Non-perturbative Collins-Soper kernel from a Coulomb-gauge-fixed quasi-TMD

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From Quarks and Gluons to the Internal Dynamics of Hadrons
May 15, 2024 → May 17, 2024

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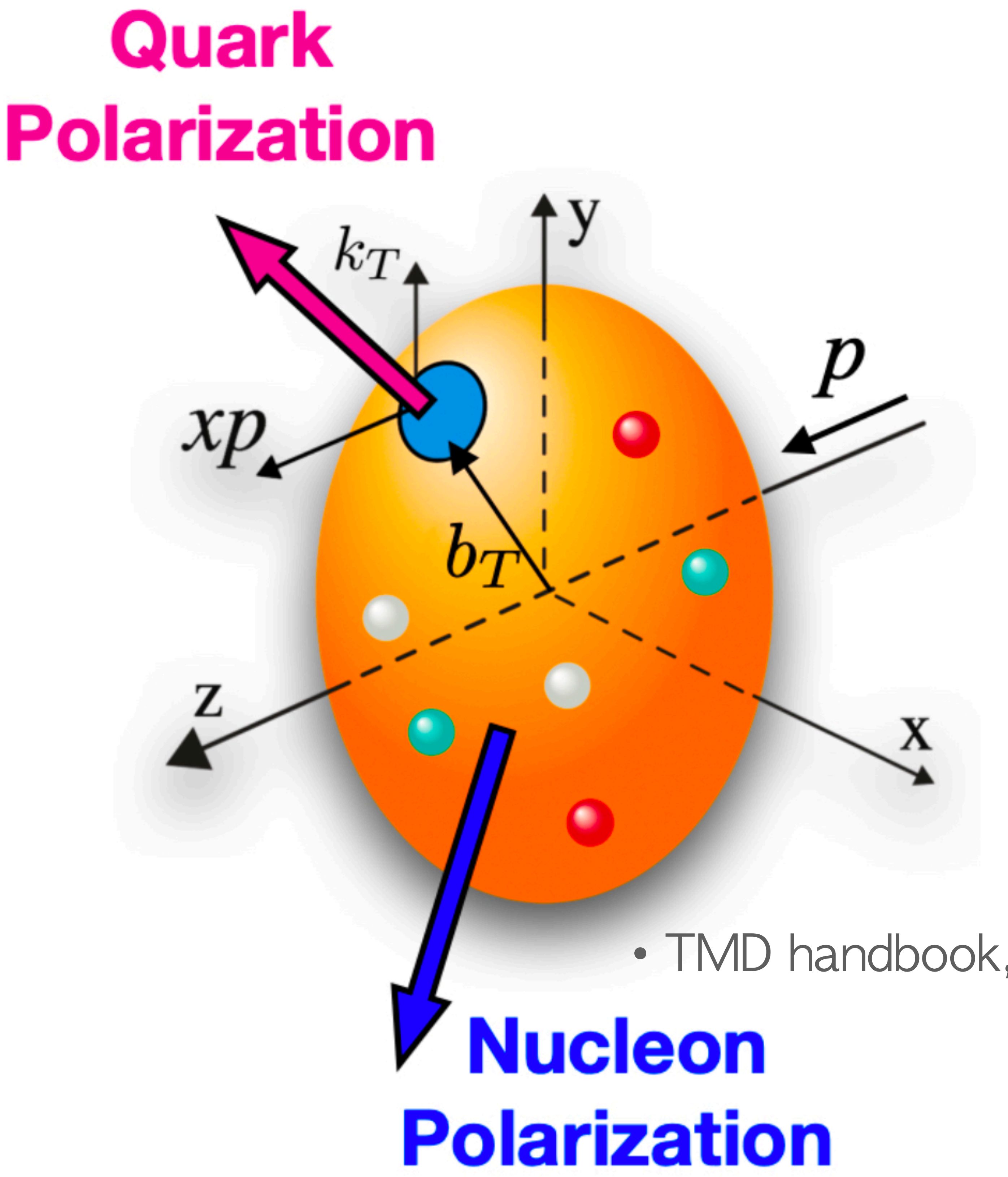
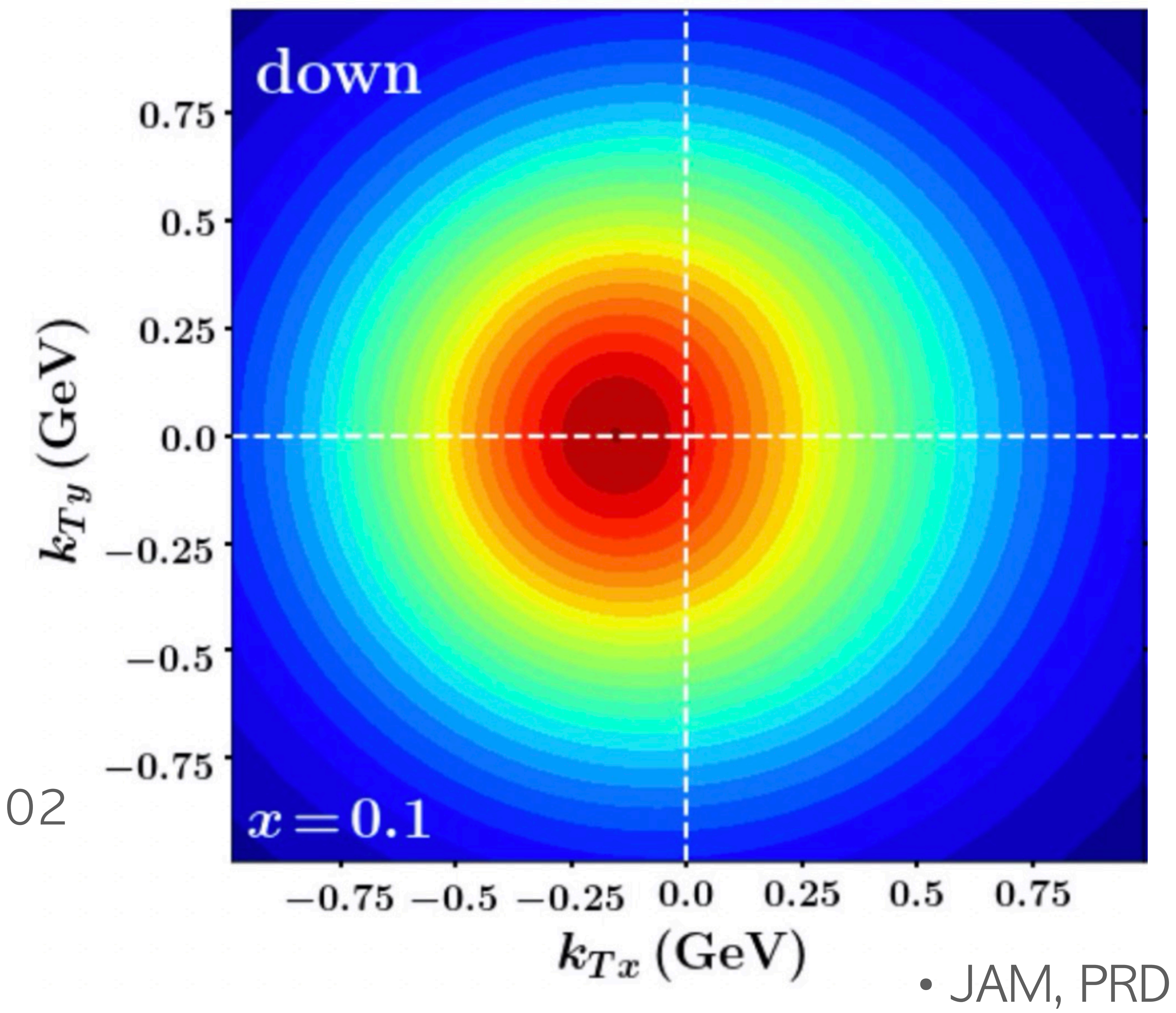
2 Transverse-momentum-dependent distributions



