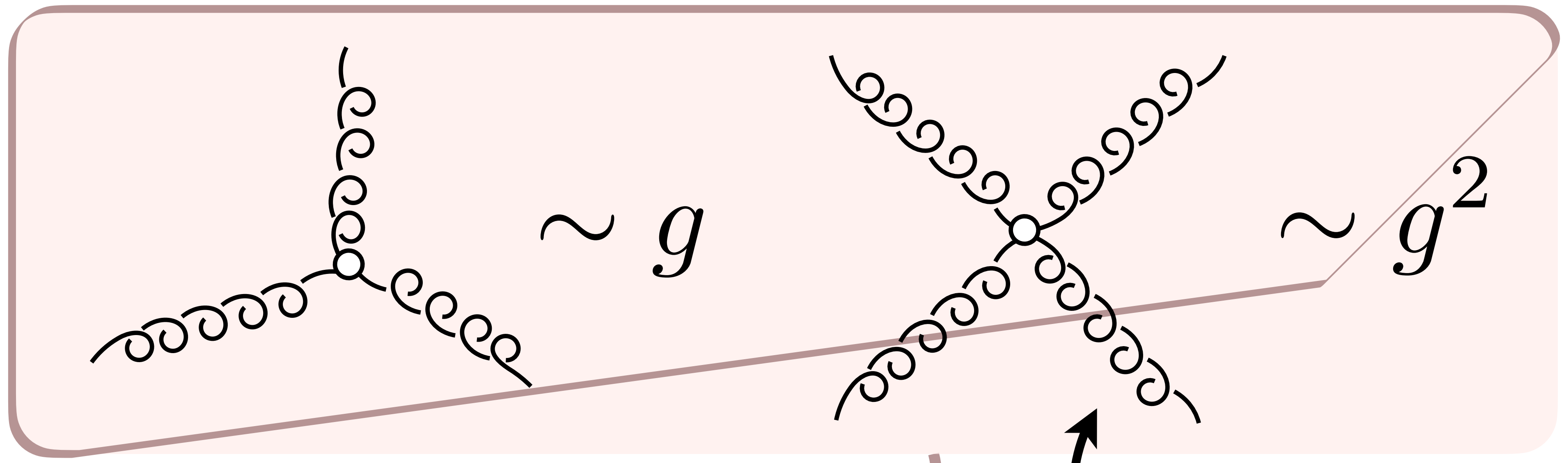


some people's idea of lattice QCD….?

Back to the basics

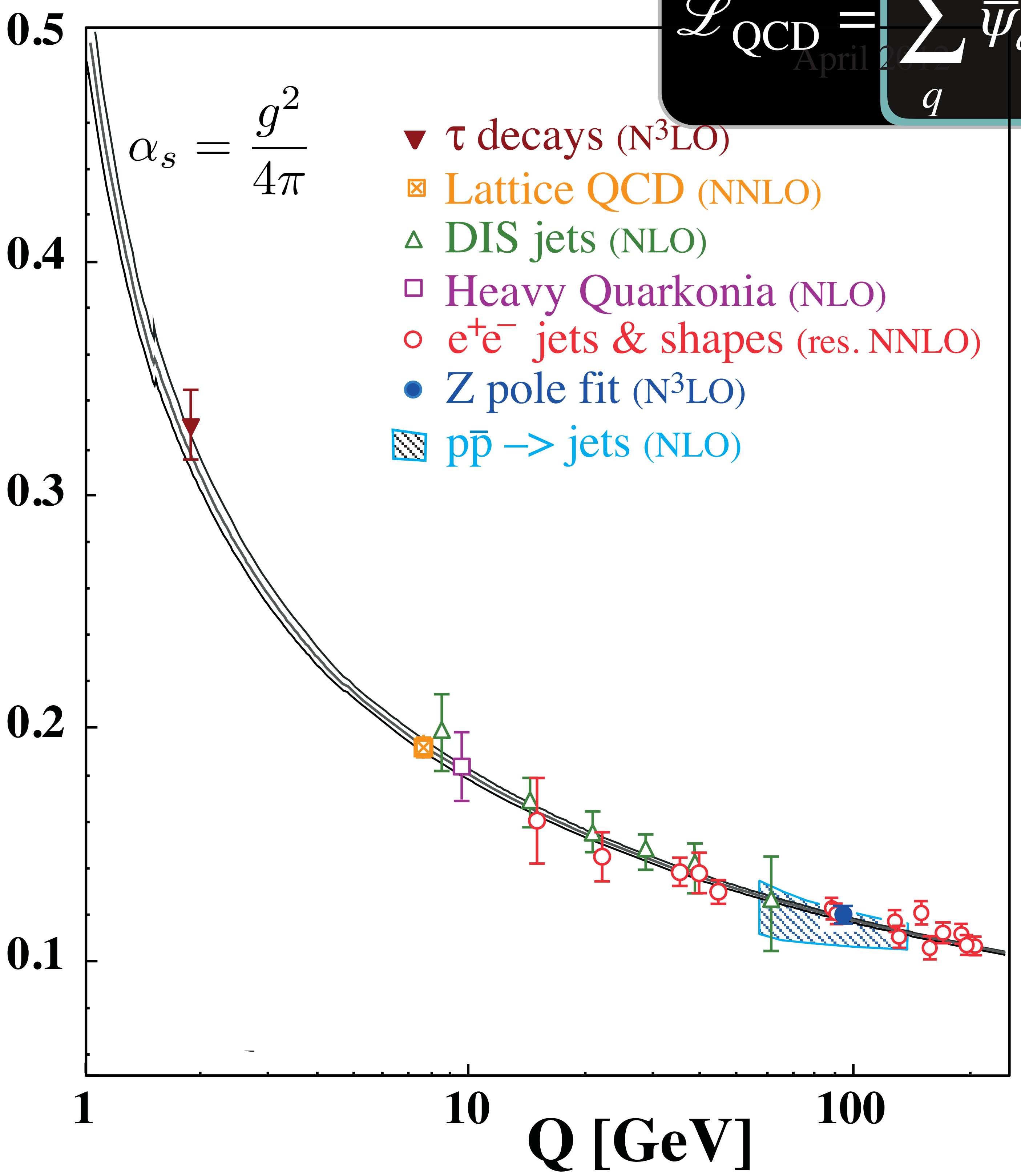
quarks



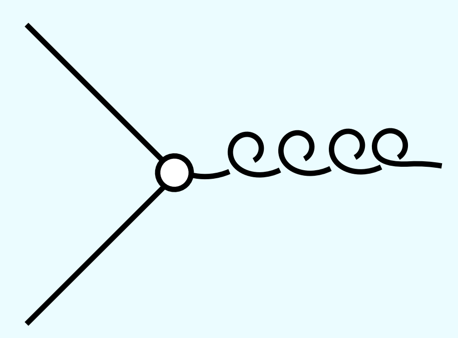


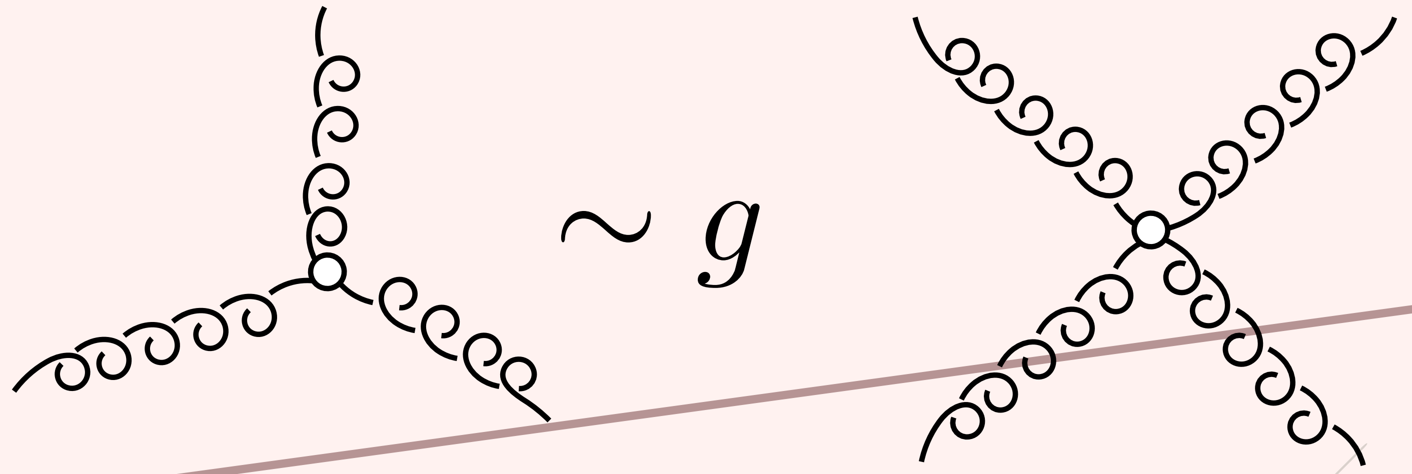
gluons

$$\mathscr{L}_{\mathrm{QCD}} = \sum_q \overline{\psi}_q \left(i\gamma^\mu D_\mu - m_q \right) \psi_q - \frac{1}{4} \mathrm{Tr}[G_{\mu\nu} G^{\mu\nu}]$$



Back to the basics





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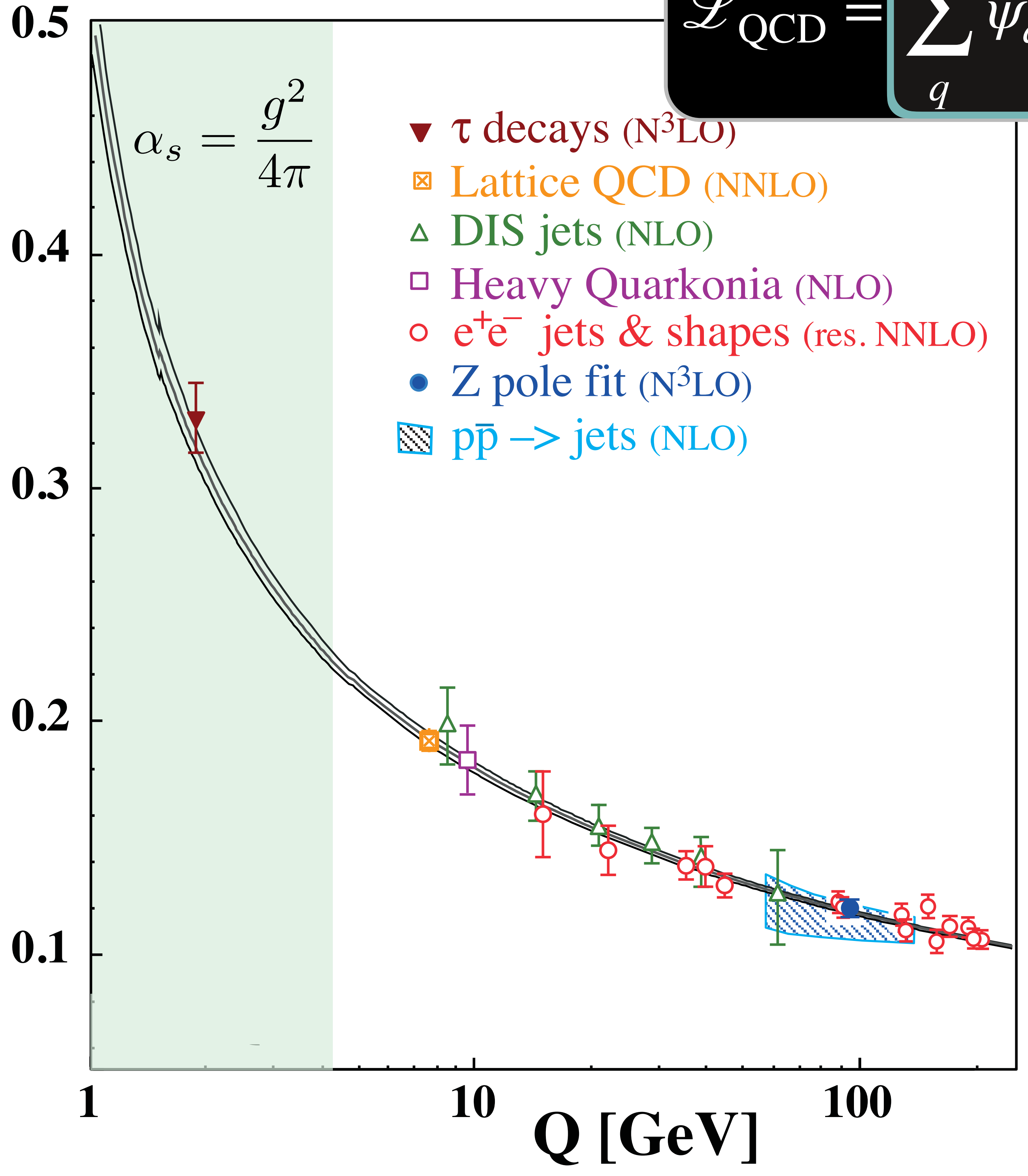
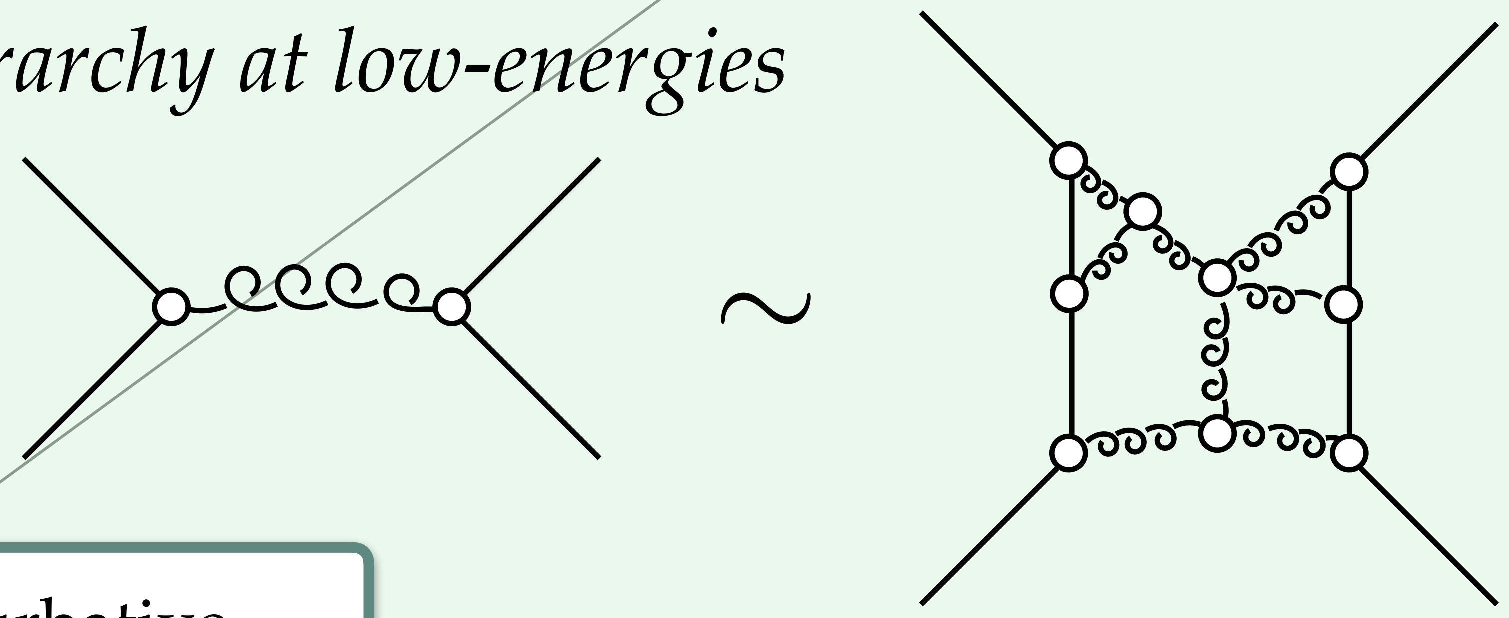


No hierarchy at low-energies



non-perturbative….

- *confinement - why are there no free quarks?*
- *origin of mass - where does mass really come from?*
- *formation of matter - how are quarks/gluons arranged?*
- …

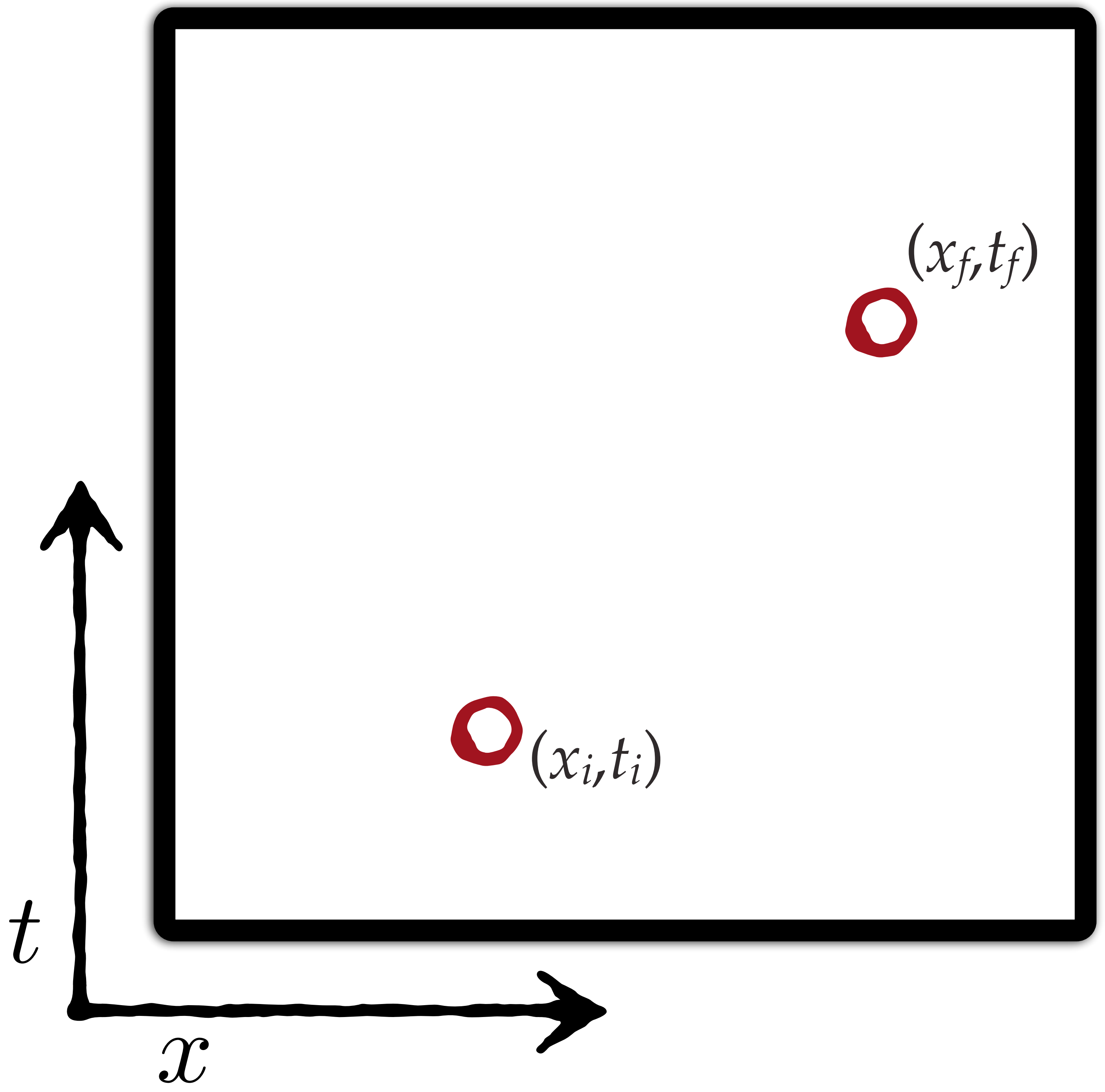
Desperate for a Nobel Prize?

Path integral in QM

- Imagine a world where quarks are free to propagate

I measured a quark at (x_i,t_i), the probability of finding it at (x_f,t_f) is:

$$\langle x_f, t_f | x_i, t_i \rangle = \langle x_f | e^{i\hat{H}t_f} e^{-i\hat{H}t_i} | x_i \rangle = \langle x_f | e^{i\hat{H}(t_f - t_i)} | x_i \rangle$$



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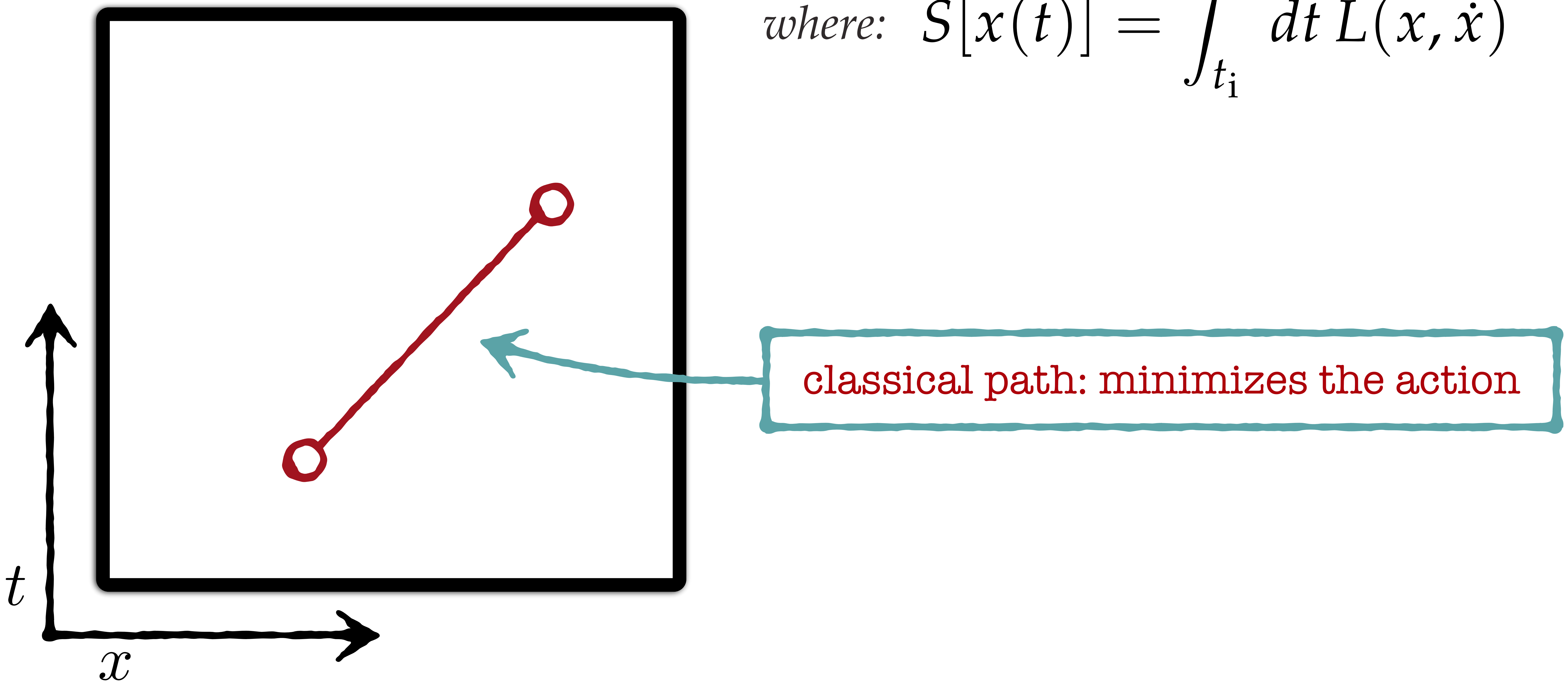
where: $S[x(t)] = \int_{t_i}^{t_f} dt \, L(x, \dot{x})$



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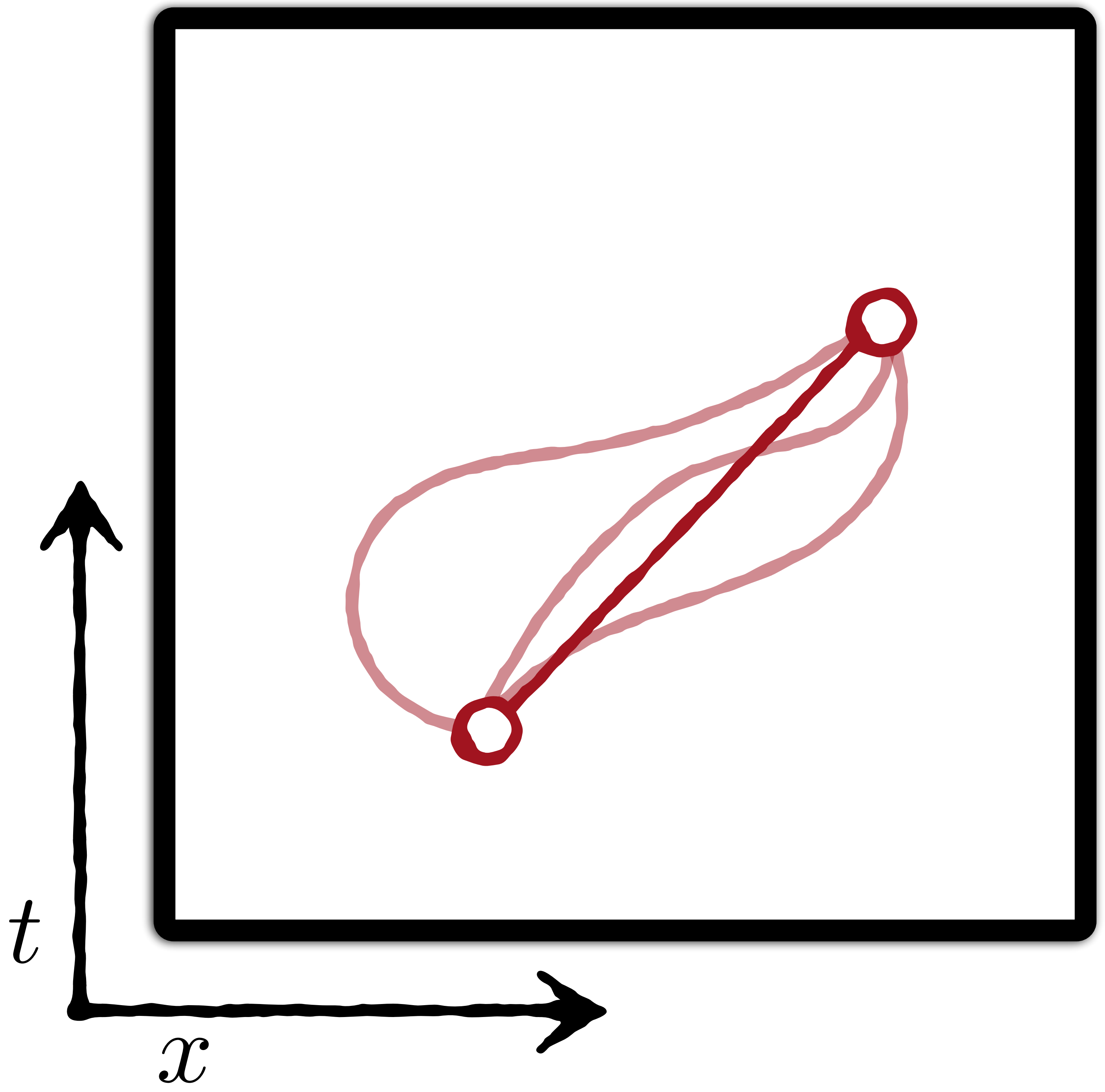
$$\langle x_f, t_f | x_i, t_i \rangle = \langle x_f | e^{i\hat{H}t_f} e^{-i\hat{H}t_i} | x_i \rangle = \langle x_f | e^{i\hat{H}(t_f - t_i)} | x_i \rangle = \int_{t_i}^{t_f} \mathcal{D}x \; e^{iS[x(t)]}$$

"sum" over all paths

weighted by a phase set by the action



Path integral in QM

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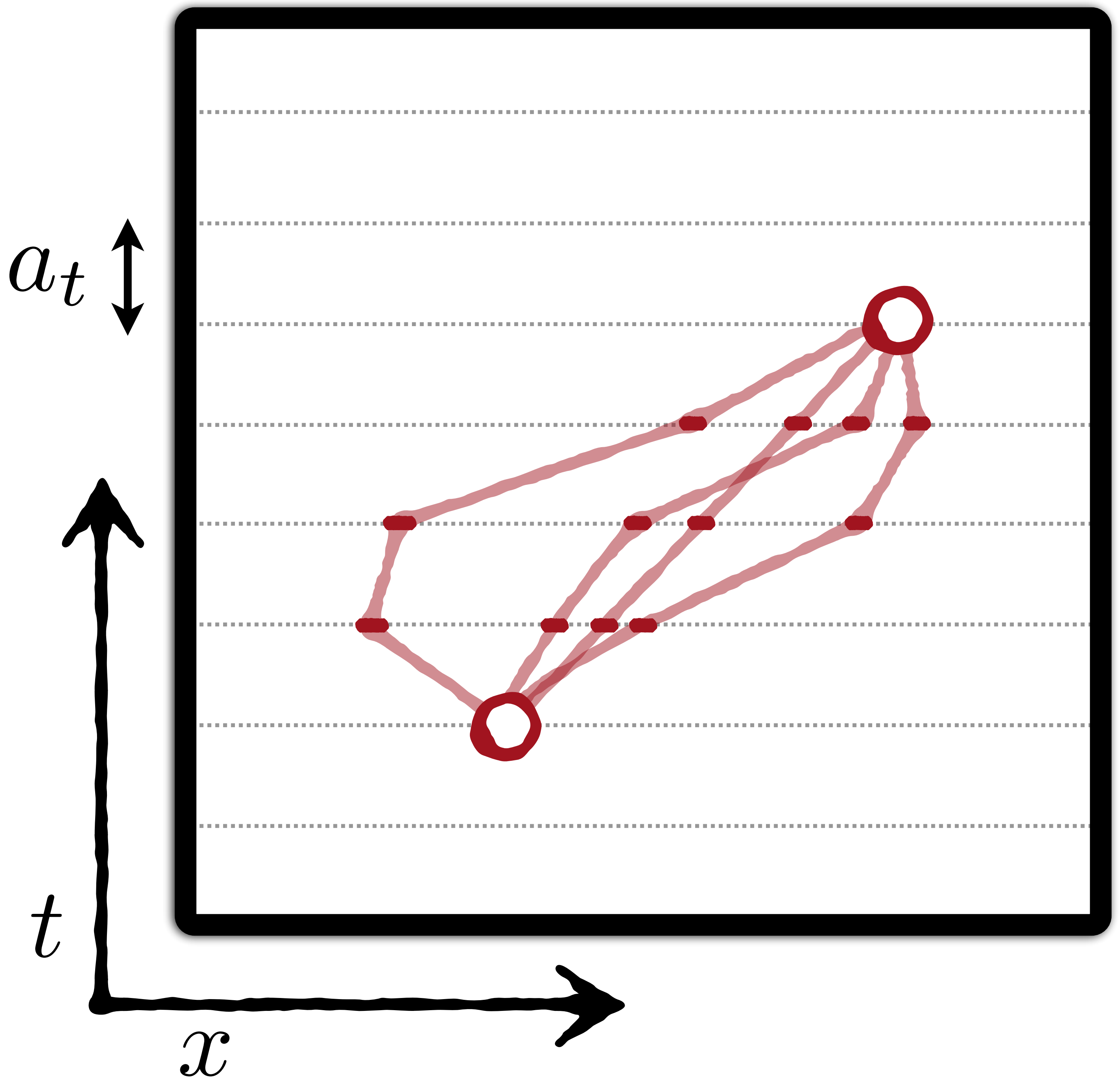
$$= \lim_{a_t \to 0} \int_{-\infty}^{\infty} dx_{i+1} \dots dx_{f-1} \; e^{iS[\{x\}, a_t]}$$



remember, this is how you derive the path integral representation

Path integral in QM

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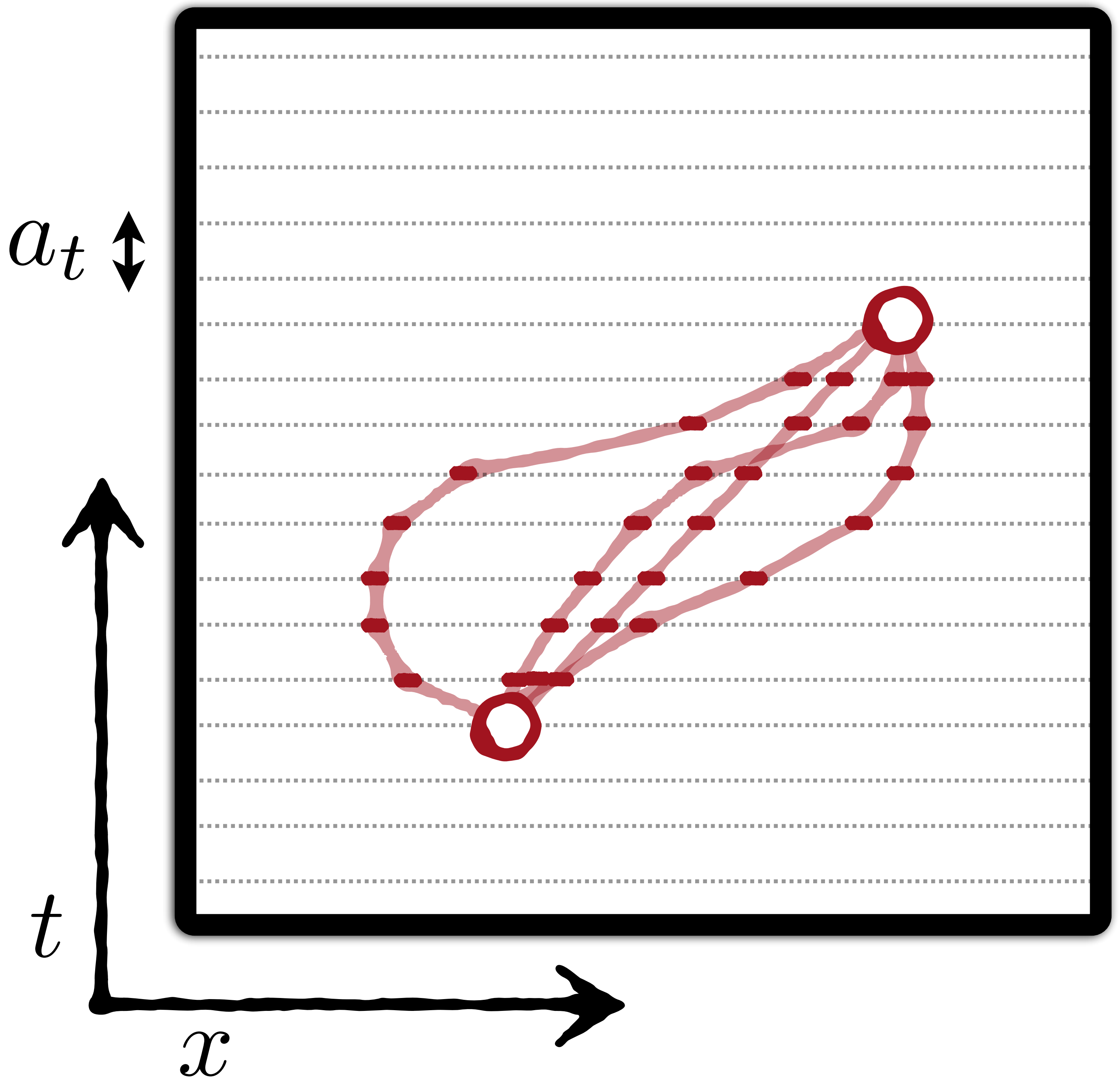

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Path integral in QM

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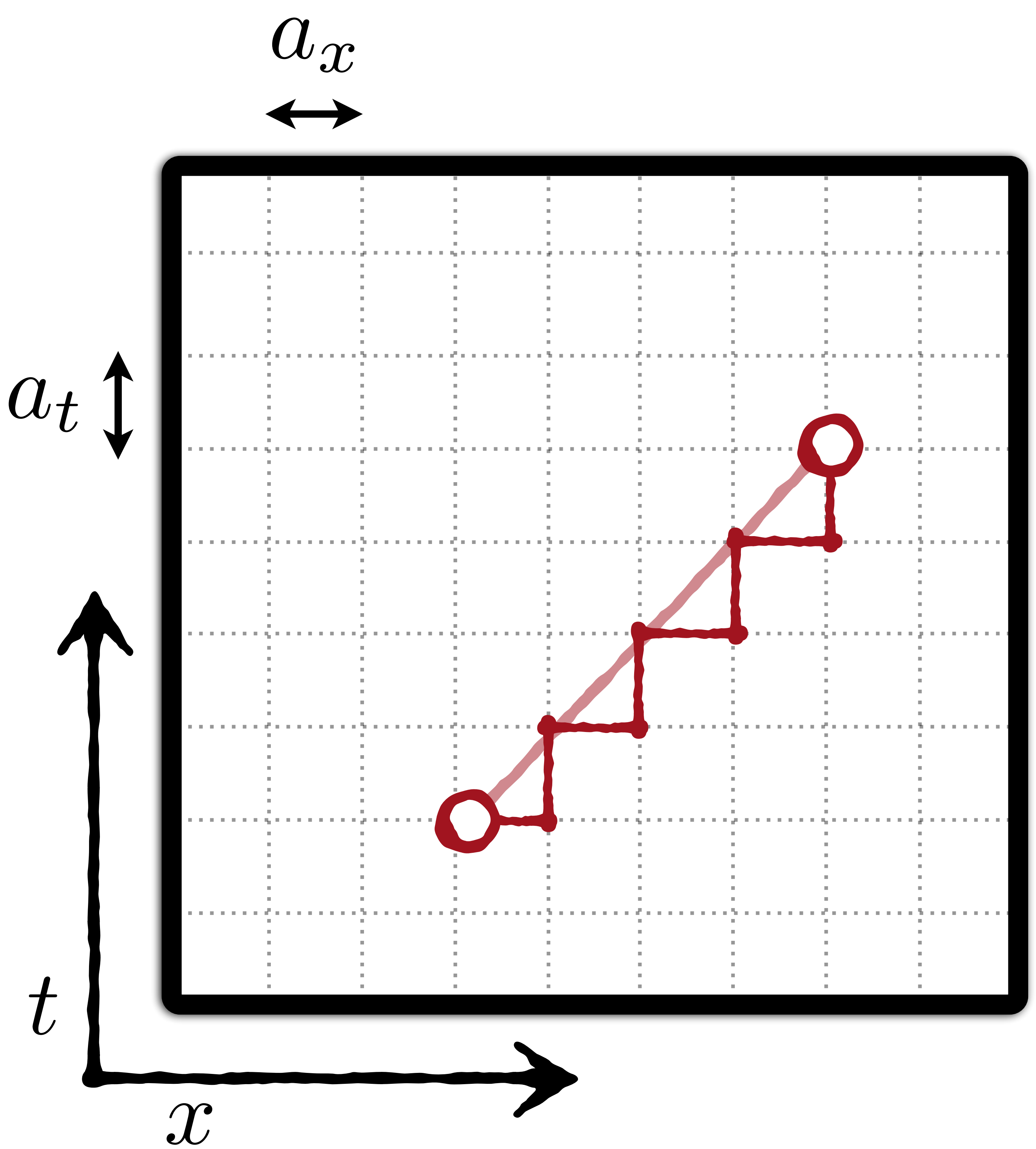
$$= \lim_{a_t \to 0} \int_{-\infty}^{\infty} dx_{i+1} \dots dx_{f-1} \; e^{iS[\{x\}, a_t]}$$

$$= \lim_{a_t \to 0} \lim_{a_x \to 0} a_x \sum_{x_{i+1}} \dots a_x \sum_{x_{f-1}} \; e^{iS[\{x\}, a_t, a_x]}$$

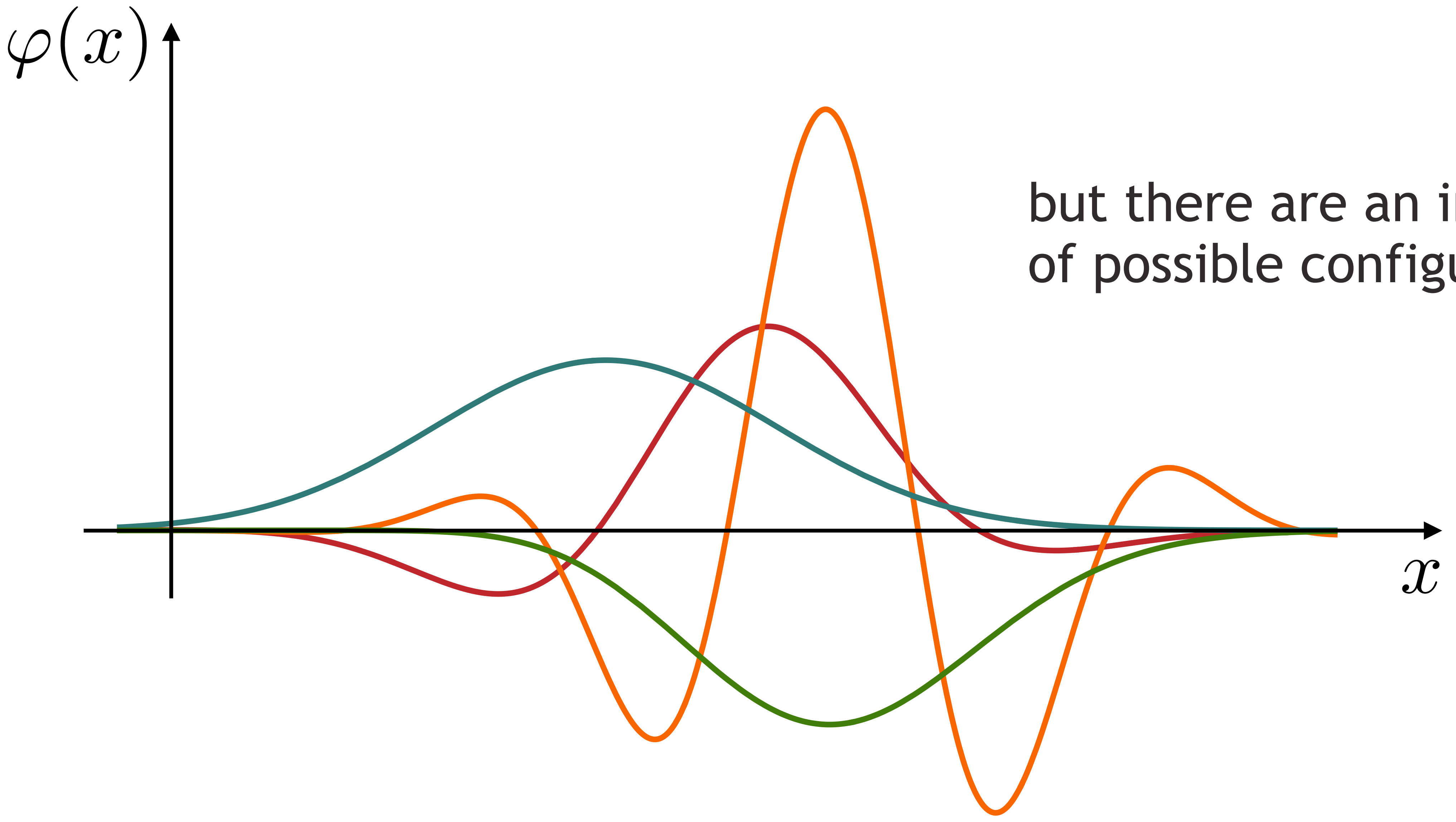


can evaluate this numerically by introducing a mesh in spacetime

Path integral in QFT

go to one dimension for simplicity of illustration

scalar field configurations



but there are an infinite number of possible configurations ...

