Baryon Stopping and the Valence Quark Distribution at Small *x*

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Outline

- Valence quark (flavor non-singlet) distributions at small *x* from a perturbative QCD calculation (Itakura, YK, McLerran, Teaney, 2003):
    - Evolution equation;
    - Small-*x* asymptotics of valence quark PDFs;
    - Baryon stopping in heavy ion collisions.

- A detour into AdS/CFT: shock wave collisions and stopping (Albacete, YK, Taliotis, 2008-2009).

Valence quarks at small *x*: evolution equation

Dipole picture of DIS

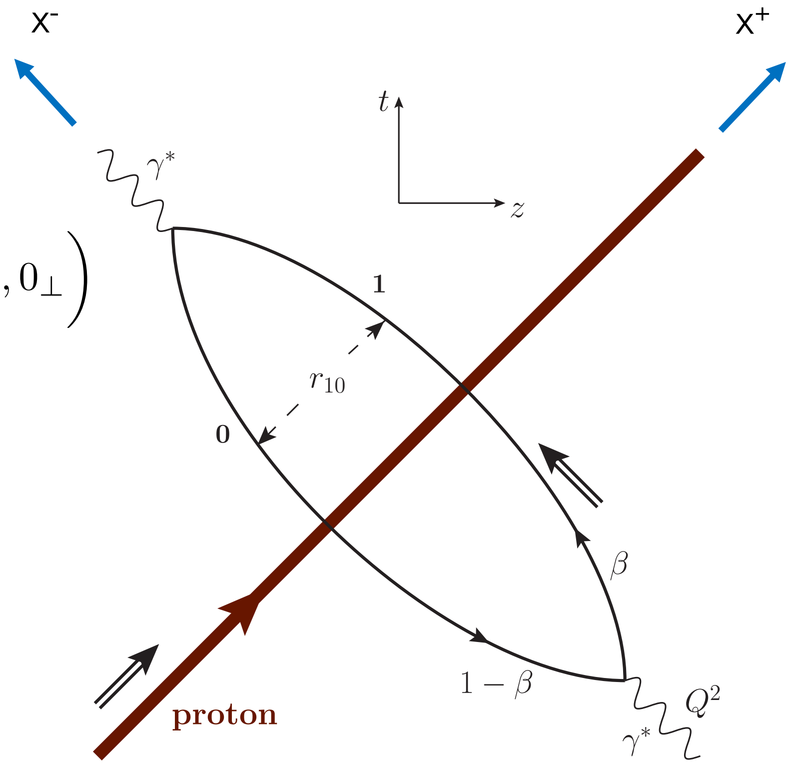
$$W^{\mu\nu} = \frac{1}{4\pi M_p} \int d^4x \, e^{iq\cdot x} \langle P | j^\mu(x) \, j^\nu(0) | P \rangle$$

$$q^\mu = \left(\frac{Q^2}{2q^-}, q^-, 0_\perp \right)$$

Large q⁻ → large x⁻ separation

$$e^{iq\cdot x} = e^{i\frac{Q^2}{2q^-}x^- + iq^- x^+}$$

$$x^\pm = \frac{t \pm z}{\sqrt{2}}$$

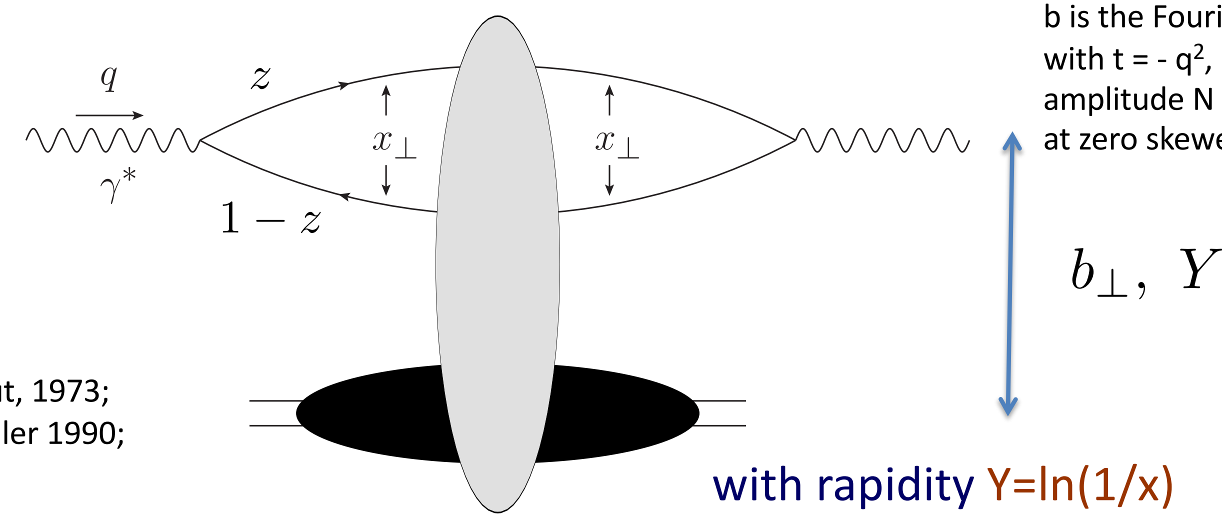


aka the "shock wave"

Dipole Amplitude

- The total DIS cross section is expressed in terms of the (Im part of the) forward quark dipole amplitude N:

$$\sigma_{tot}^{\gamma^* A} = \int \frac{d^2 x_\perp}{2\pi} d^2 b_\perp \int_0^1 \frac{dz}{z(1-z)} |\Psi^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)|^2 \; N(\vec{x}_\perp, \vec{b}_\perp, Y)$$