• TMD handbook, arXiv:2304.03302

Quark Sivers function



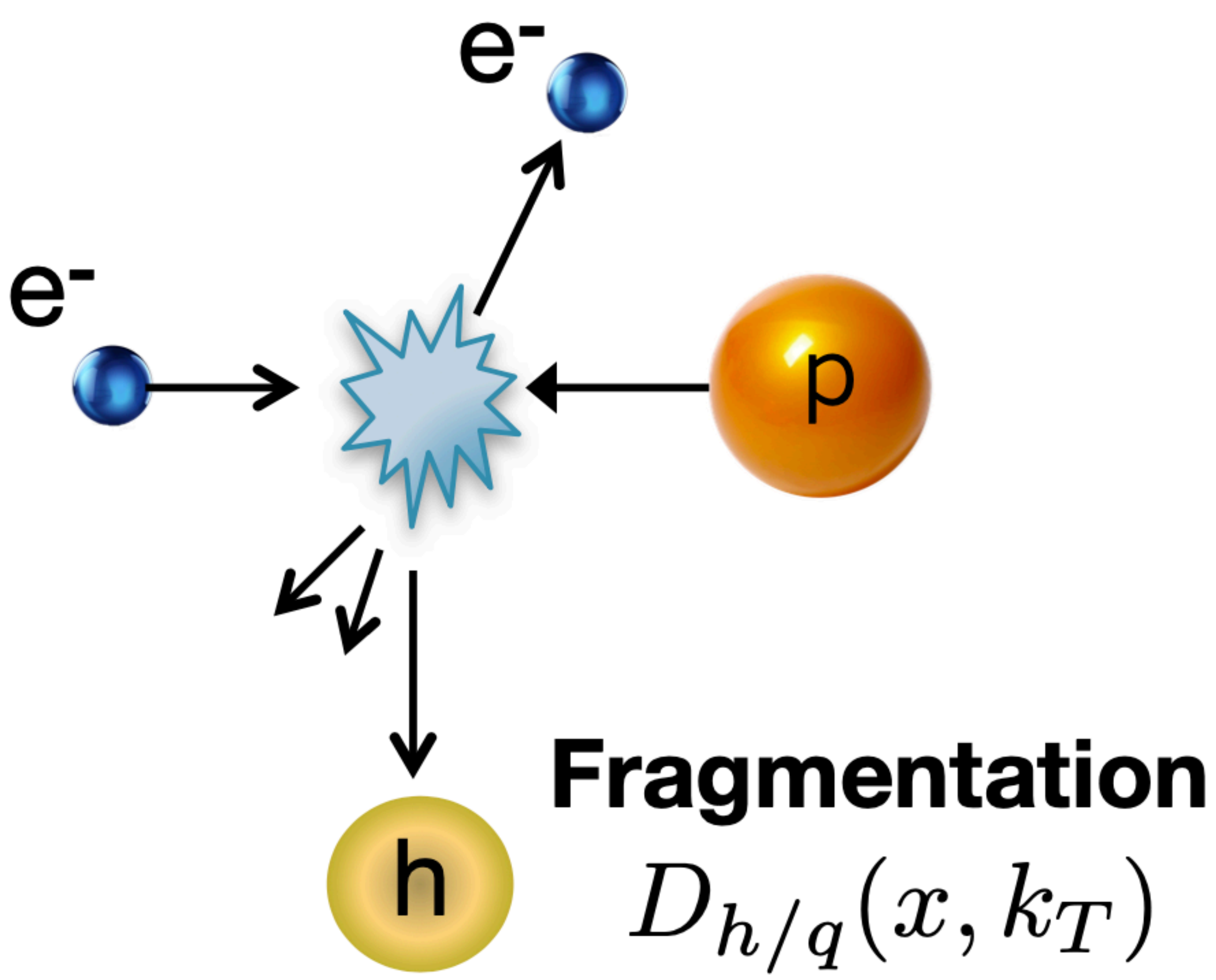
• JAM, PRD 102 (2020) 5, 054002

- 3D image: longitudinal momentum fraction x and confined motion $\vec{k}_T$.
- Spin-orbit correlations.
- QCD input for particle physics (e.g., m_W).