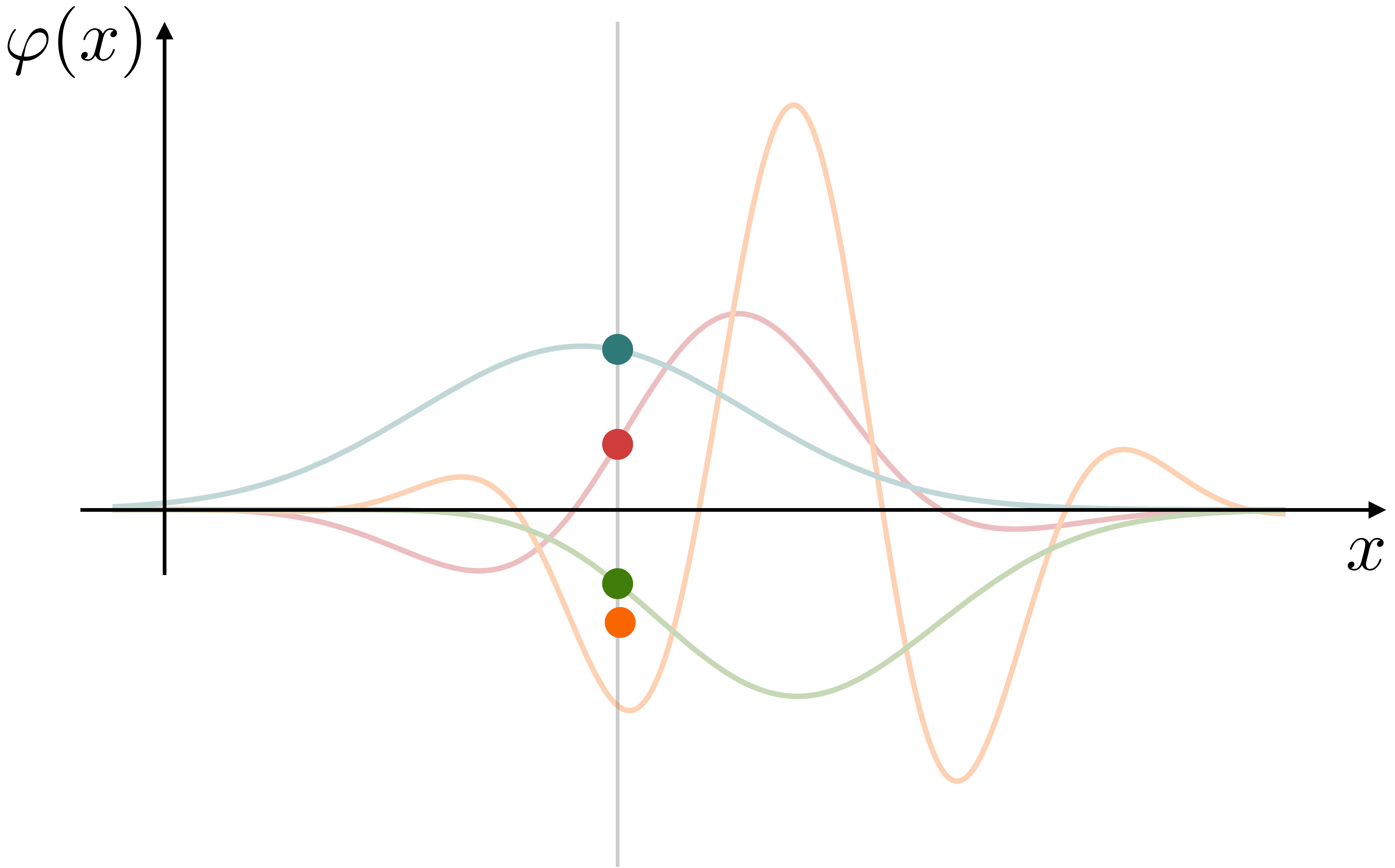
Path integral in QFT

discretize the space

$$\int \mathcal{D}\varphi(x) = \prod_x \int d\varphi_x = \int d\varphi_1 \int d\varphi_2 \int d\varphi_3 \cdots$$

an integral over all the values the field can take at x_2

scalar field configurations



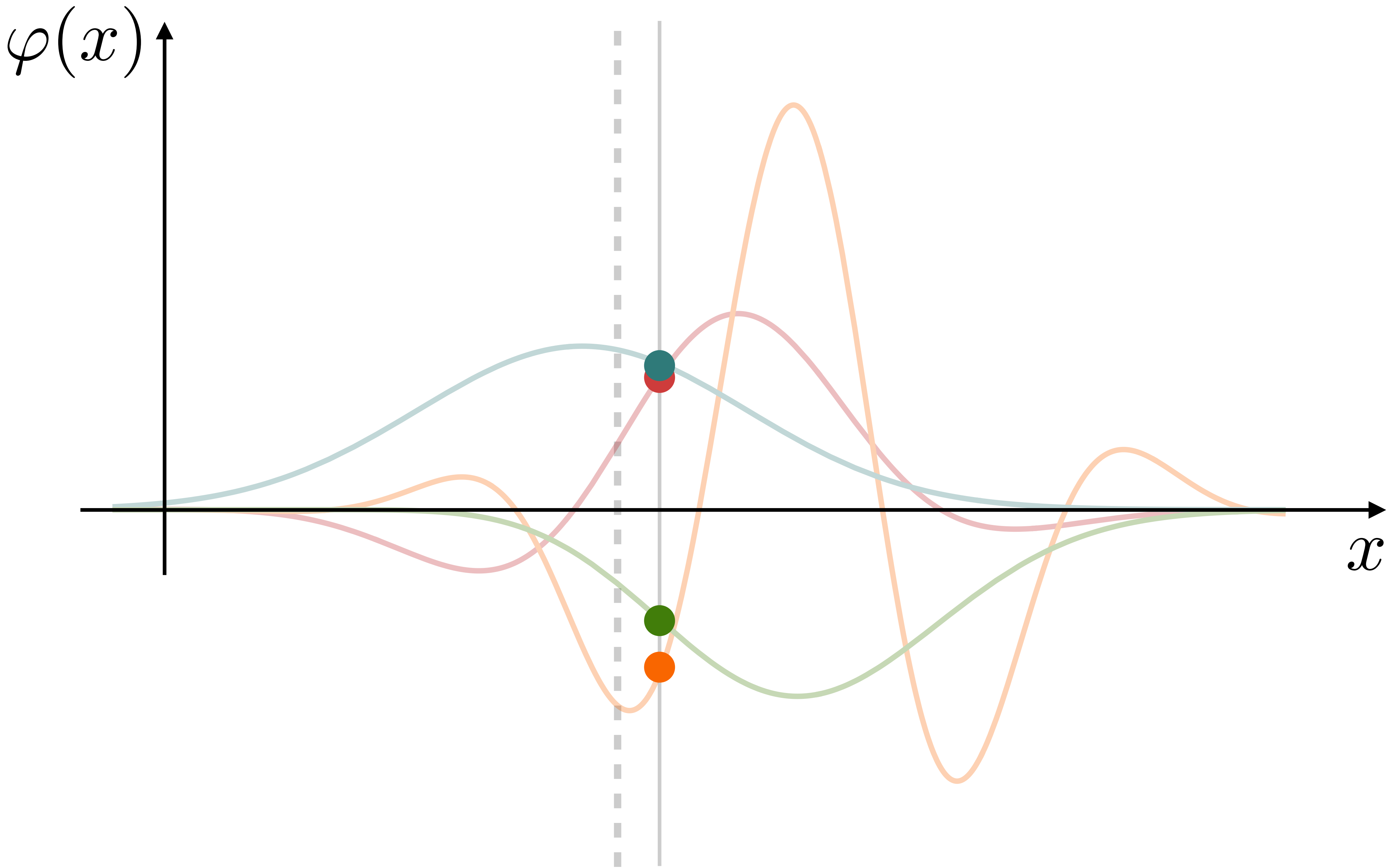
Path integral in QFT

discretize the space

$$\int \mathcal{D}\varphi(x) = \prod_x \int d\varphi_x = \int d\varphi_1 \int d\varphi_2 \int d\varphi_3 \cdots$$

an integral over all the values the field can take at x_3

scalar field configurations

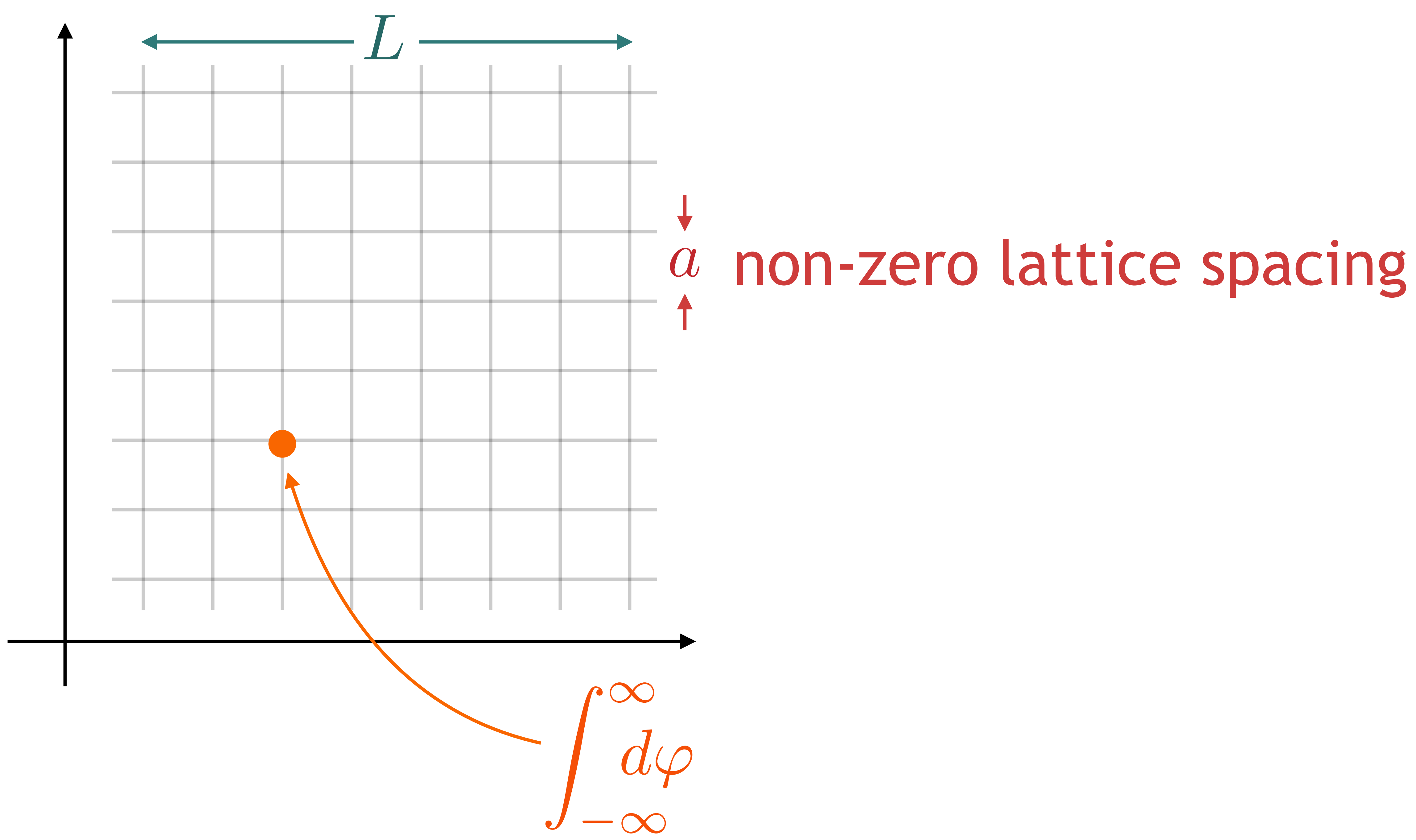


Path integral in lattice QFT

approach generally is to use a (hyper)cubic grid

$$\int \mathcal{D}\varphi(x) = \prod_x \int d\varphi_x$$

space-time grid



hiding it here,
but boundary conditions
are important

Euclidean QFT

even with the grid, still not practical:

$$Z = \int \mathcal{D}\varphi(x)\, e^{iS[\varphi(x)]}$$

a **phase** is not ideal for averaging

make a variable transform $t \to -it$ then $iS = i\int d^3x\, dt\, \mathcal{L} \to -\int d^3x\, dt\, \mathcal{L}_\mathrm{E} = -S_\mathrm{E}$

euclidean path integral

$$Z_\mathrm{E} = \int \mathcal{D}\varphi(x)\, e^{-S_\mathrm{E}[\varphi(x)]}$$

a bounded real number
⇝ a probability ?

Wick rotation [details]

- consider a 2D scalar field theory

$$\mathcal{L}_M = \frac{1}{2}\left((\partial_t\varphi)^2 - (\partial_x\varphi)^2 - m_0^2\varphi^2\right) - \mathcal{V}[\varphi]$$

- Wick rotate: $t \to -it$

$$\begin{aligned} e^{iS_M[\varphi]} = exp\left[i\int dx\, dt \mathcal{L}_M[\varphi]\right] &= exp\left[i\int dx\, dt\, \frac{1}{2}\left((\partial_t\varphi)^2 - (\partial_x\varphi)^2 - m_0^2\varphi^2 - 2\mathcal{V}[\varphi]\right)\right] \\ &\to exp\left[i\int dx\, (-idt)\, \frac{1}{2}\left(-(\partial_t\varphi)^2 - (\partial_x\varphi)^2 - m_0^2\varphi^2 - 2\mathcal{V}[\varphi]\right)\right] \\ &= exp\left[-\int dx\, dt\, \frac{1}{2}\left((\partial_t\varphi)^2 + (\partial_x\varphi)^2 + m_0^2\varphi^2 + 2\mathcal{V}[\varphi]\right)\right] \equiv e^{-S_E[\varphi]} \end{aligned}$$

- Euclidean correlation function:

$$\langle\hat{\mathcal{O}}(t)\hat{\mathcal{O}}^\dagger(0)\rangle_E = \frac{\int \mathcal{D}\varphi(x)\; e^{-S_E[\varphi]}\mathcal{O}(t)\mathcal{O}^\dagger(0)}{\int \mathcal{D}\varphi(x)\; e^{-S_E[\varphi]}} \equiv Z_E^{-1}\int \mathcal{D}\varphi(x)\; e^{-S_E[\varphi]}\mathcal{O}(t)\mathcal{O}^\dagger(0)$$

MC Sampling

euclidean path integral

$$Z_{\mathrm{E}} = \int \mathcal{D}\varphi(x)\, e^{-S_{\mathrm{E}}[\varphi(x)]}$$

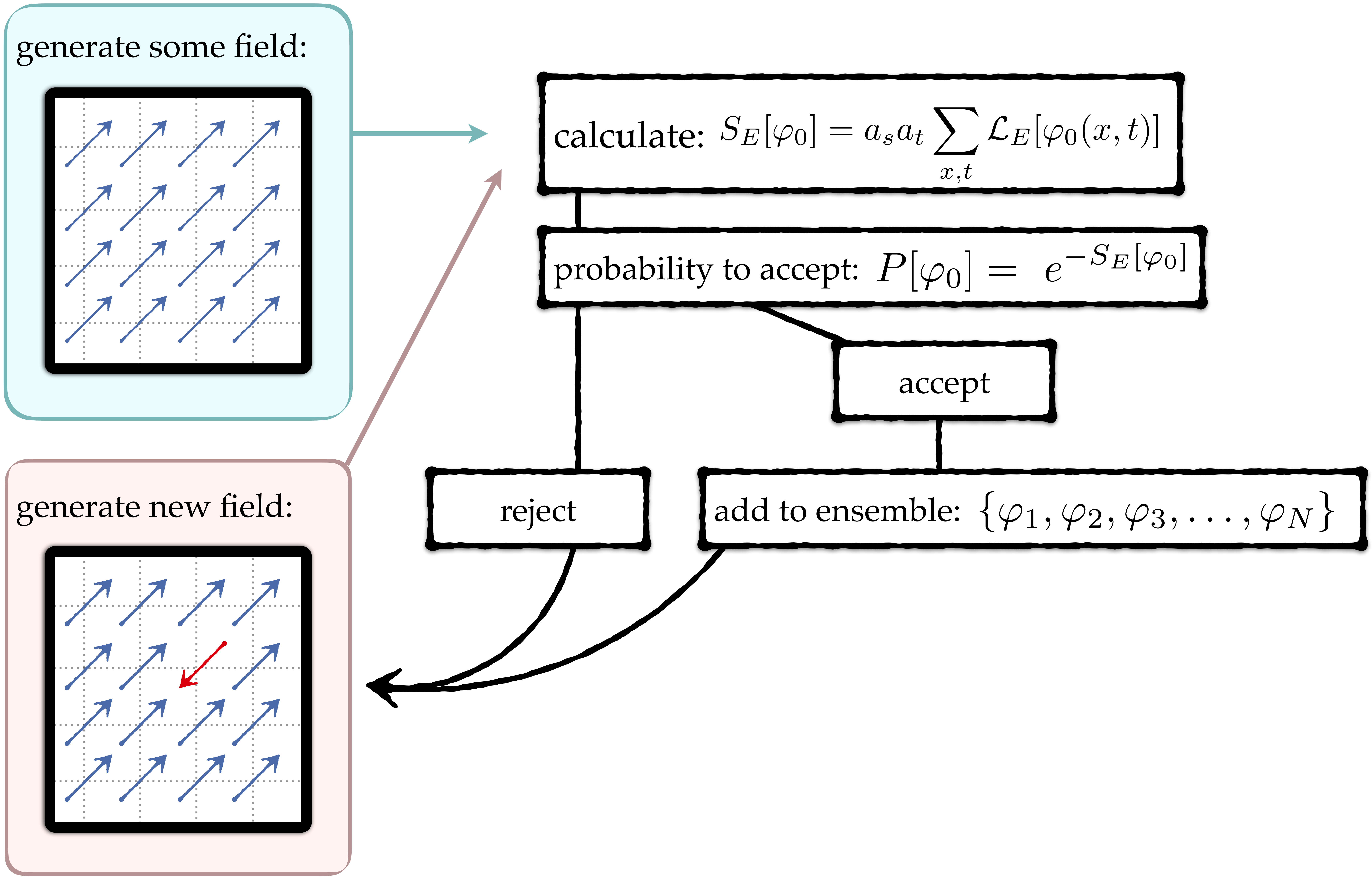
probability for a field configuration $\varphi(x)$

⇒ **importance sampled Monte Carlo generation of field configurations**

obtain an **ensemble** of configurations $\left\{\varphi_x\right\}_{i=1\dots N}$

[value of the field
at each point on the grid]

Sampling of fields



Observables

for some observable (vacuum matrix element)

$$\langle 0 | \, O[\hat{\varphi}] \, | 0 \rangle = \int \mathcal{D}\varphi \, O[\varphi] \, e^{-S_{\mathrm{E}}[\varphi]} / Z_E$$

can now be estimated as an **average over the ensemble**

$$\langle 0 | \, O[\hat{\varphi}] \, | 0 \rangle \approx \overline{O} = \frac{1}{N} \sum_{i=1}^{N} O[\varphi^{(i)}]$$

plus get an **uncertainty estimate** from the variance

$$\sigma(O) = \sqrt{\tfrac{1}{N(N-1)} \sum_{i=1}^{N} \left(O[\varphi^{(i)}] - \overline{O} \right)^2}$$

ensemble mean and error

$$\langle 0 | \, O[\hat{\varphi}] \, | 0 \rangle \approx \overline{O} \pm \sigma(O)$$

QCD isn’t a scalar field theory,
how do you handle fermions and gauge fields ?

Gauge invariance

in the continuum theory,
consider a quark field pair separated by some distance

the combination $\overline{\psi}^{j}(y)\,\delta_{ji}\,\psi^{i}(x)$ is not **gauge-invariant**

we can perform **different** local gauge transformations at locations ***x*** and ***y***

$q\bar{q}$ pair

a **gauge-invariant** combination is $\overline{\psi}^{j}(y)\left[e^{ig\int_x^y dz_\mu A^\mu(z)}\right]_{ji}\psi^{i}(x)$

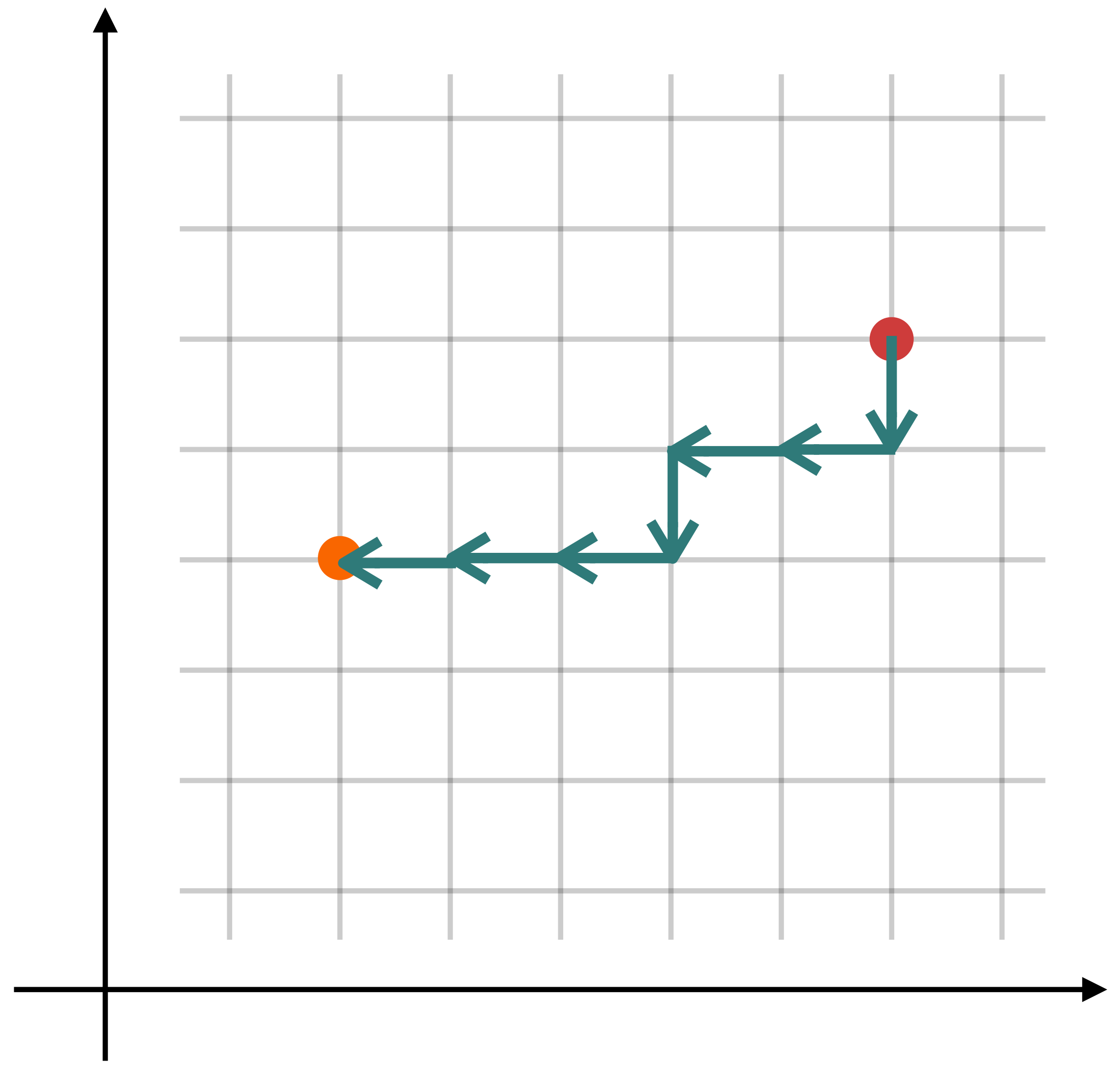
a **'Wilson line'** transports the color

$q\bar{q}$ pair with Wilson line

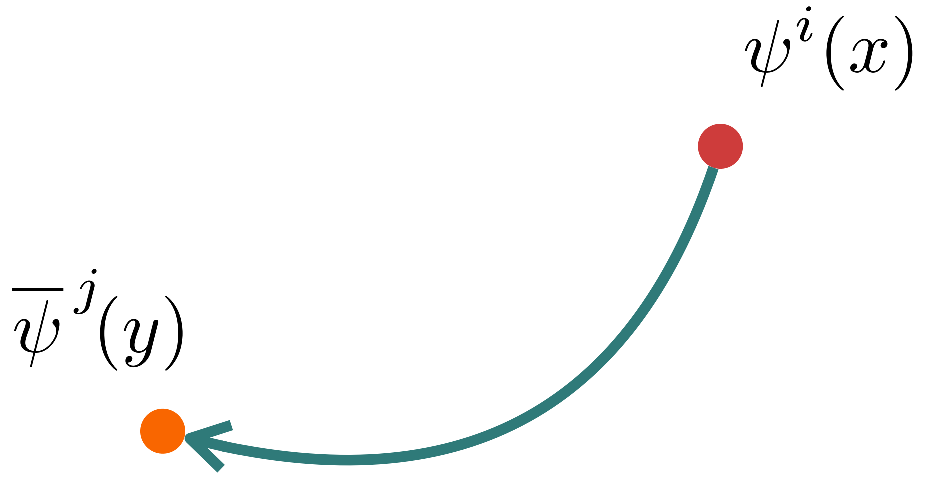
Gauge links

on a lattice, make hops to neighboring sites

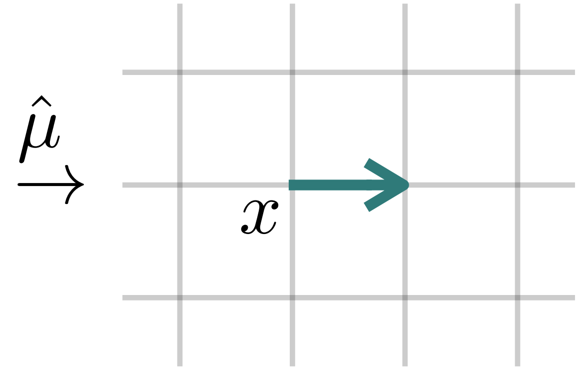
space-time grid



qq pair with Wilson line



shortest path between neighboring sites = a **'link'**



$$\left[e^{igaA^{\mu}(x)}\right]_{ji}$$

$$U_{\mu}(x) = e^{igaA^{\mu}(x)}$$

SU(3) matrix on each **link** of the lattice

Discretizing the action

can construct a gauge-invariant **finite-difference** – approximation to a derivative ?

$$\overline{\psi}(x)\,\gamma_\mu\,U_\mu(x)\,\psi(x+\hat{\mu}a) - \overline{\psi}(x)\,\gamma_\mu\,U_\mu^\dagger(x-\hat{\mu}a)\,\psi(x-\hat{\mu}a)$$

c.f. $\frac{1}{2a}\big(f(x+a)-f(x-a)\big) \xrightarrow{a\to 0} \frac{df}{dx} + O(a^2)$

$$\xrightarrow{a\to 0} 2a\,\overline{\psi}\,\gamma_\mu\big(\partial_\mu + igA_\mu\big)\,\psi + \ldots$$

and using constructions like these we can build **discretized actions**

e.g. $\int d^4x\,\overline{\psi}(\gamma_\mu D_\mu + m)\psi \quad \rightsquigarrow \quad \overline{\psi}_x^{\,i\alpha}\, M_{x,y}^{i\alpha,j\beta}[U]\,\psi_y^{j\beta}$

Dirac matrix

matrix in
color, spin, spacetime

N.B. **large** matrix, but **sparse**

e.g. for a
$24^3\times 128$ lattice,
~ 21M×21M

most of the
elements
are zero

(100 Pb !!!)

Integrating out quarks

a gauge-field ‘configuration’ is simple – it’s an SU(3) matrix on each link

but what about a quark-field configuration? **fermion fields anticommute ⇒ Grassmann variables**

actually we can do the quark field integration exactly in the path integral:

$$\int \mathcal{D}\psi \mathcal{D}\overline{\psi} \mathcal{D}U \, e^{-S_{\mathrm{E}}[\psi,\overline{\psi},U]} = \int \mathcal{D}U \, e^{-S_{\mathrm{E}}^{\mathrm{g}}[U]} \int \mathcal{D}\psi \mathcal{D}\overline{\psi} \, e^{-\overline{\psi} M[U] \psi}$$

$$= \det M[U]$$

$$= \int \mathcal{D}U \, \det M[U] \, e^{-S_{\mathrm{E}}^{\mathrm{g}}[U]}$$

interpret as the probability for configuration $U_\mu(x)$

Quark propagators

$$\langle 0 | \hat{\psi}_x^{i\alpha} \hat{\overline{\psi}}_y^{j\beta} | 0 \rangle = \int \mathcal{D}\psi \mathcal{D}\overline{\psi} \mathcal{D}U \, \psi_x^{i\alpha} \, \overline{\psi}_y^{j\beta} \, e^{-S_{\mathrm{E}}[\psi, \overline{\psi}, U]}$$

correlation between
quark at *x*, color *i*, spin *a*
and
quark at *y*, color *j*, spin *β*

$$= \int \mathcal{D}U \, e^{-S_{\mathrm{E}}^{\mathrm{g}}[U]} \int \mathcal{D}\psi \mathcal{D}\overline{\psi} \, \psi_x^{i\alpha} \overline{\psi}_y^{j\beta} \, e^{-\overline{\psi} M[U] \psi}$$

$$= \int \mathcal{D}U \left[M^{-1}[U] \right]_{x,y}^{i\alpha, j\beta} \det M[U] \, e^{-S_{\mathrm{E}}^{\mathrm{g}}[U]}$$

c.f. Wick's theorem

probability

$$= \sum_{\{U\}} \left[M^{-1}[U] \right]_{x,y}^{i\alpha, j\beta}$$

compute
'quark propagator'
on each
configuration

actually, don't do this
because it will average to zero

Correlation functions & contractions

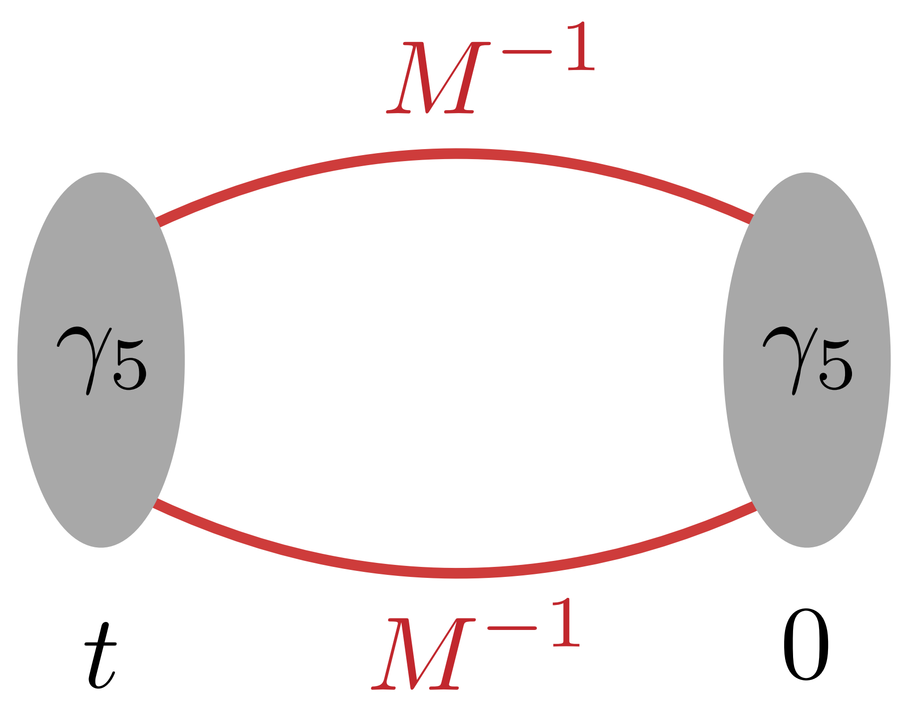
pion correlation function

$\overline{\psi}\gamma_5\psi$ pseudoscalar quantum numbers

$\sum_{\vec{x}} f(\vec{x}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} f(\vec{x})\Big|_{\vec{p}=\vec{0}}$ projection into zero momentum

$$\langle 0| \sum_{\vec{x}} (\overline{\psi}\gamma_5\psi)_{\vec{x},t} (\overline{\psi}\gamma_5\psi)_{\vec{0},0}|0\rangle$$

$$= -\sum_{\{U\}} \sum_{\vec{x}} \mathrm{tr}\Big([M^{-1}[U]]_{\vec{0}0,\vec{x}t}\gamma_5 [M^{-1}[U]]_{\vec{x}t,\vec{0}0}\gamma_5 \Big)$$



point – all propagator

$$[M[U]]_{\vec{y}t',\vec{x}t}\,\chi_{\vec{x}t} = \delta_{\vec{y},\vec{0}}\,\delta_{t',0}$$

sparse matrix — point source

$$\chi_{\vec{x}t} = [M^{-1}[U]]_{\vec{x}t,\vec{0}0}$$

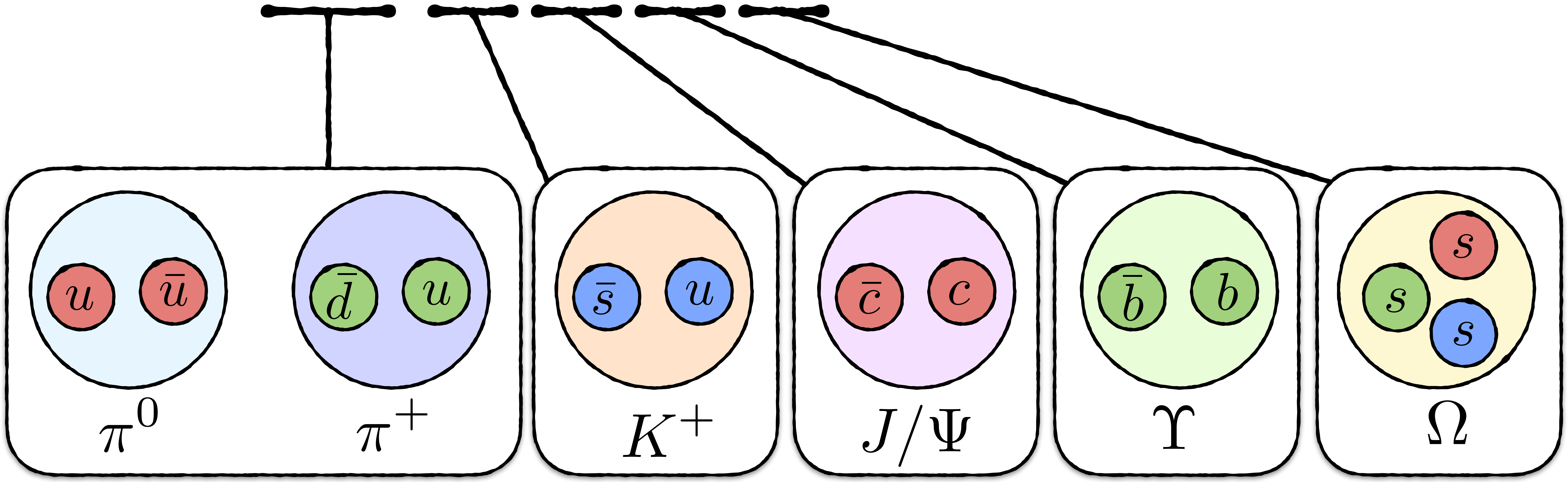
point-all propagator

solving a sparse linear system: **A** · *x* = *b*

e.g. for a $24^3\times128$ lattice, ~ 21M×12 (few Gb)

Lattice spacing & quark masses

- Parameter of QCD: m_u, m_d, m_s, m_c, m_b, m_t, g
- Dimensional transmutation: m_u/Λ_{QCD}, m_d/Λ_{QCD}, m_s/Λ_{QCD}, … , m_t/Λ_{QCD}
- QCD does not have an inherent mass scale
- QCD can predict masses of hadron in units of Λ_{QCD}
- Phenomenologically, we fix Λ_{QCD}
- Lattice QCD: am_u, am_d, am_s, am_c, am_b , am_t
- Tuning: m_u, m_d m_s m_c m_b a



Lattice workflow

select a discretization

'tune' the parameters

generate 100s of gauge-field configurations — serious parallel supercomputing

compute quark propagators — serious computing
GPUs very useful

'contract' into correlation functions — capacity computing
'bookkeeping' / memory management

•
•
•

PHYSICS ?

Outline

Lattice QCD in a nutshell **[today & tomorrow]**

- [x] Does lattice work?
- [x] why does lattice QCD work?
- [ ] what can it be used for?
- [ ] what are its limitations?

What is the cutting edge of lattice QCD? **[tomorrow]**

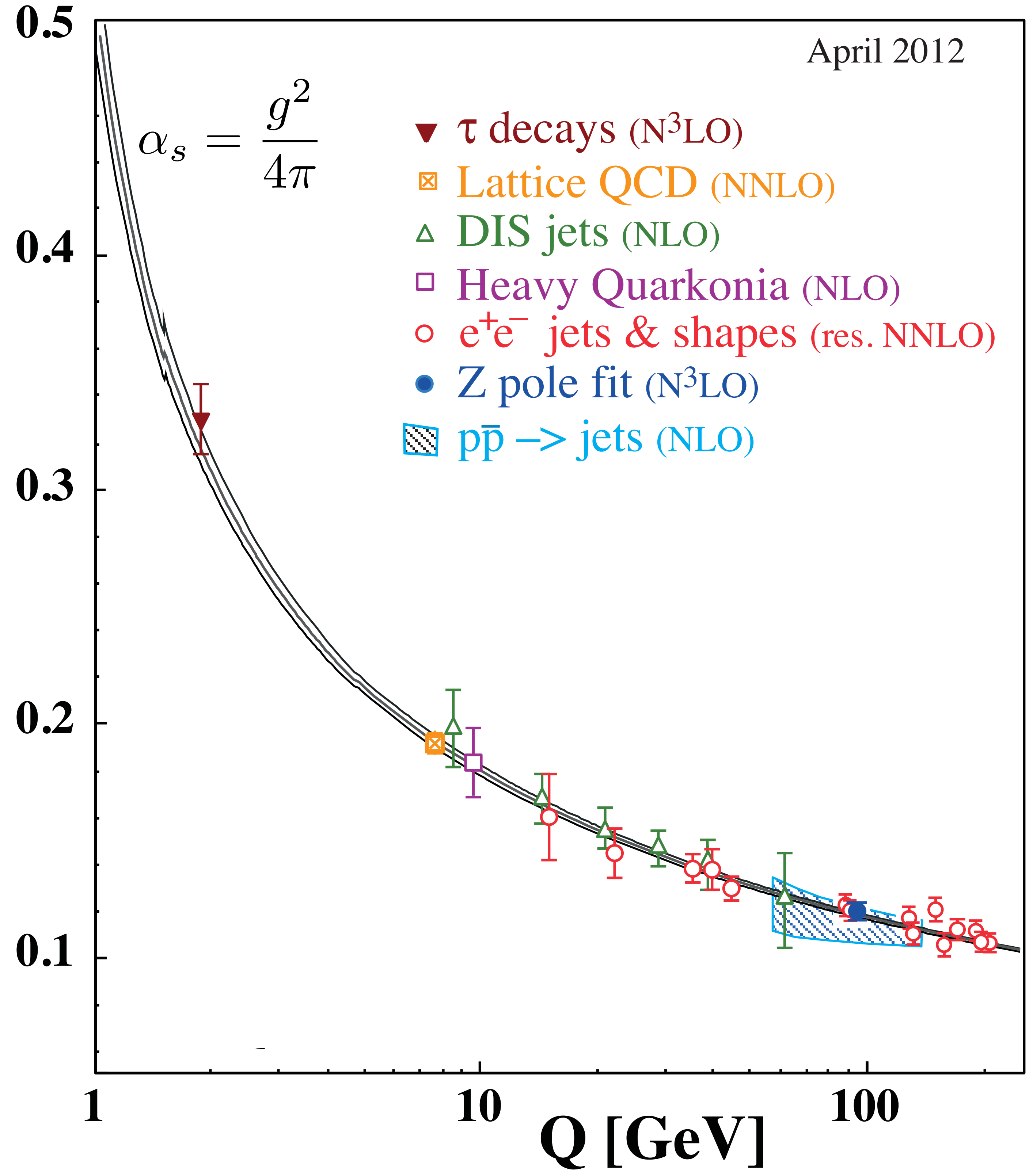
- [ ] hadron structure, fundamental symmetry,
- [ ] scattering processes,
- [ ] ….other stuff, I won't get to 🤓
  - [ ] finite-temperature, weak decays, BSM searches, …. ,

lattice spacing / UV cutoff

- acts as a UV cutoff $\Lambda \sim \frac{1}{a}$
- because the theory is perturbative in the UV region:
  - errors can [in principle] be corrected via PT
  - renomarlization dependent quantities can be matched via PT
  - renormalization-independent quantities, will suffer from discretization errors…which vanish in the continuum limit

discretization errors $X(a) = X(0) + a\,\delta X_1 + \ldots$

extrapolate $a \to 0$



PT = perturbation theory

Time evolution in Euclidean spacetime

- The time-dependence of: $\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle \equiv \langle\Omega|\mathcal{O}(t)\mathcal{O}^\dagger(0)|\Omega\rangle$

  $|\Omega\rangle$: QCD vacuum [assumed to have zero energy]

- Heisenberg picture in Minkowski spacetime

$$\mathcal{O}_M(t) = e^{it\hat{H}}\mathcal{O}(0)e^{-it\hat{H}}$$

- Wick rotation onto Euclidean spacetime: $t \to -it$

$$\mathcal{O}_E(t) = e^{t\hat{H}}\mathcal{O}(0)e^{-t\hat{H}}$$

- Euclidean correlation functions:

$$\langle\Omega|\mathcal{O}(t)\mathcal{O}^\dagger(0)|\Omega\rangle = \langle\Omega|e^{t\hat{H}}\mathcal{O}(0)e^{-t\hat{H}}\mathcal{O}^\dagger(0)|\Omega\rangle$$

Time evolution in Euclidean spacetime [cont.]