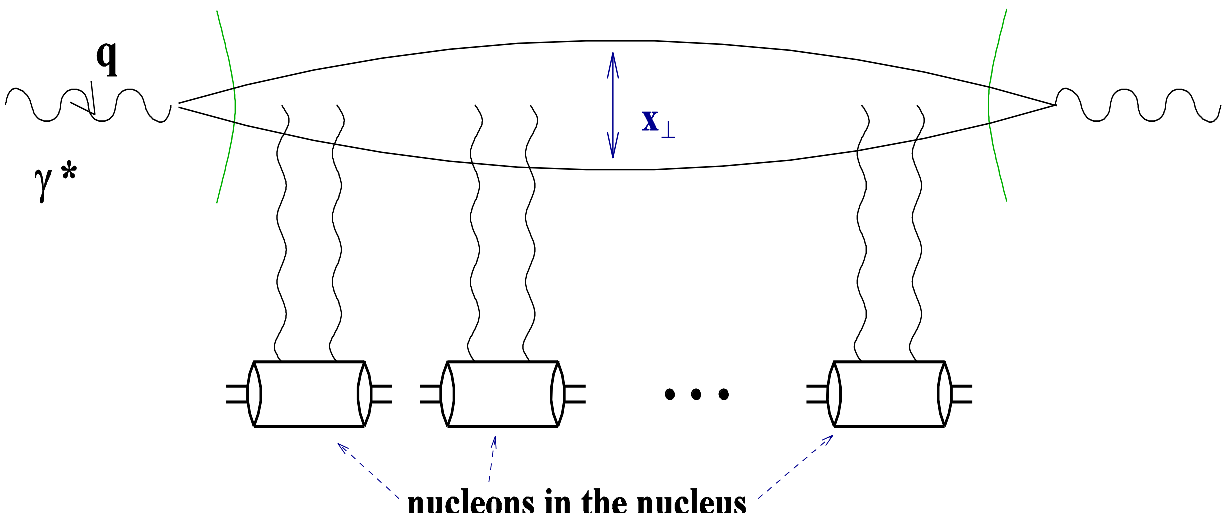
b is the Fourier conjugate to q with t = - q^2, making the dipole amplitude N similar to the GPDs at zero skewedness.

Gribov, 1970; Bjorken and Kogut, 1973; Frankfurt, Strikman 1988; Mueller 1990; Nikolaev and Zakharov 1991

with rapidity Y=ln(1/x)

DIS in the Classical Approximation

The DIS process in the rest frame of the target nucleus is shown below.



$$\sigma_{tot}^{\gamma^* A}(x_{Bj}, Q^2) = |\Psi^{\gamma^* \to q\bar{q}}|^2 \otimes N(x_\perp, Y = \ln 1/x_{Bj})$$

with rapidity Y=ln(1/x)

Valence Quarks in DIS: Multiple Rescatterings

To set up the stage for quantum evolution one has to construct initial conditions by resumming multiple rescatterings in the valence quark structure function

flavor non-singlet
F_2 structure function

$$F_2^{val}(x_{Bj}, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{EM}} \int \frac{d^2x\, dz}{4\pi}\, \Phi^{\gamma^* \to q\bar{q}}(\underline{x}, z)\; 2 \int d^2b\; R(\underline{x}, \underline{b}, z_1).$$

One has to resum the following diagrams

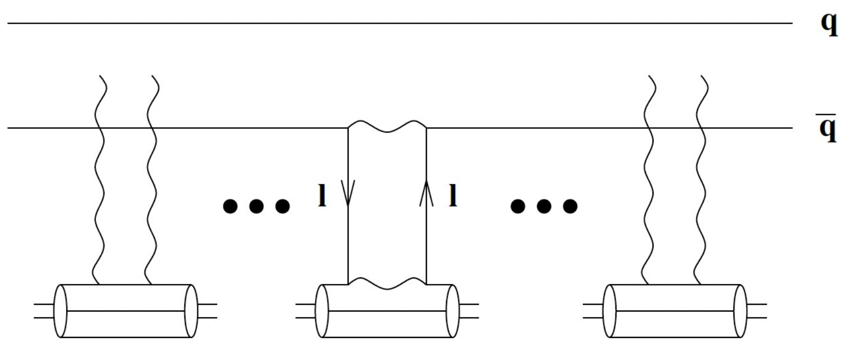


Figure 5: Forward amplitude of a $q\bar{q}$ pair interaction with the nucleus with one flavor-exchange interaction and all orders in gluon exchange rescatterings.

Non-linear Evolution Equation

Similarly to Mueller's dipole model we can write down an evolution equation in the large-N_c limit. Double line indicates a gluon.

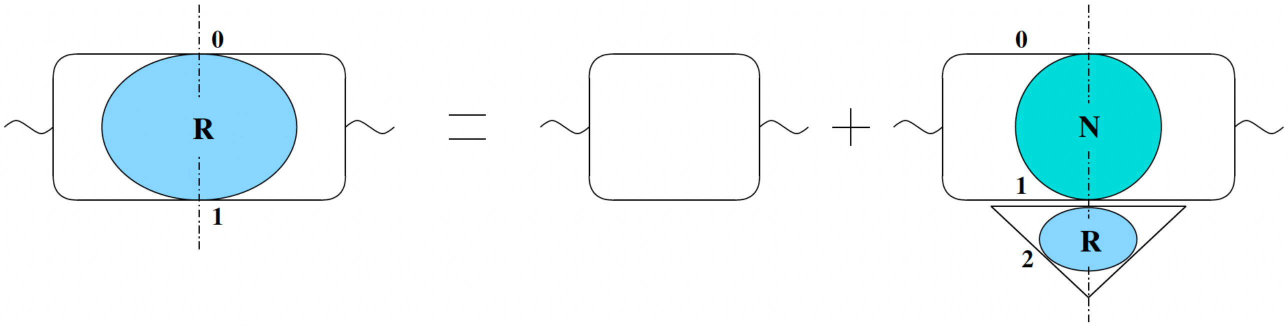
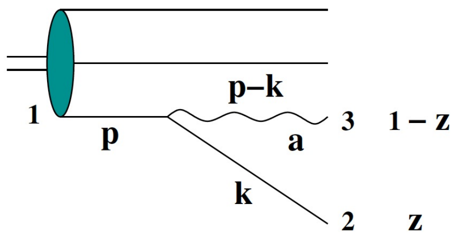


