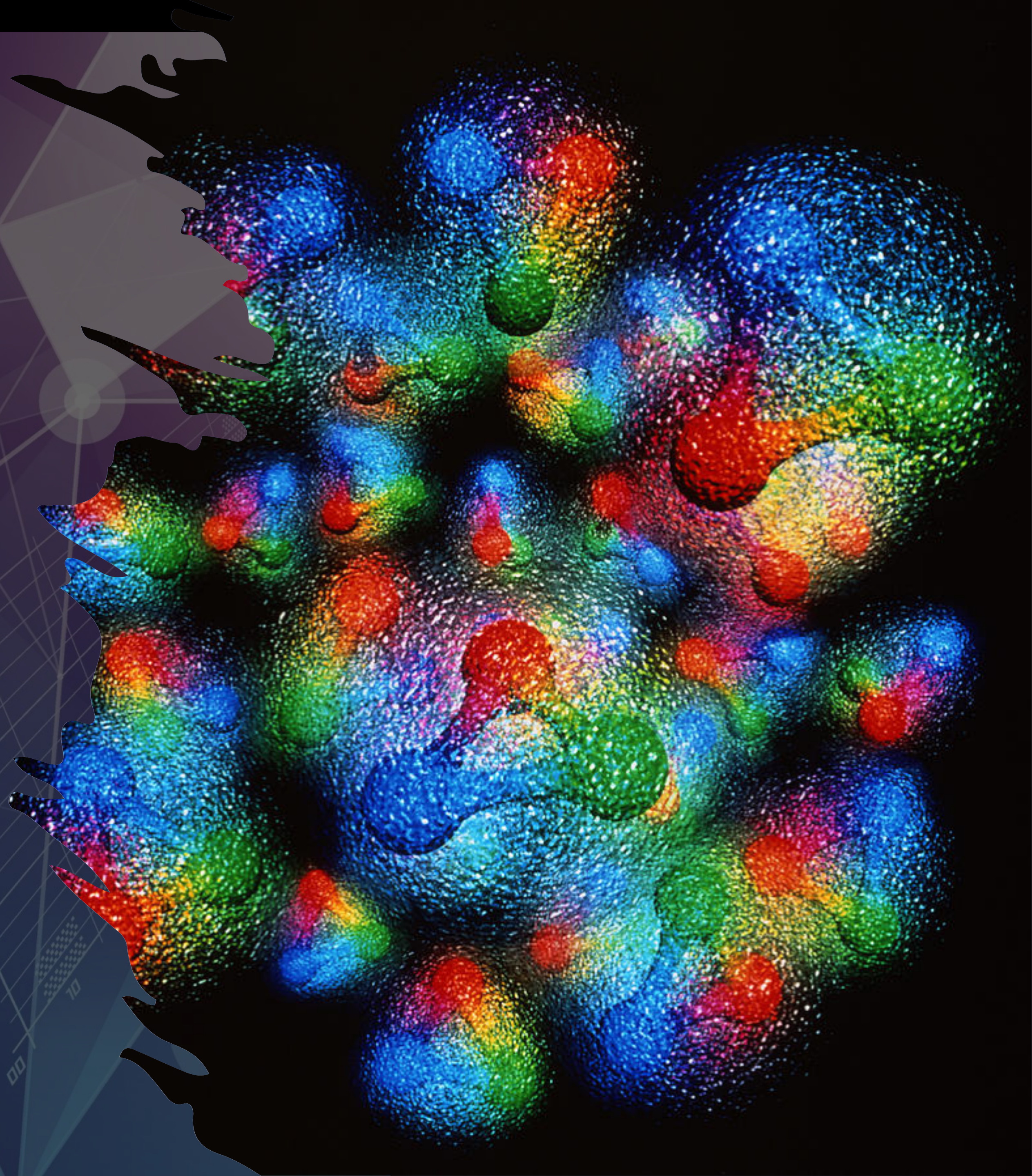
Observing the internal spatial structure of subatomic matter:

Generalized Parton Distributions

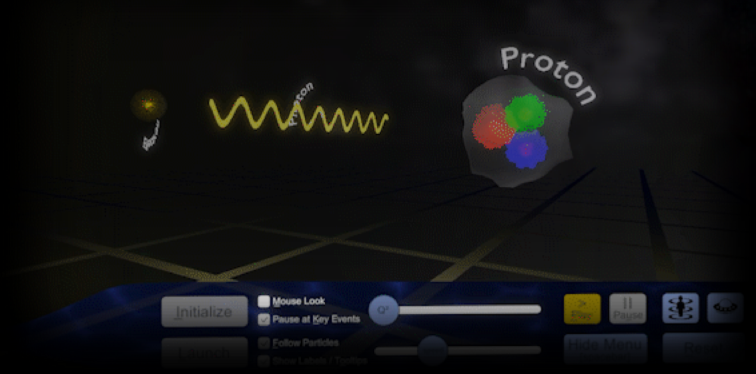
Simonetta Liuti

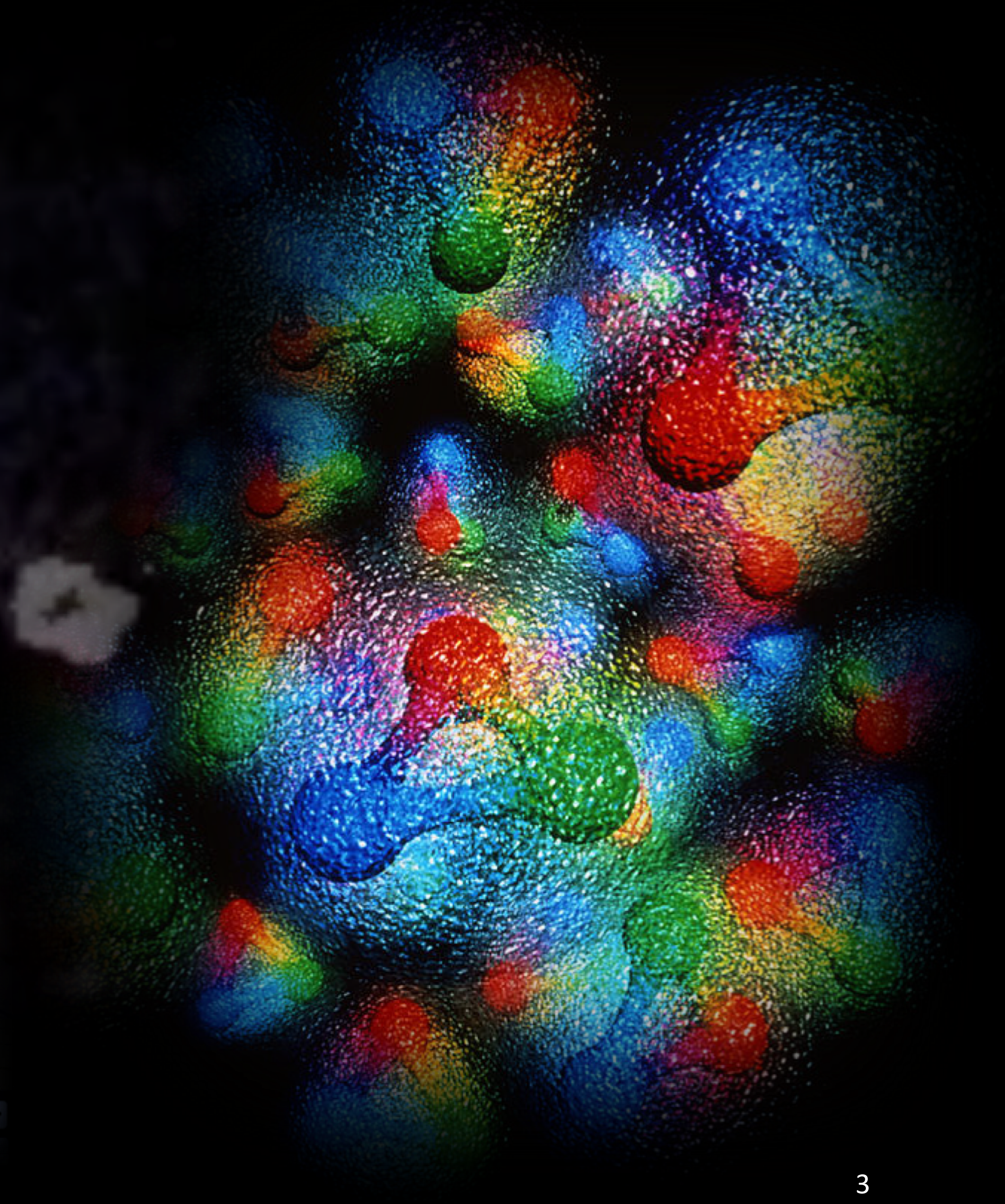


With graduate students: Joshua Bautista, Zaki Panjsheeri

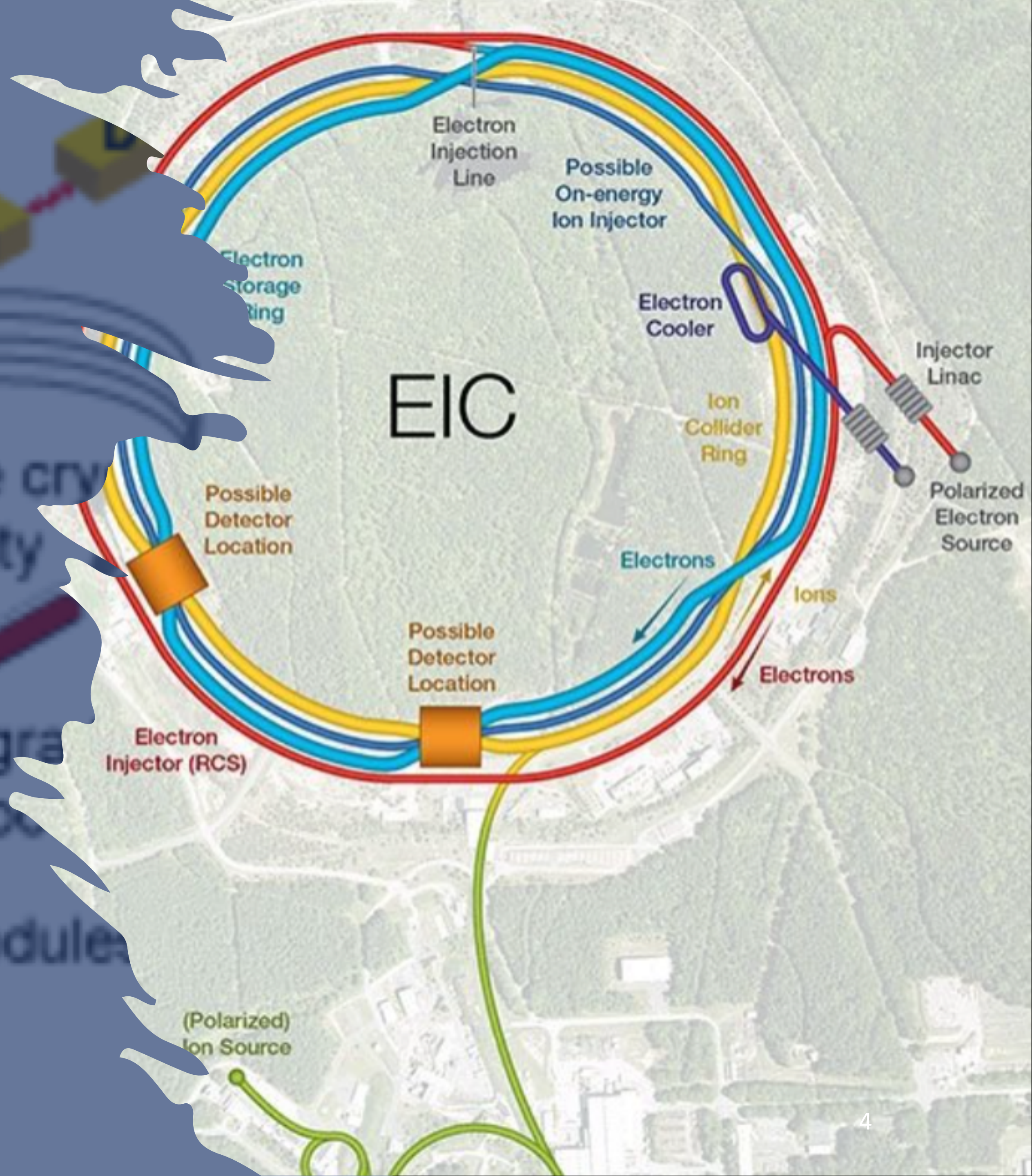
- At this workshop we are defining new paradigms to both penetrate and visualize the deep structure of visible matter