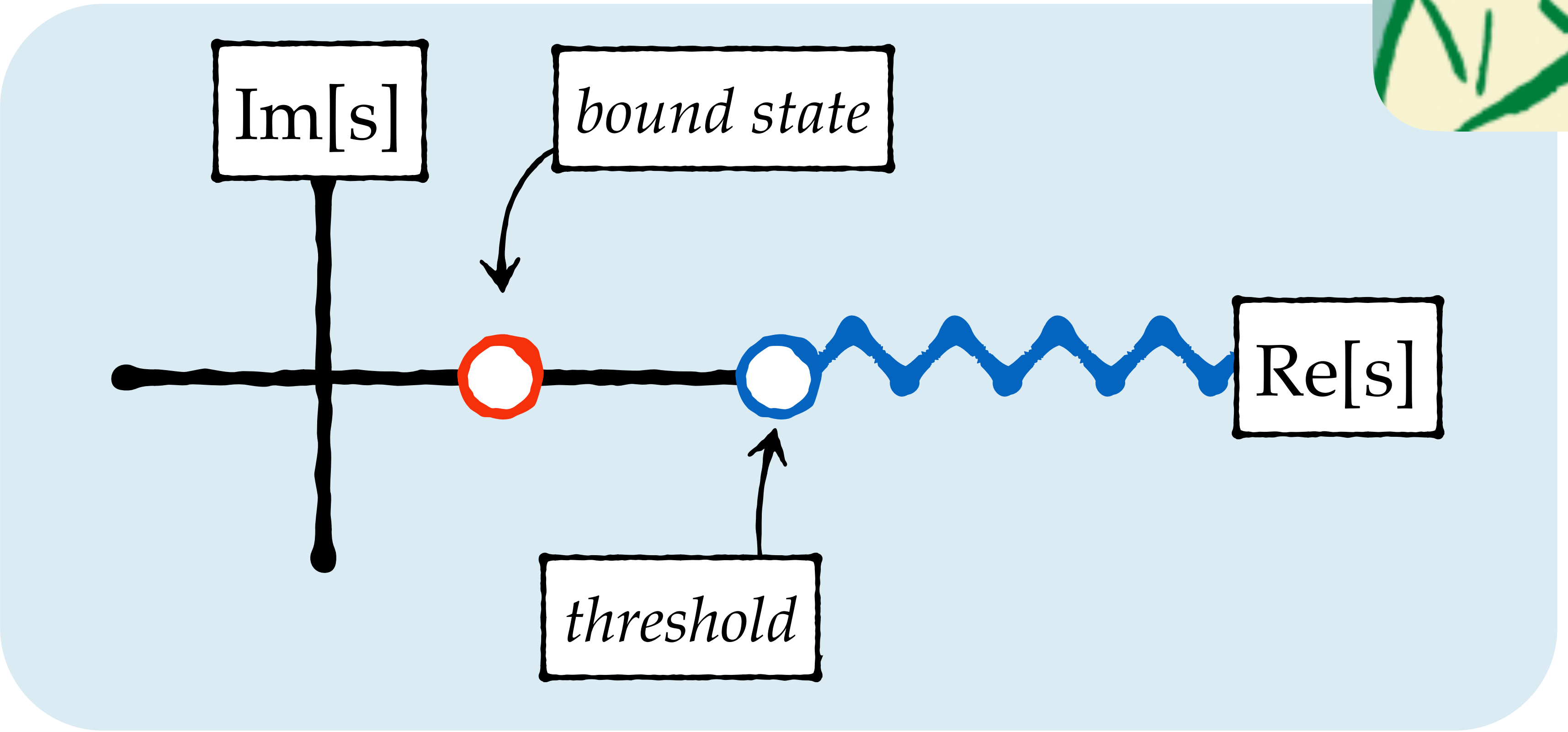
- We would like to introduce eigenstates of the Hamiltonian

$$\hat{H}|n\rangle = |n\rangle E_n$$

such that…
$$\begin{aligned} C(t) &= \langle\Omega|e^{t\hat{H}}\mathcal{O}(0)e^{-t\hat{H}}\mathcal{O}^\dagger(0)|\Omega\rangle \\ &= \sum_n e^{tE_\Omega}e^{-tE_n}\langle\Omega|\mathcal{O}(0)|n\rangle\langle n|\mathcal{O}^\dagger(0)|\Omega\rangle \\ &= \sum_n e^{-tE_n}|\langle\Omega|\mathcal{O}(0)|n\rangle|^2 \end{aligned}$$

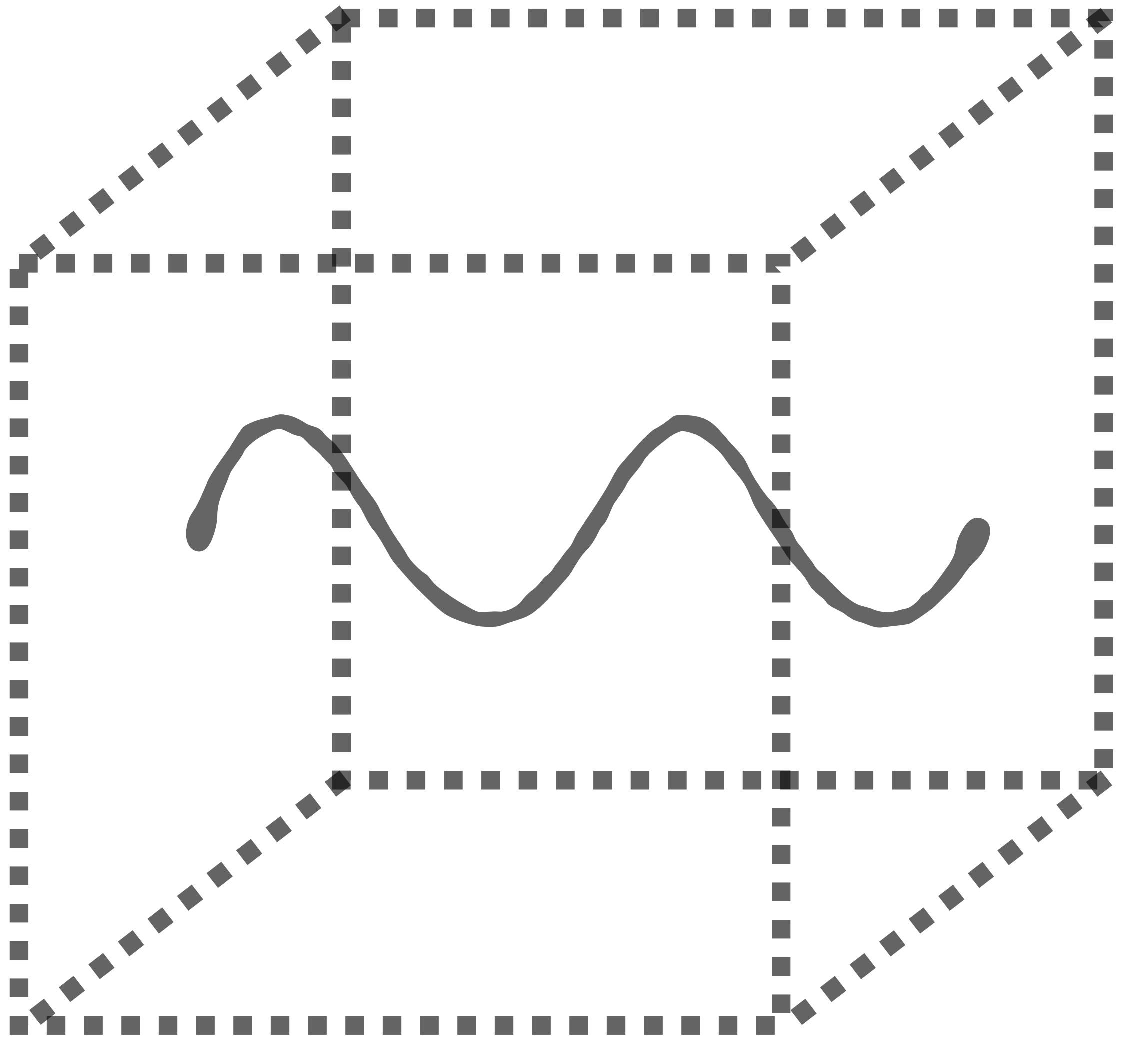
except, the spectrum is continuous!

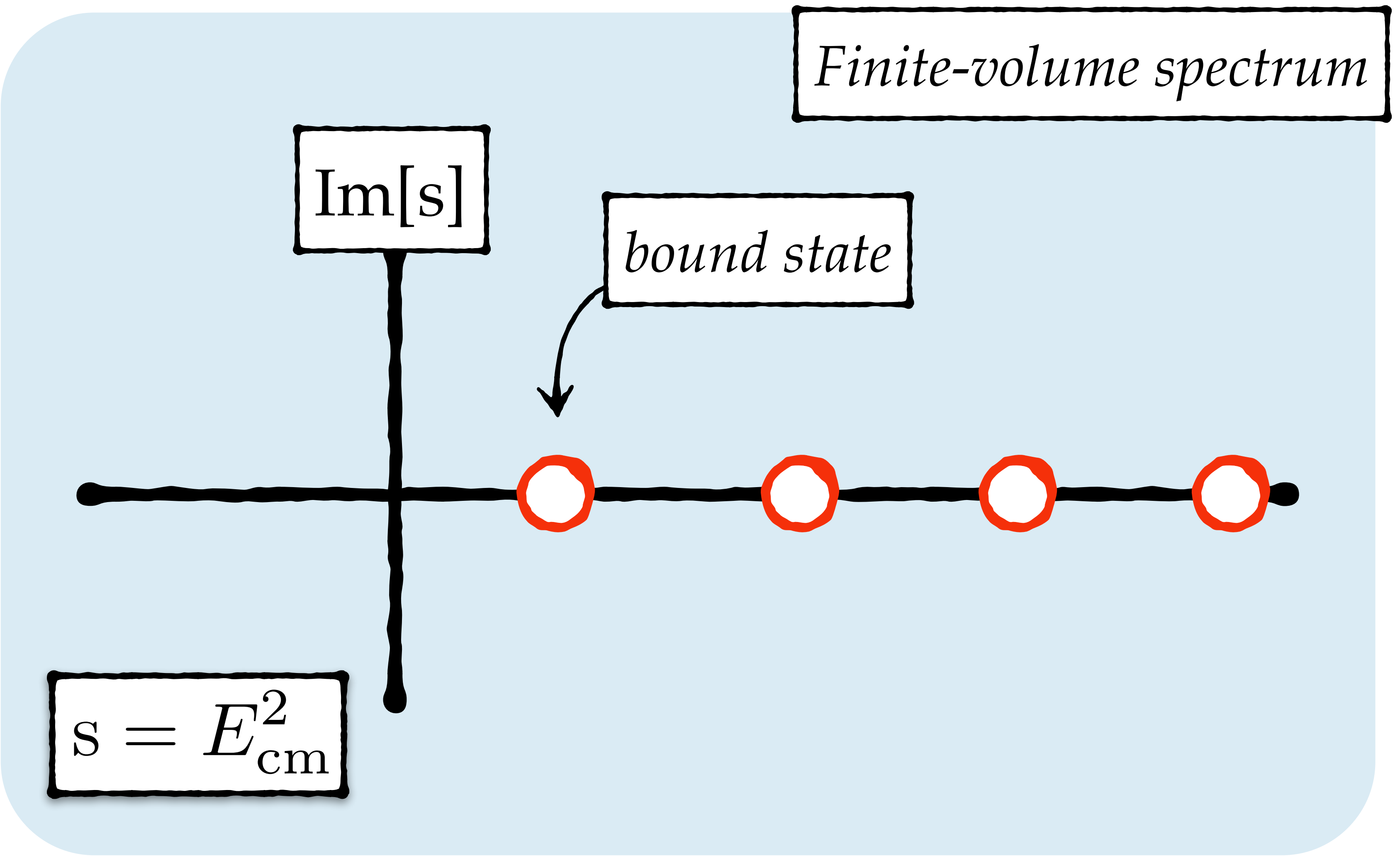
Should we be integrating…?



Time evolution in finite Euclidean spacetime [cont.]

- Remember, we have placed the theory in a finite-volume





"only a discrete number of modes can exist in a finite volume"

consequently, we can rigorously write

$$C(t) = \sum_n e^{-tE_n} |\langle \Omega | \mathcal{O}(0) | n \rangle|^2$$

Ground state masses

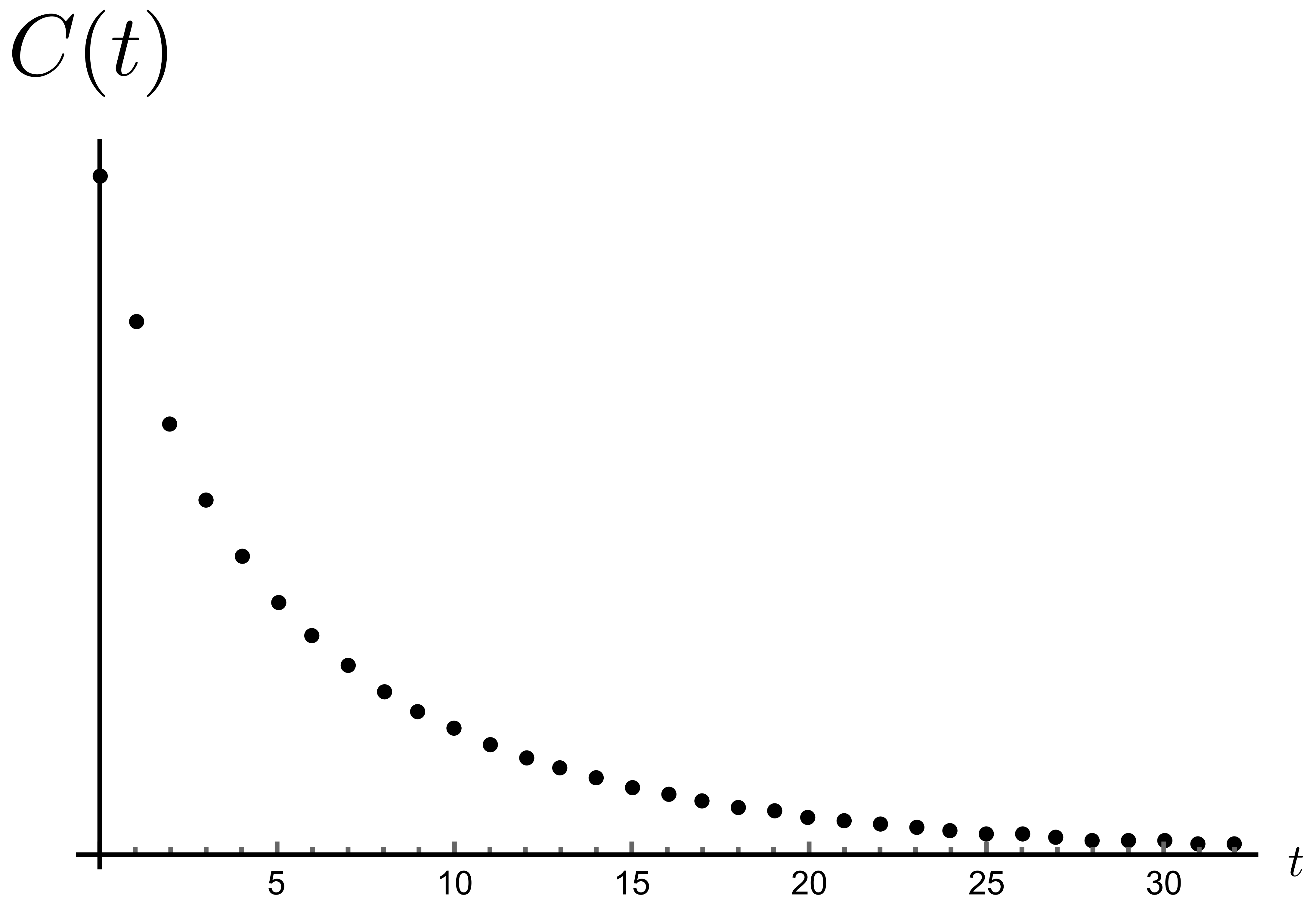
- In principle, each correlation function has access to infinite number of states
- A simple limit

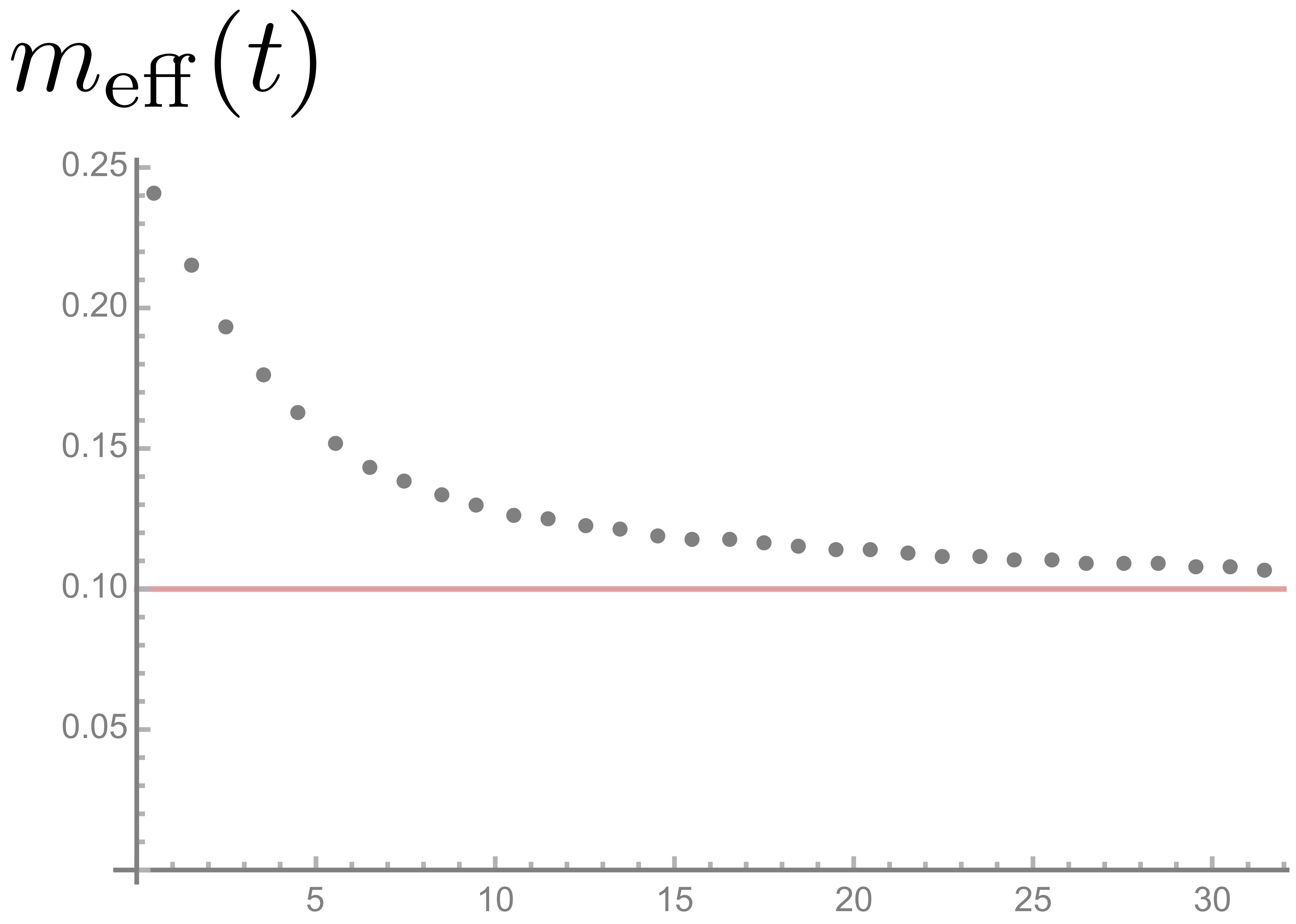
$$\lim_{t\to\infty} C(t) = \lim_{t\to\infty} \sum_n e^{-tE_n} |\langle\Omega|\mathcal{O}(0)|n\rangle|^2$$
$$= e^{-tE_0} |\langle\Omega|\mathcal{O}(0)|0\rangle|^2 + \mathcal{O}(e^{-t(E_1-E_0)})$$

- This motivates

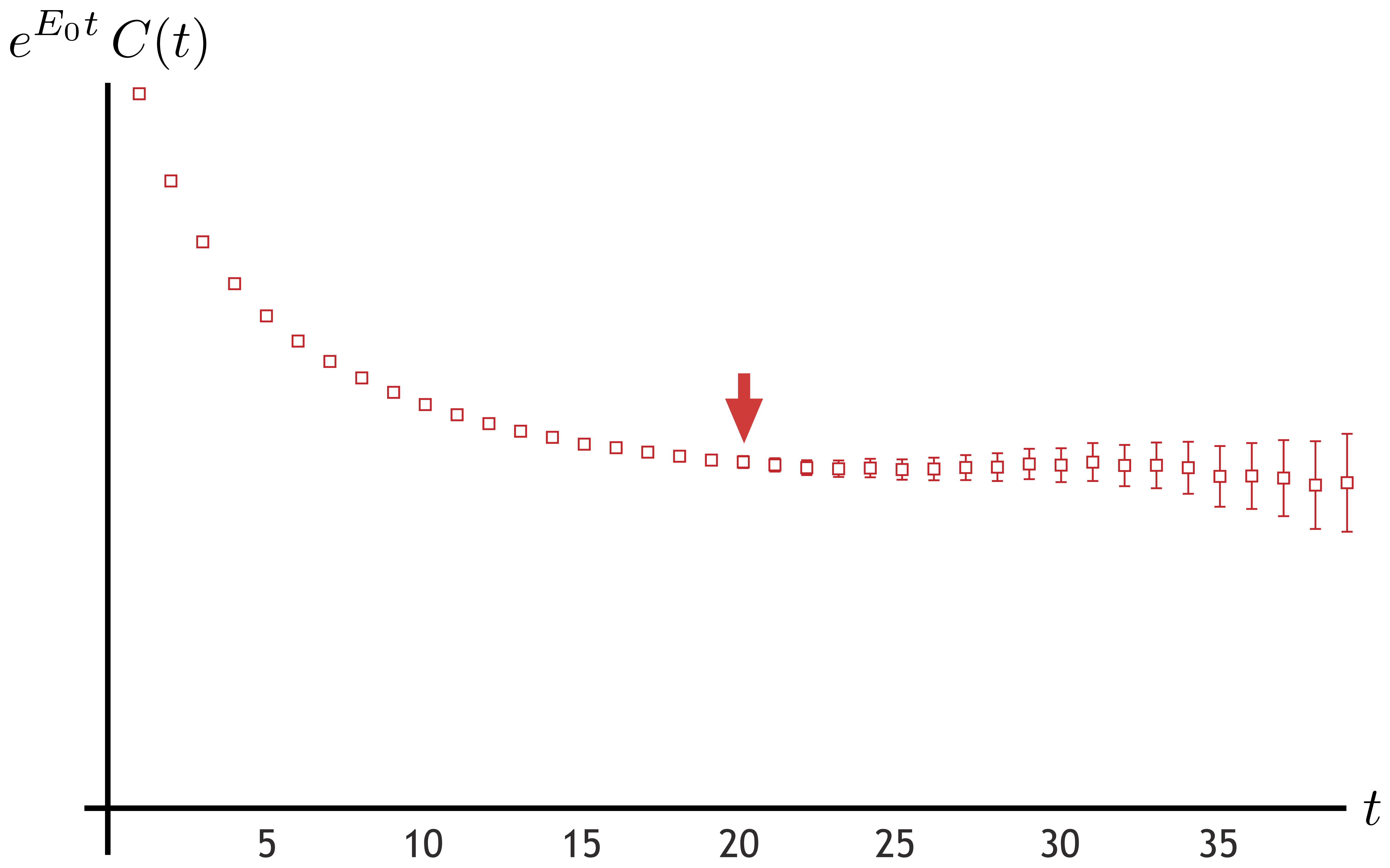
$$m_{\text{eff}}(t) = \log \frac{C(t)}{C(t+1)}$$
$$\to \log \frac{e^{-tE_0}}{e^{-(t+1)E_0}} = \log e^{E_0} = E_0$$

Toy data without errors



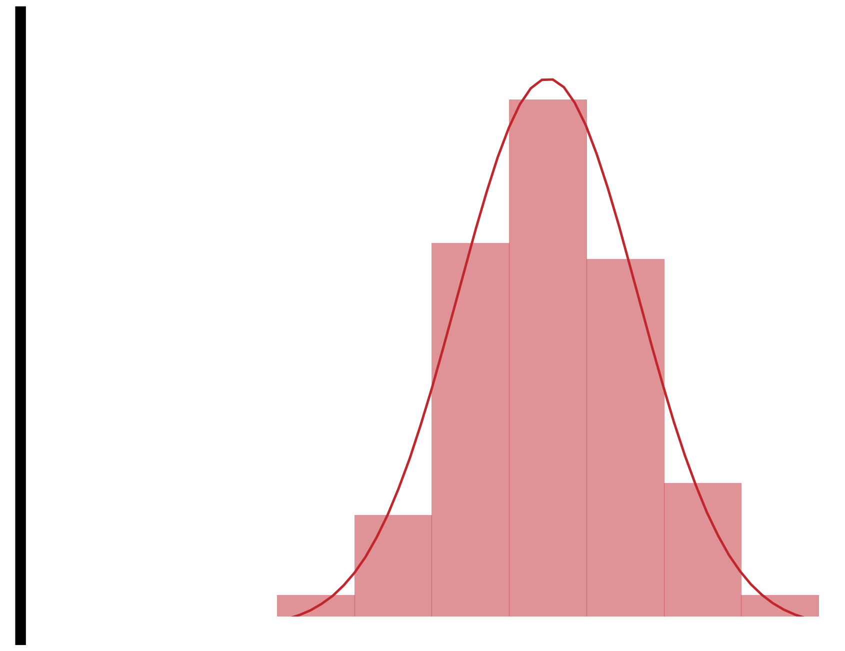


Lattice data with errors



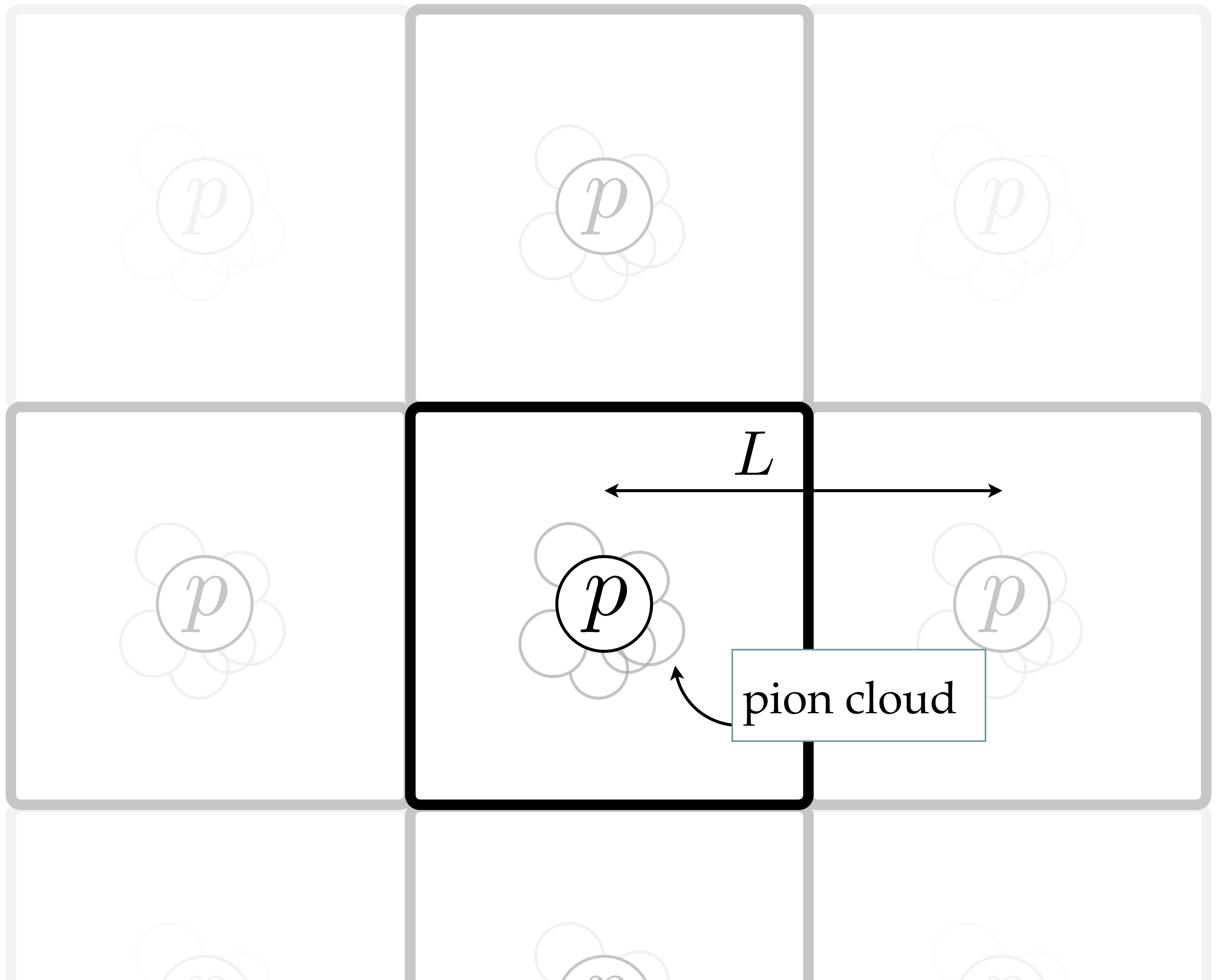
generally statistical noise
increases with increasing t

ensemble dist<u>n</u>



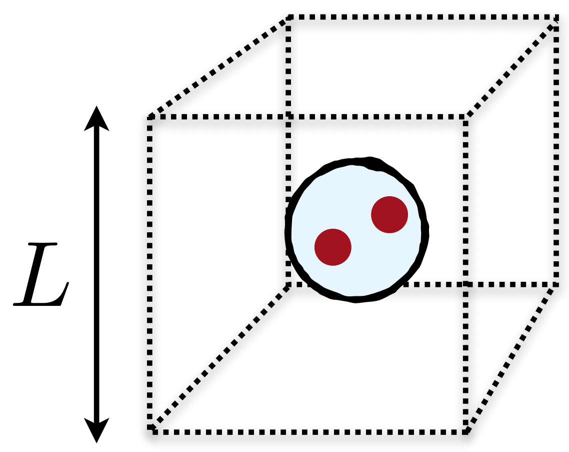
One particle in finite volume

- Stable hadron size ~ $\mathcal{O}(1/m_\pi)$
- If $L \gg m_\pi^{-1}$, finite-volume errors are suppressed

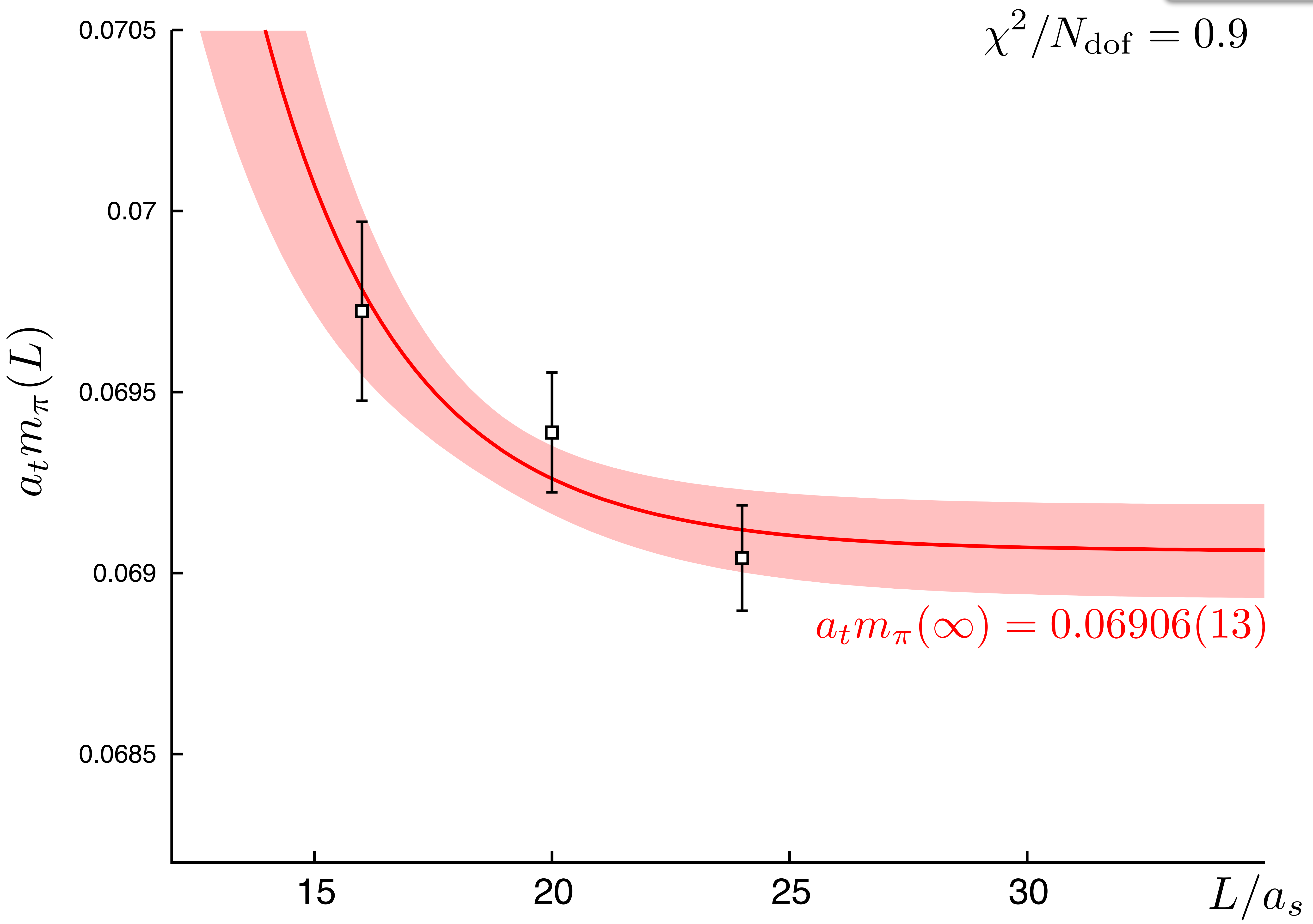


$$m_h(L) = m_h(\infty) + \mathcal{O}(e^{-m_\pi L})$$

the π in a box



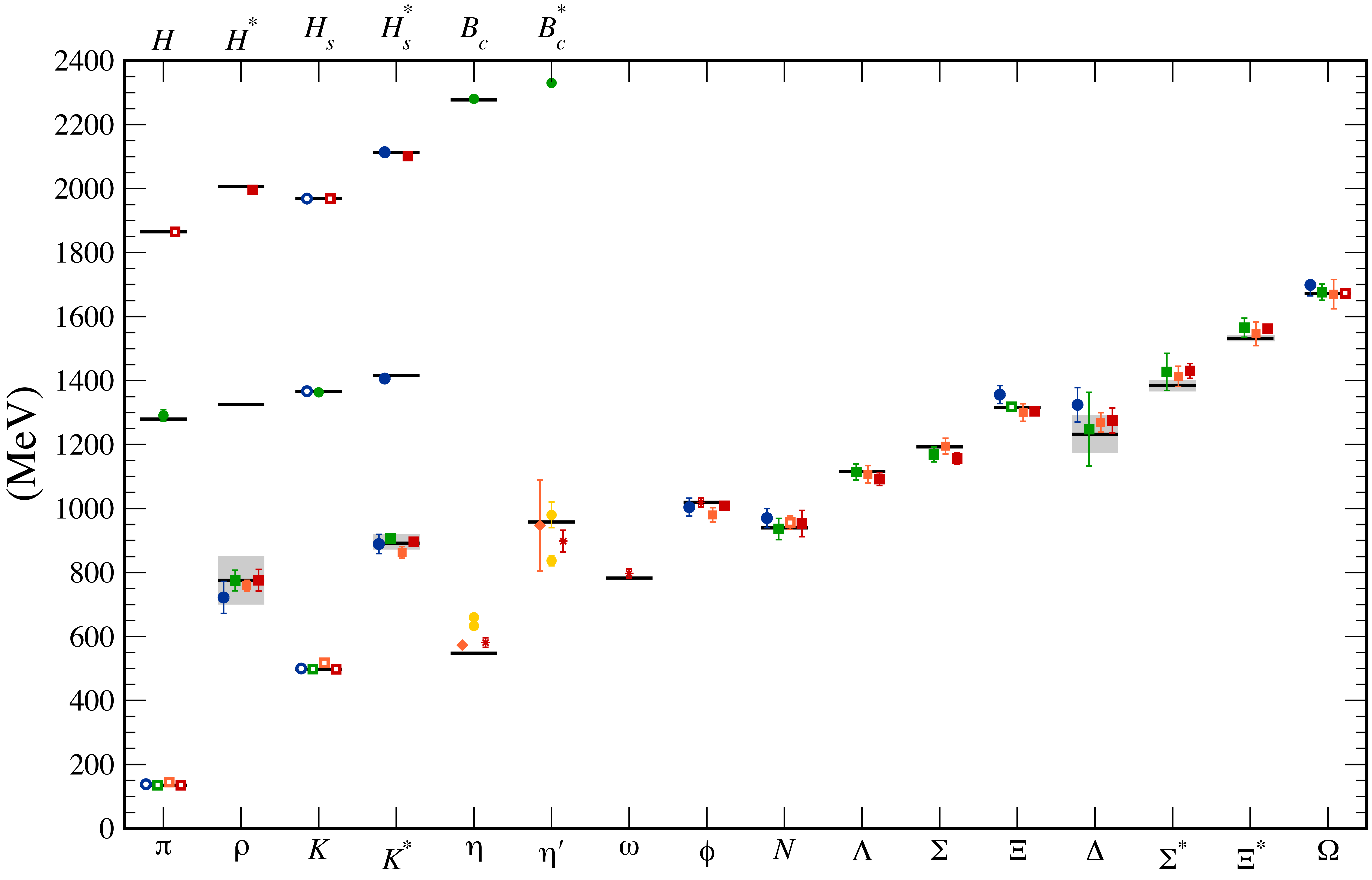
$$m_\pi(L) = m_\pi + c\frac{e^{-m_\pi L}}{(m_\pi L)^{3/2}}$$



Dudek, Edwards & Thomas (2012)

$$a_s \sim 0.12\ \text{fm} \longrightarrow m_\pi L \sim 3.8,\ 4.7,\ 5.6$$

'stable' hadron spectrum



a perfect match: QCD and lattice QCD

- Being perturbative in the UV, assures lattice artifacts are small
- Having dynamical mass geration, assures there is a mass gap!

can non-pertubatively study QCD properties of some time-independent properties of some QCD state.

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limitations?

- This is less clear…
- Many things that are naïvely impossible, have been done!
- In general, this requires new novel ideas that find correspondance between

finite, discretize
Euclidean spacetime
observables

formalism ⇄

infinite, continuous
Minkowski spacetime
observables

Outline

Lattice QCD in a nutshell **[today & tomorrow]**

- [x] Does lattice work?
- [x] why does lattice QCD work?
- [x] what can it be used for?
- [x] what are its limitations?

