Status of unpolarized TMD extractions from global fits

Matteo Cerutti

From Quarks and Gluons to the Internal Dynamics of Hadrons

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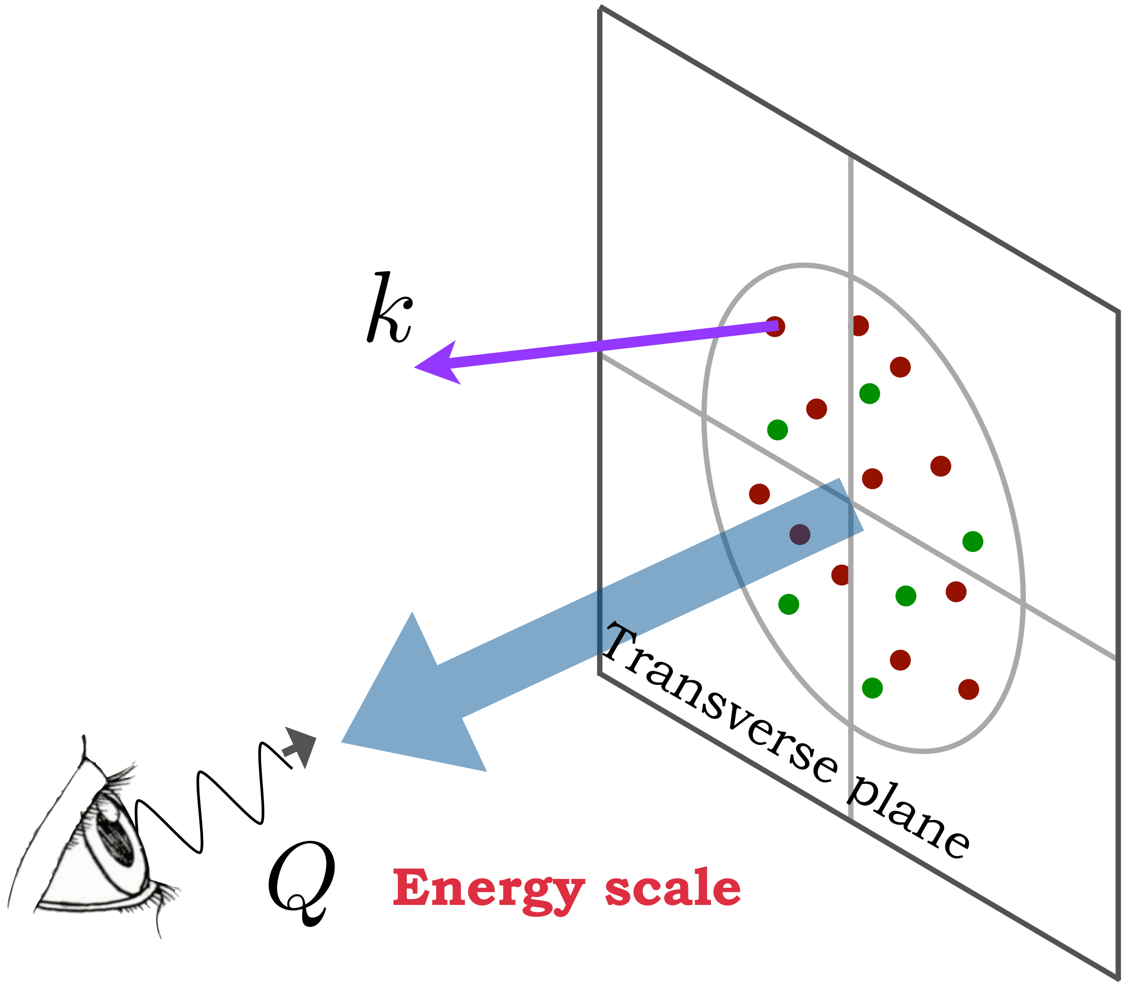
HAMPTON UNIVERSITY
1868

Transverse-Momentum Distributions (TMDs)

3-*dimensional* ***map*** of the internal structure of the nucleon



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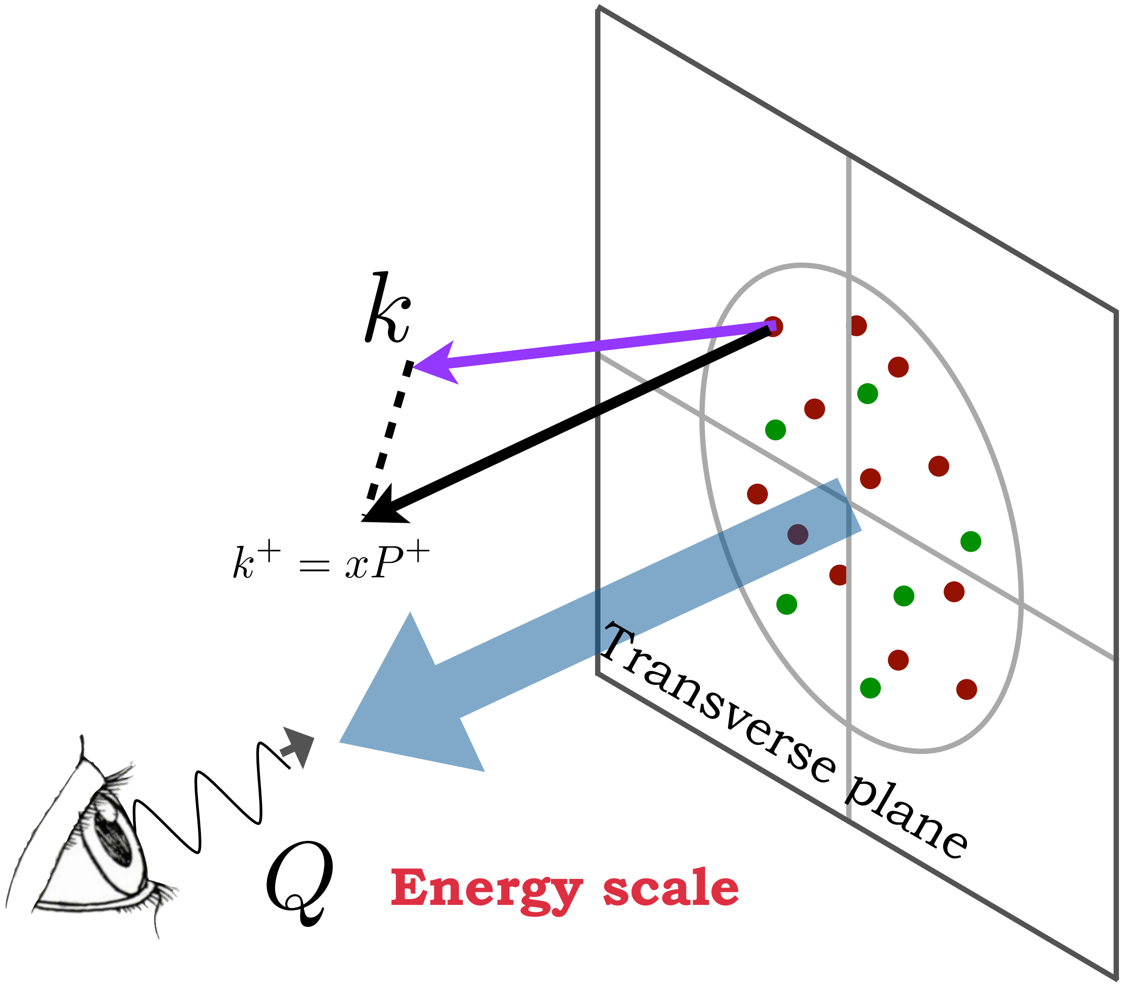

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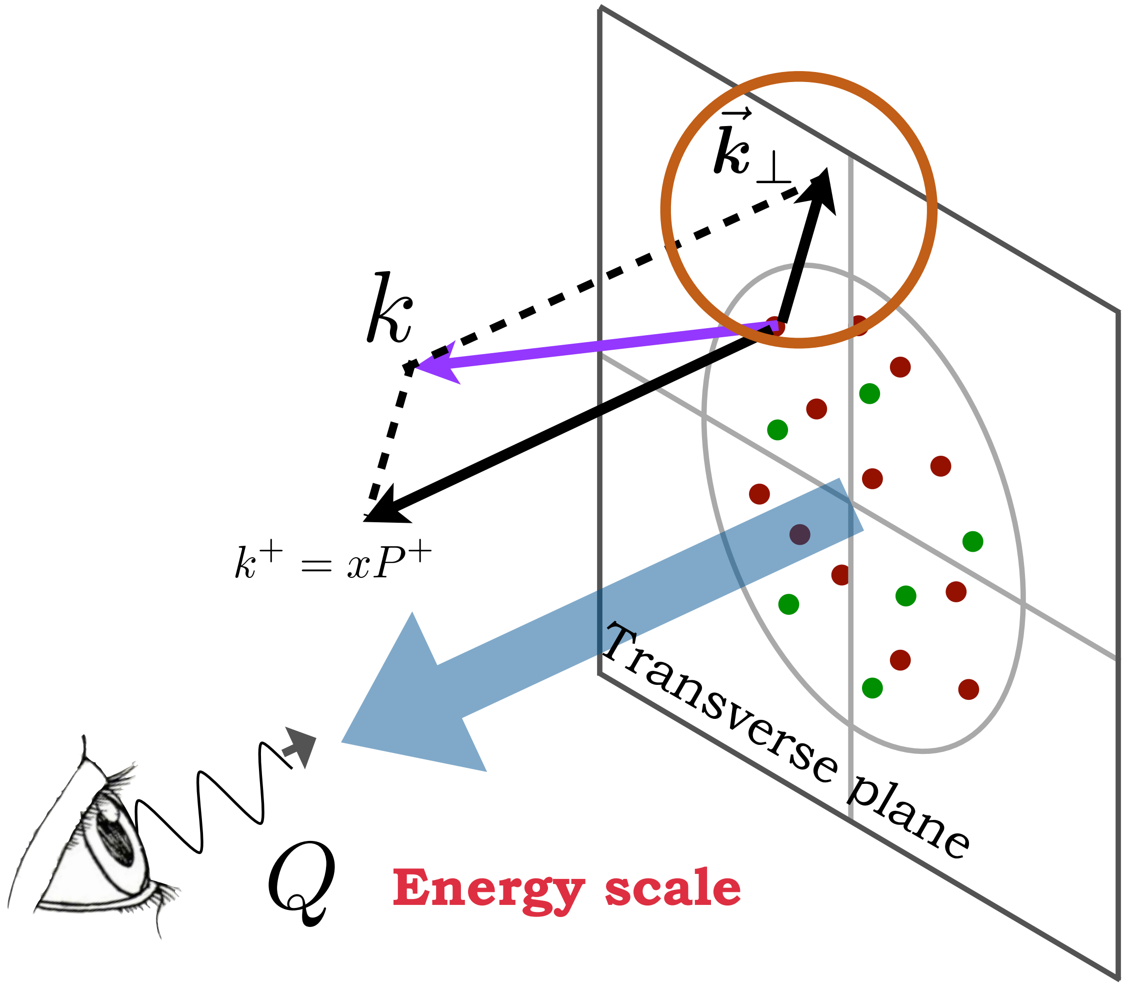

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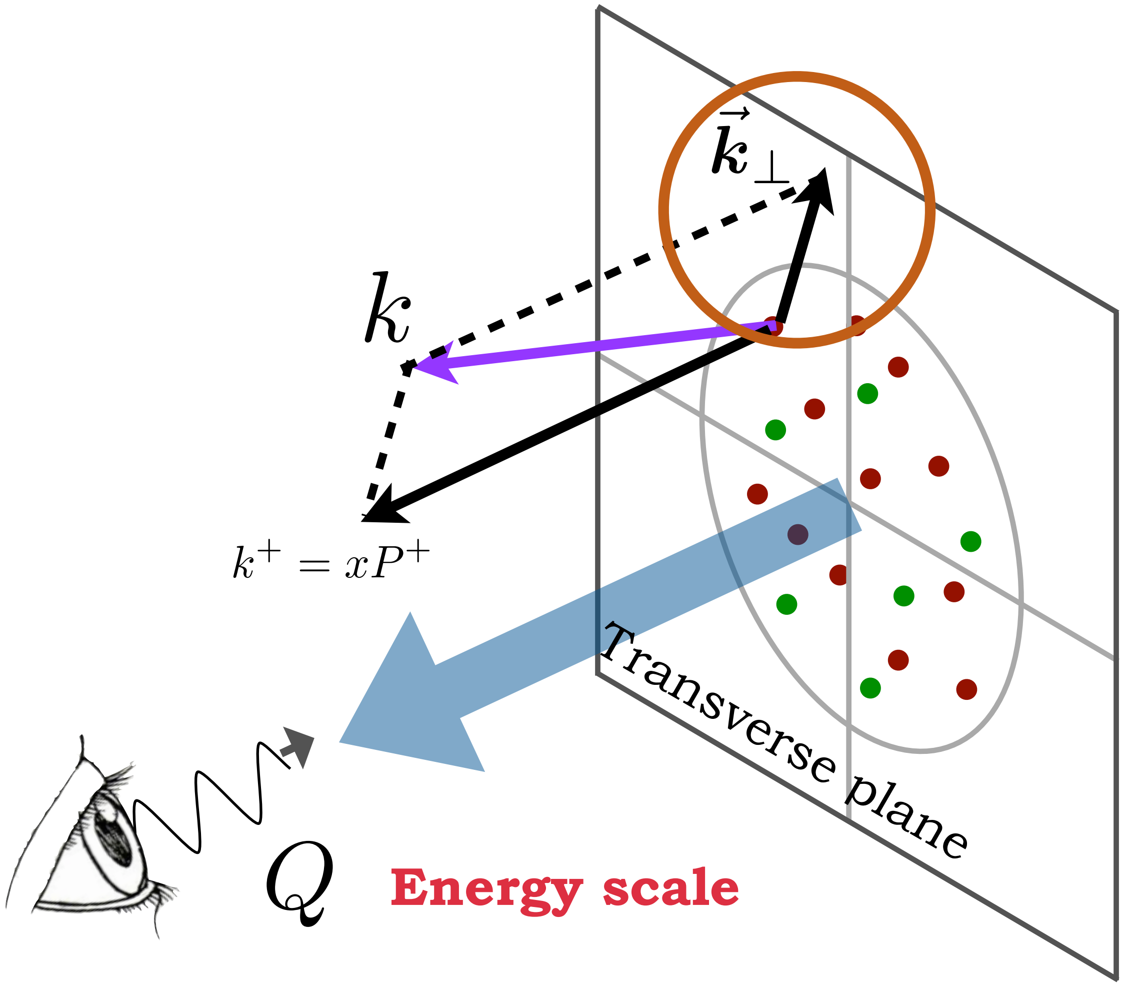
Non-collinear framework



Quark Polarization

Nucleon Pol.

	U	L	T
U	f_1		
L		g_1	
T			h_1

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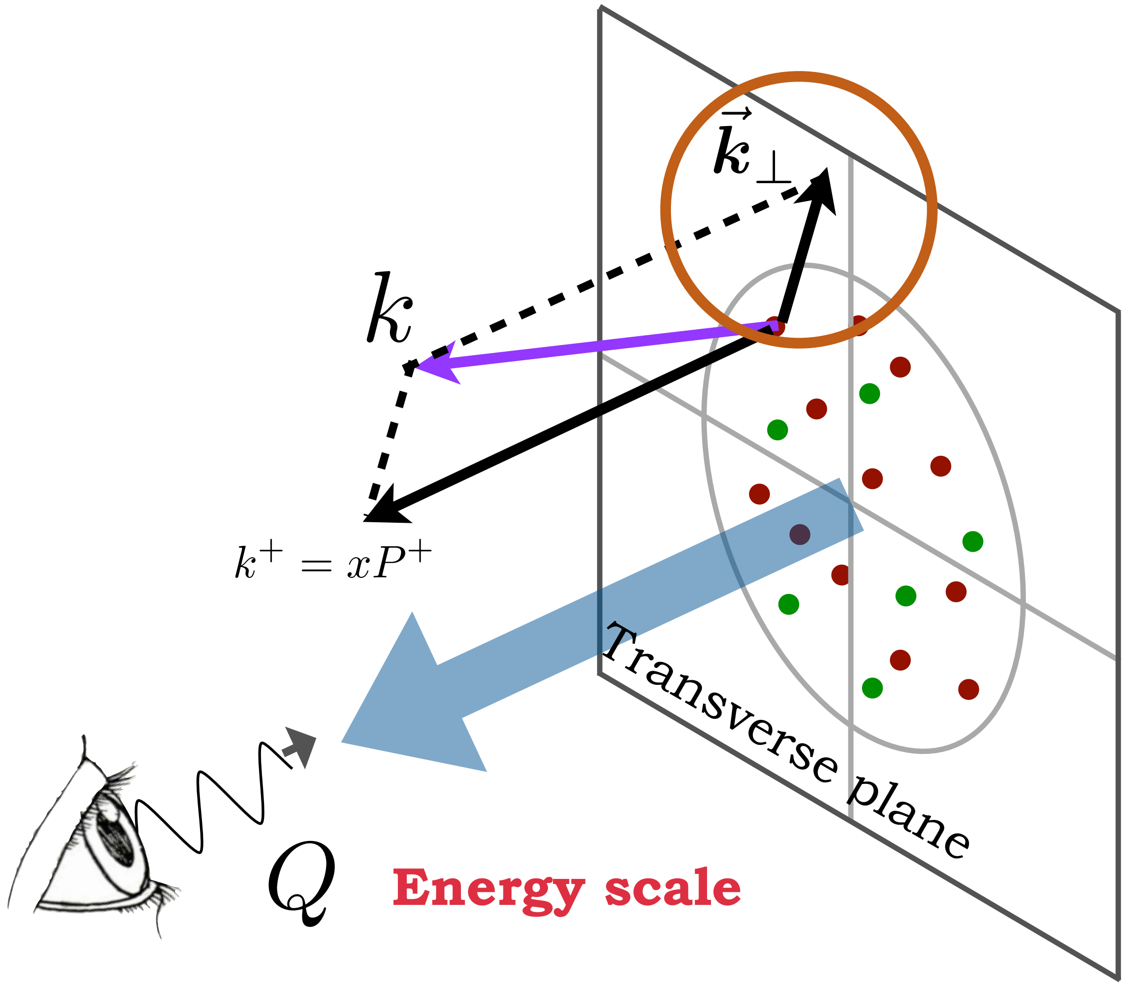

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Time-reversal odd

Time-reversal even

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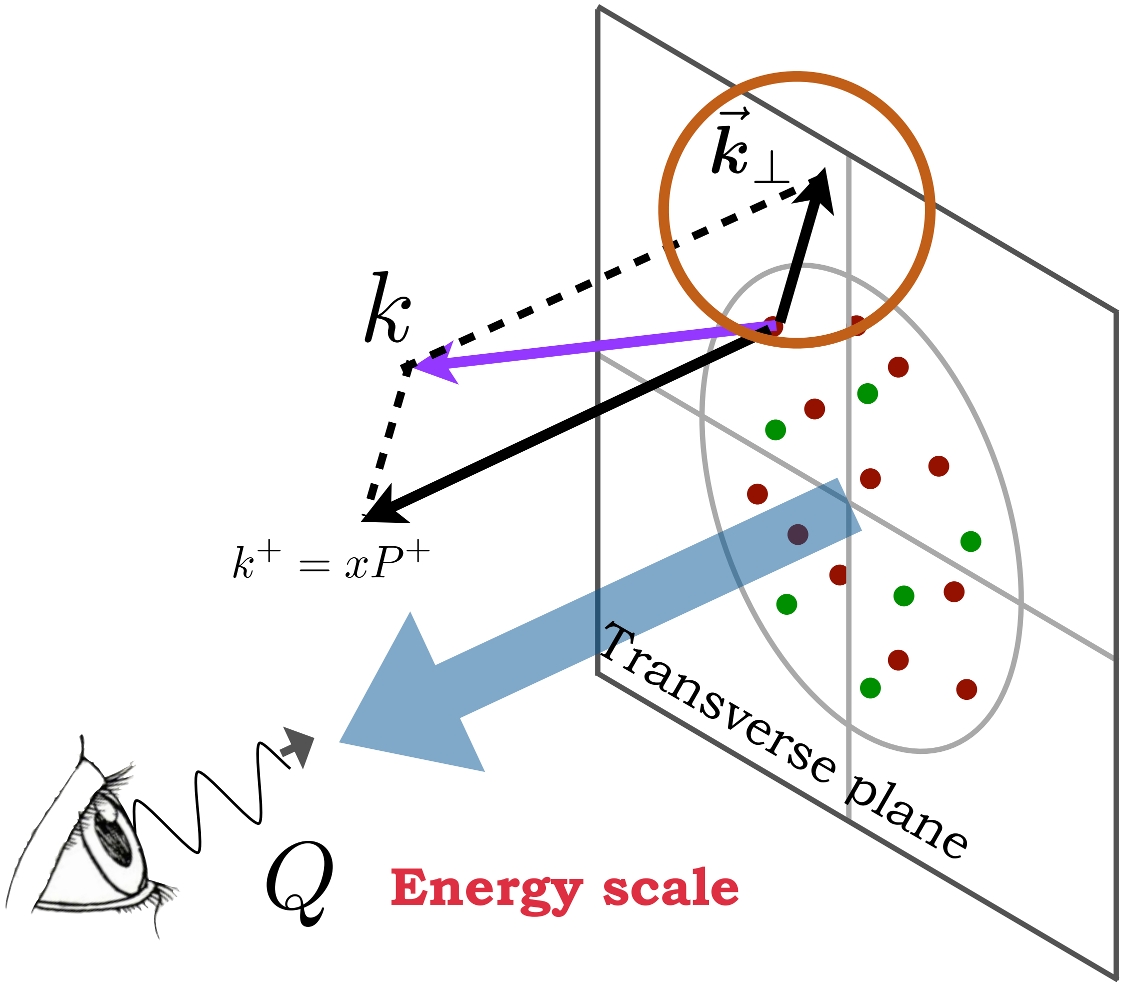

TMD PDFs

$$F(x, \boldsymbol{k}_\perp^2, \mu, \zeta)$$

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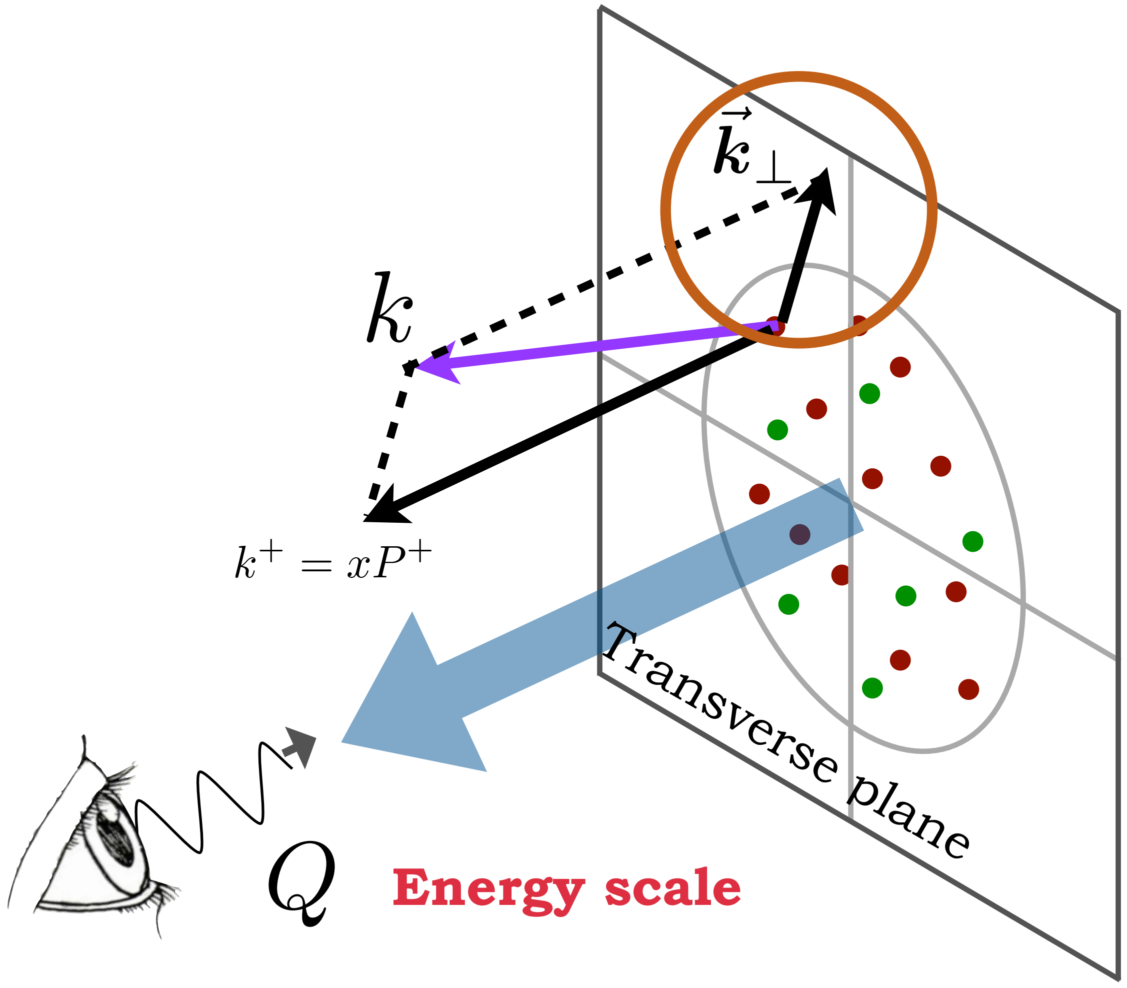

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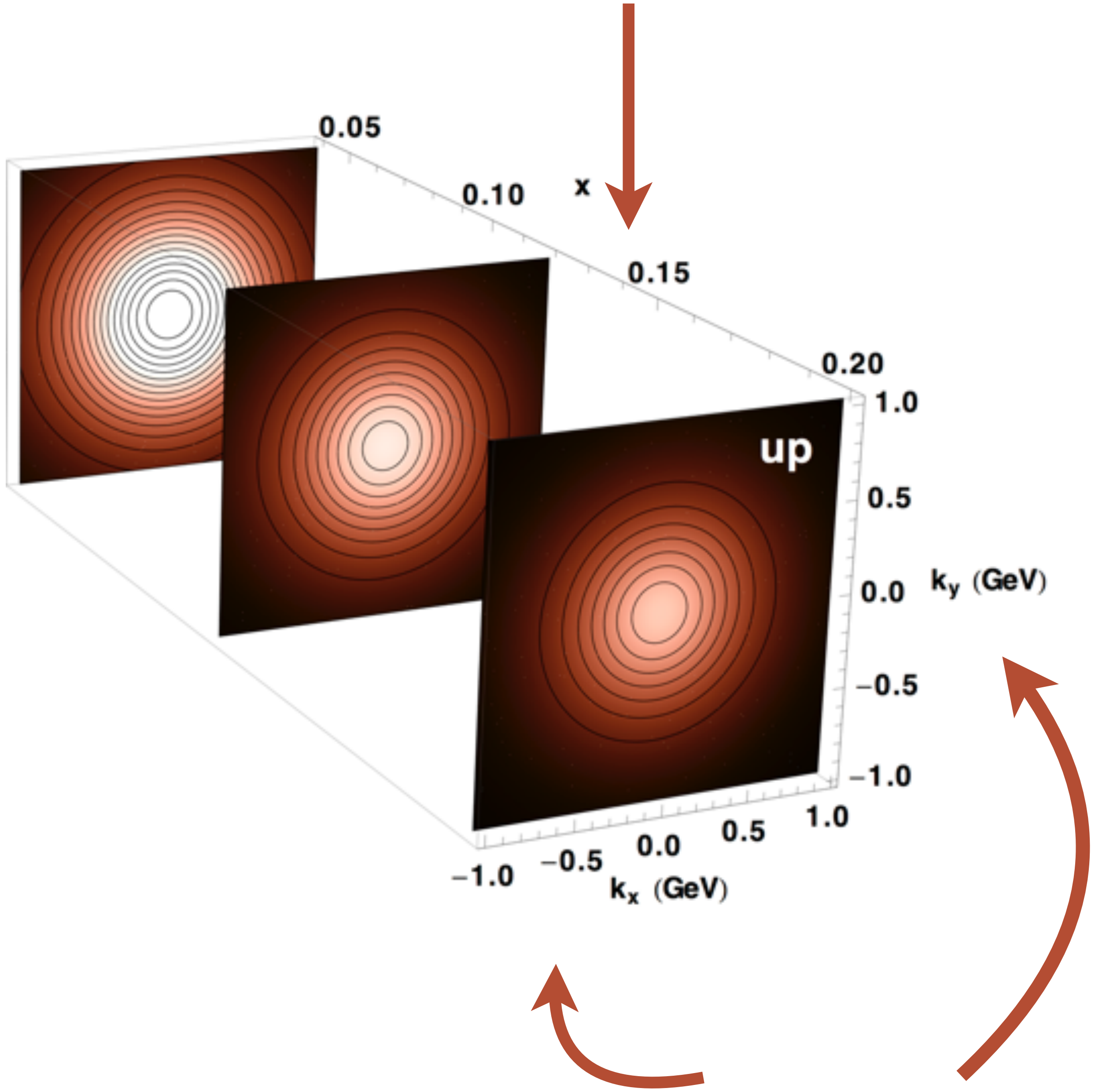
Transverse-Momentum Distributions (TMDs)

Fraction of
longitudinal momentum

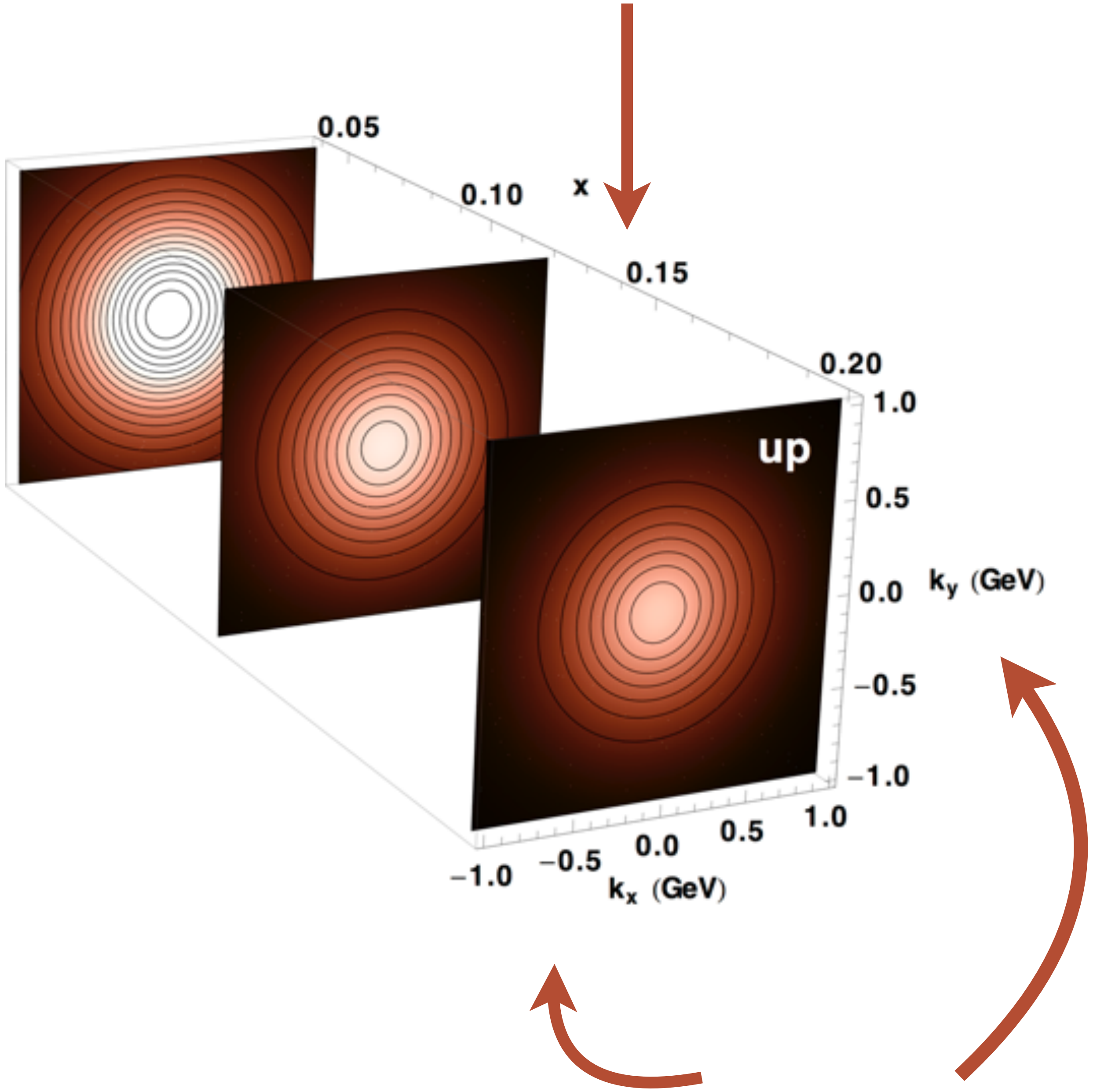


Transverse momentum

TMDs map the distribution of partons inside the nucleon in 3D in momentum space.

They can be extracted through ***global fits***
There are attempts to calculate them in lattice QCD

Transverse-Momentum Distributions (TMDs)



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Are TMDs universal?

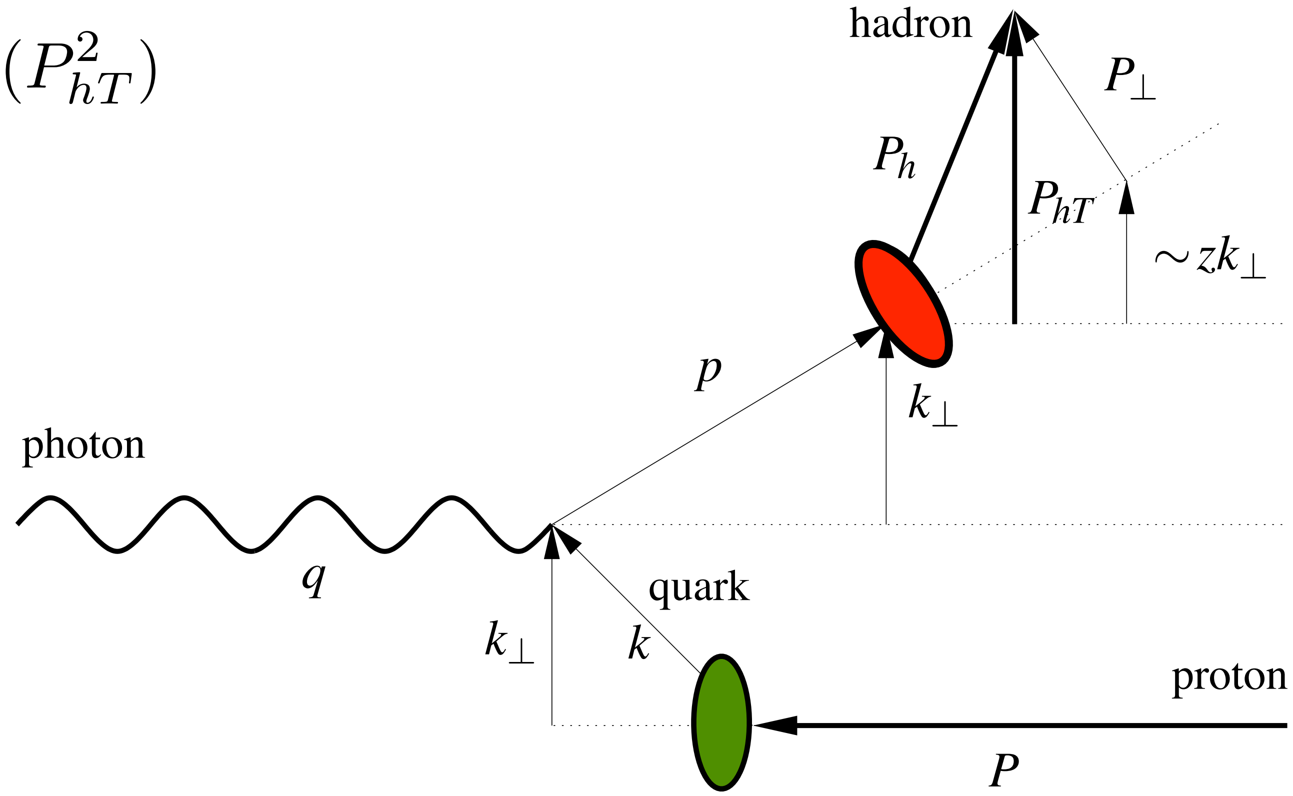
Do they depend on x?