- Addressing questions that we couldn’t even afford asking before





Made possible by key experiments from Jefferson Lab to LHC … and at the Electron Ion Collider (EIC)

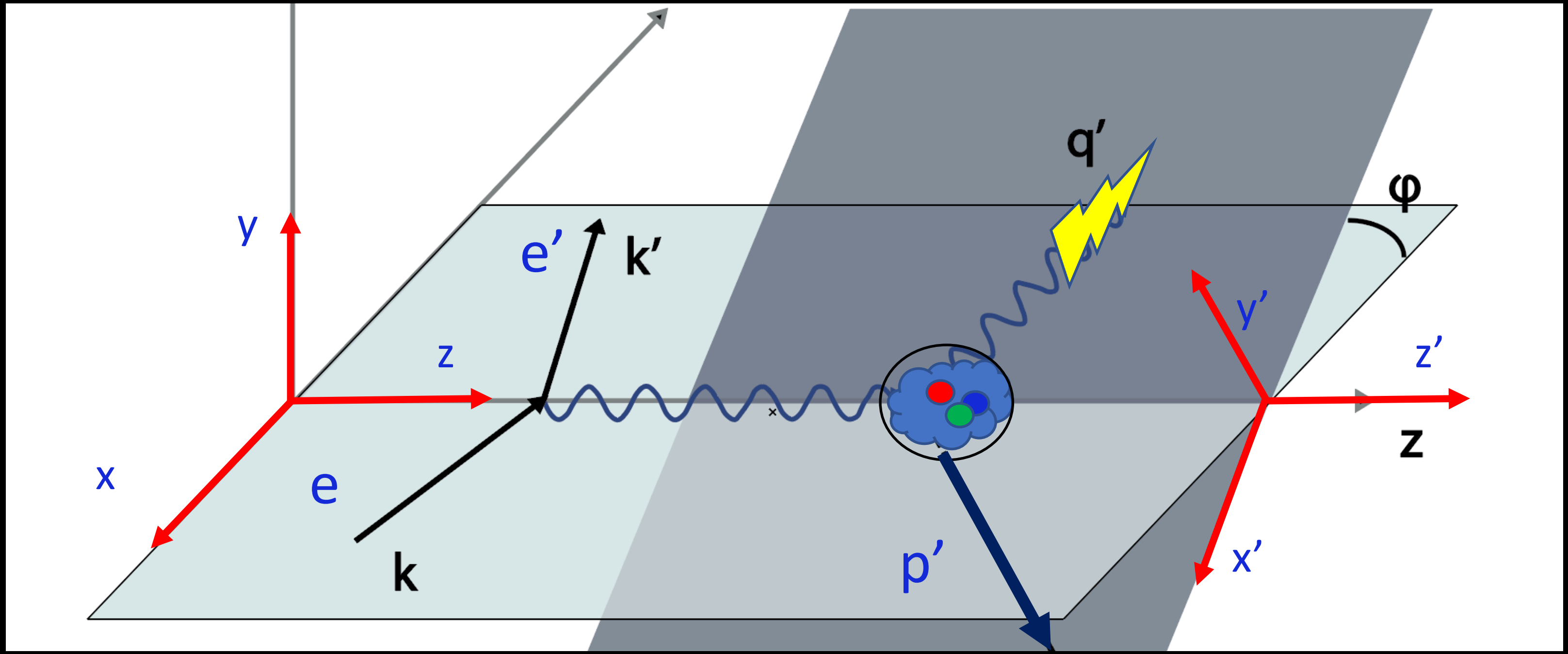


… but … what does it really take to have an image of the proton and can we really *observe* what goes on inside it?

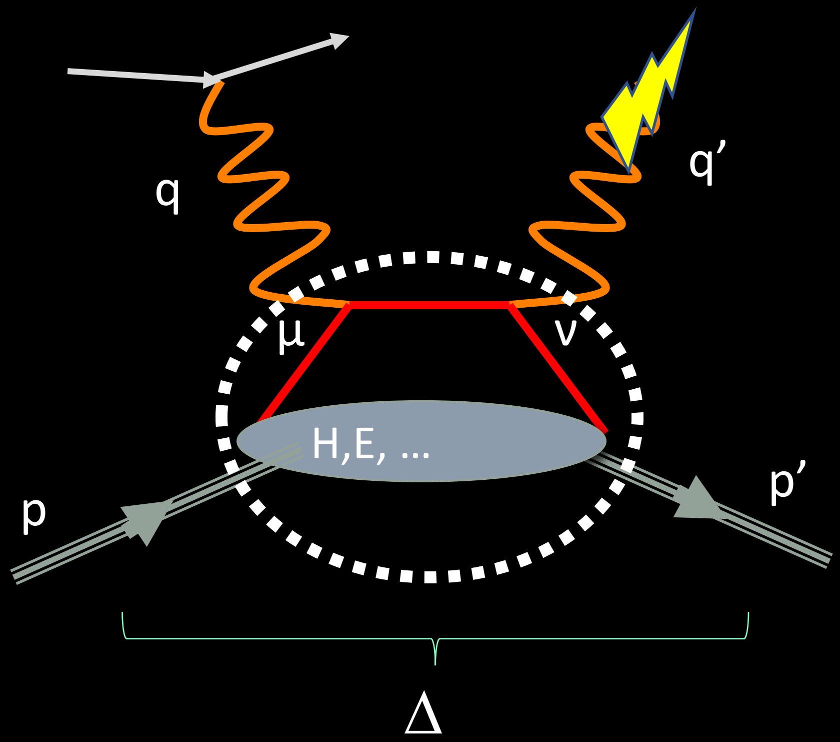
- Where are the quarks and gluons located inside the proton?
- Where are the quarks and gluons located with respect to one another?

X. Ji (1998)

A very first step: “Deeply Virtual Exclusive Scattering”

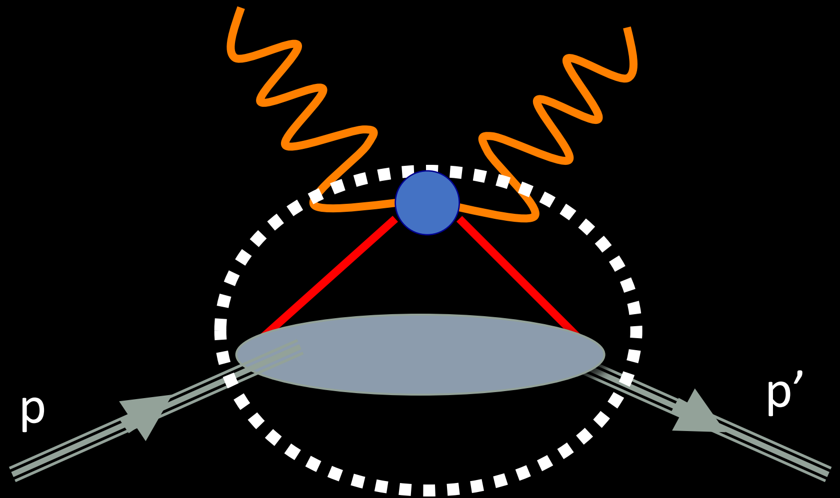


QCD matrix element



Generalized Parton Distribution

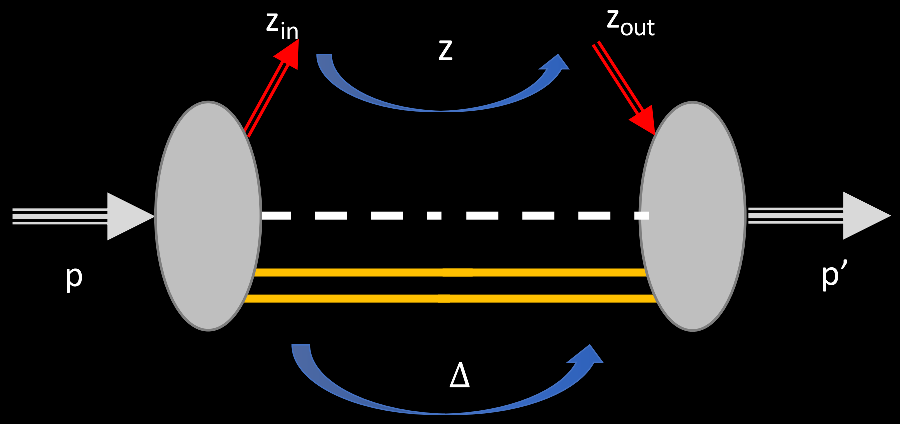
Local Operator



2^{nd} GPD Moment → (EMT) Form Factor

Additional information from measurement of final state particles, q’, p’ (Δ)

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^-_{in}}{(2\pi)} \int \frac{dz^-_{out}}{(2\pi)^2}\, e^{i(k_{in} z_{in})}\, e^{-i(k_{out} z_{out})}\, \langle p', \Lambda' | \overline{\psi}(z_{out})\, \Gamma\, \mathcal{U}(z_{in}, z_{out}) \psi(z_{in}) | p, \Lambda \rangle \Big|_{\substack{z^+_{in(out)}=0 \\ \mathbf{z}_{T,in(out)}=0}}$$



Quark-quark correlation function

*Analogous treatment for gluons

Two sets of variables/Fourier conjugates

$$\begin{cases} b = \dfrac{z_{in} + z_{out}}{2} \\[2em] \Delta = k_{in} - k_{out} = p - p' \end{cases}$$

External d.o.f., directly measurable

$$\begin{cases} z = z_{in} - z_{out} \\[2em] k = \dfrac{k_{in} + k_{out}}{2} \end{cases} \rightarrow \quad Xp^{+}$$

Loop variables

Parametrization of the (unpolarized) hadronic tensor in terms of GPDs

$$
\begin{aligned}
W^{\gamma^+}_{\Lambda,\Lambda'} &= \int \frac{dz^-}{(2\pi)}\, e^{i(X-\zeta/2)p^+z^-}\, \langle p' = p-\Delta, \Lambda' | \overline{\psi}\,(0)\; \gamma^+\, \psi\left(z^-\right) | p, \Lambda\rangle, \\
&= \frac{1}{2P^+}\left\{ \boxed{H_q(X,\zeta,t)}\,\bar{u}(p-\Delta,\Lambda')\gamma^+ u(p,\Lambda) + \boxed{E_q(X,\zeta,t)}\,\bar{u}(p-\Delta,\Lambda')\frac{\sigma^{i+}\Delta_i}{2M} u(p,\Lambda)\right\}
\end{aligned}
$$

For a leading twist object we can project out the "+" components and translate the quark operators,

$$H_q(X,0,t) = \int d^2\mathbf{k}_T^{\mathcal{X}}\, dk_{\mathcal{X}}^+\, \delta(k_{\mathcal{X}}^+ - (1-X)p^+)\, \langle p - \Delta \mid \bar{\psi}_+(0) \mid \mathcal{X}\rangle\, \langle \mathcal{X} \mid \psi_+(0) \mid p\rangle$$

$$H_q(X,0,t) = \int d^2\mathbf{k}_{T,in}\, \boxed{\phi^*(X, \mathbf{k}_{T,in} - \boldsymbol{\Delta})\phi(X, \mathbf{k}_{T,in}).}$$

one-body **non-diagonal** density in transverse momentum

$$H_q(X,0,t) = \int d^2\mathbf{b}\, e^{i\mathbf{b}\cdot\boldsymbol{\Delta}}\, \boxed{\tilde{\phi}^*(X,\mathbf{b})\, \tilde{\phi}(X,\mathbf{b})}$$

one-body **diagonal** density in transverse space

Where we define

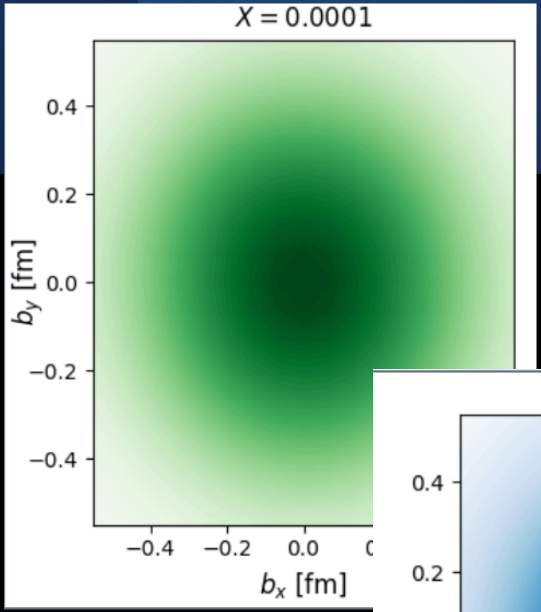
$$\phi(k_{\mathcal{X}}^+, \mathbf{k}_{T,\mathcal{X}}) \to \phi(X, \mathbf{k}_{T,in}) = \langle \mathcal{X} \mid \psi_+(0) \mid p\rangle,$$

