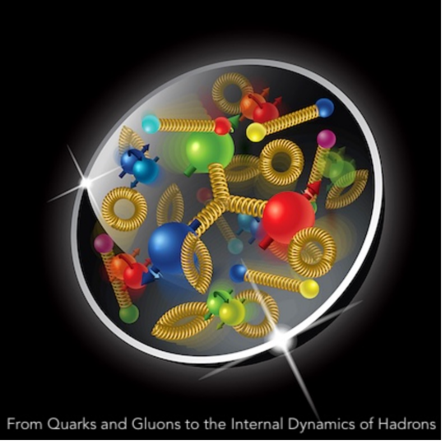


Phenomenology of transverse momentum distributions of the pion and proton

Patrick Barry, Leonard Gamberg, Wally Melnitchouk, Eric Moffat, Daniel Pitonyak, Alexei Prokudin, Nobuo Sato

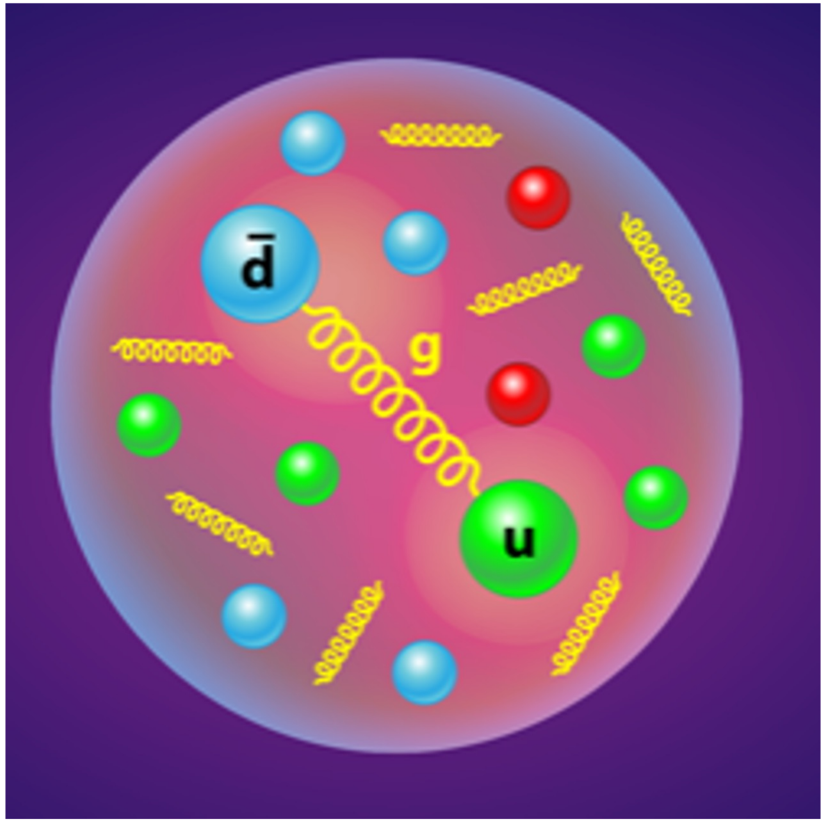
Based on: Phys. Rev. D **108**, L091504 (2023).

Argonne NATIONAL LABORATORY

Pions

- To give insights into color confined systems, we shouldn't limit ourselves to only proton structures
- Pion presents itself as a dichotomy

1. It is the Goldstone boson associated with spontaneous symmetry breaking of chiral $SU(2)_L \times SU(2)_R$ symmetry
2. Made up of quark and antiquark constituents



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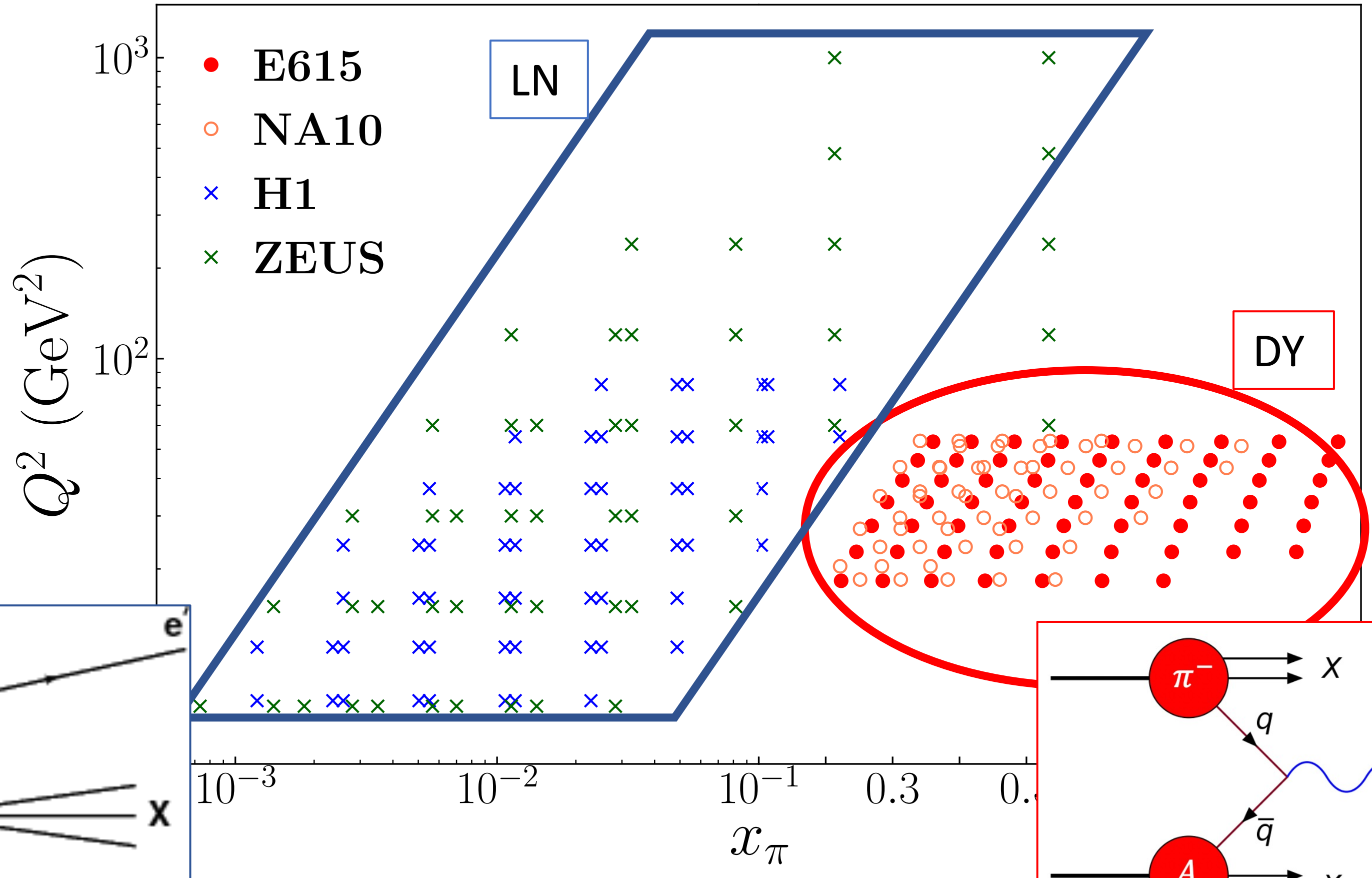
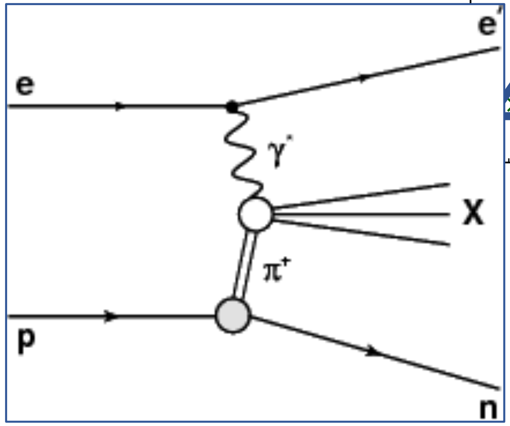
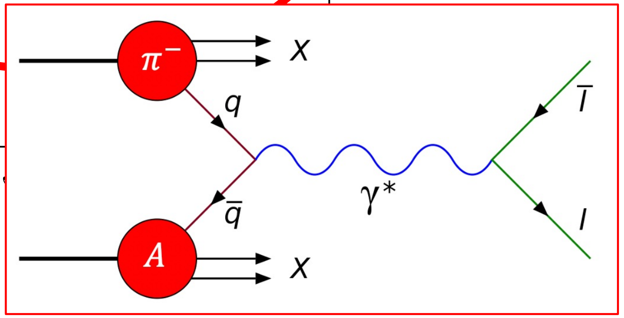
Available datasets for pion structures