Do they depend on the quark flavor?

TMD factorization: SIDIS

Semi-Inclusive Deep-Inelastic Scattering

If $Q^2 \gg M^2$ and $Q^2 \gg q_T^2(P_{hT}^2)$



$$F_{UU,T}\big(x,z,|\boldsymbol{q}_T|,Q\big) = \frac{x}{2\pi}\,\mathcal{H}^{SIDIS}(Q,\mu)\sum_{a=q,\bar{q}} e_a^2 \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T|J_0\big(|\boldsymbol{b}_T||\boldsymbol{q}_T|\big)\hat{f}_1^a(x,b_T^2;\mu,\zeta_A)\hat{D}_1^{a\to h}(z,b_T^2;\mu,\zeta_B)$$

$$+Y_{UU,T}(Q^2,\mathbf{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

Bacchetta, Diehl, et al., JHEP 02 (2007)

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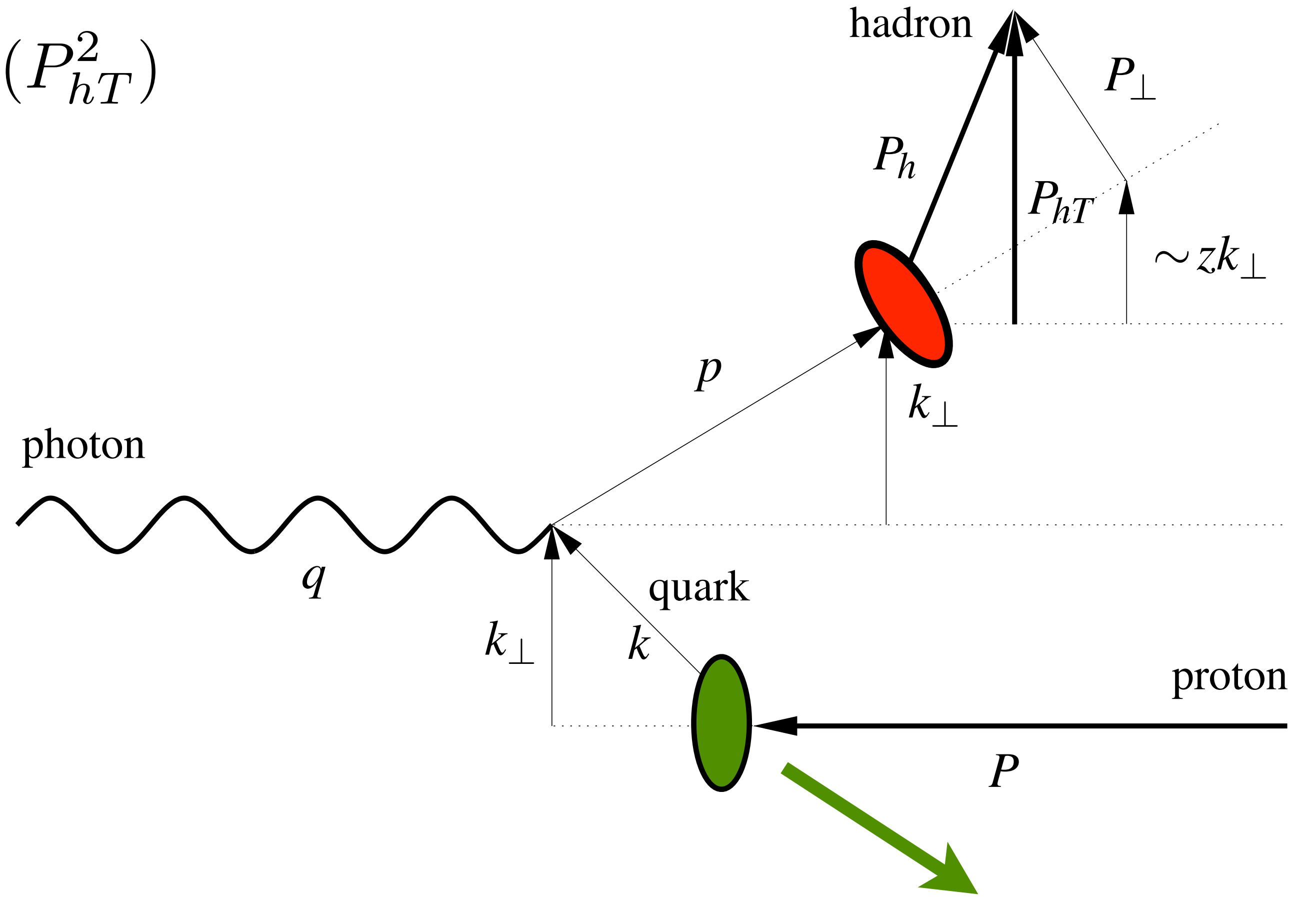

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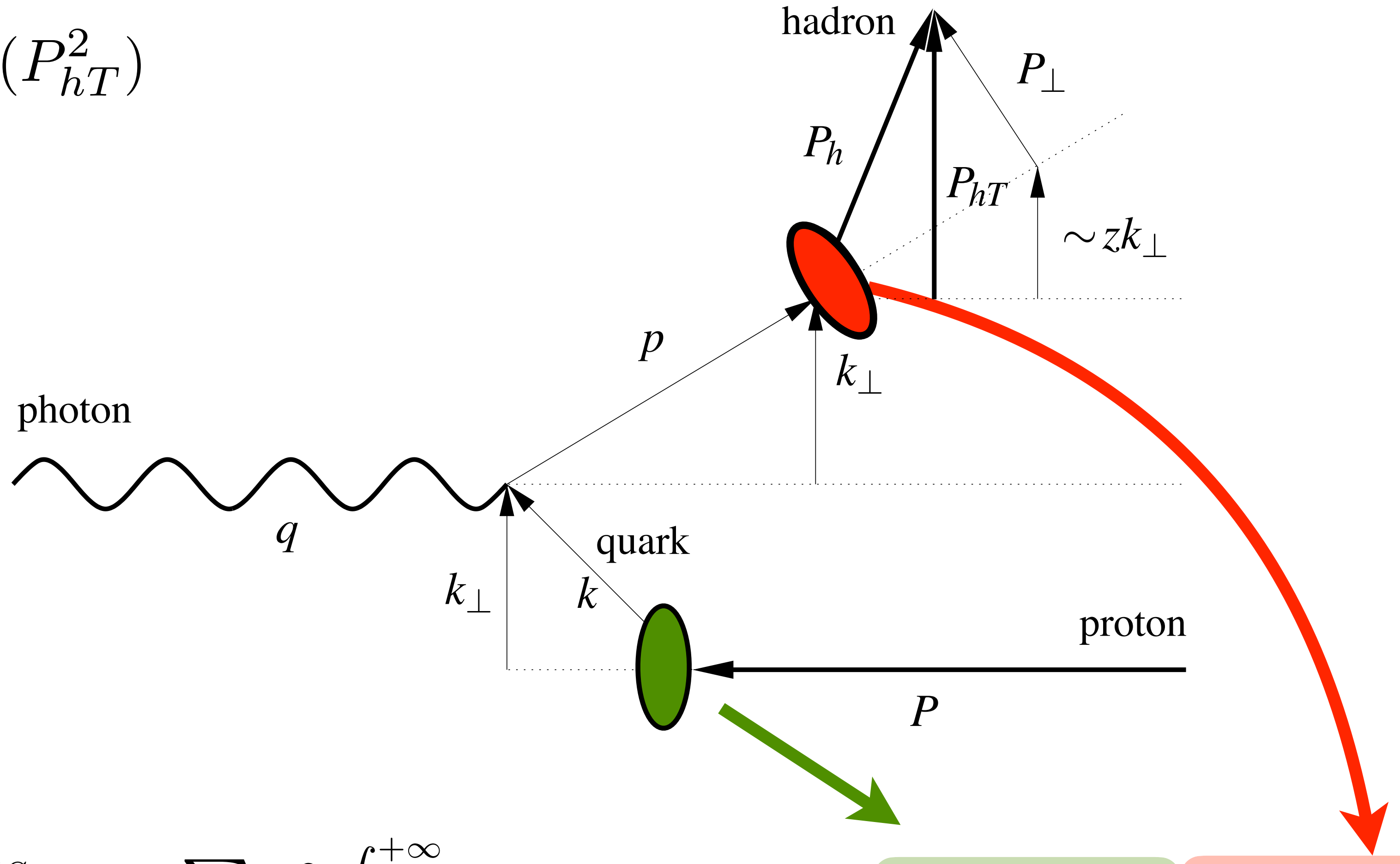

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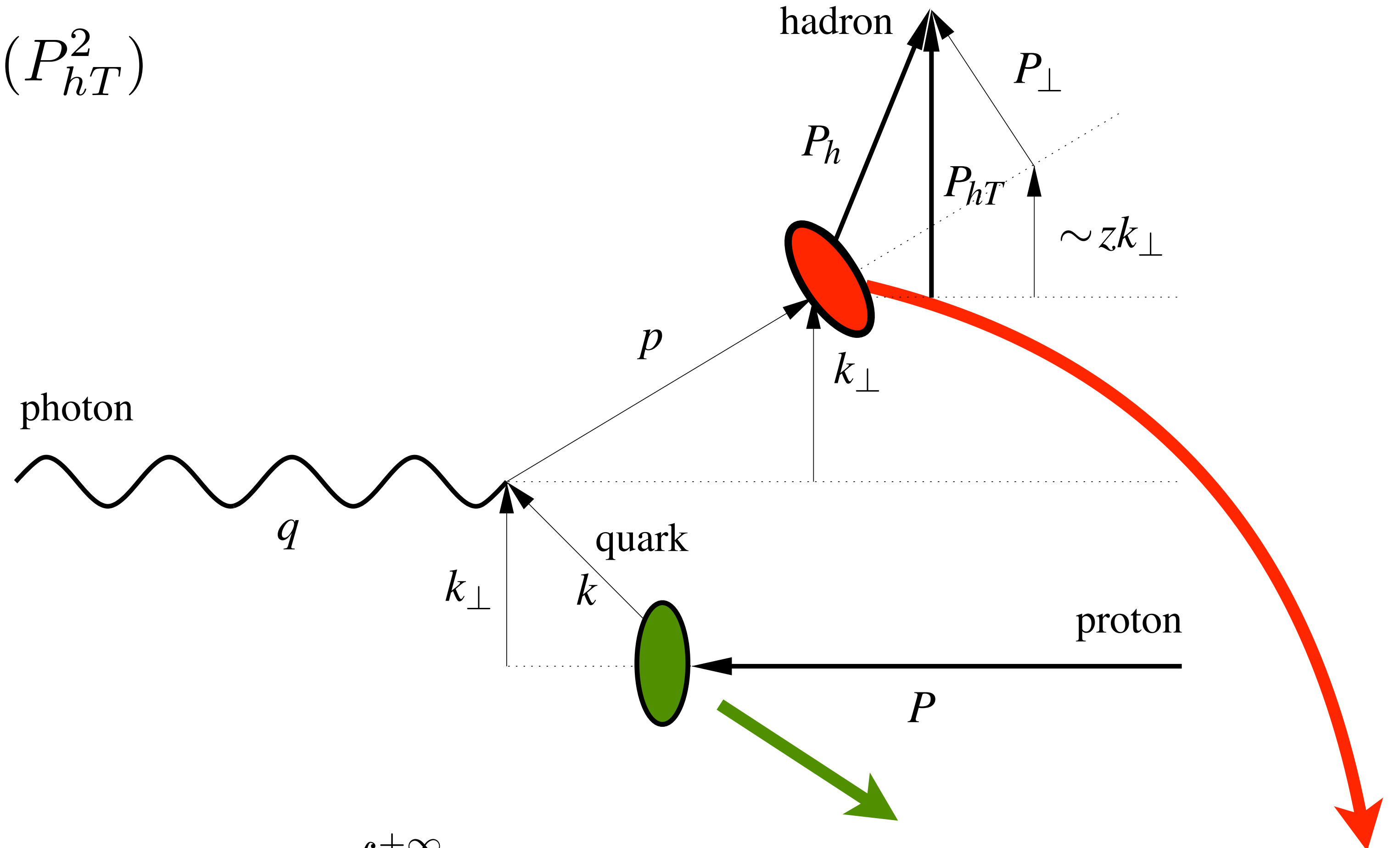

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- The **W term** dominates in the region where **$q_T \ll Q$**
- The Y term has been excluded in the analysis

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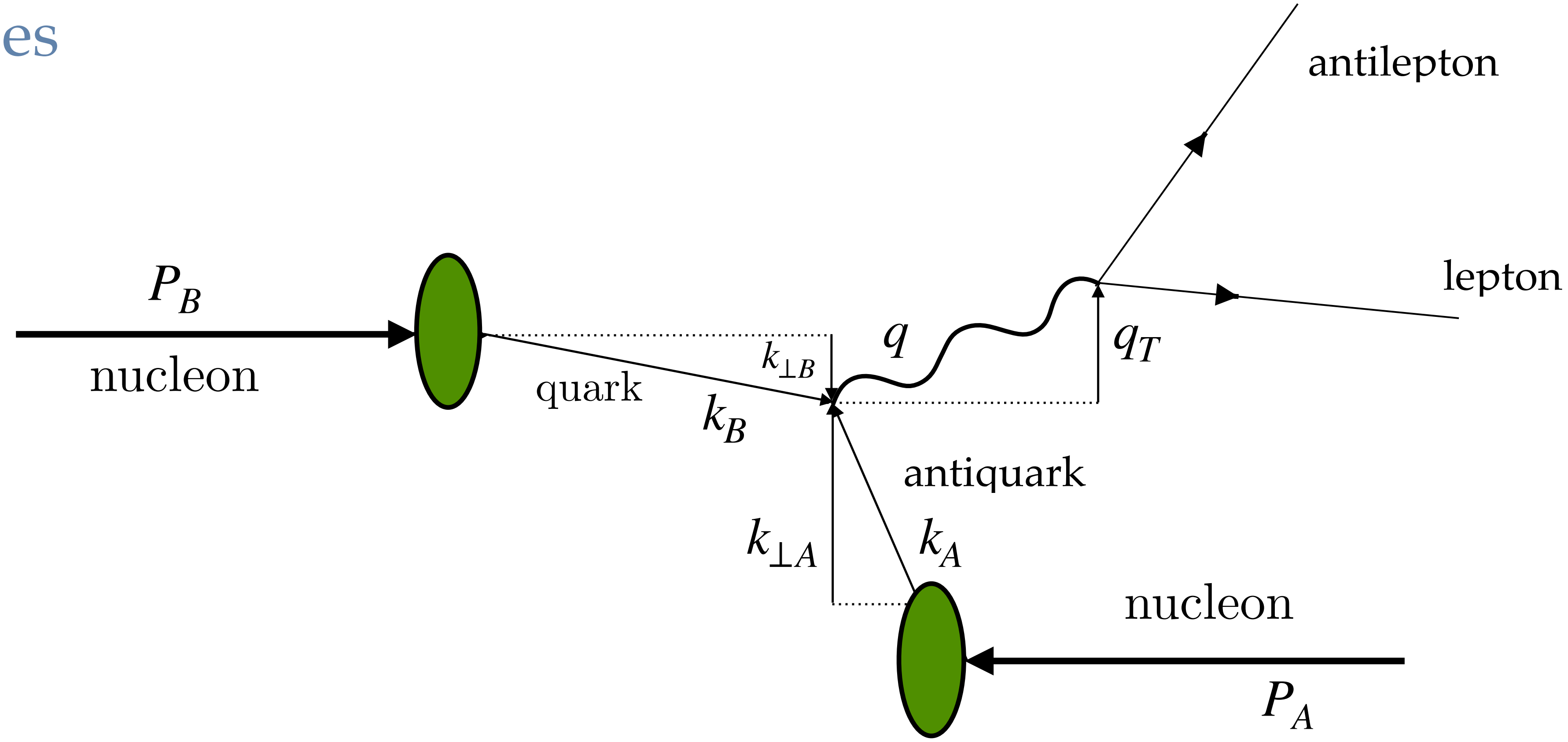
TMD factorization: Drell-Yan

Drell-Yan processes



Collins, Metz, PRL 93 (2004)

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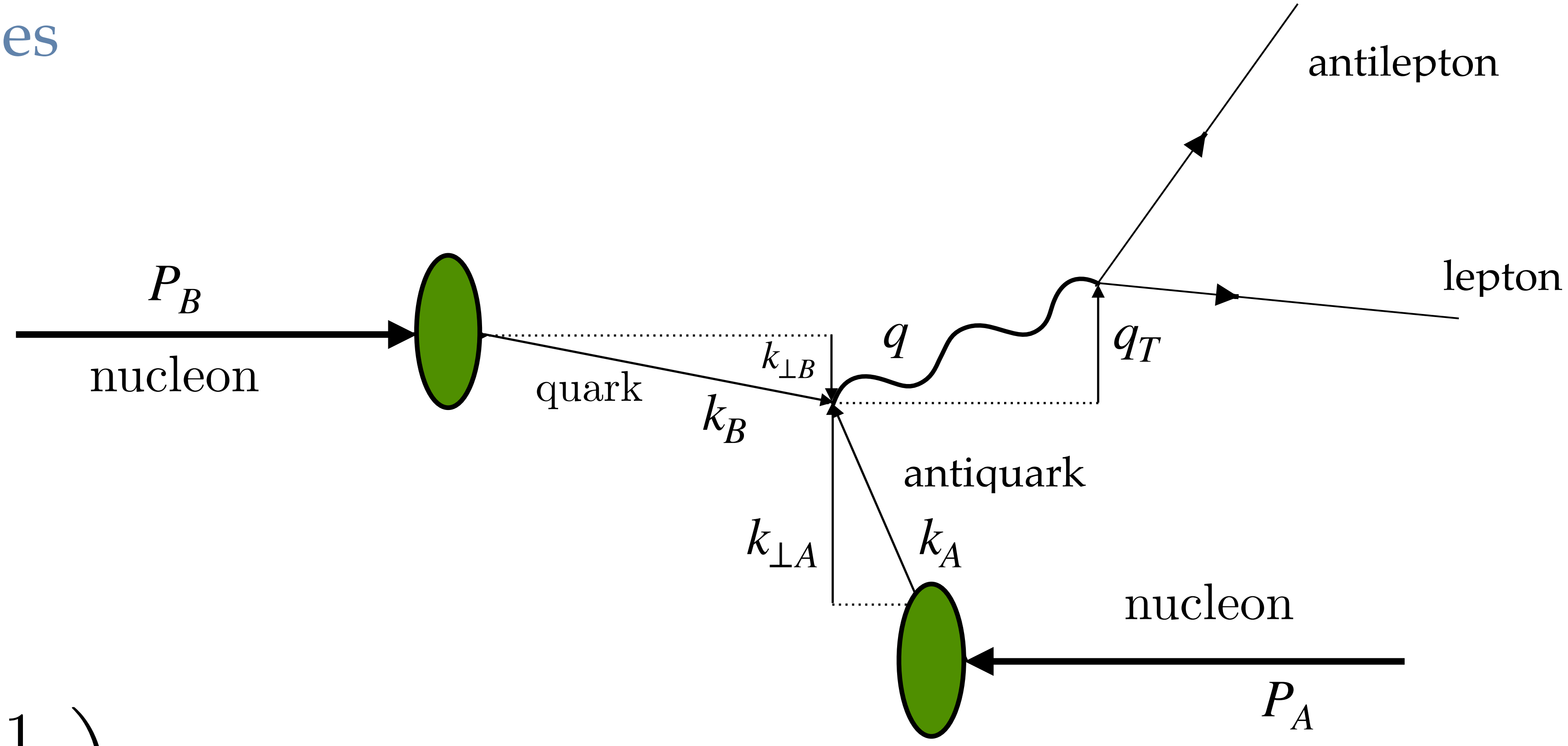

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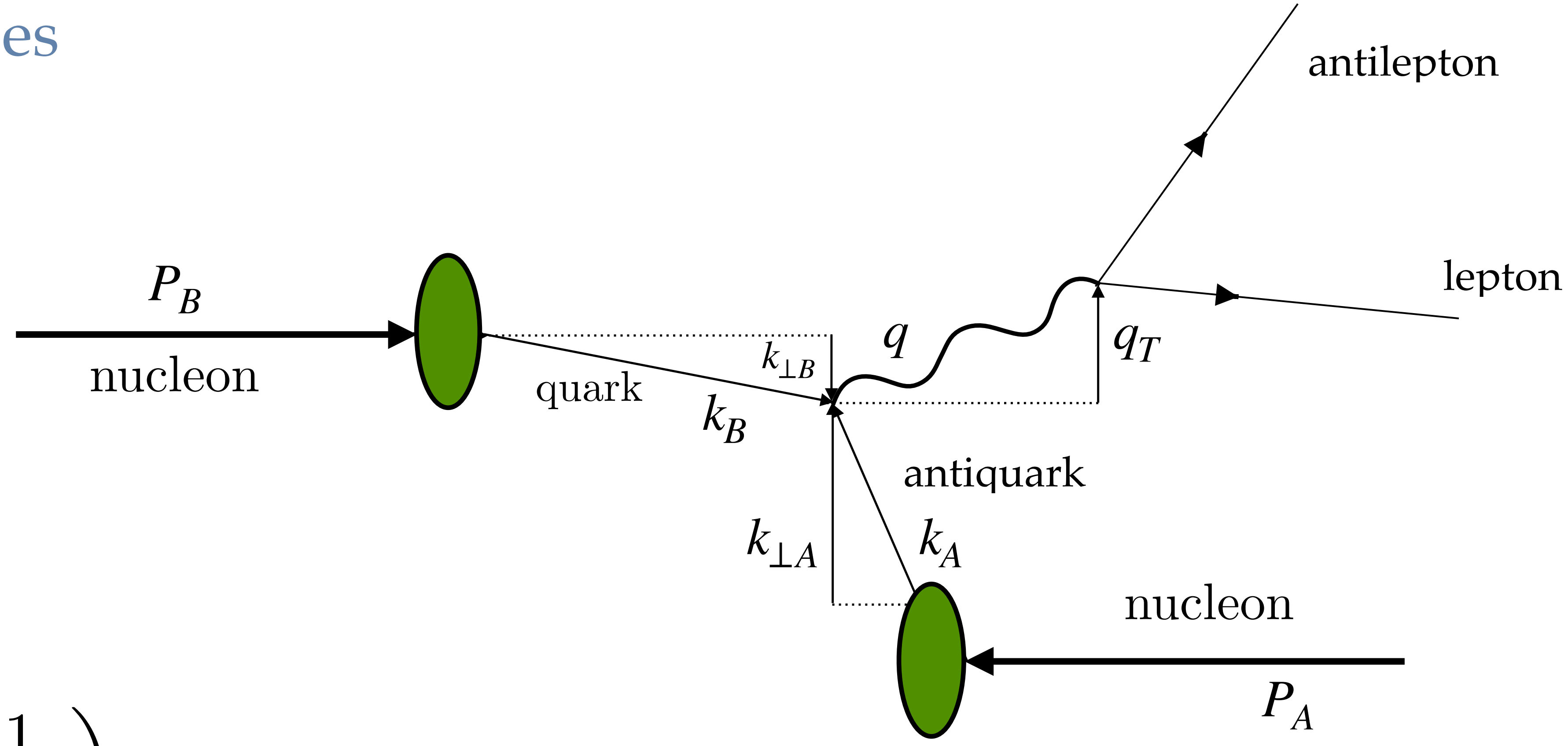

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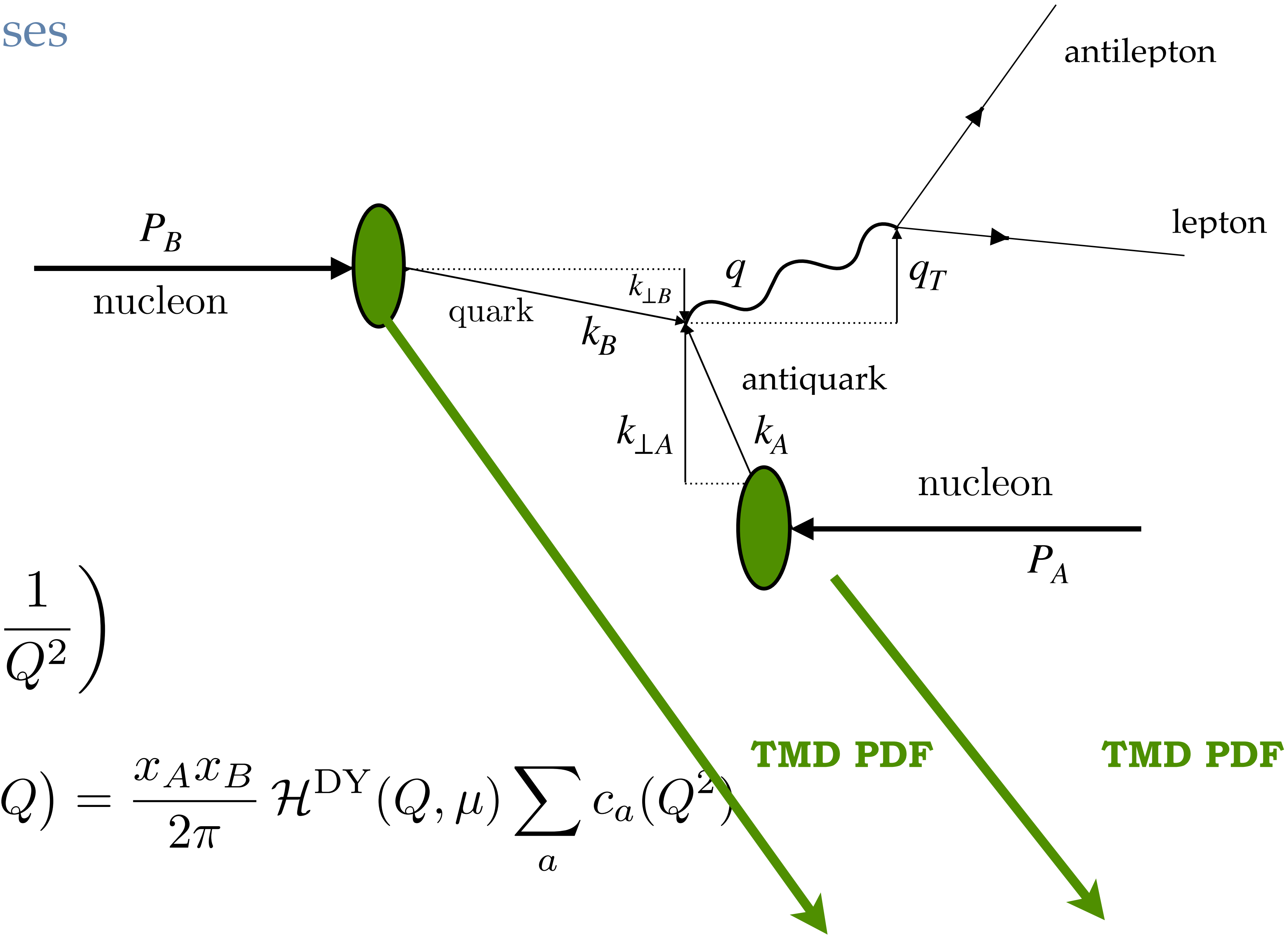

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Structure of a TMD

TMD in Fourier space

$$\hat{F}(x, b_T^2; \mu, \zeta) = \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp} F(x, k_\perp^2; \mu, \zeta)$$

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: A

Perturbative TMD at the initial scale

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Perturbative TMD at the initial scale

$$\times \exp\left\{ K(b_*; \mu_{b_*}) \ln \frac{\sqrt{\zeta}}{\mu_{b_*}} + \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} \qquad : \mathrm{B}$$

Evolution to final scale (of the process)

J. Collins, “Foundation of Perturbative QCD”

Structure of a TMD

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Non-perturbative part of the TMD

J. Collins, “Foundation of Perturbative QCD”

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Perturbative TMD at the initial scale

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Parameterization

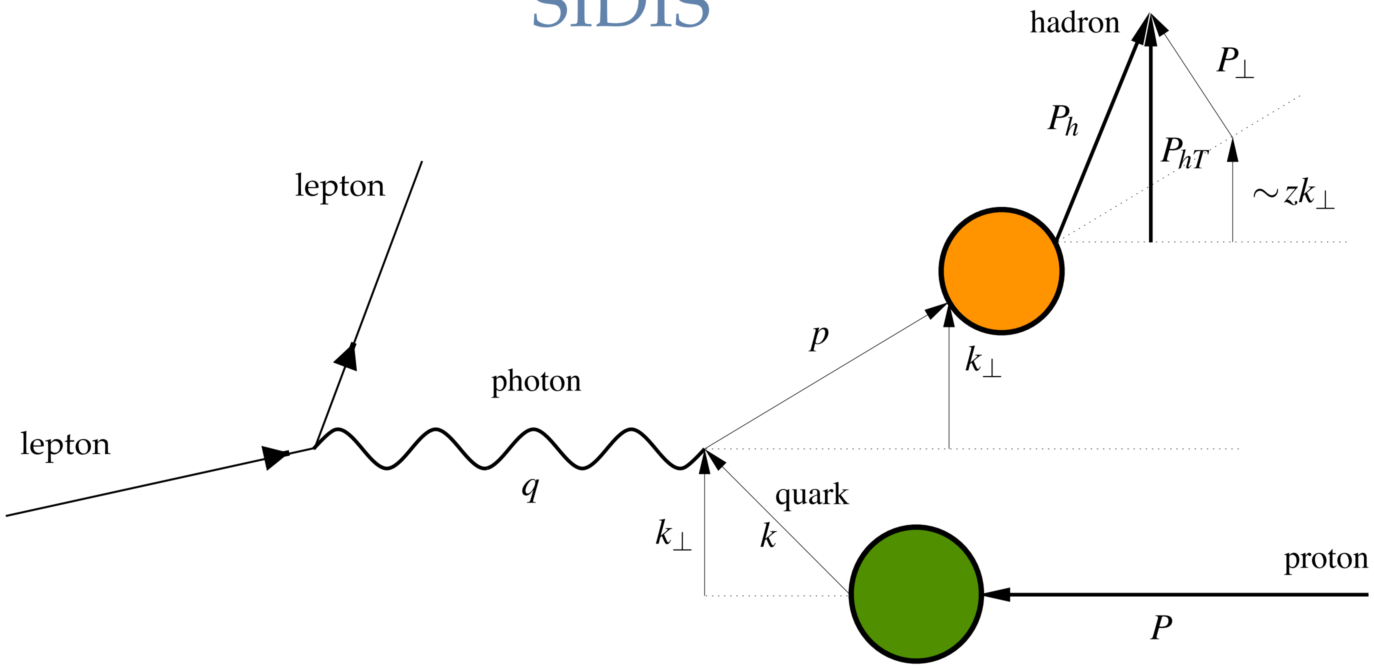
Non-perturbative part of the TMD

J. Collins, “Foundation of Perturbative QCD”

TMD factorization: Universality

TMD factorization: Universality

SIDIS



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TMD factorization: Universality

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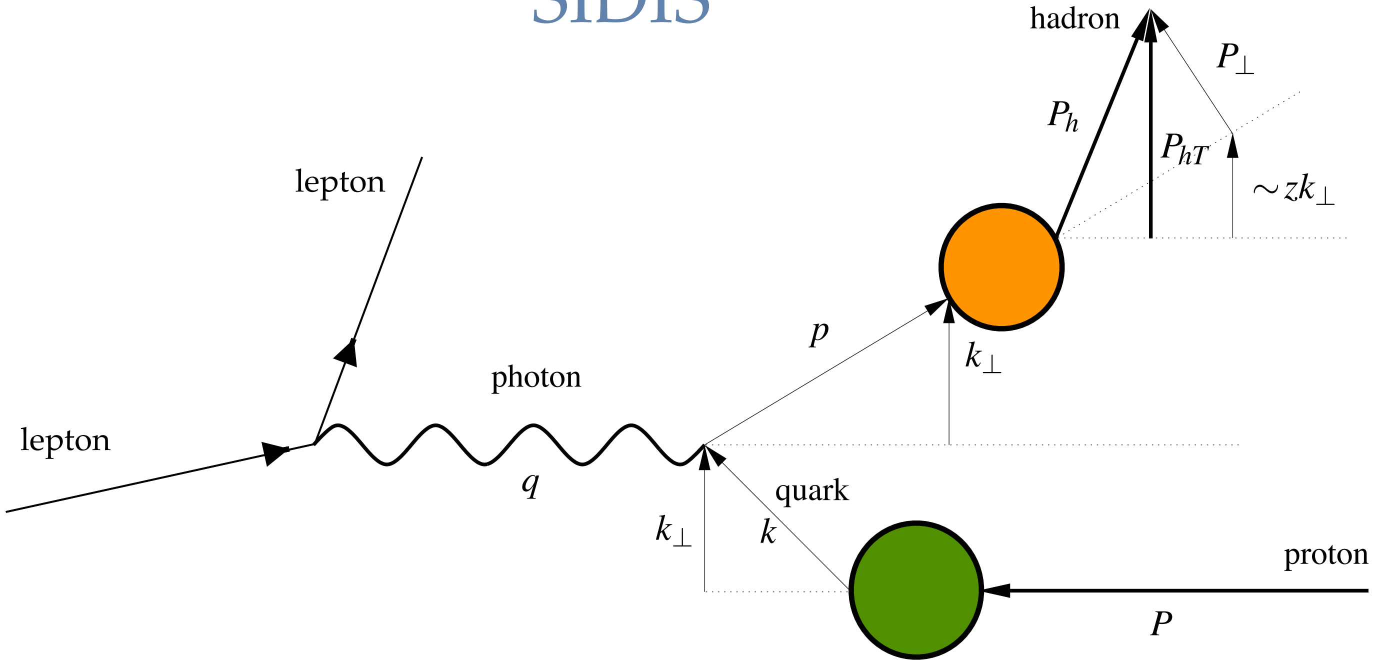
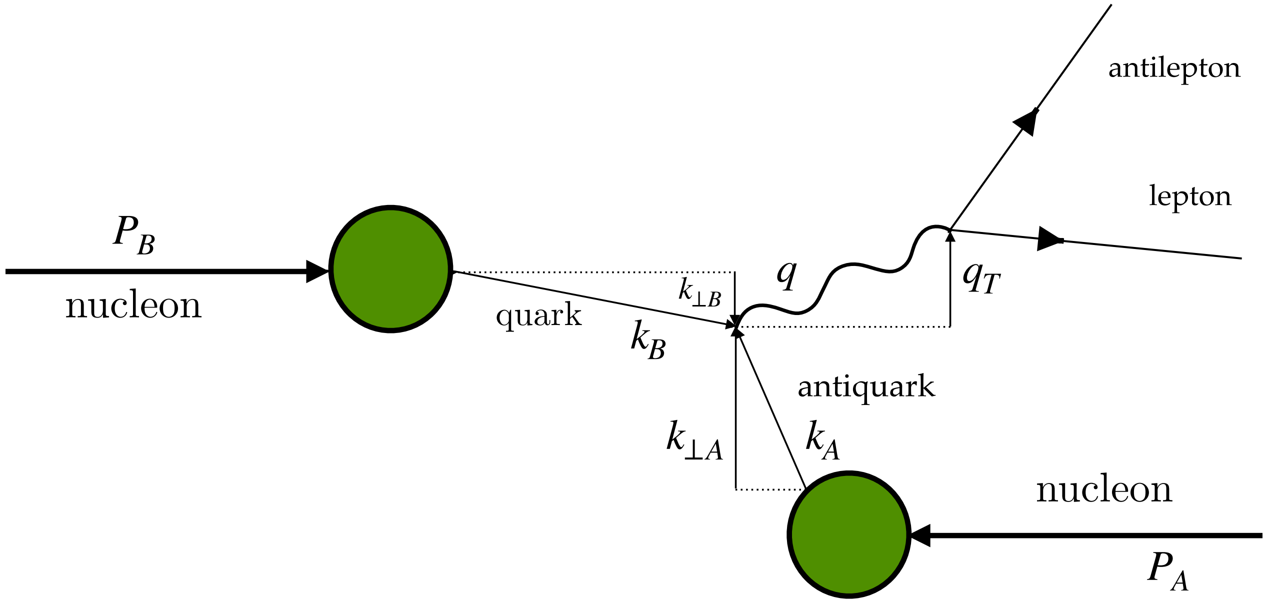