3 TMDs from global analyses of experimental data

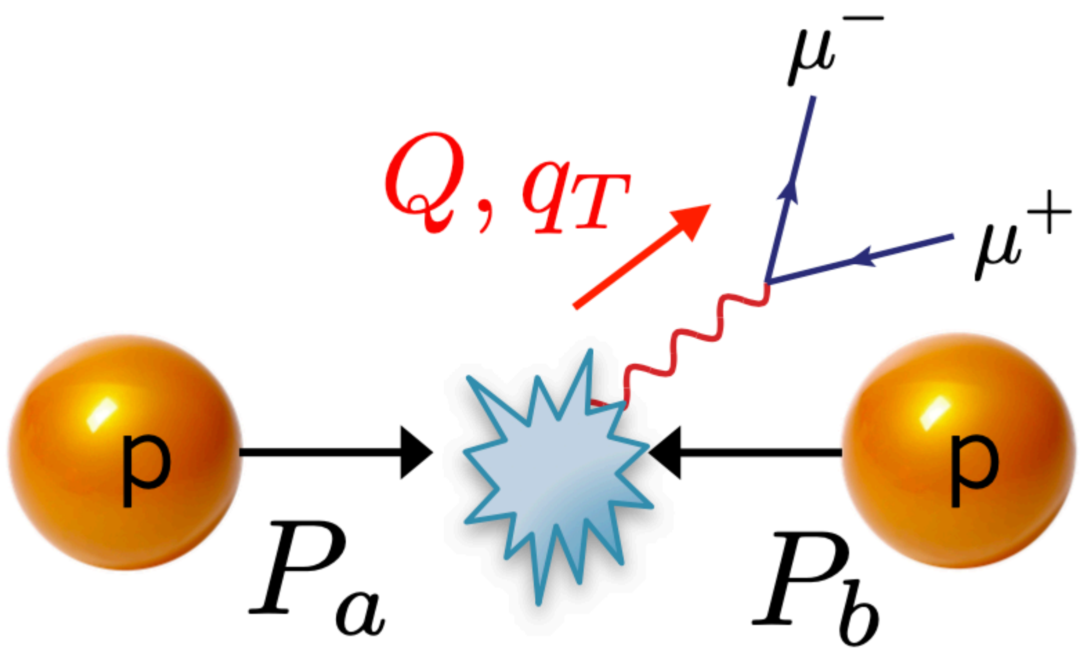
Semi-Inclusive DIS

$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$



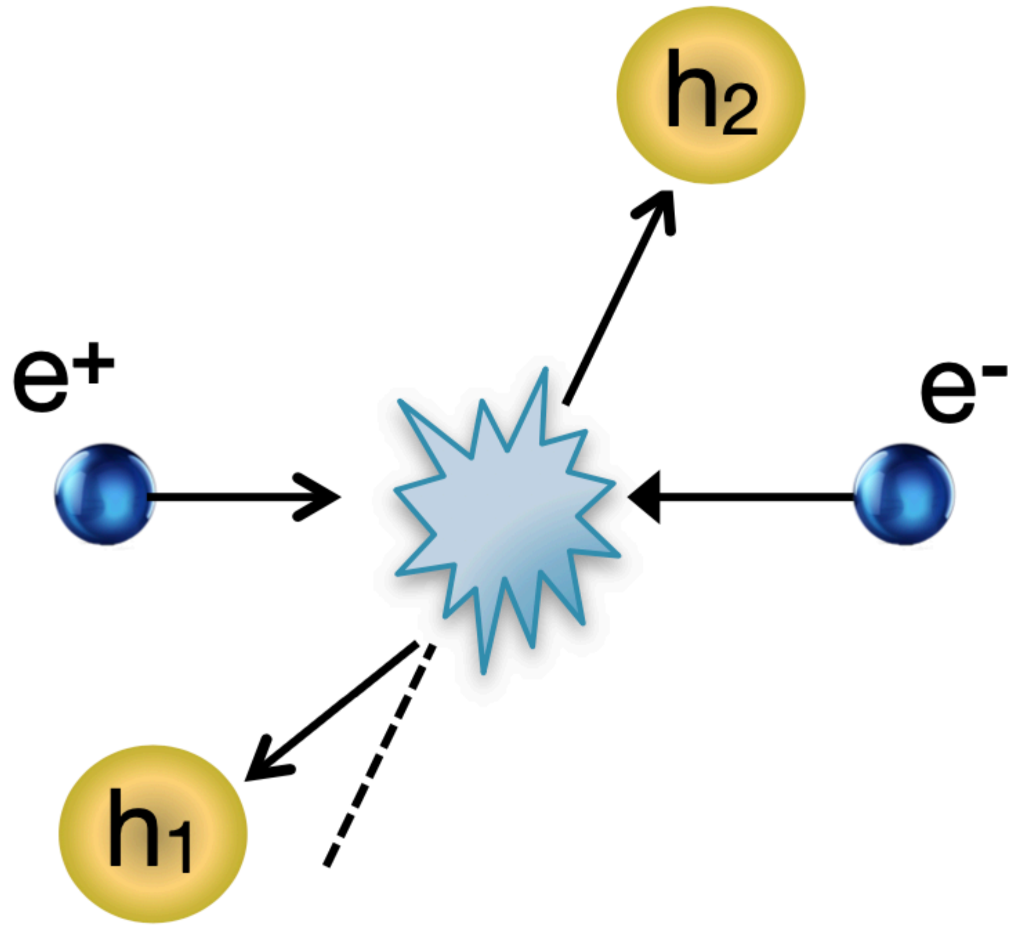
Drell-Yan

$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$



Dihadron in e+e-

$\sigma \sim D_{h_1/q}(x, k_T) D_{h_2/q}(x, k_T)$



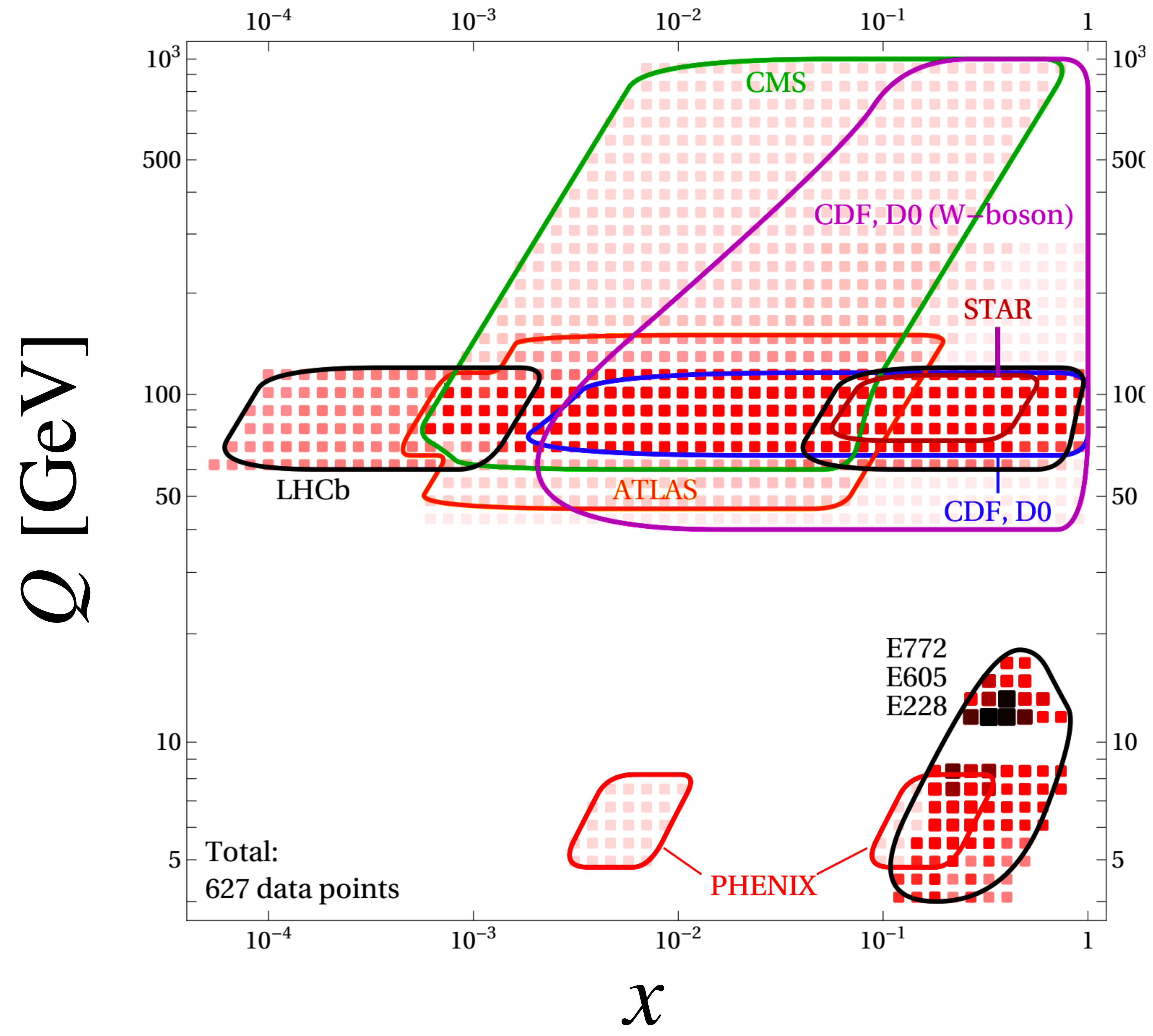
$$\frac{d\sigma_{\mathrm{DY}}}{dQ\,dY\,dq_T^2} = H(Q,\mu) \int d^2\vec{b}_T e^{i\vec{q}_T\cdot\vec{b}_T} f_q(x_a, \vec{b}_T, \mu, \zeta_a)\, f_q(x_b, \vec{b}_T, \mu, \zeta_b)\left[1 + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)\right]$$