- Much less available data than in the proton case

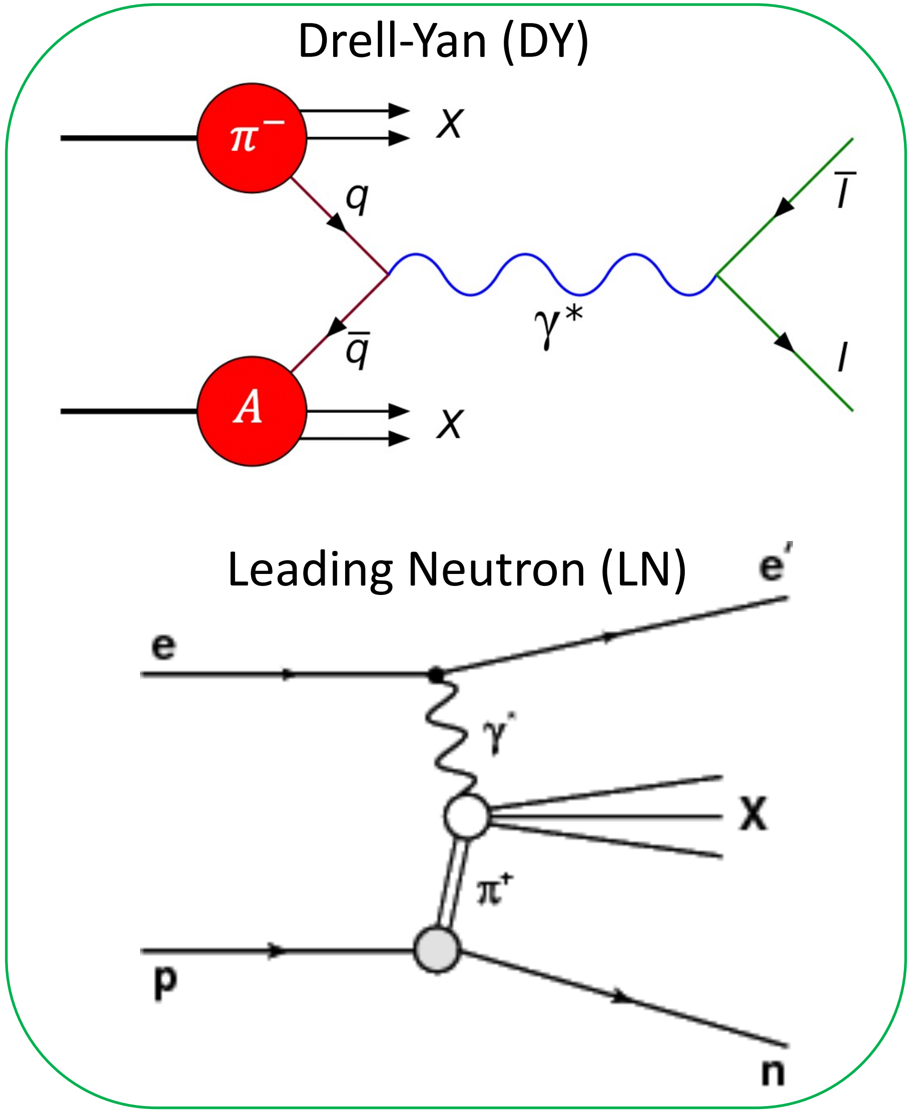
- Still valuable to study







Pion PDFs in JAM

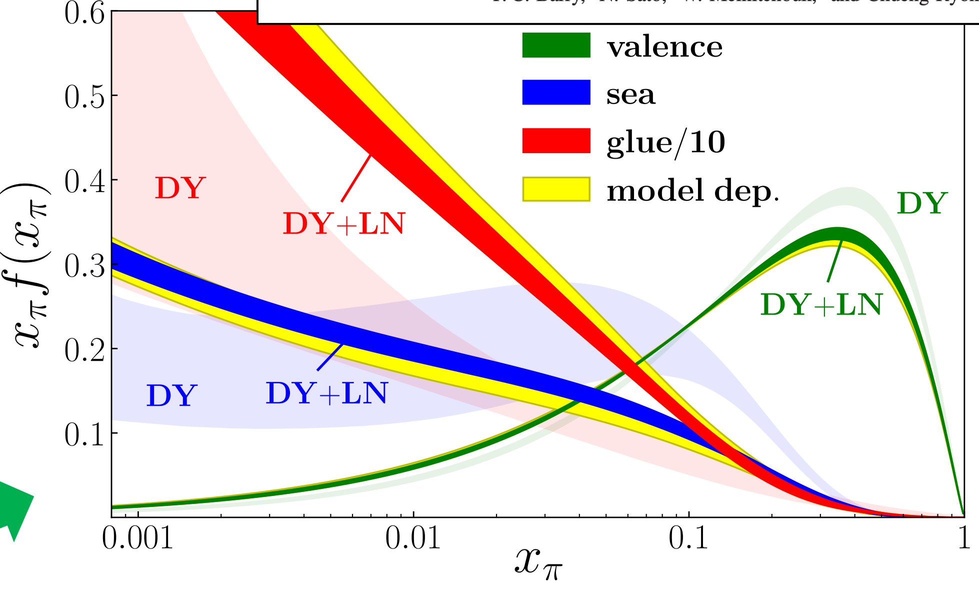


PHYSICAL REVIEW LETTERS **121**, 152001 (2018)

Featured in Physics

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

P. C. Barry,[1] N. Sato,[2] W. Melnitchouk,[3] and Chueng-Ryong Ji[1]

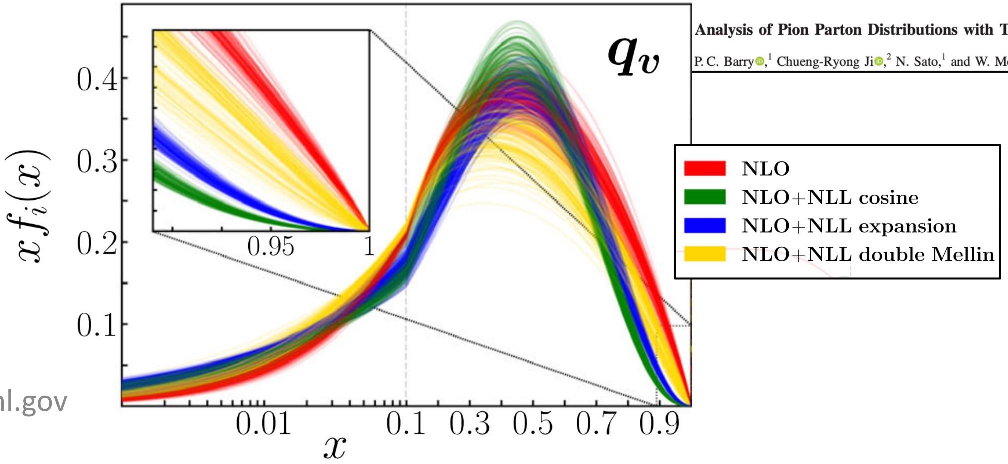


Threshold resummation in DY

PHYSICAL REVIEW LETTERS **127**, 232001 (2021)

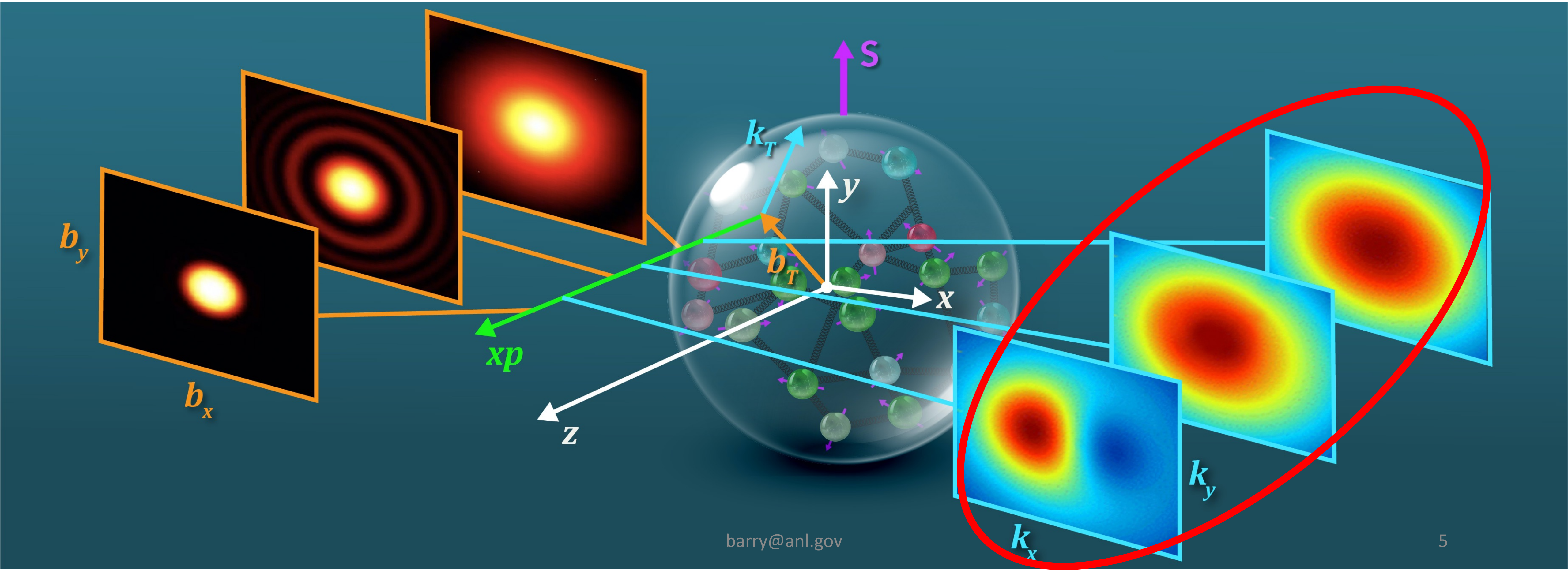
Analysis of Pion Parton Distributions with Threshold Resummation