Figure 8: Evolution equation for the reggeon amplitude $R(\underline{x}_{01}, \underline{b}, z_1)$ in the double logarithmic approximation. $R(\underline{x}_{01}, \underline{b}, z_1)$ is the forward amplitude of a $q\bar{q}$ dipole interacting with the nucleus by a single $\bar{q}$ exchange and many gluon exchanges, while $N(\underline{x}_{01}, \underline{b}, z_1)$ is the forward amplitude of a $q\bar{q}$ dipole interacting with the nucleus by gluon exchanges only.



Defining

$$\tilde{R}(\underline{x}_{01}, \underline{b}, z_1) \equiv z_1 s \, R(\underline{x}_{01}, \underline{b}, z_1)$$

we write the nonlinear equation

$$\tilde{R}(\underline{x}_{01}, \underline{b}, z_1) = \tilde{R}_0(\underline{x}_{01}, \underline{b}, z_1) +$$

$$+ \frac{\alpha_S \, C_F}{2\,\pi^2} \int\limits_{z_i}^{z_1} \int^{\min\{1, x_{01}^2/x_{21}^2\}} \frac{dz_2}{z_2} \frac{d^2 x_2}{x_{21}^2} \tilde{R}(\underline{x}_{12}, \underline{b} + \frac{1}{2}\underline{x}_{20}, z_2)$$

$$\times \left[1 - N(\underline{x}_{01}, \underline{b}, z_1)\right].$$

K. Itakura, YK,
L. McLerran, D. Teaney,
2003

N = unpolarized dipole scattering amplitude, obeys BK/JIMWLK evolution

Double-logarithmic evolution at large N_c.

Resummation Parameter

- For helicity evolution the leading resummation parameter is different from BFKL, BK or JIMWLK, which resum powers of leading logarithms (LLA or SLA)

$$\alpha_s \ln(1/x)$$

- Reggeon evolution resummation parameter is double-logarithmic (DLA):

$$\alpha_s \ln^2 \frac{1}{x}$$

- The second logarithm of *x* arises due to transverse momentum (or transverse coordinate) integration being logarithmic both in the UV and IR.

- This was known before: Kirschner and Lipatov ’83; Kirschner ’84; Bartels, Ermolaev, Ryskin ‘95, ‘96; Griffiths and Ross ’99; Bartels and Lublinsky ‘03. It has been recently utilized in small-x evolution for helicity (YK, Pitonyak, Sievert ‘15-’18; Cougoulic, YK, Tarasov, Tawabutr ‘22), Sivers and Boer-Mulders functions (YK, Santiago, ’21-’22), transversity (Kirschner et al, ‘96; YK, Sievert ‘18), and OAM distributions (Boussarie, Hatta, Yuan ‘19; YK, Manley ’23; Manley ‘24).

Operator treatment

The same calculation can be done using the light-cone operator treatment (LCOT) formalism. The corresponding operator probably is

$$V_{\underline{x}}^{\mathrm{q}[2]} = - \frac{g^2 P^+}{2\, s} \int\limits_{-\infty}^{\infty} d x_1^- \int\limits_{x_1^-}^{\infty} d x_2^- \, V_{\underline{x}}[\infty, x_2^-] \, t^b \, \psi_\beta(x_2^-, \underline{x}) \, U_{\underline{x}}^{ba}[x_2^-, x_1^-] \, \left[\gamma^+ \right]_{\alpha\beta} \, \bar{\psi}_\alpha(x_1^-, \underline{x}) \, t^a \, V_{\underline{x}}[x_1^-, -\infty]$$

with the flavor non-singlet Reggeon dipole amplitude (note the relative minus sign):

$$R_{10}(zs) \equiv \frac{1}{2N_c} \, \mathrm{Re} \left\langle\!\!\left\langle \mathrm{T} \, \mathrm{tr} \left[V_{\underline{0}} \, V_{\underline{1}}^{\mathrm{q}[2] \, \dagger} \right] - \mathrm{T} \, \mathrm{tr} \left[V_{\underline{1}}^{\mathrm{q}[2]} \, V_{\underline{0}}^{\dagger} \right] \right\rangle\!\!\right\rangle$$

Here

$$\langle\langle \ldots \rangle\rangle = z \, s \, \langle \ldots \rangle$$

Flavor Non-Singlet Observables

- In the flavor non-singlet case, all helicity observables again depend on the polarized dipole amplitude:

$$g_1^{NS}(x,Q^2) = \frac{N_c}{2\,\pi^2 \alpha_{EM}} \int\limits_{z_i}^{1} \frac{dz}{z^2(1-z)} \int dx_{01}^2 \left[\frac{1}{2} \sum_{\lambda\sigma\sigma'} |\psi^T_{\lambda\sigma\sigma'}|^2_{(x_{01}^2,z)} + \sum_{\sigma\sigma'} |\psi^L_{\sigma\sigma'}|^2_{(x_{01}^2,z)} \right] G^{NS}(x_{01}^2, z),$$

$$\Delta q^{NS}(x,Q^2) = \frac{N_c}{2\pi^3} \int\limits_{z_i}^{1} \frac{dz}{z} \int\limits_{\frac{1}{zs}}^{\frac{1}{zQ^2}} \frac{dx_{01}^2}{x_{01}^2} \, G^{NS}(x_{01}^2, z),$$