Perturbative hard kernels

Nonperturbative TMDs

$q_T^2 \ll Q^2$

4 TMDs from global analyses of experimental data



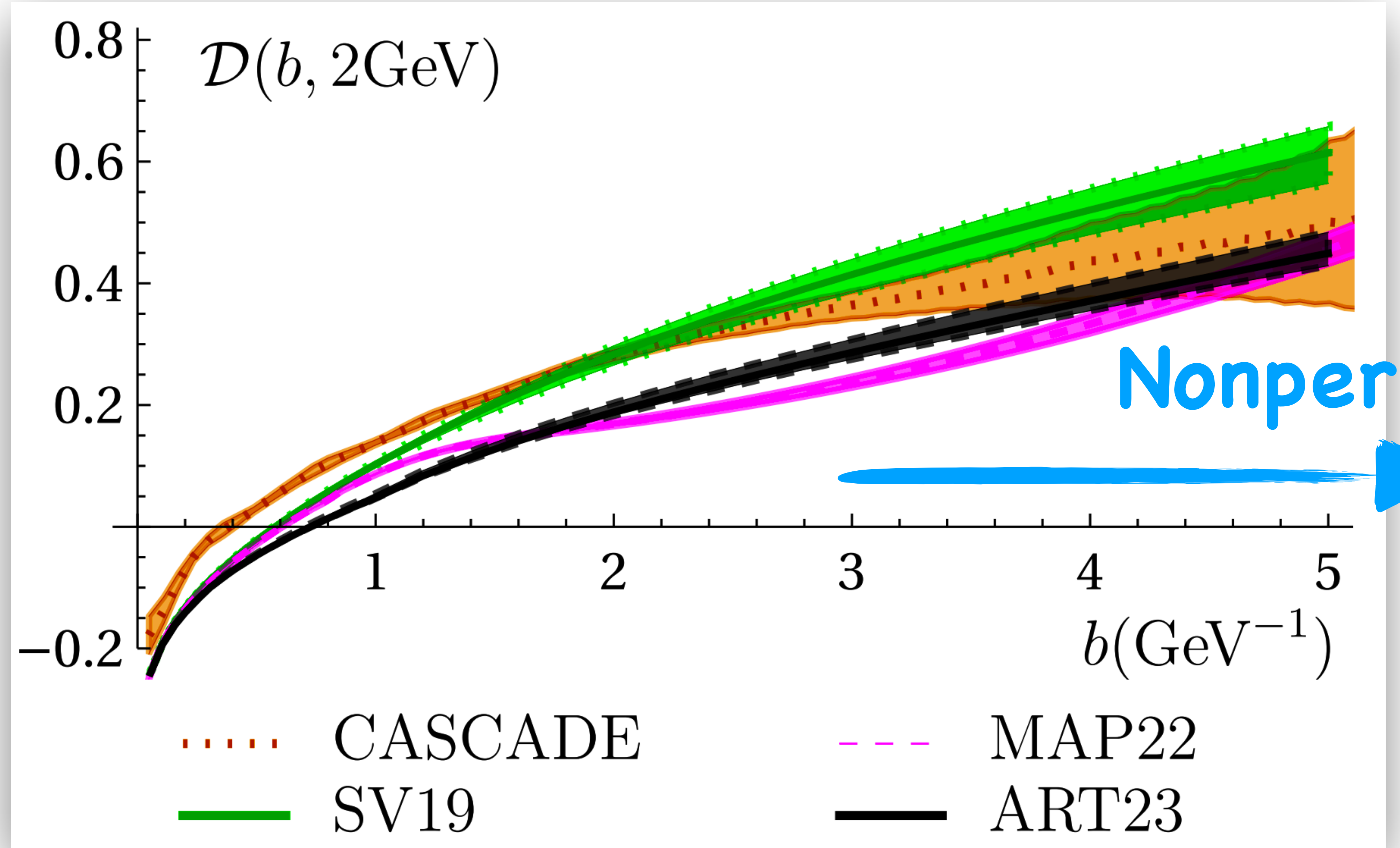
• V. Moos, et. al. (ART23), JHEP 05 (2024) 036