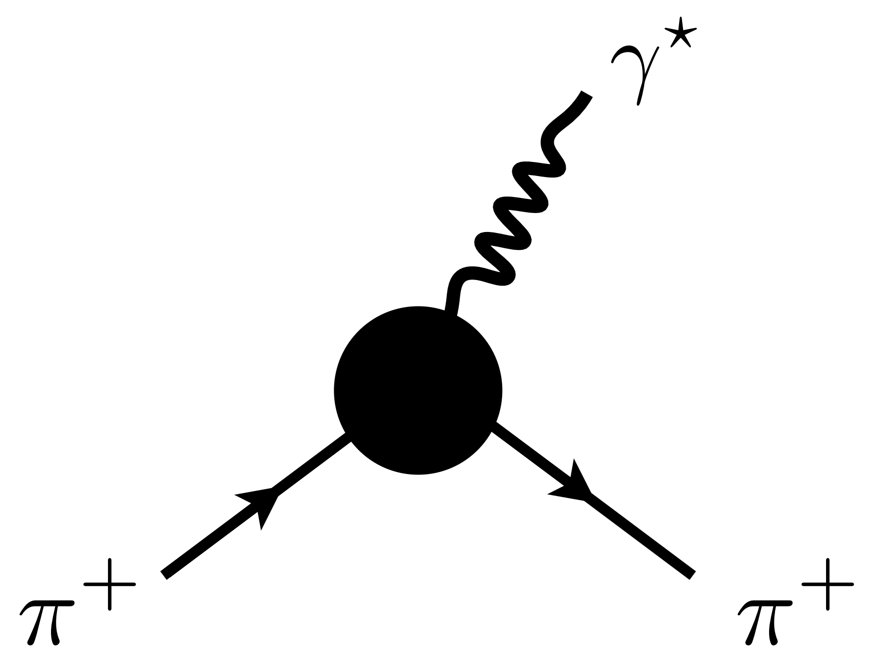
What is the cutting edge of lattice QCD? **[tomorrow]**

- [ ] hadron structure, fundamental symmetry,
- [ ] scattering processes,
- [ ] ….other stuff, I won't get to 🤓
  - [ ] finite-temperature, weak decays, BSM searches, …. ,

Electroweak form factors

Electroweak probes can be studied perturbatively by evaluating matrix elements of currents,

e.g. the pion electromagnetic form-factor

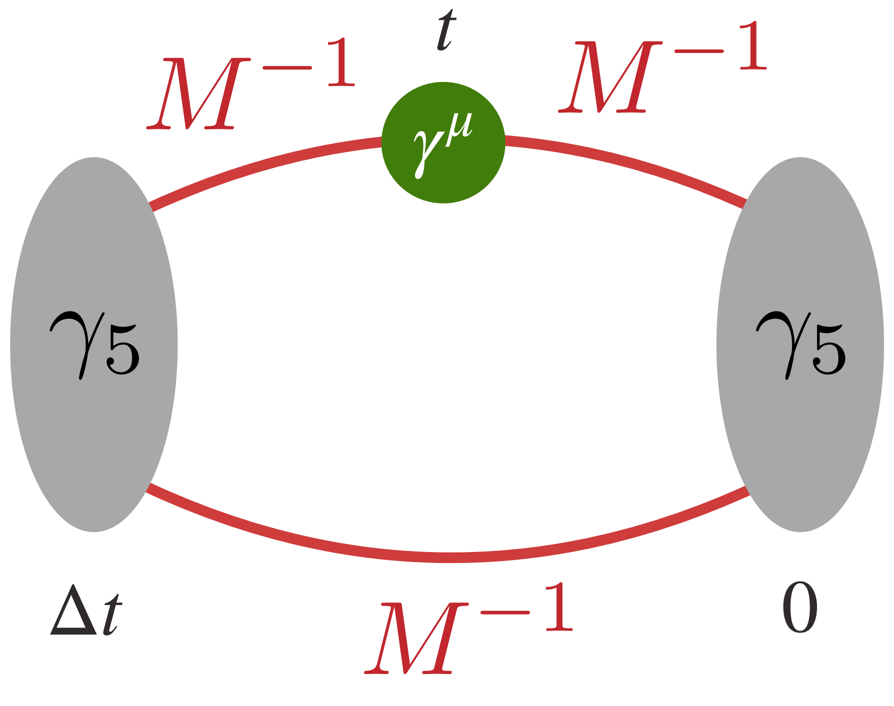


$$\langle \pi^+(\vec{p}\,') \,|\, \overline{\psi}\gamma^\mu\psi(0) \,|\, \pi^+(\vec{p}) \rangle = (p+p')^\mu \, F_\pi(Q^2)$$

access through a three-point correlation function

$$\langle 0 \,|\, \underbrace{\overline{\psi}\gamma_5\psi(\Delta t)}_{\text{annihilate pion q.n.}} \, \underbrace{\overline{\psi}\gamma^\mu\psi(t)}_{\text{vector current}} \, \underbrace{\overline{\psi}\gamma_5\psi(0)}_{\text{create pion q.n.}} \,|\, 0 \rangle$$

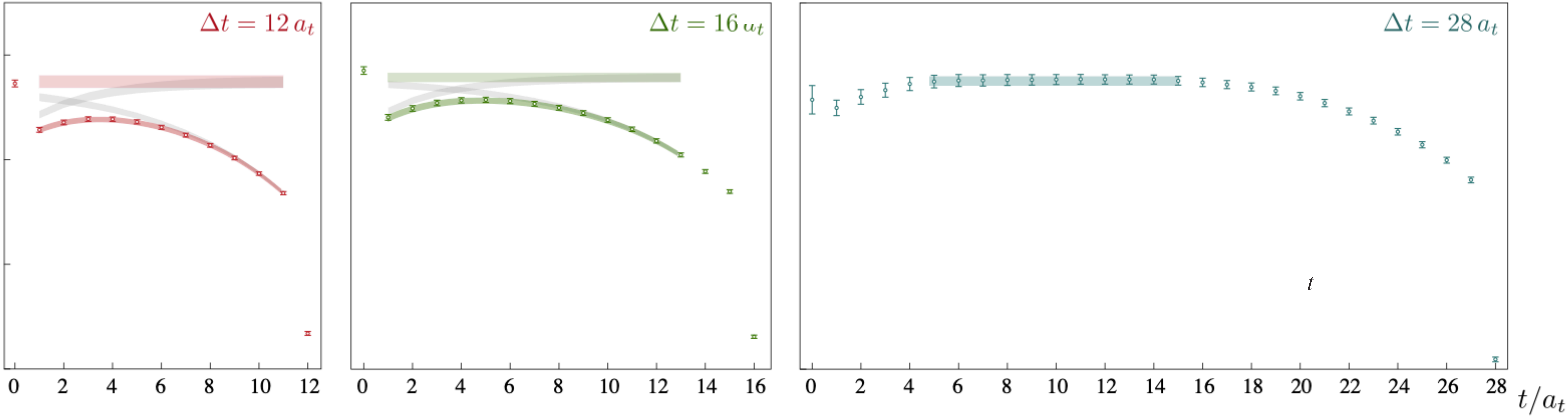
$$= \langle 0 \,|\, \overline{\psi}\gamma_5\psi \,|\, \pi(\vec{p}\,') \rangle \, e^{-E_\pi(\vec{p}\,')(\Delta t - t)} \, \underbrace{\langle \pi(\vec{p}\,') \,|\, \overline{\psi}\gamma^\mu\psi \,|\, \pi(\vec{p}) \rangle}_{\text{desired matrix element}} \, e^{-E_\pi(\vec{p})t} \, \langle \pi(\vec{p}) \,|\, \overline{\psi}\gamma_5\psi \,|\, 0 \rangle \, + \ldots$$



Extracting matrix elements

$$\langle 0 \,|\, \overline{\psi}\gamma_5\psi(\Delta t)\, \overline{\psi}\gamma^\mu\psi(t)\, \overline{\psi}\gamma_5\psi(0) \,|\, 0 \rangle$$

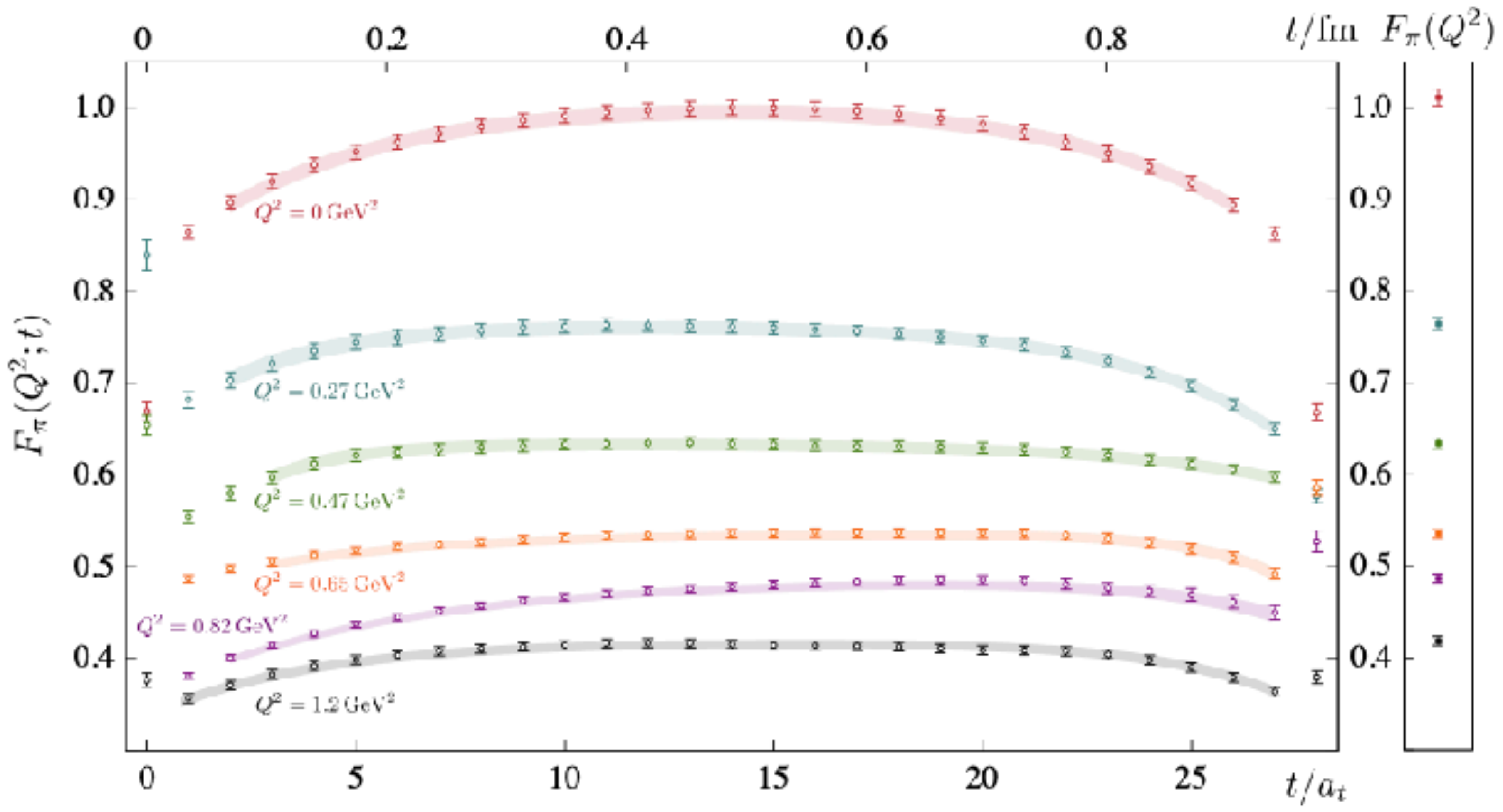
$$= \langle 0 \,|\, \overline{\psi}\gamma_5\psi \,|\, \pi(\vec{p}\,') \rangle\, e^{-E_\pi(\vec{p}\,')(\Delta t - t)}\, \langle \pi(\vec{p}\,') \,|\, \overline{\psi}\gamma^\mu\psi \,|\, \pi(\vec{p}) \rangle\, e^{-E_\pi(\vec{p})t}\, \langle \pi(\vec{p}) \,|\, \overline{\psi}\gamma_5\psi \,|\, 0 \rangle \;+ \ldots$$



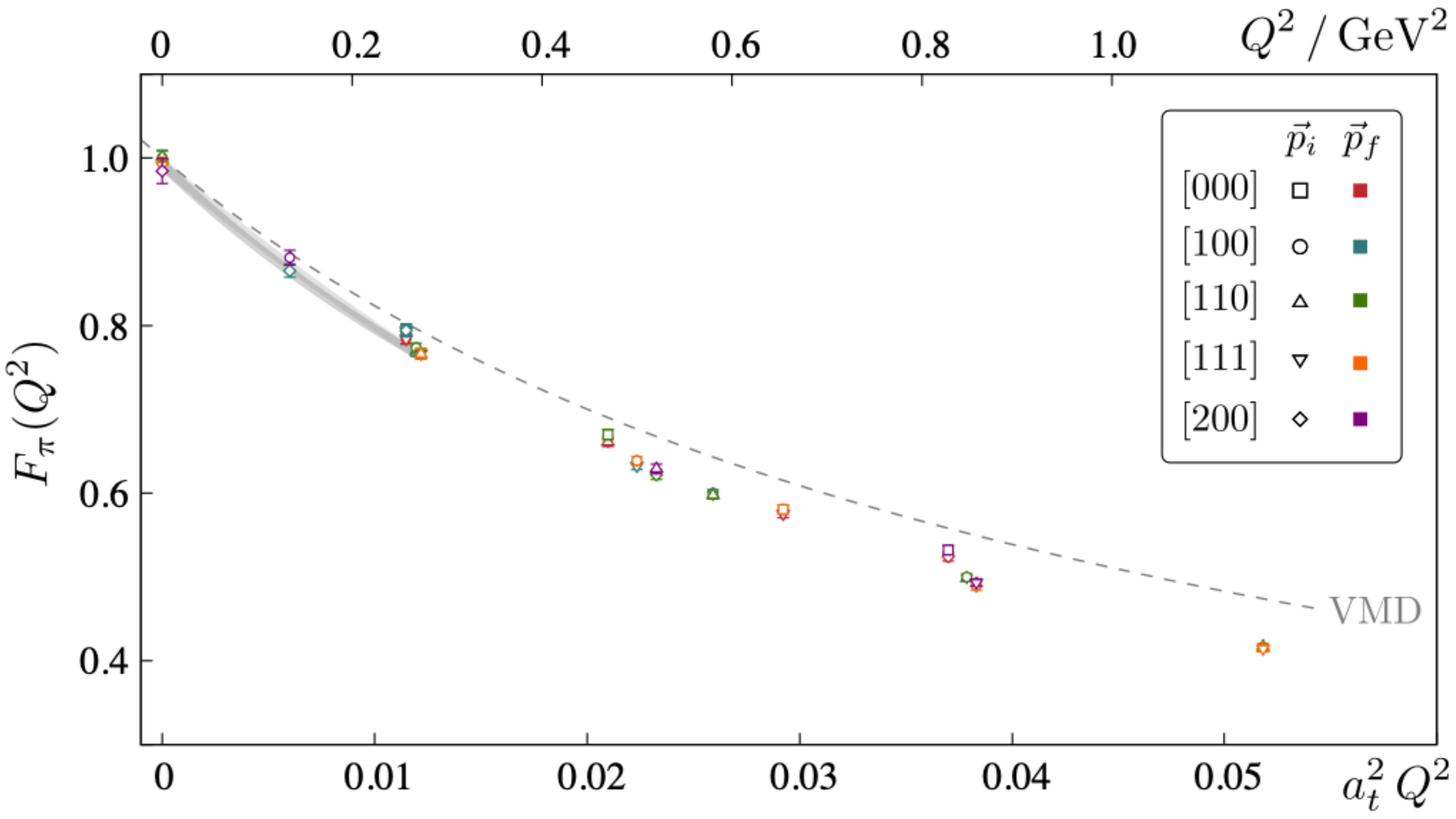
Form factors

$$\langle 0 \,|\, \overline{\psi}\gamma_5\psi(\Delta t)\, \overline{\psi}\gamma^\mu\psi(t)\, \overline{\psi}\gamma_5\psi(0) \,|\, 0 \rangle$$

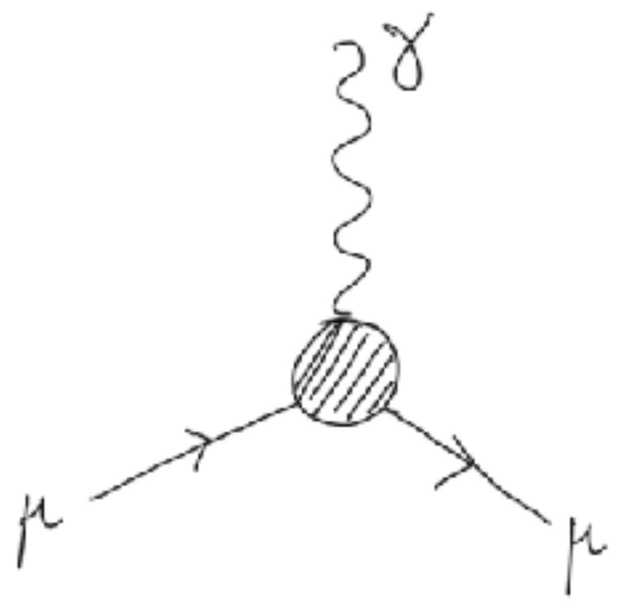
$$= \langle 0 \,|\, \overline{\psi}\gamma_5\psi \,|\, \pi(\vec{p}\,') \rangle \, e^{-E_\pi(\vec{p}\,')(\Delta t - t)} \langle \pi(\vec{p}\,') \,|\, \overline{\psi}\gamma^\mu\psi \,|\, \pi(\vec{p}) \rangle \, e^{-E_\pi(\vec{p})t} \langle \pi(\vec{p}) \,|\, \overline{\psi}\gamma_5\psi \,|\, 0 \rangle \; + \ldots$$



$$\vec{p} = \frac{2\pi}{L}[n_x, n_y, n_z]$$

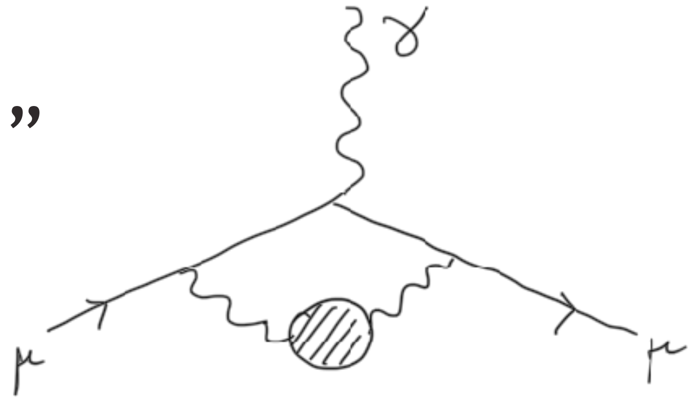


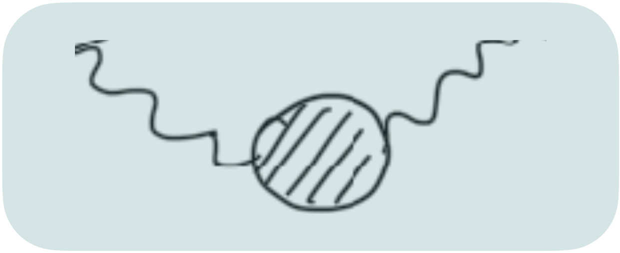
g-2



are there beyond the standard model particles in  ?

need to precisely determine the standard model contribution & this includes QCD

e.g. the “vacuum polarization” 

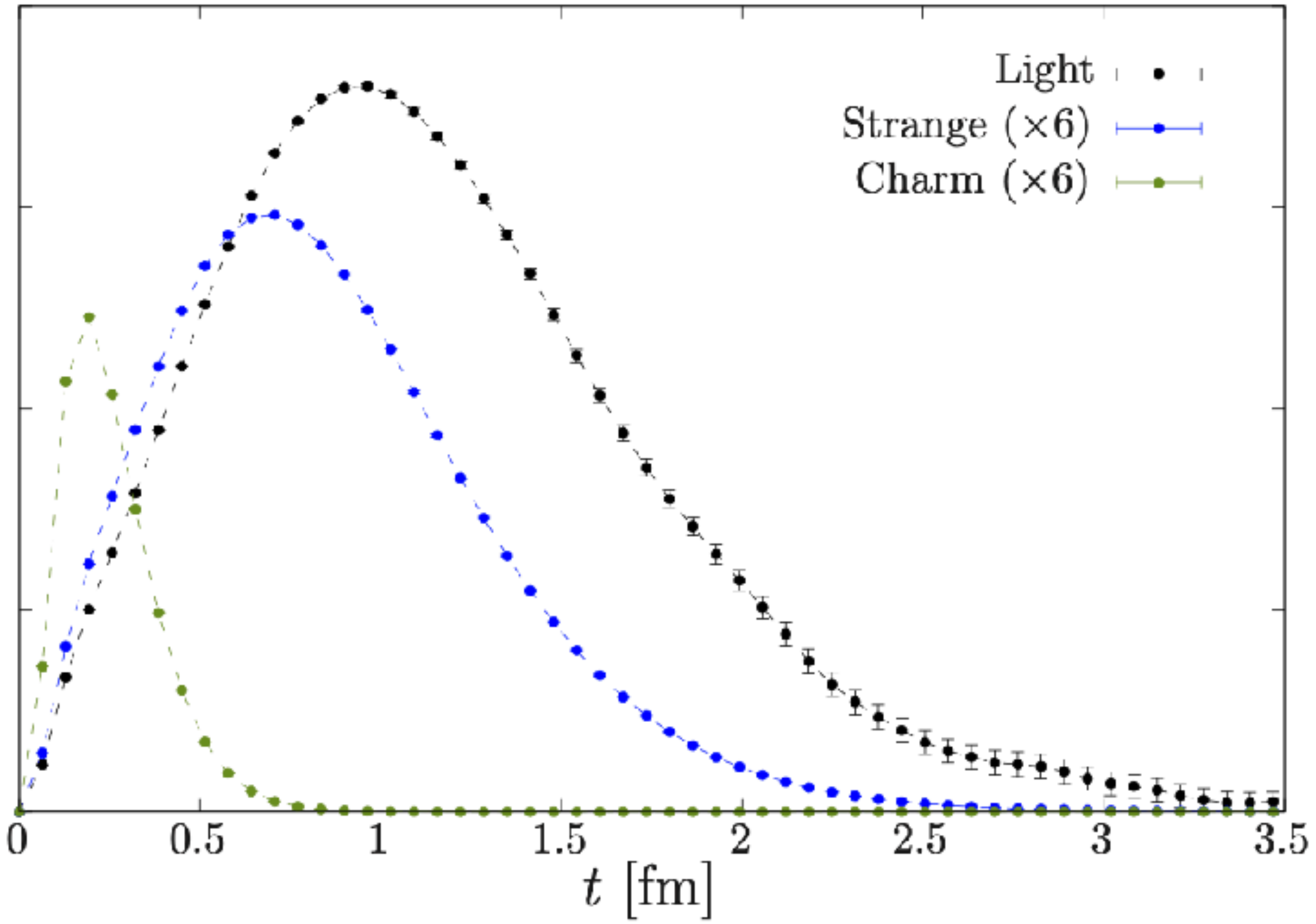


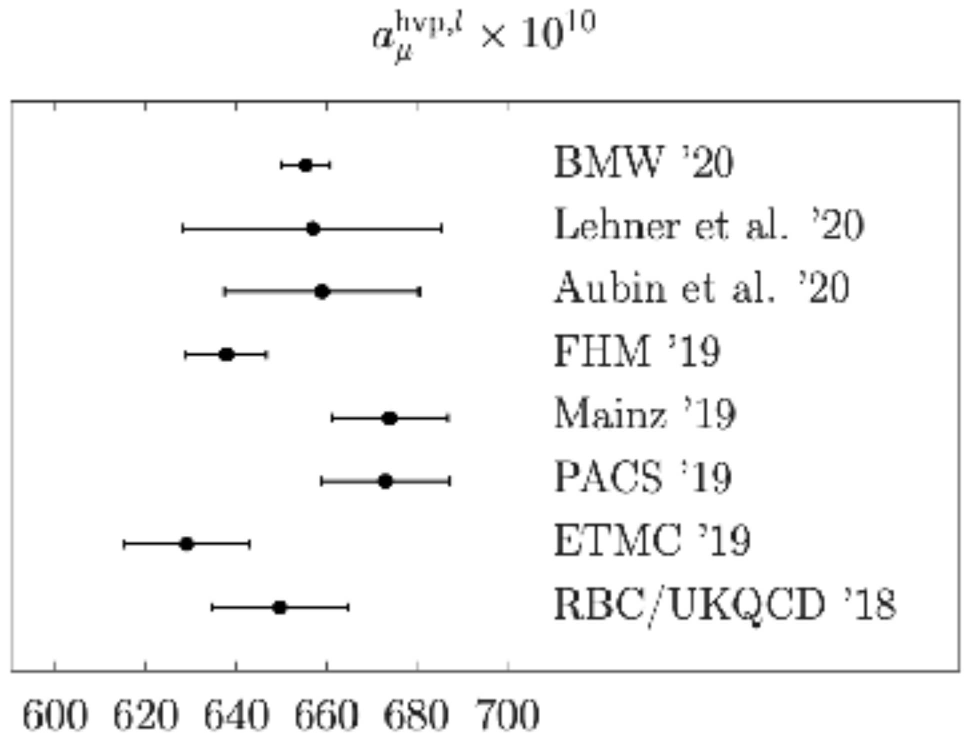
hadronic contribution:

$$a_\mu = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{K}(t)\, G(t)$$

$$G(t) = \frac{1}{3} \sum_{k=1}^{3} \int d^3x\, \langle 0 \,|\, J_k(x,t)\, J_k(0,0) \,|\, 0 \rangle$$

$m_\mu^{-1}\, \tilde{K}(t)\, G(t)$





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- [ ] ….other stuff, I won't get to 🤓
  - [ ] finite-temperature, weak decays, BSM searches, …. ,

2 minimum requirements

Two "musts" for few-body systems:

- ☑ Generalized eigenvalue problem (GEVP),
  - ☑ large basis of ops,

$$\mathcal{O}_b \sim \bar{q}\,\Gamma_b\, q, \pi\pi, K\overline{K}, \ldots,$$

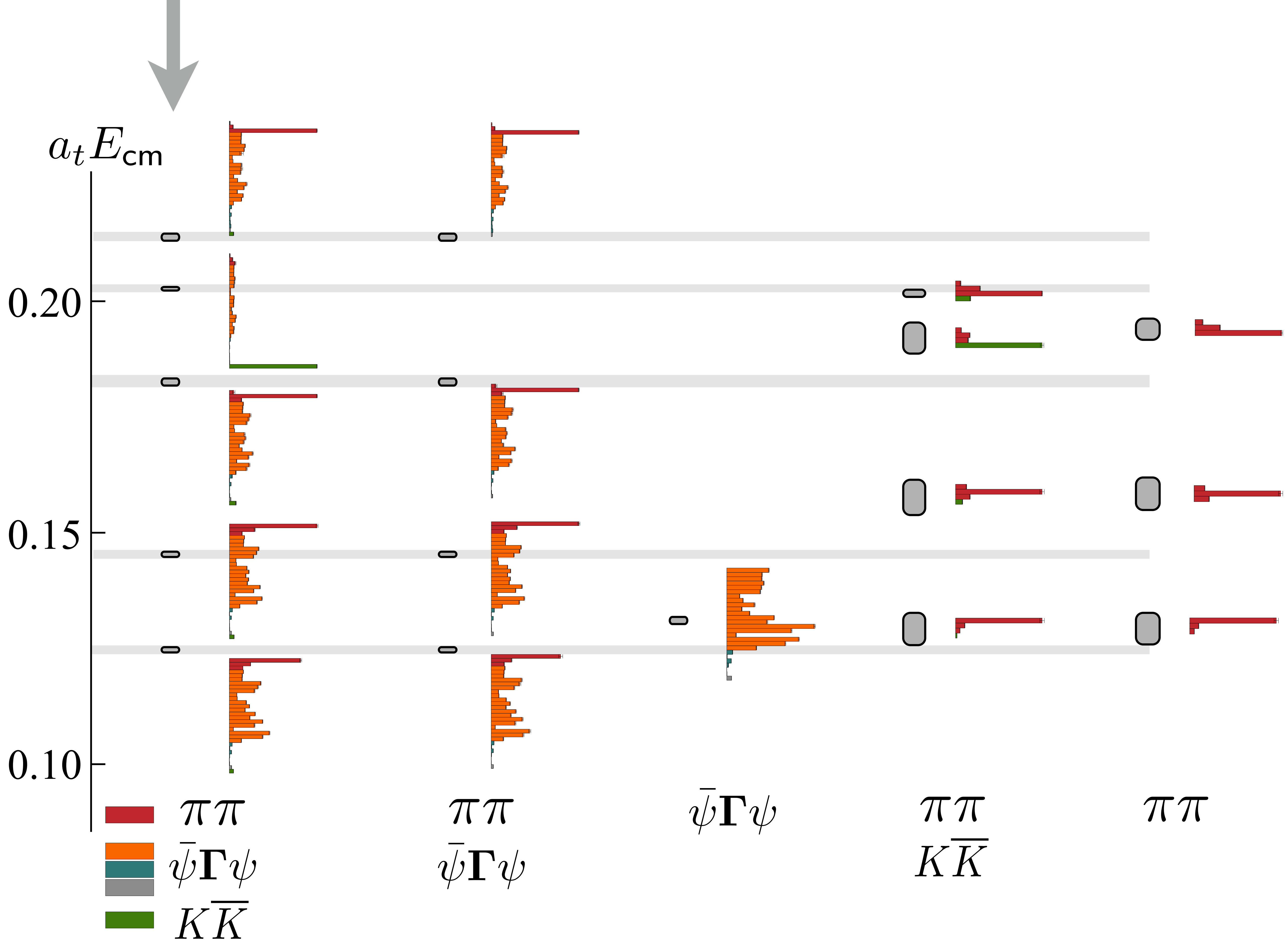
  - ☑ "diagonalization",

$$C_{ab}^{2pt.}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,\mathbf{P})|0\rangle = \sum_n Z_{b,n}Z_{a,n}^* e^{-E_n t}$$

$$C(t)\vec{v}^{(n)}(t,t_0) = \lambda_n(t,t_0)C(t_0)\vec{v}^{(n)}(t,t_0)$$

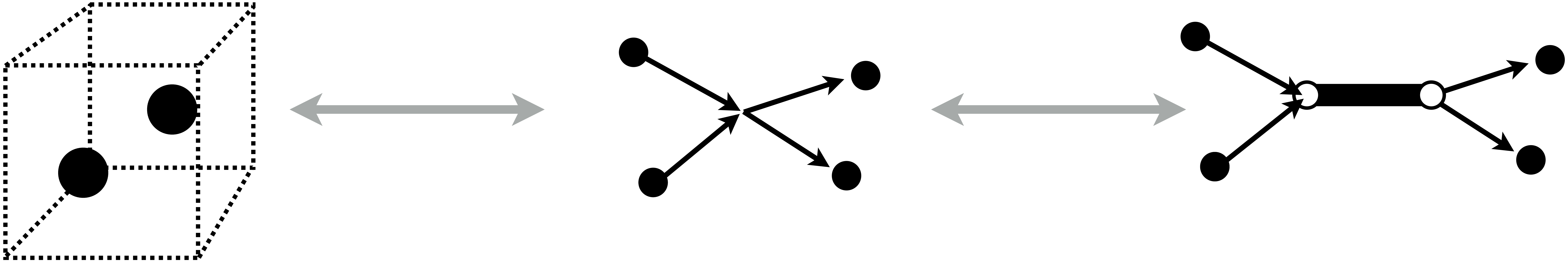
- ☑ Finite-volume formalisms.

GEVP with large basic is **necessary** but **not sufficient** to trust spectrum!



Wilson, RB, Dudek, Edwards, & Thomas (2015)

Two-hadron systems



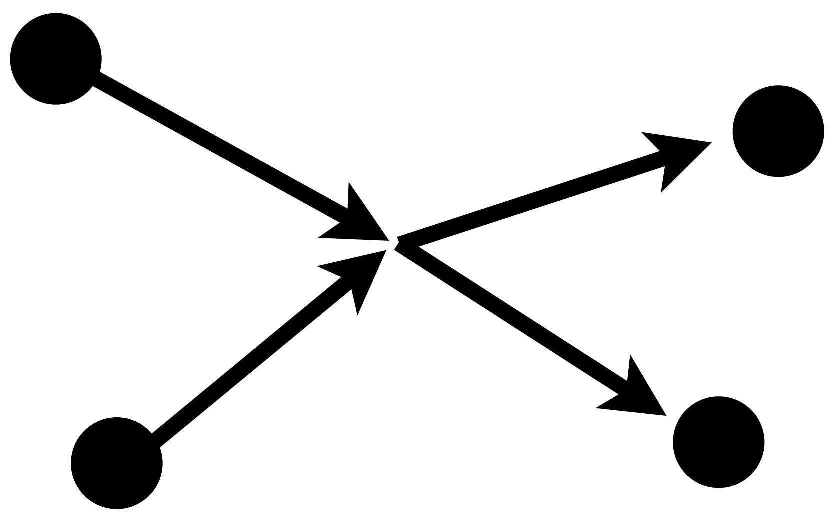
finite-volume
spectroscopy

infinite-volume
scattering amplitudes

bound state and
resonance poles

Two-hadron systems

finite-volume
spectroscopy



infinite-volume
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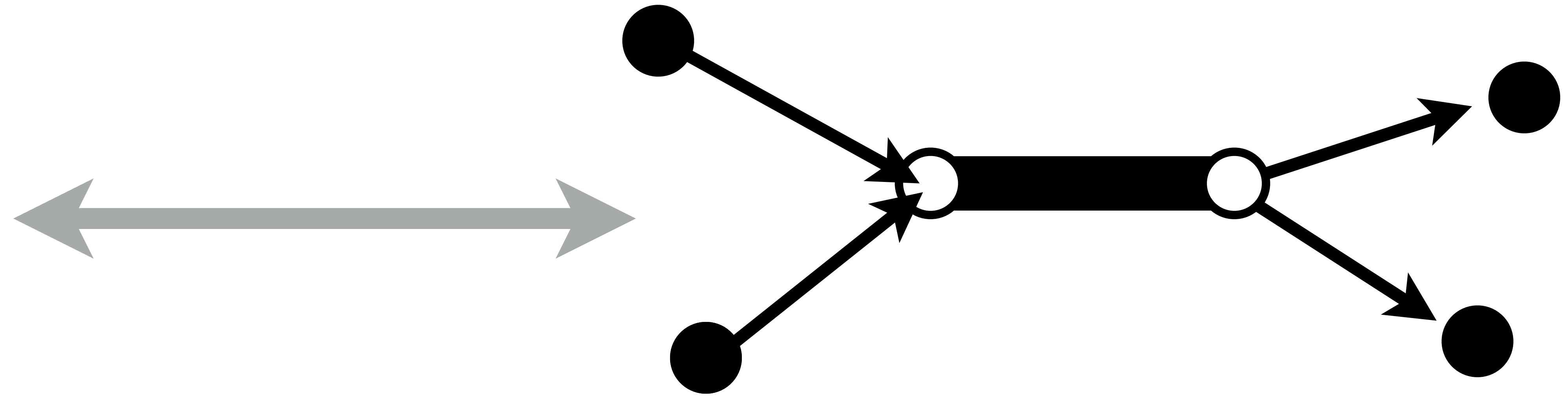
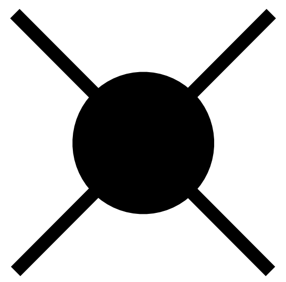


bound state and
resonance poles

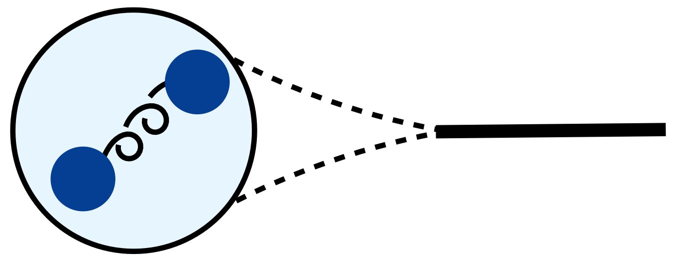
Two-hadron scattering

$i\mathcal{M} =$ 

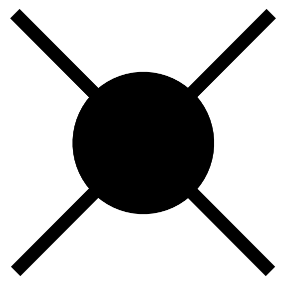
Goal: *isolated ALL kinematic singularities of amplitudes in a kinematic region.*

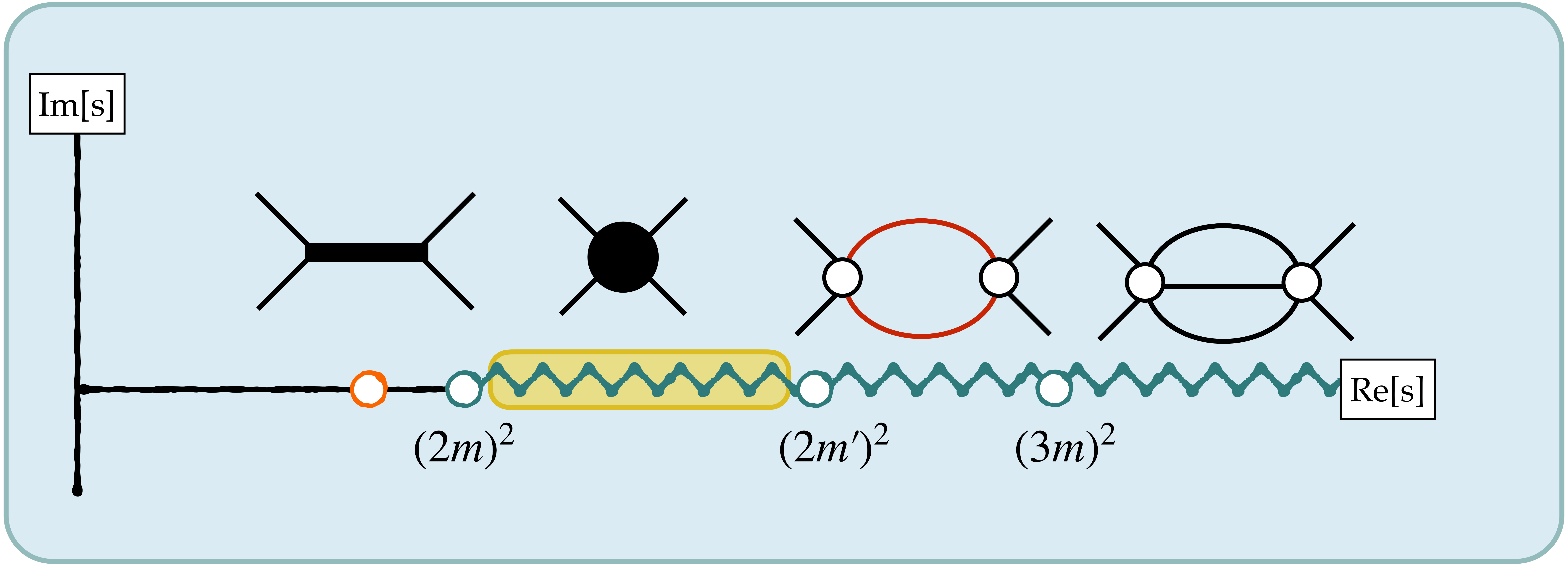
Observation: *kinematic singularities are due to intermediate particles goin on-shell.*

IR limit of QCD, only interested in hadronic d.o.f.



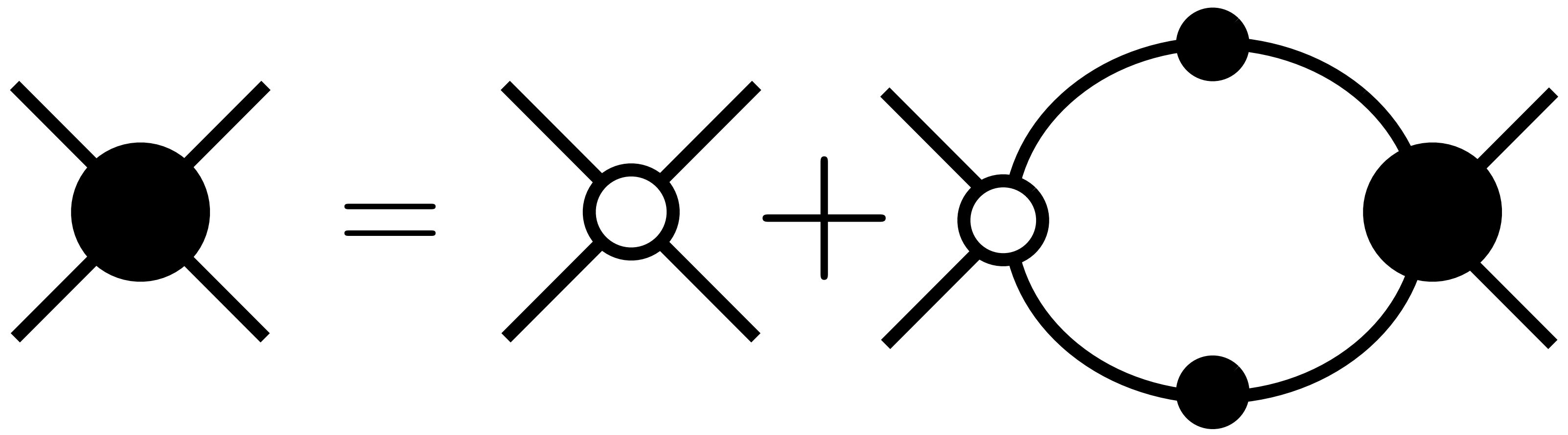
Two-hadron scattering

$i\mathcal{M} =$ 



Two-hadron scattering

The sum over all two-to-two connected diagrams…

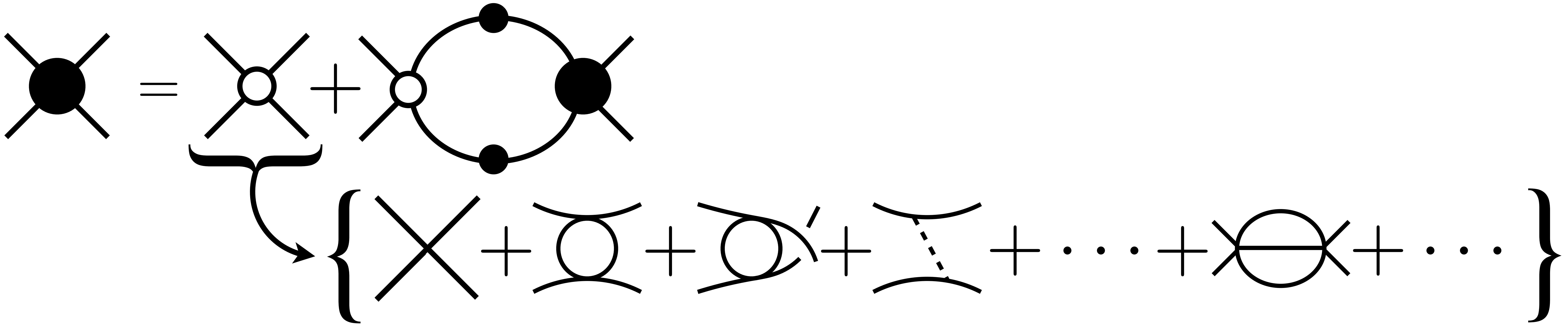
$i\mathcal{M} =$ 

{ *the off-shell amplitude satisfies a self-consistent integral equation [Schwinger-Dyson]* }

Two-hadron scattering

The sum over all two-to-two connected diagrams…

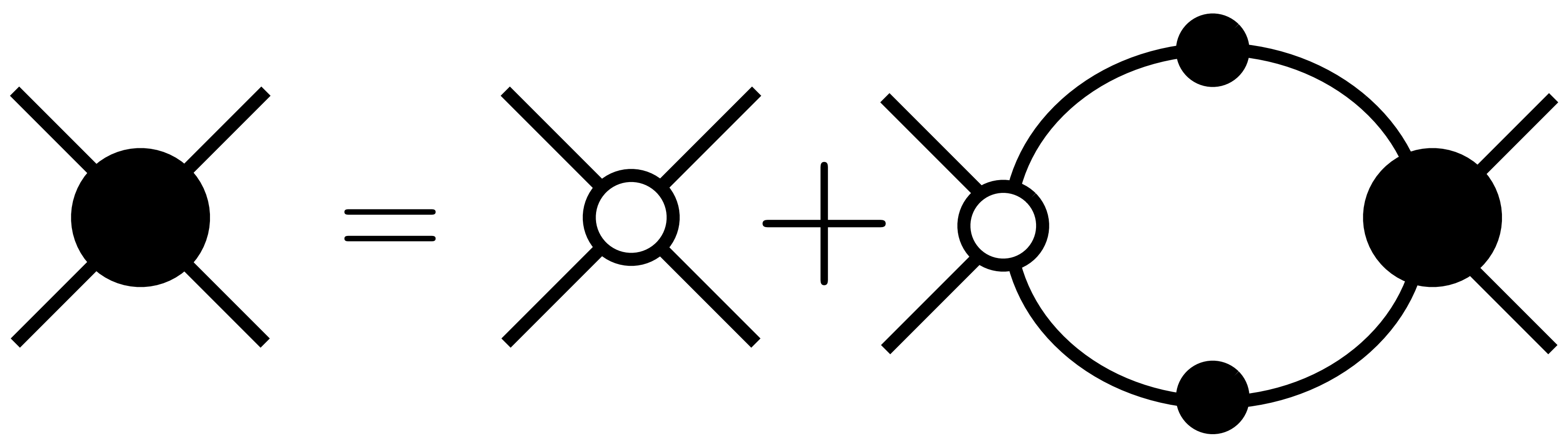
$i\mathcal{M} =$

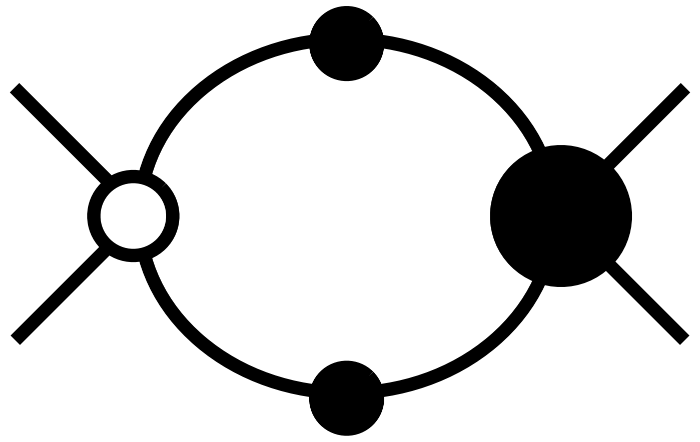


{ non-perturbative kernel including all diagrams not shown [2-PI s-channel diagrams] }