YK, D. Pitonyak, M. Sievert, ‘16

$$g_{1L}^{NS}(x,k_T^2) = \frac{8\,N_c}{(2\pi)^6} \int\limits_{z_i}^{1} \frac{dz}{z} \int d^2 x_{01}\, d^2 x_{0'1}\, e^{-i\underline{k}\cdot(\underline{x}_{01} - \underline{x}_{0'1})} \, \frac{\underline{x}_{01} \cdot \underline{x}_{0'1}}{x_{01}^2 x_{0'1}^2} \, G^{NS}(x_{01}^2, z)$$

- Polarized dipole amplitude is different (difference instead of sum):

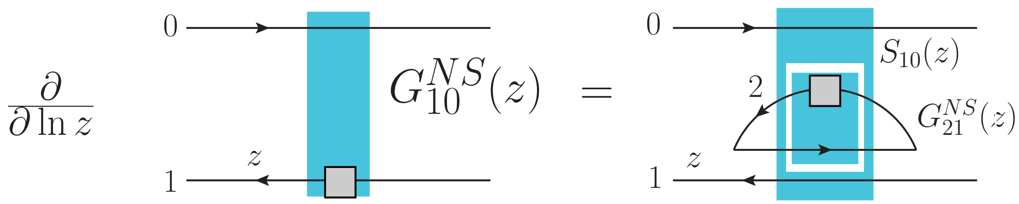
$$G_{10}^{NS}(z) \equiv \frac{1}{2N_c} \left\langle\!\!\left\langle \mathrm{tr}\left[V_{\underline{0}} V_{\underline{1}}^{pol\,\dagger} \right] - \mathrm{tr}\left[V_{\underline{1}}^{pol} V_{\underline{0}}^{\dagger} \right] \right\rangle\!\!\right\rangle (z)$$

- This is related to the definition

$$\Delta q^{NS}(x,Q^2) \equiv \Delta q^f(x,Q^2) - \Delta \bar{q}^f(x,Q^2)$$

Flavor Non-Singlet Evolution

- Evolution equations end up being much simpler in the non-singlet case:



YK, D. Pitonyak, M. Sievert, '16

$$G_{10}^{NS}(z) = G_{10}^{NS\,(0)}(z) + \frac{\alpha_s N_c}{4\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2 \frac{z}{z'}} \frac{dx_{21}^2}{x_{21}^2} \, S_{10}(z') \, G_{21}^{NS}(z')$$

- Analytical solution (in the DLA case, S=1) leads to (in agreement with Bartels et al, '95)

$$g_1^{NS}(x, Q^2) \sim \Delta q^{NS}(x, Q^2) \sim g_{1L}^{NS}(x, k_T^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^{NS}} \approx \left(\frac{1}{x}\right)^{\sqrt{\frac{\alpha_s N_c}{\pi}}}$$

- The resulting intercept is smaller than the flavor-singlet intercept.

Valence quarks at small *x*: asymptotics

High energy asymptotics of the Reggeon amplitude

Solution of the linear DLA equation

The linear part of our non-linear equation can be solved to give (cf. Kirschner and Lipatov, '83)

$$R \sim s^{\alpha_R - 1} \sim s^{\sqrt{2\frac{\alpha_S C_F}{\pi}} - 1}.$$

K. Itakura, YK,
L. McLerran, D. Teaney,
2003

Order–α_s correction to the intercept (the power) was found by R. Kirschner '95.

Small-x asymptotics of the valence quark PDFs

- The above translates into

$$q_{\text{val}}(x, Q^2) \sim \left(\frac{1}{x}\right)^{\sqrt{\frac{2\alpha_s C_F}{\pi}}} \sim \left(\frac{1}{x}\right)^{\sqrt{\frac{\alpha_s N_c}{\pi}}}$$

  outside the saturation region.

- Inside the saturation region q_{val} is negligibly small.

- Note the same intercept as for Δq^{NS} , the flavor non-singlet quark helicity PDF.

Baryon stopping

Relation to RHIC data on Baryon Stopping

At RHIC people measure the # of baryons — of anti-baryons at mid-rapidity. This gives the total baryon number at mid-rapidity - "baryon stopping":

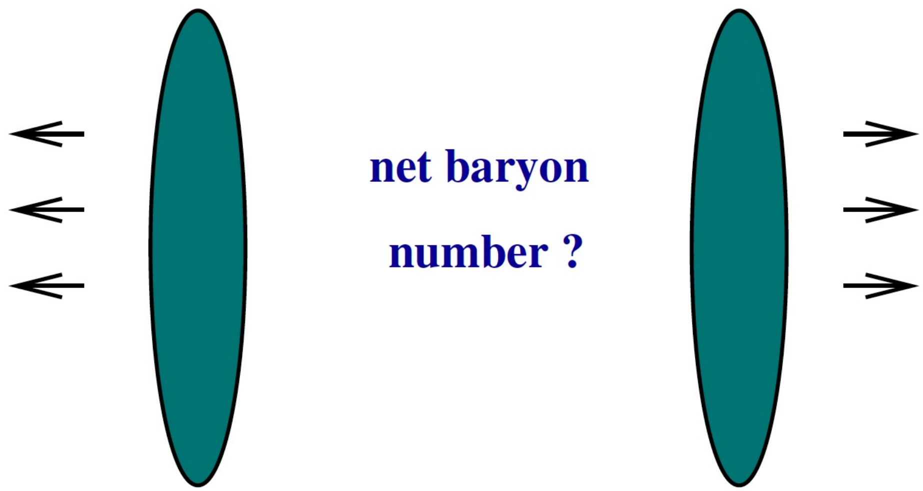


Figure 10: Heavy ion collision.