- Relate TMDs at different energy scales

$$\mu\frac{d}{d\mu}\ln f_q(x,\vec{b}_T,\mu,\zeta) = \gamma_\mu^q(\mu,\zeta)$$

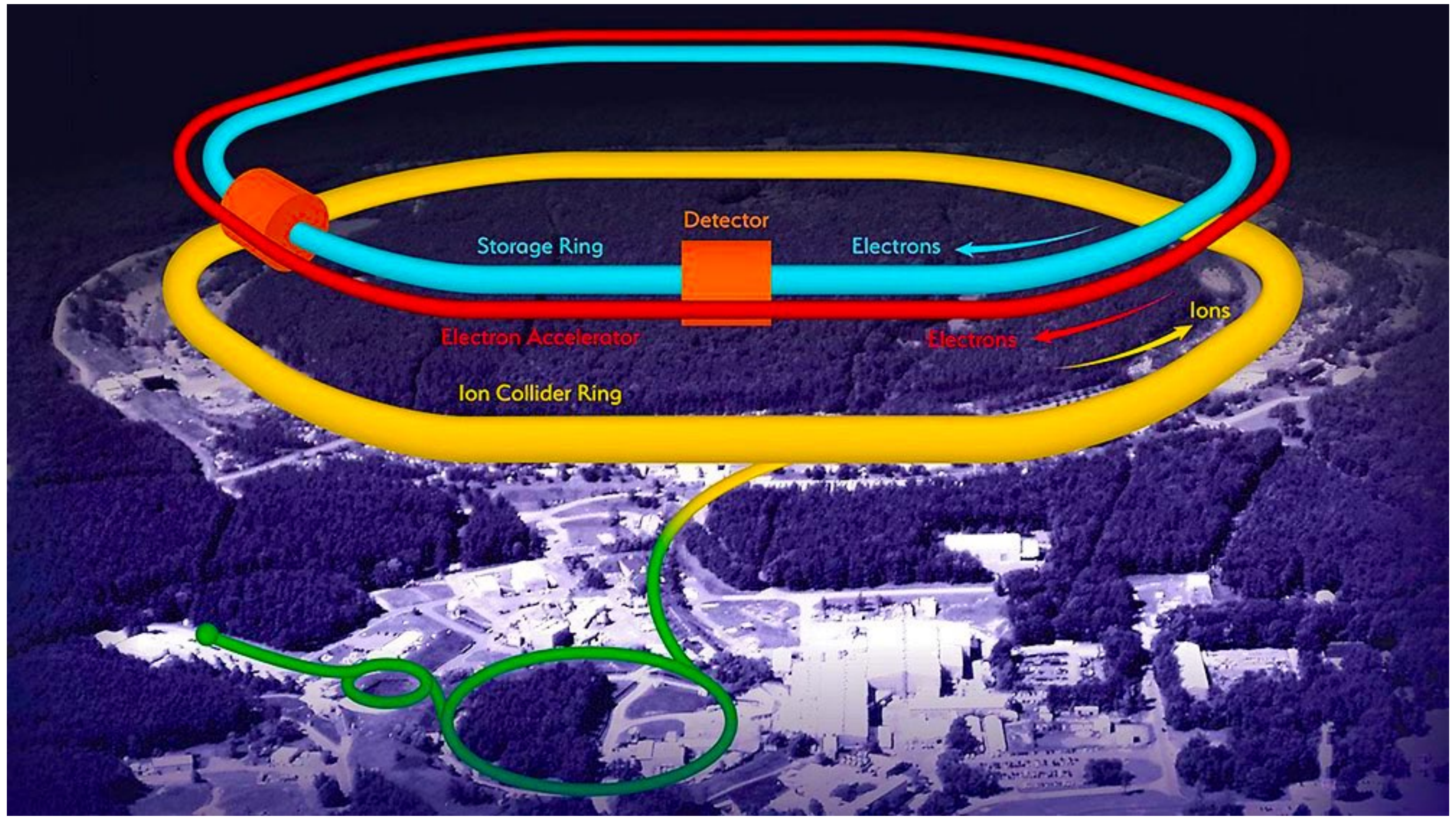
$$\mu\frac{d}{d\zeta}\ln f_q(x,\vec{b}_T,\mu,\zeta) = \gamma_\zeta^q(\mu,b_T)$$