Two-hadron scattering

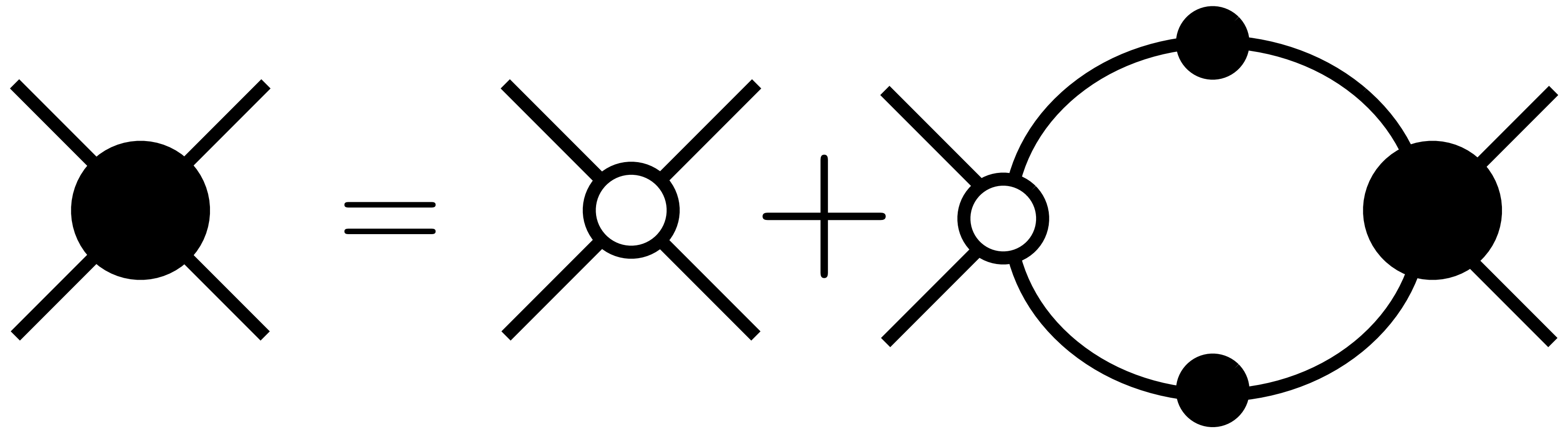
The sum over all two-to-two connected diagrams…

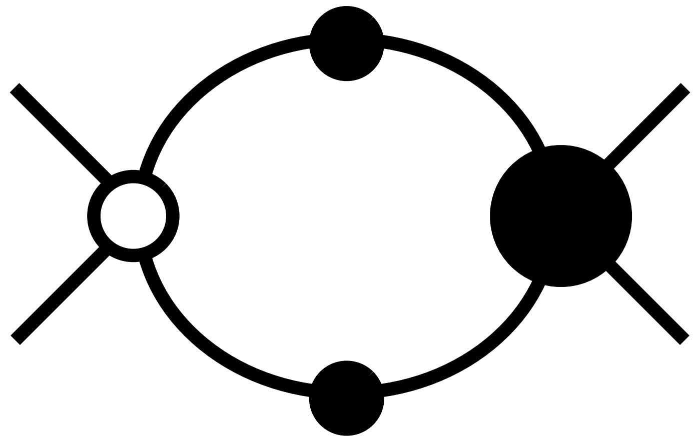
$i\mathcal{M} =$ 

 $= \int \frac{d^4k}{(2\pi)^4} \frac{B(k,P)}{k^2 - m^2 + i\epsilon} \frac{\mathcal{M}(P,k)}{(P-k)^2 - m^2 + i\epsilon}$

Two-hadron scattering

The sum over all two-to-two connected diagrams…

$i\mathcal{M} =$ 

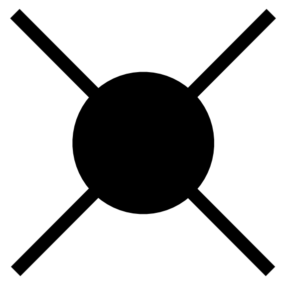
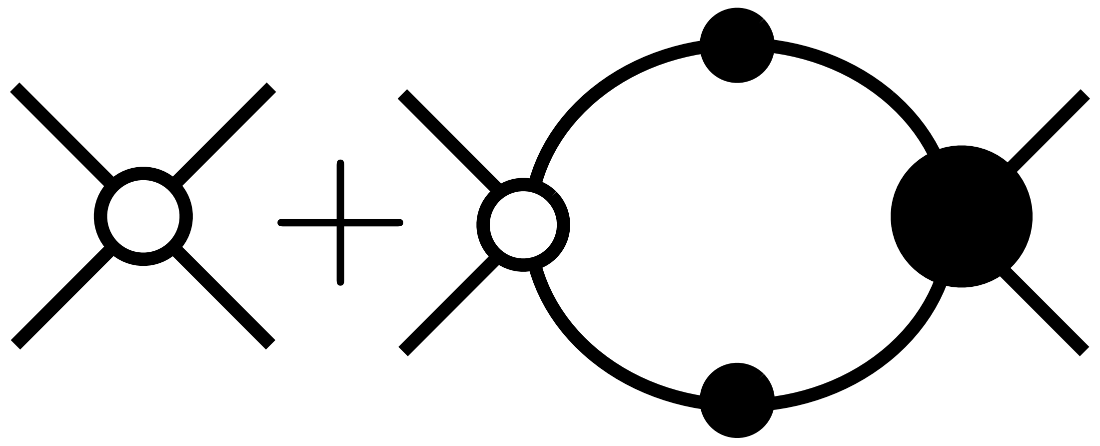


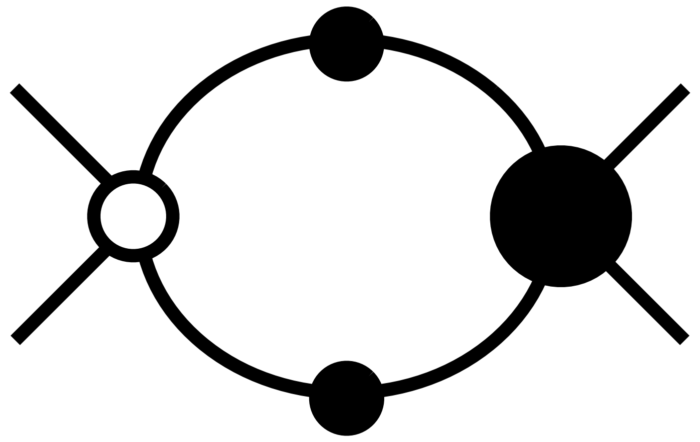
$$= \int \frac{d^4k}{(2\pi)^4} \frac{B(k,P)}{k^2 - m^2 + i\epsilon} \frac{\mathcal{M}(P,k)}{(P-k)^2 - m^2 + i\epsilon}$$

$$= -\int \frac{d^3k}{(2\pi)^3} \frac{B(k,P)\,\mathcal{M}(P,k)}{(2\omega_k)^2} \pi\delta(E - 2\omega_k) + \text{“PV integral”}$$

Two-hadron scattering

The sum over all two-to-two connected diagrams…

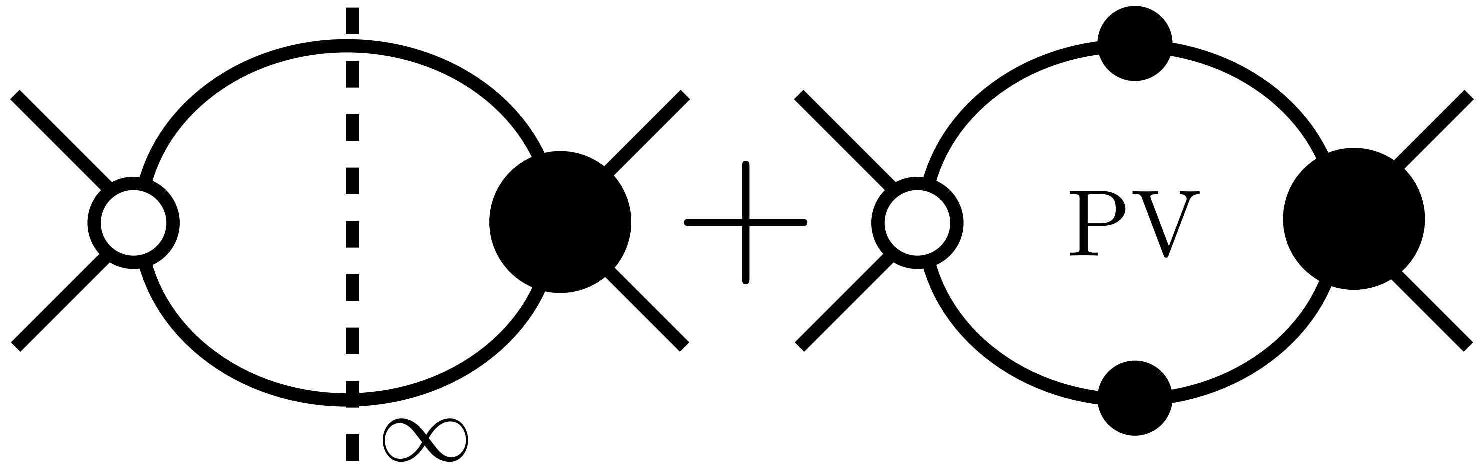
$i\mathcal{M} =$  $=$ 



$$= \int \frac{d^4k}{(2\pi)^4} \frac{B(k,P)}{k^2 - m^2 + i\epsilon} \frac{\mathcal{M}(P,k)}{(P-k)^2 - m^2 + i\epsilon}$$

$$= -\int \frac{d^3k}{(2\pi)^3} \frac{B(k,P)\,\mathcal{M}(P,k)}{(2\omega_k)^2} \pi\delta(E - 2\omega_k) + \text{“PV integral”}$$

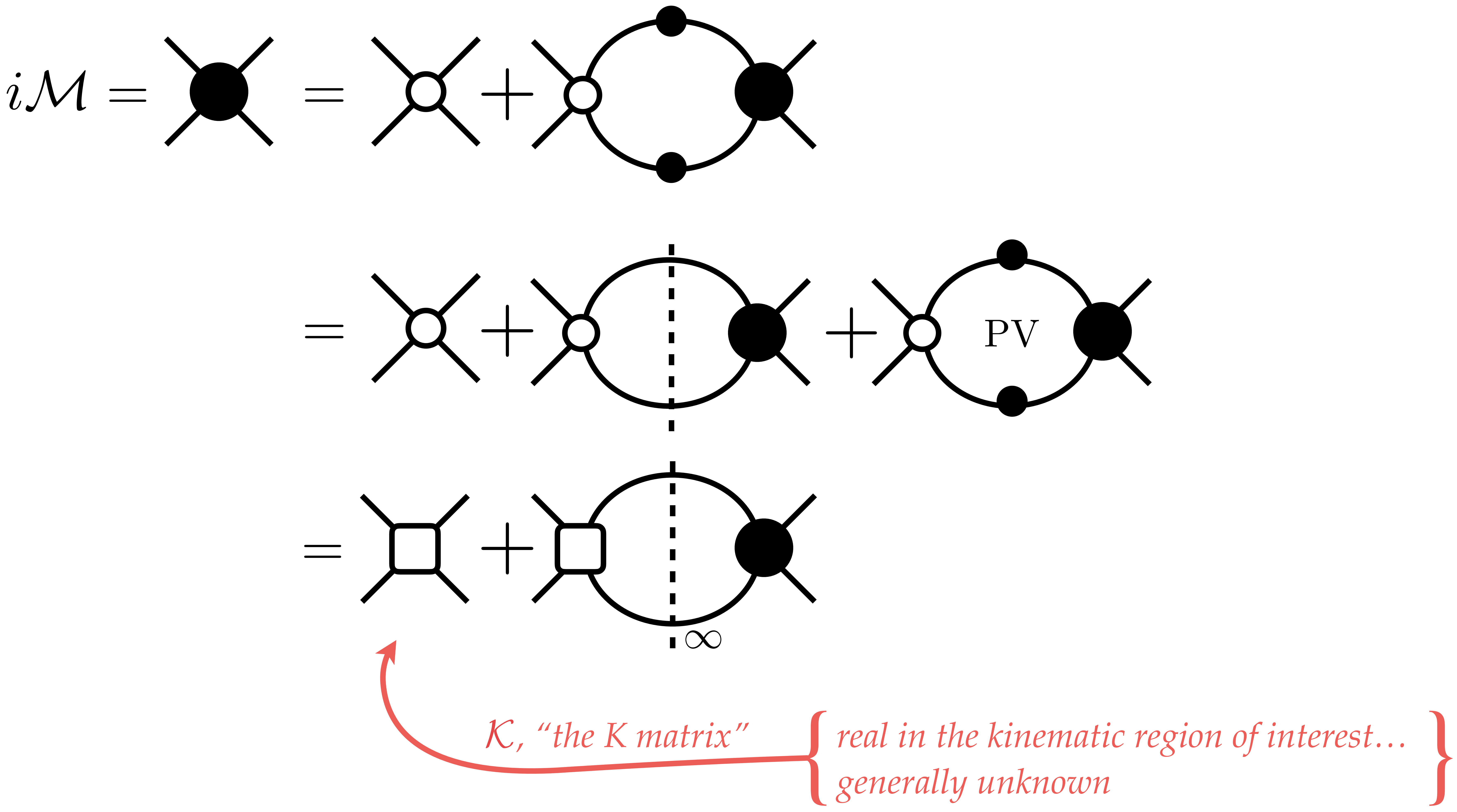
$$= [iB_{\text{off,on}}]\rho\,[i\mathcal{M}_{\text{on,off}}] + \text{“PV integral”}$$

$=$ 

$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$ { *square root singularity* }

Two-hadron scattering

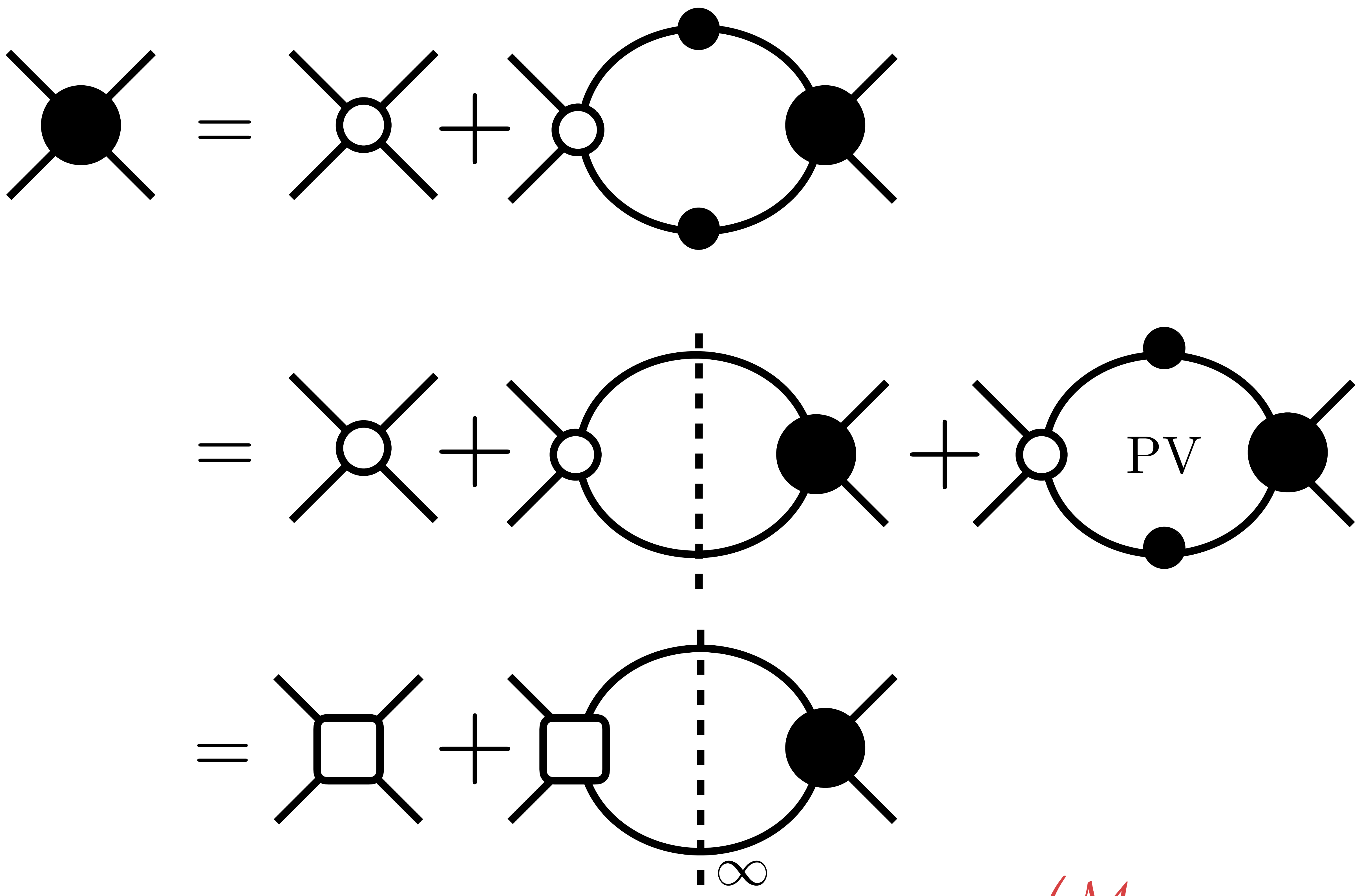
The sum over all two-to-two connected diagrams…



Two-hadron scattering

The sum over all two-to-two connected diagrams…

$i\mathcal{M} =$



placing all legs on-shell
& partial-wave projecting

$$\longrightarrow \frac{i}{\mathcal{K}^{-1} - i\rho} = \begin{pmatrix} \mathcal{M}_0 & & & & \\ & \mathcal{M}_1 & & & \\ & & \mathcal{M}_1 & & \\ & & & \mathcal{M}_1 & \\ & & & & \ddots \end{pmatrix}$$

diagonal matrix over partial waves…

Unitarity check

Diagrammatic result: $\mathcal{M} = \dfrac{1}{\mathcal{K}^{-1} - i\rho}$

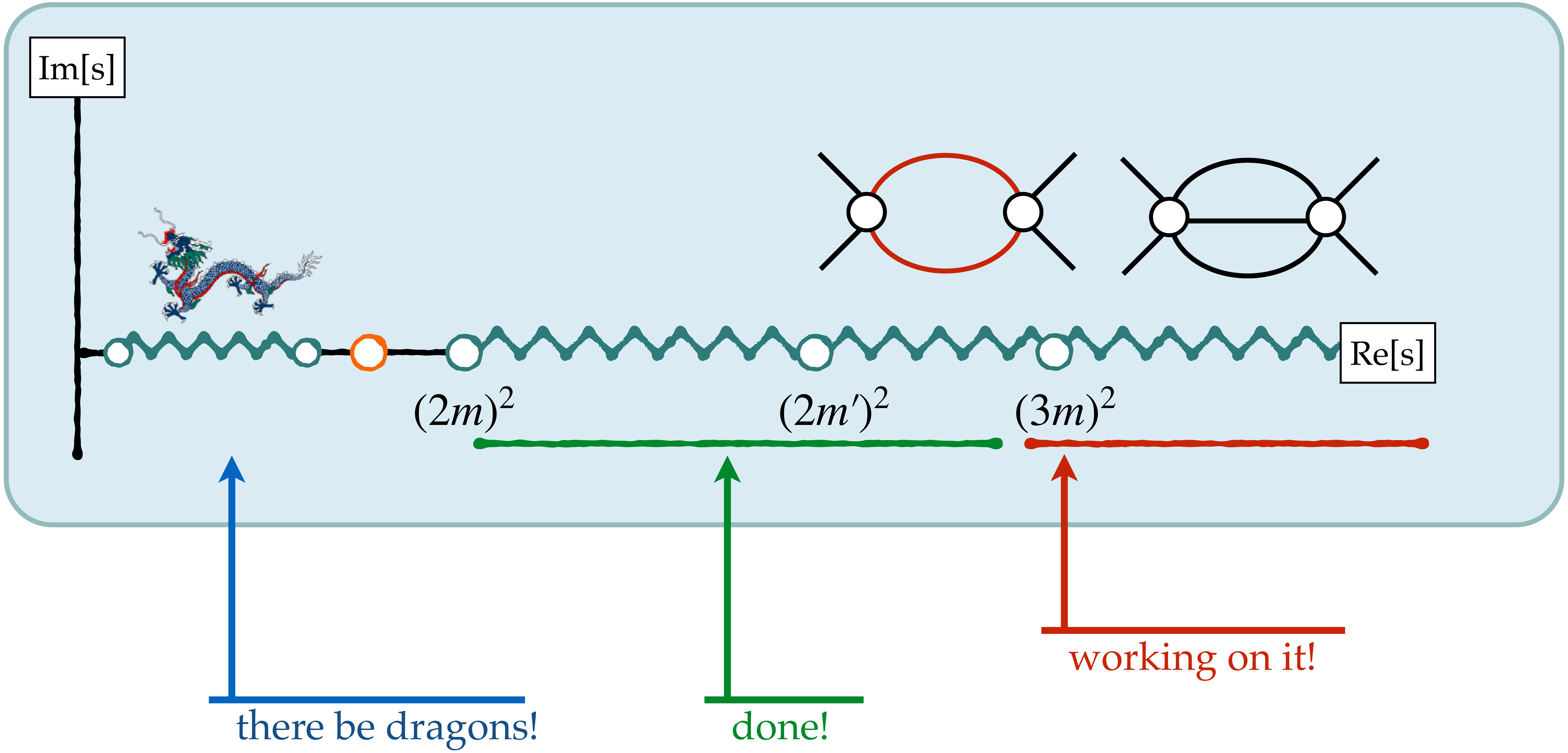
…satisfies the optical theorem: $\mathrm{Im}\,\mathcal{M} = \mathcal{M}^*\,\rho\,\mathcal{M}$

From unitarity…. $S = e^{2i\delta} = 1 + 2i\rho\mathcal{M}$

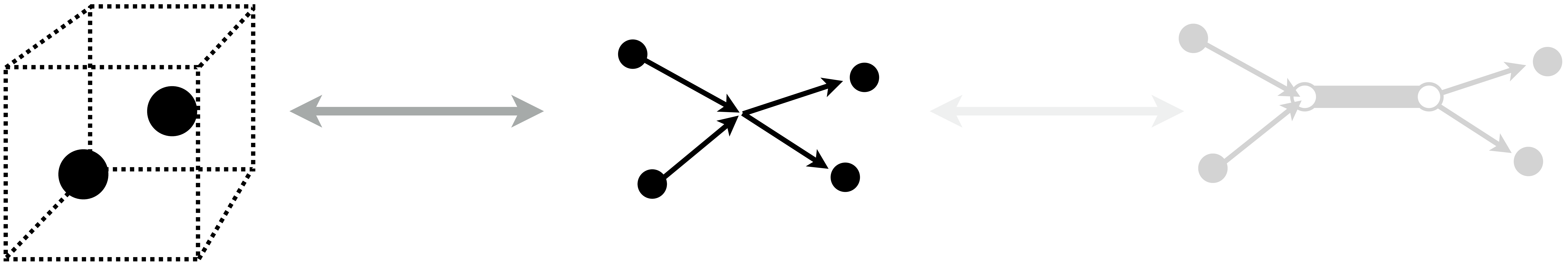
…we get: $\mathcal{K}^{-1} = \rho\cot\delta$

now we just need to constrain the phase-shift or, equivalently, the K matrix…from lattice QCD

Going to higher energies



Two-hadron systems



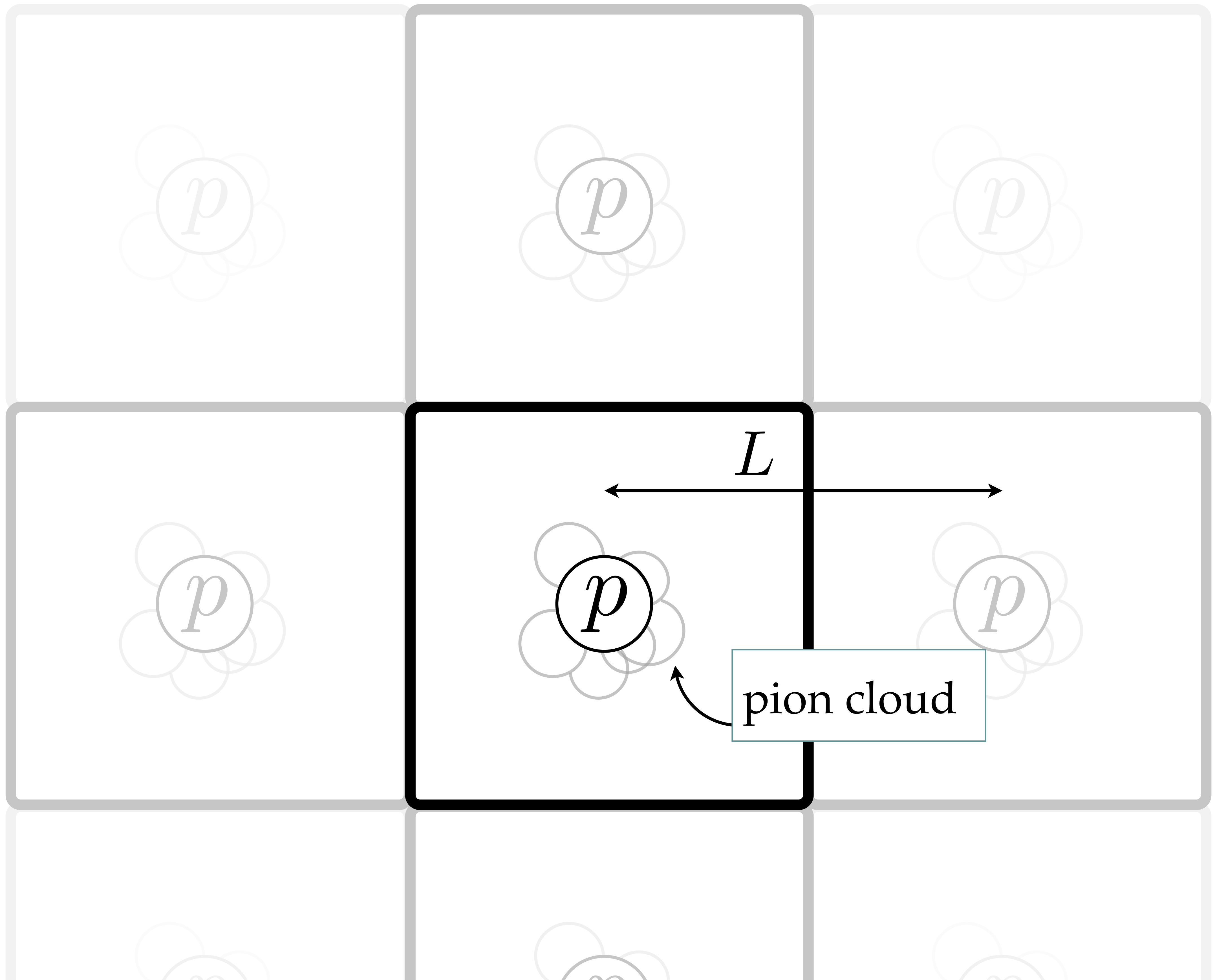
finite-volume
spectroscopy

infinite-volume
scattering amplitudes

bound state and
resonance poles

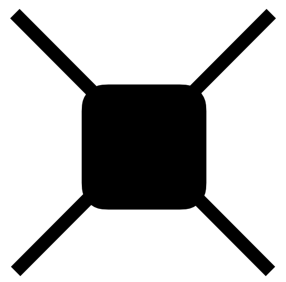
One particle in finite volume

- Stable hadron size ~ $\mathcal{O}(1/m_\pi)$
- If $L \gg m_\pi^{-1}$, finite-volume errors are suppressed



$$m_h(L) = m_h(\infty) + \mathcal{O}(e^{-m_\pi L})$$

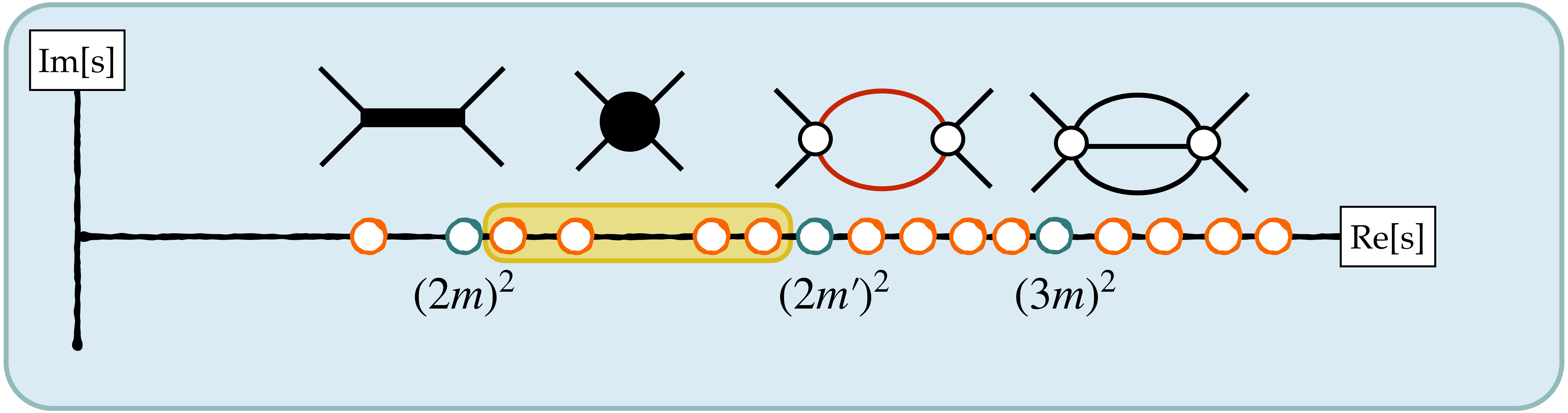
Two particle in finite volume

$i\mathcal{M}_L =$ 

<u>Goal:</u> find the condition that the finite-volume spectrum must satisfy.

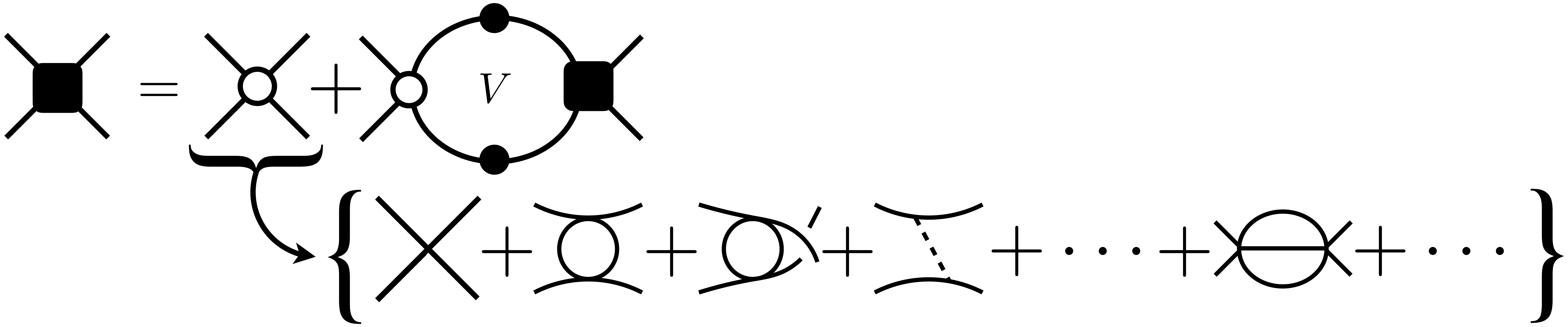
<u>Observation #1:</u> the finite-volume states are real-valued poles of correlation functions

<u>Observation #2:</u> power-law finite-volume effects are due to intermediate particles goin on-shell.



Two particle in finite volume

Similar story as before…except momenta are discrete $\vec{k} = 2\pi\vec{n}/L$

$i\mathcal{M}_L =$ 

$\left\{ \textit{same as before up to } \mathcal{O}(e^{-m_\pi L}) \right\}$

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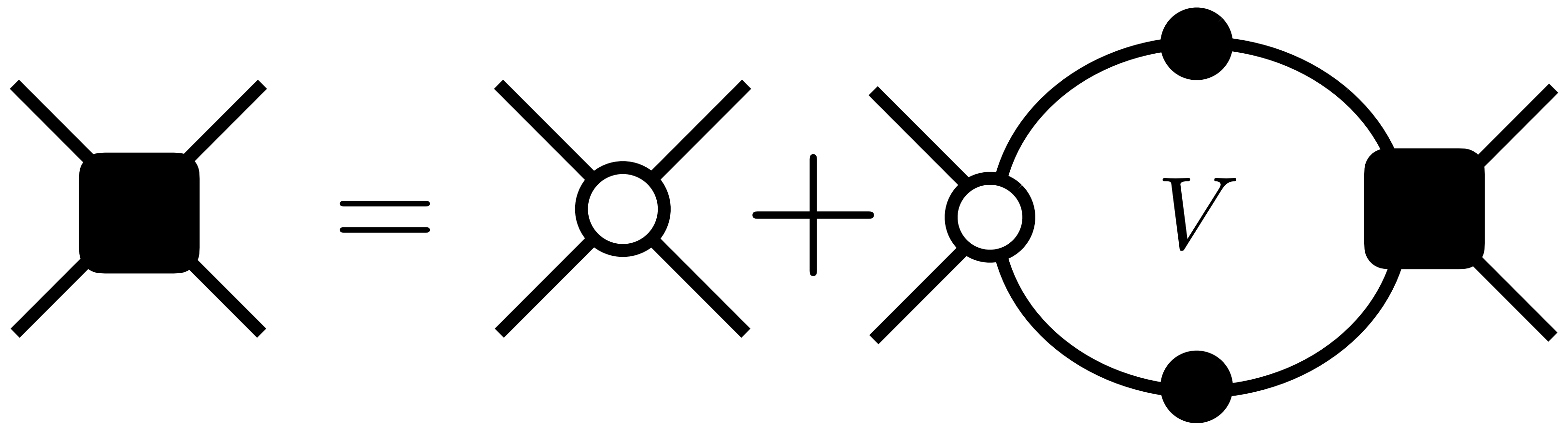
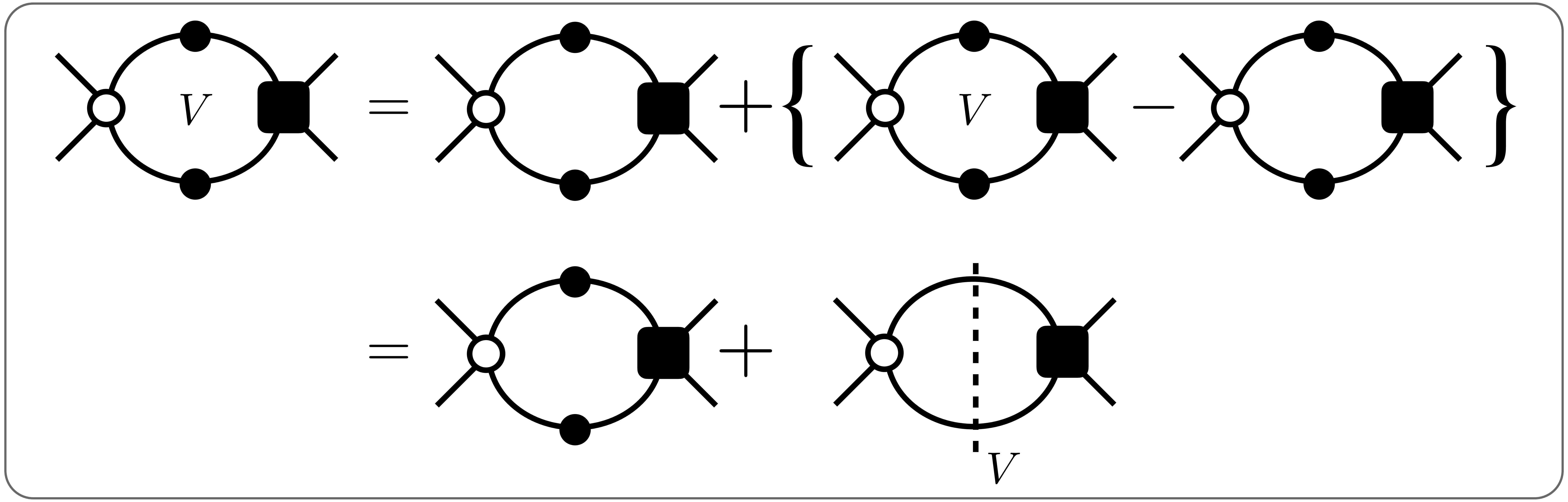
$i\mathcal{M}_L =$ 



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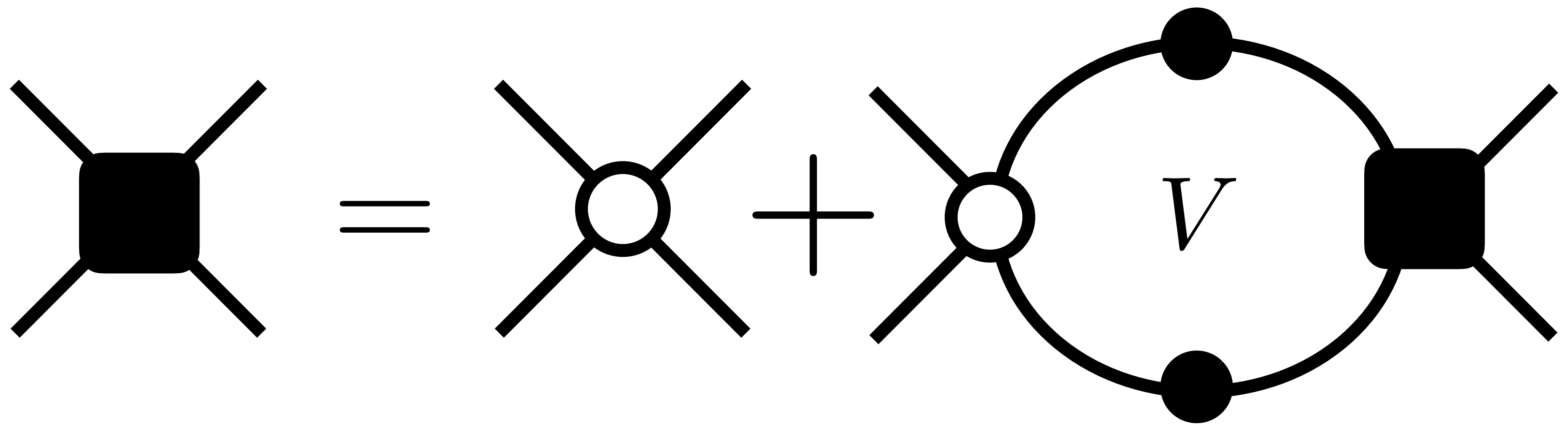
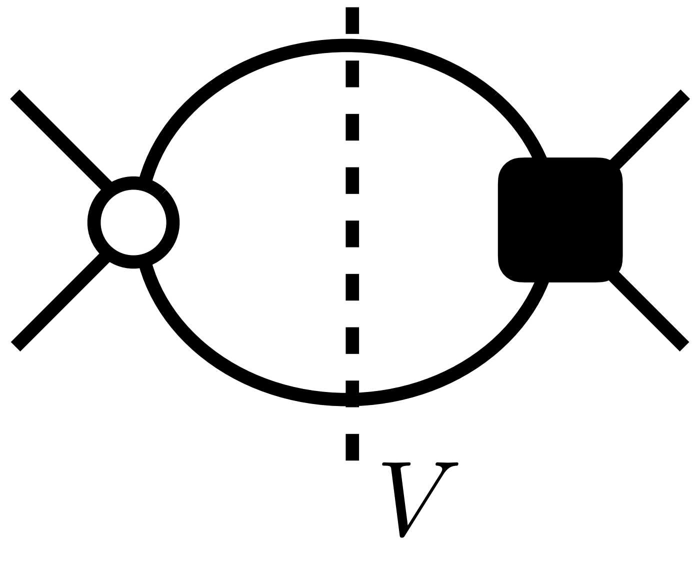
$i\mathcal{M}_L =$ 



$$= [iB]_{\ell' m'} \left(\left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int \frac{d^3k}{(2\pi)^3} \right] \frac{1}{2\omega_k)} \frac{i\mathcal{Y}_{\ell' m'}(\hat{k})\, \mathcal{Y}^*_{\ell m}(\hat{k})}{(P-k)^2 - m^2 + i\epsilon} \right) [i\mathcal{M}_L]_{\ell m}$$

$$\equiv [iB]\, iF\, [iB]$$

$$F = \begin{pmatrix} F_{00;00} & F_{00;11} & F_{00;10} & & \\ F_{11;00} & F_{11;11} & F_{11;10} & & \\ F_{10;00} & F_{10;11} & F_{10;10} & & \\ & & & \ddots & \\ & & & & \ddots \end{pmatrix}$$

non-diagonal matrix over partial waves… because angular momentum is not a good quantum number