Perturbative baryon transport is proportional to distribution functions

$$\frac{dN_B^{net}}{dk^2dy} \sim x_R f_{val}(x_R, Q_s^2) + x_L f_{val}(x_L, Q_s^2)$$

so that

$$\frac{dN_B^{net}}{dk^2dy} \sim e^{-\Delta_R(Y_B-y)} + e^{-\Delta_R(Y_B+y)} \ .$$

with

+/- Y_B is the beam rapidity

$$\Delta_R \equiv 1 - 2\sqrt{\frac{\alpha_s C_F}{2\pi}} \ .$$

Taking $\alpha_S = 0.33$ gives $\Delta_R = 0.47$, which gives a good fit of the existing data:

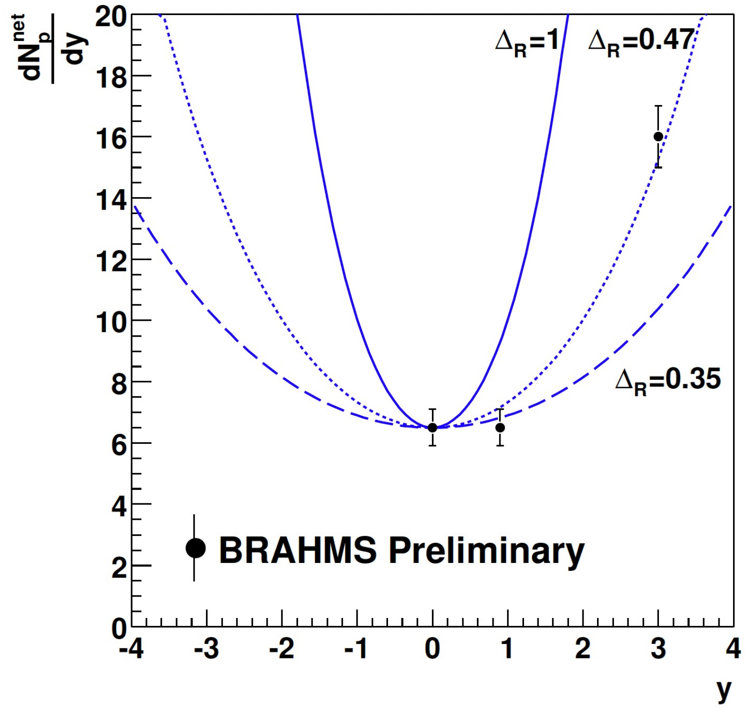
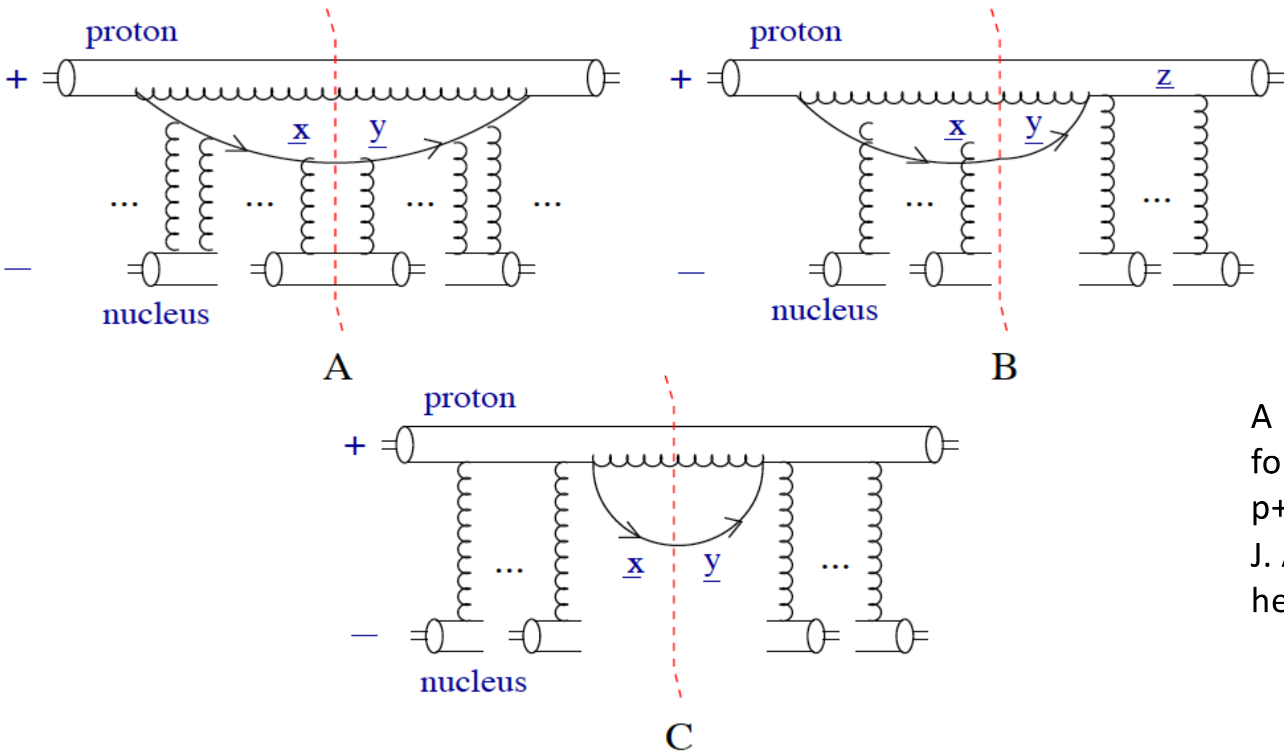


Figure 11: A comparison to preliminary BRAHMS data on net-proton rapidity distributions.