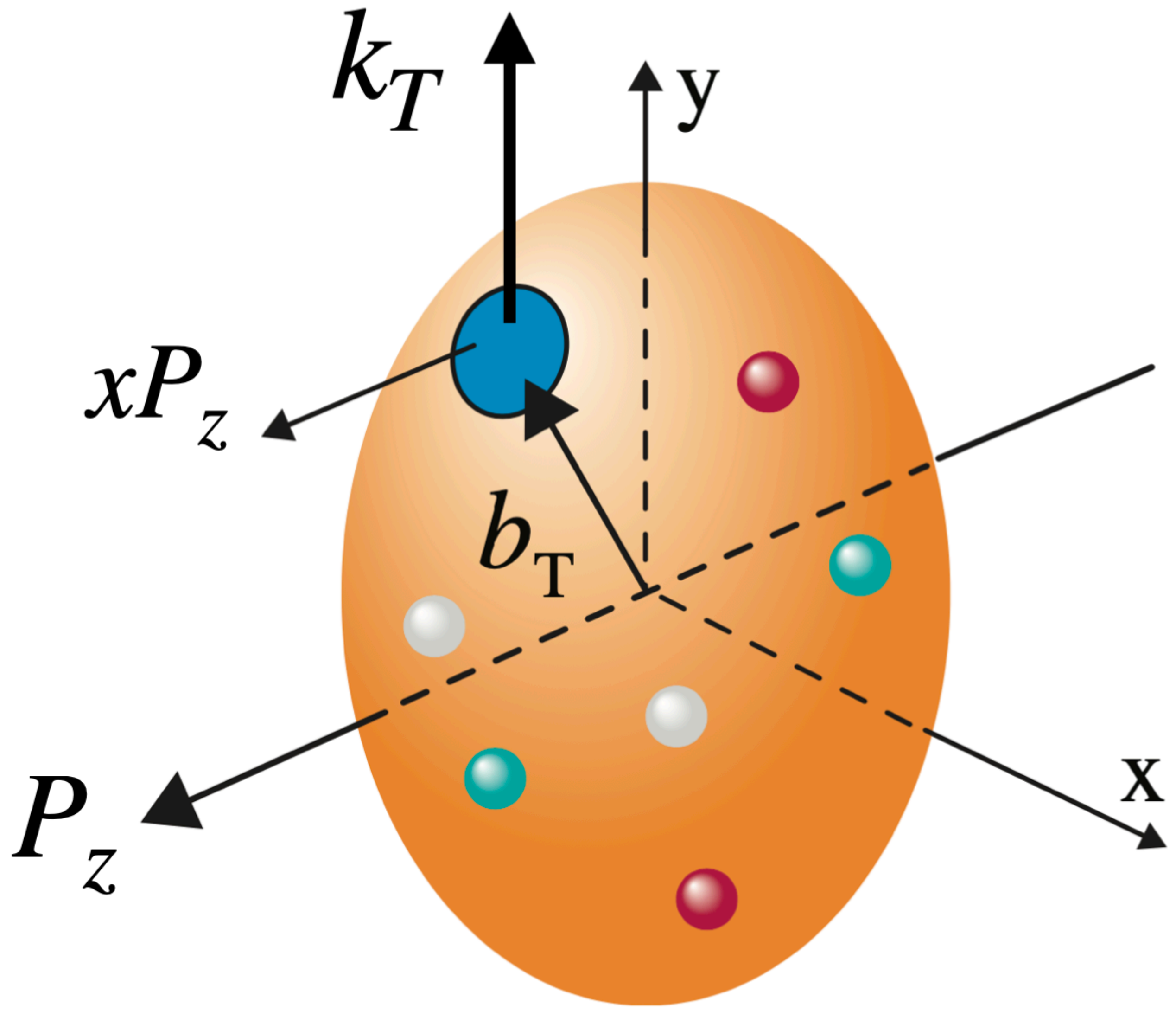
Collins-Soper kernel



Nonperturbative

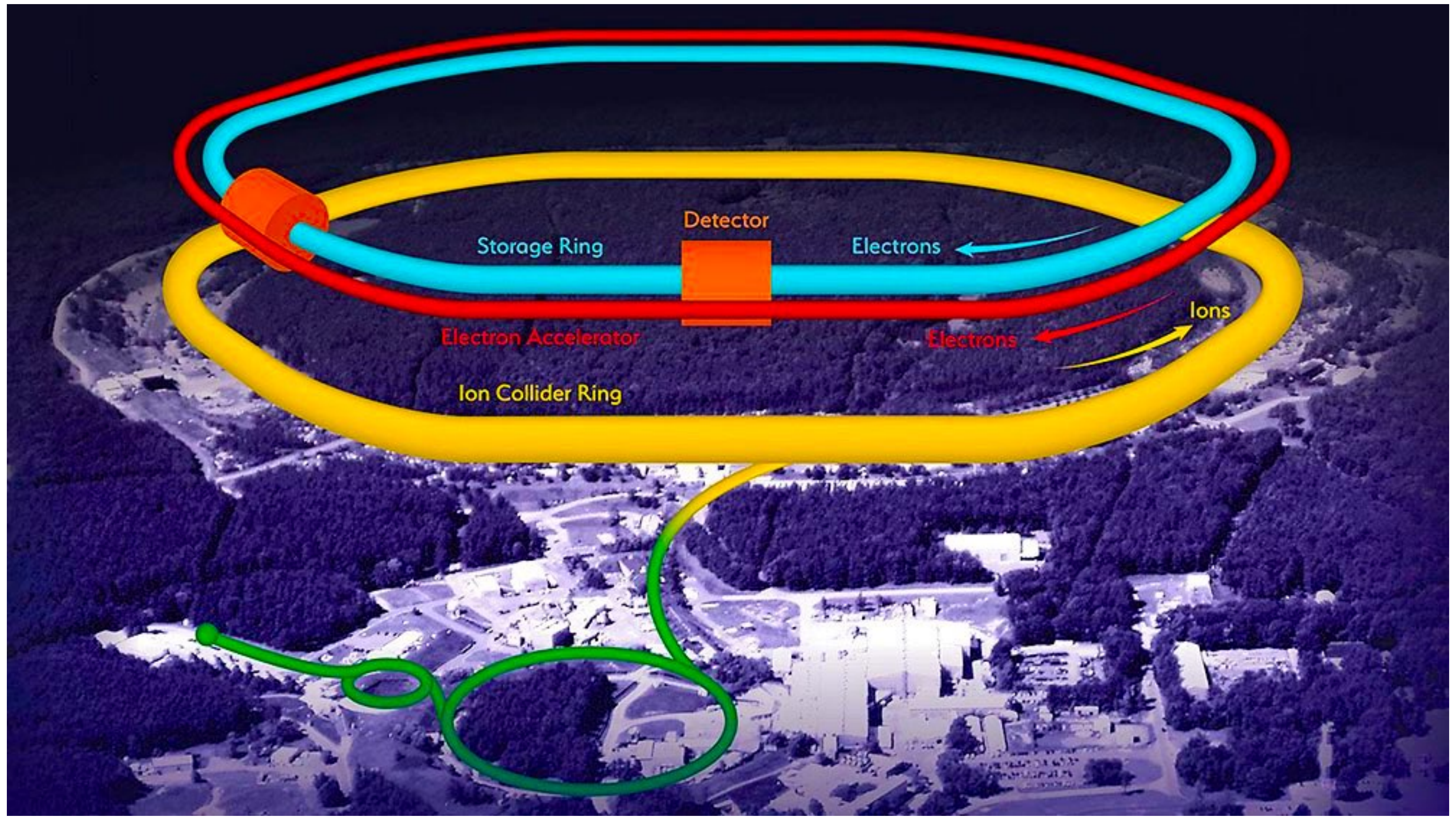
5 Determination of TMDs

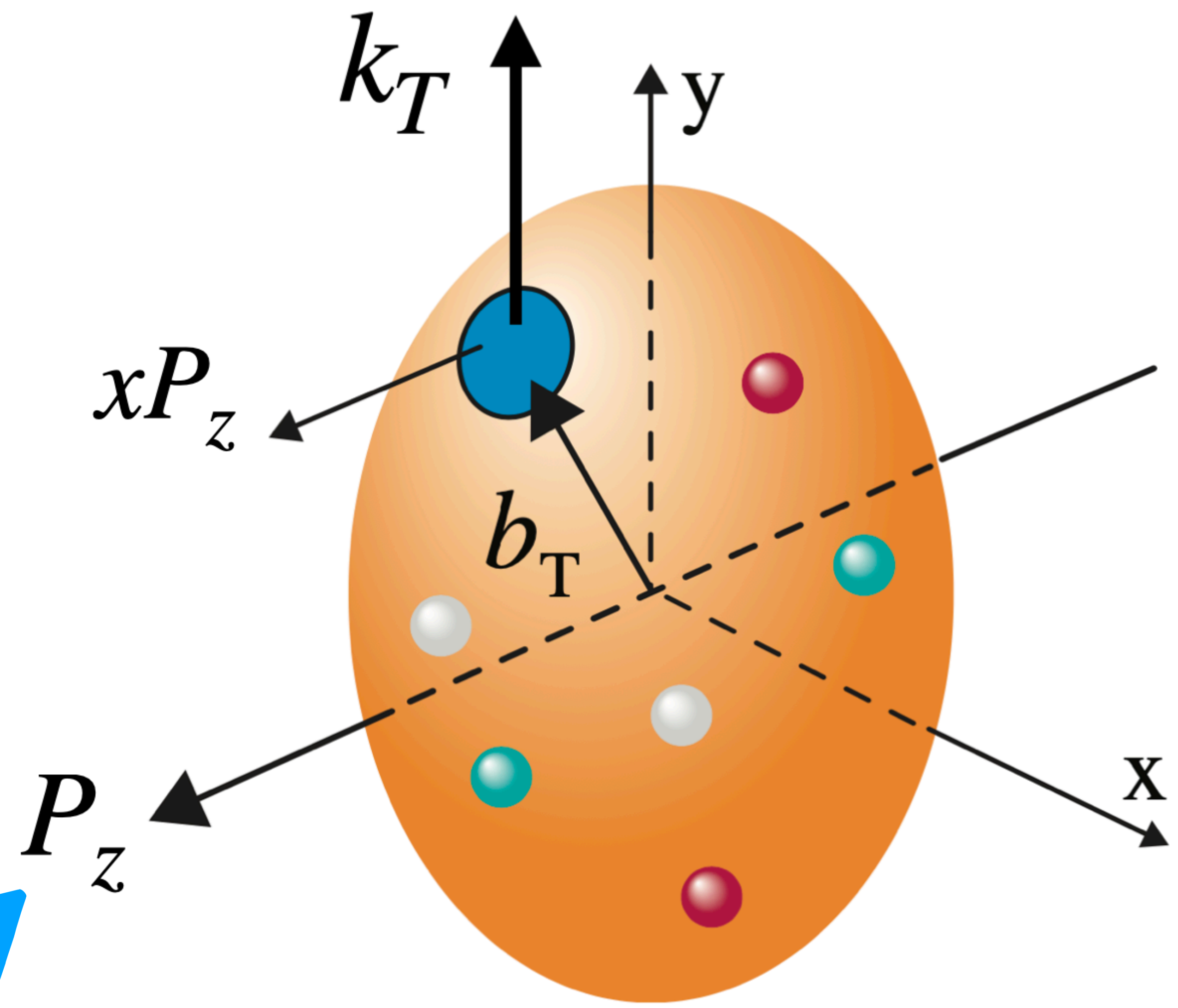