3D Coordinate Space Representation

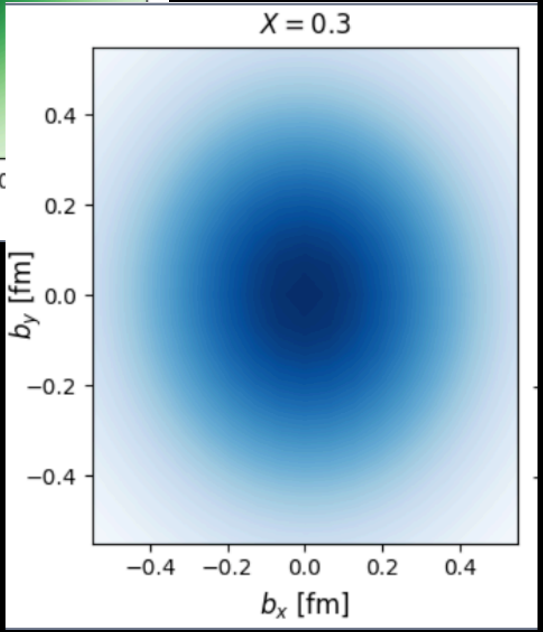
GPDs can be Fourier transformed from momentum space into coordinate space, providing insight into the spatial distributions of quarks and gluons inside the proton, besides matter and charge distributions.

$$\underbrace{\boxed{\mathcal{H}^q(X,0,b_T)}}_{\text{Wigner phase space distribution}} = \int \frac{d^2\Delta_T}{(2\pi)^2} \underbrace{\boxed{H^q(X,0,\Delta_T)}}_{\text{GPD}} e^{-i\Delta_T \cdot b_T}$$

gluon

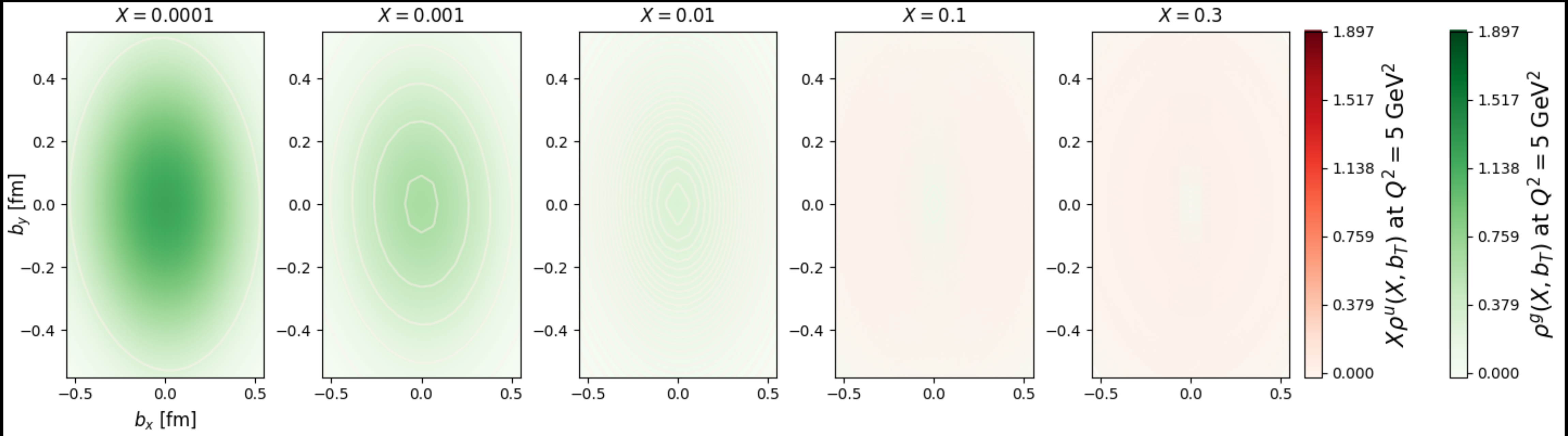


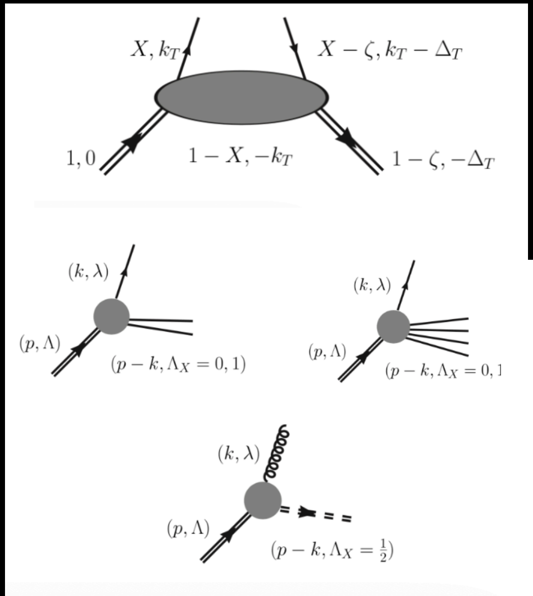
d-quark



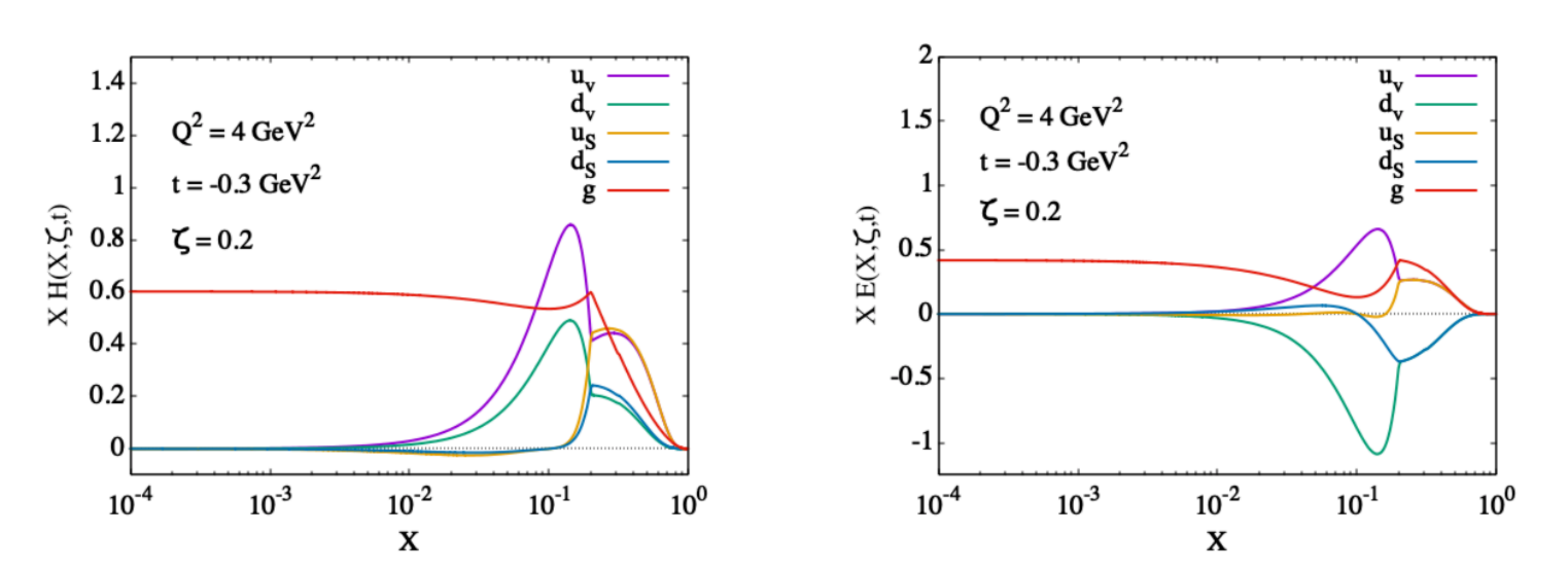
With Z. Panjsheeri and J. Bautista

Relative weight of u and g distributions





Calculations are based on a reggeized spectator model-based parametrization for all "constrainable" flavor components + gluons

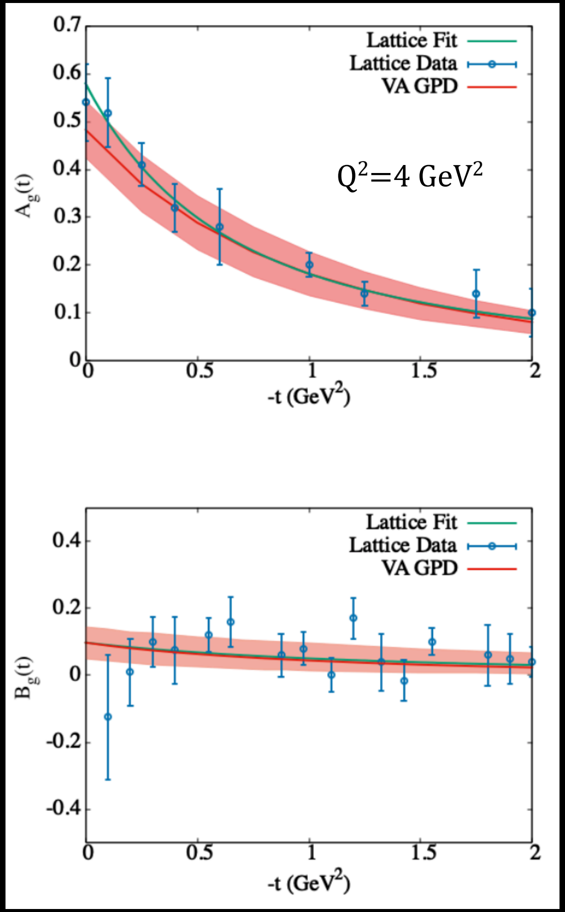


B. Kriesten. P. Velie, E. Yeats, F. Y. Lopez, & S. Liuti,
Phys.Rev.D 105 (2022) 5, 056022 and references therein

More work in progress with A. Khawaja

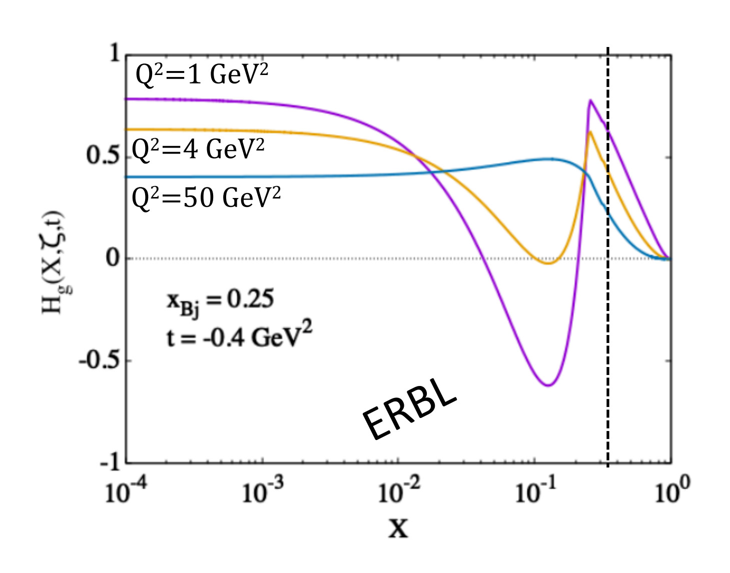
Constraint from lattice moments

$$\int_0^1 dx H^g(x,\xi,t;Q^2) = A_g(t) + (2\xi)^2 C_g(t)$$
$$\int_0^1 dx E^g(x,\xi,t;Q^2) = B_g(t) - (2\xi)^2 C_g(t)$$



Lattice points from P. Shanahan and W. Detmold, Phys. Rev. D 99, 014511(2019), 1810.04626