K. Itakura, YK,
L. McLerran, D. Teaney,
2003

Production Cross Section



A more detailed calculation for baryon production in p+A collisions was done in J. Albacete, YK, hep-ph/0605053.

AdS/CFT Detour

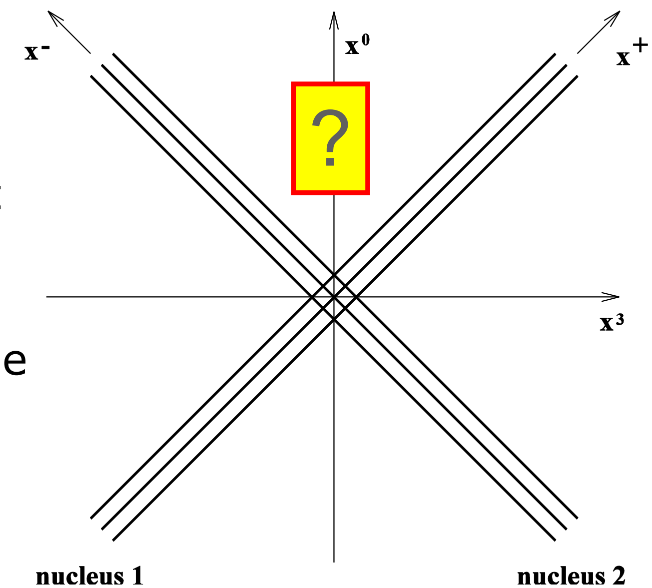
Colliding shock waves in AdS

Considered by Nastase; Shuryak, Sin, Zahed; Kajantie, Louko, Tahkokkalio; Grumiller, Romatshcke; Gubser, Pufu, Yarom.

I will follow J. Albacete, A. Taliotis, Yu.K.
arXiv:0805.2927 [hep-th], arXiv:0902.3046 [hep-th]

Model of heavy ion collisions in AdS

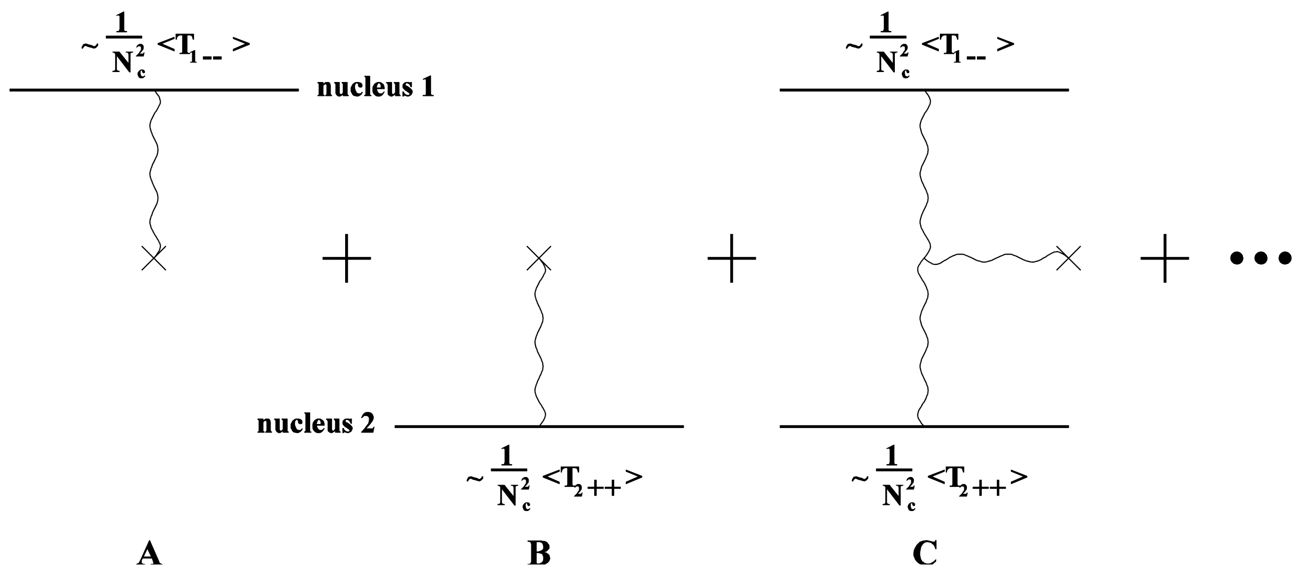
- Imagine a collision of two shock waves in AdS:
- We know the metric of both shock waves, and know that nothing happens before the collision.
- Need to find the metric in the forward light cone!
(cf. classical fields in CGC)



$$ds^2 = \frac{L^2}{z^2}\left[\underbrace{-2\,dx^+dx^- + dx_\perp^2 + dz^2}_{\text{empty AdS}_5} + \underbrace{\frac{2\pi^2}{N_C^2}\langle T_{1--}(x^-)\rangle z^4\,dx^{-2} + \frac{2\pi^2}{N_C^2}\langle T_{2++}(x^+)\rangle z^4\,dx^{+2}}_{\text{1-graviton part}} + \ldots\right]$$