P. C. Barry,[1] Chueng-Ryong Ji,[2] N. Sato,[1] and W. Melnitchouk[1]



3D structures of hadrons

- Even more challenging is the 3d structure through GPDs and TMDs



Unpolarized TMD PDF

$$\tilde{f}_{q/\mathcal{N}}(x, b_T) = \int \frac{\mathrm{d}b^-}{4\pi} e^{-ixP^+b^-} \mathrm{Tr}\left[\langle \mathcal{N} \,|\, \bar{\psi}_q(b)\gamma^+ \mathcal{W}(b,0)\psi_q(0) \,|\, \mathcal{N} \rangle\right]$$

$$b \equiv (b^-, 0^+, \boldsymbol{b_T})$$

- $\boldsymbol{b_T}$ is the Fourier conjugate to the intrinsic transverse momentum of quarks in the hadron, $\boldsymbol{k_T}$
- We can learn about the coordinate space correlations of quark fields in hadrons
- Modification needed for UV and rapidity divergences; acquire regulators: $\tilde{f}_{q/\mathcal{N}}(x, b_T) \to \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)$

Factorization for low-q_T Drell-Yan

- Triply differential cross section, dependent on $\tau = Q^2/S$, rapidity Y, and transverse momentum q_T
- Cross section has hard part and two functions that describe structure of beam and target
- So called "W"-term, optimized at low-q_T

$$\frac{d^3\sigma}{d\tau dY dq_T^2} = \frac{4\pi^2\alpha^2}{9\tau S^2} \sum_q H_{q\bar{q}}(Q^2, \mu) \int d^2 b_T \, e^{i b_T \cdot q_T}$$

$$\times \tilde{f}_{q/\pi}(x_\pi, b_T, \mu, Q^2) \, \tilde{f}_{\bar{q}/A}(x_A, b_T, \mu, Q^2) + \mathcal{O}\left(\frac{q_T}{Q}\right)$$

TMD PDF within the b_* prescription

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}.$$

Low-b_T: perturbative
high-b_T: non-perturbative

$$\tilde{f}_{q/\mathcal{N}(A)}(x, b_T, \mu_Q, Q^2) = (C \otimes f)_{q/\mathcal{N}(A)}(x; b_*)$$
$$\times \exp\left\{-g_{q/\mathcal{N}(A)}(x, b_T) - g_K(b_T) \ln\frac{Q}{Q_0} - S(b_*, Q_0, Q, \mu_Q)\right\}$$

Relates the TMD at small-b_T to the **collinear** PDF
⇒ TMD is sensitive to collinear PDFs

$g_{q/\mathcal{N}(A)}$: intrinsic non-perturbative TMD structure of the hadron $\mathcal{N}(A)$
g_K: universal non-perturbative Collins-Soper kernel – same in all hadrons

Controls the perturbative evolution of the TMD

Details on the analysis

- Focus on the low-energy fixed target Drell-Yan data
  - Regime available for pion physics
- Introduce proton TMDs and A-dependent TMD parameter to understand the nuclear background Phys. Rev. Lett., **129**, 242001 (2022)
- We use the MAP collaboration's parametrization for non-perturbative TMDs JHEP 10 (2022) 127
  - Only tested parametrization flexible enough to capture features of Q bins
- Perform a **simultaneous global analysis** of pion TMD and collinear PDFs, with proton (nuclear) TMDs

Note about E615 πA Drell-Yan data

- Provides both $\frac{d\sigma}{dx_F d\sqrt{\tau}}$ (p_T-integrated) **and** $\frac{d\sigma}{dx_F dp_T}$ (p_T-dependent)
  - Large constraints on π collinear PDFs from p_T-integrated
  - Large constraints on π TMD PDFs from p_T-dependent
- Projections of same events $\Rightarrow$ correlated measurements
- They have the **same luminosity** uncertainty, so they have the **same** overall **normalization** uncertainty
- To account for this, we *equate* the fitted normalizations of the two otherwise independent measurements
  - No other guidance from experiment how the uncertainties are correlated

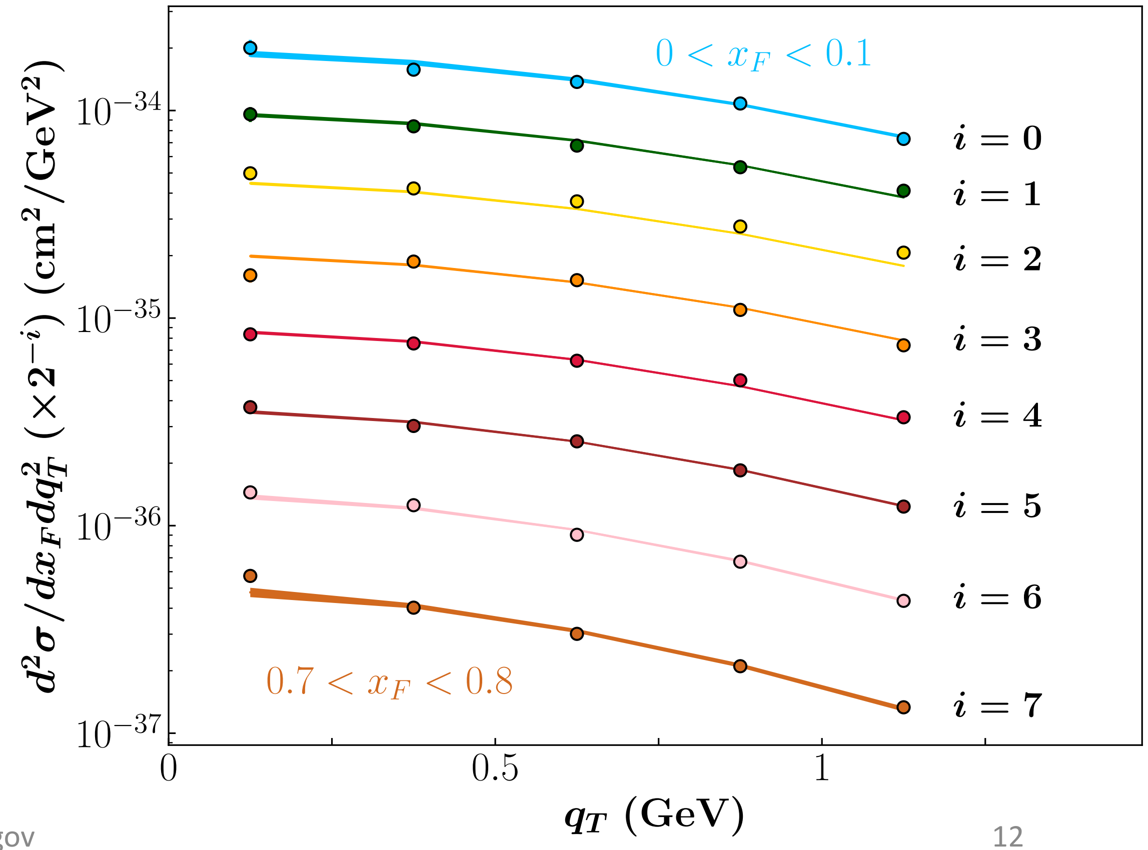
Note on collinear DY theory

- When equating the normalizations, we found
  - ***Agreement*** when using **NLO** theory on the collinear observables
  - ***Tension*** when using **NLO+NLL** threshold resummed theory on the collinear observables
- We note that in the OPE part of the TMD formalism, we use **NLO** accuracy
  - We do not use any *threshold enhancements* on the p_T-dependent observables