NLO Evolution



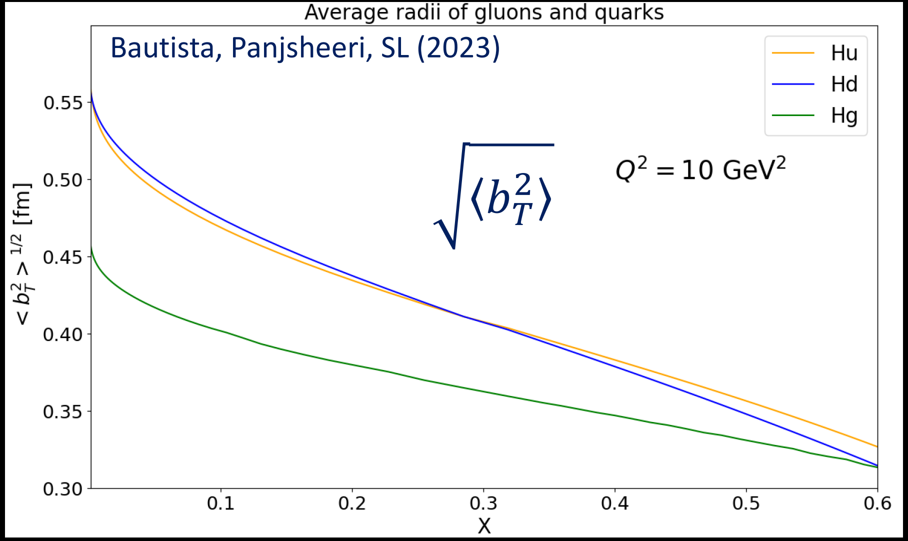
$$\frac{\partial}{\partial \ln Q^2} F_{q_v}(X,\zeta,Q^2) = \frac{\alpha_S}{2\pi} P_{qq}\left(\frac{X}{Z}, \frac{X-\zeta}{Z-\zeta}, \alpha_S\right) \otimes F_{q_v}(Z,\zeta,Q^2) \tag{96}$$

$$\frac{\partial}{\partial \ln Q^2} F^{\Sigma}(X,\zeta,Q^2) = \frac{\alpha_S}{2\pi}\left[P_{qq}\left(\frac{X}{Z}, \frac{X-\zeta}{Z-\zeta}, \alpha_S\right) \otimes F^{\Sigma}(Z,\zeta,Q^2) + 2N_f P_{qg}\left(\frac{X}{Z}, \frac{X-\zeta}{Z-\zeta}, \alpha_S\right) \otimes F_g(Z,\zeta,Q^2)\right] \tag{97}$$

$$\frac{\partial}{\partial \ln Q^2} F_g(X,\zeta,Q^2) = \frac{\alpha_S}{2\pi}\left[P_{gq}\left(\frac{X}{Z}, \frac{X-\zeta}{Z-\zeta}, \alpha_S\right) \otimes F^{\Sigma}(Z,\zeta,Q^2) + P_{gg}\left(\frac{X}{Z}, \frac{X-\zeta}{Z-\zeta}, \alpha_S\right) \otimes F_g(Z,\zeta,Q^2)\right] \tag{98}$$

Gluon and quark matter density radius

$$< b_T^2 >^q (X) = \frac{\int_0^\infty d^2 b_T \boxed{b_T^2} \mathcal{H}^q(X,0,b_T)}{\int_0^\infty d^2 b_T \mathcal{H}^q(X,0,b_T)}$$

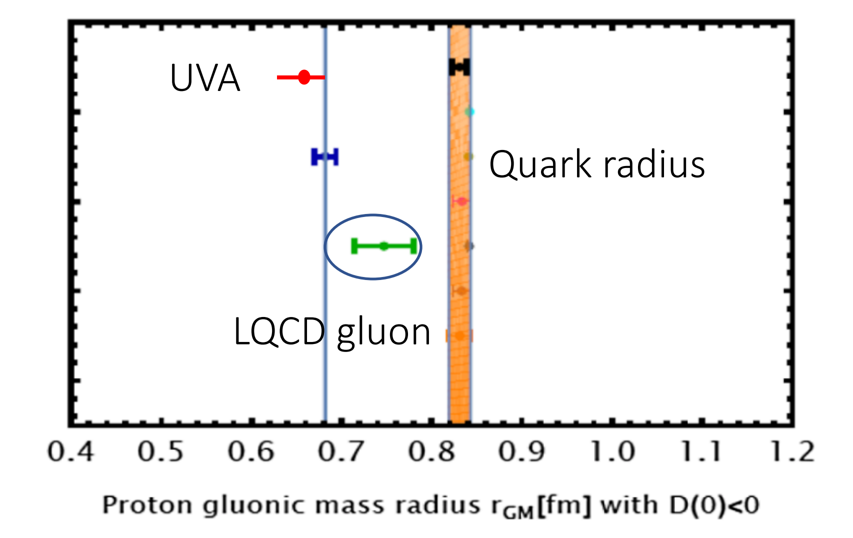


Compare to lattice and AdS/CFT integrated value

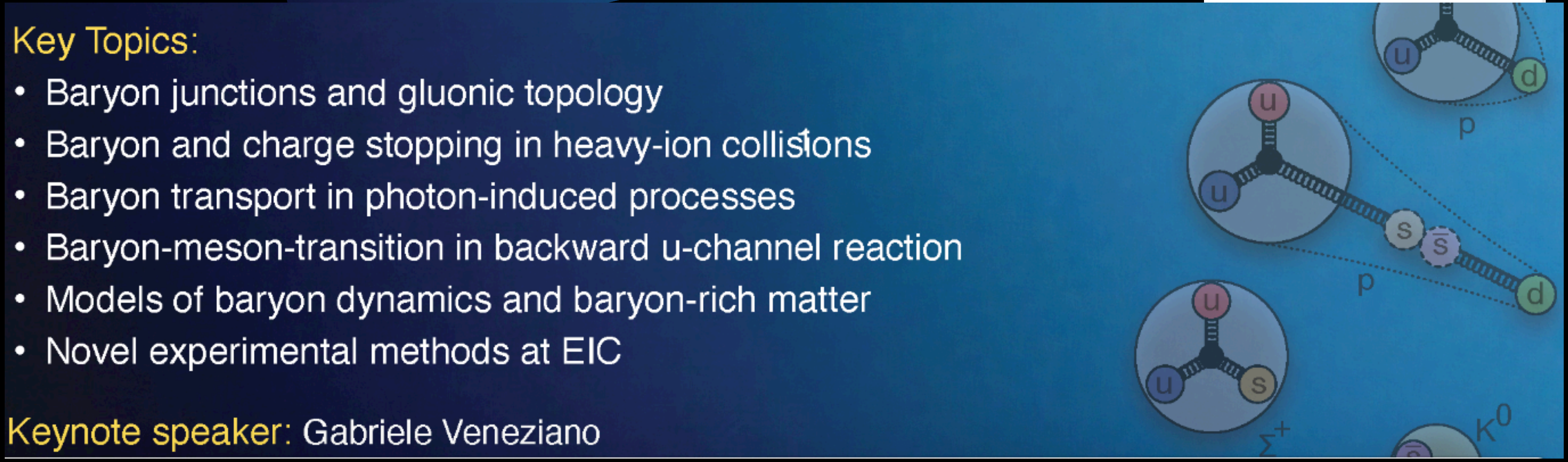
K. Mamo and I. Zaeed

PRD106, 086004 (2022)

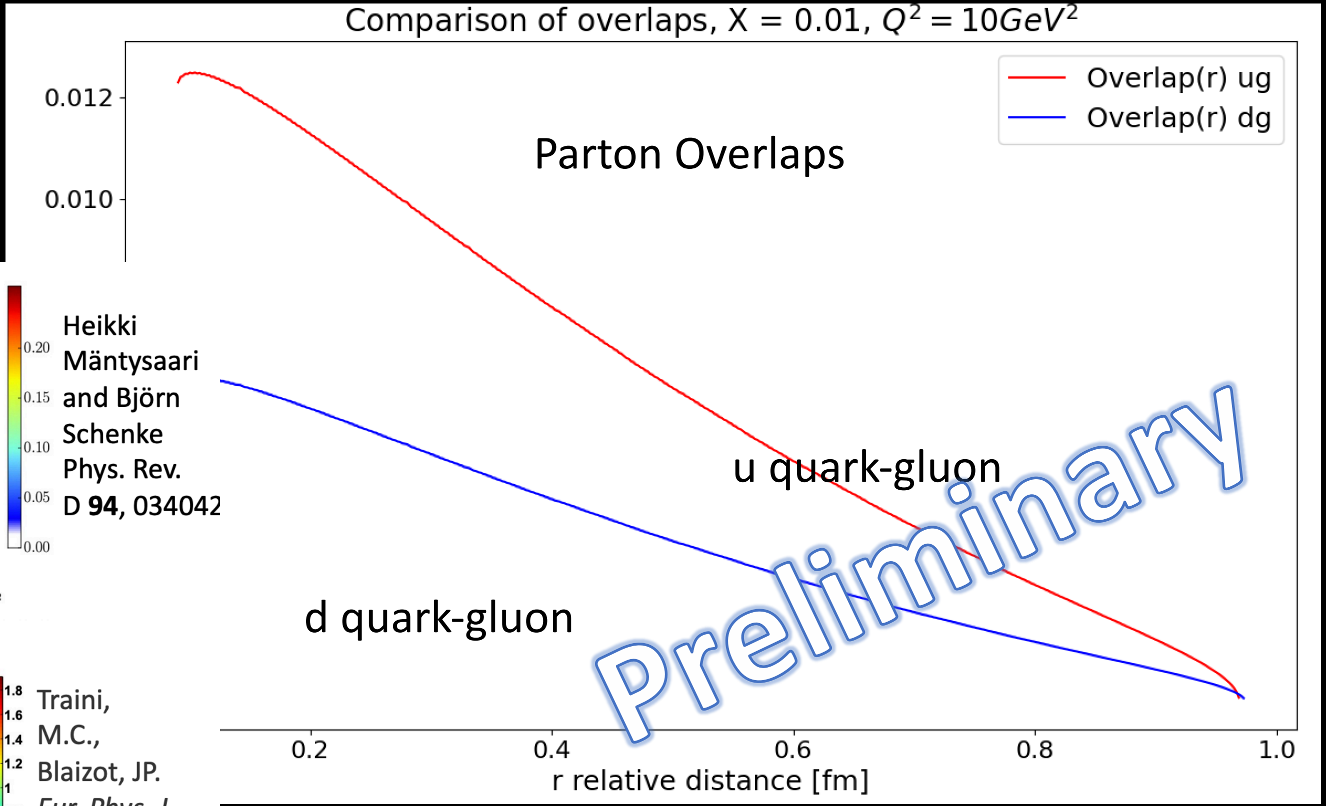
LQCD: Detmold and Shanahan

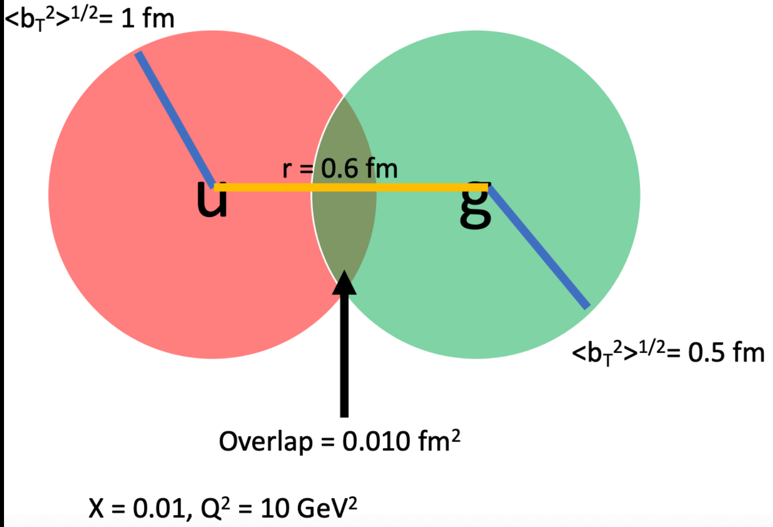


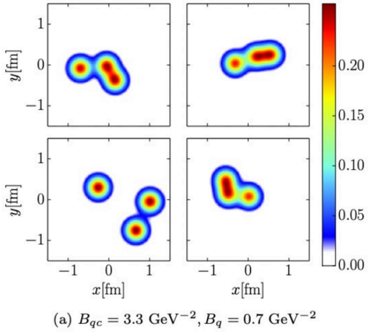
- This emerging picture supports the idea of the gluons being at the core of the nucleon and carrying baryo
number