Two particle in finite volume

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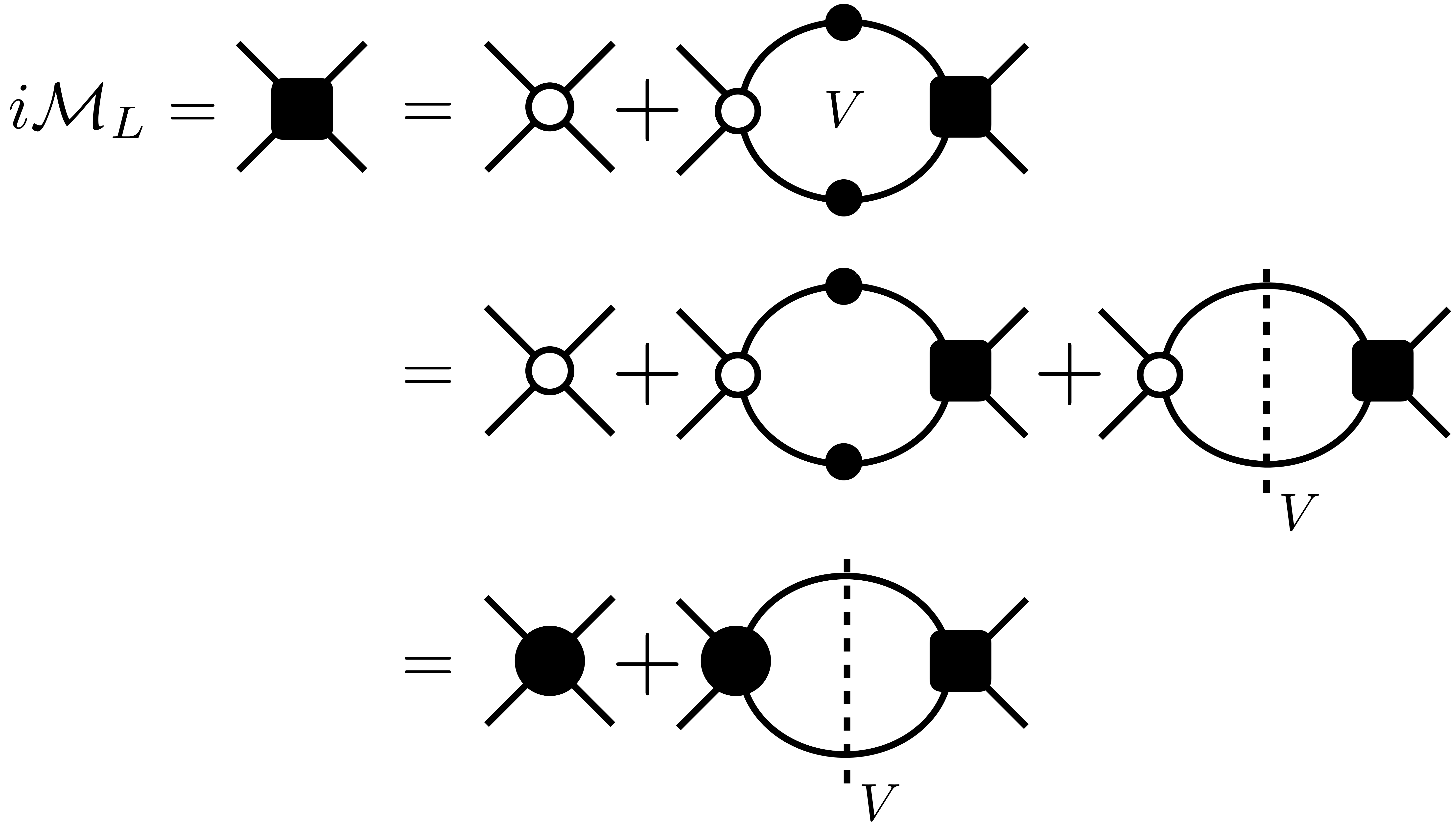

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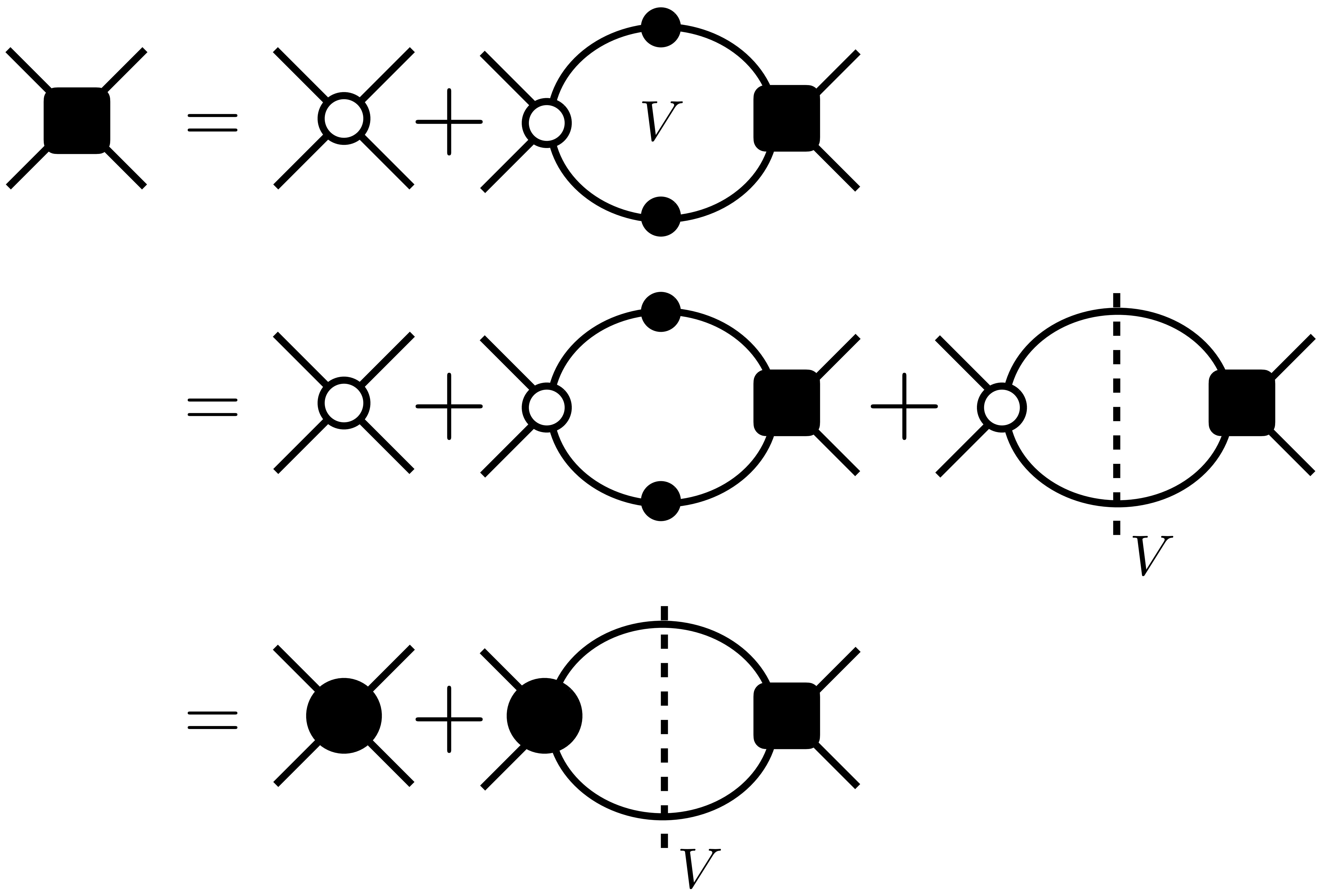
$i\mathcal{M}_L =$



placing all legs on-shell
& partial-wave projecting →

$$i\mathcal{M}\frac{1}{1+F\mathcal{M}}$$

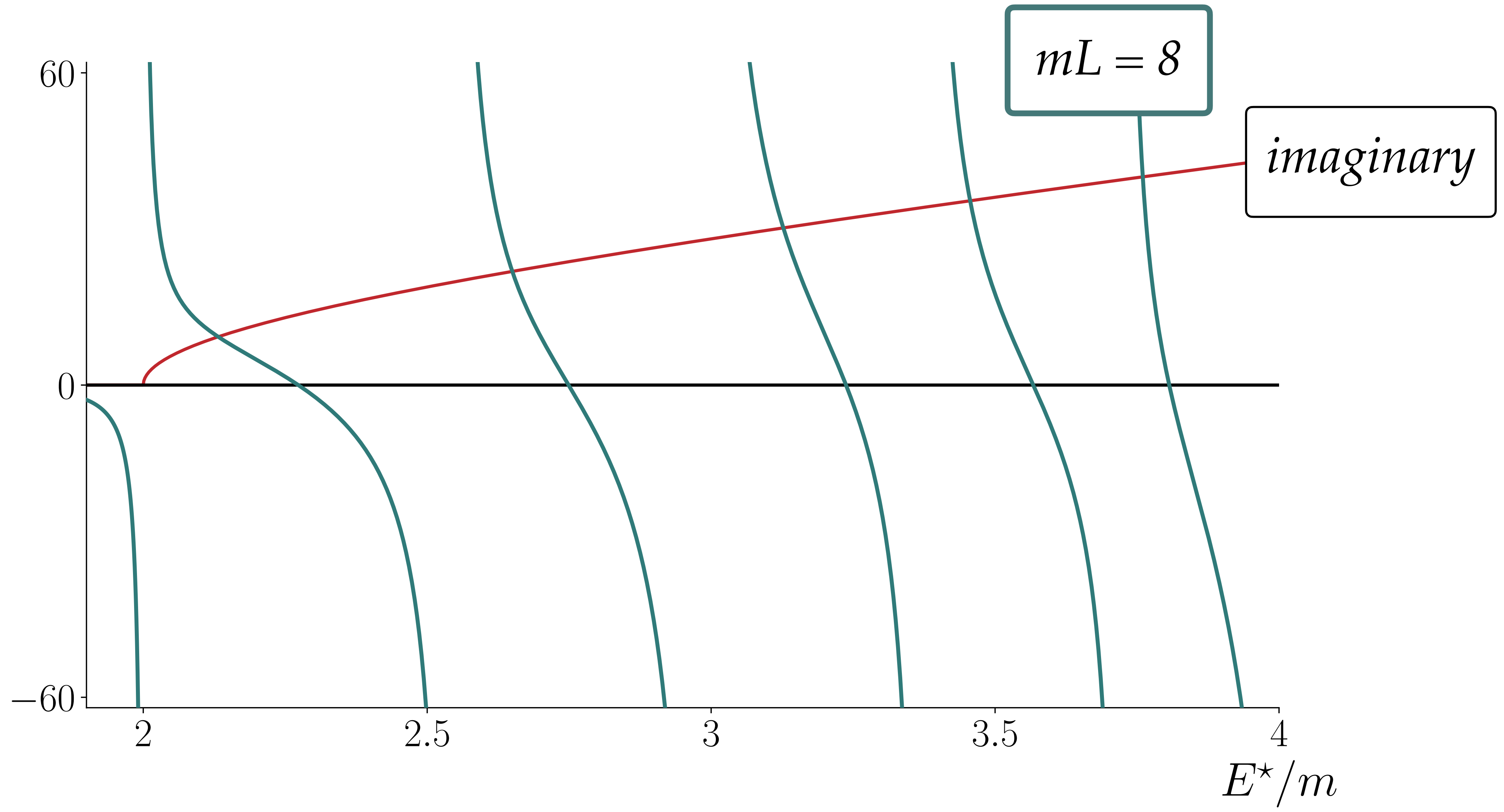
$$\det[F^{-1}+\mathcal{M}]=0$$

poles satisfy…

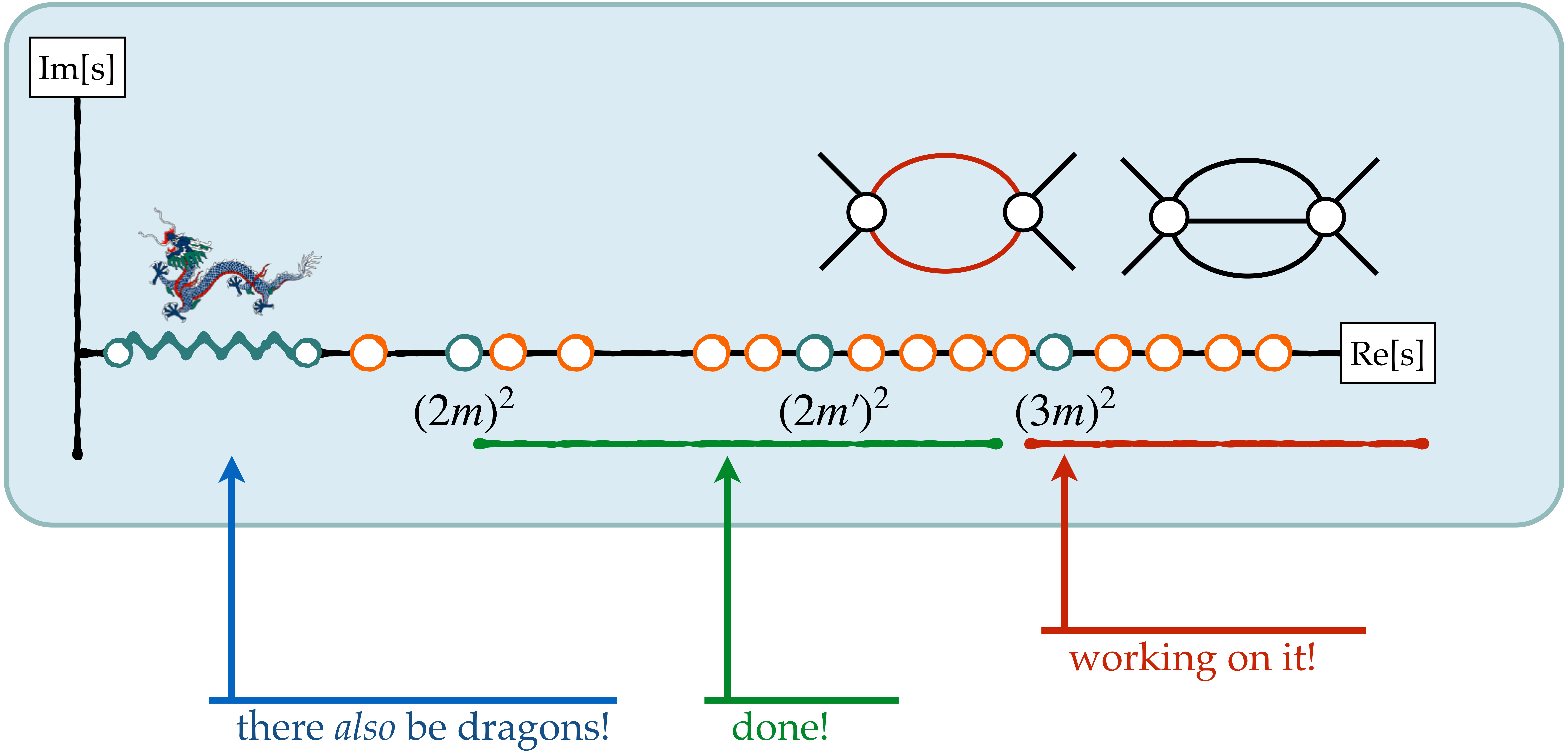
Some comments

$$\det[F^{-1}(P,L) + \mathcal{M}(P^2)] = 0$$

- ☑ exact up to $\mathcal{O}(e^{-m_\pi L})$,
- ☑ Mapping, not an extrapolationg,
- ☑ Not one-to-one [no asymptotic states & angular momentum is not a good quantum number],
- ☑ For moderate energies, low partial waves saturate the amplitude,
- ☑ We know F arbitrary boost, so we can further constraint the amplitude by considered boosted systems.

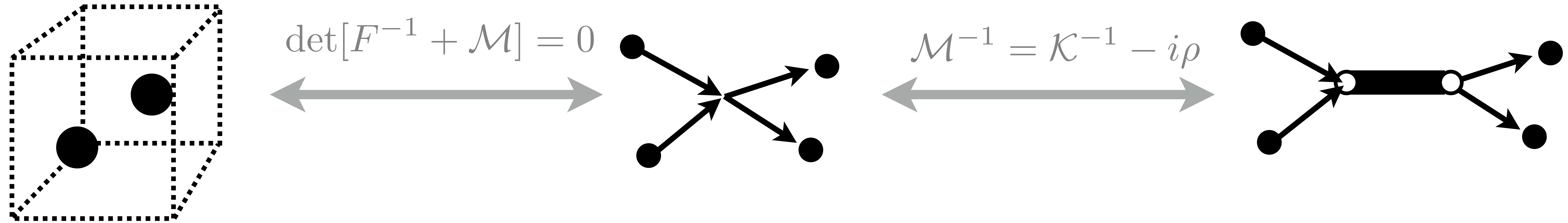


Going to higher energies

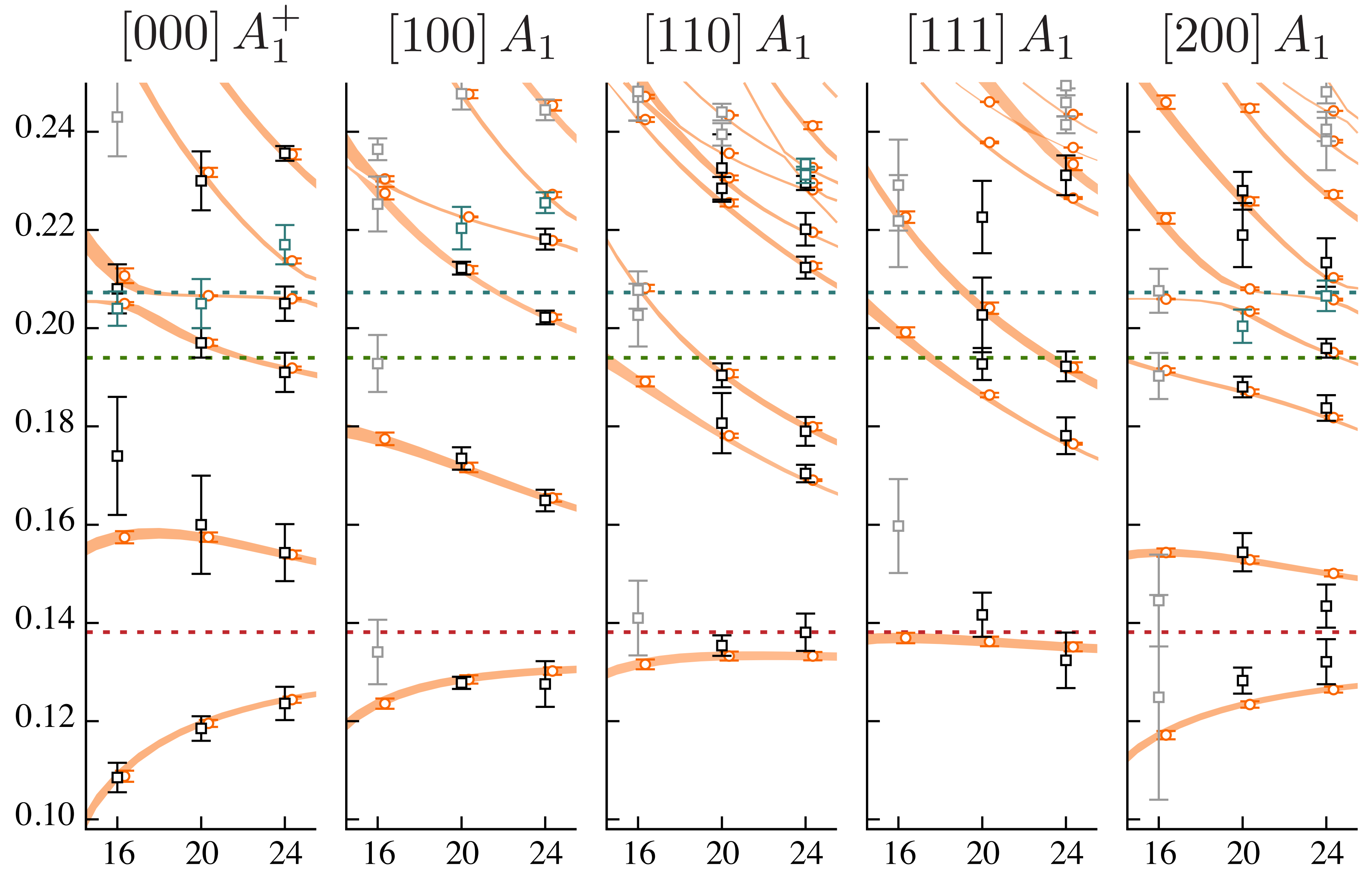


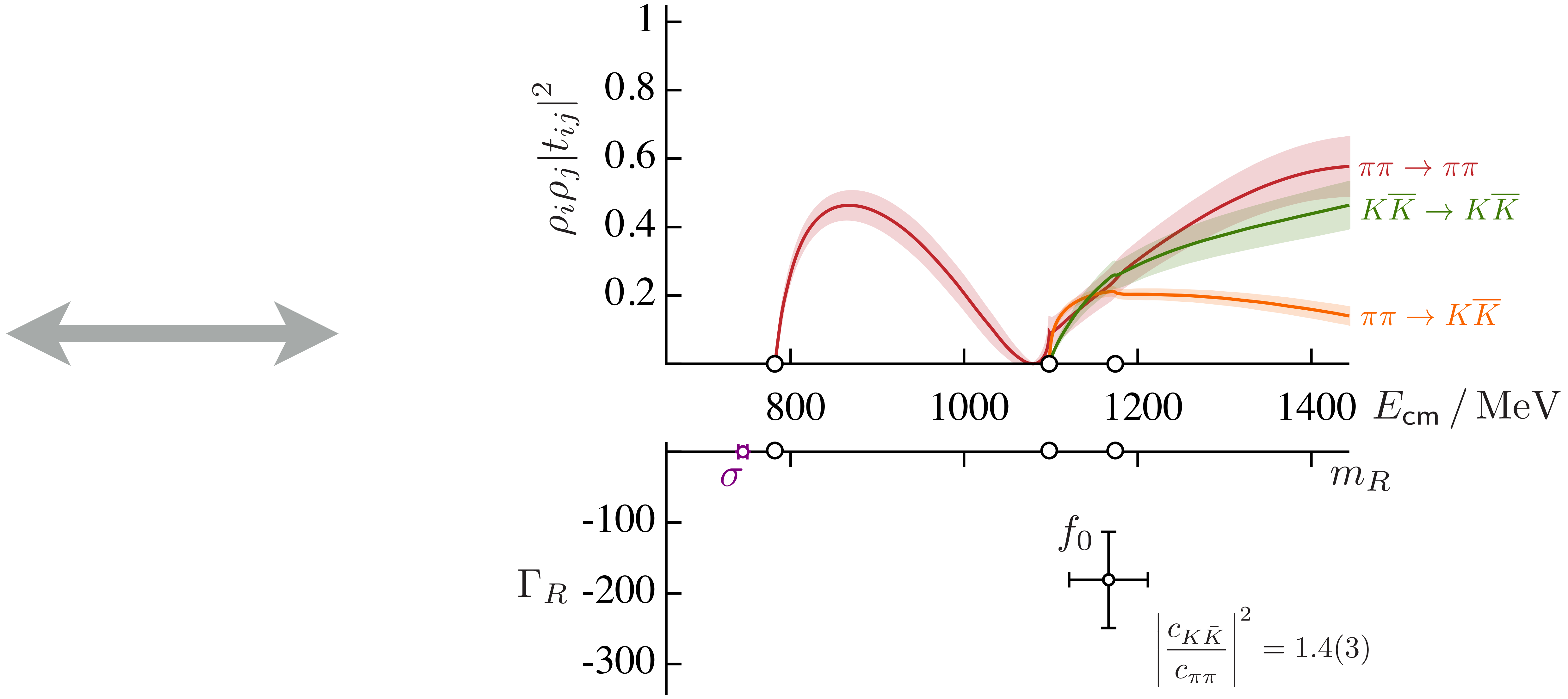
Outline

☑ Formalism



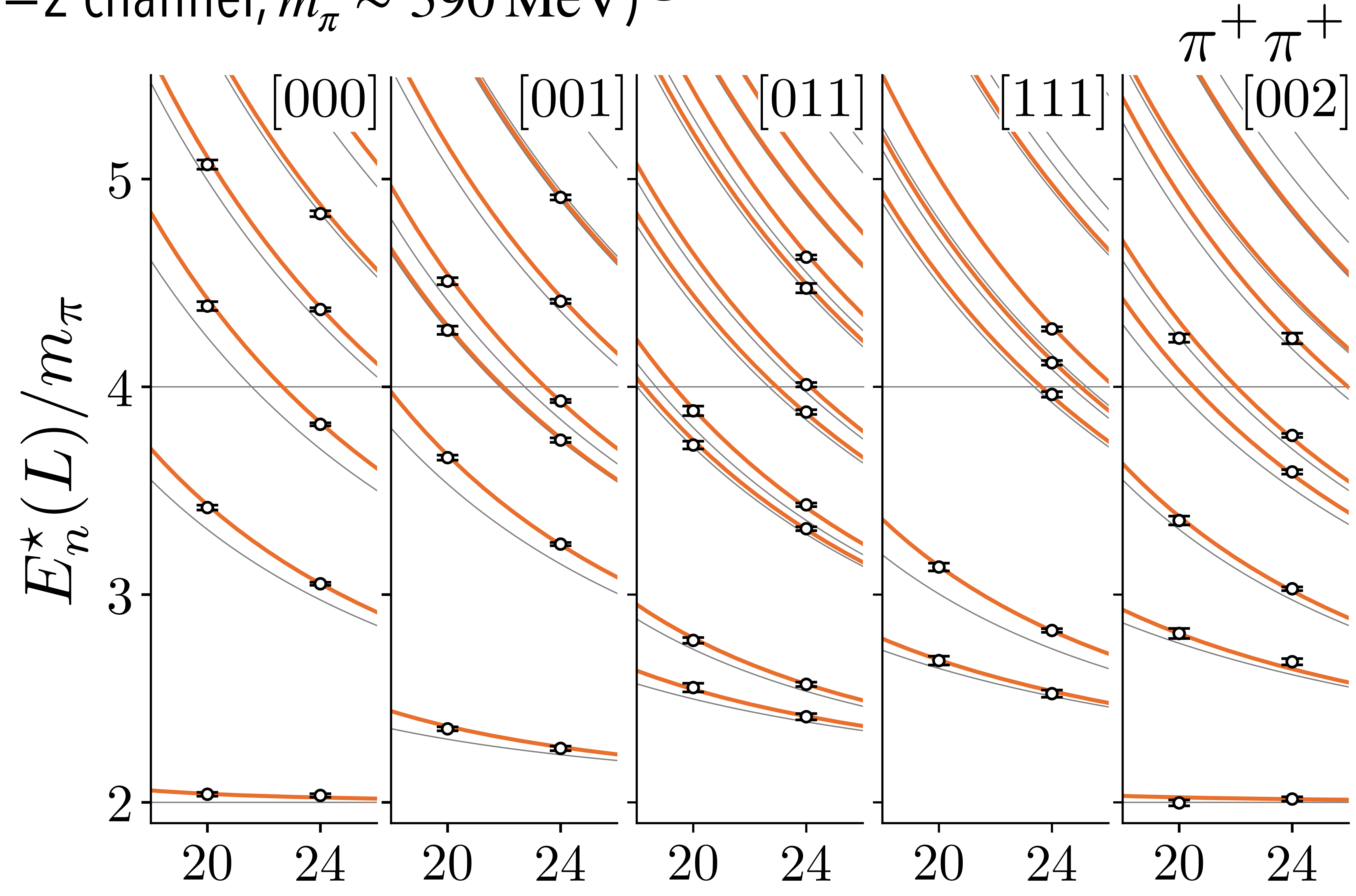
☐ Lattice QCD calculations





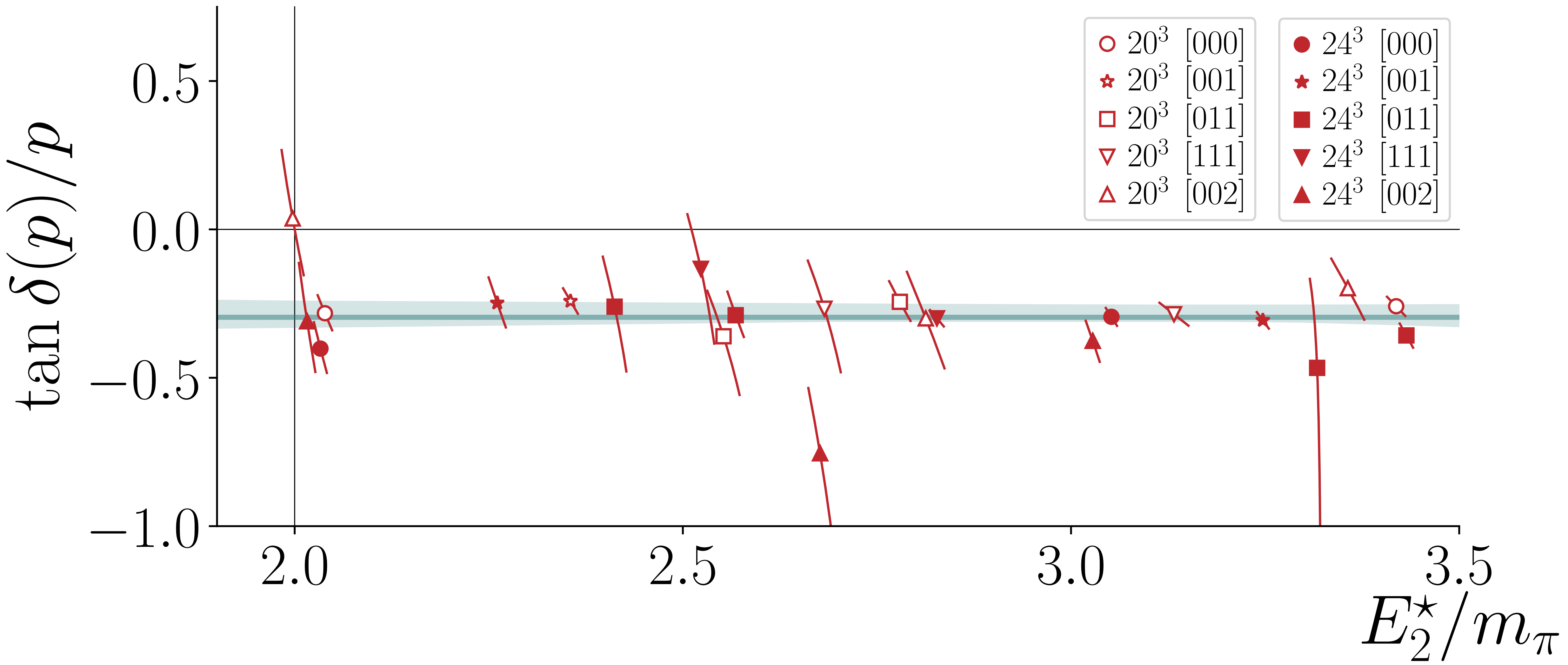
$\pi\pi$ scattering

(I=2 channel, $m_\pi \sim 390\,\mathrm{MeV}$)



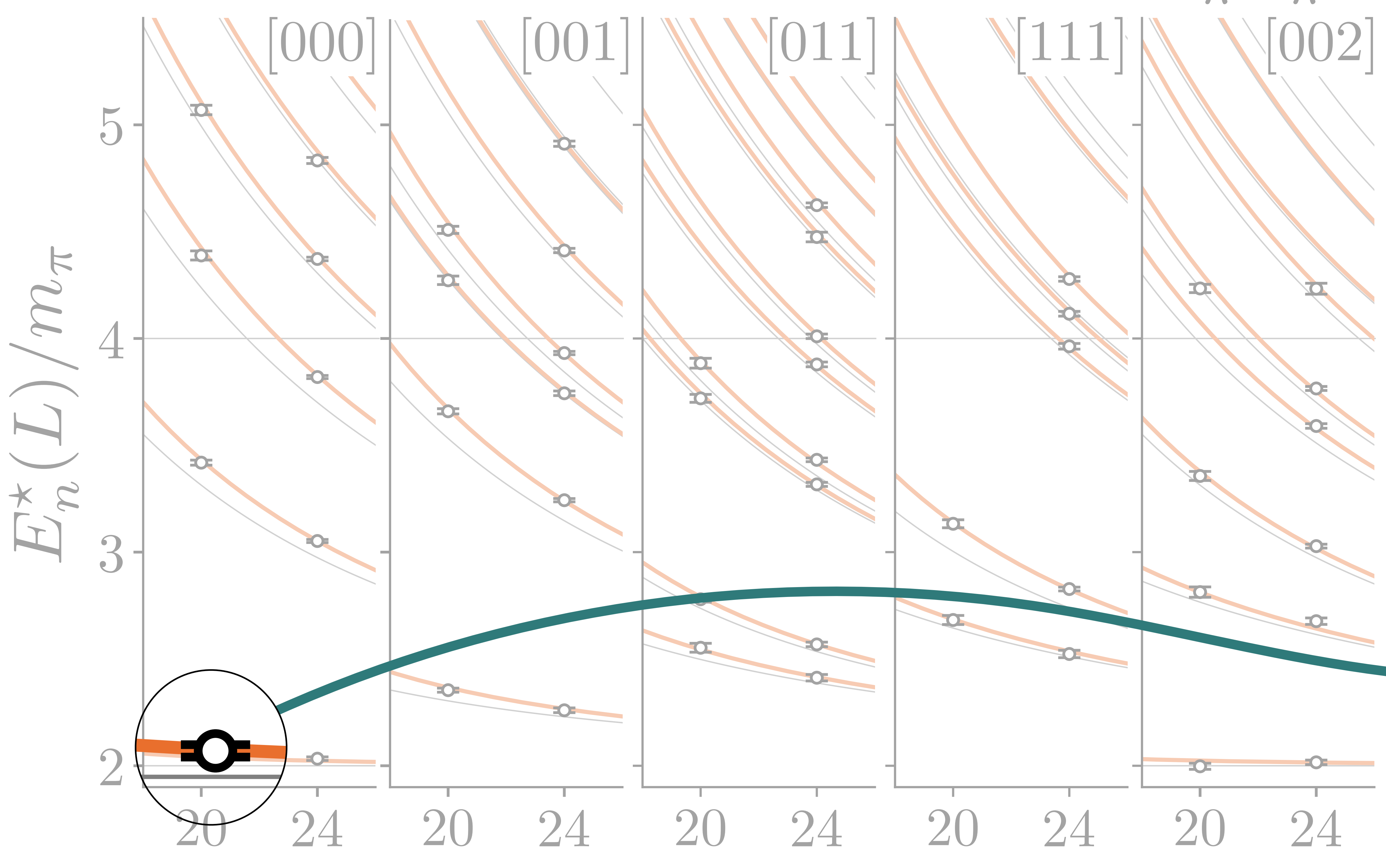
$$\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$$

$$\mathcal{M} \sim \frac{1}{p\cot\delta - ip}$$



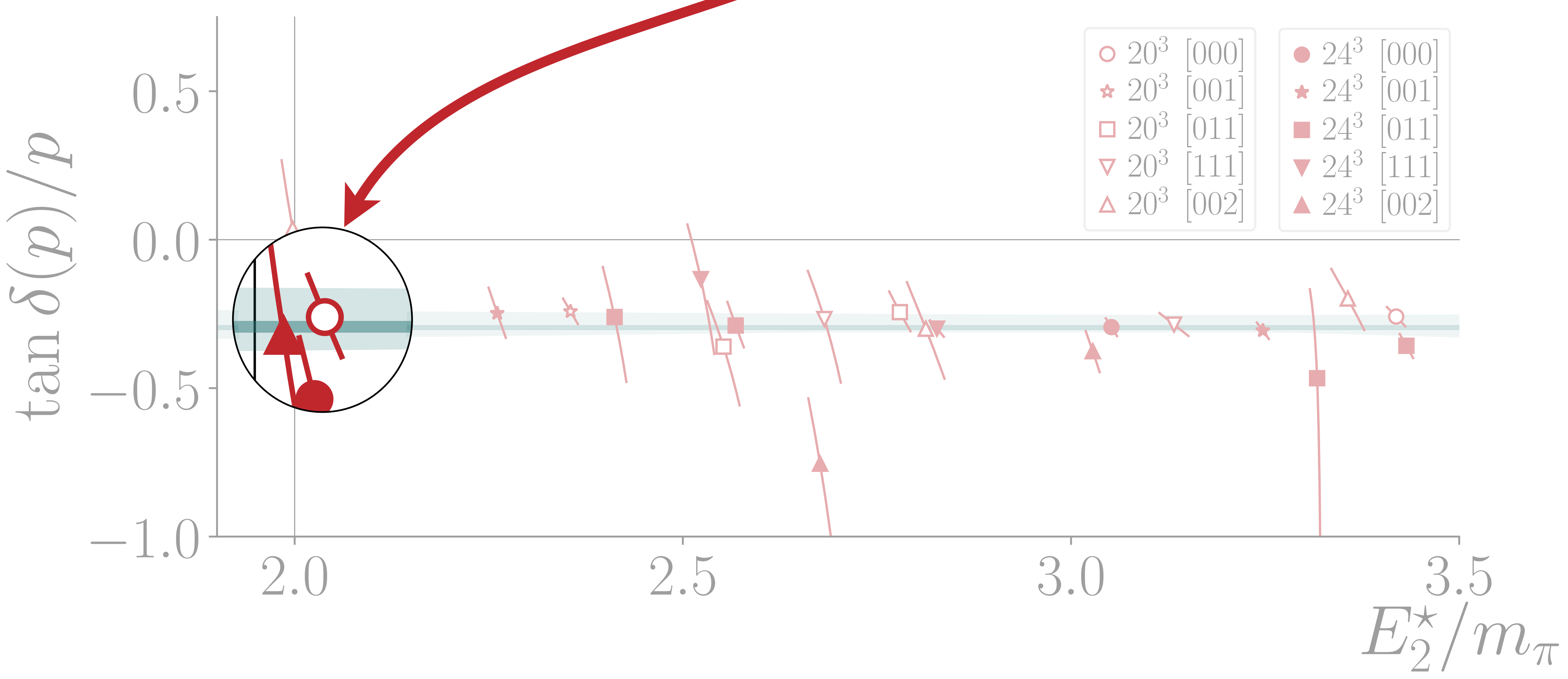
Hansen, RB, Edwards, Thomas, & Wilson (2020)

$\pi\pi$ scattering
(I=2 channel, $m_\pi \sim 390\,\mathrm{MeV}$)



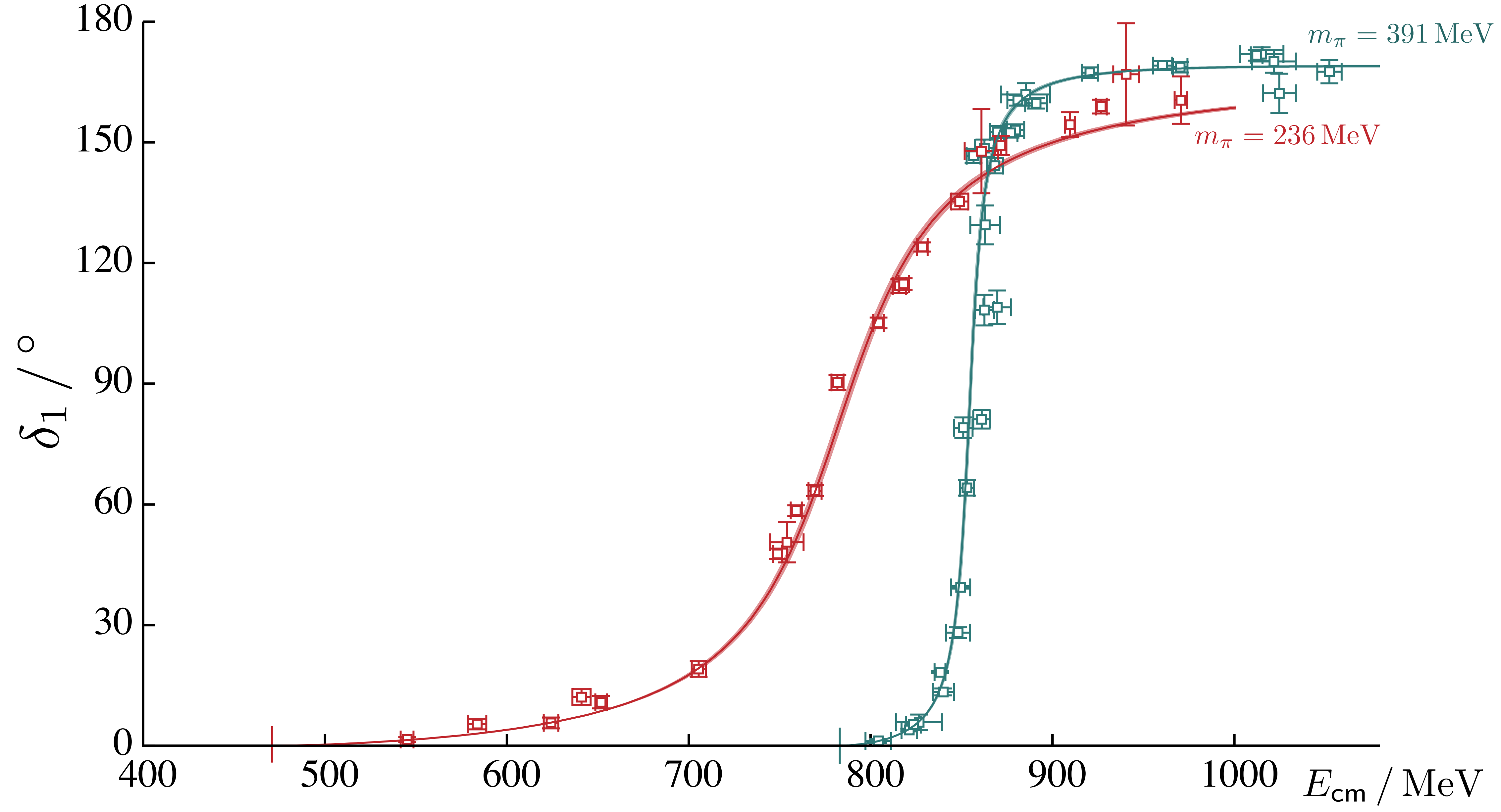
$$\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$$

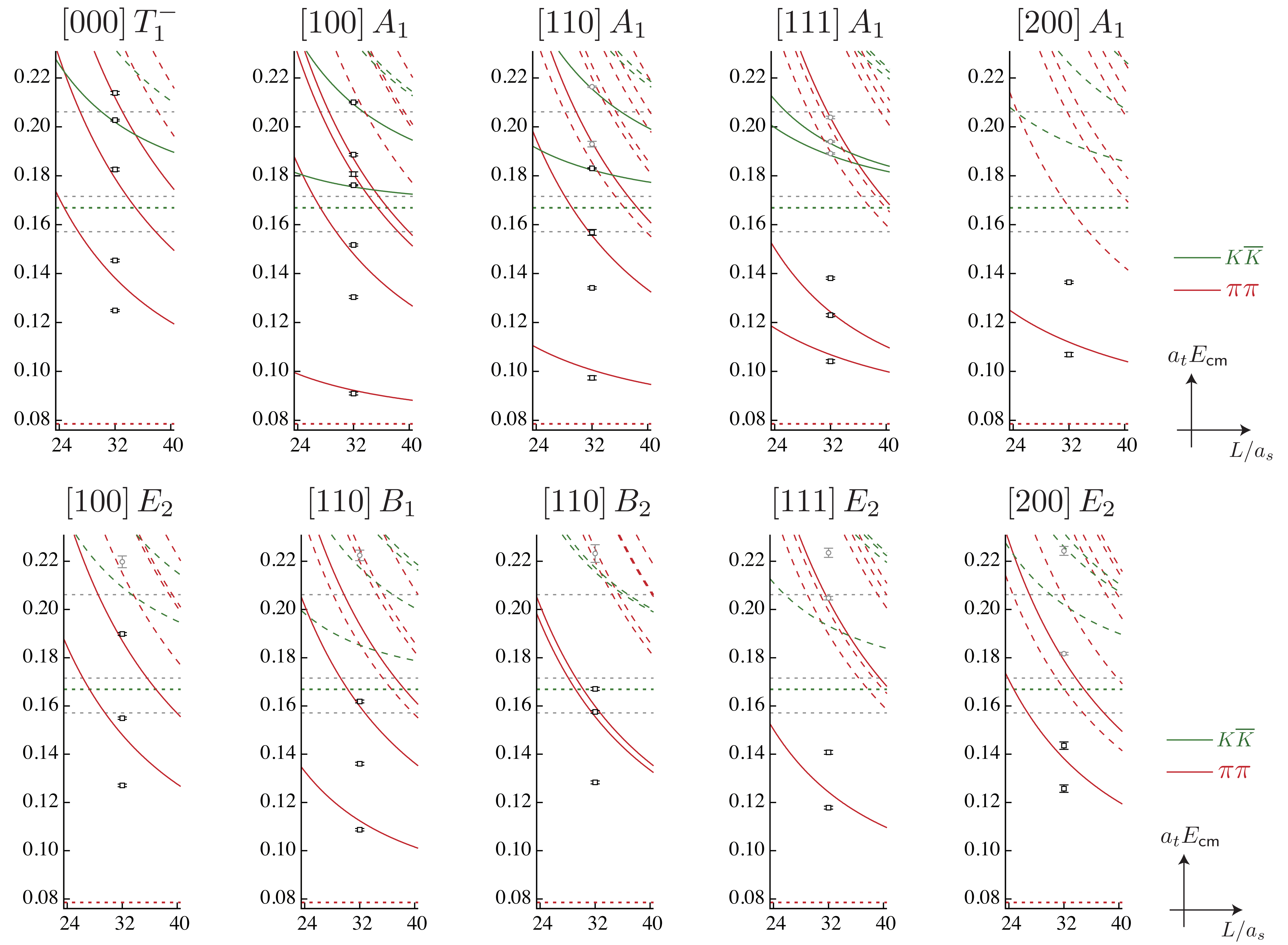
$$\mathcal{M} \sim \frac{1}{p\cot\delta - ip}$$