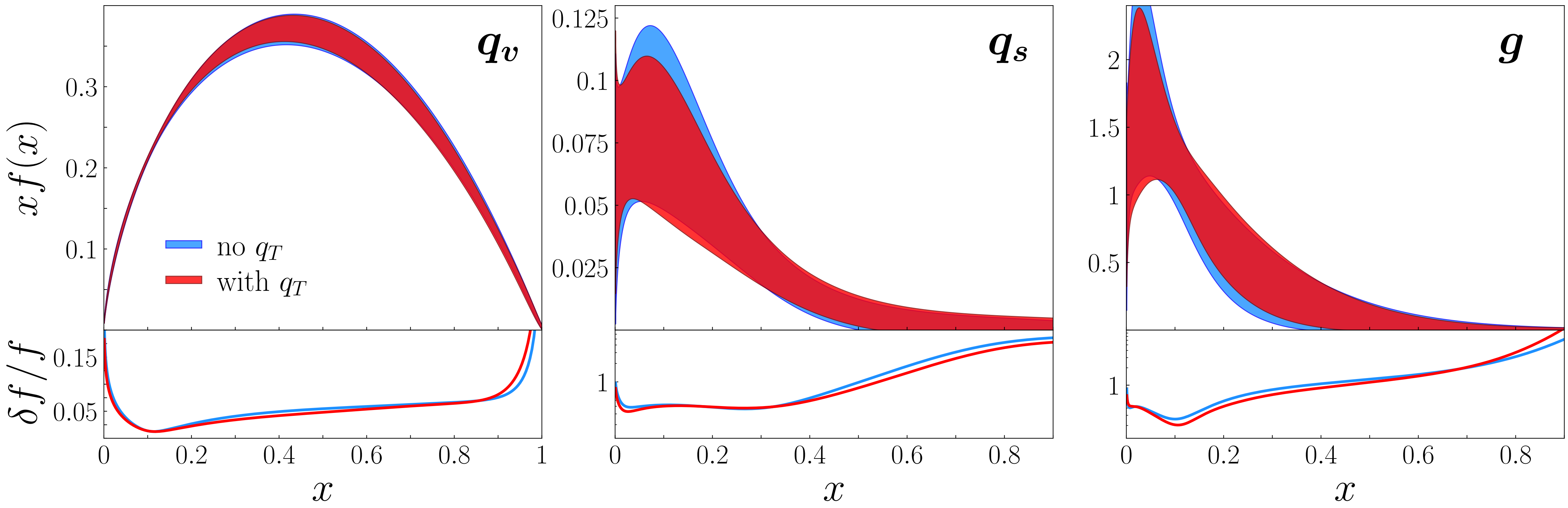
Data and theory agreement

- Fit both pA and πA DY data and achieve good agreement to both

Process	Experiment	$\sqrt{s}$ (GeV)	χ^2/N	Z-score
TMD				
q_T-dep. pA DY	E288 [90]	19.4	1.07	0.34
$pA \to \mu^+\mu^- X$	E288 [90]	23.8	0.99	0.05
	E288 [90]	24.7	0.82	0.99
	E605 [91]	38.8	1.22	1.03
	E772 [92]	38.8	2.54	5.64
(Fe/Be)	E866 [93]	38.8	1.10	0.36
(W/Be)	E866 [93]	38.8	0.96	0.15
q_T-dep. πA DY	E615 [94]	21.8	1.45	1.85
$\pi W \to \mu^+\mu^- X$	E537 [95]	15.3	0.97	0.03
collinear				
q_T-integr. DY	E615 [94]	21.8	0.90	0.48
$\pi W \to \mu^+\mu^- X$	NA10 [96]	19.1	0.59	1.98
	NA10 [96]	23.2	0.92	0.16
leading neutron	H1 [97]	318.7	0.36	4.59
$ep \to enX$	ZEUS [98]	300.3	1.48	2.15
Total			1.12	1.86



Extracted pion PDFs

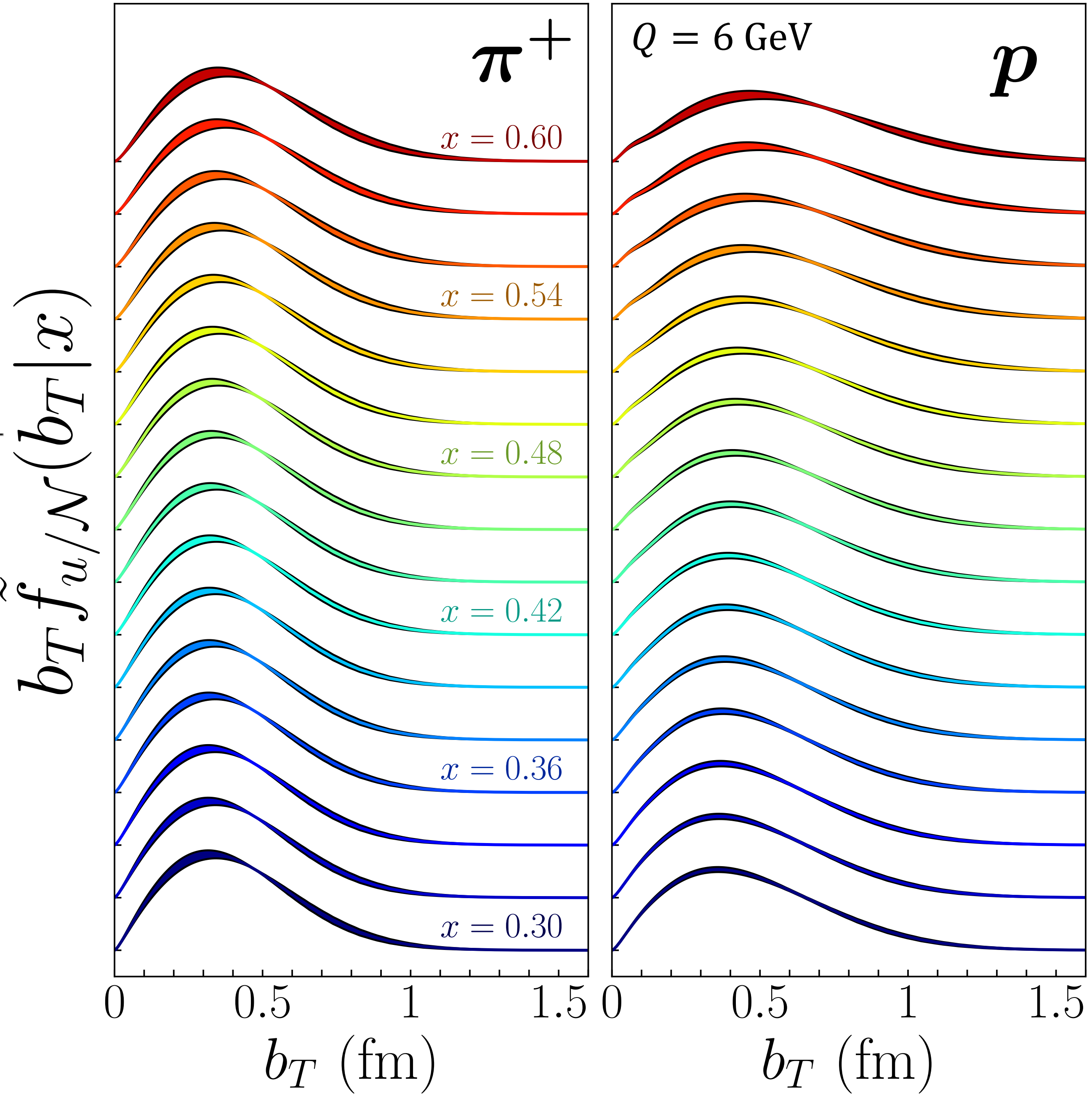


- The small-q_T data do not constrain much the PDFs

Resulting TMD PDFs of proton and pion

$$\tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2) \equiv \frac{\tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}{\int \mathrm{d}^2\boldsymbol{b}_T \tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}$$

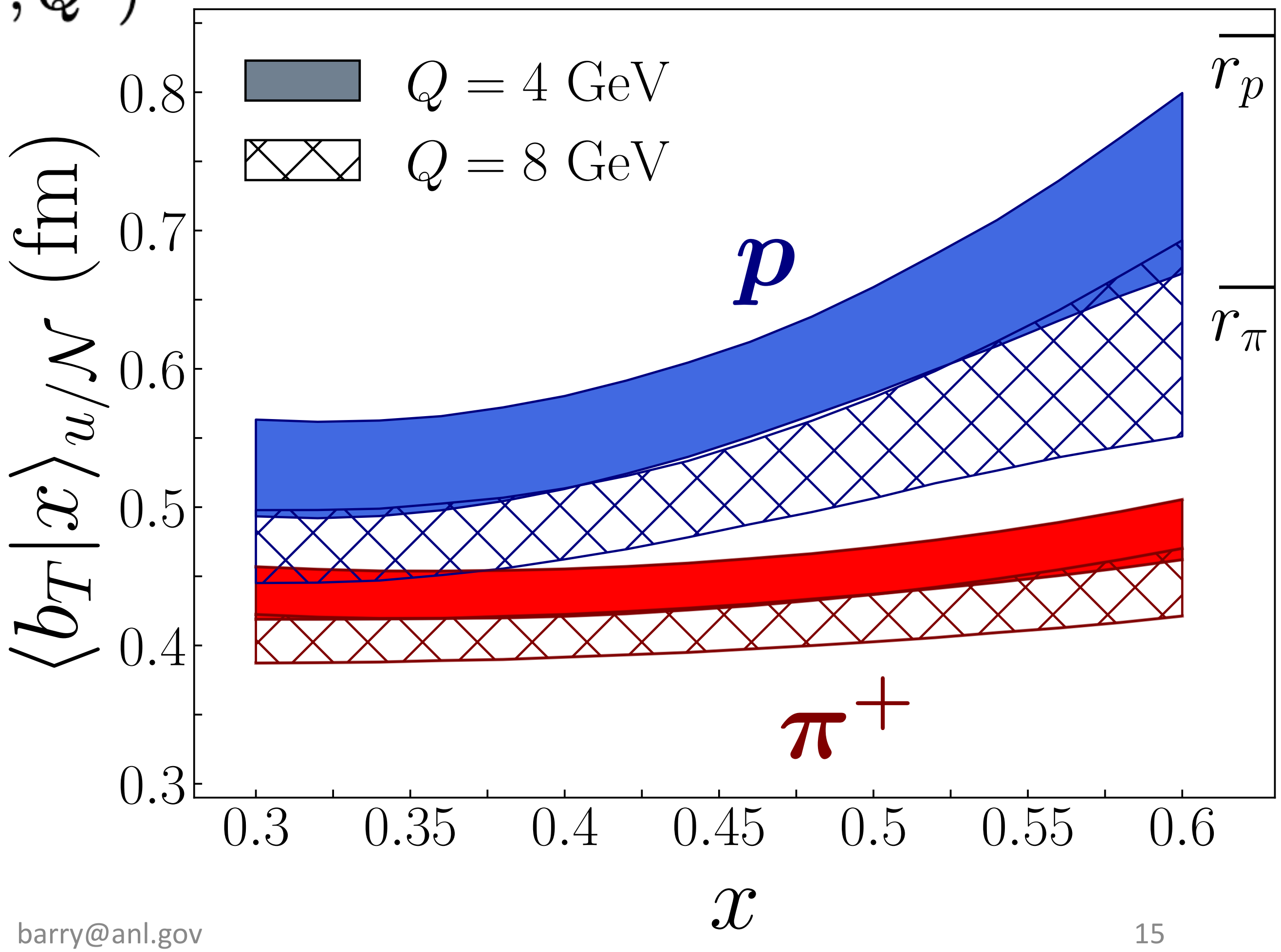
- Broadening appearing as x increases
- Up quark in pion is narrower than up quark in proton