Drell-Yan



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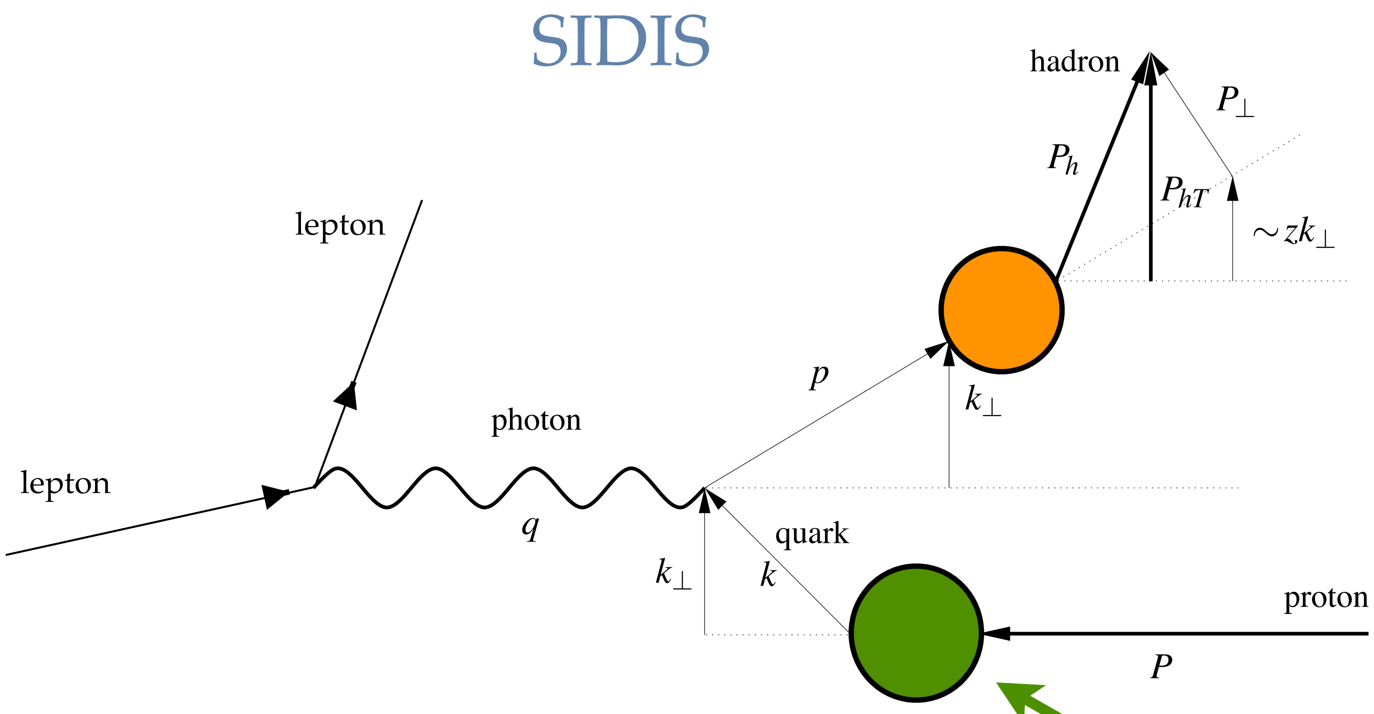

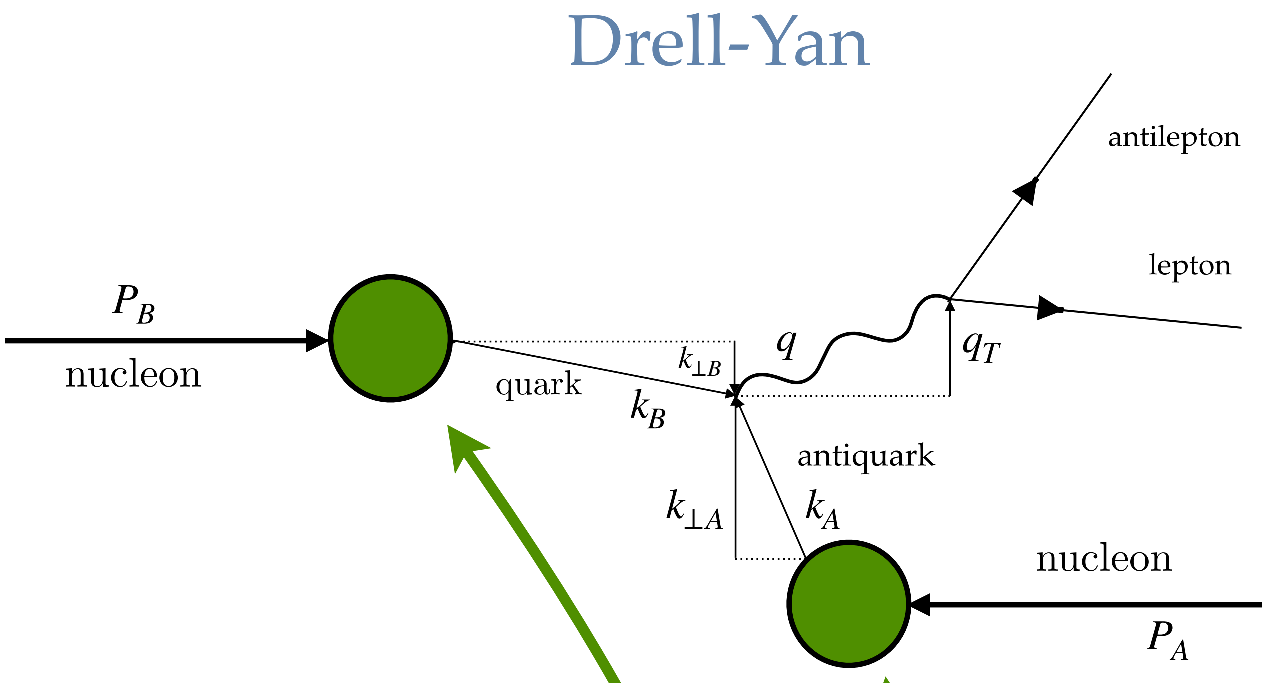

Same functions

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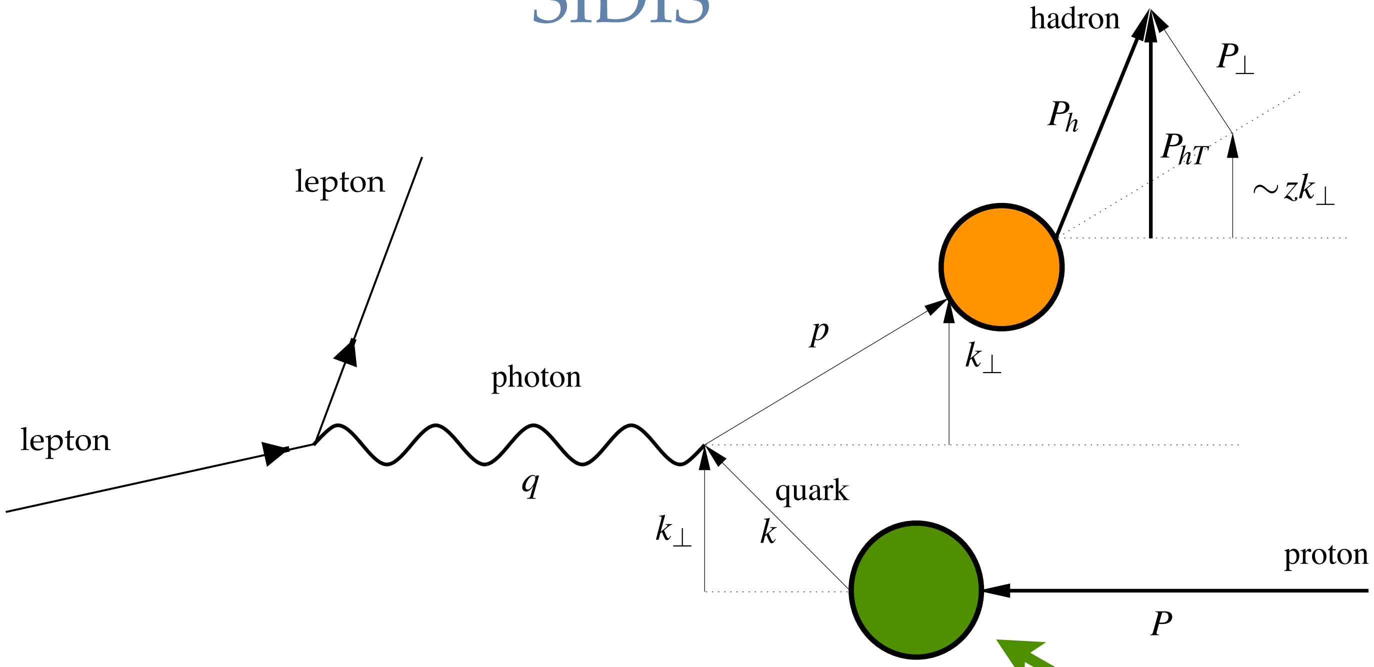

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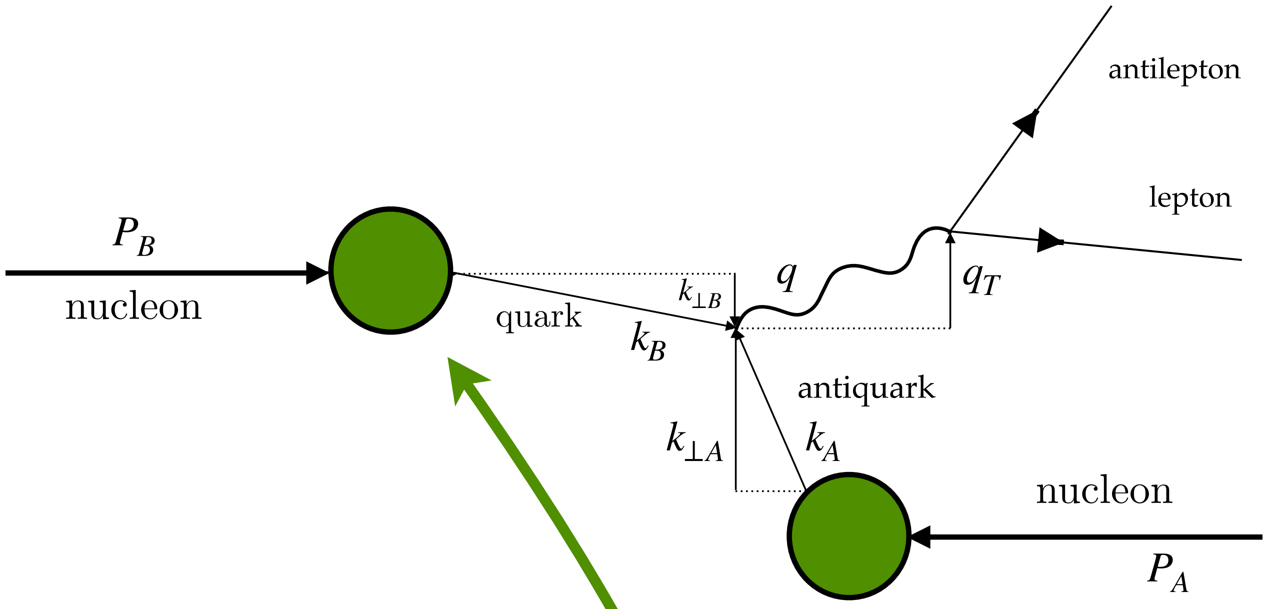


Same functions

$$F_{UU,T}\big(x, z, |\boldsymbol{q}_T|, Q\big) \sim \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0\big(|\boldsymbol{b}_T||\boldsymbol{q}_T|\big) \boxed{\hat{f}_1^a(x, b_T^2; \mu, \zeta_A)} \hat{D}_1^{a\to h}(z, b_T^2; \mu, \zeta_B)$$

$$F_{UU}^1\big(x_A, x_B, |\boldsymbol{q}_T|, Q\big) \sim \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0\big(|\boldsymbol{b}_T||\boldsymbol{q}_T|\big) \boxed{\hat{f}_1^a(x_A, b_T^2; \mu, \zeta_A)} \boxed{\hat{f}_1^{\bar{a}}(x_B, b_T^2; \mu, \zeta_B)}$$

GLOBAL FITs

Available TMD fitting frameworks

https://github.com/MapCollaboration/NangaParbat

README.md

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

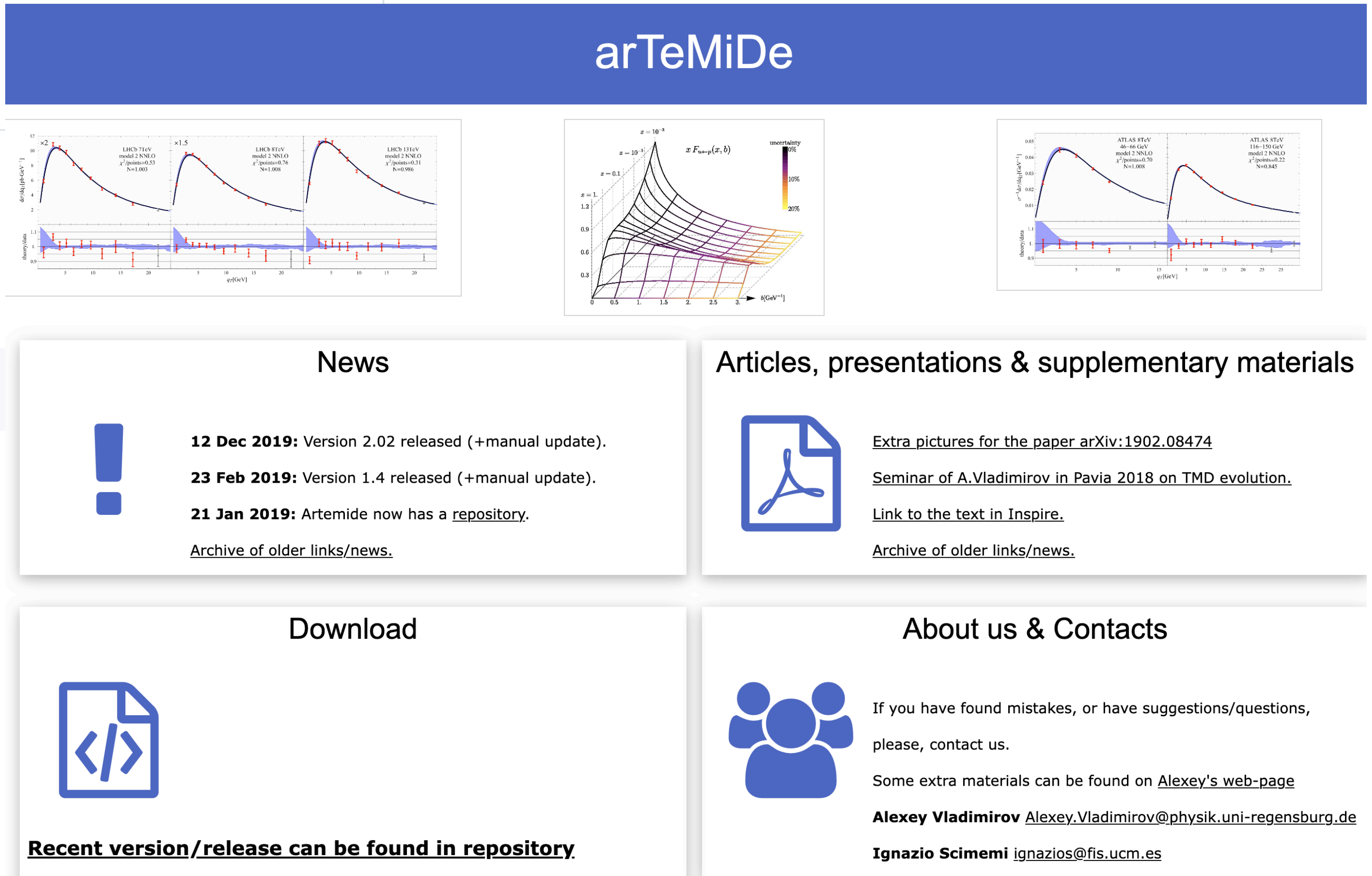
You can obtain NangaParbat directly from the github repository:

https://github.com/MapCollaboration/NangaParbat

For the last development branch you can clone the master code:

```
git clone git@github.com:MapCollaboration/NangaParbat.git
```

https://teorica.fis.ucm.es/artemide/



arTeMiDe

News

12 Dec 2019: Version 2.02 released (+manual update).

23 Feb 2019: Version 1.4 released (+manual update).

21 Jan 2019: Artemide now has a repository.

Archive of older links/news.

Articles, presentations & supplementary materials

Extra pictures for the paper arXiv:1902.08474

Seminar of A.Vladimirov in Pavia 2018 on TMD evolution.

Link to the text in Inspire.

Archive of older links/news.

Download

Recent version/release can be found in repository

About us & Contacts

If you have found mistakes, or have suggestions/questions, please, contact us.

Some extra materials can be found on Alexey's web-page

Alexey Vladimirov Alexey.Vladimirov@physik.uni-regensburg.de

Ignazio Scimemi ignazios@fis.ucm.es

Available Global Fits

	Accuracy	SIDIS	DY	N of points	χ^2/N_{data}
Pavia 2017 Bacchetta, Delcarro, et al., JHEP 06 (2017)	NLL	✔	✔	8059	1.55
SV 2019 Scimemi, Vladimirov, JHEP 06 (2020)	N^{3}LL$^-$	✔	✔	1039	1.06
MAPTMD22 Bacchetta, Bertone, et al., JHEP 10 (2022)	***N^{3}LL$^-$***	✔	✔	***2031***	***1.06***

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MAPTMD22 Bacchetta, Bertone, et al., JHEP 10 (2022)	***N^3LL^-***	✔	✔	***2031***	***1.06***

MAPTMD22 global fit

- Global analysis of Drell-Yan and SIDIS data sets: ***2031*** data points
- Perturbative accuracy: N^3LL^-
- Number of fitted parameters: ***21***
- Extremely good description: $\chi^2/N_{data} = 1.06$

MAP Collaboration, JHEP 10 (2022)

Differences in recent global fits

MAPTMD22 vs **SV19**

Differences in recent global fits

MAPTMD22 vs **SV19**

- Criteria of data selection

  2031 vs **1039** included data

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  2031 vs **1039** included data

- Implementation of TMD evolution

  CSS framework vs **zeta-prescription**

Differences in recent global fits

MAPTMD22 vs **SV19**

- Criteria of data selection

  2031 vs **1039** included data

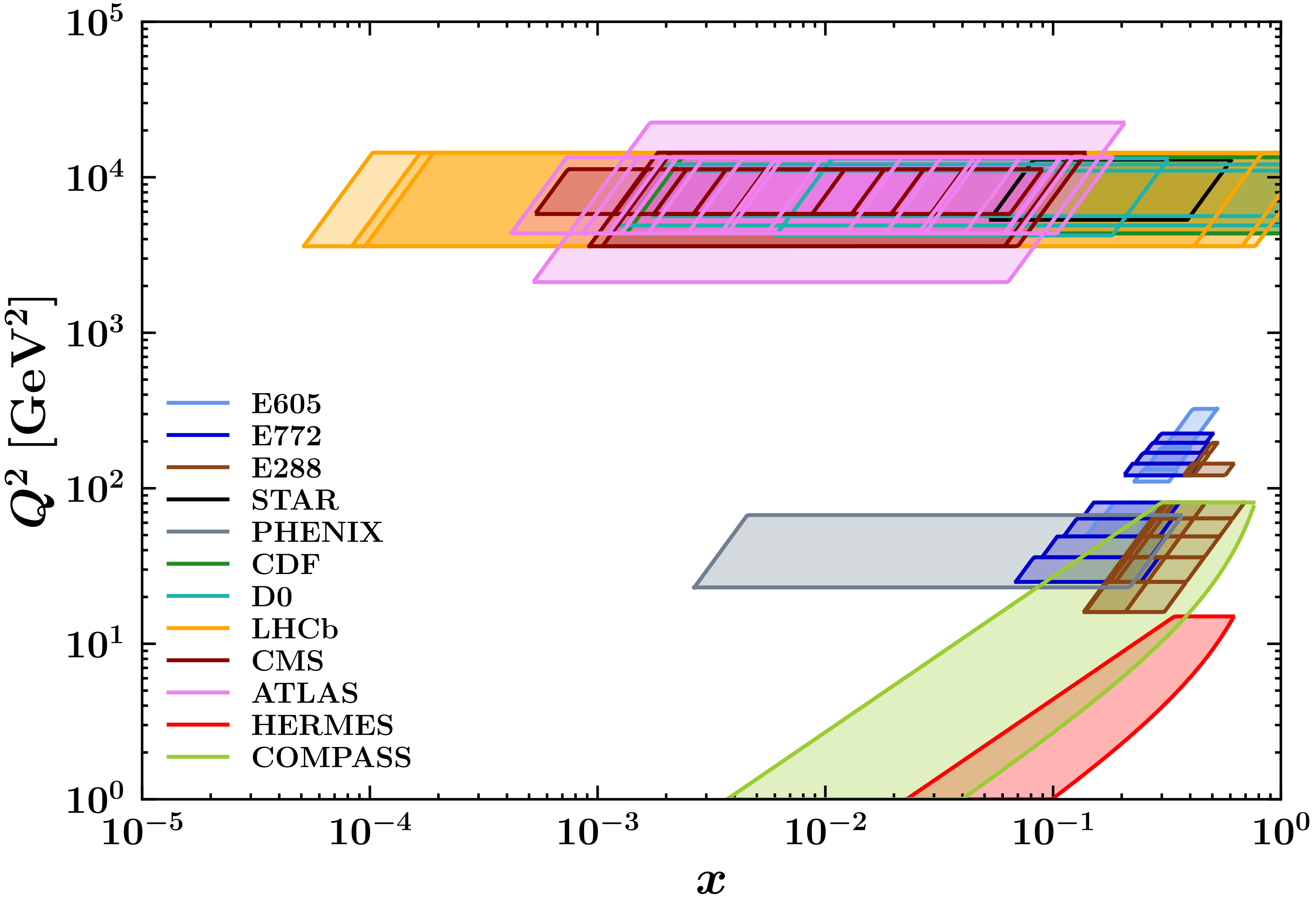
- Implementation of TMD evolution

  CSS framework vs **zeta-prescription**

- Nonperturbative parameterization

  Gaussian + wGaussian vs **complicated function**

MAP22: included data sets



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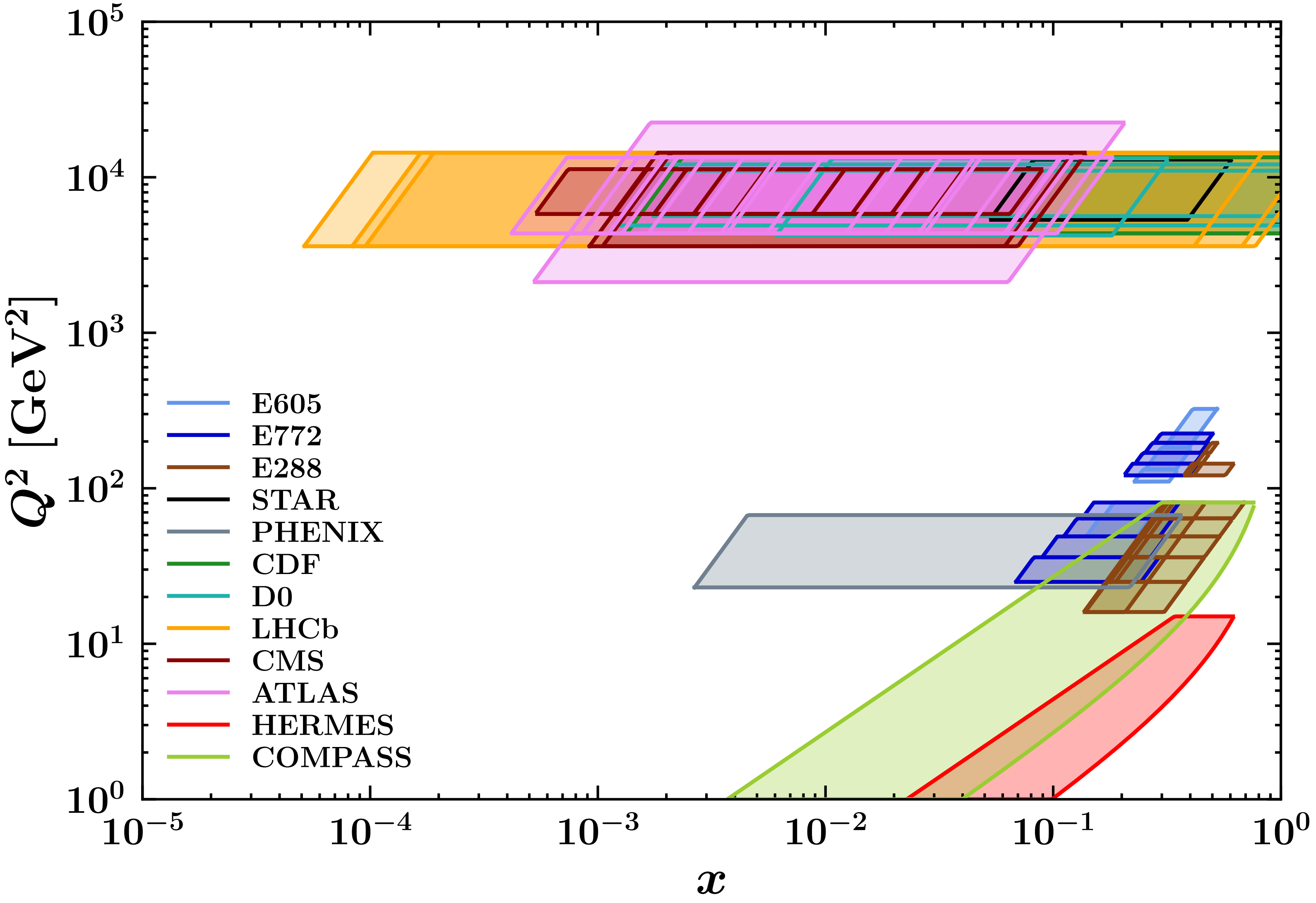
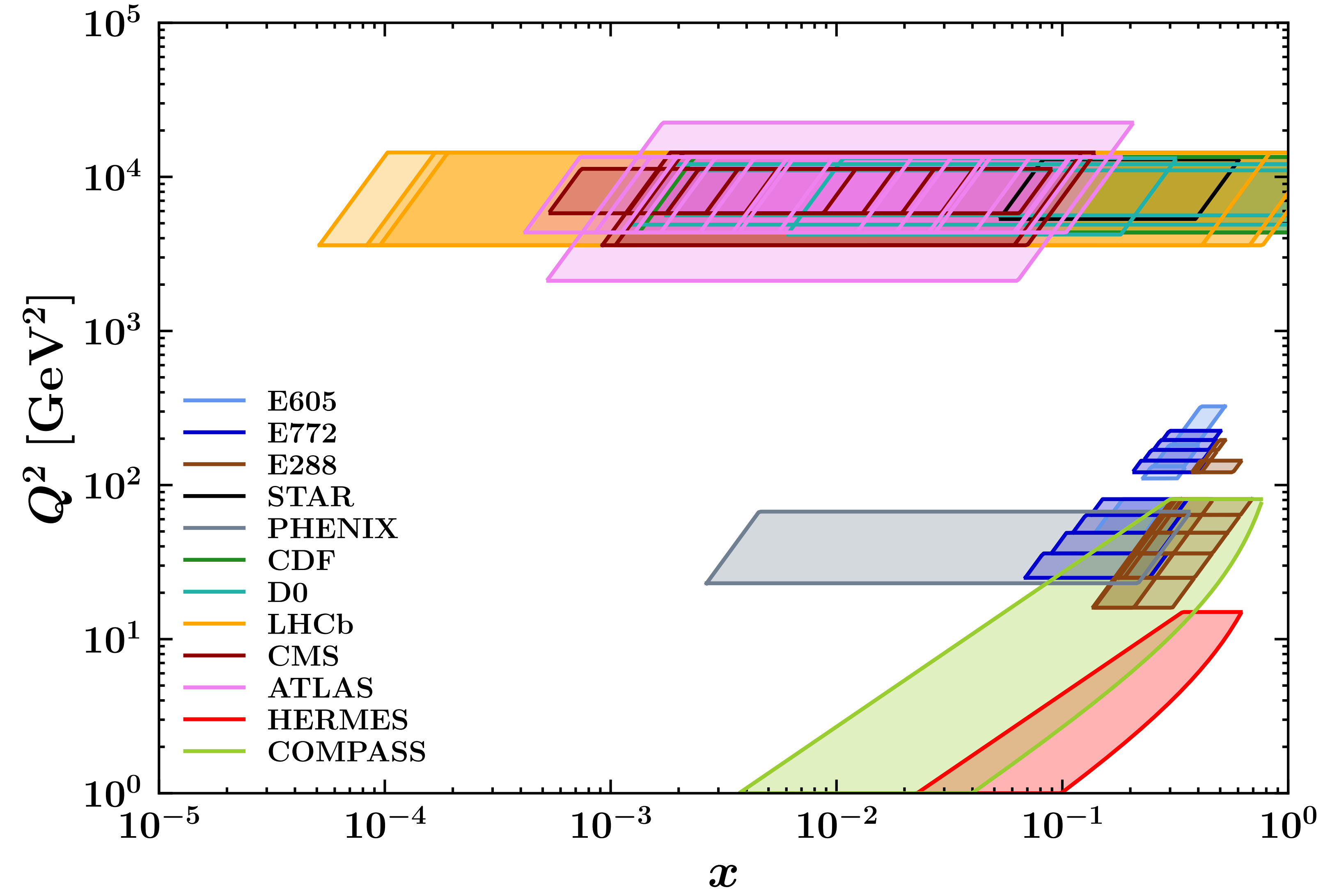


Drell-Yan data	484	DY fixed-target + collider

MAP22: included data sets



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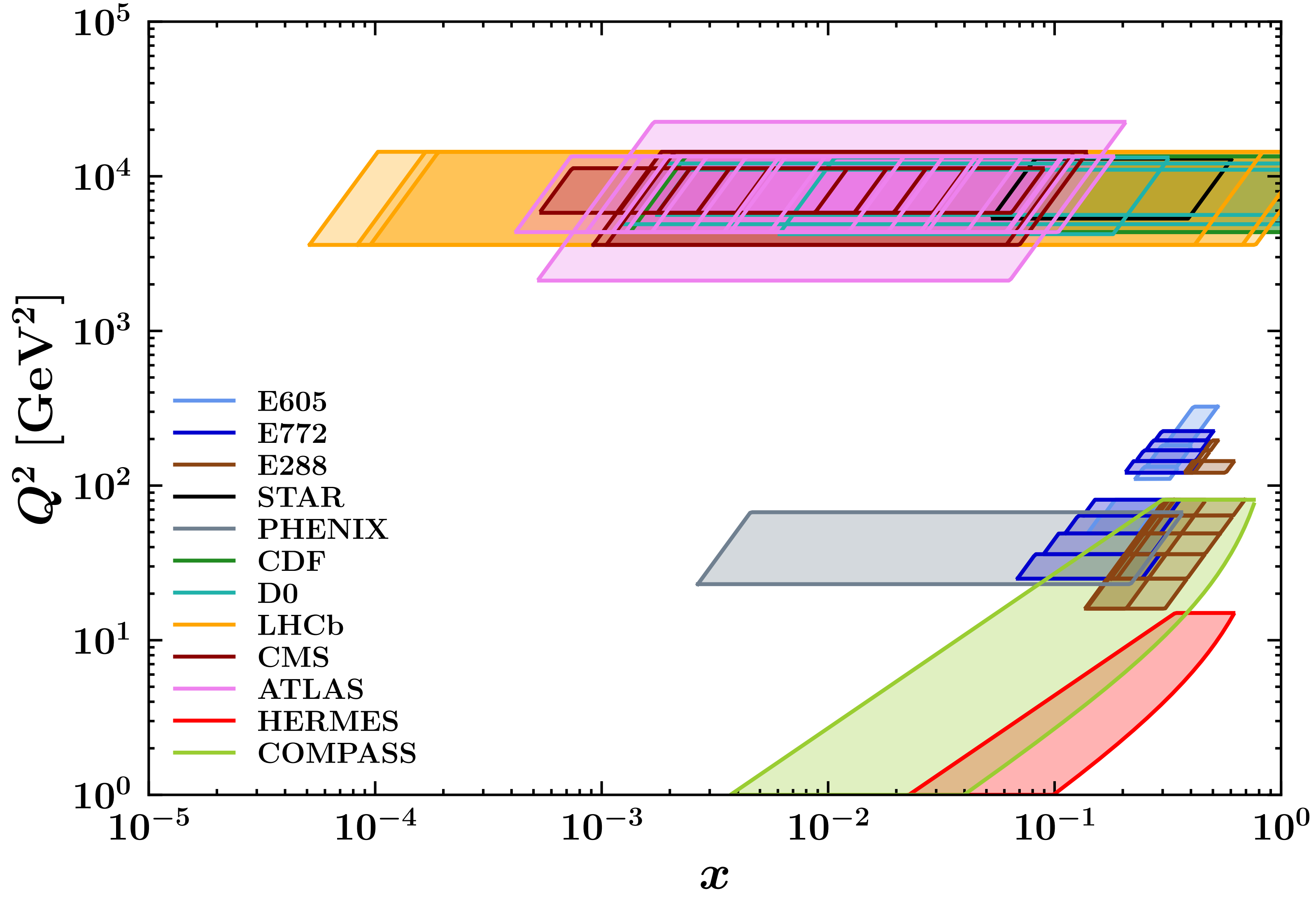

Drell-Yan data	484	DY fixed-target + collider
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Total number of data	2031	

MAP22: Collinear input

Perturbative TMD at the initial scale

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_j C_{q/j}(x, b_*; \mu_{b_*}, \mu_{b_*}^2) \otimes f_1^j(x, \mu_{b_*}) \qquad : \mathrm{A}$$

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Collinear distributions

MAP22: Collinear input

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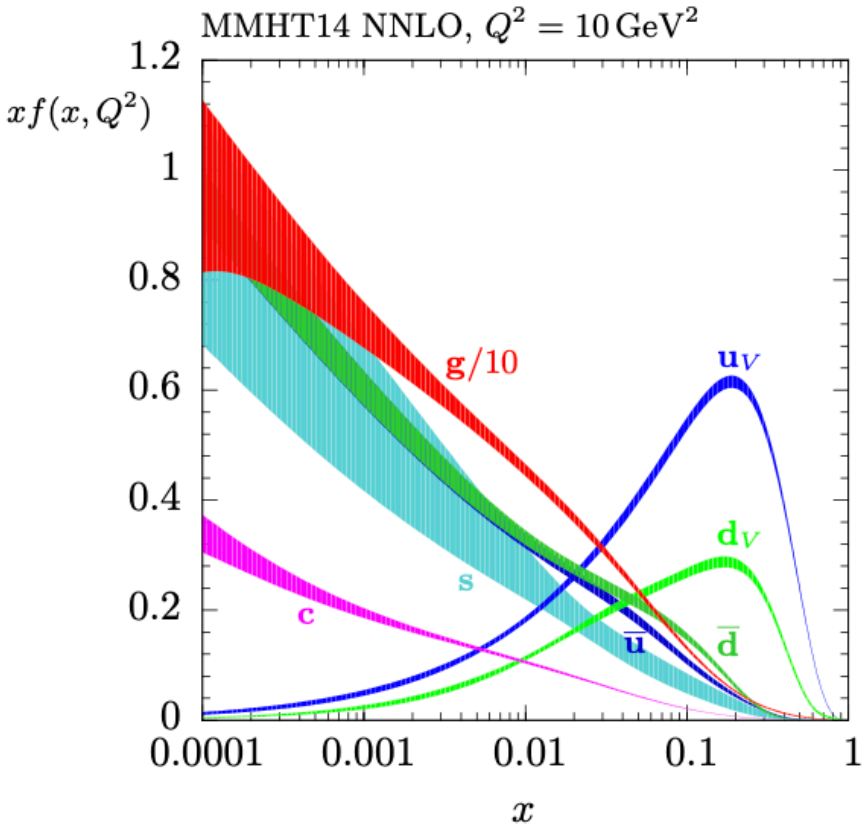
Collinear distributions