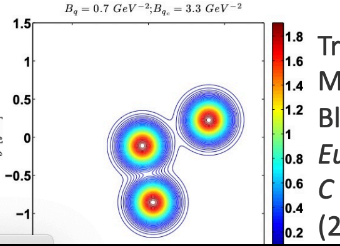
Beyond one-body densities: two body densities and parton overlaps







Heikki Mäntysaari and Björn Schenke Phys. Rev. D **94**, 034042



Traini, M.C., Blaizot, JP. *Eur. Phys. J. C* **79**, 327 (2019)

J. Bautista, Z. Panjsheeri, SL, in preparation

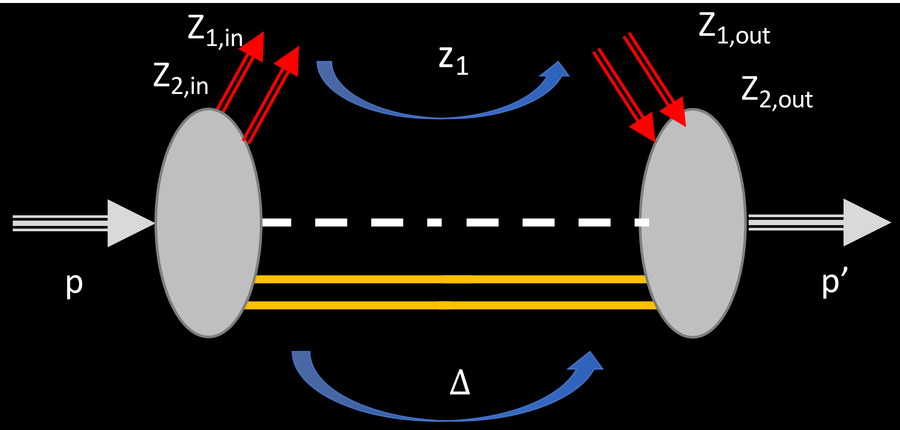
Z. Panjsheeri, SPIN 2023

Beyond one-body densities: two body densities and parton overlaps

B. Two-body correlation function

The two-body correlation function is defined by a bilinear expression [17],

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^-_{1,in} d\mathbf{z}_{1,T,in}}{(2\pi)^3} \frac{dz^-_{2,in} d\mathbf{z}_{2,T,in}}{(2\pi)^3} \int \frac{dz^-_{1,out} d\mathbf{z}_{1,T,out}}{(2\pi)^3} \frac{dz^-_{2out} d\mathbf{z}_{2,T,out}}{(2\pi)^3}$$
$$\times\, e^{i(k_{1,in} z_{1,in} + k_{2,in} z_{2,in})}\, e^{-i(k_{1,out} z_{1,out} + k_{2,out} z_{2,out})} \langle p', \Lambda' | \overline{\psi}(z_{1,out})\, \Gamma \psi(z_{1,in})\, \overline{\psi}(z_{2,out})\, \Gamma \psi(z_{2,in}) | p, \Lambda \rangle \Big|_{z_1^+ = z_2^+ = 0}$$



…manipulate the hadronic tensor similarly to the one-body case

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz_1^- d\mathbf{z}_{1,T}}{(2\pi)^3} \frac{dz^- d\mathbf{z}_T}{(2\pi)^3} \int \frac{dy^- d\mathbf{y}_T}{(2\pi)^3} \times e^{i\Delta_2 y} e^{i(k_1+\Delta/2+k_2)z_1} e^{i(k_2 z)} \sum_{\mathcal{X}} \langle p', \Lambda' | \boxed{\overline{\psi}_+(0)\, \overline{\psi}_+(y - z/2)} | \mathcal{X} \rangle \langle \mathcal{X} | \boxed{\psi_+(z_1)\, \psi_+(y + z/2 + z_1)} | p, \Lambda \rangle$$

qq scattering

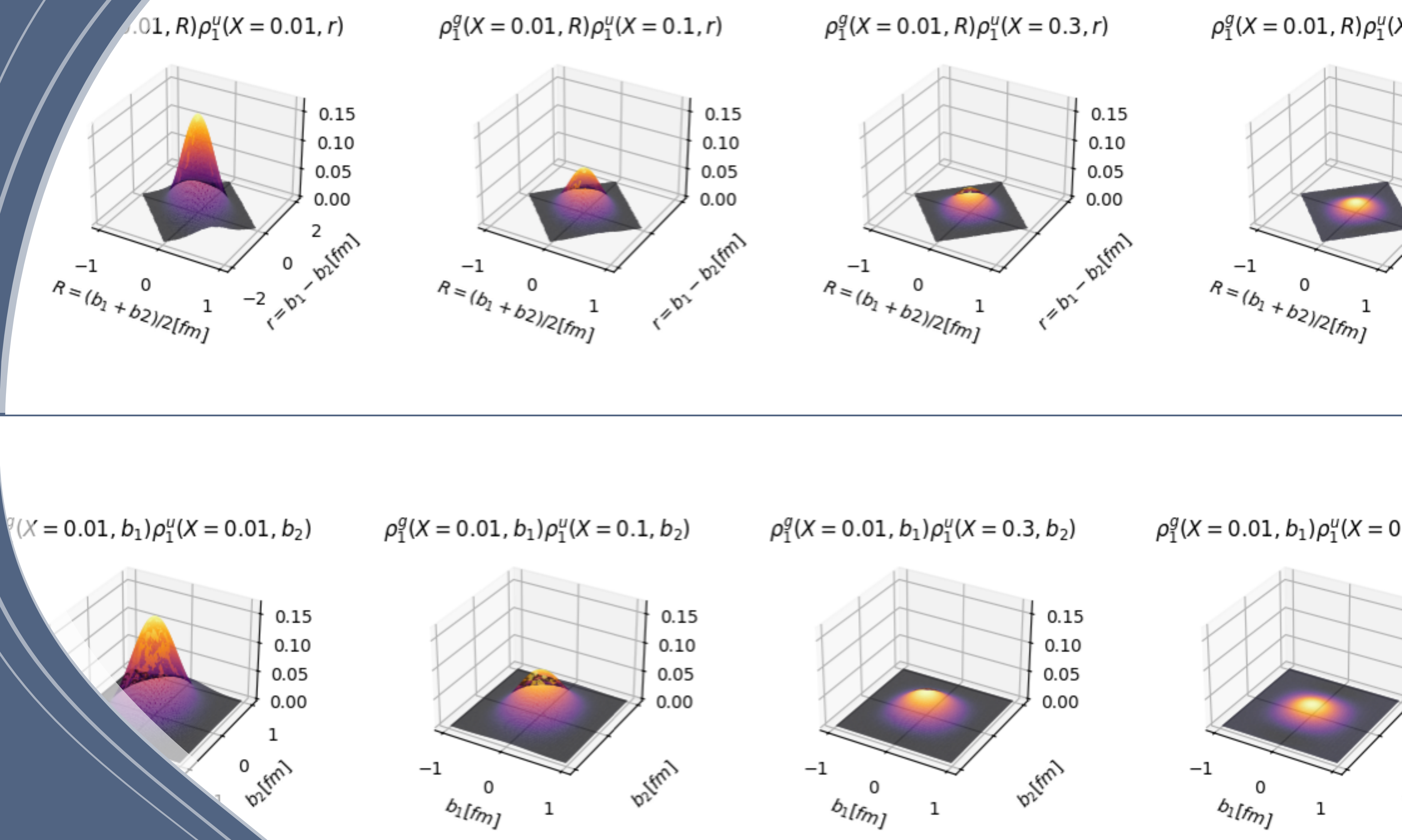
Double GPD

$$H_{qq}(X_1, X_2, 0, t_1, t_2) = \int d^2\mathbf{k}_T \int \frac{dy^- d\mathbf{y}_T}{(2\pi)^3} e^{i(\Delta_2 - k_2)y} \langle p', \Lambda' | \overline{\psi}_+(0)\, \overline{\psi}_+(y) | \mathcal{X} \rangle \langle \mathcal{X} | \psi_+(0)\, \psi_+(y) | p, \Lambda \rangle$$

Relevant distances

$y = b_1 - b_2$ ➔ relative position of parton 1 and 2

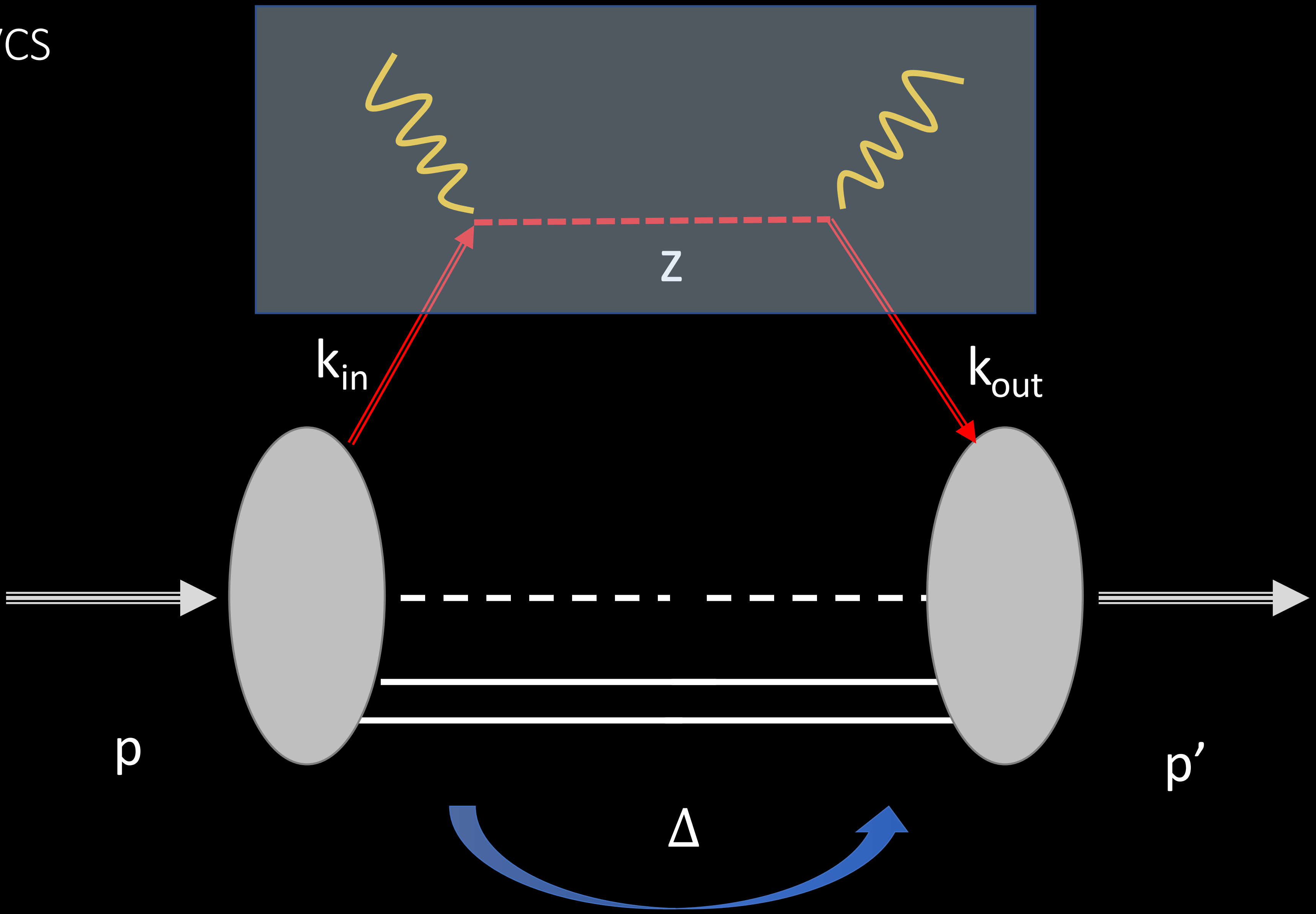
z_1, z_2 ➔ LC distance between "in" and "out" partons