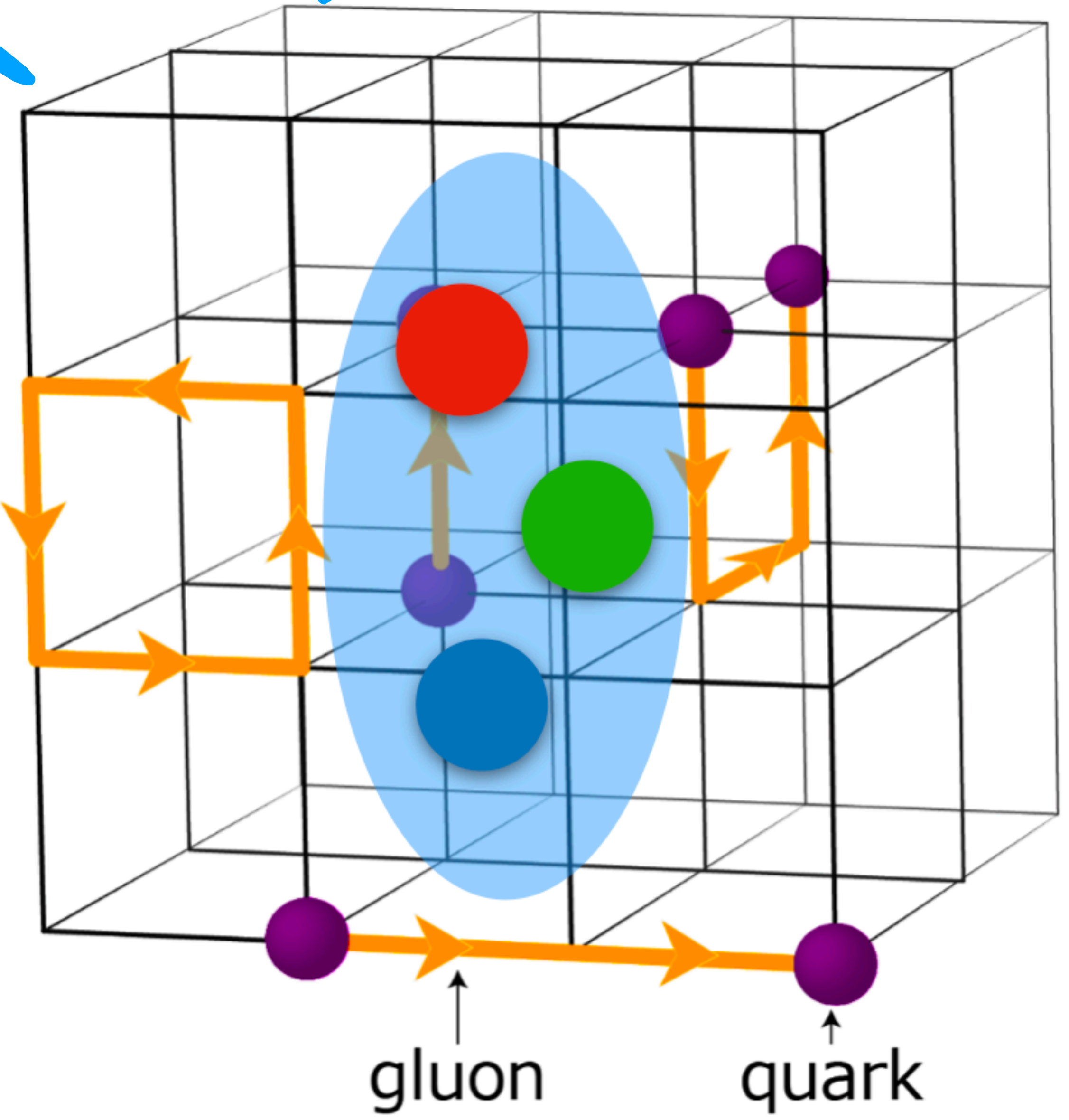


- Global analysis of experimental data.

6 Determination of TMDs







- Global analysis of experimental data.
- Complementary knowledge from lattice QCD is essential.