Input for PDFs: MMHT2014



Harland-Lang, Martin, Motylinski, Thorne, EPJ C 75 (2015)

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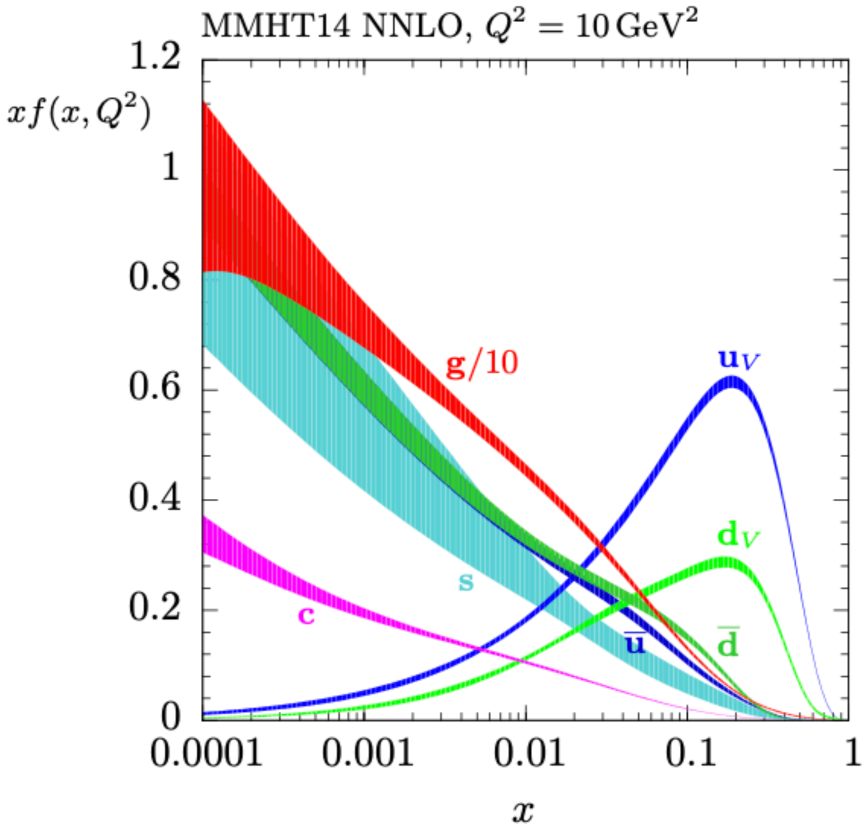
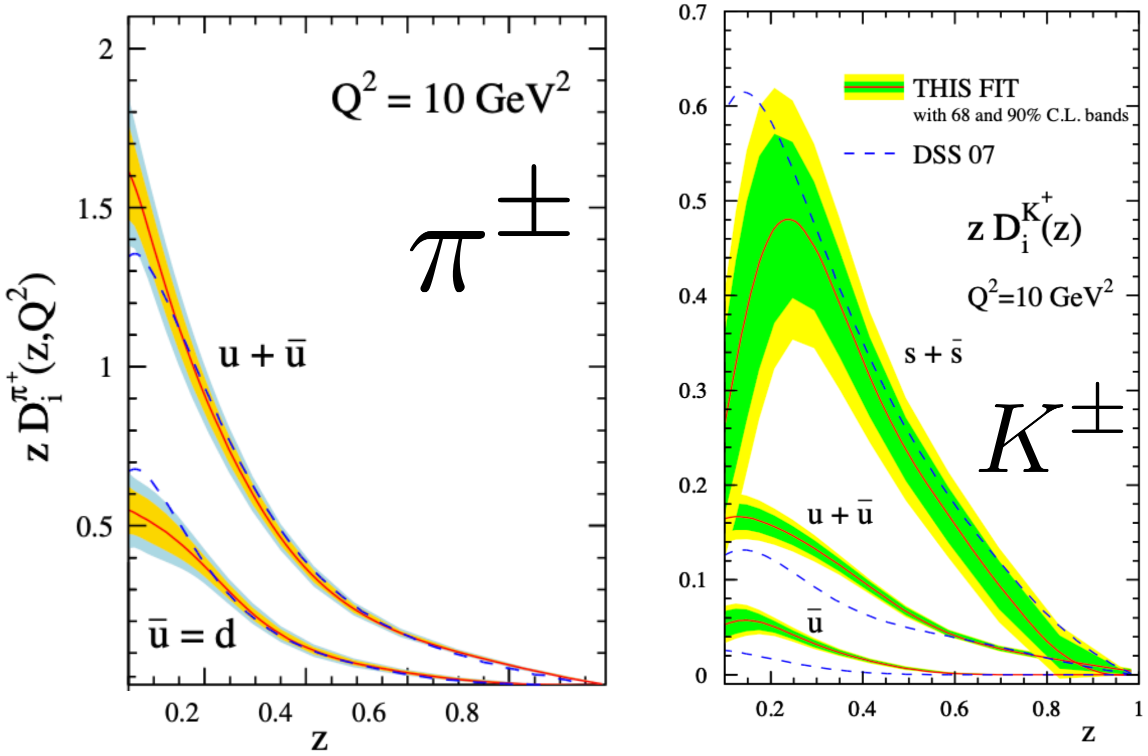


Harland-Lang, Martin, Motylinski, Thorne, EPJ C 75 (2015)

Input for FFs: DSS14 - DSS17



De Florian, Sassot, Hepele, Hernandez-Pinto, Stratmann, PRD 91 (2015)
De Florian, Hepele, Hernandez-Pinto, Sassot, Stratmann, PRD 95 (2017)

MAP22: NP parametrization

$$f_{NP}(x, b_T^2) \exp \left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\}$$

Bacchetta, Gamberg, Goldstein, et al., PLB 659 (2008)
Bacchetta, Conti, Radici, PRD 78 (2008)
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Matevosyan, Bentz, Cloet, Thomas, PRD 85 (2012)
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Grewal, Kang, Qiu, Signori, PRD 101 (2020)

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\}$$

$$f_{1\mathrm{NP}}(x, b_T^2) \propto \mathrm{F.T.\ of} \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

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$$D_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

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11 parameters for TMD PDF

+ 1 for NP evolution + 9 for TMD FF

= 21 free parameters

MAP22: SIDIS normalization

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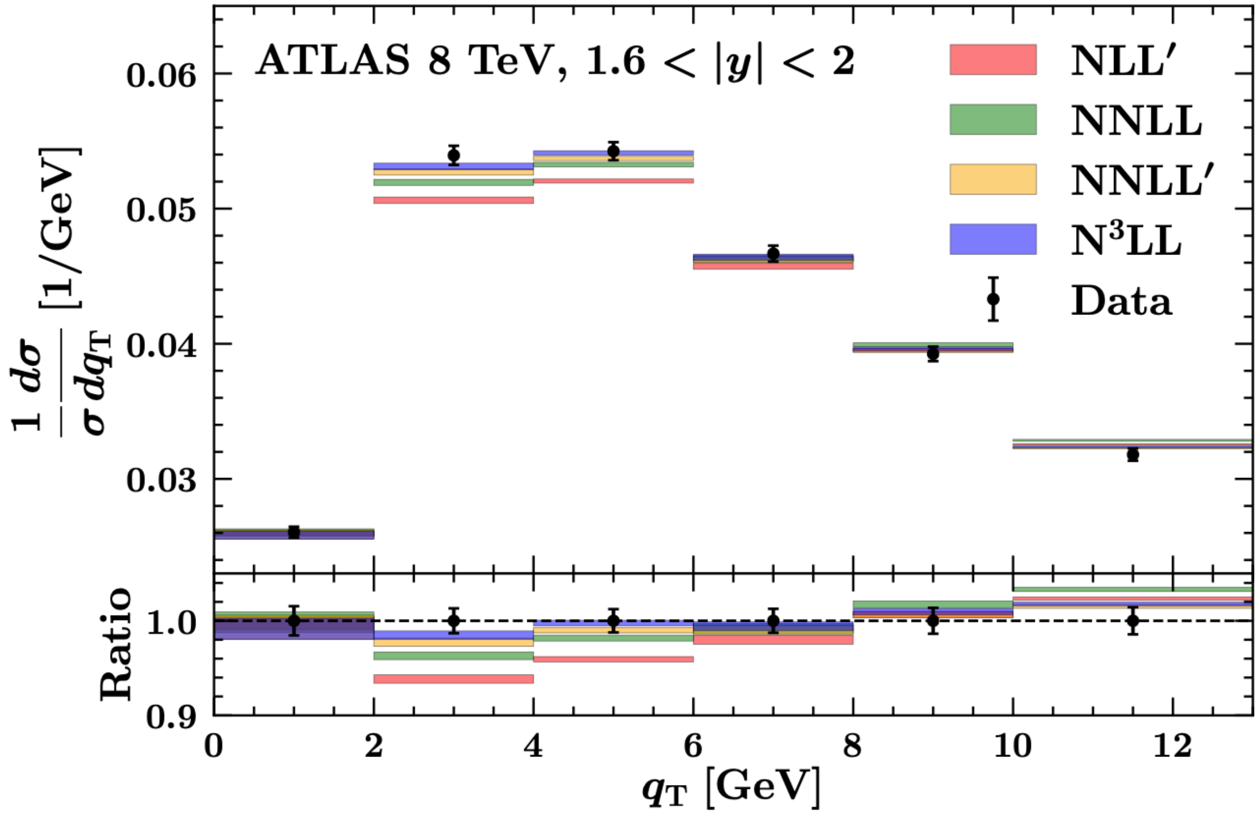
High-Energy Drell-Yan beyond NLL

$\boxed{Q \sim 100 \text{ GeV}}$



Bacchetta, Bertone, Bissolotti, et al., JHEP 07 (2020)

MAP22: SIDIS normalization

SIDIS observables beyond NLL

High-Energy Drell-Yan beyond NLL

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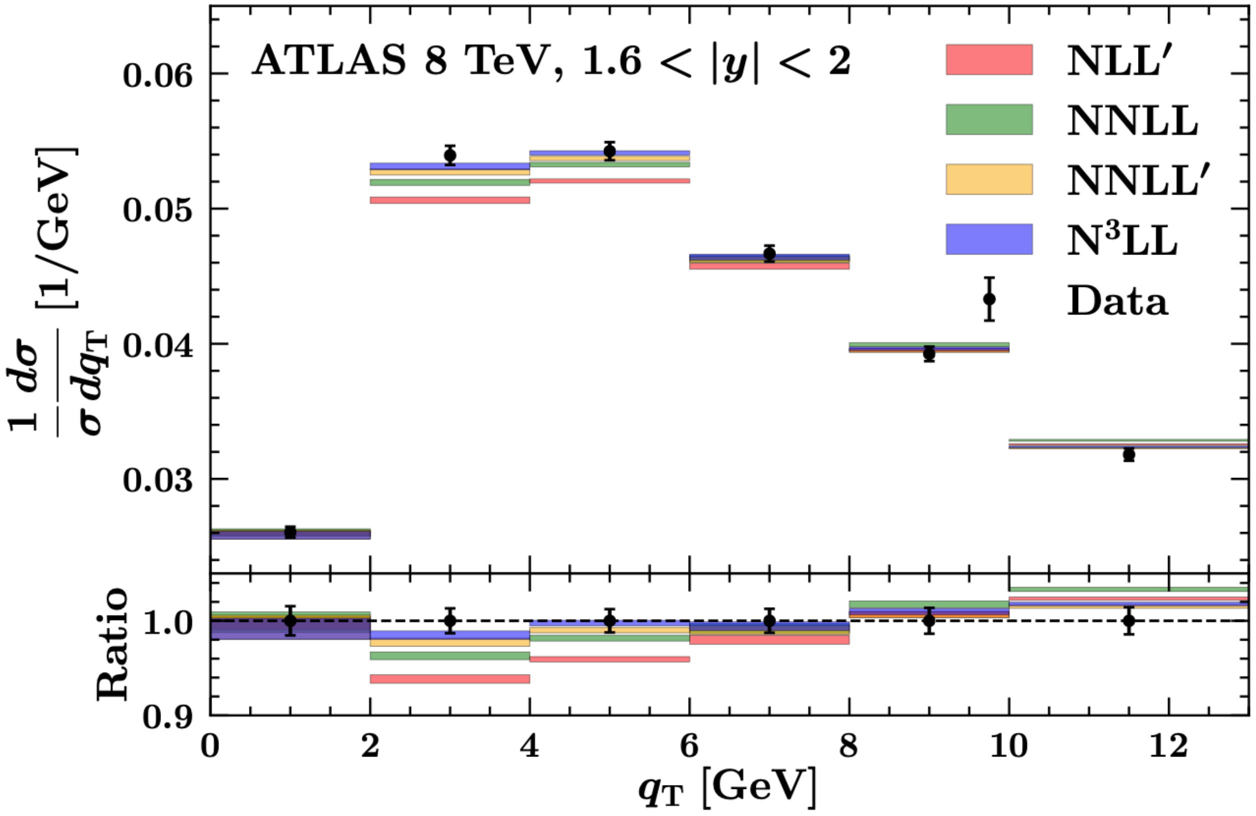


Bacchetta, Bertone, Bissolotti, et al., JHEP 07 (2020)

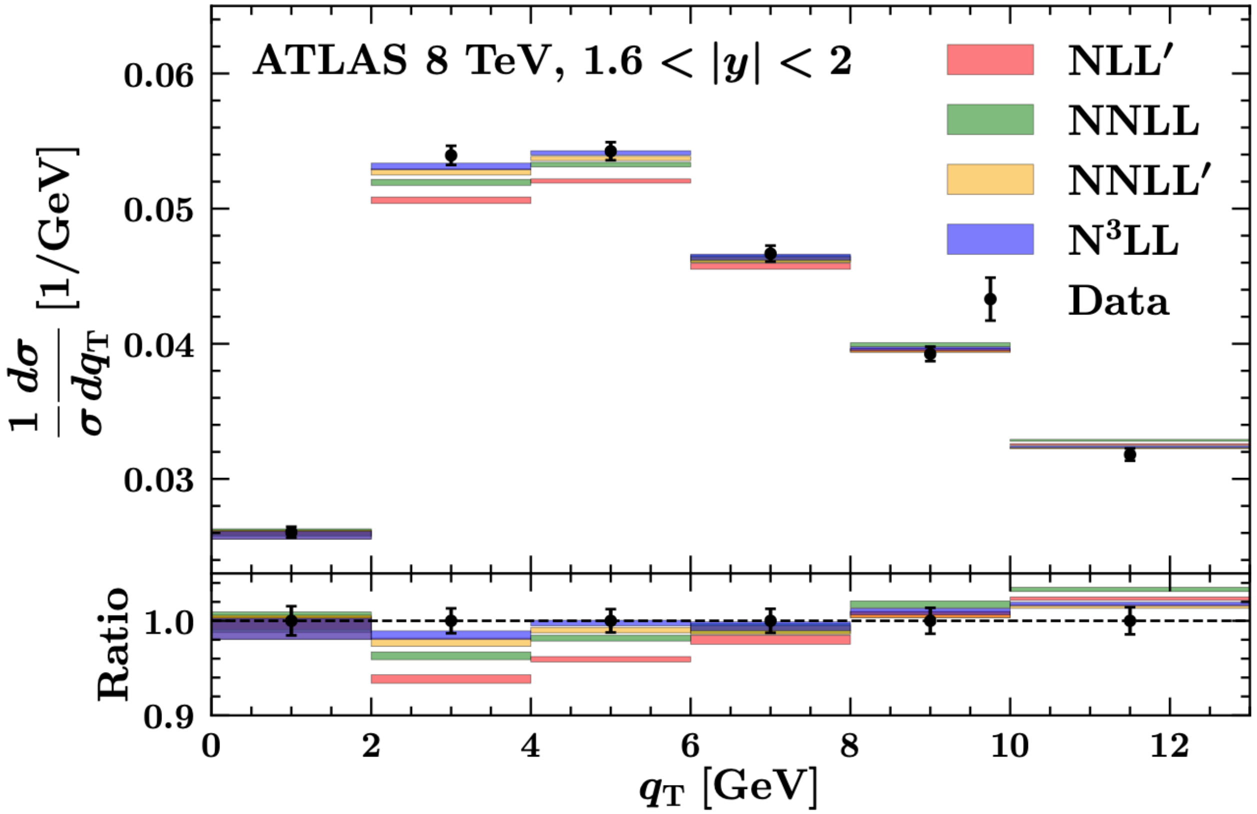
MAP22: SIDIS normalization

SIDIS observables beyond NLL

$\boxed{Q \sim 2 \text{ GeV}}$

High-Energy Drell-Yan beyond NLL

$\boxed{Q \sim 100 \text{ GeV}}$



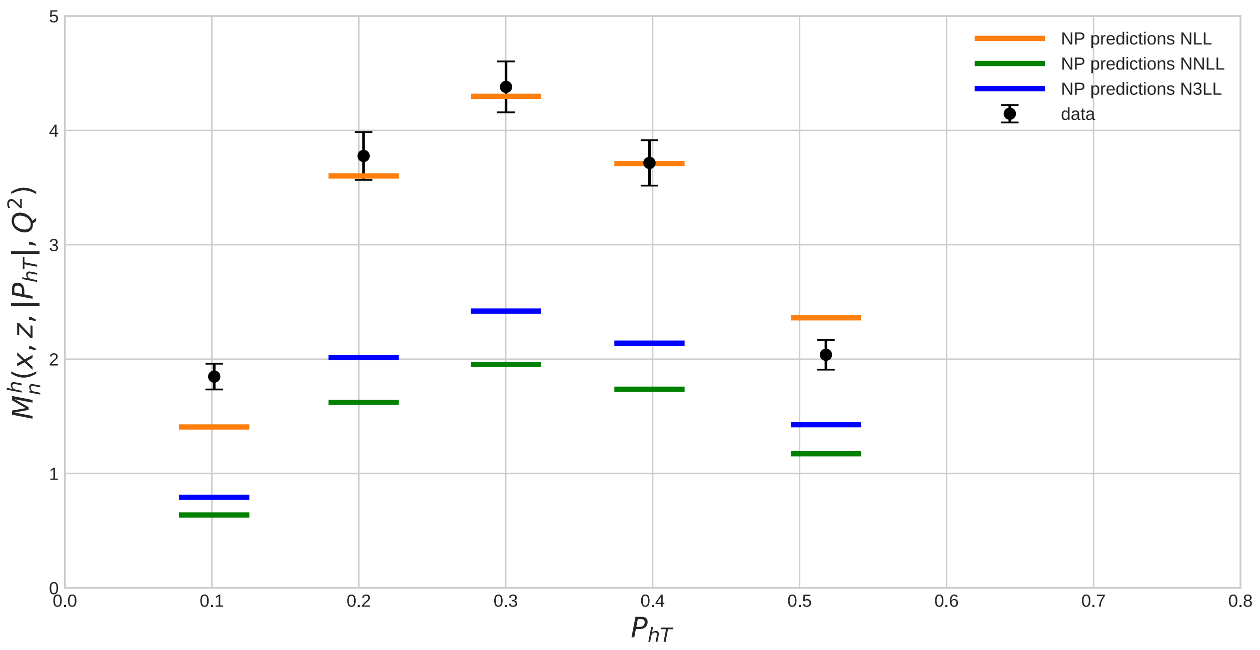
Bacchetta, Bertone, Bissolotti, et al., JHEP 07 (2020)

MAP22: SIDIS normalization

<u>SIDIS observables beyond NLL</u>

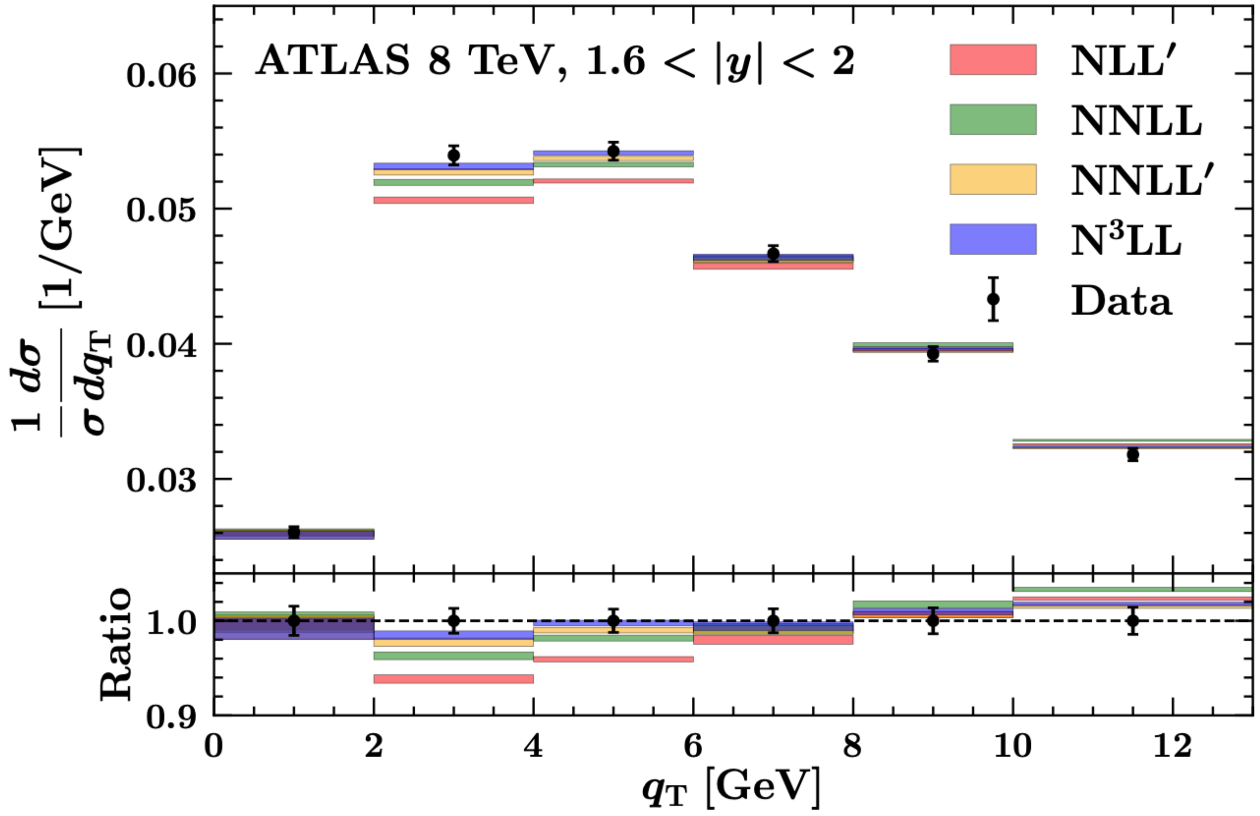
$Q \sim 2$ GeV

HERMES



<u>High-Energy Drell-Yan beyond NLL</u>

$Q \sim 100$ GeV



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MAP22: SIDIS normalization

SIDIS observables beyond NLL

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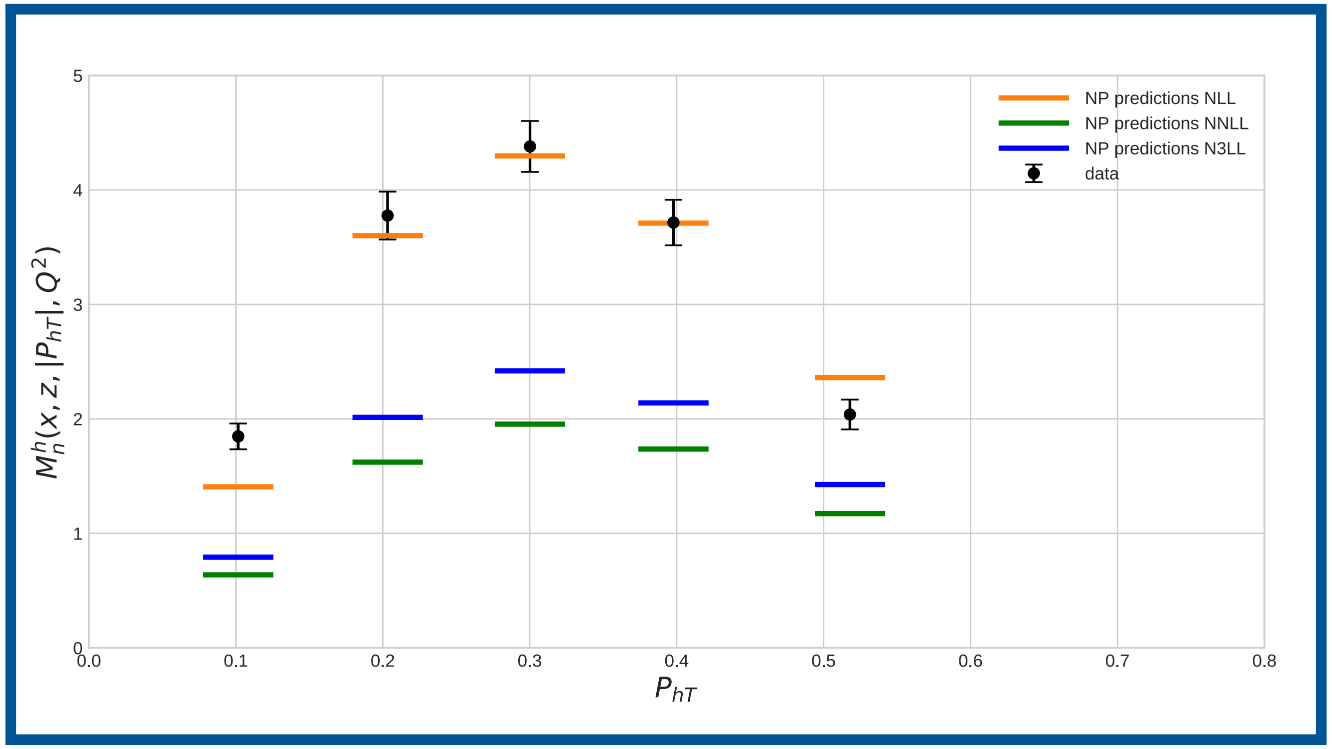
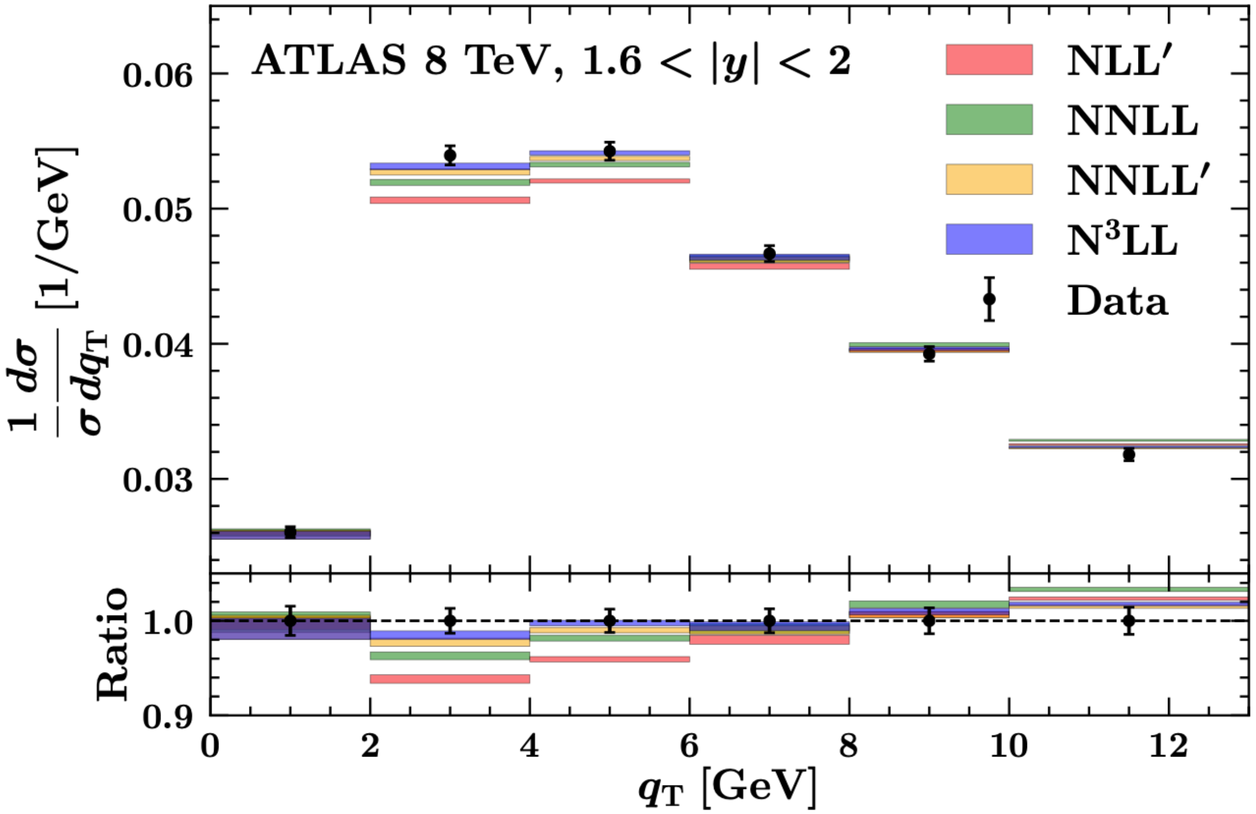
HERMES



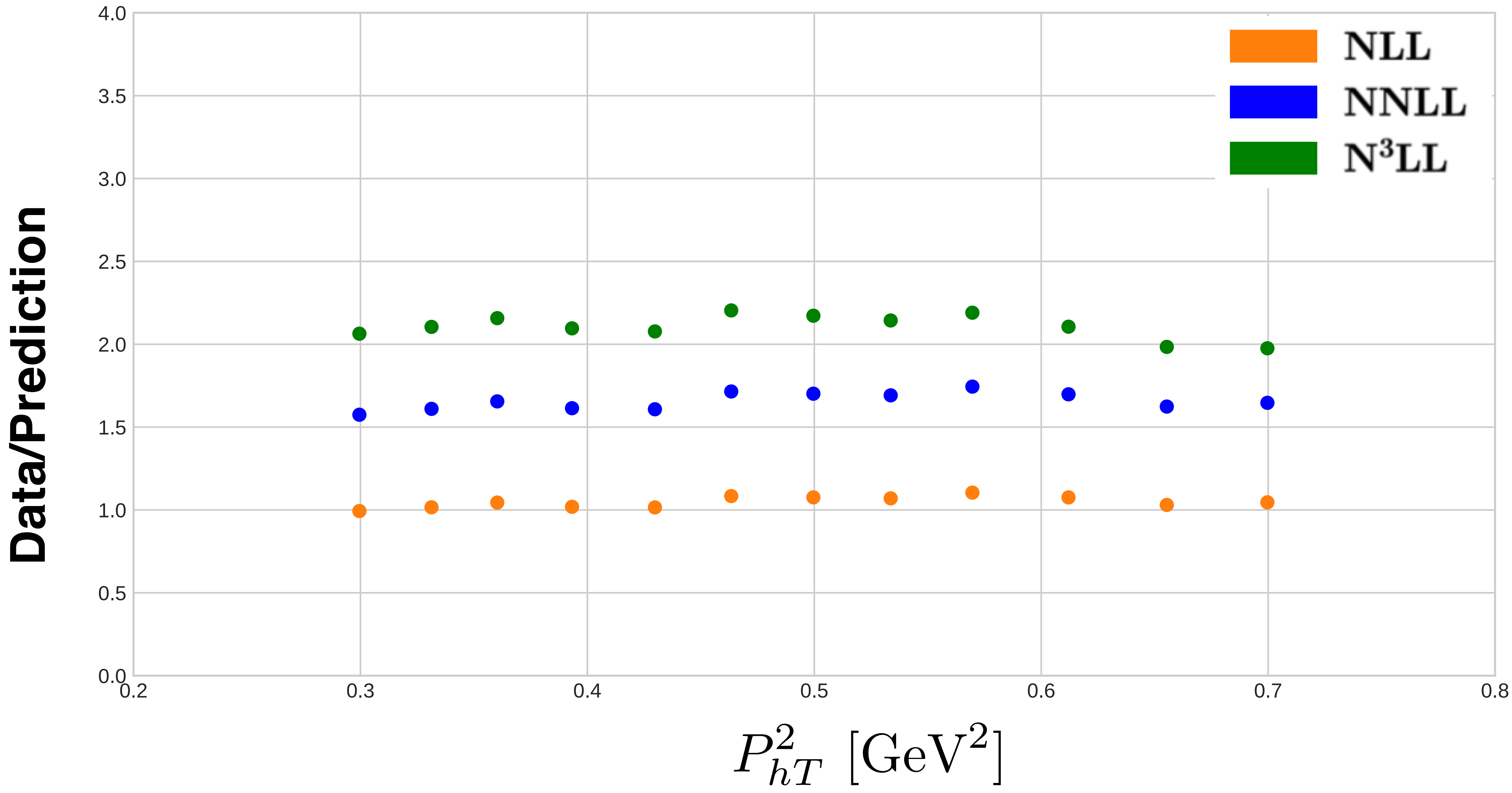


Bacchetta, Bertone, Bissolotti, et al., JHEP 07 (2020)

The description considerably worsens at higher orders!!