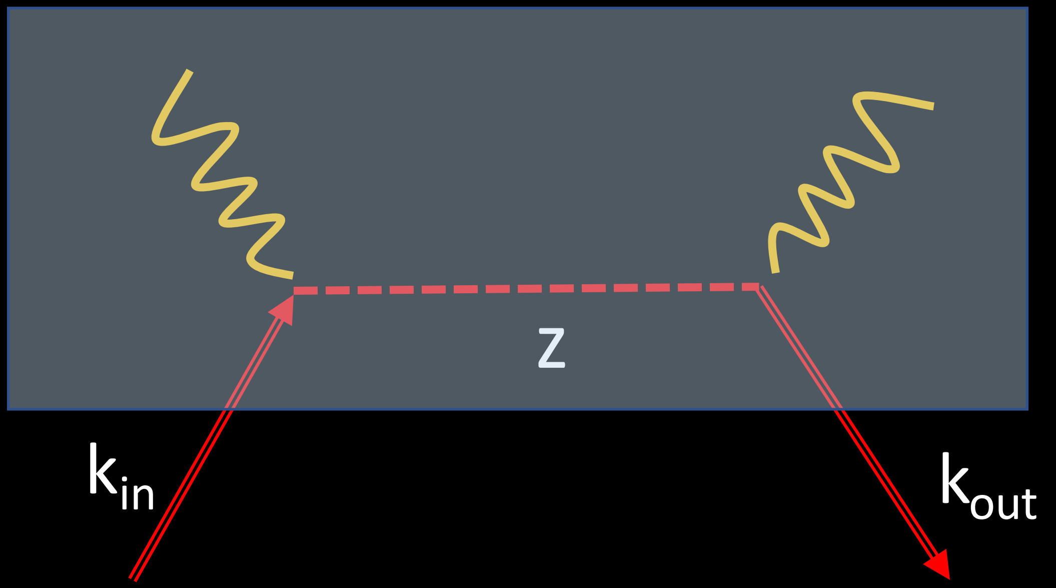


.01, R)ρ1u(X = 0.01, r)
ρ1g(X = 0.01, R)ρ1u(X = 0.1, r)
ρ1g(X = 0.01, R)ρ1u(X = 0.3, r)
ρ1g(X = 0.01, R)ρ1u(X
0.15
0.10
0.05
0.00
R = (b1 + b2)/2[fm]
r = b1 − b2[fm]
(X = 0.01, b1)ρ1u(X = 0.01, b2)
ρ1g(X = 0.01, b1)ρ1u(X = 0.1, b2)
ρ1g(X = 0.01, b1)ρ1u(X = 0.3, b2)
ρ1g(X = 0.01, b1)ρ1u(X = 0
b1[fm]
b2[fm]

How to probe all this

Back to DVCS





At leading order in pQCD

$$\int_{-1}^{1} dX \, \frac{1}{X - \zeta + i\epsilon} = P.V. \int_{-1}^{1} dX \, \frac{1}{X - \zeta} - i\pi\delta(X - \zeta)$$

At Jlab kinematics

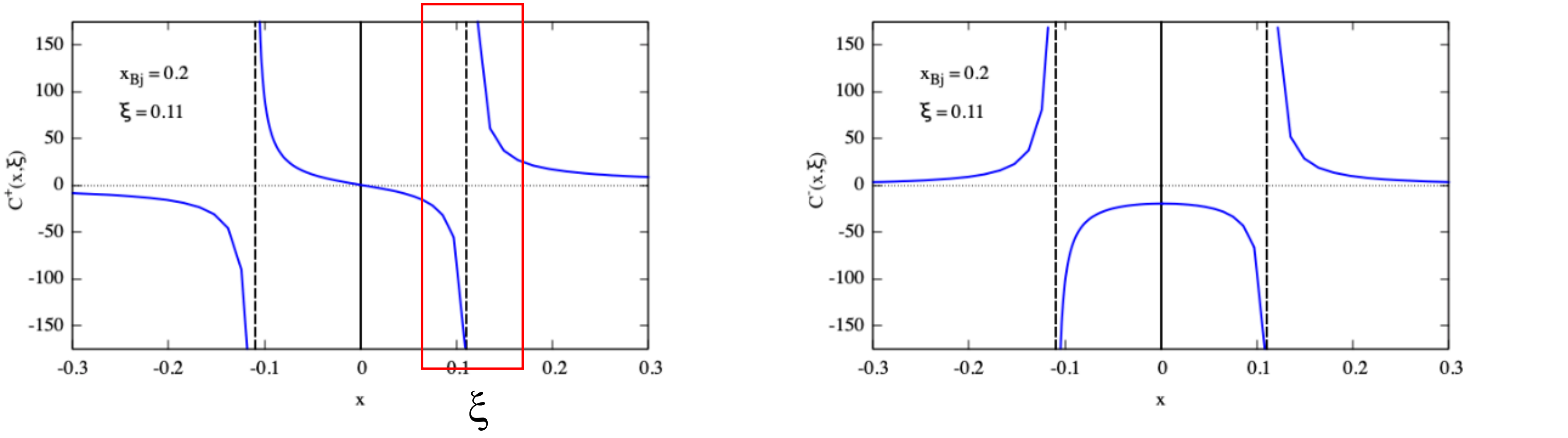


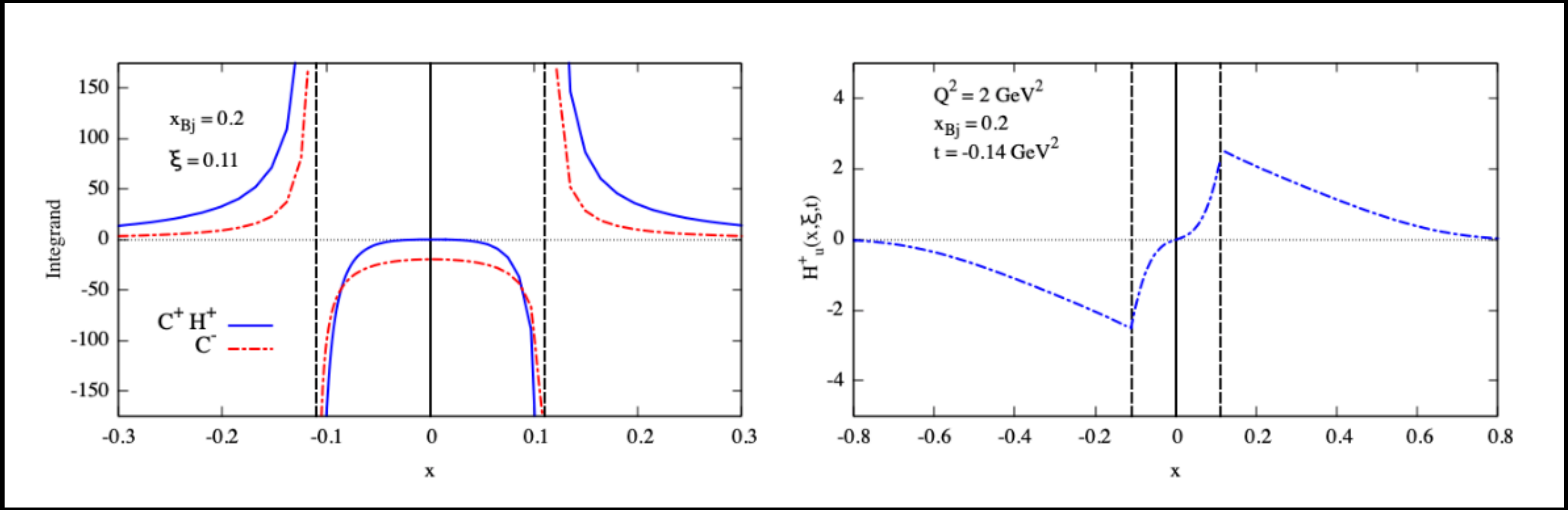
FIG. 7. Coefficients of the Compton form factors, C^+ (left) and C^- (right) (Eqs.(44)). The figure visualizes the symmetries around the $x = 0$, displayed in Eqs.(47).

$$\Re e \mathcal{H}_q = P.V. \int_{-1}^{1} dx\, H_q^+(x,\xi,t) \left(\frac{1}{x-\xi} + \frac{1}{x+\xi} \right) = P.V. \int_0^1 dx\, \frac{H_q^+(x,\xi,t)}{x-\xi} + \int_0^1 dx\, \frac{H_q^+(x,\xi,t)}{x+\xi} \tag{57}$$

$$\Re e \widetilde{\mathcal{H}}_q = P.V. \int_{-1}^{1} dx\, \widetilde{H}_q(x,\xi,t) \left(\frac{1}{x-\xi} - \frac{1}{x+\xi} \right) = P.V. \int_0^1 dx\, \frac{\widetilde{H}_q^+(x,\xi,t)}{x-\xi} - \int_0^1 dx\, \frac{\widetilde{H}_q^+(x,\xi,t)}{x+\xi} \tag{58}$$

With B. Kriesten et al., in preparation

DVCS experiments are sensitive to the anti-symmetric (flavor singlet) part of GPDs only



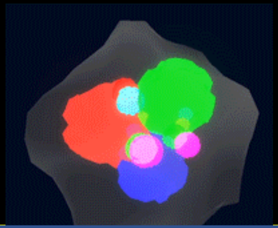
integrand

Anti-symmetric GPD component

- ✓ Because of many intricate reasons, despite the efforts at HERMES, HERA, Jlab, COMPASS, no GPD let alone image of the quark gluon structure is yet available

- ✓ New, alternative and/or more refined numerical methods are a must

The EXCLAIM project (EXCLusives with Artificial Intelligence and Machine learning)



OUR PEOPLE

Computer Science/Machine Learning: Douglas Adams, Tareq Alghamdi, GiaWei Chern, Brandon Kriesten, Yaohang Li, RA2

Experiment: Marie Boer, Debaditya Biswas

Lattice QCD: Michael Engelhardt, Huey Wen Lin, (Postdoc)

Phenomenology/Theory: Joshua Bautista, Marija Cuic, Andrew Dotson, Gary Goldstein, Carter Gustin, Adil Khawaja, SL, Zaki Panjsheeri, Kiara Ruffin, Matt Sievert, Dennis Sivers, RA2 NMSU, (Saraswati Pandey)

OUR PLAN

EXCLAIM is developing ***physics aware*** networks by using ***theory constraints*** in ***deep learning*** models (not PINN)

1. ML is not treated as a set of “black boxes” whose working is not fully controllable

2. Utilize concepts in *information theory and quantum information theory* to interpret the working of ML algorithms necessary to extract information from data

3. At the same time, use ML methods as a testing ground for the working of quantum information theory in a large class of deeply virtual scattering processes

Step 2. Extracting the observables from the cross section

Background

$$|T^{BH}_{UU}|^2 = \frac{\Gamma}{t}\left[A^{BH}_{UU}\boxed{(F_1^2 + \tau F_2^2)} + B^{BH}_{UU}\boxed{\tau G_M^2(t)}\right]$$

Physics to extract

$$|T^{\mathcal{I}}_{UU}|^2 = \frac{\Gamma}{Q^2 t}\left[A^{\mathcal{I}}_{UU}\Re e\boxed{(F_1\mathcal{H} + \tau F_2\mathcal{E})} + B^{\mathcal{I}}_{UU}G_M\Re e\boxed{(\mathcal{H}+\mathcal{E})} + C^{\mathcal{I}}_{UU}\boxed{G_M\Re e\widetilde{\mathcal{H}}}\right]$$

$$|T^{\mathcal{I}}_{LU}|^2 = \frac{\Gamma}{Q^2 t}\left[A^{\mathcal{I}}_{LU}\Im m\boxed{(F_1\mathcal{H} + \tau F_2\mathcal{E})} + B^{\mathcal{I}}_{LU}G_M\Im m\boxed{(\mathcal{H}+\mathcal{E})} + C^{\mathcal{I}}_{LU}\boxed{G_M\Im m\widetilde{\mathcal{H}}}\right]$$

$$|T^{DVCS}_{UU}|^2 = \frac{\Gamma}{Q^2}\frac{2}{1-\epsilon}\Big[(1-\xi^2)\left[(\Re e\mathcal{H})^2 + (\Im m\mathcal{H})^2 + (\Re e\widetilde{\mathcal{H}})^2 + (\Im m\widetilde{\mathcal{H}})^2\right]$$
$$+ \frac{t_o - t}{4M^2}\left[(\Re e\mathcal{E})^2 + (\Im m\mathcal{E})^2 + \boxed{\xi^2(\Re e\widetilde{\mathcal{E}})^2 + \xi^2(\Im m\widetilde{\mathcal{E}})^2}\right]$$
$$- 2\xi^2\left(\Re e\mathcal{H}\,\Re e\mathcal{E} + \Im m\mathcal{H}\Im m\mathcal{E} + \Re e\widetilde{\mathcal{H}}\,\Re e\widetilde{\mathcal{E}} + \Im m\widetilde{\mathcal{H}}\Im m\widetilde{\mathcal{E}}\right)\Big]$$