Hansen, RB, Edwards, Thomas, & Wilson (2020)

$\pi\pi$ scattering
(I=1 channel)



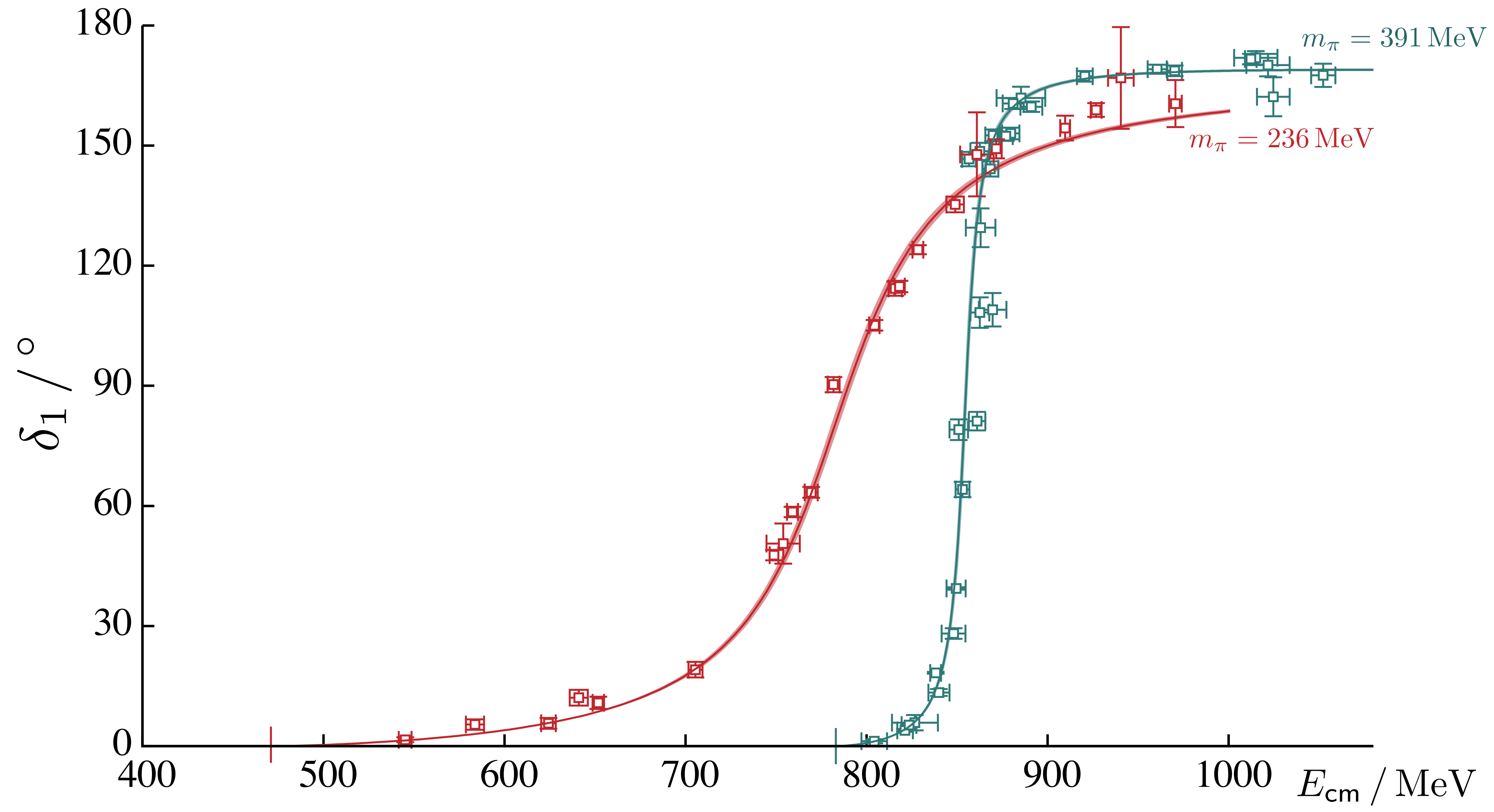


$$\mathcal{M} \sim \frac{1}{p \cot \delta - ip}$$

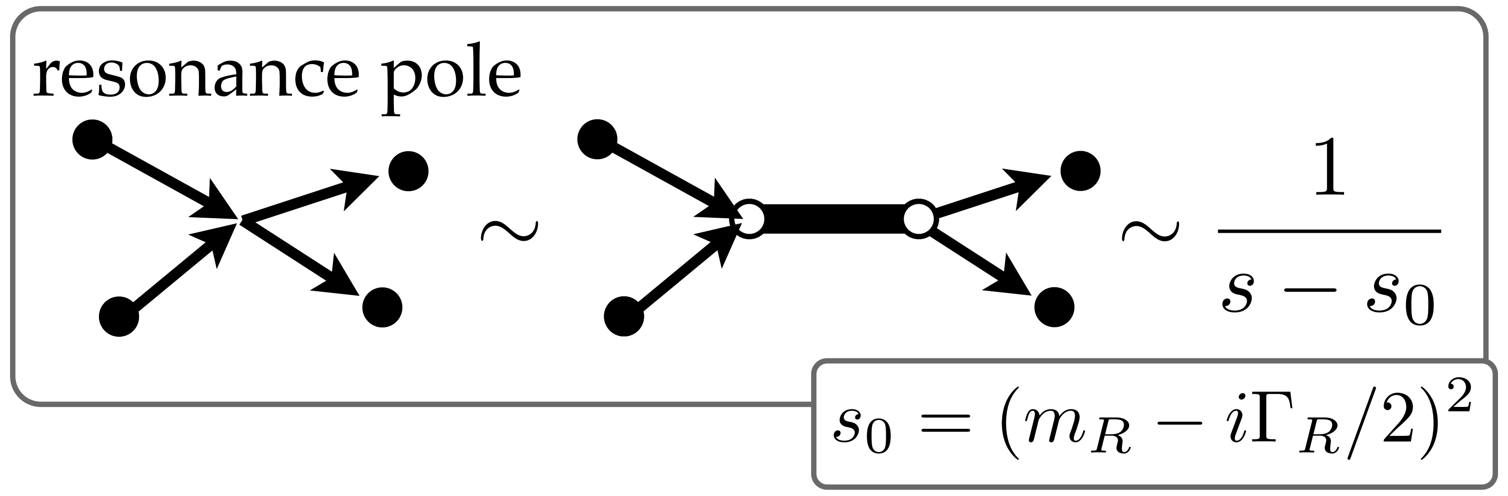
Dudek, Edwards, & Thomas (2012)

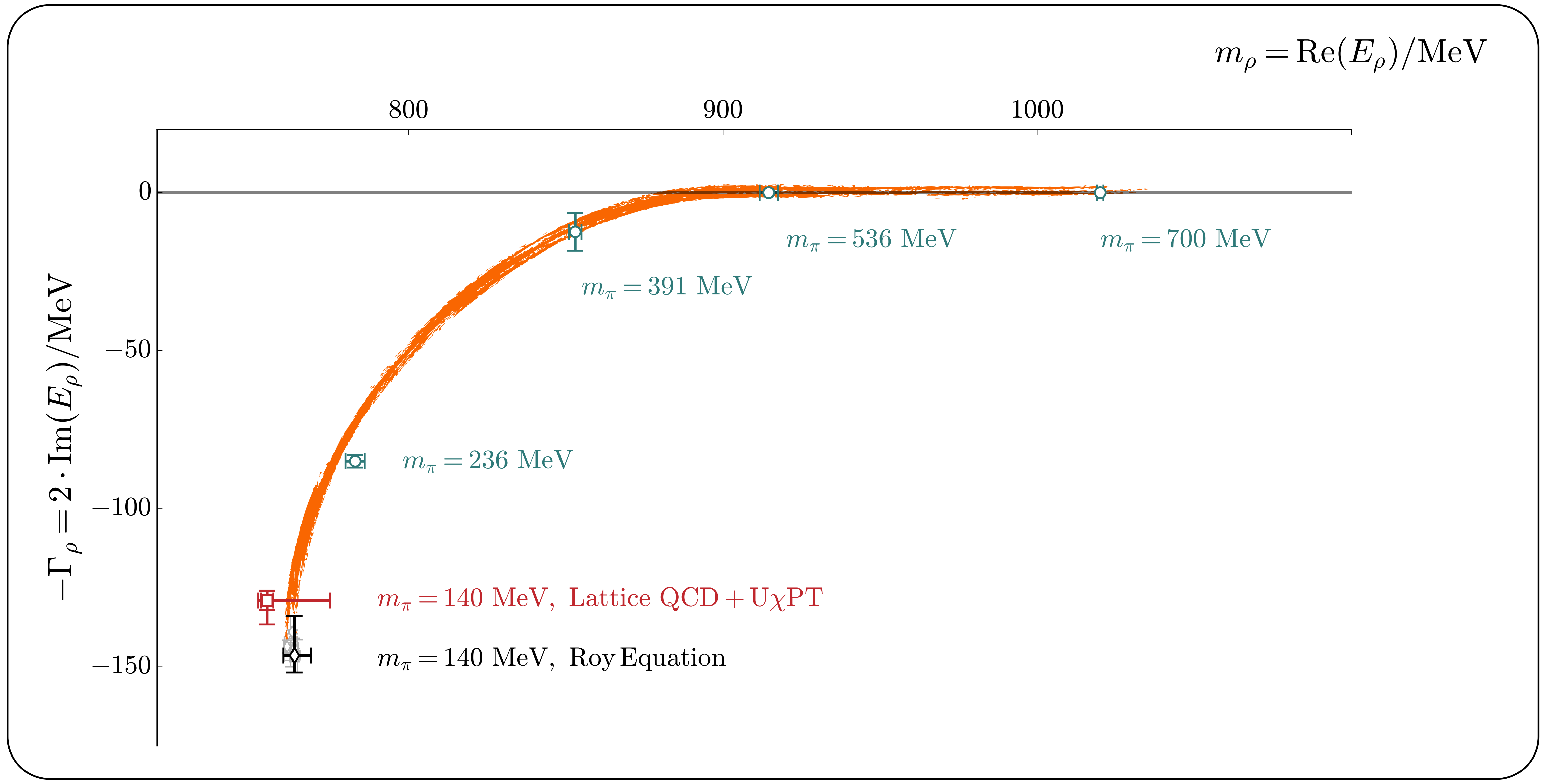
Wilson, RB, Dudek, Edwards, & Thomas (2015)

$\pi\pi$ scattering
(I=1 channel)



$$\mathcal{M} \sim \frac{1}{p \cot\delta - ip}$$

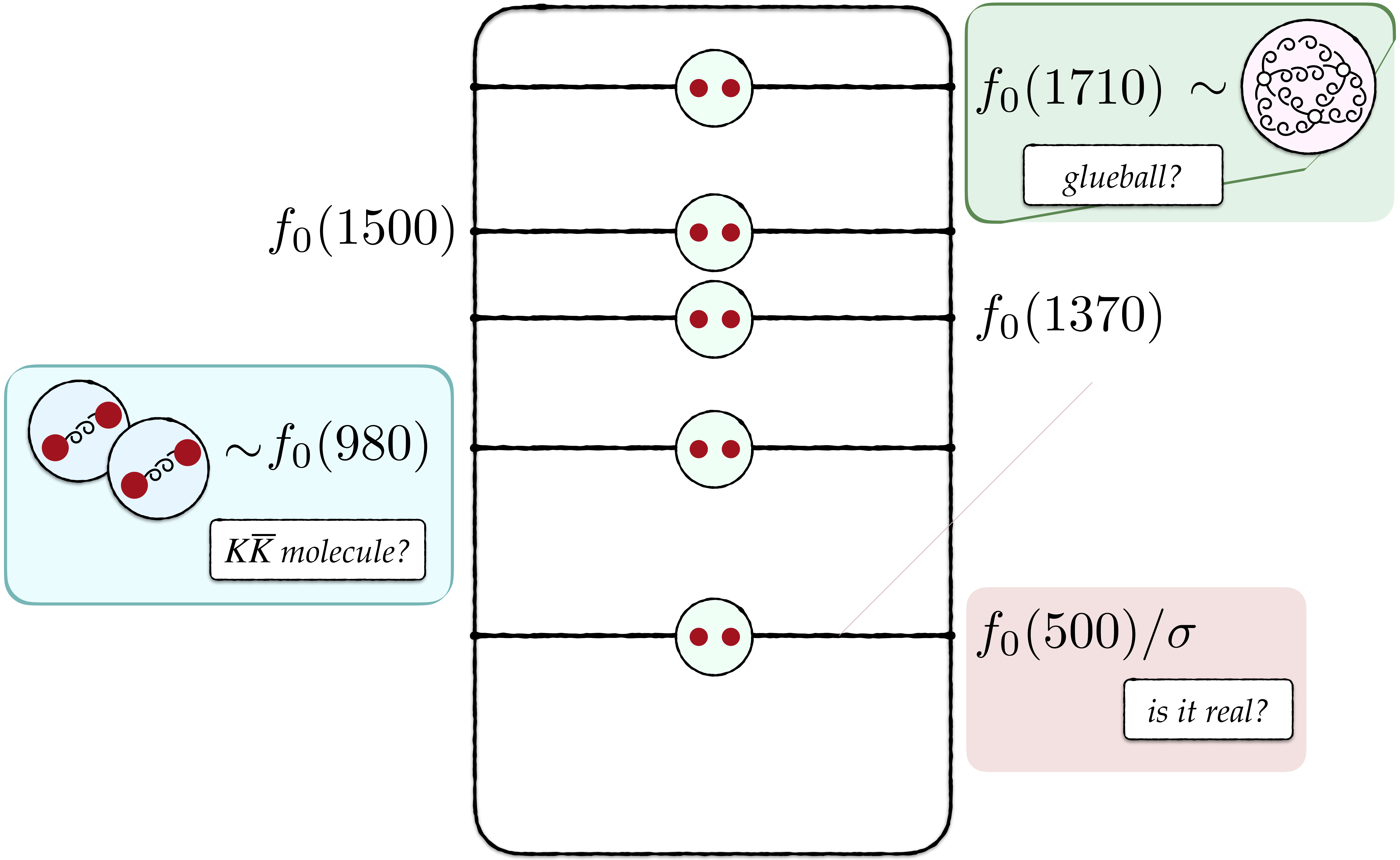




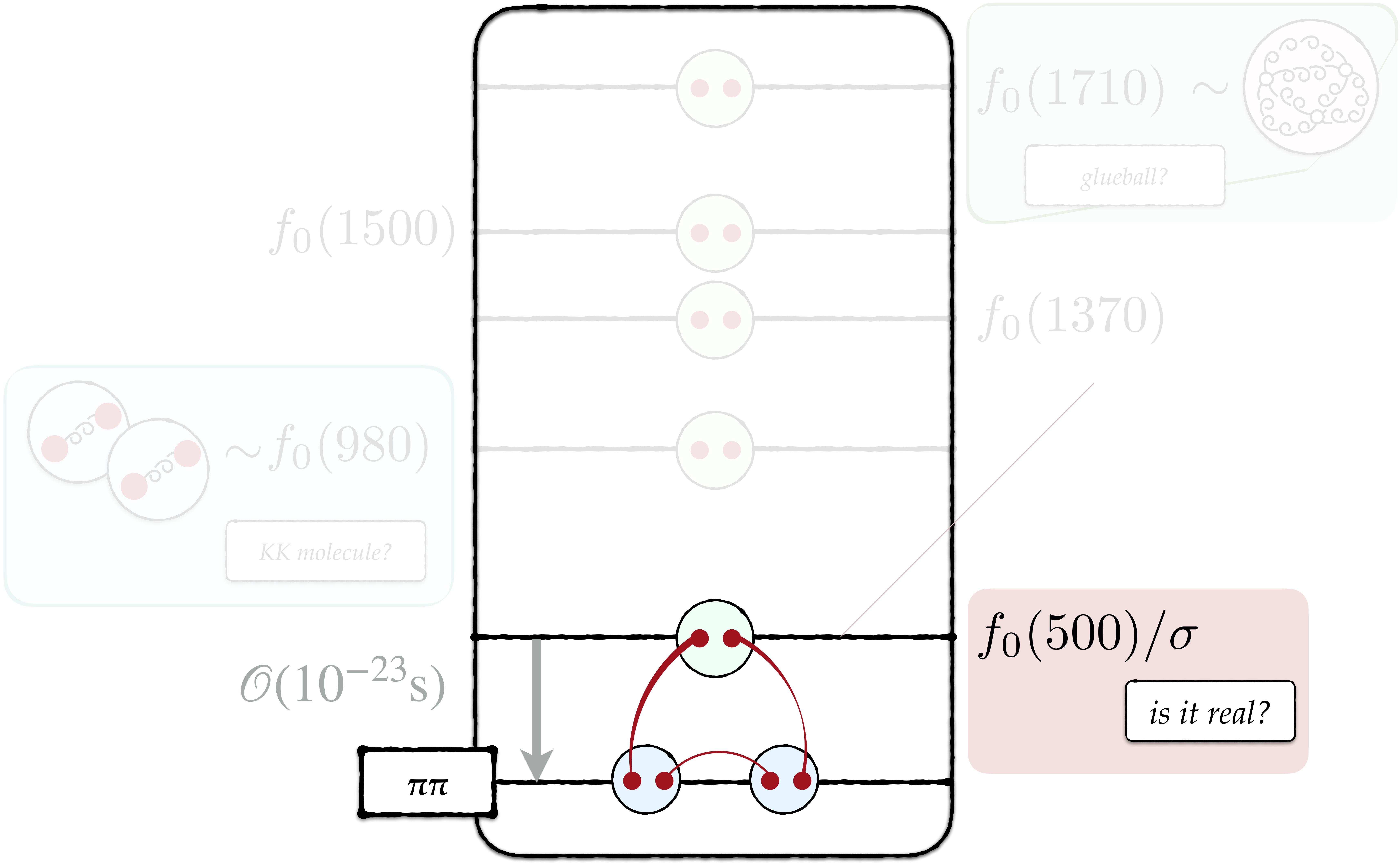
Dudek, Edwards, & Thomas (2012)

Wilson, RB, Dudek, Edwards, & Thomas (2015)

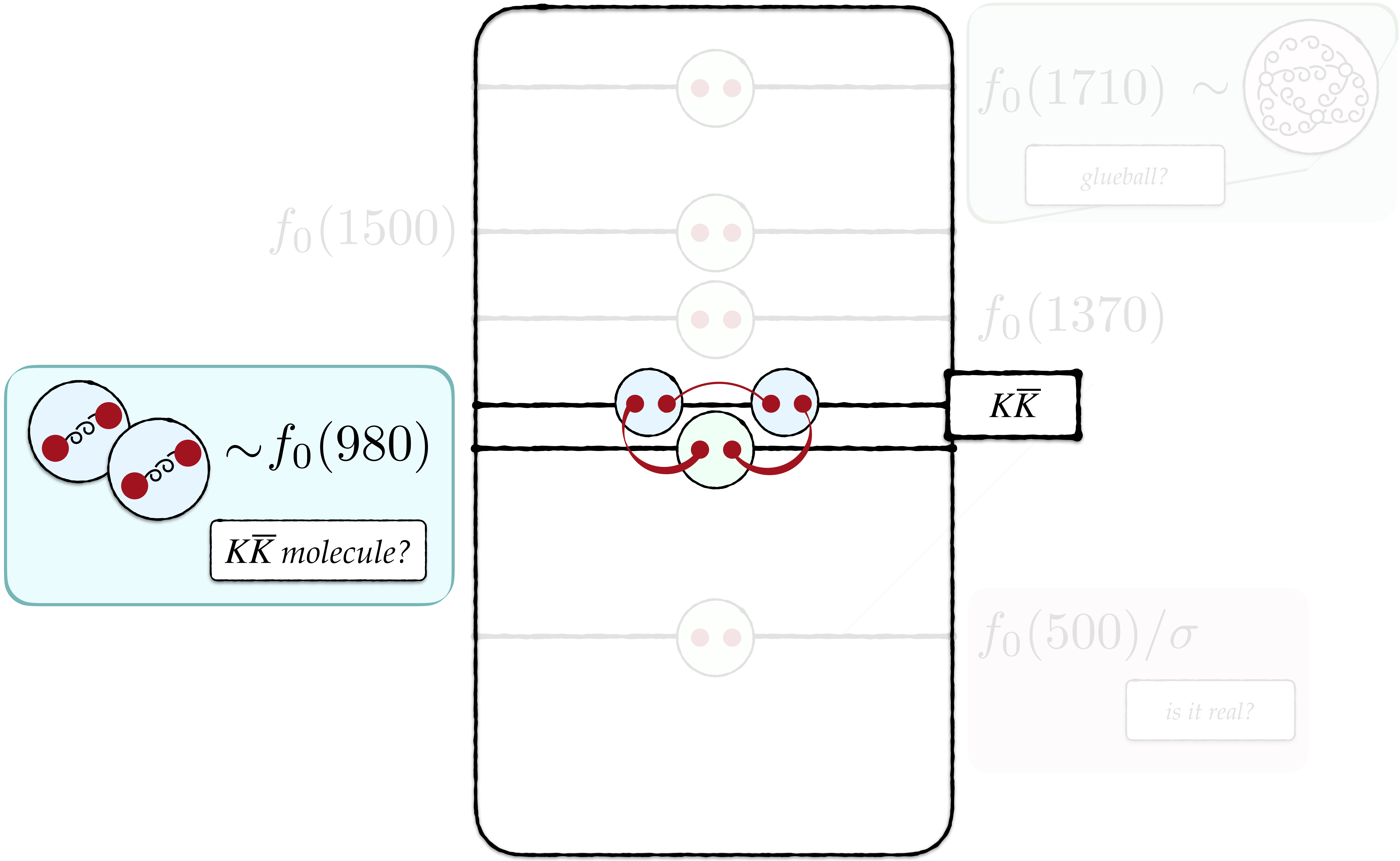
The vacuum channel



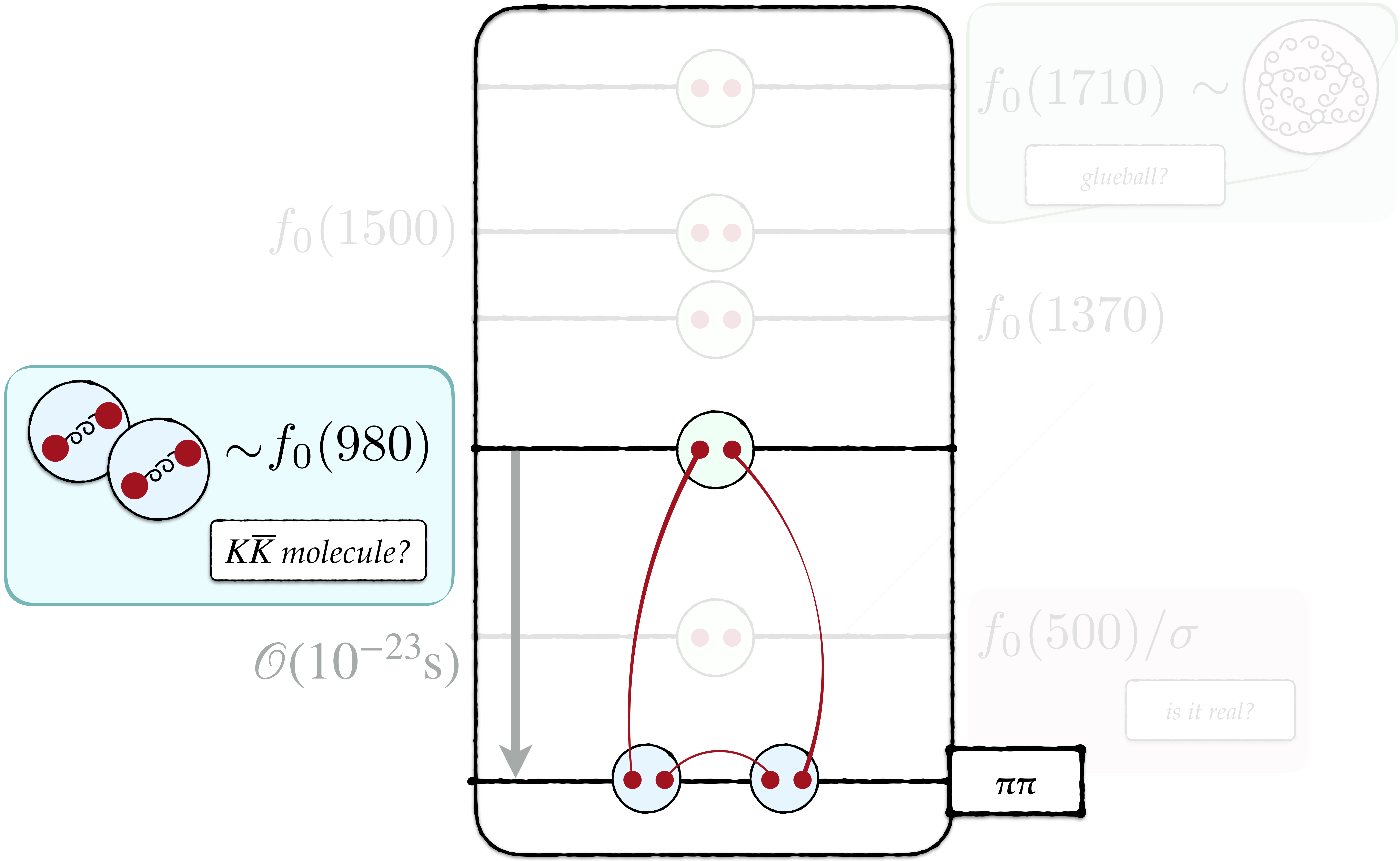
The vacuum channel



The vacuum channel

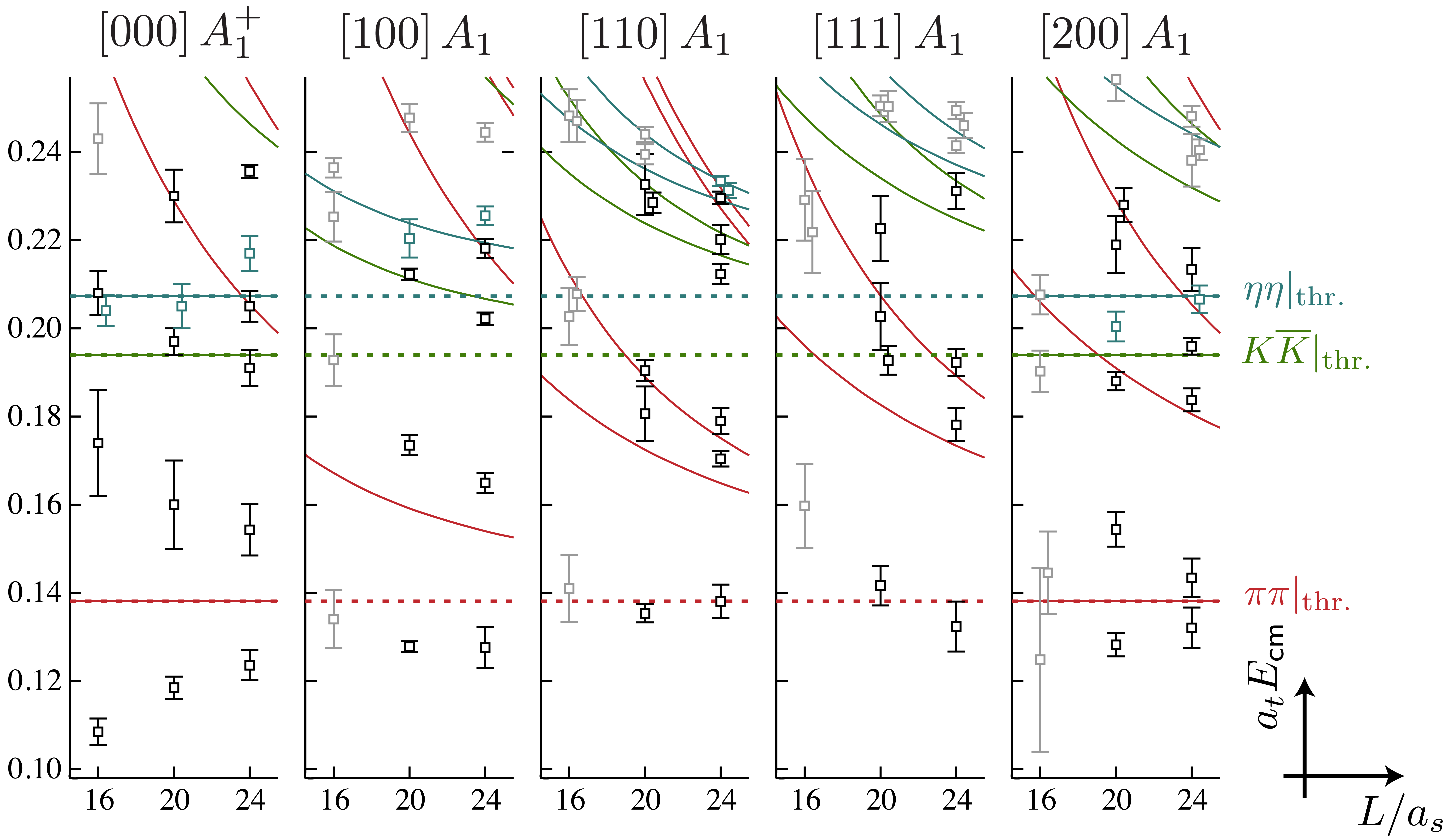


The vacuum channel



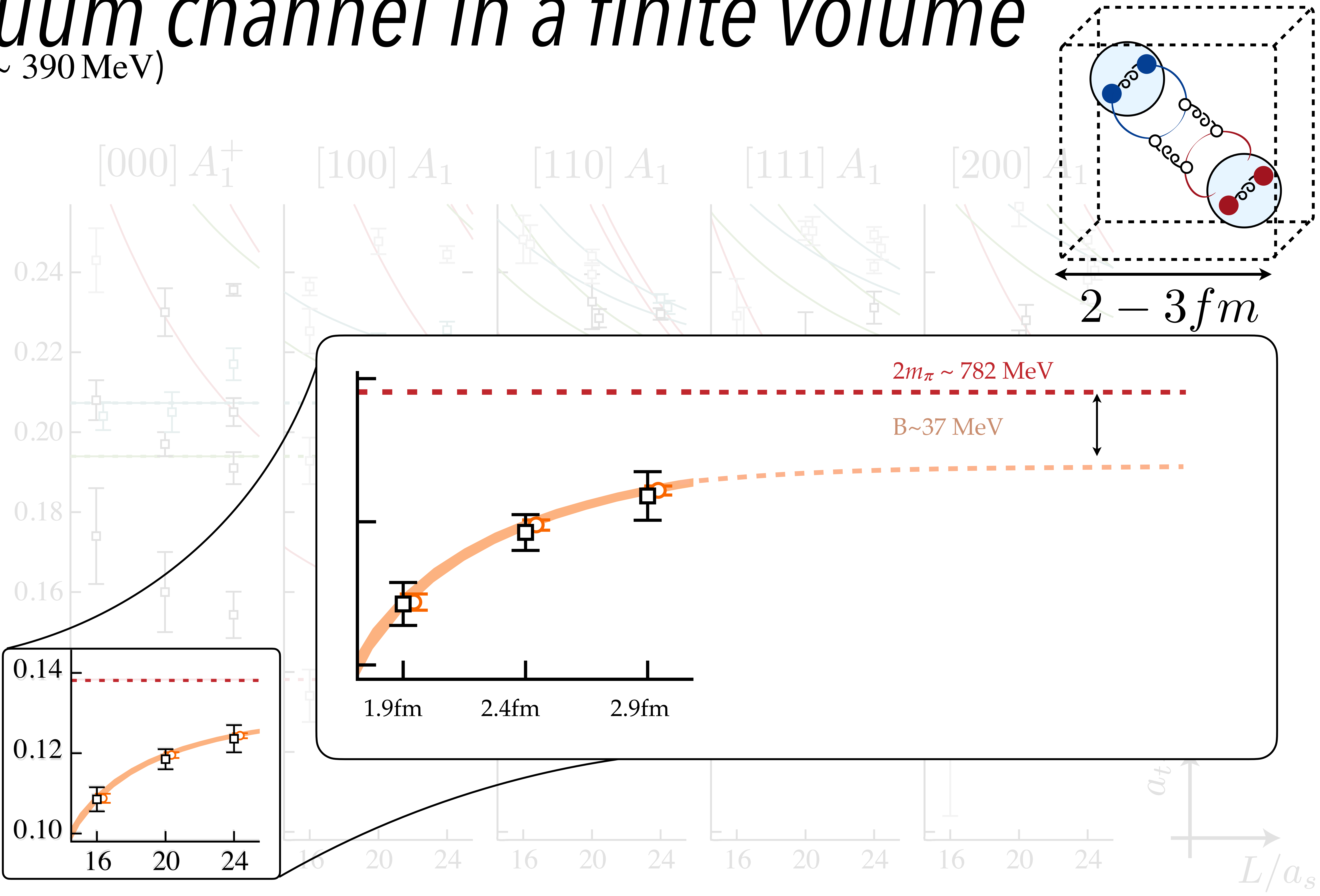
The vacuum channel in a finite volume

(I=0 channel, $m_\pi \sim 390\,\mathrm{MeV}$)



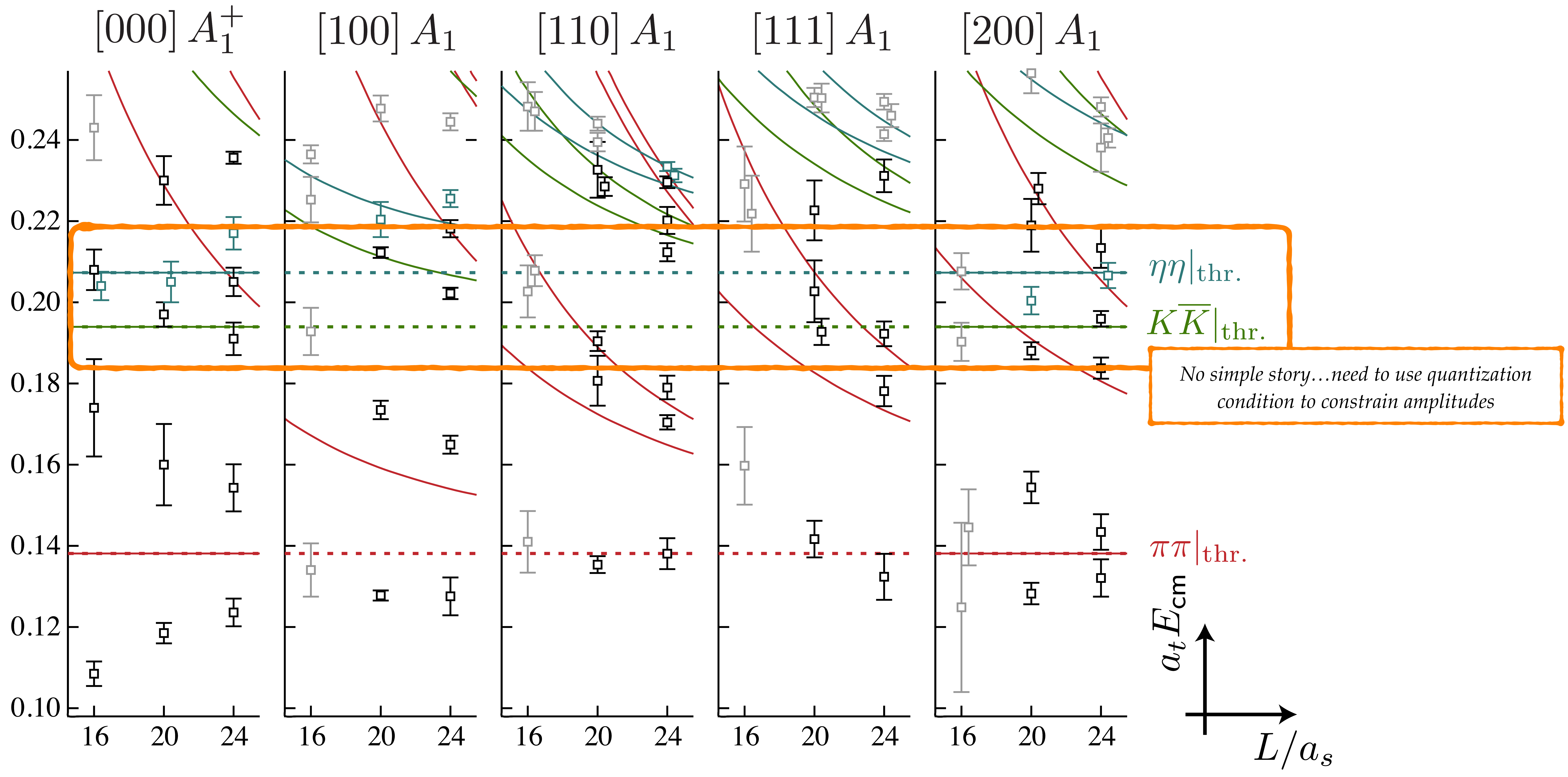
The vacuum channel in a finite volume

(I=0 channel, $m_\pi \sim 390\,\mathrm{MeV}$)



The vacuum channel in a finite volume

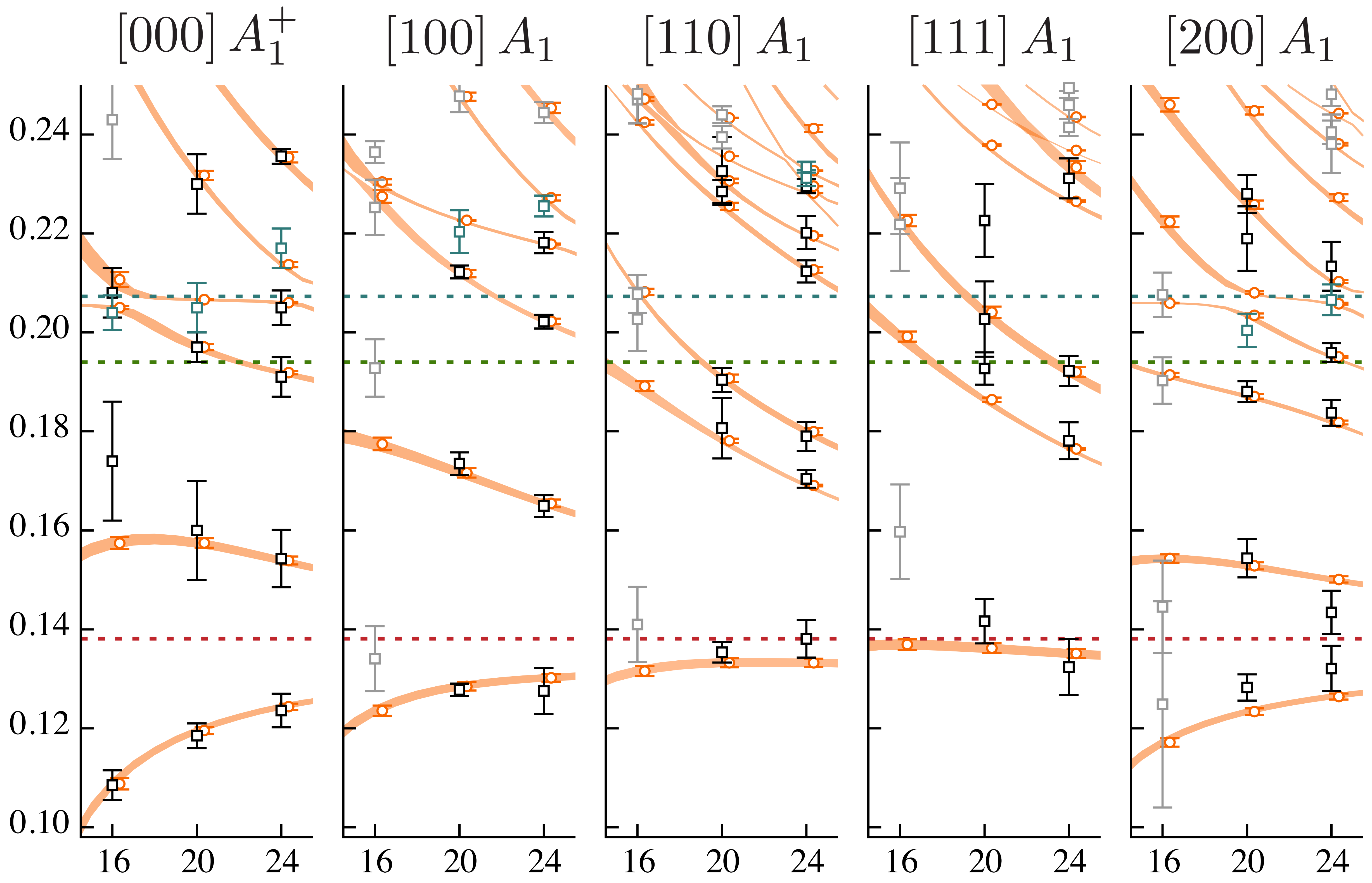
(I=0 channel, $m_\pi \sim 390\,\mathrm{MeV}$)