MAP22: SIDIS normalization

COMPASS multiplicities (one of many bins)



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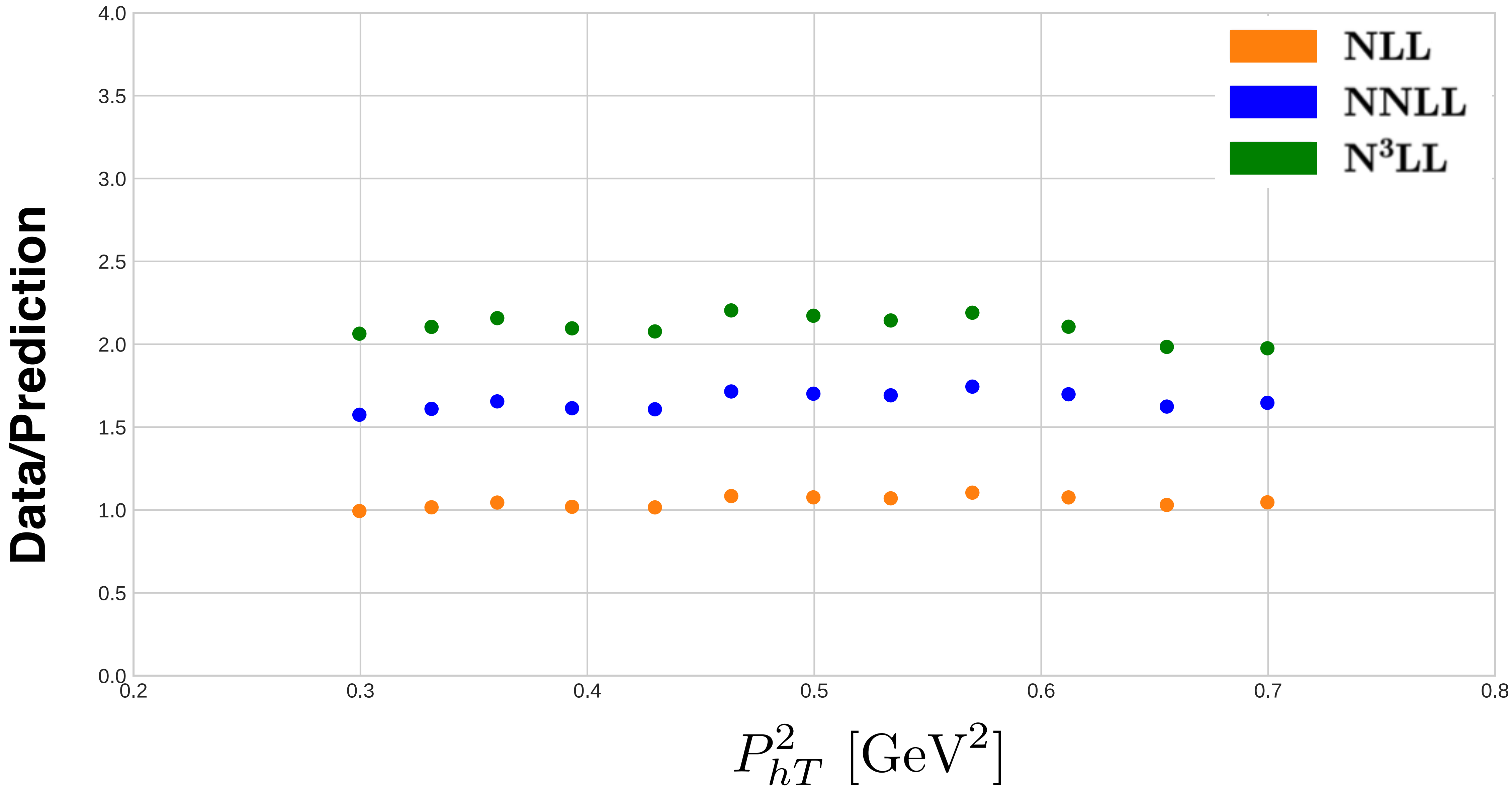
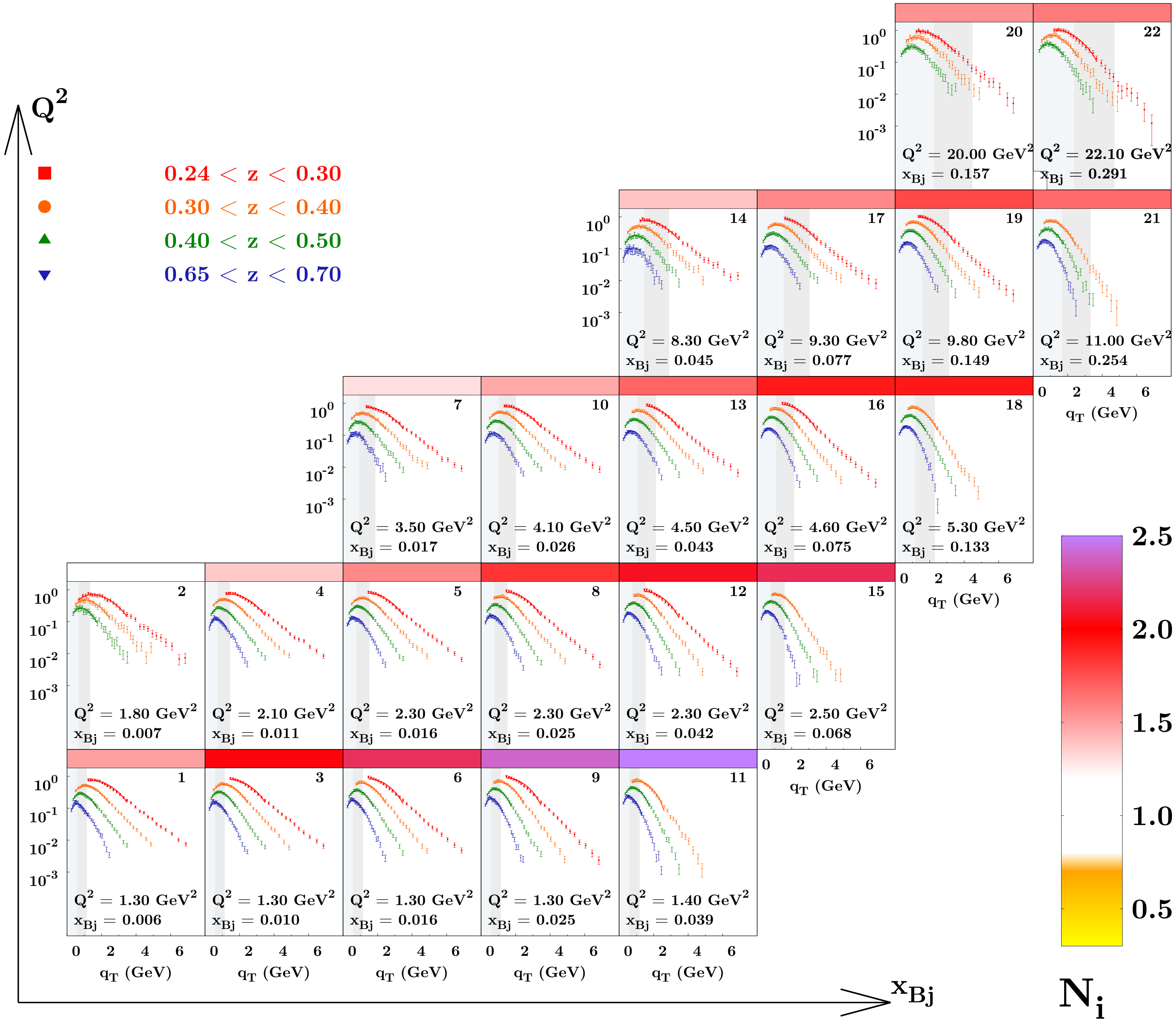


Discrepancy of an almost constant factor

Normalization of SIDIS calculation

Normalization issue confirmed also in other analyses from different collaborations



Gonzalez-Hernandez, PoS DIS2019 (2019)

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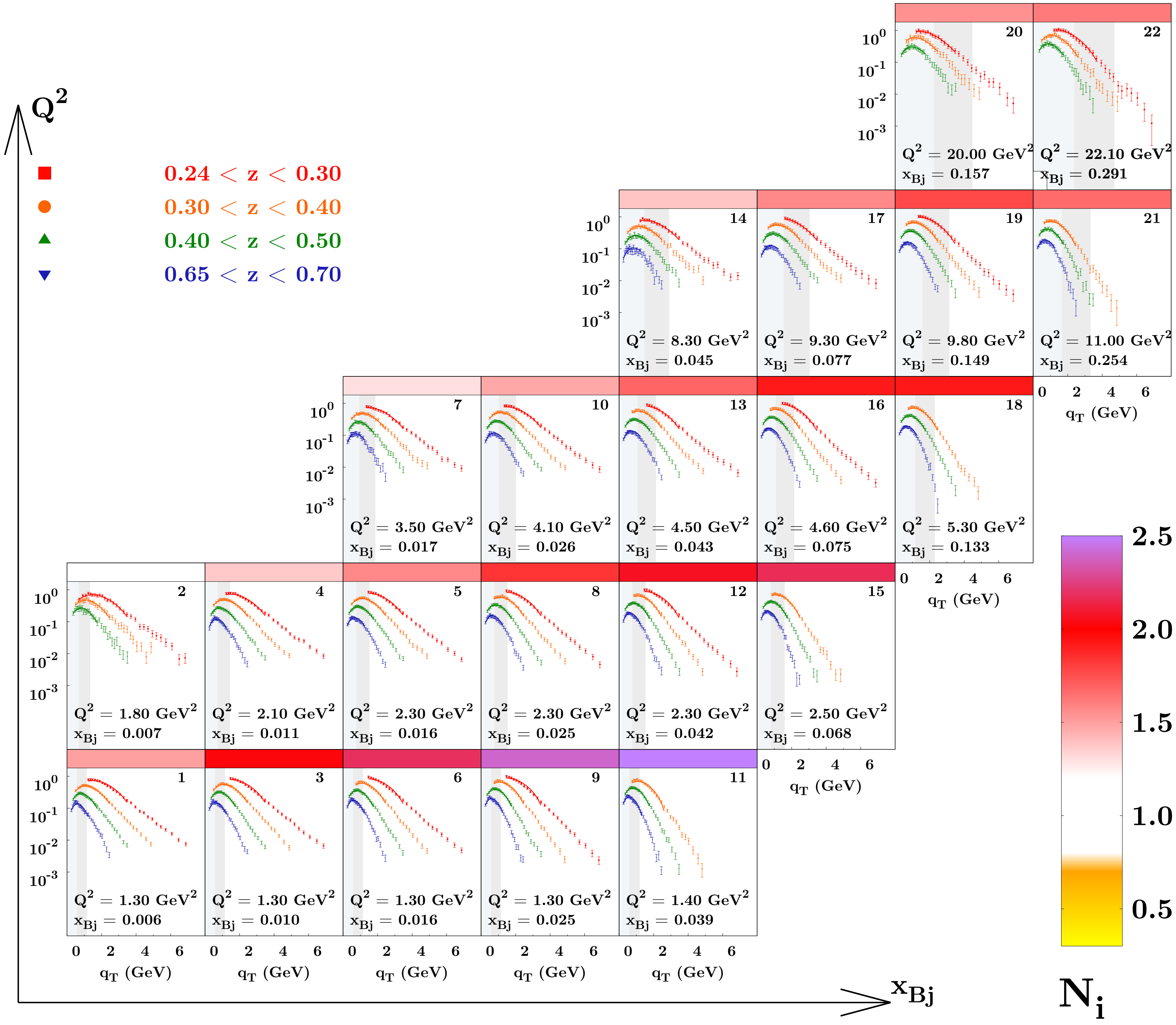

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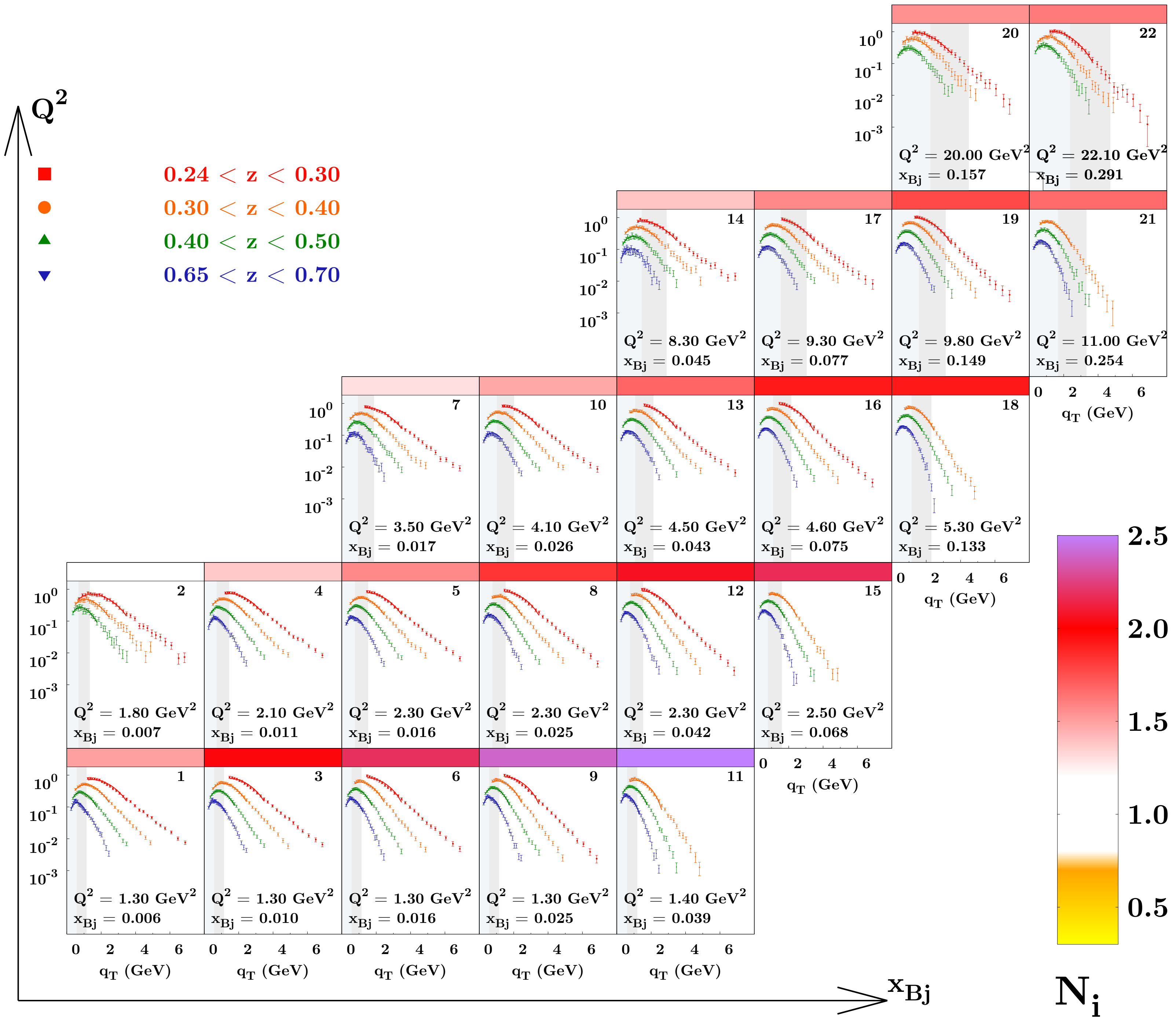

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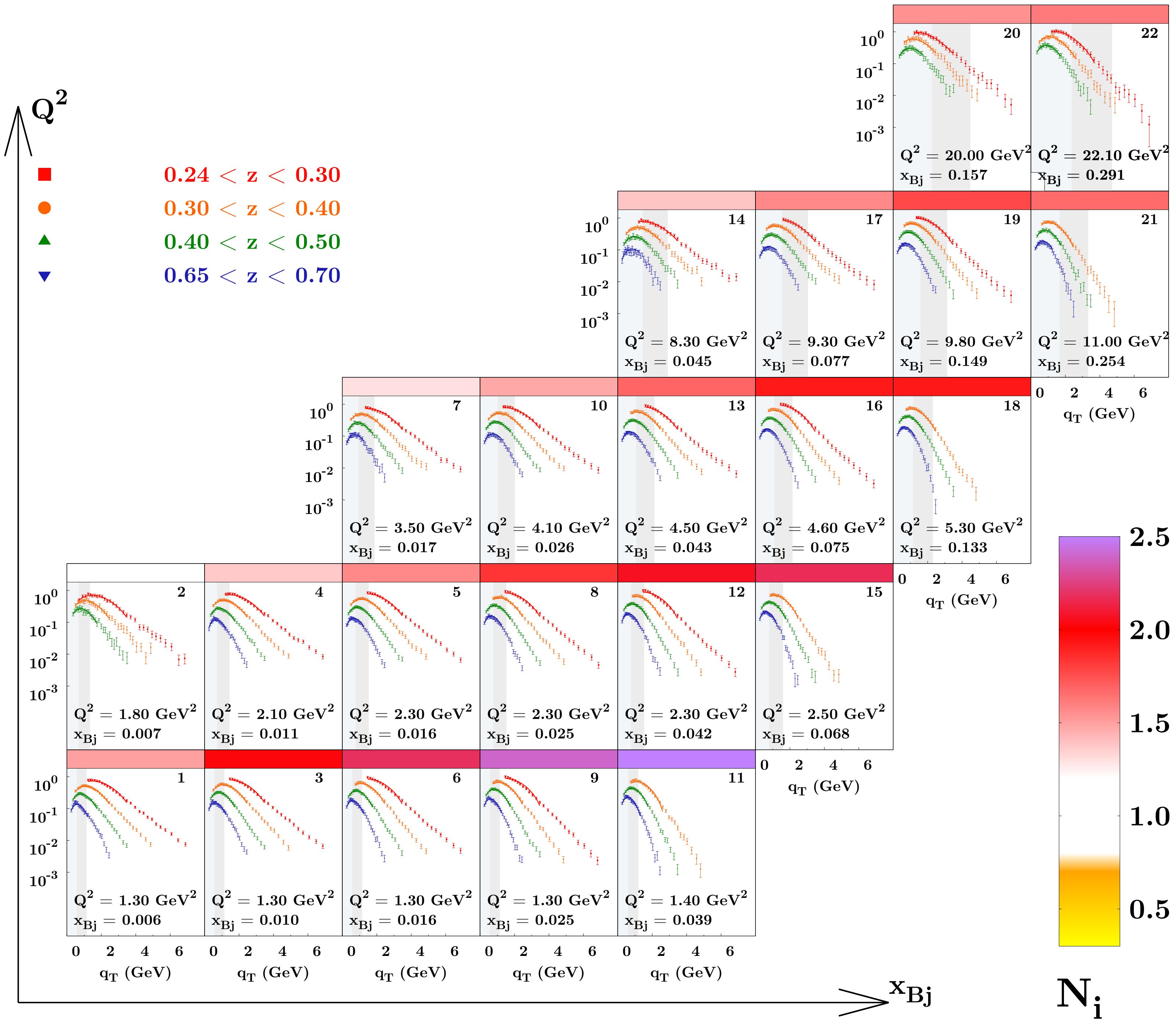
In contrast with SV19 analysis



Gonzalez-Hernandez, PoS DIS2019 (2019)

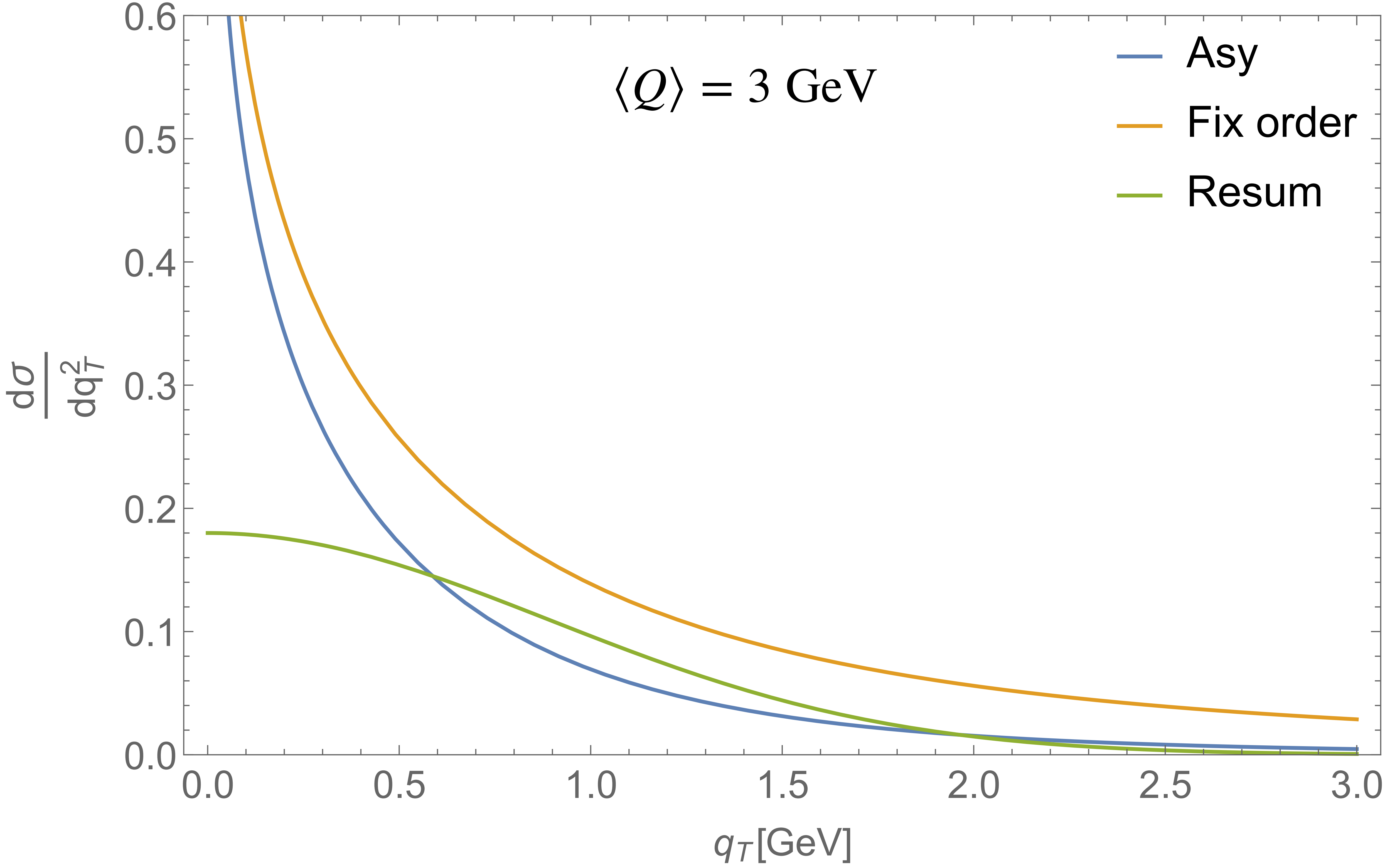
Normalization of SIDIS calculation

situation at low energy scale

$$\langle Q \rangle = 3\ \mathrm{GeV}$$

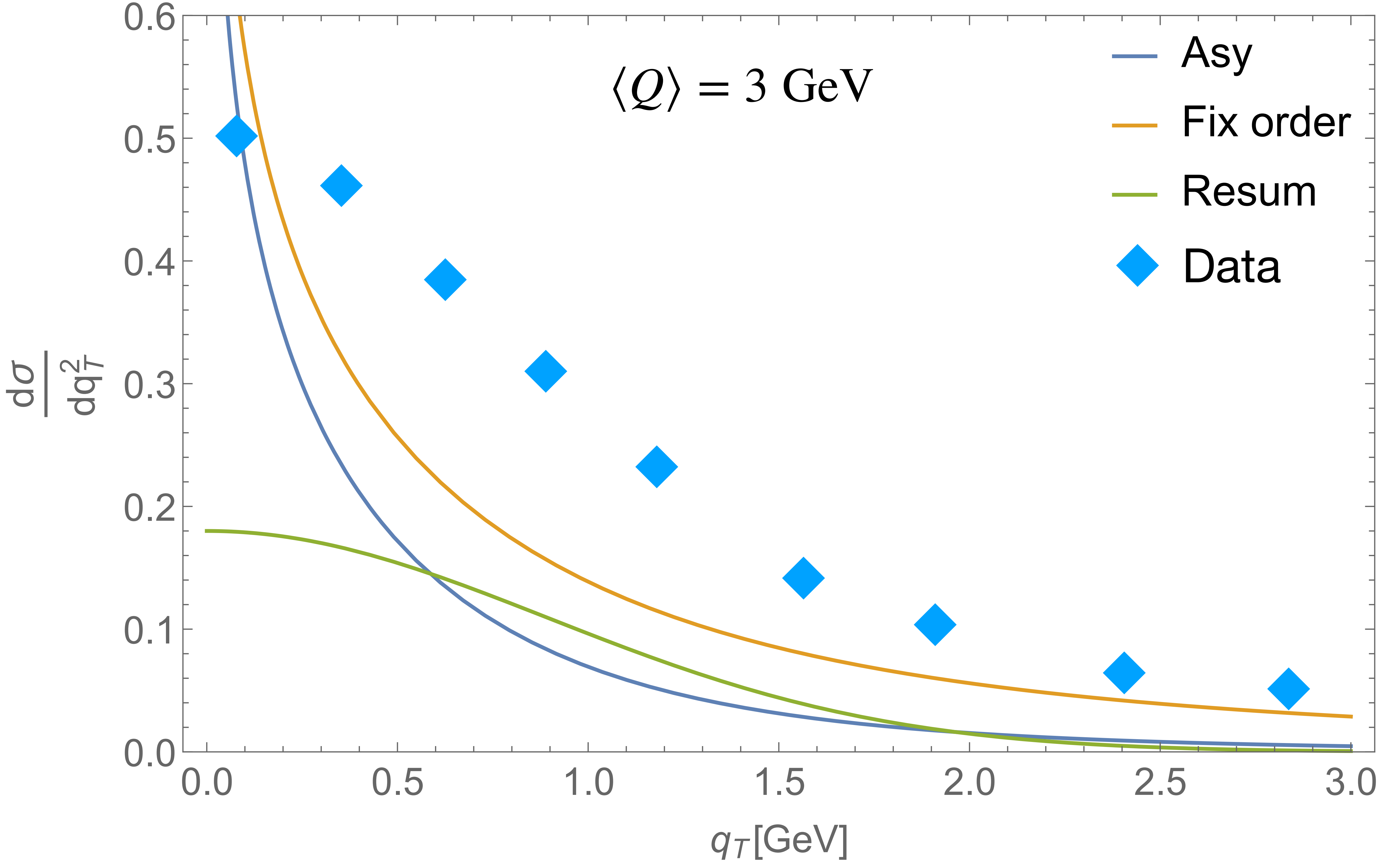
Normalization of SIDIS calculation

situation at low energy scale



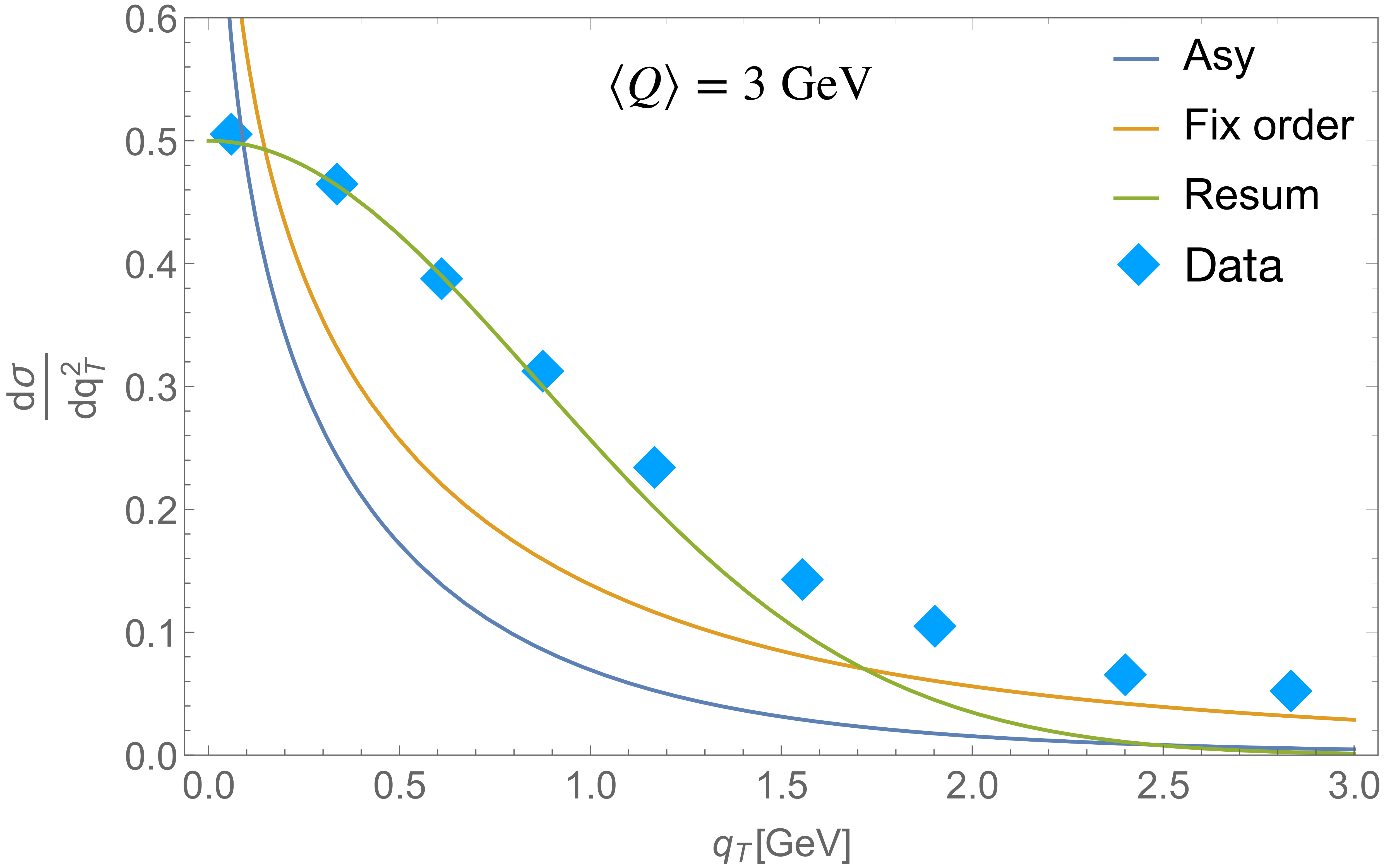
Normalization of SIDIS calculation

situation at low energy scale



Normalization of SIDIS calculation

ENHANCEMENT OF TMD CONTRIBUTION



Normalization of SIDIS calculation

MAP22 work solution

Normalization of SIDIS calculation

MAP22 work solution

SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dxdQ\,dzdP_{hT}} \Bigg/ \frac{d\sigma}{dxdQ}$$

Normalization of SIDIS calculation

MAP22 work solution

SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \Big/ \frac{d\sigma}{dx dQ}$$

Collinear SIDIS cross section

$$\frac{d\sigma}{dx dQ dz}$$

Normalization of SIDIS calculation

MAP22 work solution

SIDIS multiplicity

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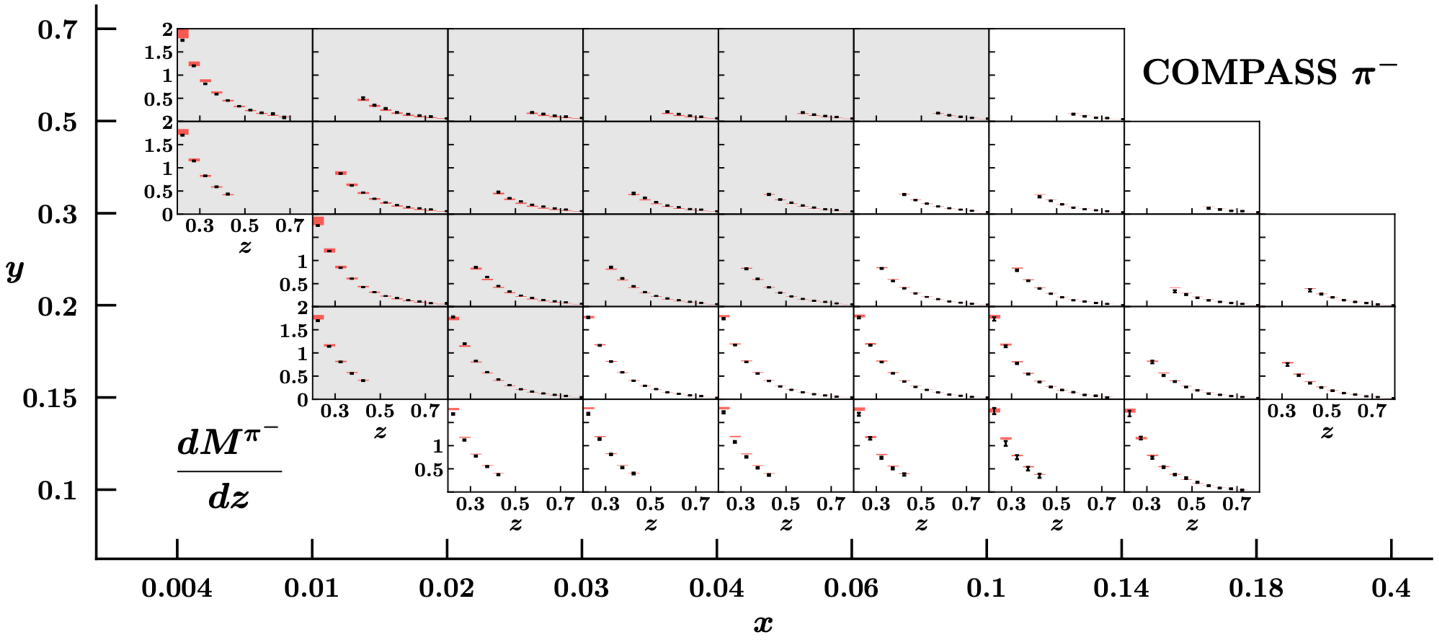
Collinear SIDIS cross section

$$\frac{d\sigma}{dx dQ {\color{red}dz}}$$

Good agreement theory/data



Khalek, Bertone, Nocera, et al., PRD 104 (2021)

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MAP22 work solution

SIDIS multiplicity
$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ \color{red}{dz dP_{hT}}} \Big/ \frac{d\sigma}{dx dQ}$$

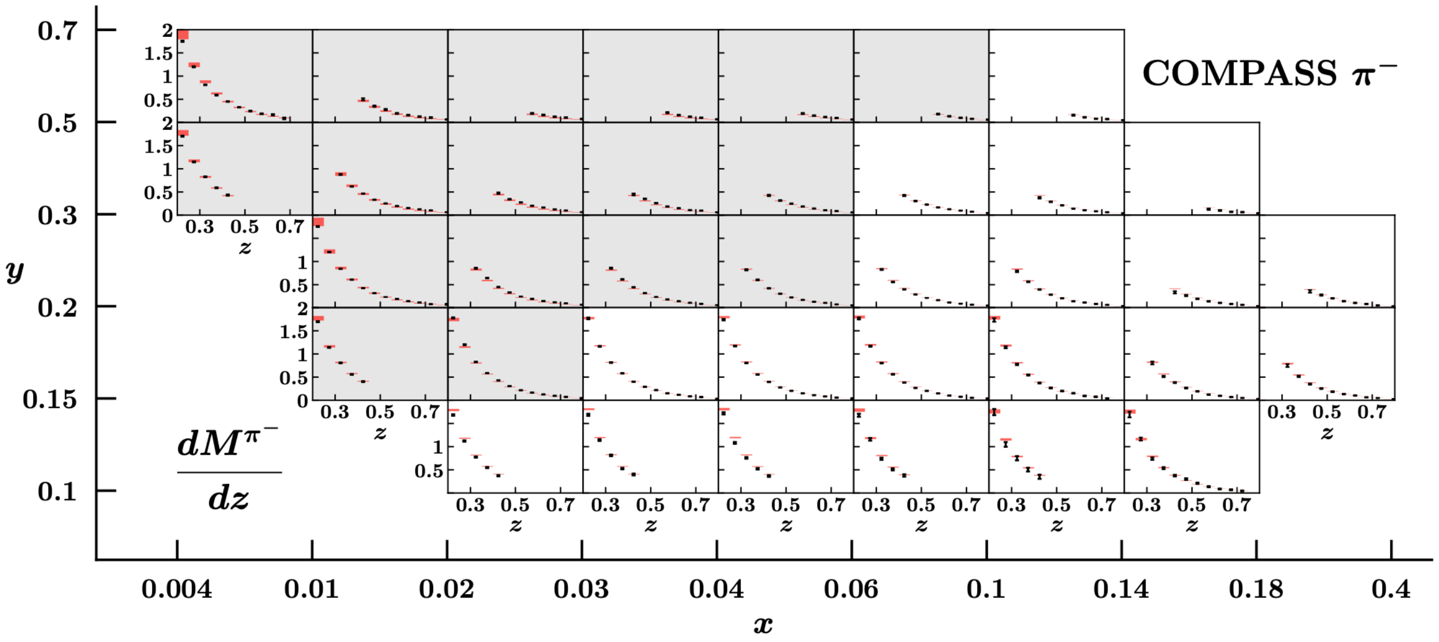
Collinear SIDIS cross section
$$\frac{d\sigma}{dx dQ \color{red}{dz}}$$

Normalization of prediction such that

$$\int d\color{red}{P_{hT}}\color{black} W(x, z, Q, \color{red}{P_{hT}}\color{black}) = \frac{d\sigma}{dx dQ dz}$$

Piacenza, PhD thesis (2020)

Good agreement theory/data



Khalek, Bertone, Nocera, et al., PRD 104 (2021)

Normalization of SIDIS calculation

MAP22 work solution

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Piacenza, PhD thesis (2020)