Compton Form Factors

- B. Kriesten et al, *Phys.Rev. D* 101 (2020)
- B. Kriesten and S. Liuti, *Phys.Rev. D105 (2022),* arXiv 2004.08890
- B. Kriesten and S. Liuti, Phys. Lett. B829 (2022), arXiv:2011.04484

Compton form factors

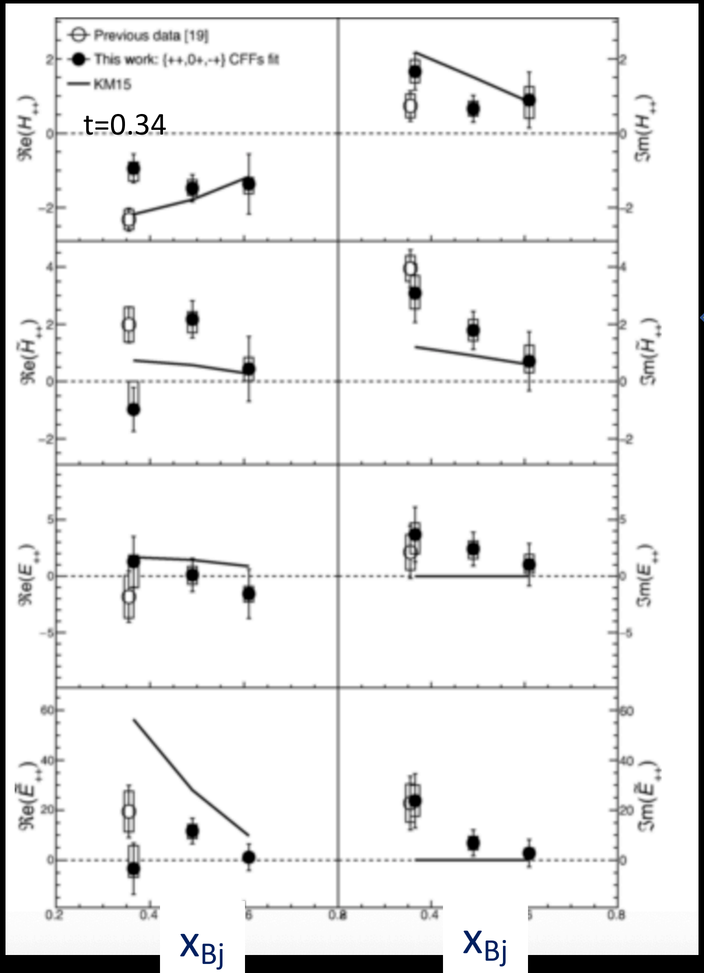
Hessian based

H

Ht

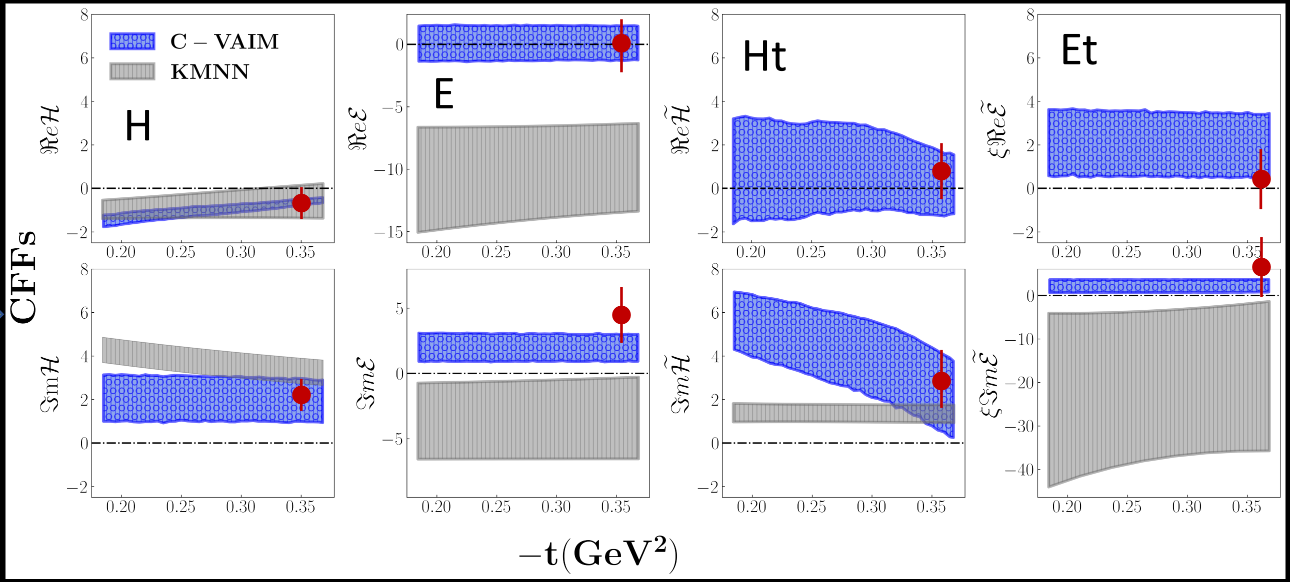
E

Et

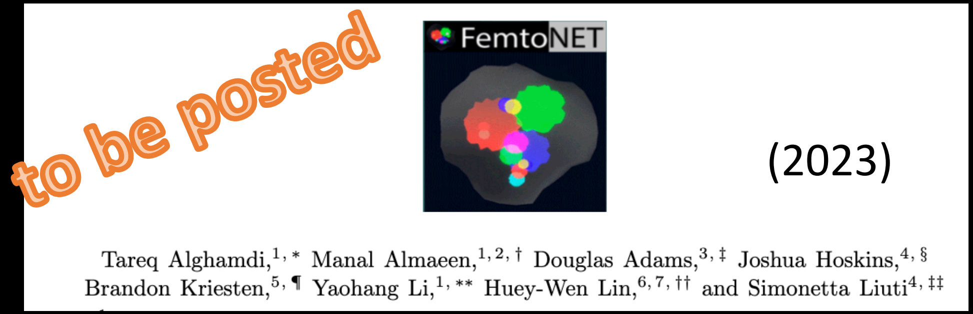


Hall A, https://arxiv.org/pdf/2201.03714.pdf

ML based



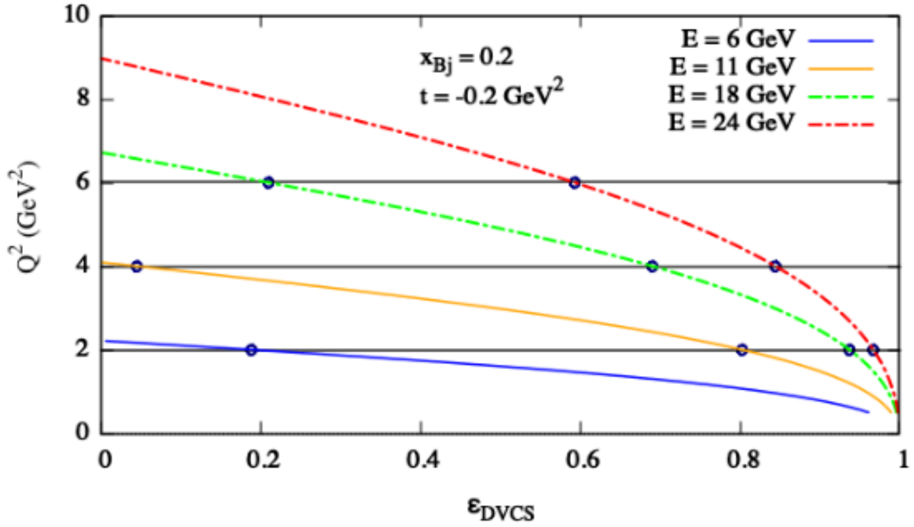
- KMNN, Cuic, Kumericki, Schaefer, https://arxiv.org/abs/2007.00029
- C-VAIM -: **A variational autoencoder inverse mapper solution to Compton form factor extraction from deeply virtual exclusive reactions**



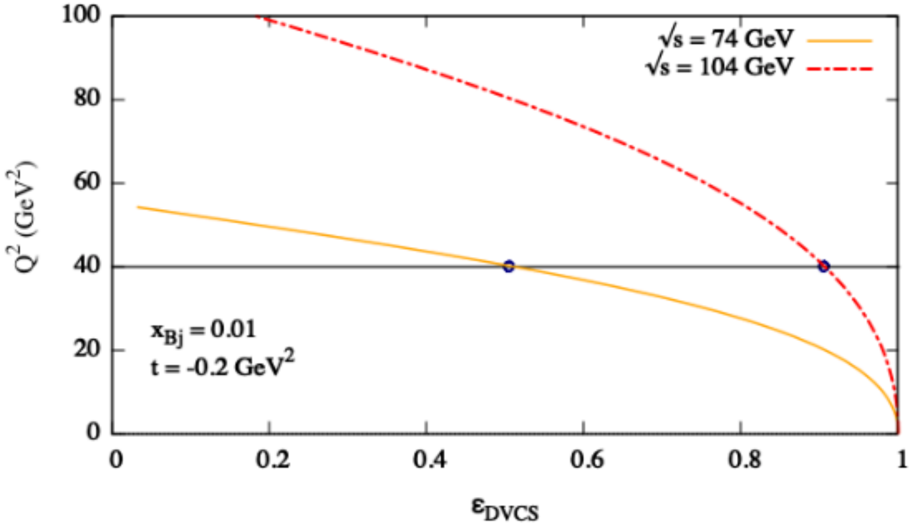
Unpolarized DVCS

$$| T_{DVCS} |^2 = F_T + \epsilon F_L + \sqrt{\epsilon(1-\epsilon)} F_{LT} \cos\phi + \epsilon F_{TT} \cos 2\phi$$

Fixed target



EIC



B. Kriesten and S. Liuti, Phys. Rev. D 105, 016015 (2022), arXiv:2004.08890 [hep-ph],

In preparation

B. Kriesten, S. Liuti, L. Calero-Diaz, D. Keller, A. Meyer, G. R. Goldstein, and J. Osvaldo Gonzalez-Hernandez, Phys. Rev. D 101, 054021 (2020), arXiv:1903.05742 [hep-ph]