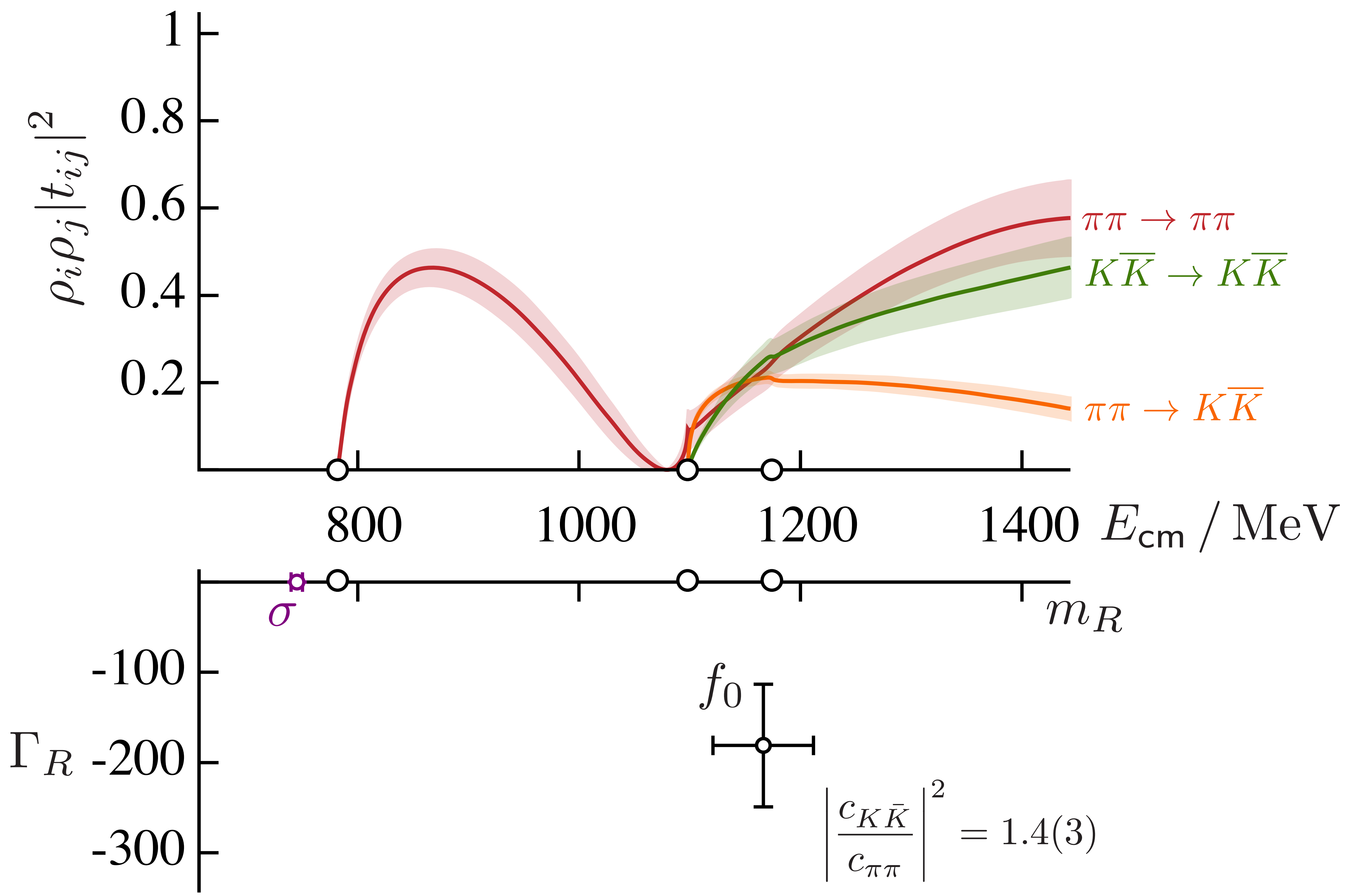


Coupled $\pi\pi$, $K\overline{K}$ and the f_0's

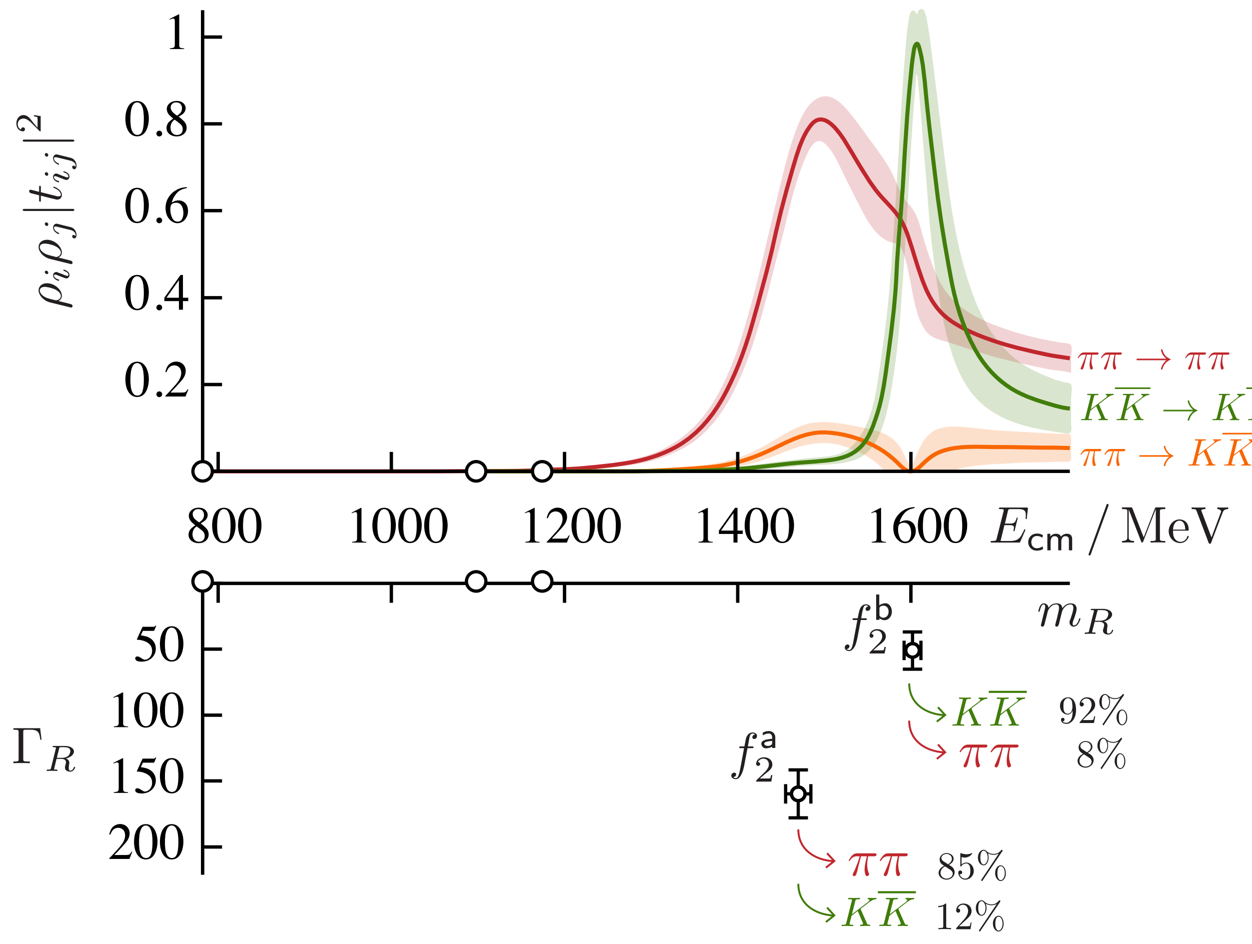
- ☑ Above $K\overline{K}$-threshold, spectrum satisfies:
- ☑ No one-to-one correspondence,
- ☑ Parameterize amplitude and perform global fit.

$$\det \begin{bmatrix} F_{\pi\pi}^{-1} + \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\overline{K}} \\ \mathcal{M}_{\pi\pi,K\overline{K}} & F_{K\overline{K}}^{-1} + \mathcal{M}_{K\overline{K},K\overline{K}} \end{bmatrix} = 0$$

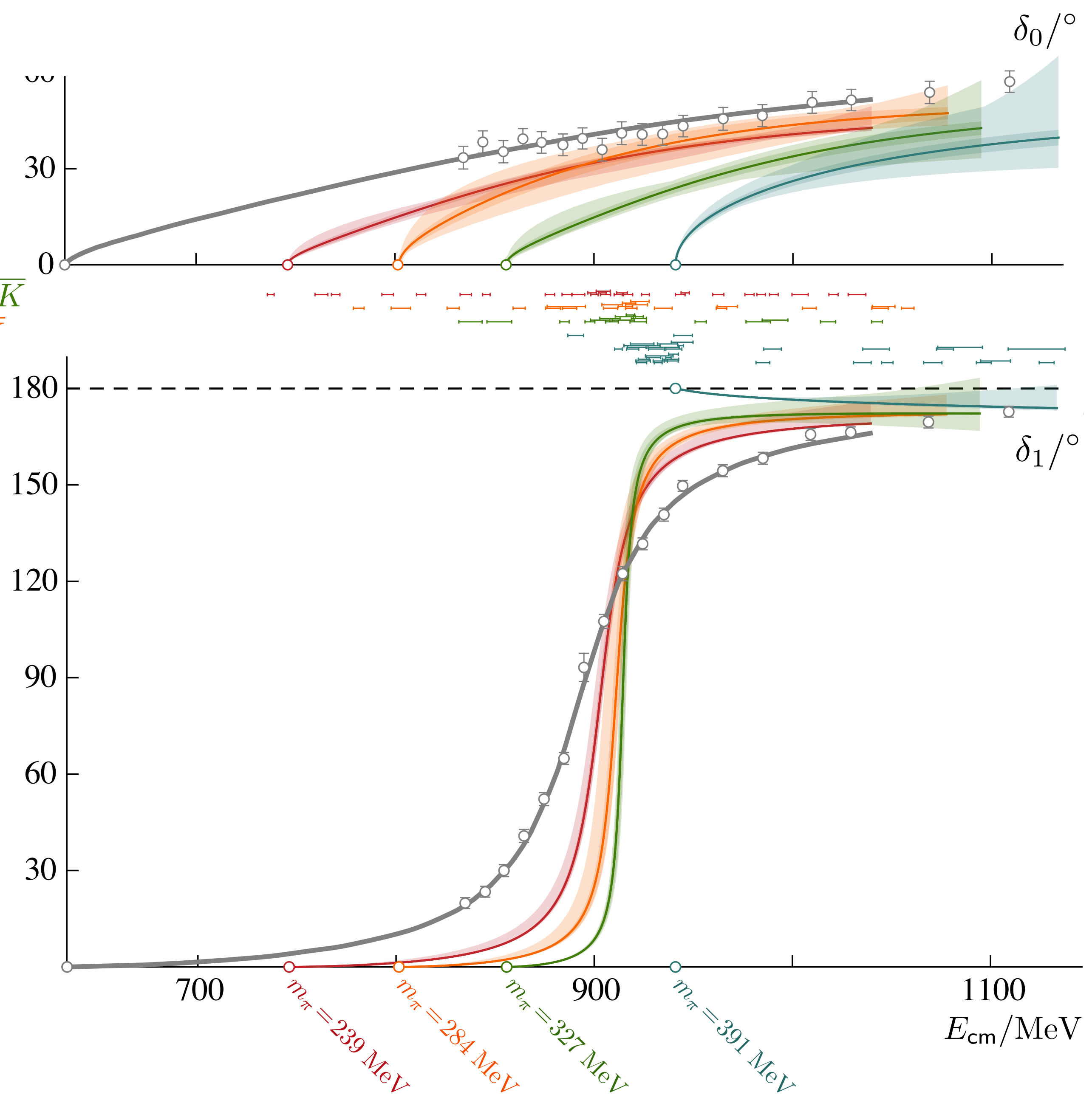




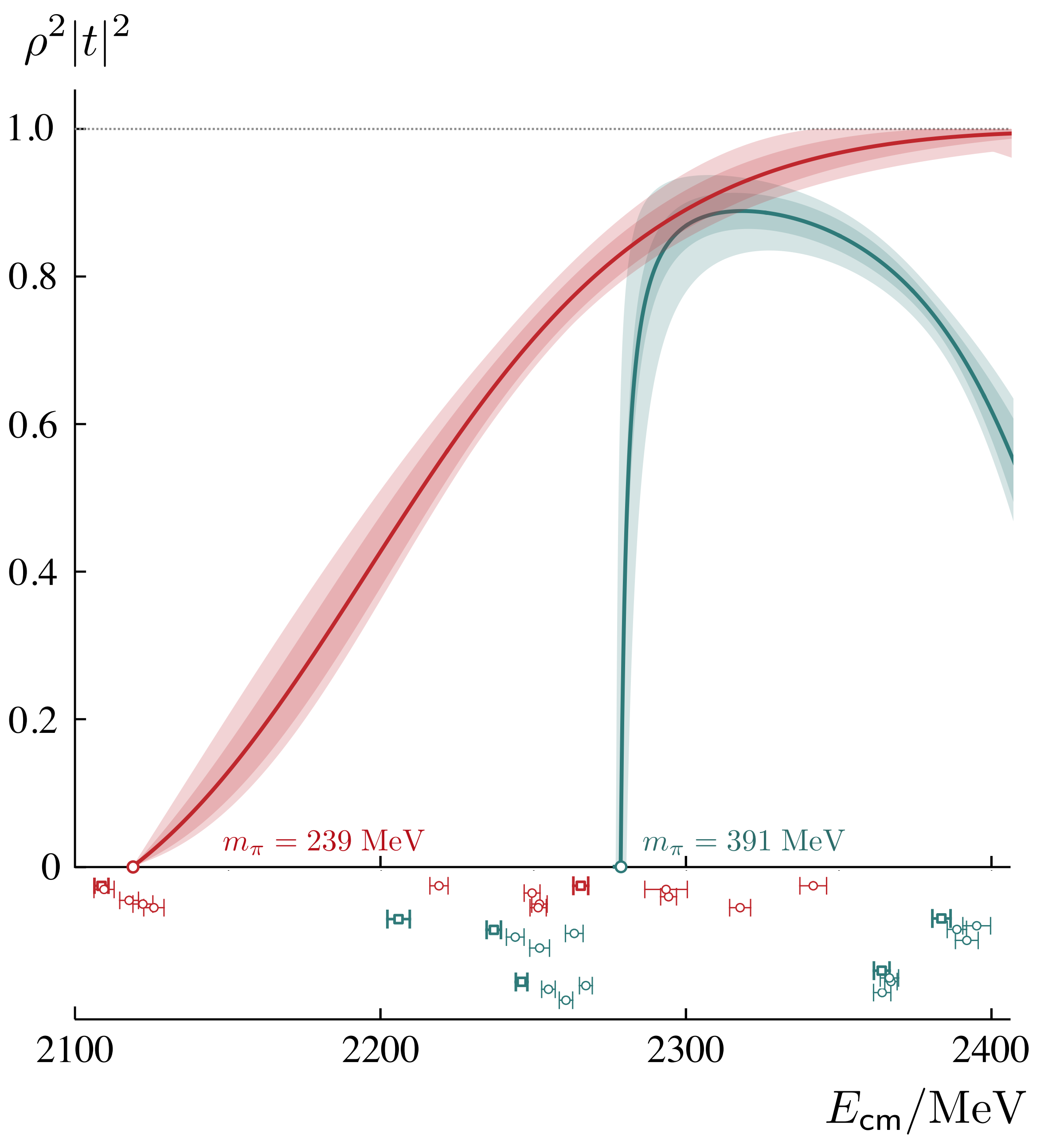
Many other calculations



D-wave $\pi\pi$-KK



$K\pi$ in the κ and $K^\star$

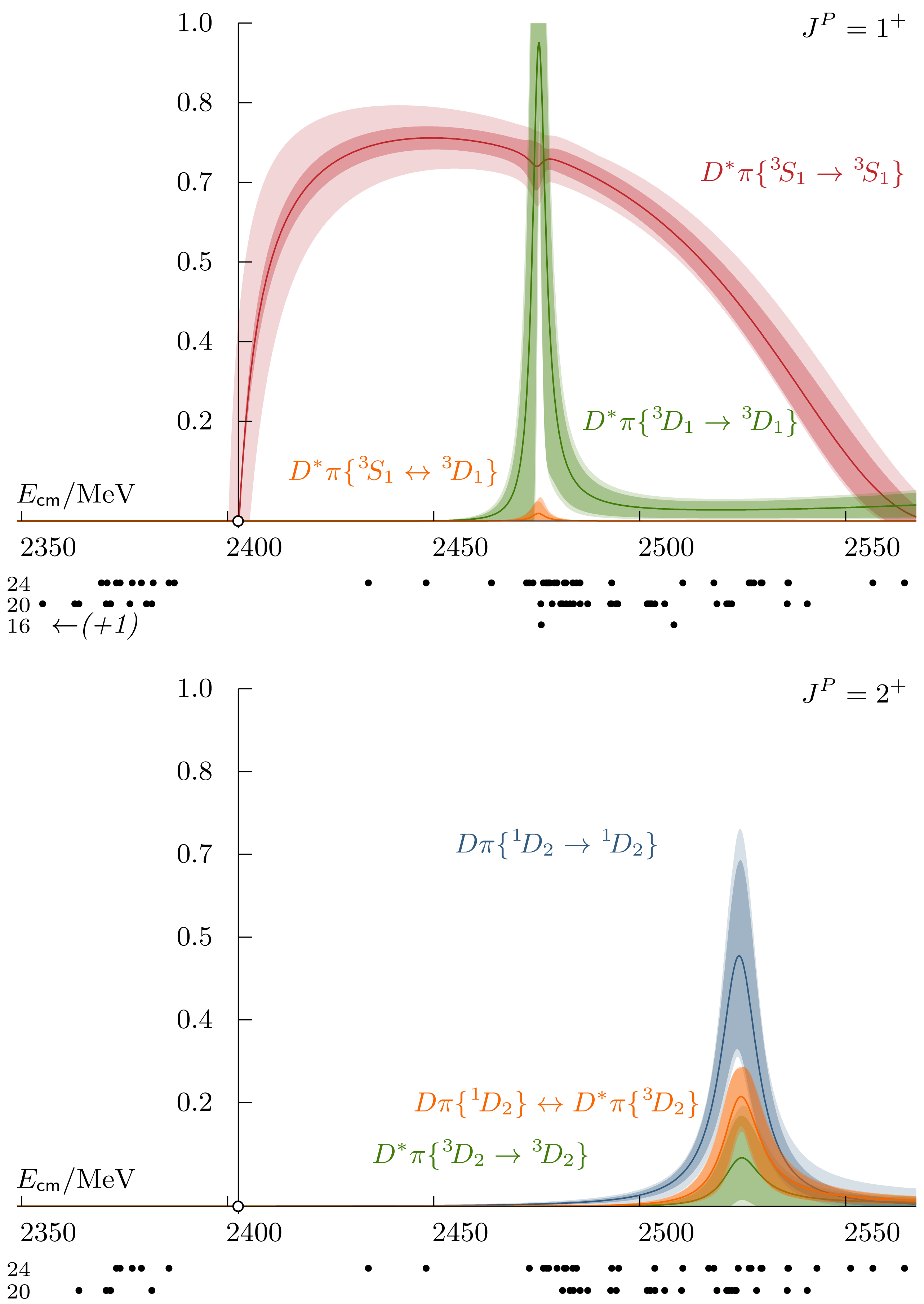


$D\pi$ and the $D_0^\star$

RB

RB

$D^\star\pi$ & the D_1 and the D_2

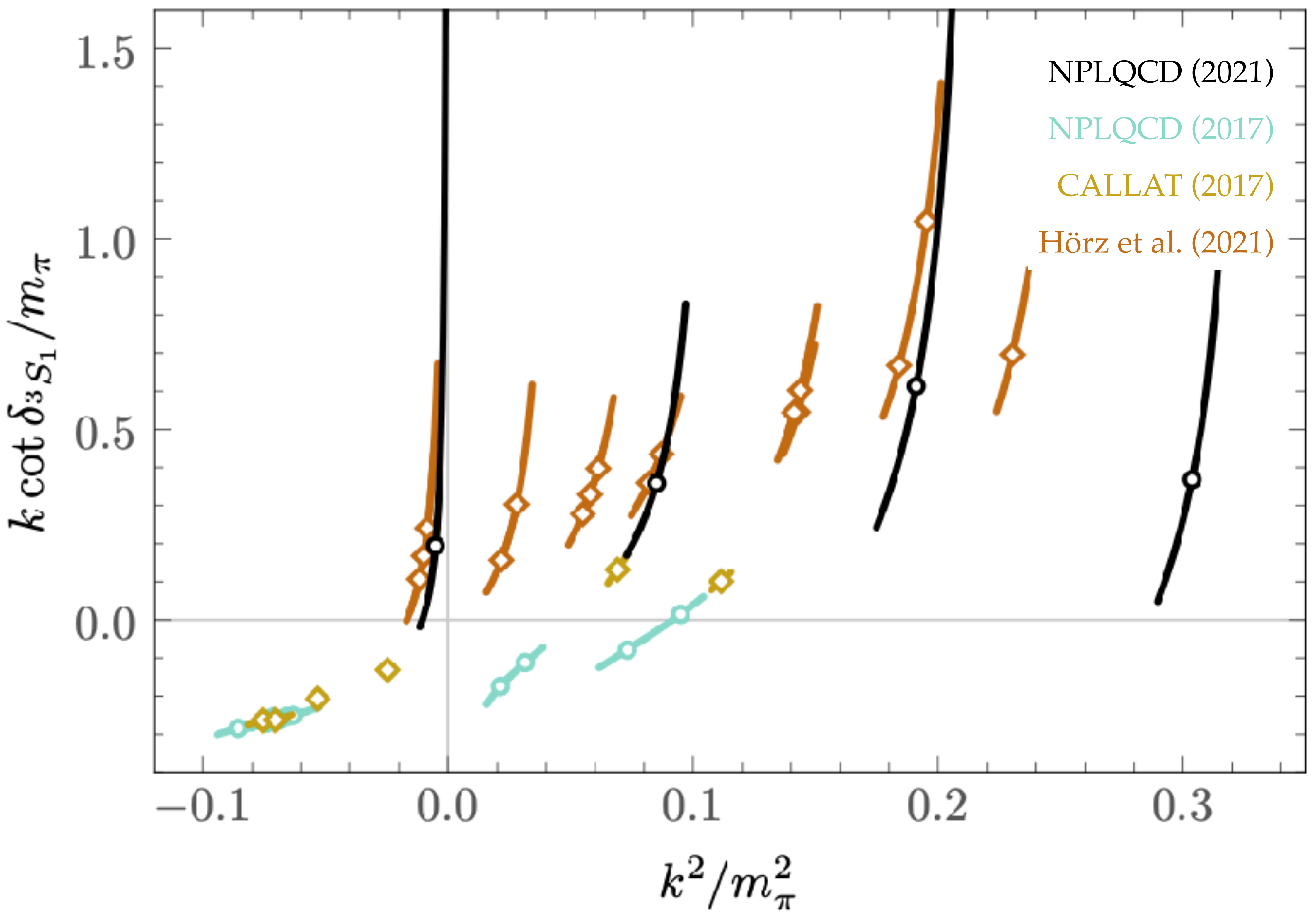


hadspec

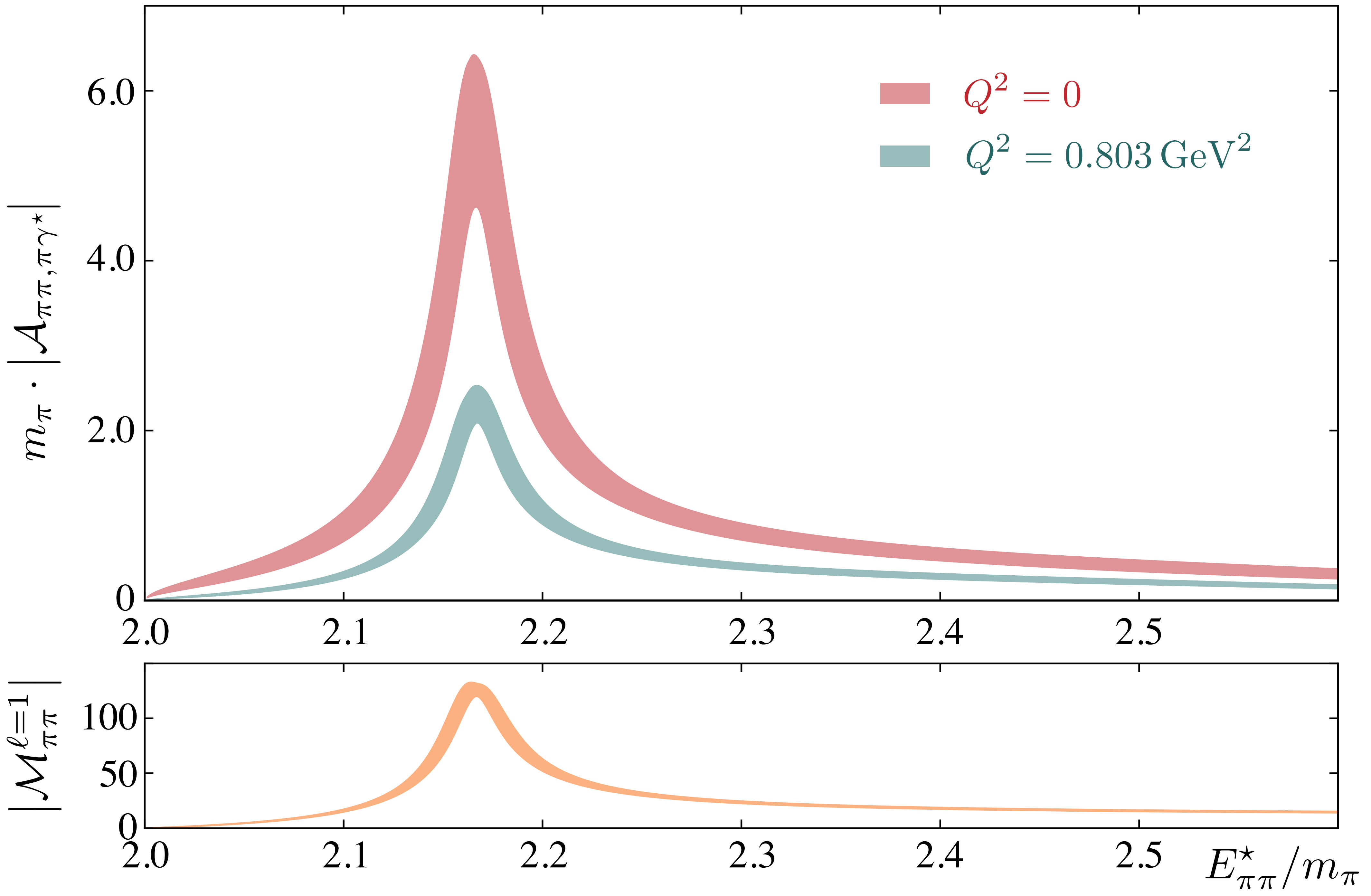
The few-body frontier

- ☑ two-nucleon systems,
- ☑ three-particle systems,
- ☑ few-body systems with electroweak probes.

NN $m_\pi \sim 700$ MeV



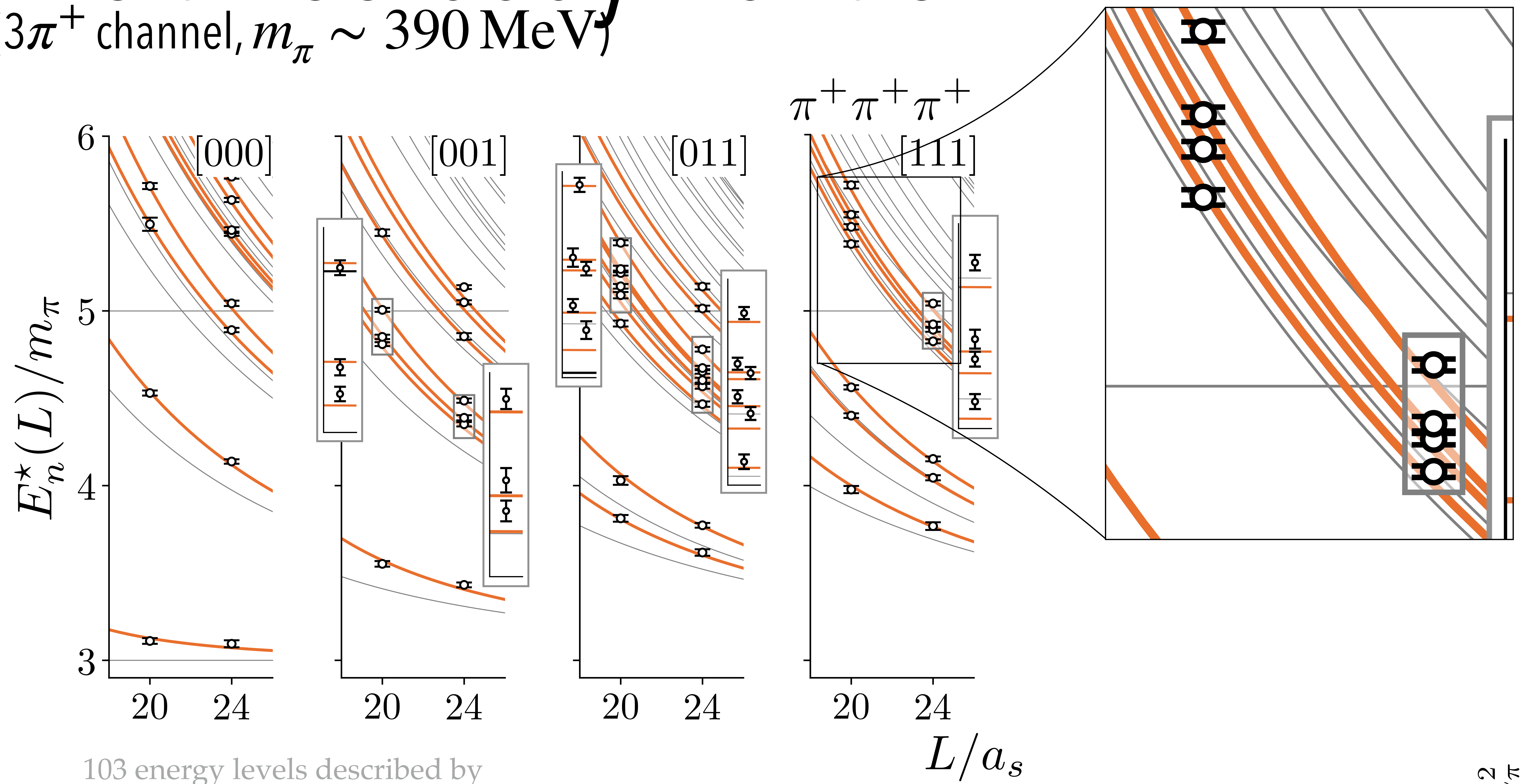
$\pi\gamma^\star \to \pi\pi$ $m_\pi \sim 400$ MeV



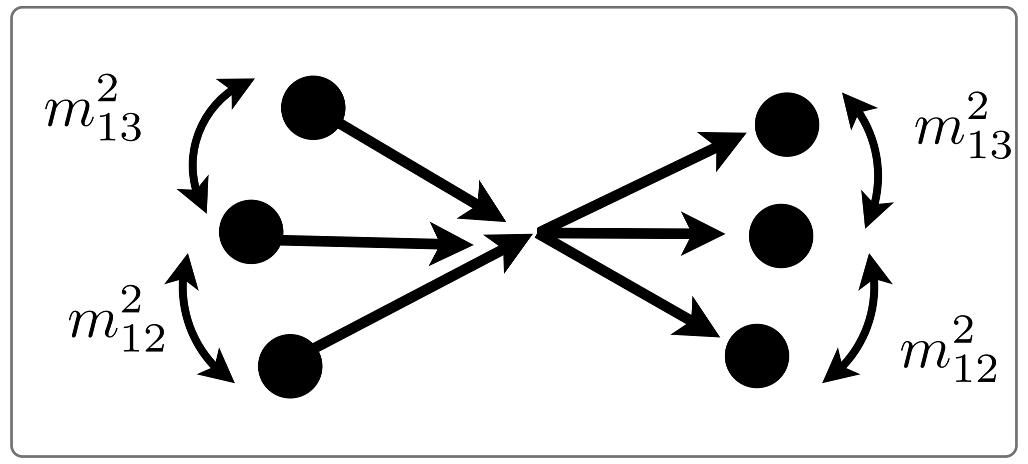
RB, Dudek, Edwards, Schultz, Thomas, Wilson (2016)

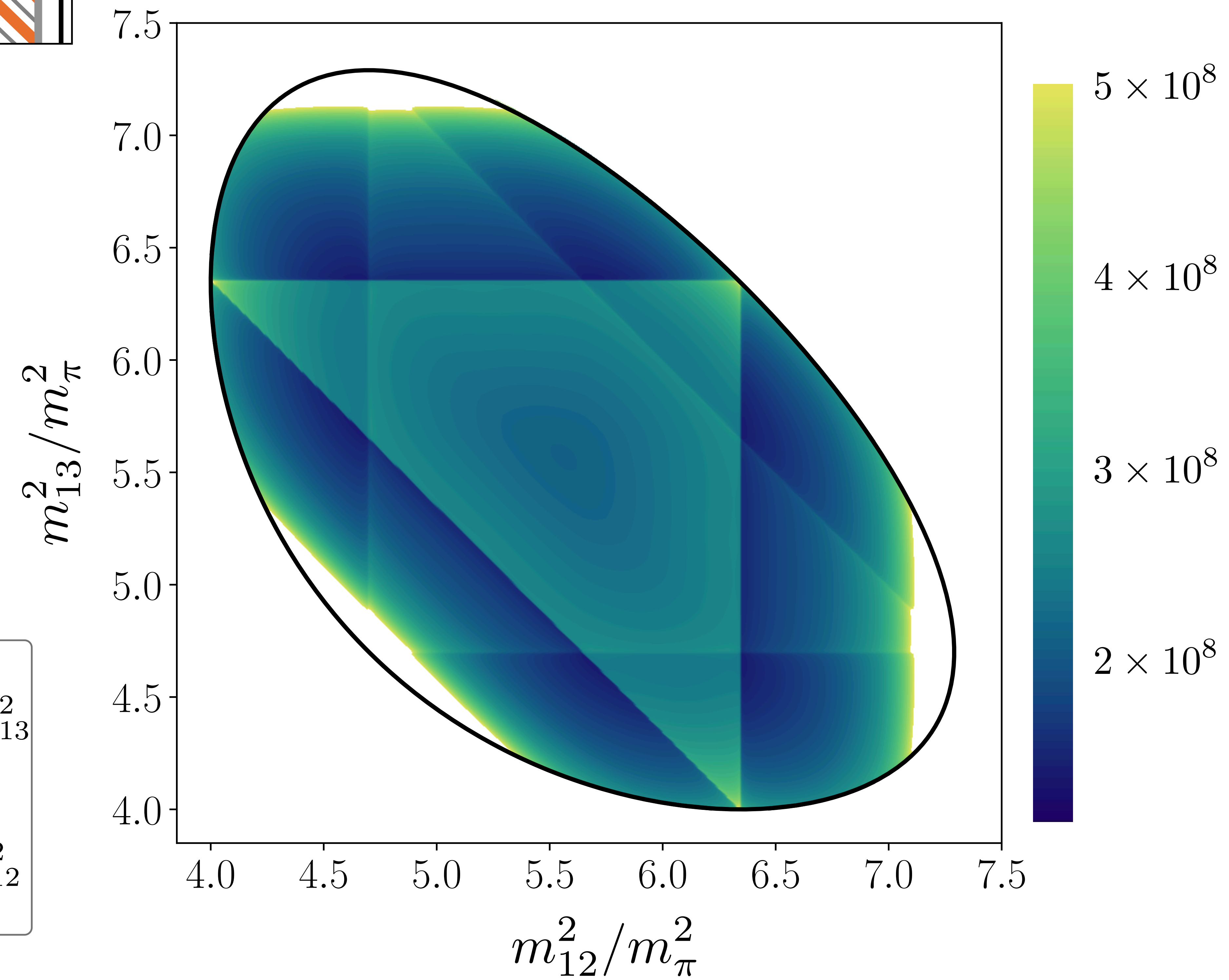
The three-body frontier

($3\pi^+$ channel, $m_\pi \sim 390\,\mathrm{MeV}$)



103 energy levels described by three numbers: m_π, $a_{\pi\pi}$, $\mathcal{K}_{3,\mathrm{iso}}$





Hansen, RB, Edwards, Thomas, & Wilson (2020)

Outline

Lattice QCD in a nutshell **[today & tomorrow]**

- [x] Does lattice work?
- [x] why does lattice QCD work?
- [x] what can it be used for?
- [x] what are its limitations?

What is the cutting edge of lattice QCD? **[tomorrow]**

- [x] hadron structure, fundamental symmetry,
- [x] scattering processes,
- [ ] ….other stuff, I won't get to 🤓
  - [ ] finite-temperature, weak decays, BSM searches, …. ,

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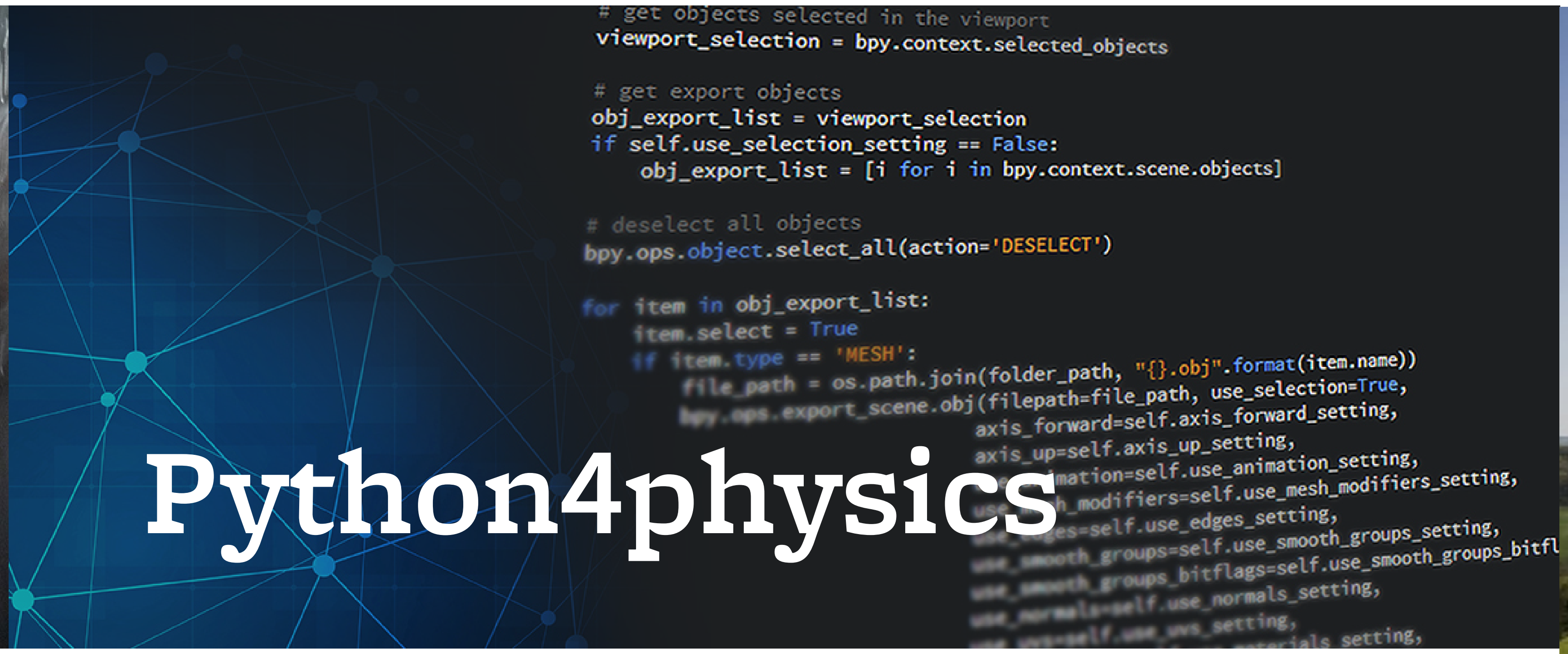

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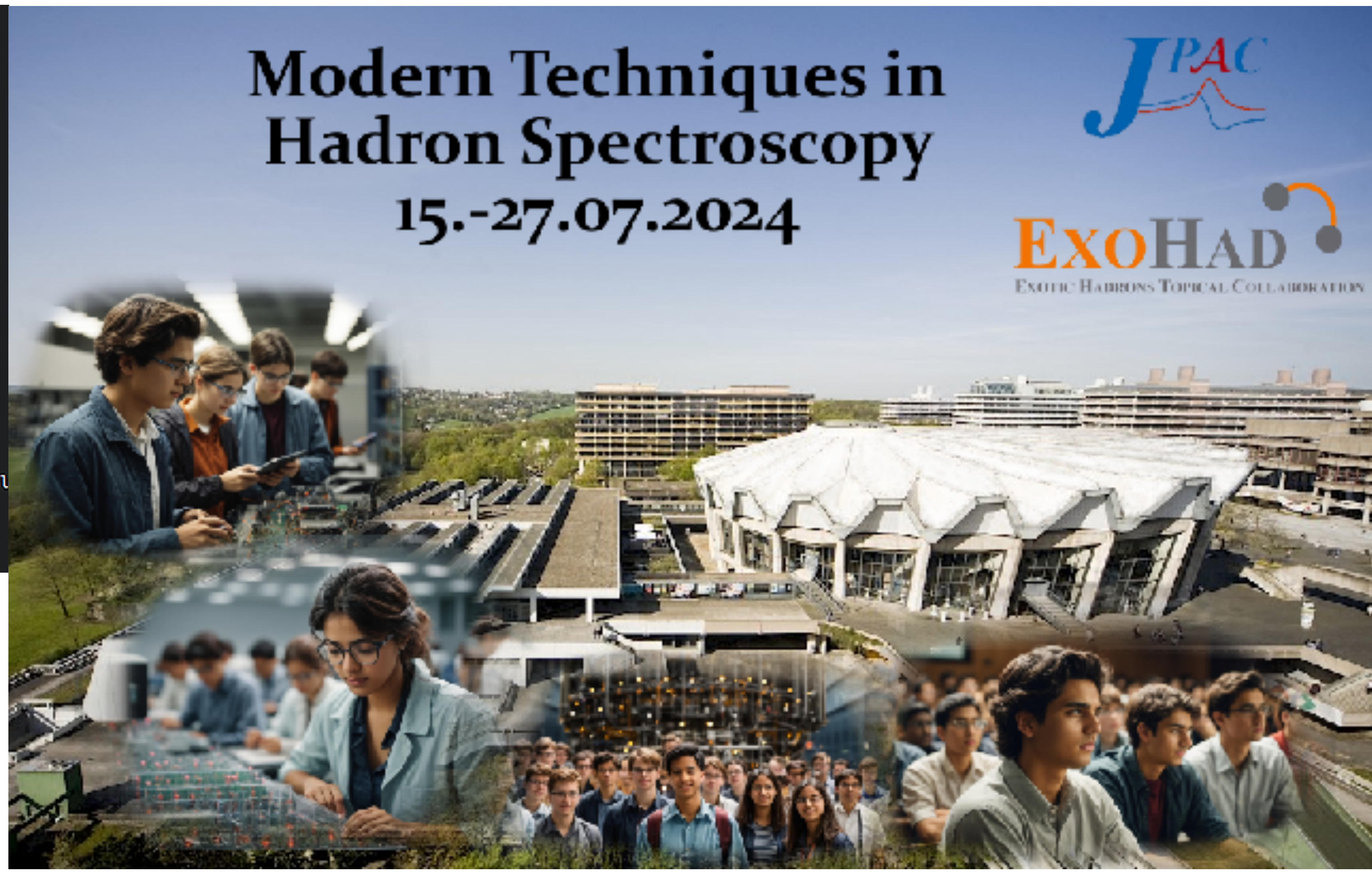

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