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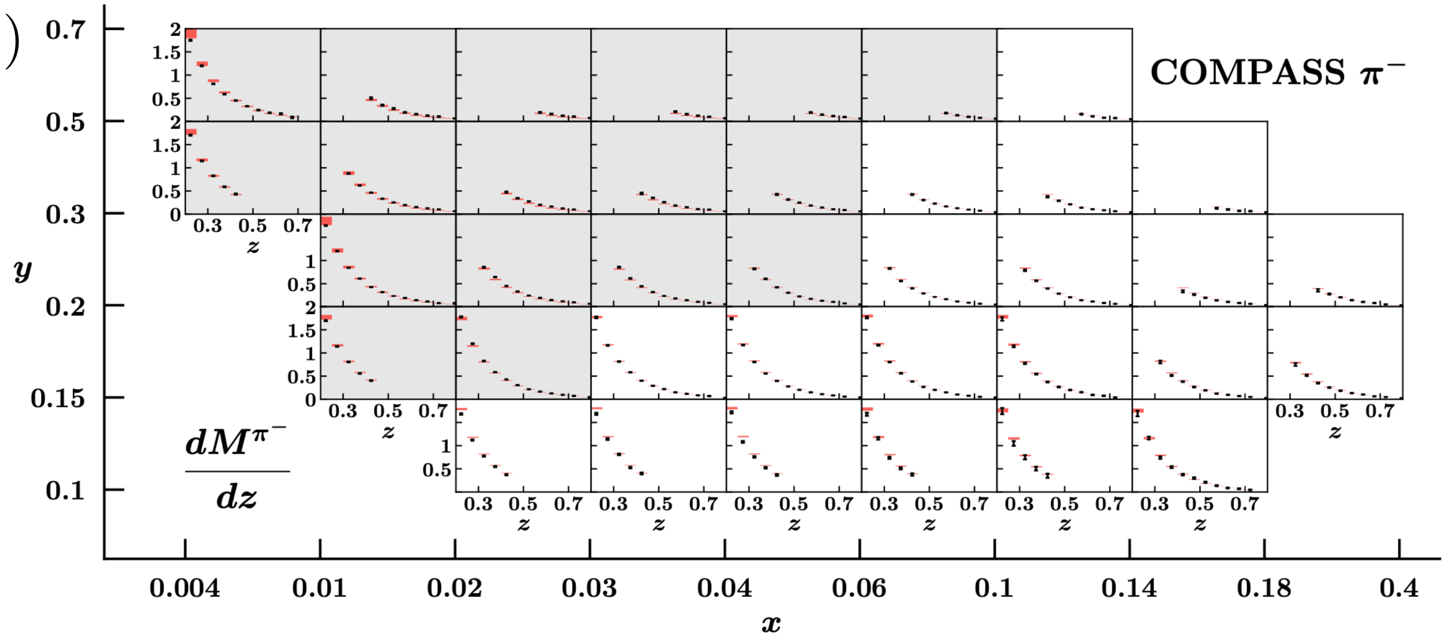

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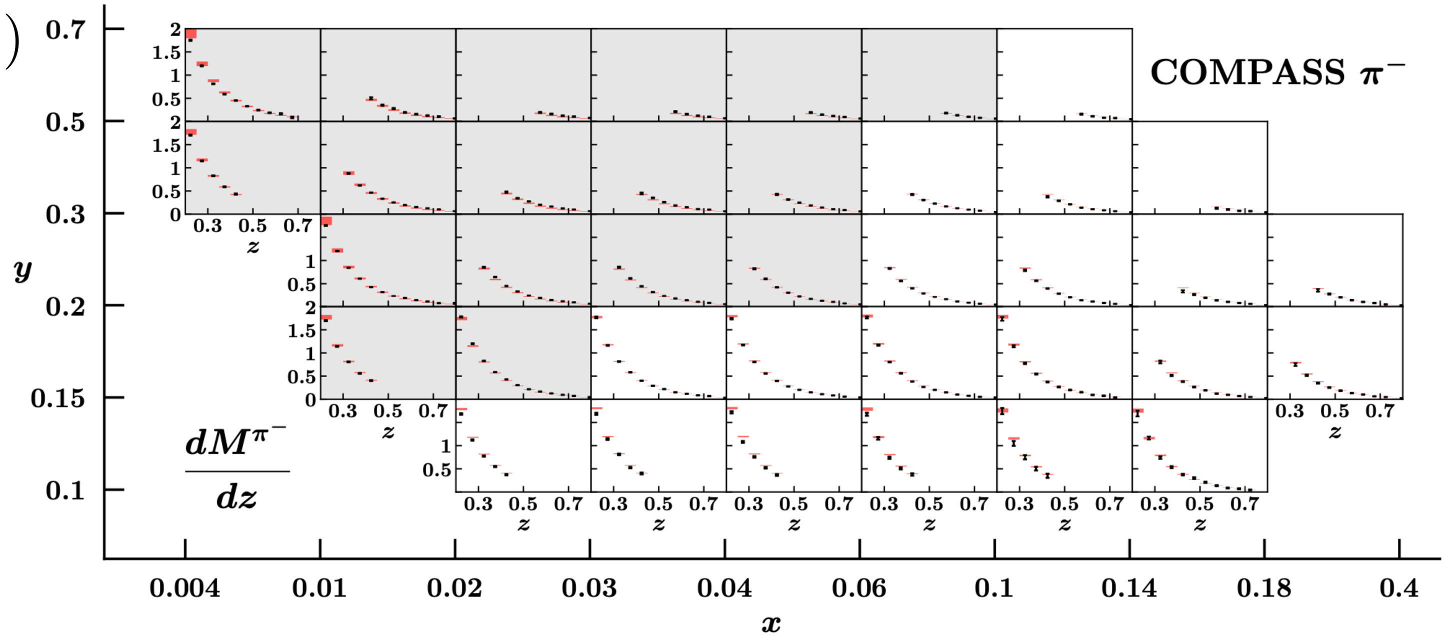

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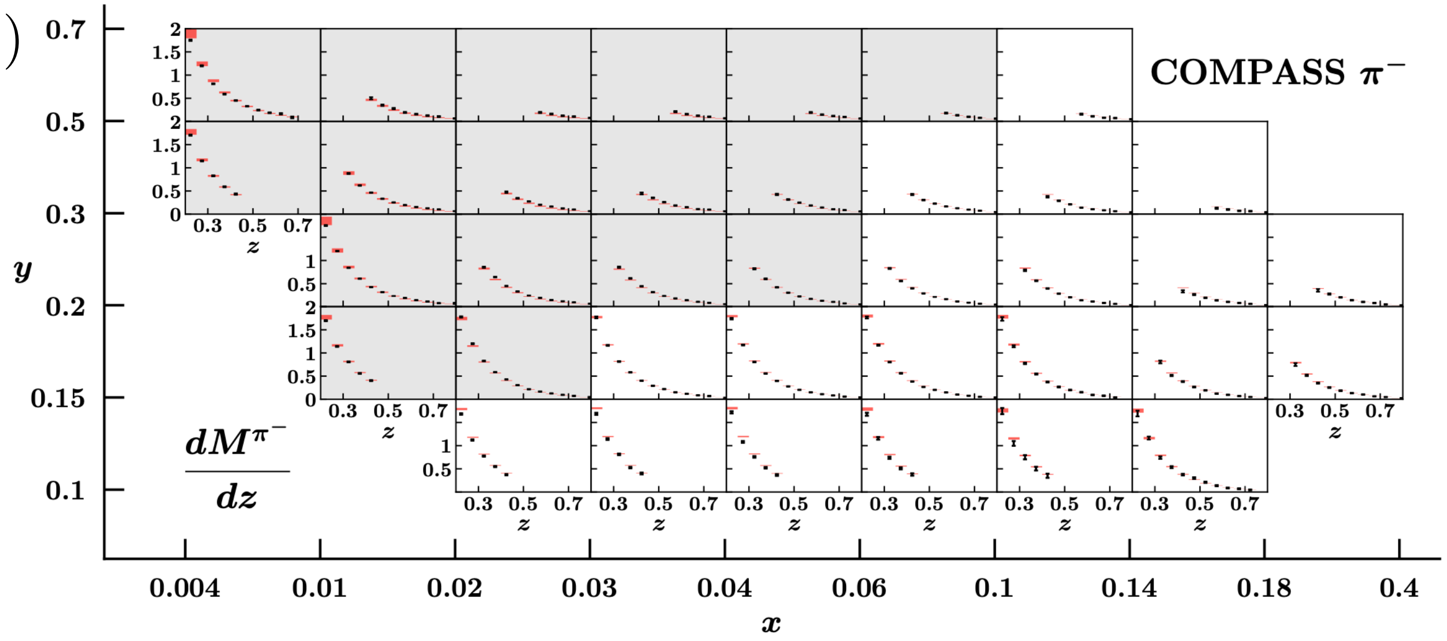
Piacenza, PhD thesis (2020)

$$M(x,z,P_{hT},Q) = \boxed{n(x,z,Q)} W(x,z,Q,P_{hT}) \Big/ \frac{d\sigma}{dxdQ}$$

Calculable before the fit

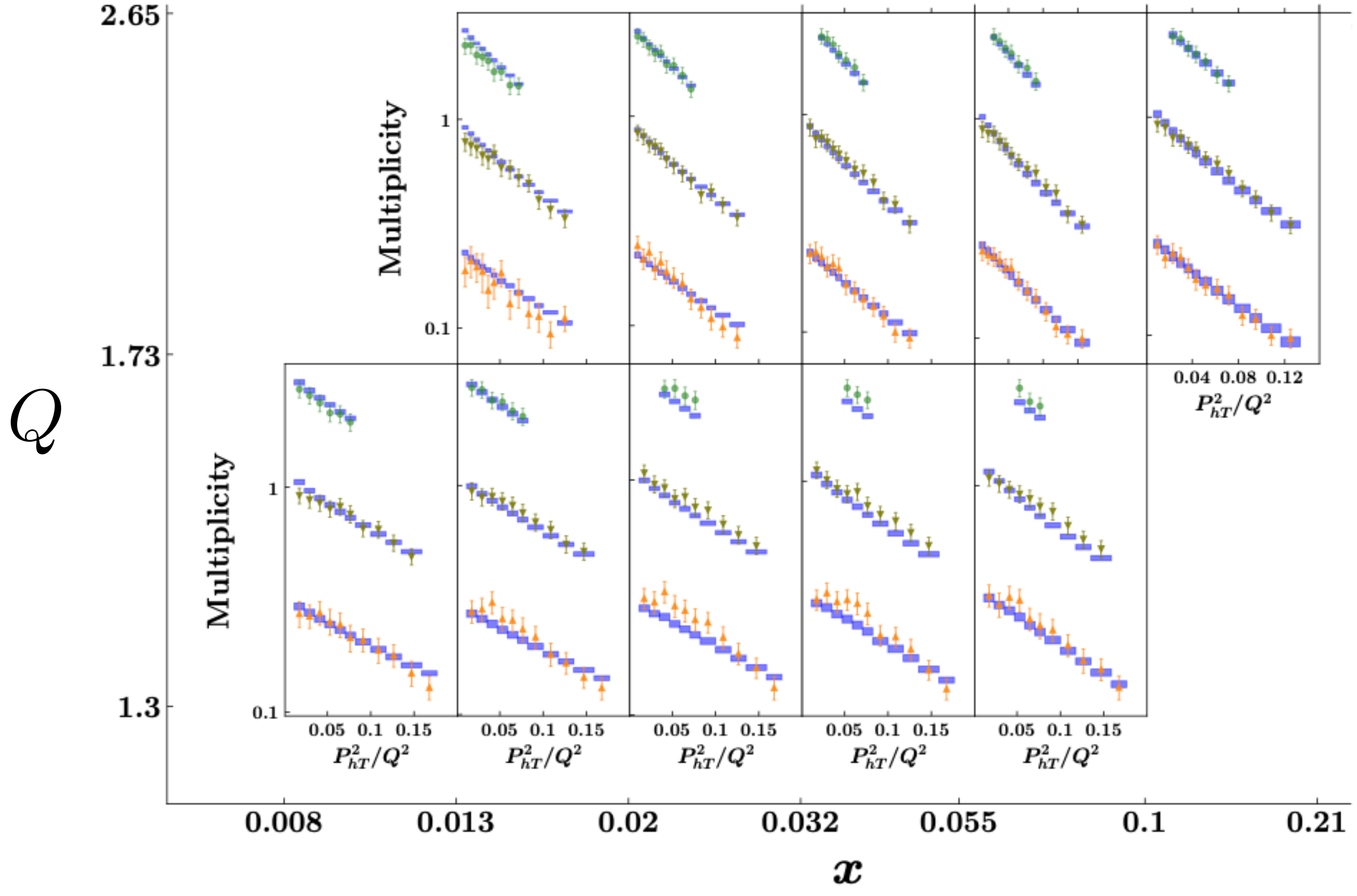
Good agreement theory/data



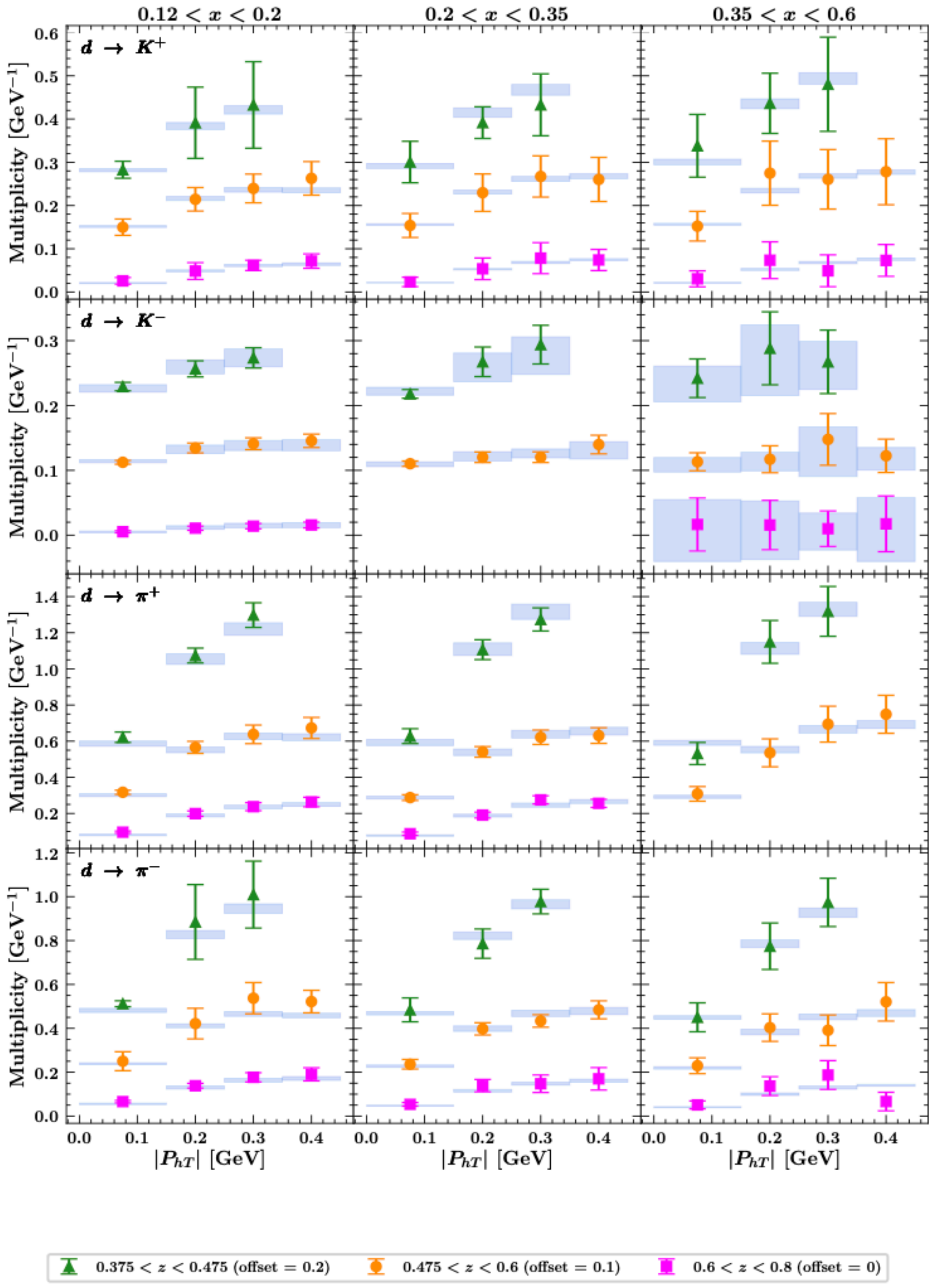
Khalek, Bertone, Nocera, et al., PRD 104 (2021)

MAP22: Results for SIDIS data

COMPASS



HERMES

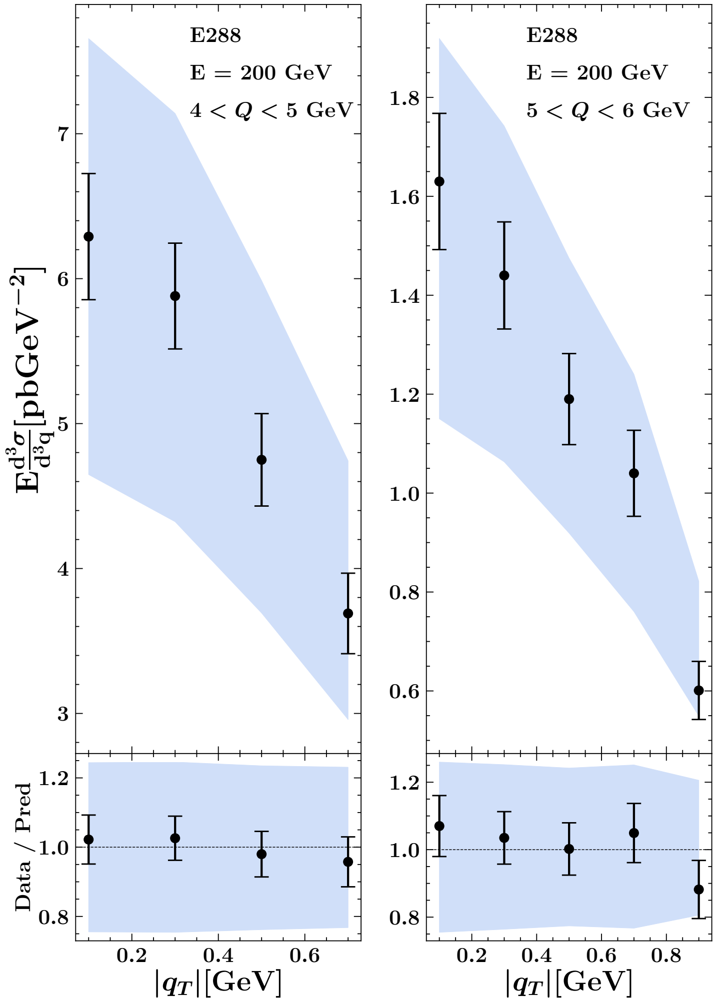


Good agreement for almost all bins

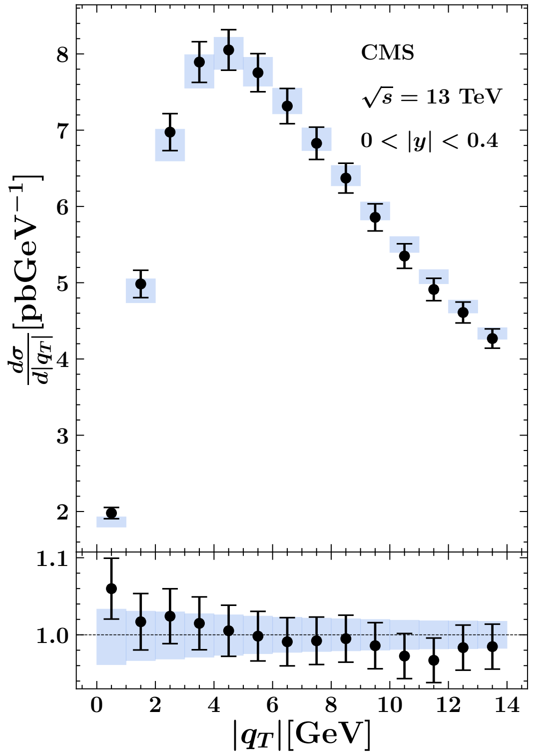
$\chi^2/N_{data} = 0.87$ (SIDIS total)

MAP Coll., Bacchetta, Bertone, et al., JHEP 10 (2022)

MAP22: Results for DY data

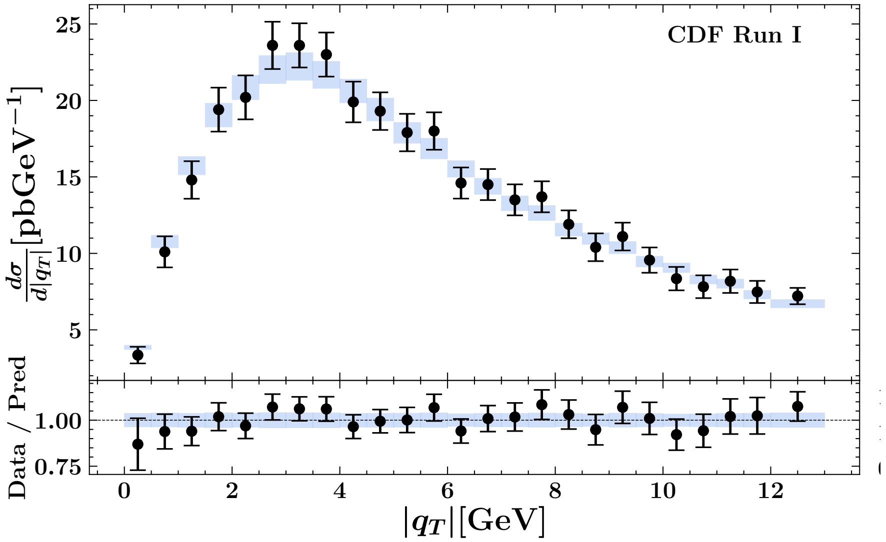


χ^2/N_{data} = 1.24
(DY fixed-target)

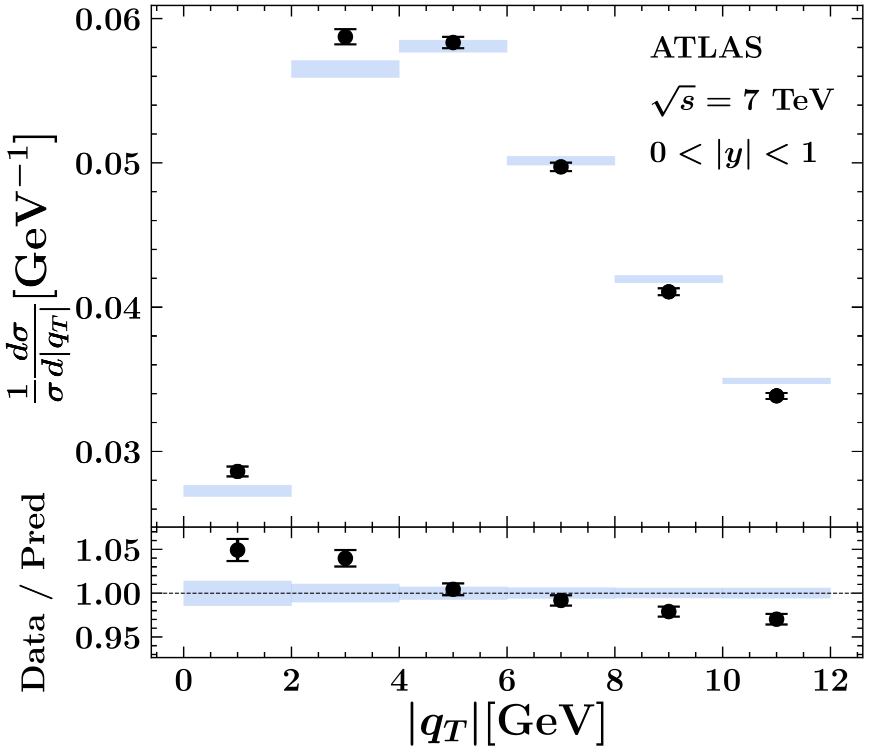


χ^2/N_{data} = 0.55
(DY CMS)

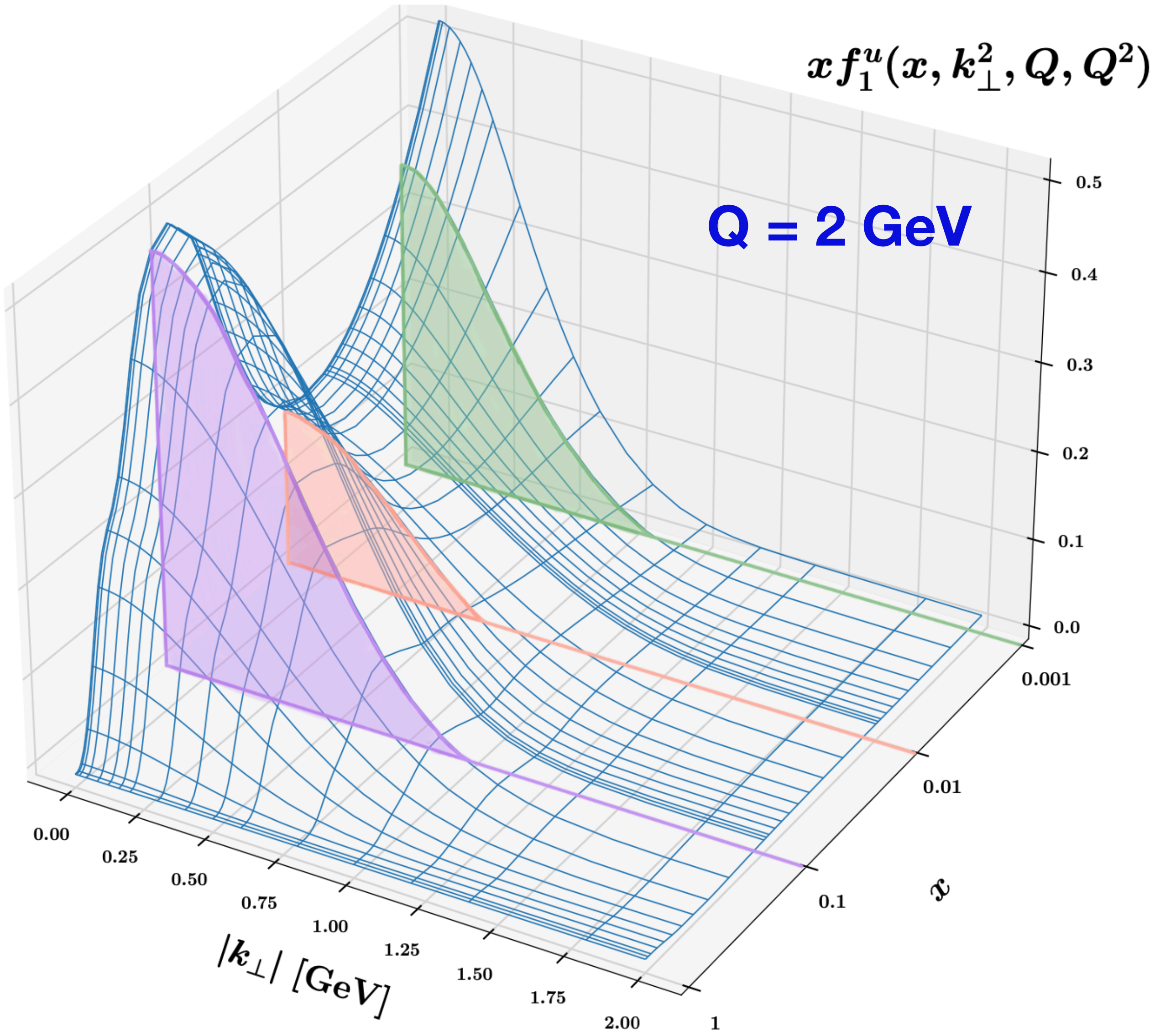
χ^2/N_{data} = 5.05
(DY ATLAS)

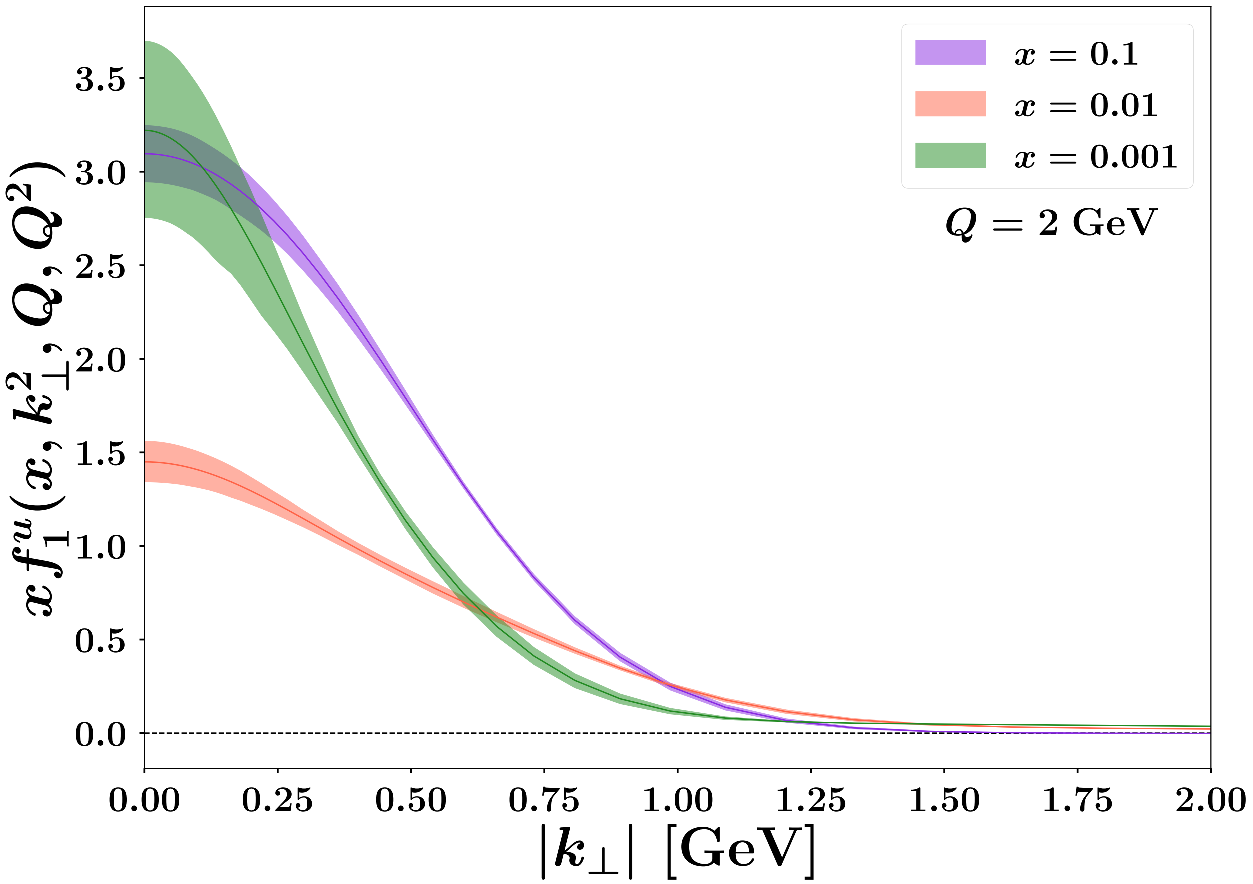


χ^2/N_{data} = 0.93
(DY Tevatron)



MAP22: Extracted TMD PDFs

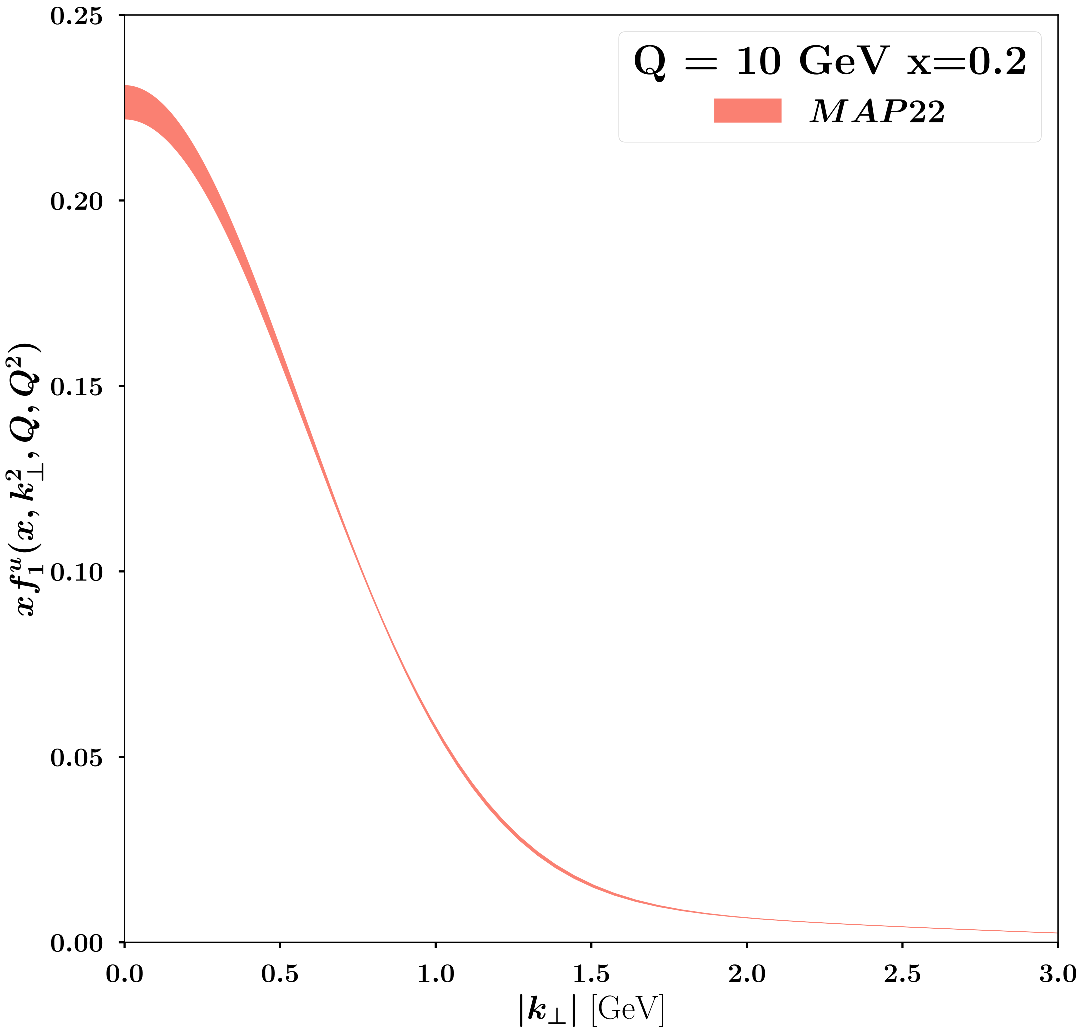


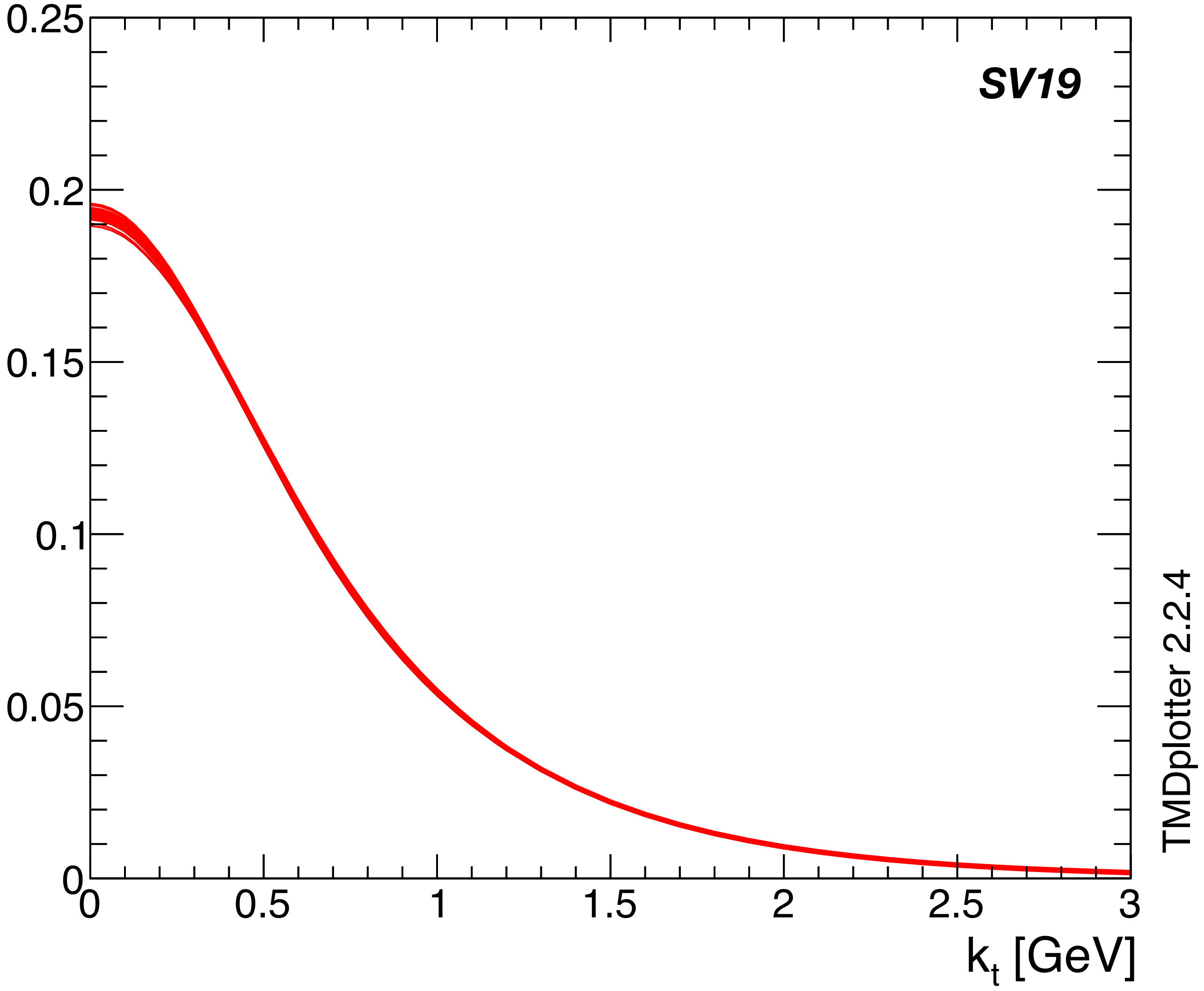


- Non-trivial dependence on the variable x
- Need more data to better constrain small-x region (EIC?)