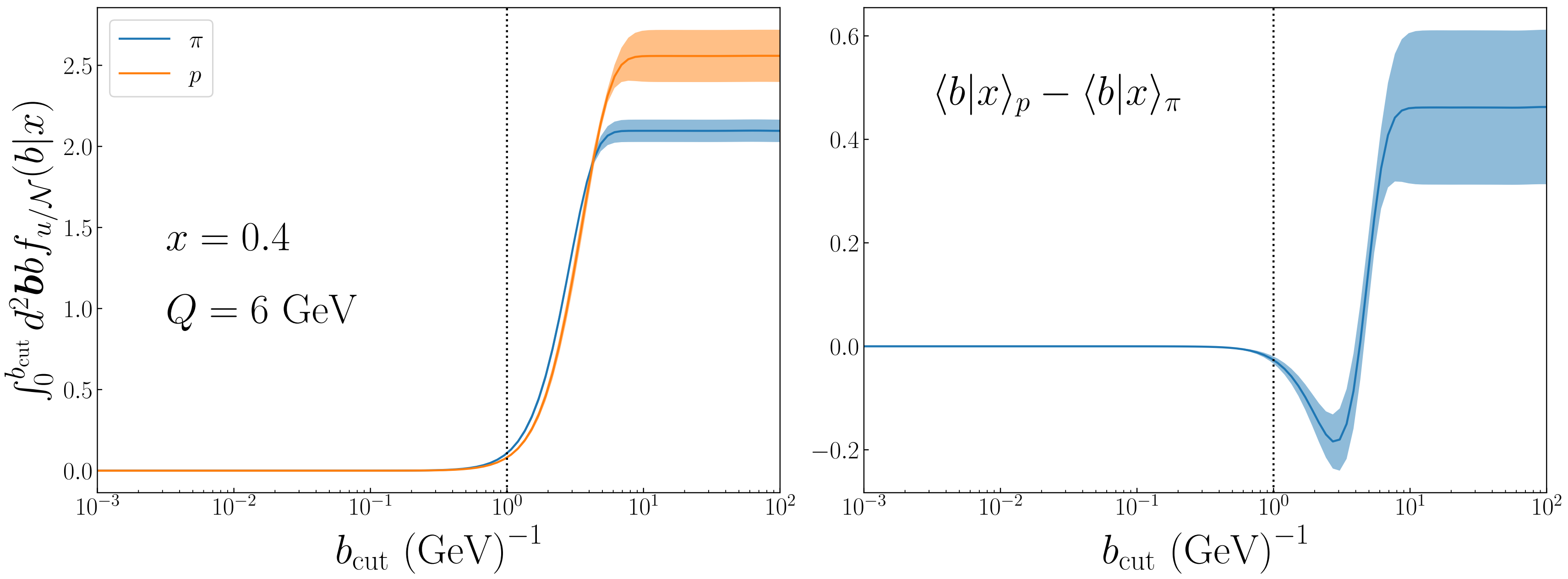
Resulting average b_T

$$\langle b_T|x\rangle_{q/\mathcal{N}} = \int \mathrm{d}^2\boldsymbol{b}_T\, b_T\, \tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2)$$

- Average transverse spatial correlation of the up quark in proton is ~ 1.2 times bigger than that of pion
- Pion's $\langle b_T|x\rangle$ is $4-5.2\sigma$ smaller than proton in this range
- Decreases as x decreases



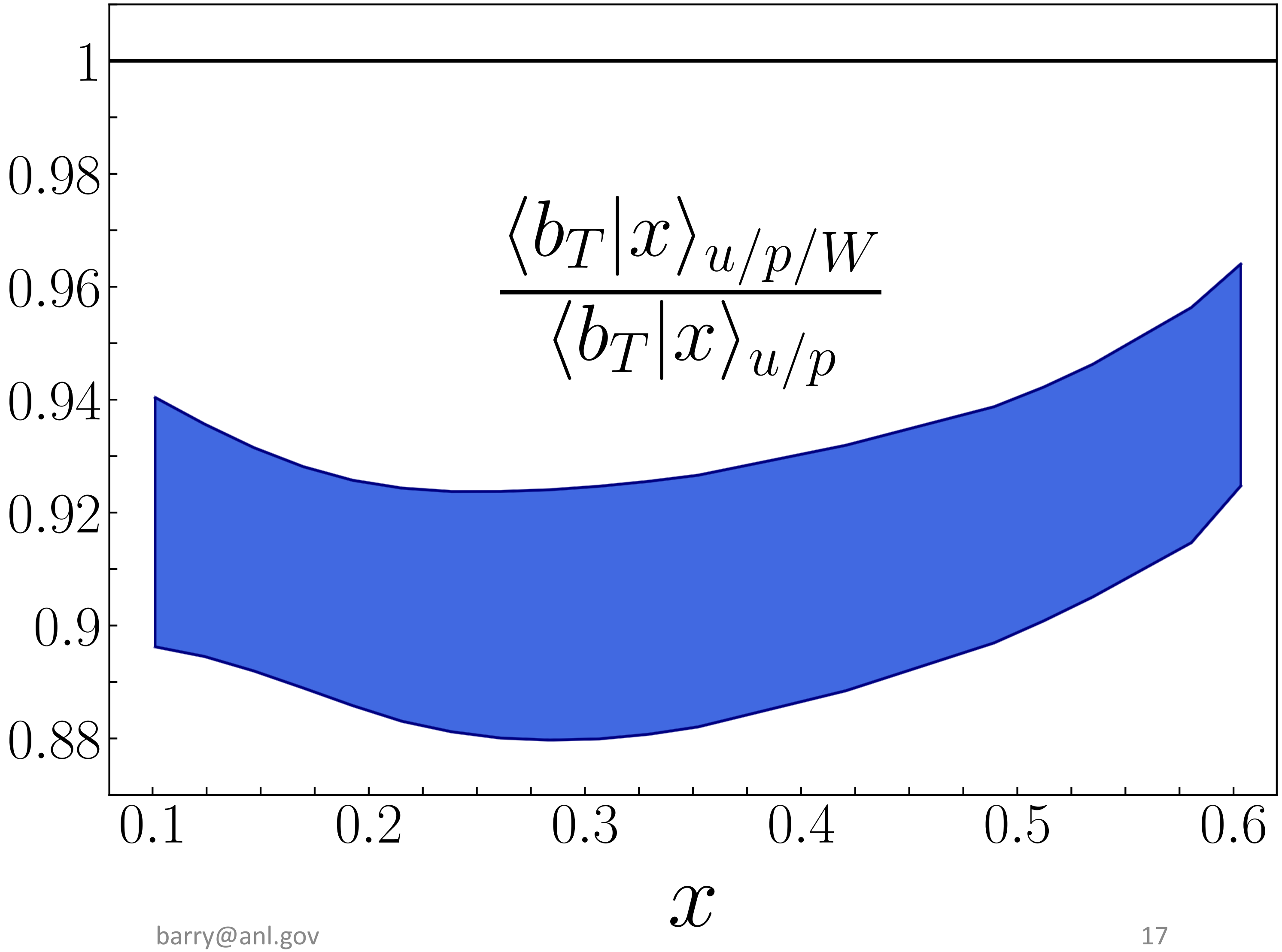
Emphasis on nonperturbative effects



- The $\langle b_T|x\rangle$ grows appreciably in the large-b_T region
- Saturation well beyond a perturbative scale
- Differences between proton and pion are in the nonperturbative region

Transverse EMC effect

- Compare the average b_T given x for the up quark in the bound proton to that of the free proton
- Less than 1 by $\sim 5 - 12\%$ over the x range



Outlook

- High energy data from the TeVatron and LHC provide further constraints on the proton TMDs
  - Potentially also to the collinear PDFs JHEP 01 (2022) 110
  - Explore various prescriptions for TMD treatment beyond CSS
- Combine the low-q_T and large-q_T pion-induced E615 Drell-Yan data, utilizing $W + Y$
  - Build upon our success to fit the large-q_T pion data Phys. Rev. D, **103**, 114014 (2021).
- Future pion-induced and kaon-induced Drell-Yan data will be available from COMPASS and AMBER
  - Also additional tagged DIS processes from JLab and EIC