higher order graviton exchanges

Heavy ion collisions in AdS

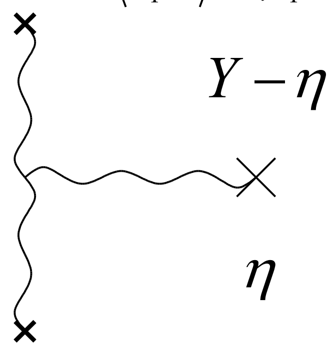


$$ds^2 = \frac{L^2}{z^2}\left[\underbrace{-2\,dx^+ dx^- + dx_\perp^2 + dz^2}_{\text{empty AdS}_5} + \underbrace{\frac{2\pi^2}{N_C^2}\left\langle T_{1--}(x^-)\right\rangle z^4\, dx^{-2} + \frac{2\pi^2}{N_C^2}\left\langle T_{2++}(x^+)\right\rangle z^4\, dx^{+2}}_{\text{1-graviton part}} + \ldots\right]$$

higher order
graviton exchanges

Physical shock waves

$\mathbf{t_1} \sim \langle T_{1--} \rangle \sim \mu_1\, \delta(x^-)$



$\mathbf{t_2} \sim \langle T_{2++} \rangle \sim \mu_2\, \delta(x^+)$

Simple dimensional analysis:

$$\varepsilon \sim \mu_1\, \mu_2\, \tau^2$$

The same result comes out of detailed calculations.

Grumiller, Romatschke '08
Albacete, Taliotis, Yu.K. '08

Each graviton gives e^{η}, hence get no rapidity dependence:

$$e^{\eta}\, e^{Y-\eta} = \eta - \text{independent}$$

Physical shock waves: problem 2

- Delta-functions are unwieldy. We will smear the shock wave:

$$\mu\,\delta(x^-) \to \frac{\mu}{a}\,\theta(x^-)\,\theta(a - x^-)$$

- Look at the energy-momentum tensor of a nucleus after collision:

$$\boxed{\left\langle T_{--}(x^+ >> a, x^- = a/2)\right\rangle = \frac{\mu}{a} - 4\pi^2\mu^2 x^{+2}}$$

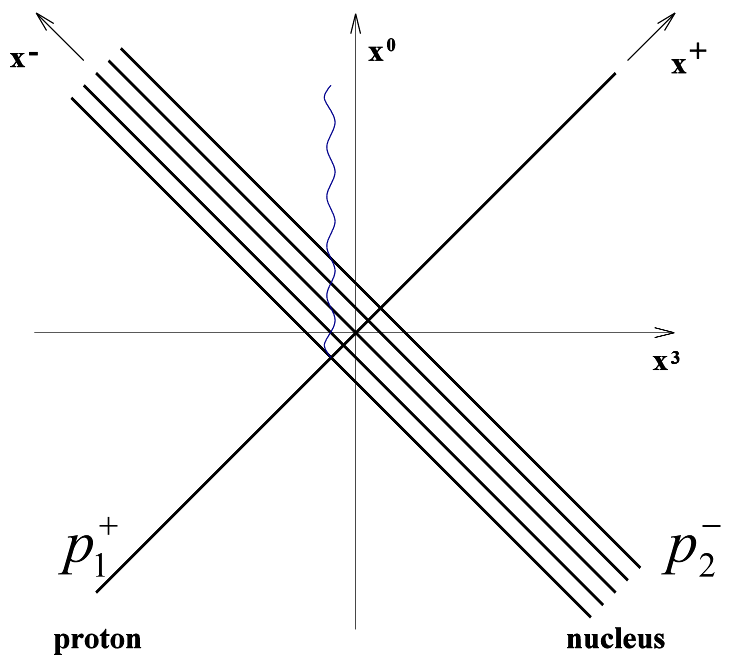
- Looks like by the light-cone time $x^+ \sim \frac{1}{\sqrt{\mu\,a}} \sim \frac{1}{\Lambda\,A^{1/3}}$

  the nucleus will run out of momentum and **stop**!

Proton-Nucleus Collisions in AdS

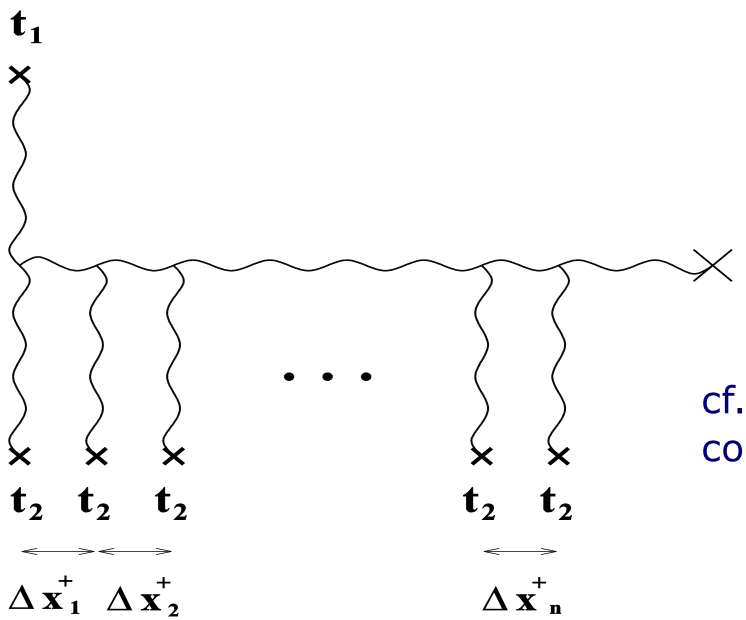
pA Setup

- Consider pA collisions:



pA Setup

- In terms of graviton exchanges need to resum diagrams like this:



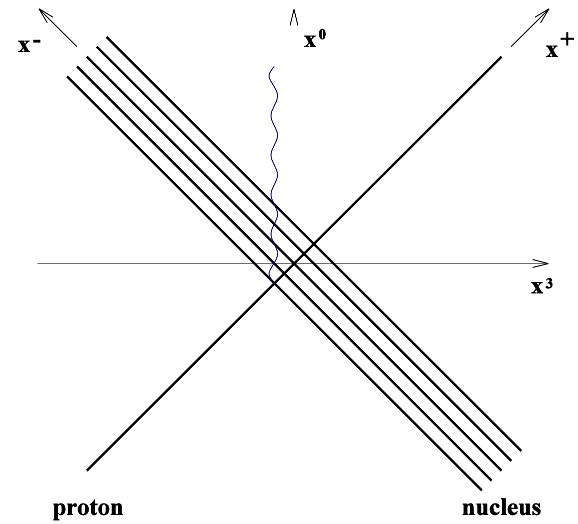
cf. gluon production in pA collisions in CGC!

Proton Stopping

- What about the proton? Due to our earlier result about shock wave stopping

$$\left\langle T_{--}(x^+ >> a, x^- = a/2)\right\rangle = \frac{\mu}{a} - 4\pi^2\mu^2 x^{+2}$$

  we should be able to see how it stops.



- And we do:

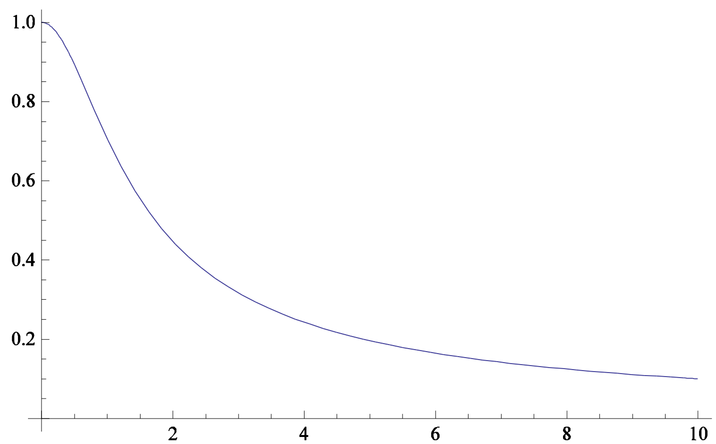
$$\langle T^{++}_{tot}\rangle = \langle T^{++}_{orig}\rangle + \langle T^{++}_{prod}\rangle = \frac{N_c^2}{2\,\pi^2}\,\frac{\mu_1}{a_1}\,\frac{1}{\sqrt{1+8\,\mu_2\,(x^+)^2\,x^-}}, \quad \text{for} \quad 0 < x^- < a_1$$

T^{++} goes to zero as x^+ grows large!

Proton Stopping

- We get complete proton stopping (arbitrary units):

T^{++}
of the proton

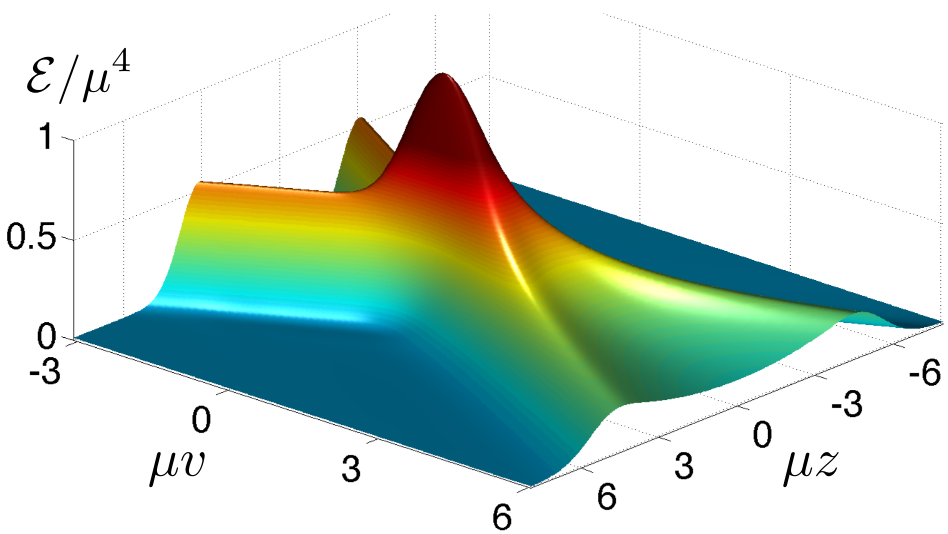


X^{+}

Albacete, Taliotis, Yu.K. '09

Exact Numerical Solution

- Exact numerical solution of Einstein equations for two colliding shock waves in AdS_5 also exhibits stopping of the colliding “nuclei” (P. Chesler, L. Yaffe, ‘11):



Note that this is not just baryon stopping: this is ‘everything stopping’.
Still, may be relevant…?

Conclusions

- Perturbative QCD at small *x*:

$$q_{\mathrm{val}}(x, Q^2) \sim \left(\frac{1}{x}\right)^{\sqrt{\frac{2\alpha_S C_F}{\pi}}} \sim \left(\frac{1}{x}\right)^{\sqrt{\frac{\alpha_S N_C}{\pi}}}$$

- Non-perturbative approaches: AdS/CFT correspondence exhibits strong shock-wave stopping, translating into nuclear stopping, though not just for the baryon number.