Comparison between recent global fits

MAPTMD22 vs **SV19**





- Contrary to collinear extractions, we are still far from getting a good compatibility between two different TMD extractions

Comparison between recent global fits

Collins-Soper kernel:

Comparison between recent global fits

Collins-Soper kernel: kernel of the rapidity evolution equation

$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = K(b_T, \mu)$$

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to be fitted

perturbatively calculable

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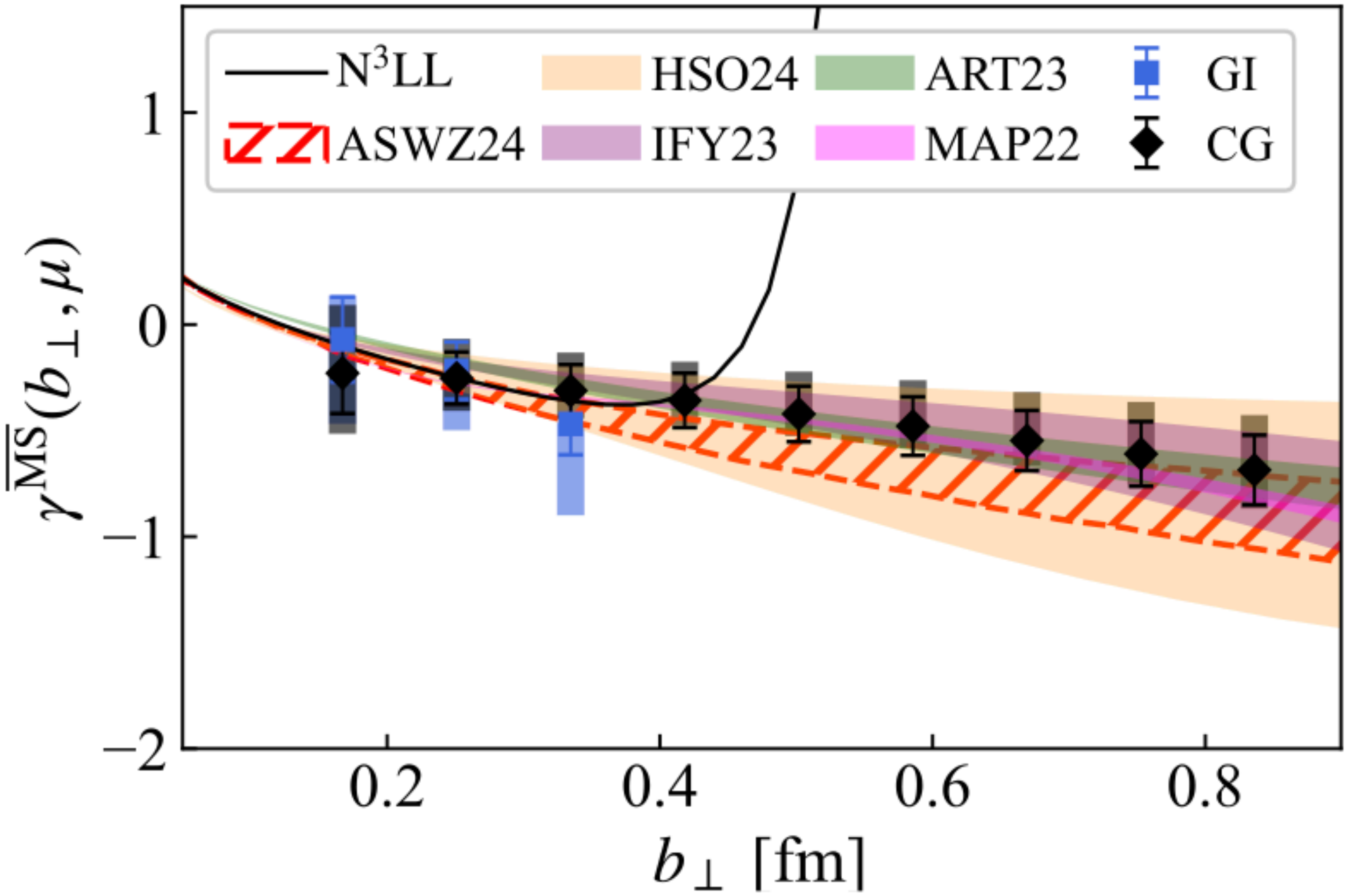

Bollweg, Gao, Mukherjee, et al., PLB 852 (2024)

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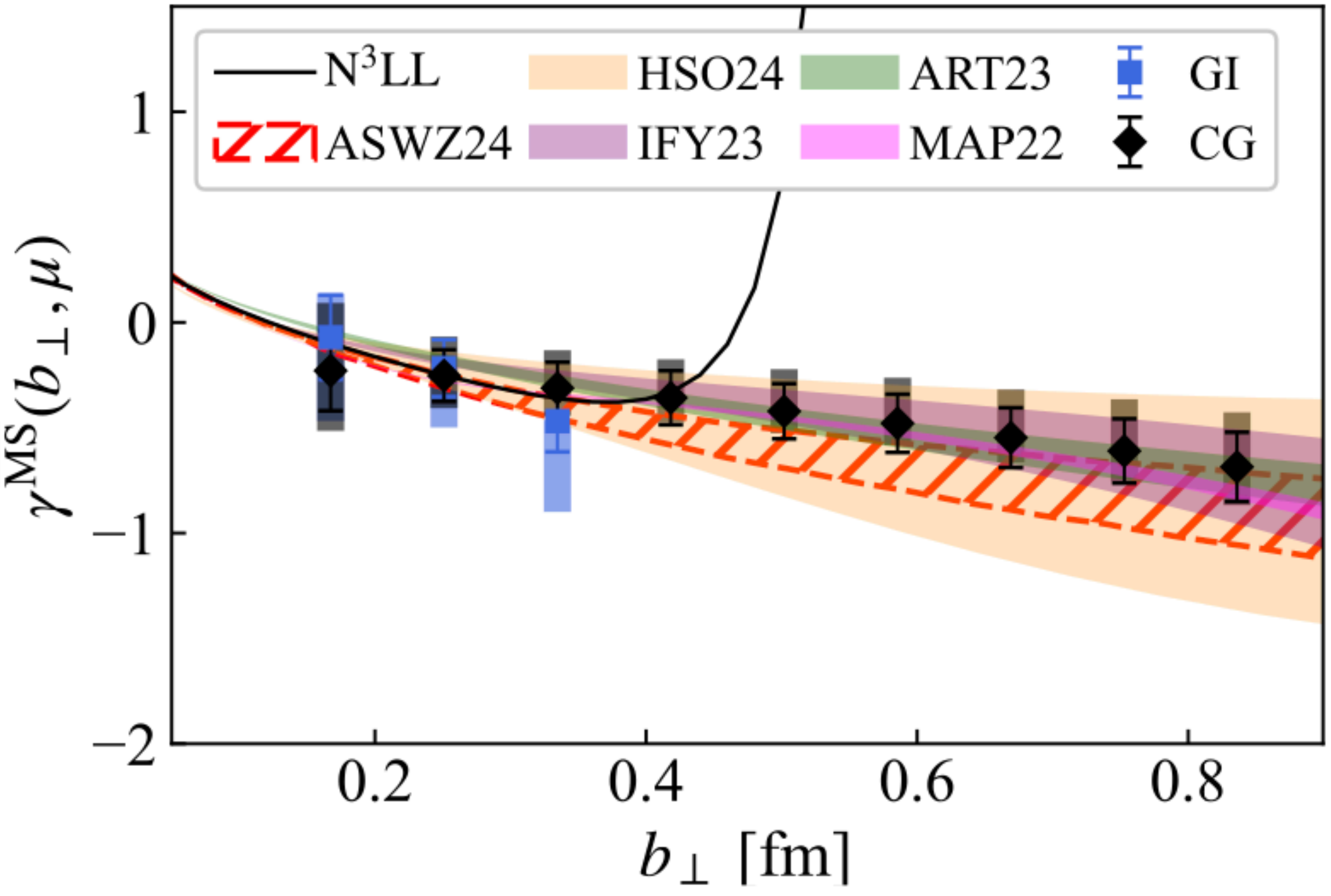

Bollweg, Gao, Mukherjee, et al., PLB 852 (2024)

- Compatibility between the most recent extractions

What happened in the last 2 years?

The New-York Times.

VOL. X—NO. 2990.

NEW-YORK, SUNDAY, APRIL 21, 1861

PRICE TWO CENTS.

	Accuracy	SIDIS	DY	N of points	χ^2/N_{data}	Flavor Dependence
MAPTMD22 Bacchetta, Bertone, et al., JHEP 10 (2022)	*N^3LL^-*	✓	✓	***2031***	***1.06***	✗

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ART23 Moss, Scimemi, JHEP 05 (2024)	N^{4}LL$^-$	✗	✓	627	0.97	✓

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HSO approach Aslan, Boglione, preprint: 2401.14266	NLO	✗	Low Q only	130, 52	1.04, 1.68	✗

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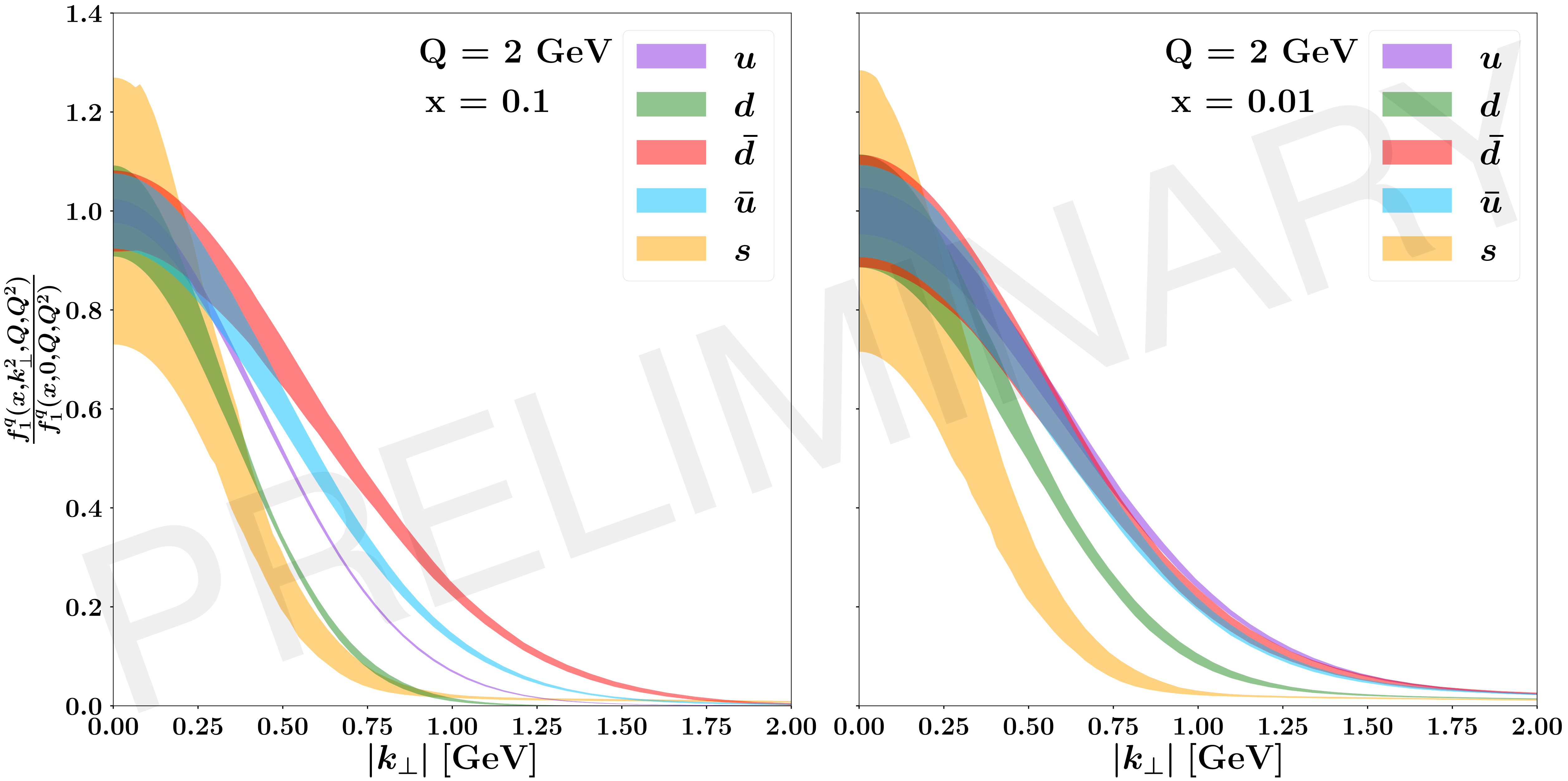
MAPTMD24?	N^{3}LL	✓	✓	2031	?	✓

MAPTMD24?

Study of flavor-dependent behavior of TMDs through a **global** fit



Conclusions and Outlook

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- SIDIS theoretical calculations are affected by a normalization issue (MAPTMD22: first attempt of work solution)

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- SIDIS theoretical calculations are affected by a normalization issue (MAPTMD22: first attempt of work solution)

- Nowadays, studies on the extraction of flavor-dependent TMDs are in progress (up to now, only on Drell-Yan data)

Backup

Structure of a TMD: Evolution

Resummation of large logs

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Resummation of large logs

$$S_{\text{pert}}(\mu_b, \mu) = 1 + \sum_{k=0}^{\infty} \boxed{R_{\text{N}^k\text{LL}}}$$

$$S_{\text{pert}}(\mu_b, \mu) = 1 + \sum_{k=0}^{\infty} \sum_{n=1+[k/2]}^{\infty} \left(\frac{\alpha_S(\mu)}{4\pi}\right)^n \sum_{k=1}^{2n} L^{2n-k} R^{(n,2n-k)}$$

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Accuracy	H and C	K and γ_F	γ_K	PDF and α_S evol.
LL	0	-	1	-
NLL	0	1	2	LO
NLL’	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL’	2	2	3	NNLO
N^{3}LL	2	3	4	NNLO
N^{3}LL’	3	3	4	N^{3}LO

Bacchetta, Bertone, Bissolotti, et al., JHEP 07 (2020)

TMD handbook, Boussarie, et al., 2023

Structure of a TMD: NP content

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

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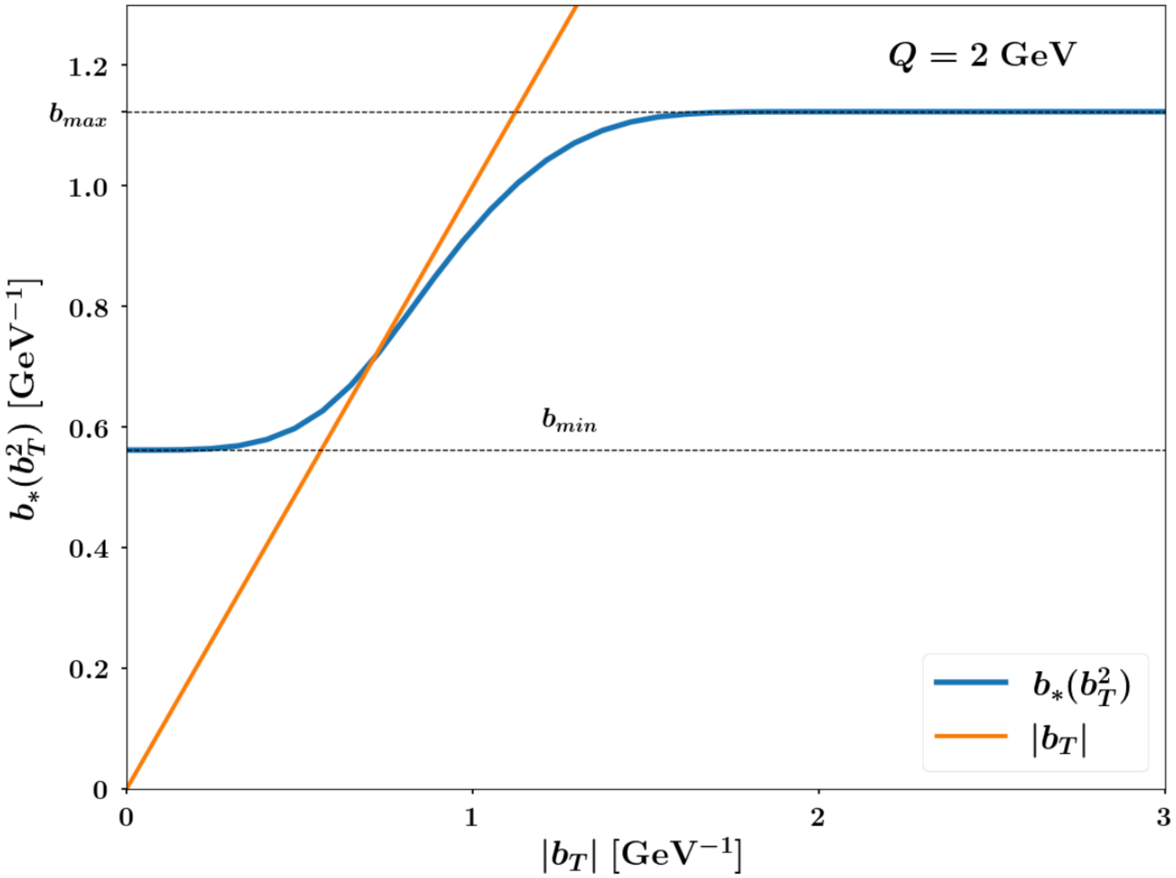
b$_*$-prescription



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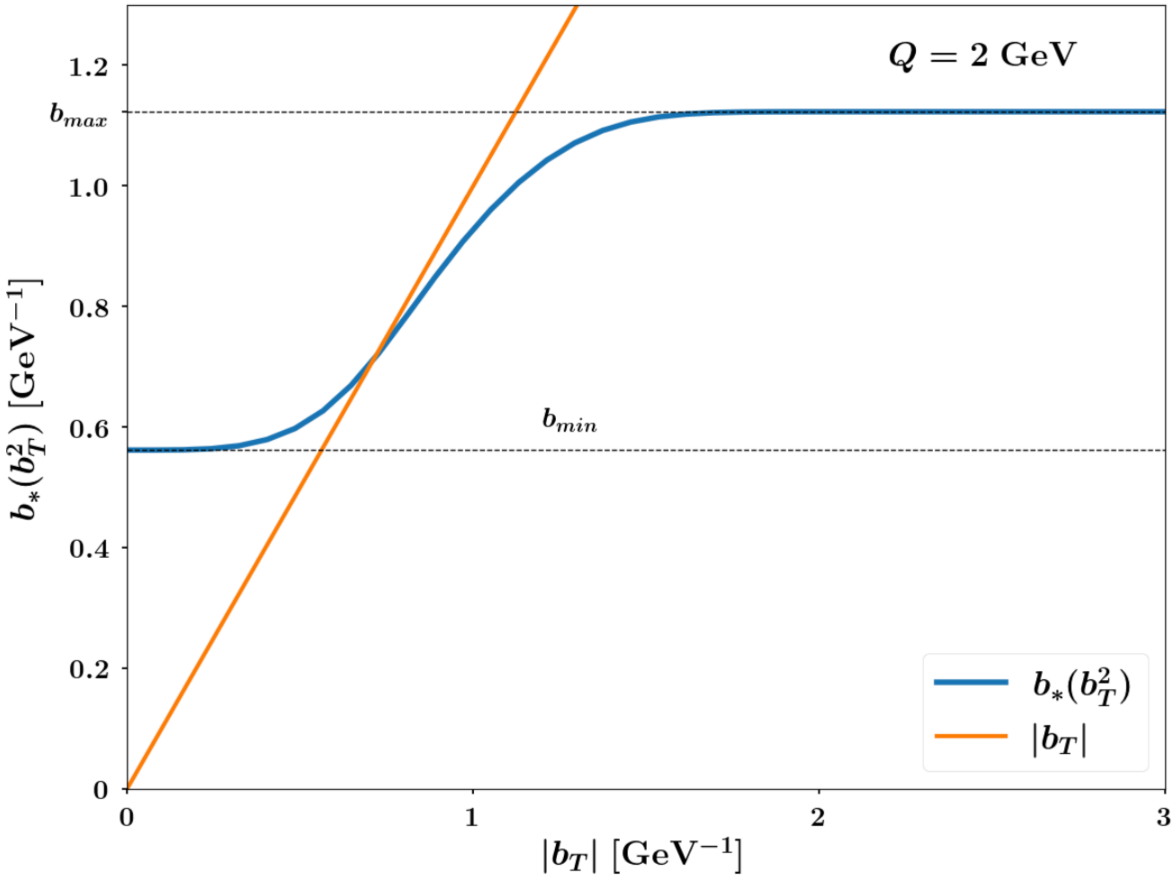
Collins, Soper, Sterman, Nucl. Phys. B250 (1985)

Collins, Gamberg, et al., PRD (2016)

Bacchetta, Echevarria, Mulders, et al., JHEP 11 (2015)



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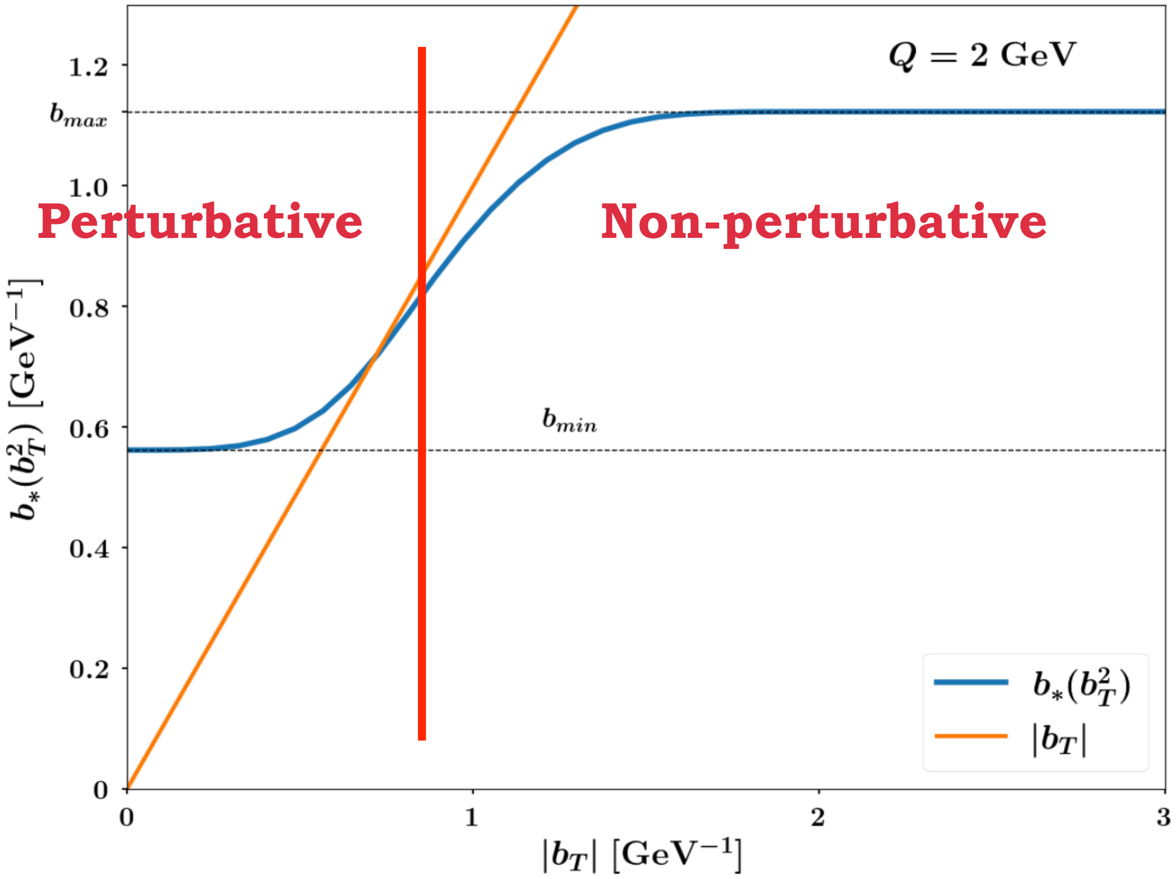

$$\hat{f}_1(x, b_T^2; \mu, \zeta) = \left[\frac{\hat{f}_1(x, b_T^2; \mu, \zeta)}{\hat{f}_1(x, b_*(b_T^2); \mu, \zeta)} \right] \hat{f}_1(x, b_*(b_T^2); \mu, \zeta) \equiv f_{\mathrm{NP}}(x, b_T^2; \zeta) \hat{f}_1(x, b_*(b_T^2); \mu, \zeta)$$

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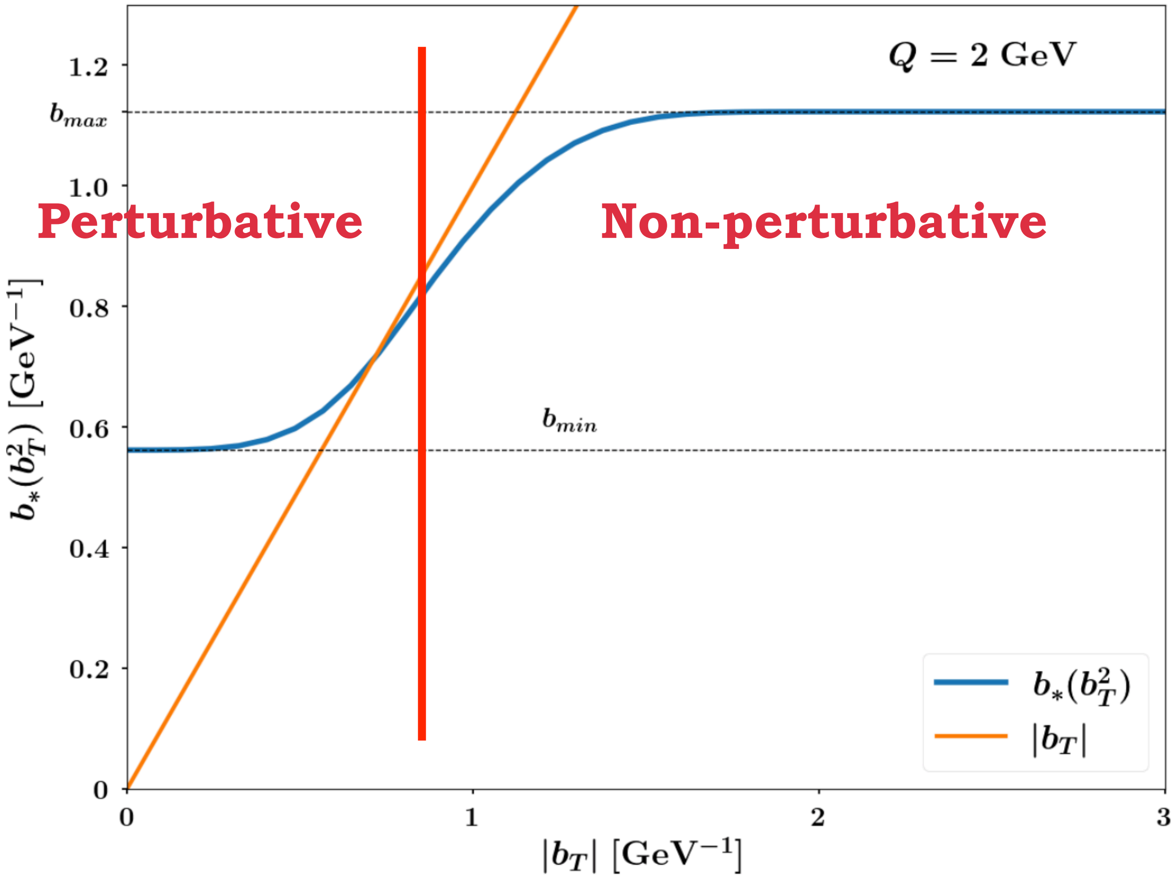

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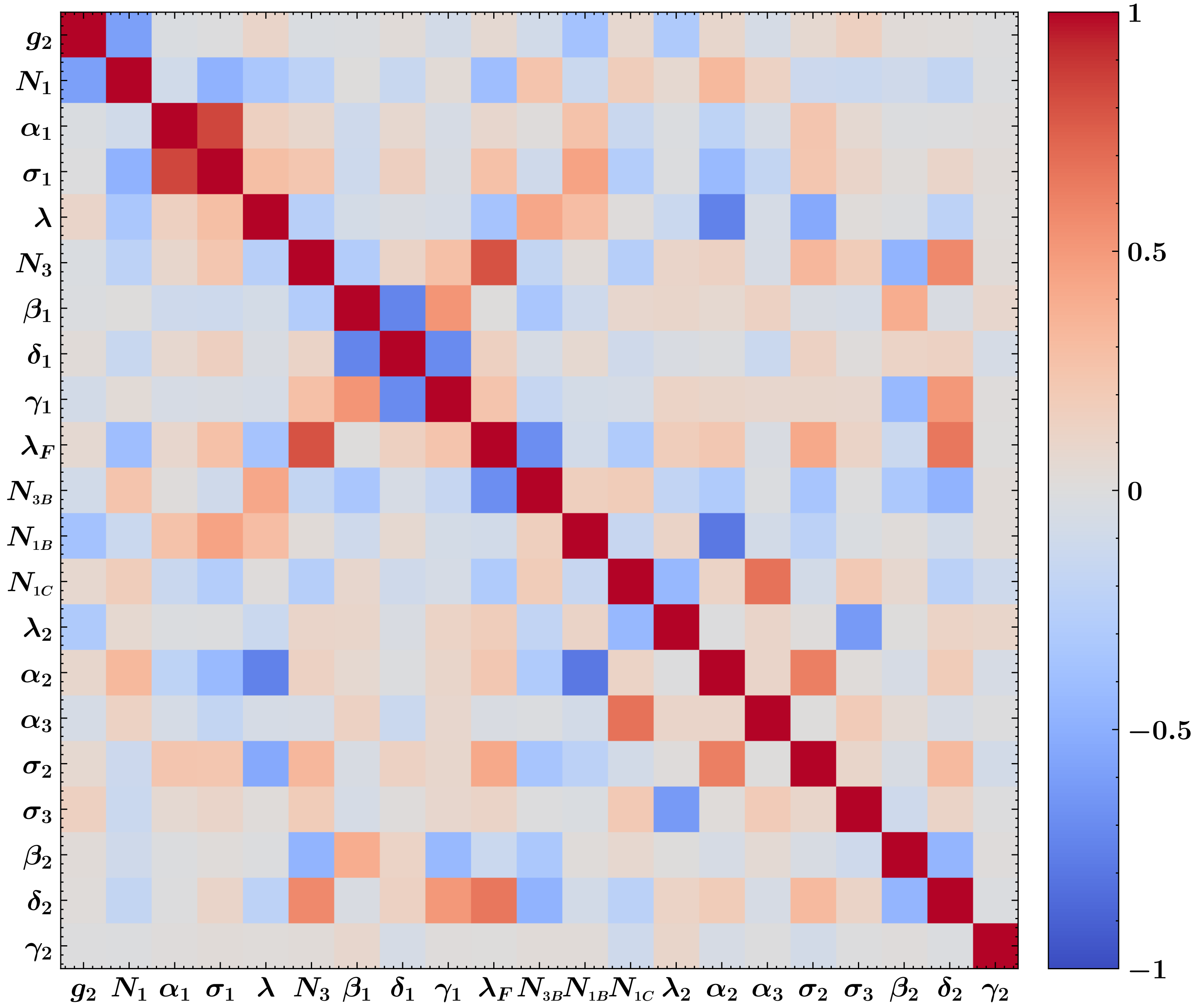
MAPTMD22 — Error analysis