Backup

Introduction of non-perturbative functions

- Because $b_* \neq b_T$, have to non-perturbatively describe large b_T behavior

Completely general – independent of quark, hadron, PDF or FF

$$g_K(b_T; b_{\max}) = -\tilde{K}(b_T, \mu) + \tilde{K}(b_*, \mu)$$

Non-perturbative function dependent in principle on flavor, hadron, etc.

$$e^{-g_{j/H}(x, \boldsymbol{b}_{\mathrm{T}}; b_{\max})} = \frac{\tilde{f}_{j/H}(x, \boldsymbol{b}_{\mathrm{T}}; \zeta, \mu)}{\tilde{f}_{j/H}(x, \boldsymbol{b}_*; \zeta, \mu)} e^{g_K(b_{\mathrm{T}}; b_{\max}) \ln(\sqrt{\zeta}/Q_0)}.$$

MAP parametrization

- A recent work from the MAP collaboration (arXiv:2206.07598) used a complicated form for the non-perturbative function

$$f_{1\,NP}(x, \boldsymbol{b}_T^2; \zeta, Q_0) = \frac{g_1(x)\, e^{-g_1(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda^2\, g_{1B}^2(x) \left[1 - g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}\right] e^{-g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda_2^2\, g_{1C}(x)\, e^{-g_{1C}(x)\frac{\boldsymbol{b}_T^2}{4}}}{g_1(x) + \lambda^2\, g_{1B}^2(x) + \lambda_2^2\, g_{1C}(x)} \left[\frac{\zeta}{Q_0^2}\right]^{g_K(\boldsymbol{b}_T^2)/2}, \quad (38)$$

$$g_{\{1,1B,1C\}}(x) = N_{\{1,1B,1C\}} \frac{x^{\sigma_{\{1,2,3\}}} (1-x)^{\alpha^2_{\{1,2,3\}}}}{\hat{x}^{\sigma_{\{1,2,3\}}} (1-\hat{x})^{\alpha^2_{\{1,2,3\}}}},$$

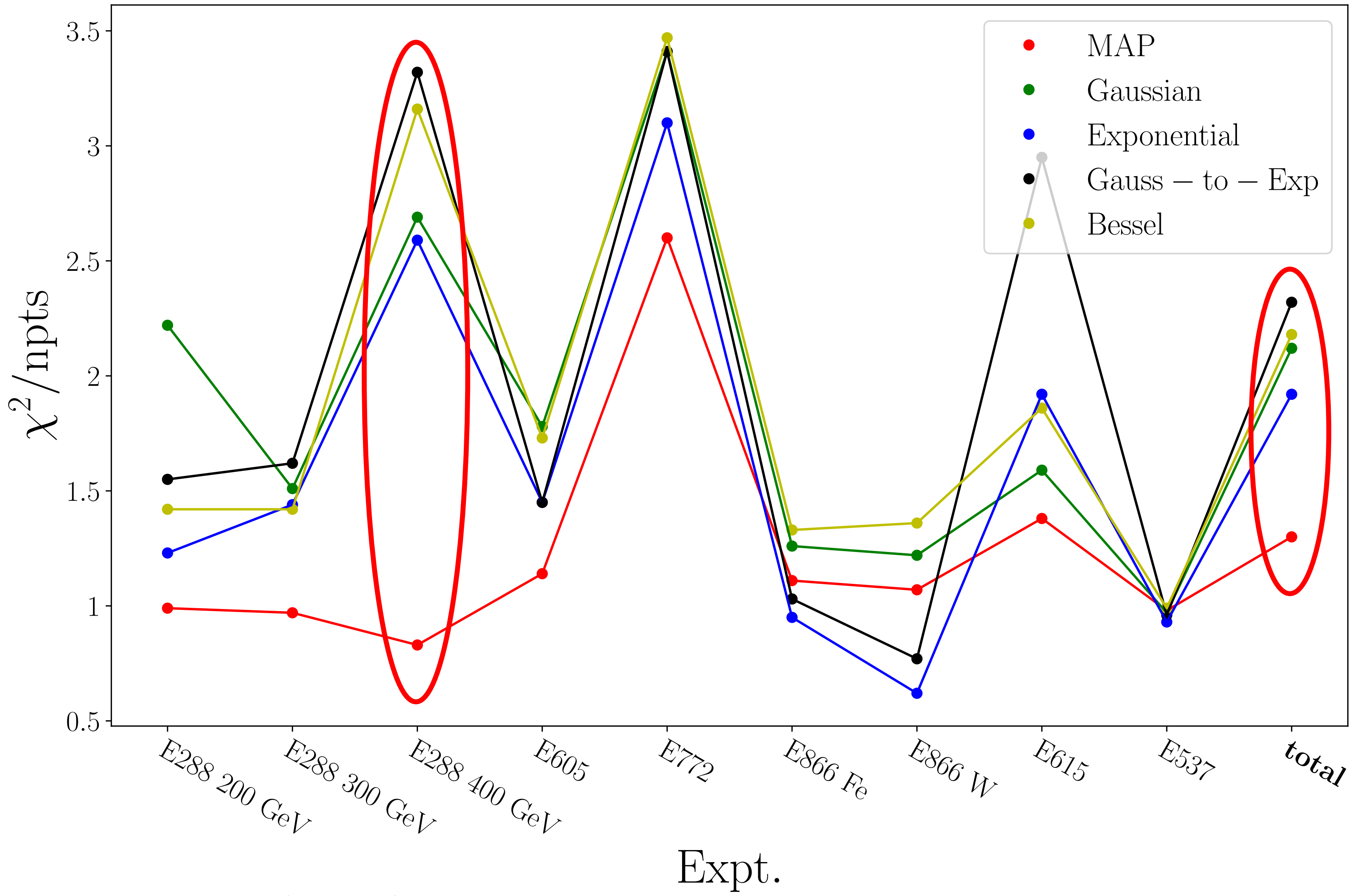
$$g_K(\boldsymbol{b}_T^2) = -g_2^2 \frac{\boldsymbol{b}_T^2}{2}$$

Universal CS kernel

- 11 free parameters for each hadron! (flavor dependence not necessary) (12 if we include the nuclear TMD parameter)

Resulting χ^2 for each parametrization

- Tried multiple parametrizations for non-perturbative TMD structures
- MAP parametrization is able to describe better all the datasets



Nuclear TMD PDFs – working hypothesis

- We must model the nuclear TMD PDF from proton

$$\tilde{f}_{q/A}(x, b_T, \mu, \zeta) = \frac{Z}{A}\tilde{f}_{q/p/A}(x, b_T, \mu, \zeta) + \frac{A-Z}{A}\tilde{f}_{q/n/A}(x, b_T, \mu, \zeta)$$

- Each object on the right side independently obeys the CSS equation
  - **Assumption** that the bound proton and bound neutron follow TMD factorization
- Make use of isospin symmetry in that $u/p/A \leftrightarrow d/n/A$, etc.

Building of the nuclear TMD PDF

- Then taking into account the intrinsic non-perturbative, we model the flavor-dependent pieces of the TMD PDF as

$$(C \otimes f)_{u/A}(x) e^{-g_{u/A}(x, b_T)} \rightarrow \frac{Z}{A} (C \otimes f)_{u/p/A}(x) e^{-g_{u/p/A}(x, b_T)} + \frac{A - Z}{A} (C \otimes f)_{d/p/A}(x) e^{-g_{d/p/A}(x, b_T)}$$

and

$$(C \otimes f)_{d/A}(x) e^{-g_{d/A}(x, b_T)} \rightarrow \frac{Z}{A} (C \otimes f)_{d/p/A}(x) e^{-g_{d/p/A}(x, b_T)} + \frac{A - Z}{A} (C \otimes f)_{u/p/A}(x) e^{-g_{u/p/A}(x, b_T)}.$$

Nuclear TMD parametrization

- Specifically, we include a parametrization similar to Alrashed, et al., Phys. Rev. Lett **129**, 242001 (2022).

$$g_{q/\mathcal{N}/A} = g_{q/\mathcal{N}} \left(1 - a_{\mathcal{N}} \left(A^{1/3} - 1\right)\right)$$

- Where $a_{\mathcal{N}}$ is an additional parameter to be fit

Bayesian Inference

- Minimize the χ^2 for each replica

Normalization parameter

$$\chi^2(\boldsymbol{a}, \mathrm{data}) = \sum_e \left(\sum_i \left[\frac{d_i^e - \sum_k r_k^e \, \beta_{k,i}^e - t_i^e(\boldsymbol{a})/n_e}{\alpha_i^e} \right]^2 + \left(\frac{1 - n_e}{\delta n_e} \right)^2 + \sum_k \left(r_k^e \right)^2 \right)$$