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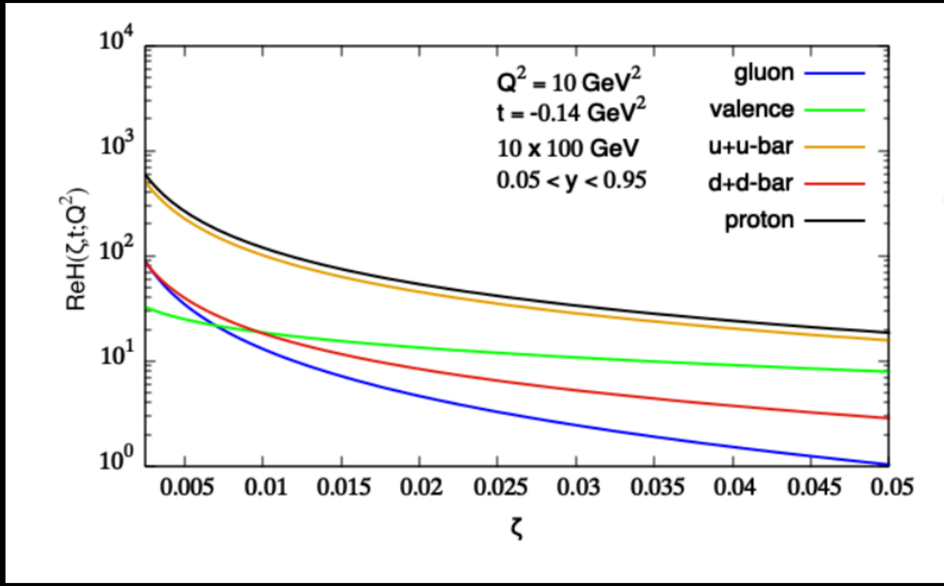
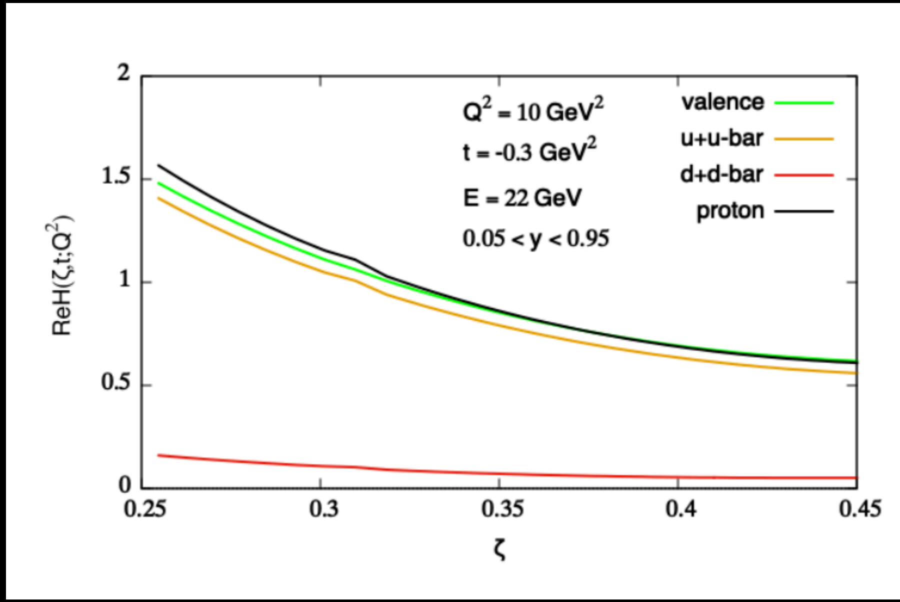
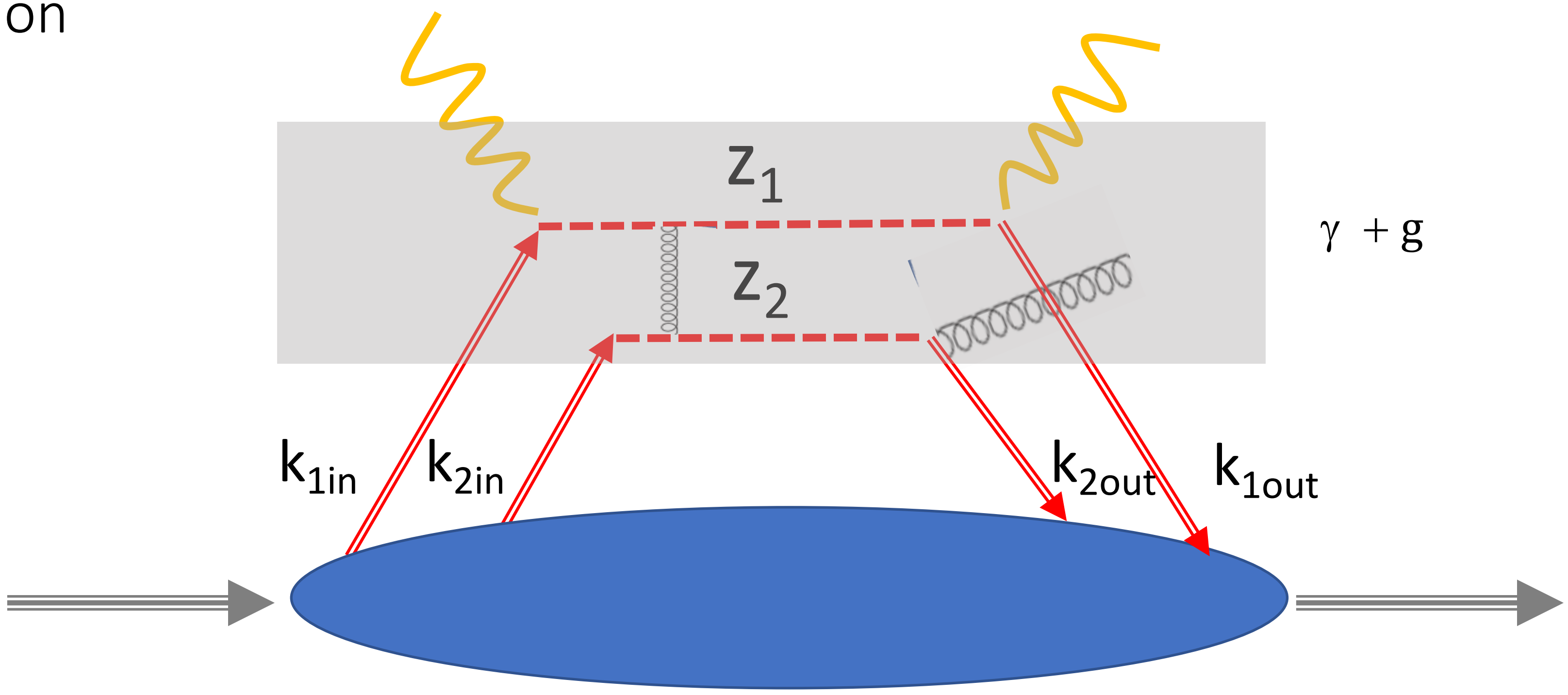


Jlab 12 GeV+

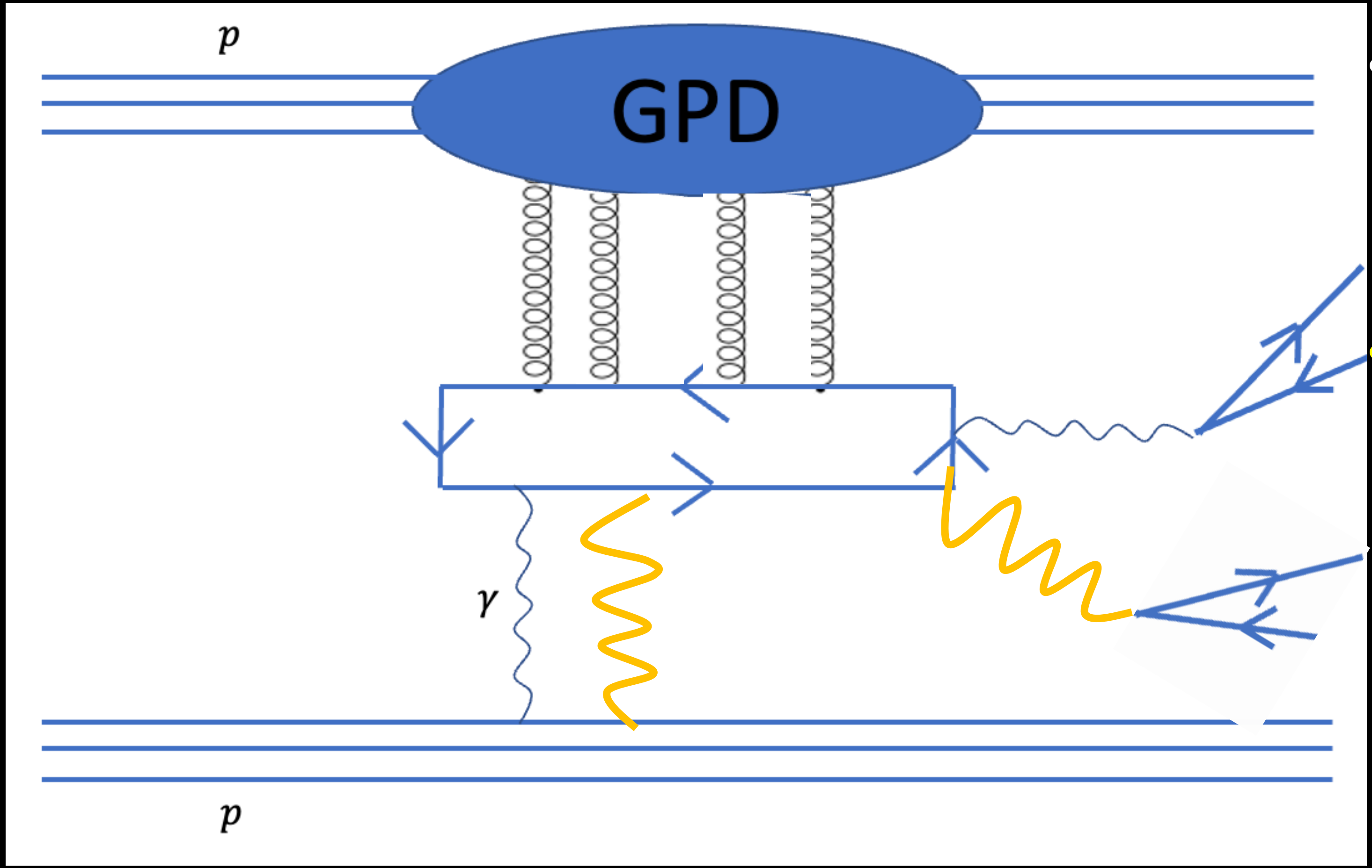


Beyond one-body exclusive detection

Two jets detection

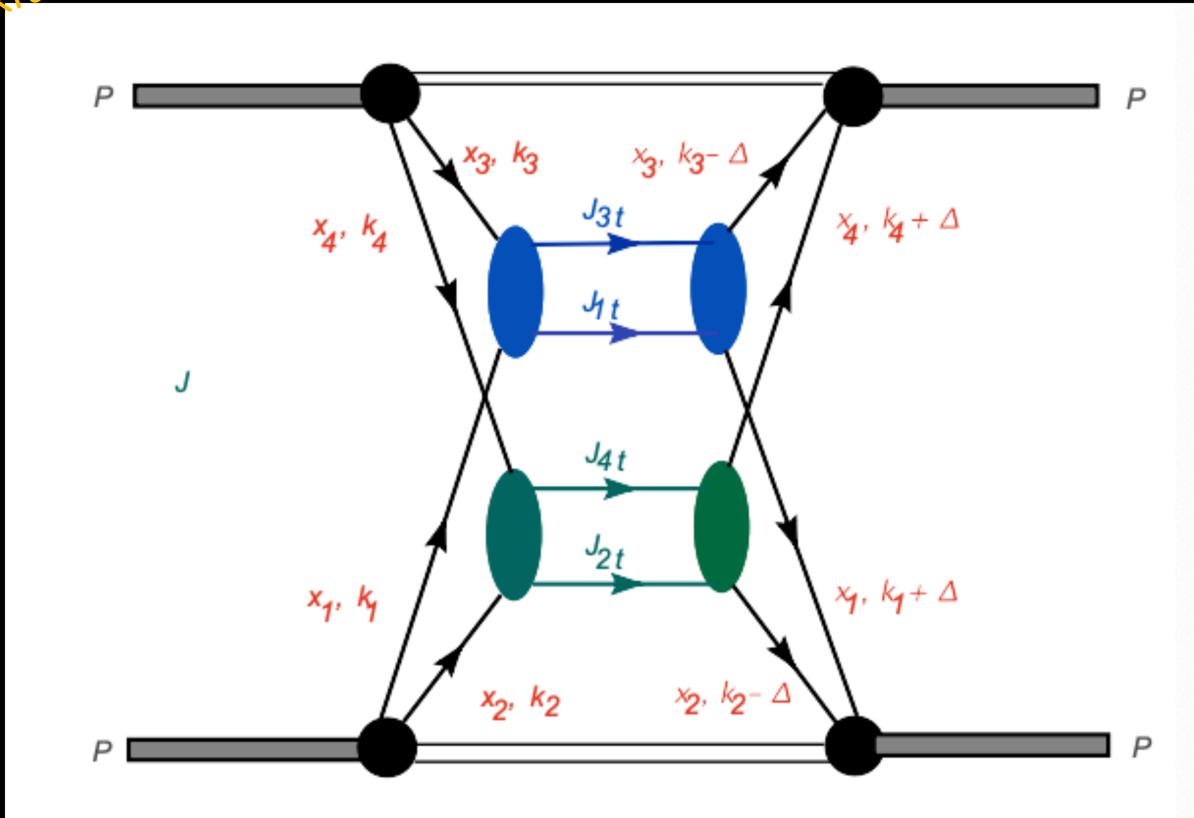


Connection to UPCs



- GPDs can be observed in UPCs as in this diagram
- This figure is equivalent to time-like Compton scattering (TCS) because a lepton pair is created in the end
- We can obtain DPDs through UPCs by observing two such scatterings

The four jet production at LHC and Tevatron in QCD.
B. Blok, Yu. Dokshitzer, L. Frankfurt, M.Strikman (PRD 2011)



Conclusions

- Extracting spatial information from data is an unprecedented challenging problem which is uniquely highly-dimensional with respect to what done so far
- Two-body densities area must to investigate the relative distance among particles: this informationis needed to locate the gluons
- How to extract it from data:
  - design experiments testing beyond "one-body" DVCS-type scenarios
  - keep developing refined numerical/ML-based approaches, as the complexity of the problems increase
  - build a platform with benchmarks for the community to compare results using consistent treatments of the uncertainties
- I left out several topics: spin configurations, nuclei are most important