Error propagation

↓

250 Montecarlo replicas



MAPTMD22 — Error analysis

Error propagation

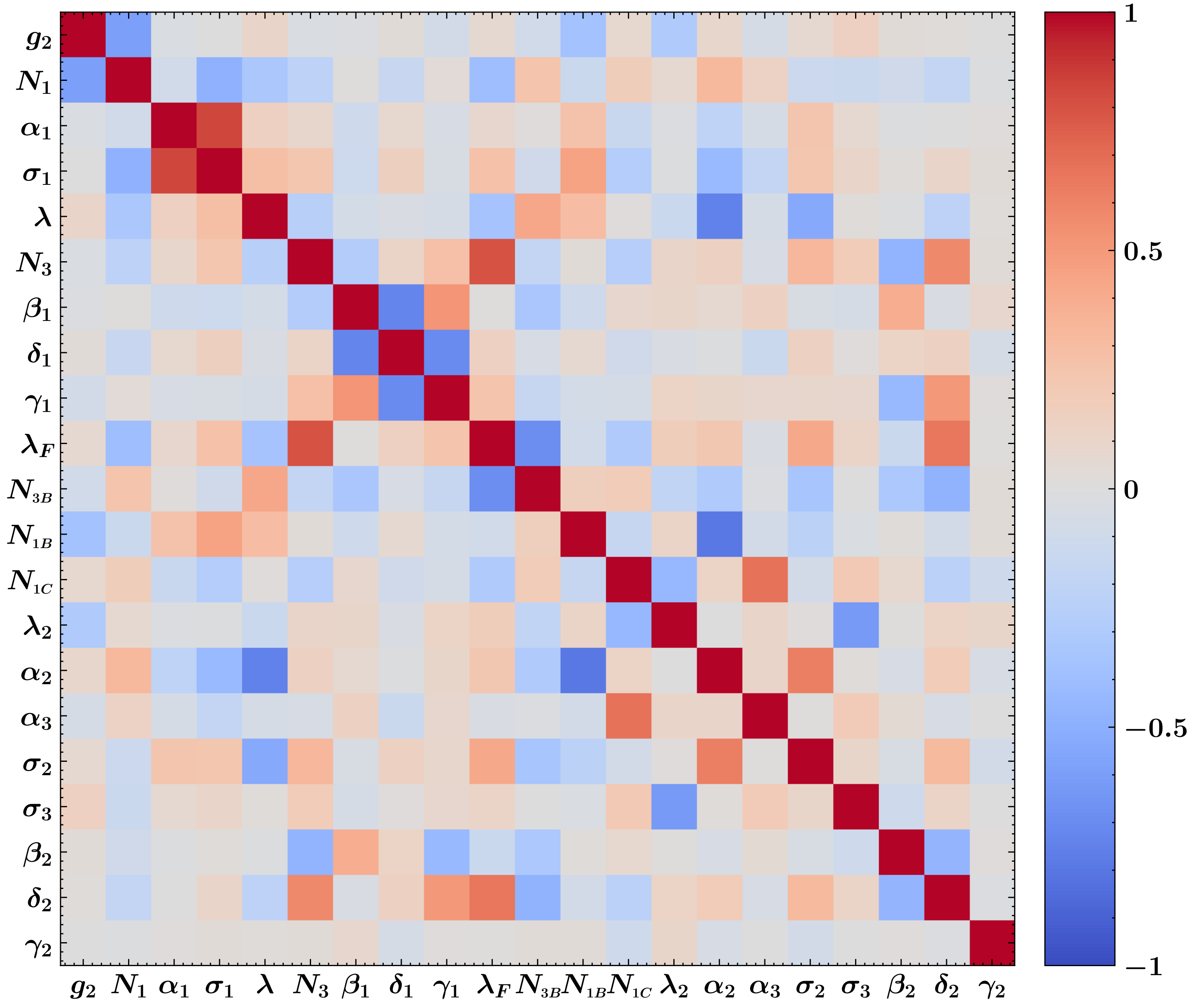
↓

250 Montecarlo replicas

Correlation matrix

↓

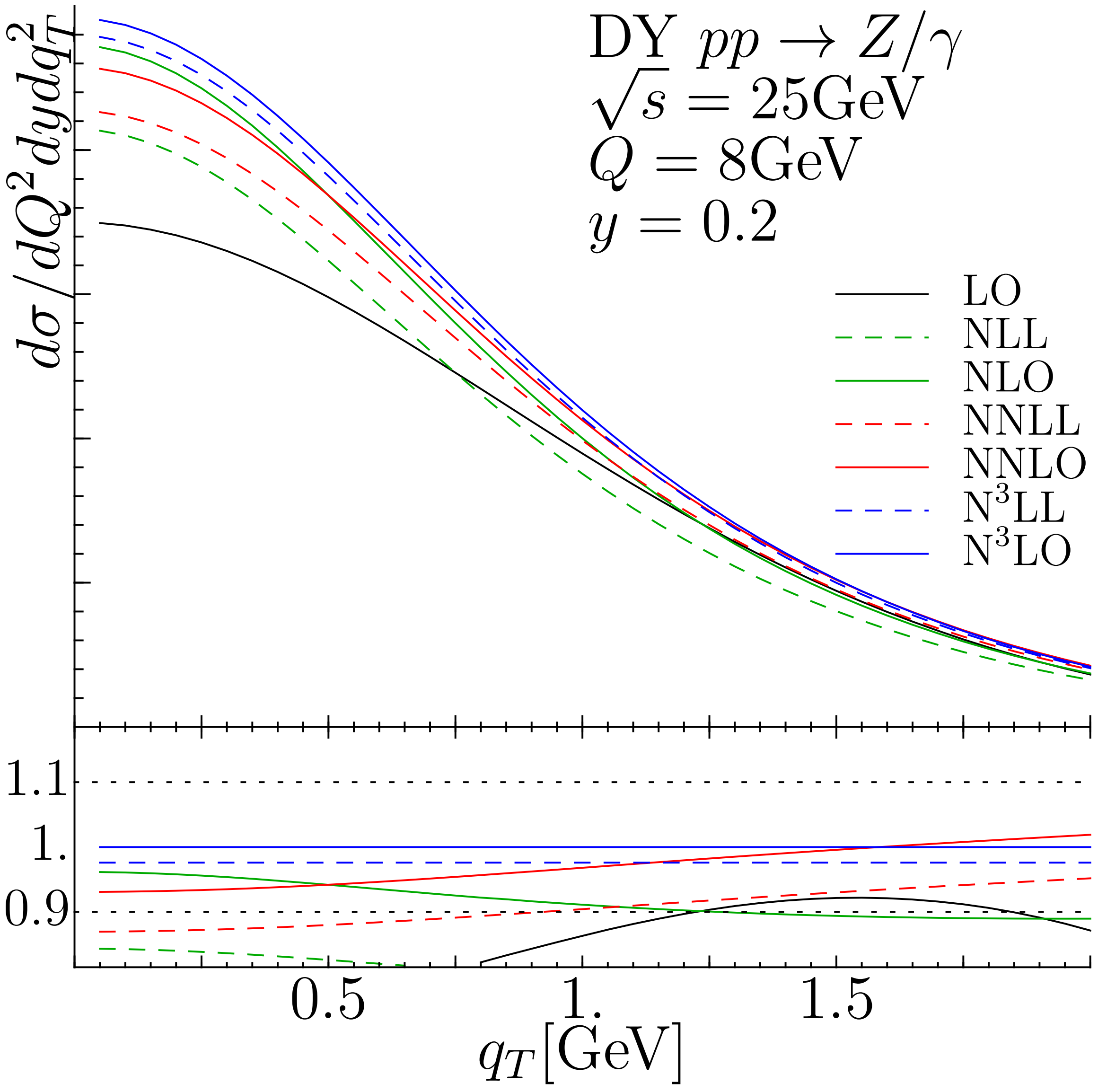
Hints of the appropriateness of the chosen functional form



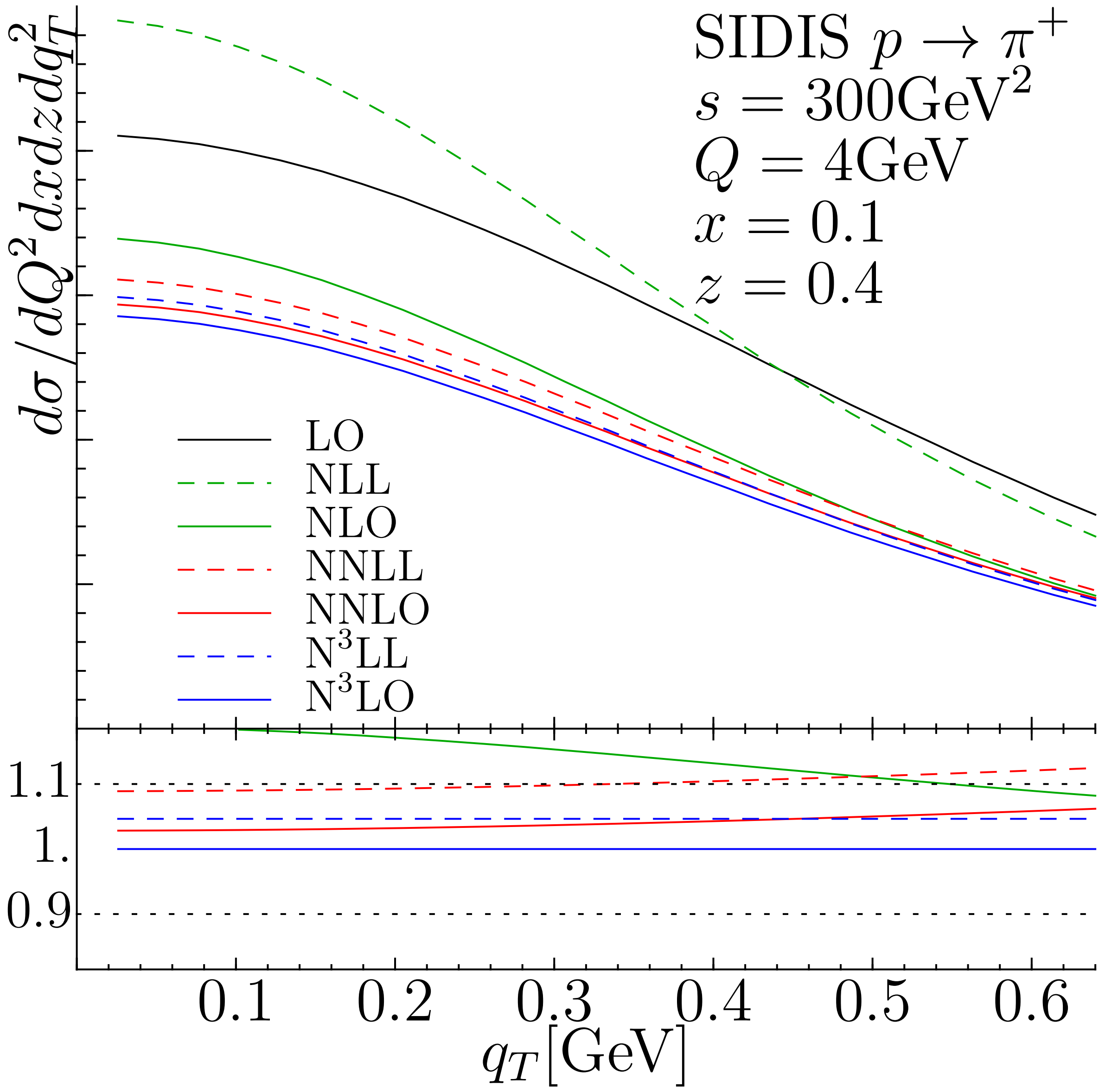
Comparison with SV19

Scimemi, Vladimirov, arXiv:1912.06532

Drell-Yan



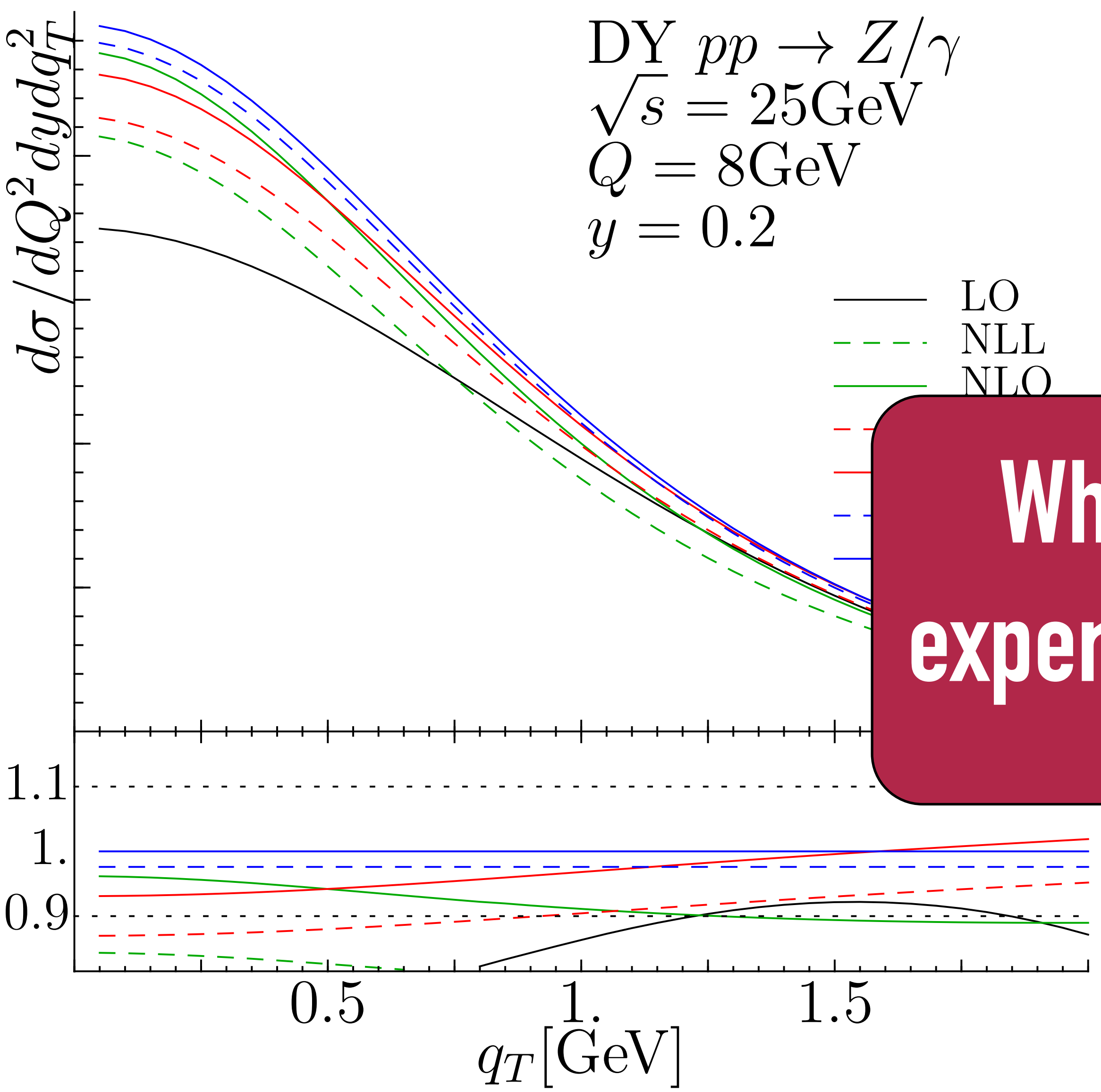
SIDIS



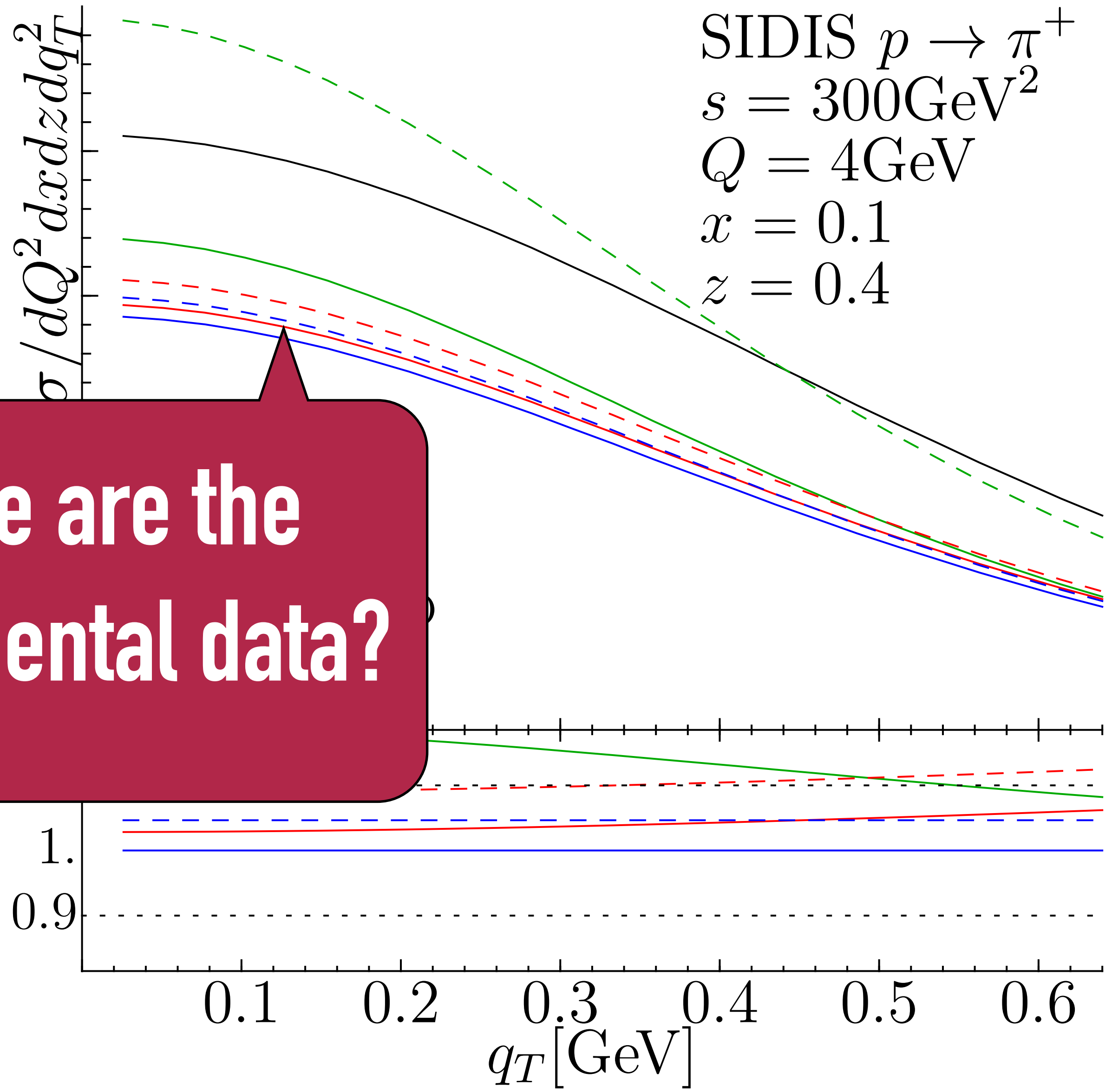
Comparison with SV19

Scimemi, Vladimirov, arXiv:1912.06532

Drell-Yan



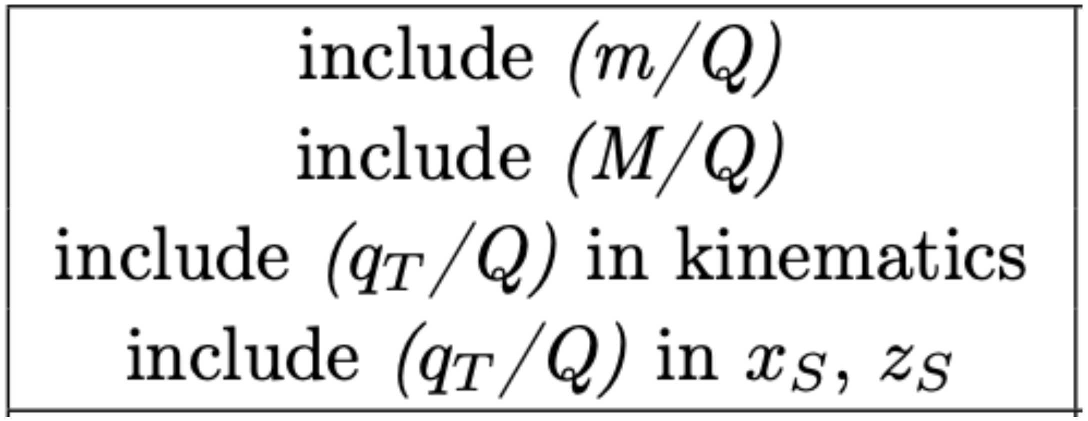
SIDIS



Where are the experimental data?

Estimate of the impact of power corrections

Results obtained within the arTeMiDe framework

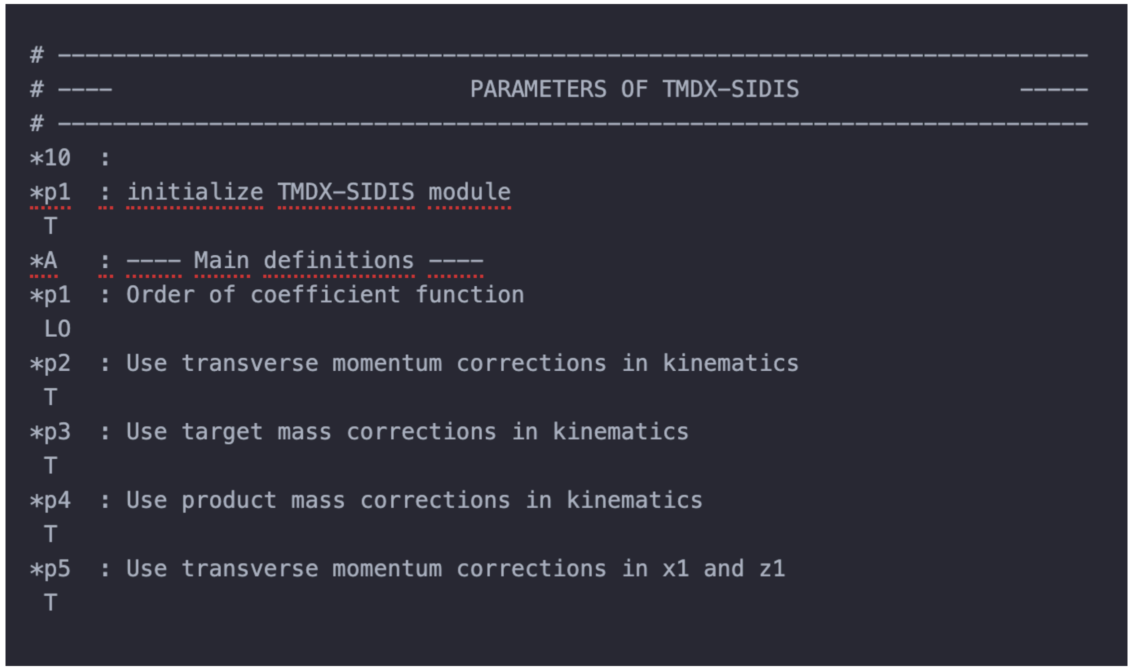


include (m/Q)
include (M/Q)
include (q_T/Q) in kinematics
include (q_T/Q) in x_S, z_S

Scimemi, Vladimirov, arXiv:1912.06532

Estimate of the impact of power corrections

Results obtained within the arTeMiDe framework

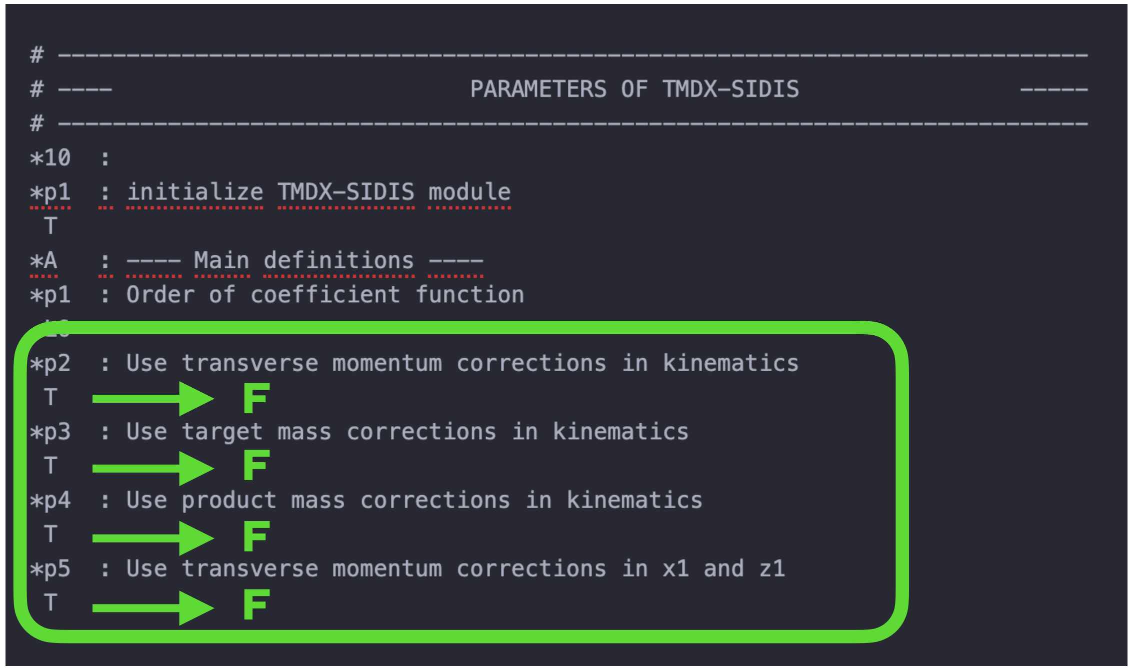


```
# ----------------------------------------------------------------------------
# ----                         PARAMETERS OF TMDX-SIDIS                  -----
# ----------------------------------------------------------------------------
*10  :
*p1  : initialize TMDX-SIDIS module
 T
*A   : ---- Main definitions ----
*p1  : Order of coefficient function
 LO
*p2  : Use transverse momentum corrections in kinematics
 T
*p3  : Use target mass corrections in kinematics
 T
*p4  : Use product mass corrections in kinematics
 T
*p5  : Use transverse momentum corrections in x1 and z1
 T
```

Scimemi, Vladimirov, arXiv:1912.06532

Estimate of the impact of power corrections

Results obtained within the arTeMiDe framework



```
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Scimemi, Vladimirov, arXiv:1912.06532

Estimate of the impact of power corrections

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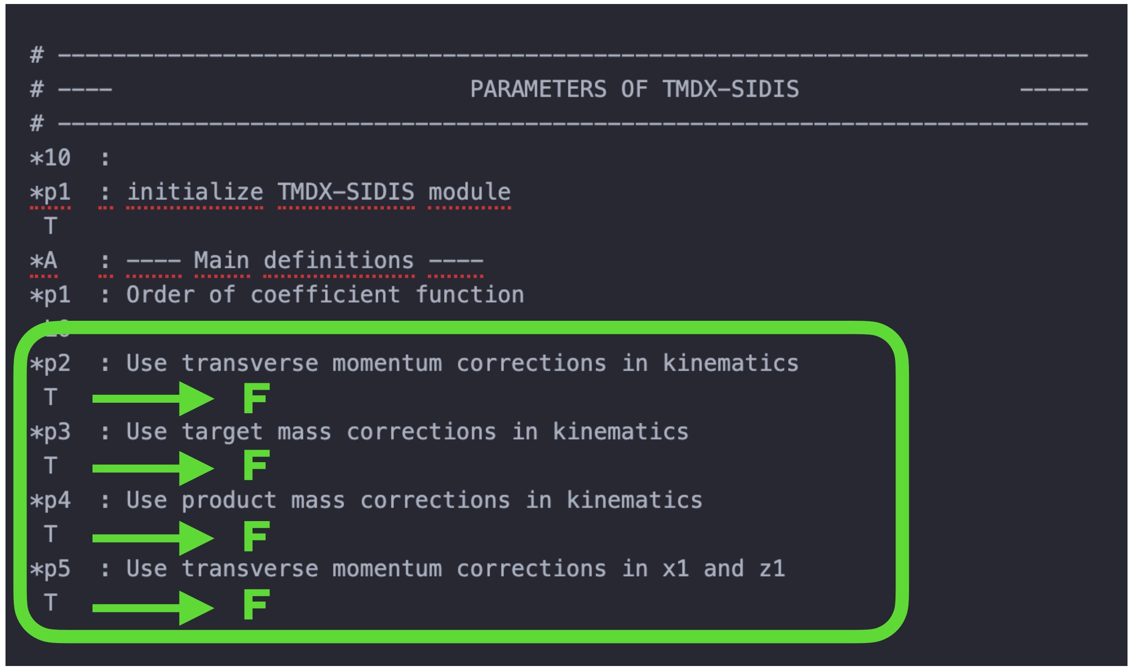

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 T
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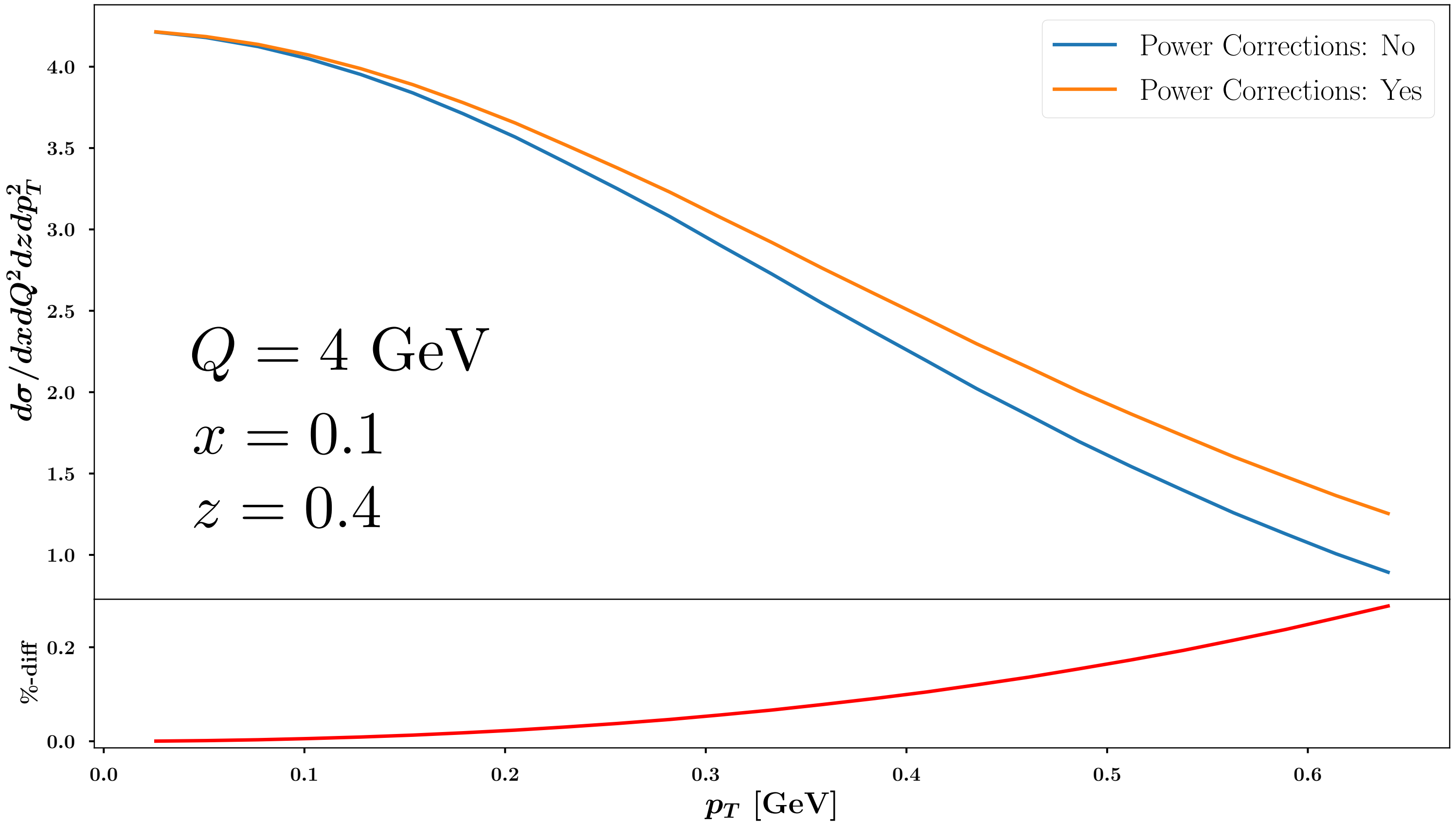
 T
```

Scimemi, Vladimirov, arXiv:1912.06532



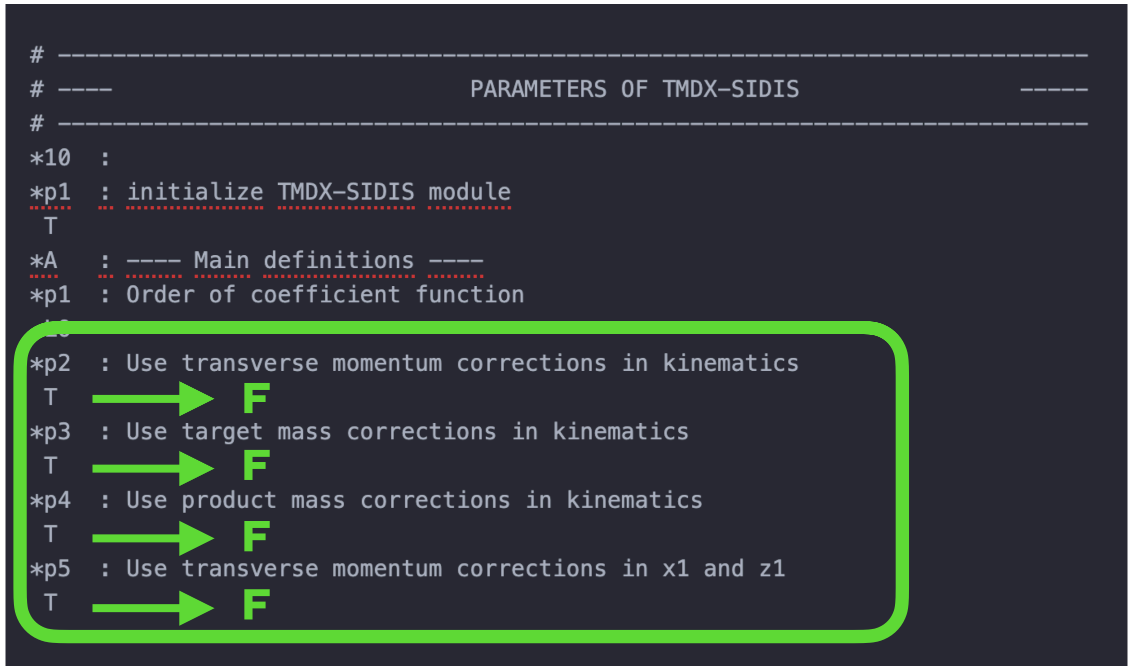
$Q = 4$ GeV

$x = 0.1$

$z = 0.4$

Estimate of the impact of power corrections

Results obtained within the arTeMiDe framework



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# --------------------------------------------------------------------------
# ----                       PARAMETERS OF TMDX-SIDIS                  -----
# --------------------------------------------------------------------------
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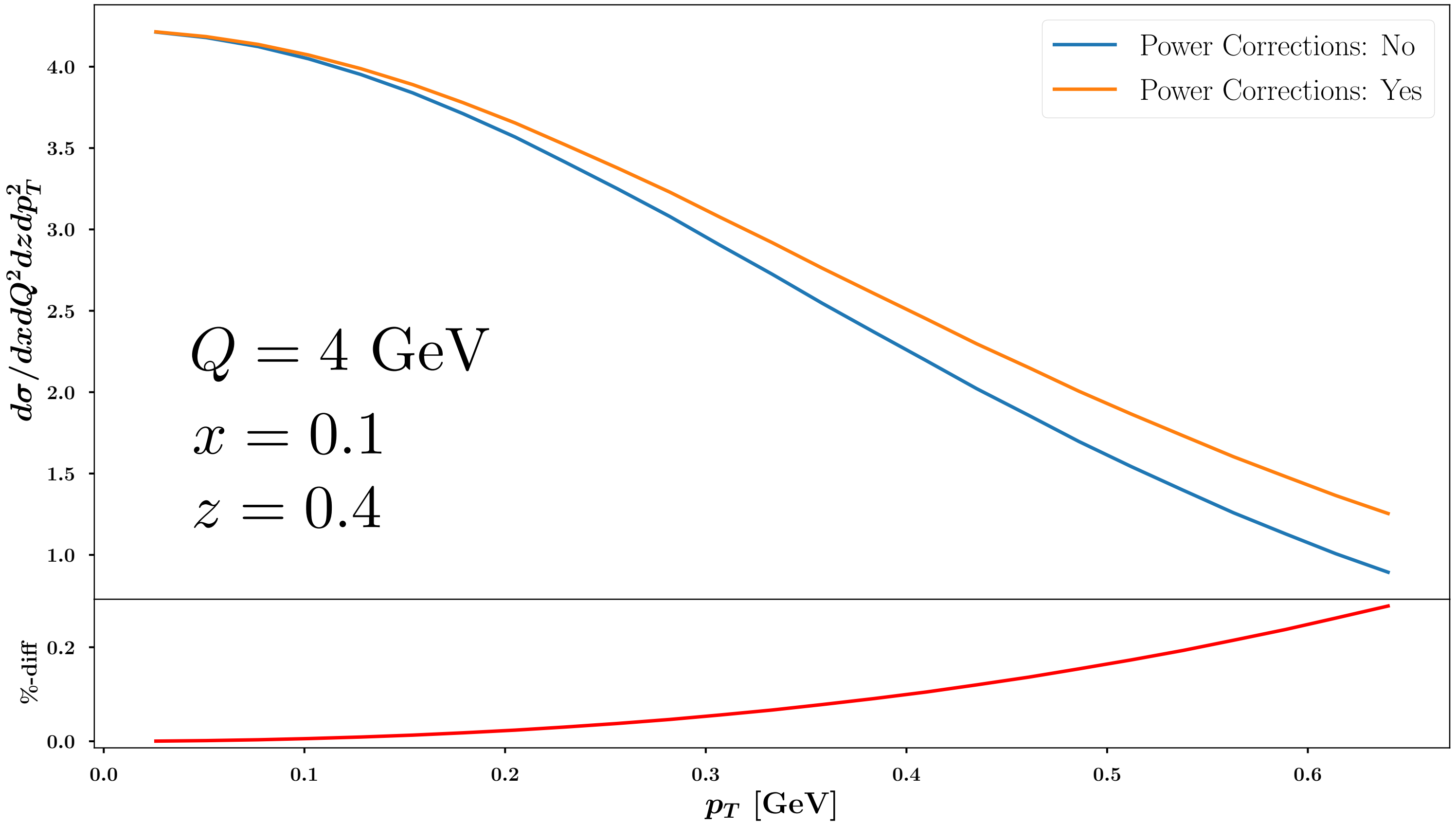
*p1  : initialize TMDX-SIDIS module
 T
*A   : ---- Main definitions ----
*p1  : Order of coefficient function
*p2  : Use transverse momentum corrections in kinematics
 T  →  F
*p3  : Use target mass corrections in kinematics
 T  →  F
*p4  : Use product mass corrections in kinematics
 T  →  F
*p5  : Use transverse momentum corrections in x1 and z1
 T  →  F
```

Scimemi, Vladimirov, arXiv:1912.06532



This is NOT a constant factor