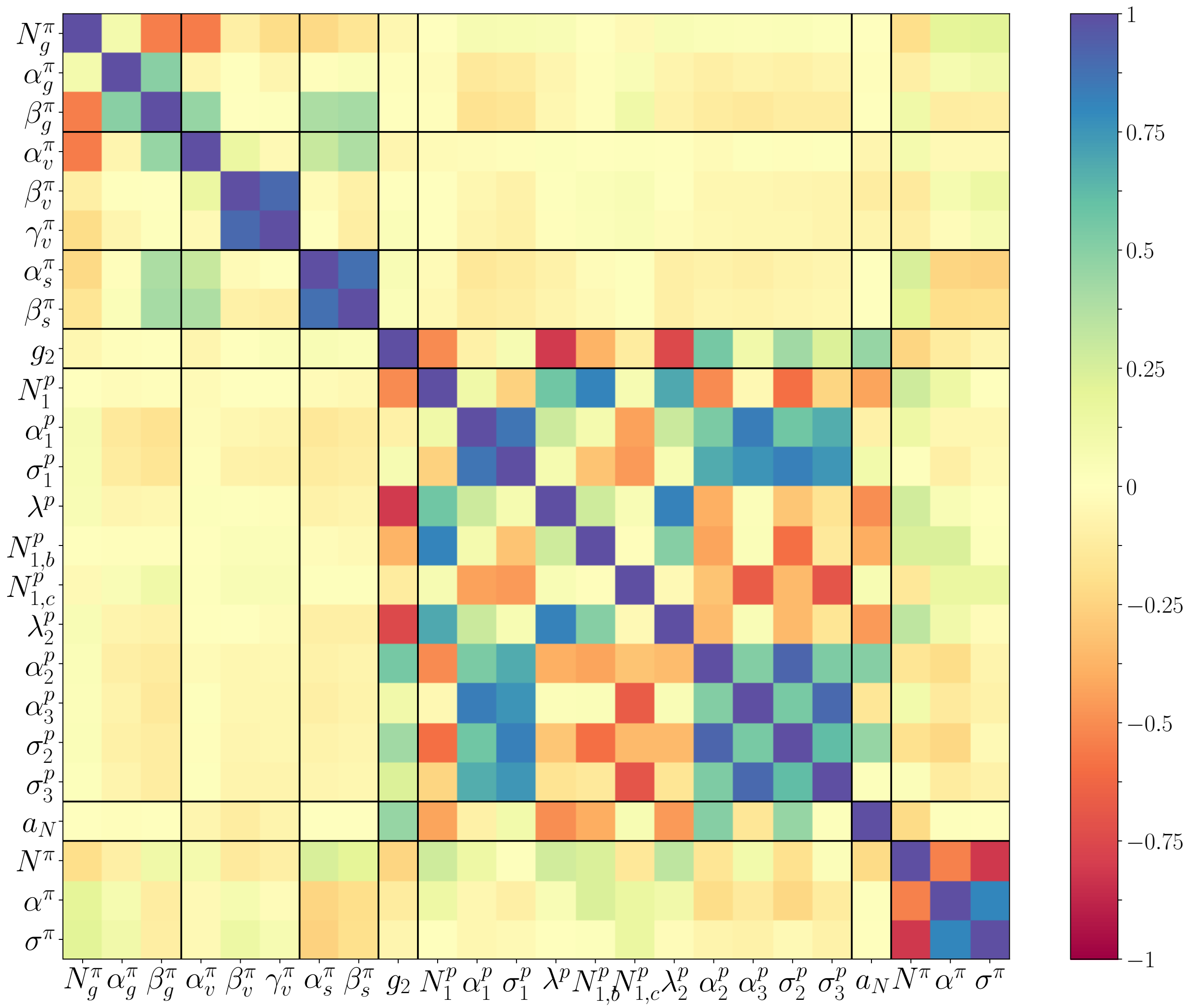
- Perform N total χ^2 minimizations and compute statistical quantities

Expectation value $$\mathrm{E}[\mathcal{O}] = \frac{1}{N} \sum_k \mathcal{O}(\boldsymbol{a}_k),$$

Variance $$\mathrm{V}[\mathcal{O}] = \frac{1}{N} \sum_k \left[\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}] \right]^2,$$

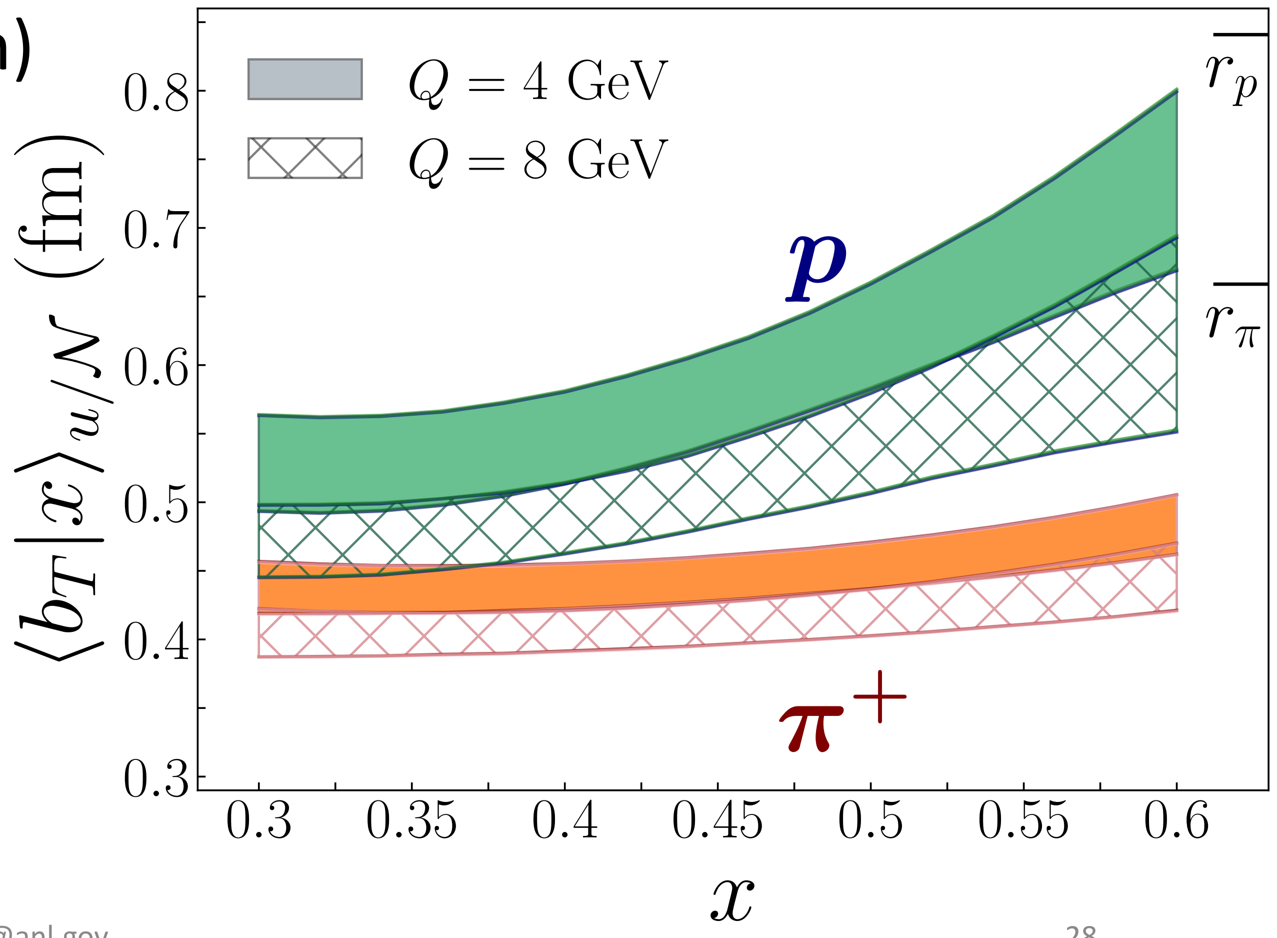
Correlations

- Level at which the distributions are correlated with each other
- Different distributions are largely correlated only within themselves



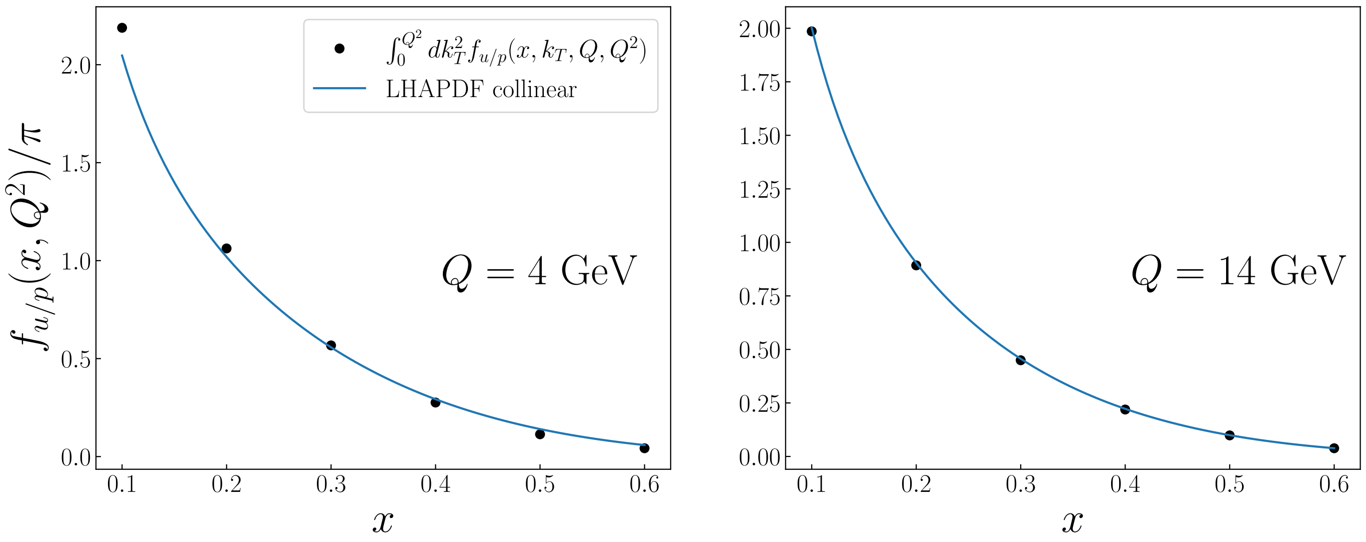
Emphasis on nonperturbative effects

- We vary the collinear PDFs
  p: CT14nlo (blue) → MMHT14 (green)
  π: JAM (red) → xFitter (orange)
- No change in the quantity!



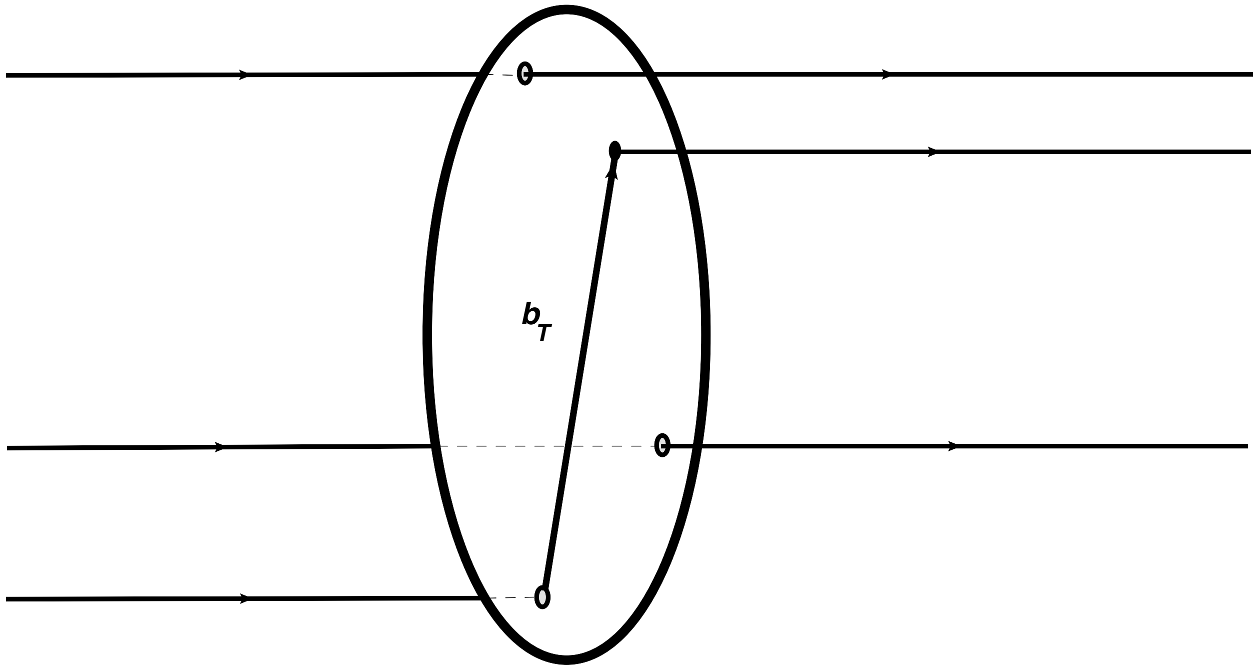
Collinear relation

- The TMD formalism requires that the integral over k_T^2 of the TMD gives the collinear PDF up to higher order corrections
- We demonstrate this for example in the proton case
- At larger Q, the power corrections are less important



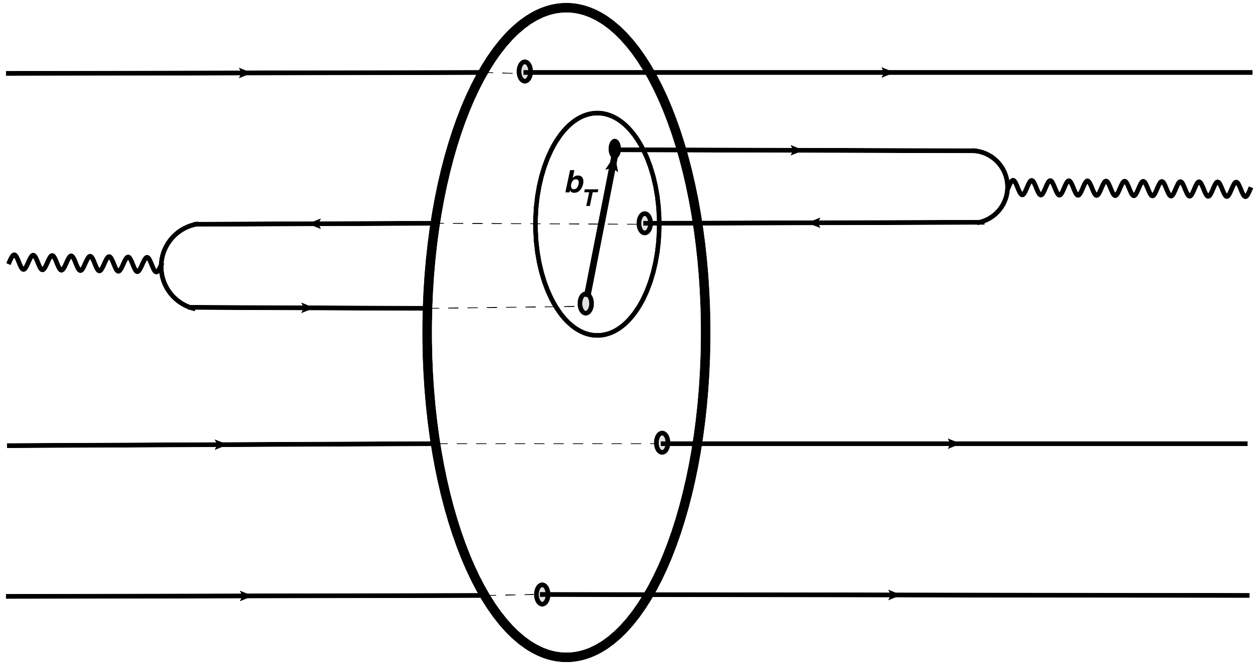
Possible explanation

- At large x, we are in a valence region, where only the valence quarks are populating the momentum dependence of the hadron



Possible explanation

- At small x, sea quarks and potential $q\bar{q}$ bound states allowing only for a smaller bound system



Deriving resummation expressions – MF

$$z \equiv \frac{Q^2}{\hat{S}} = \frac{\tau}{\hat{x}_\pi \hat{x}_A}$$

Claim: yellow terms give rise to the resummation expressions

$$y = \frac{\frac{\hat{x}_\pi}{\hat{x}_A} e^{-2Y} - z}{(1-z)(1+\frac{\hat{x}_\pi}{\hat{x}_A} e^{-2Y})}$$

$$\frac{C_{q\bar{q}}}{e_q^2} = \delta(1-z)\,\frac{\delta(y)+\delta(1-y)}{2}\left[1+\frac{C_F\alpha_s}{\pi}\left(\frac{3}{2}\ln\frac{M^2}{\mu_f^2}+\frac{2\pi^2}{3}-4\right)\right]$$
$$+\frac{C_F\alpha_s}{\pi}\left\{\frac{\delta(y)+\delta(1-y)}{2}\left[(1+z^2)\left[\frac{1}{1-z}\ln\frac{M^2(1-z)^2}{\mu_f^2 z}\right]_+ +1-z\right]\right.$$
$$\left.+\frac{1}{2}\left[1+\frac{(1-z)^2}{z}y(1-y)\right]\left[\frac{1+z^2}{1-z}\left(\left[\frac{1}{y}\right]_+ + \left[\frac{1}{1-y}\right]_+\right)-2(1-z)\right]\right\}$$

Claim: Red terms are power suppressed in $(1-z)$ and wouldn't contribute to the same order as the yellow terms

Generalized Threshold resummation

G. Lustermans, J. K. L. Michel, and F. J. Tackmann, arXiv:1908.00985 [hep-ph].

- Write the (z, y) coefficients in terms of (z_a, z_b), and for the red terms, you get:

$$z_a = \frac{x_\pi^0}{\hat{x}_\pi}$$

$$\mathrm{d}z\,\mathrm{d}y\,\frac{1}{1-z}\left(\frac{1}{y}+\frac{1}{1-y}\right) = \mathrm{d}z_a \mathrm{d}z_b\,\frac{1}{(1-z_a)(1-z_b)}\left[1+\mathcal{O}(1-z_a, 1-z_b)\right].$$

$$z_b = \frac{x_A^0}{\hat{x}_A}$$

- This is *not* power suppressed in $(1 - z_a)$ or $(1 - z_b)$ but instead the same order as the leading power in the soft limit
- Generalized threshold resummation in the soft limit does not agree with the MF methods