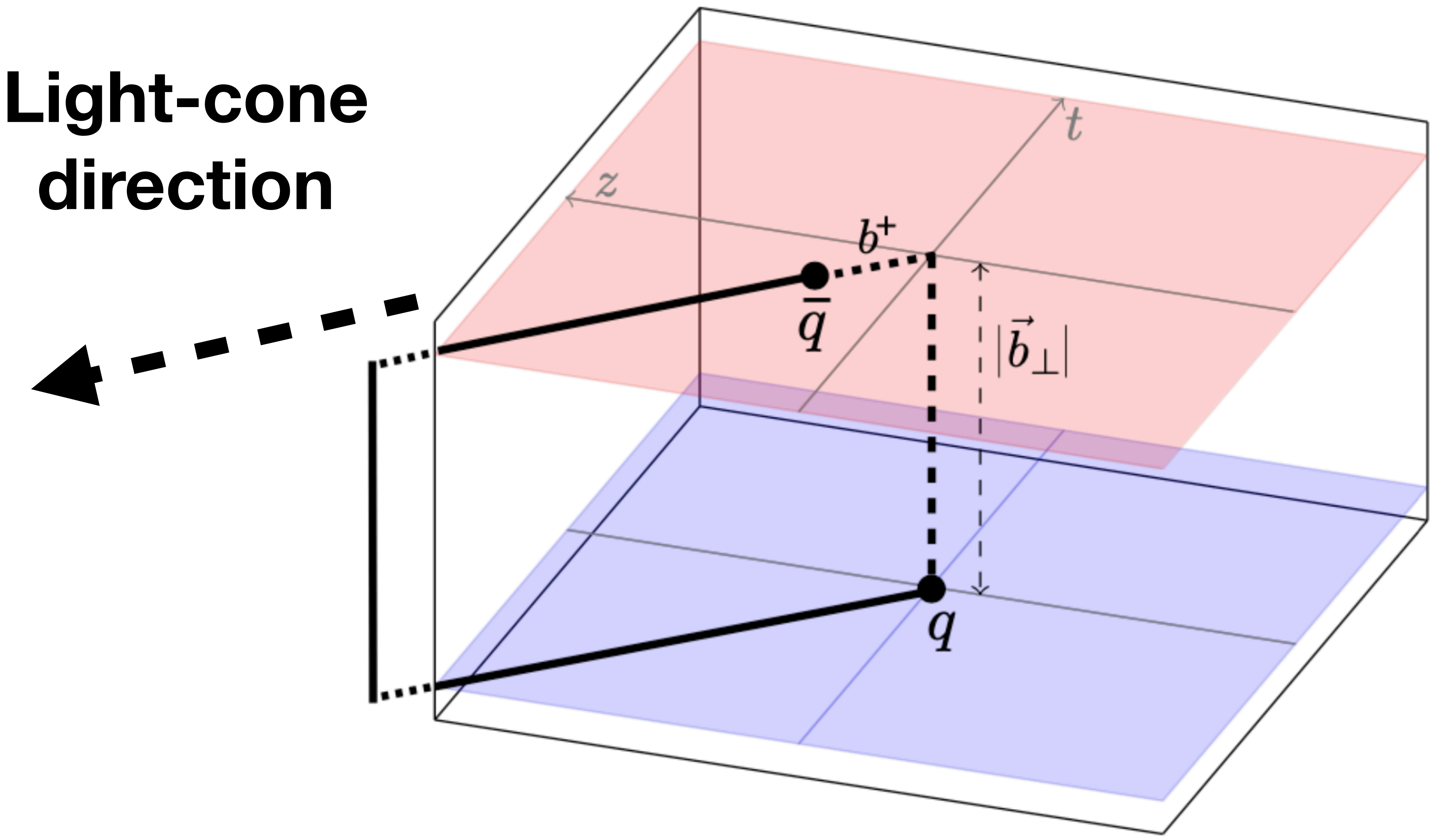
7 The definition of TMDs

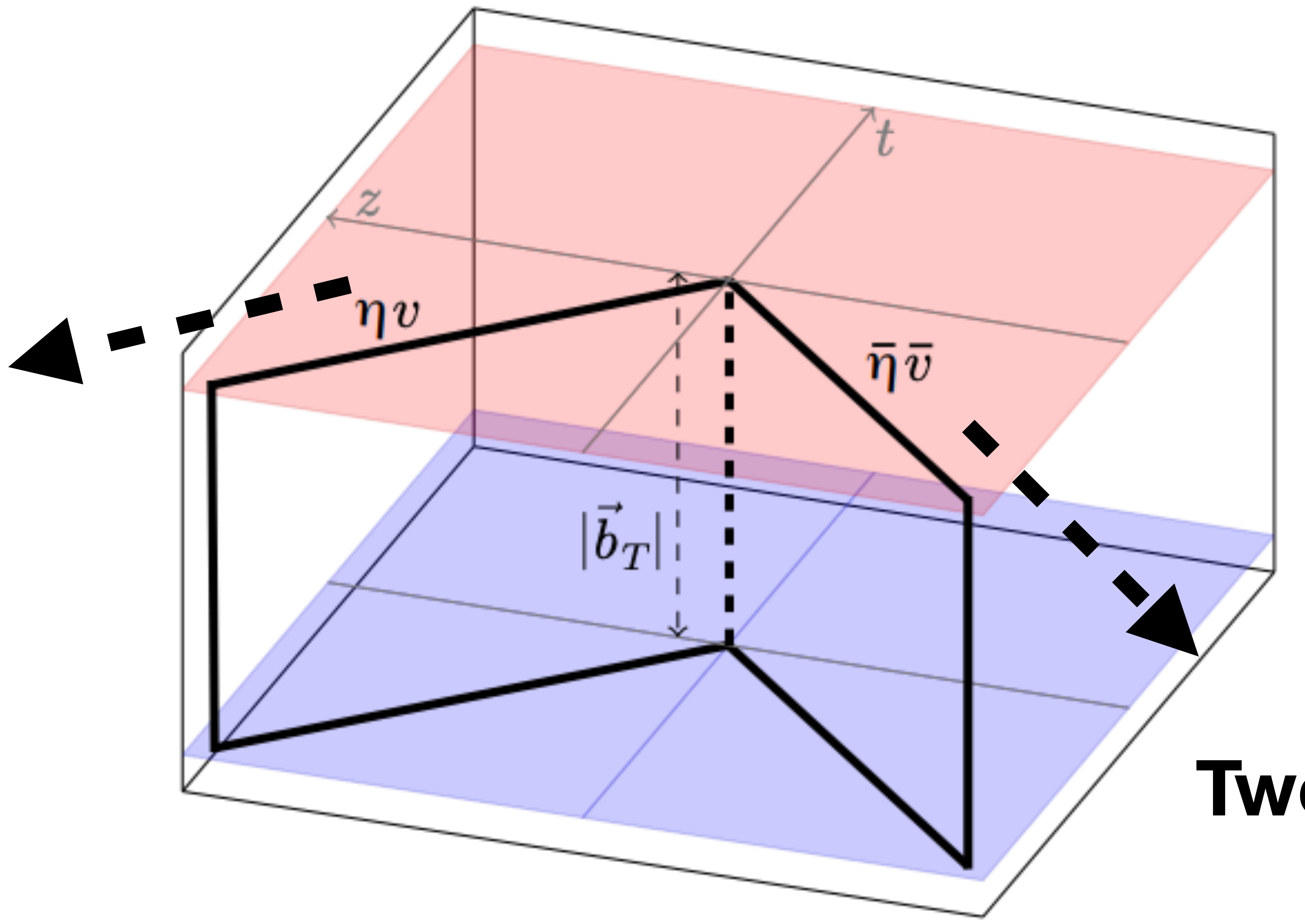
Beam function

$$f_q(x, \vec{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{\mathrm{UV}}(\epsilon, \mu, \zeta) \lim_{\tau \to 0} \frac{B_q(x, \vec{b}_T, \epsilon, \tau, \zeta)}{\sqrt{S_q(\vec{b}_T, \epsilon, \tau)}}$$

UV regulator ($\epsilon \to 0$) Rapidity regulator ($\tau \to 0$) **Soft function**

Light-cone direction





Two Light-cone direction

Hadron matrix elements B_q

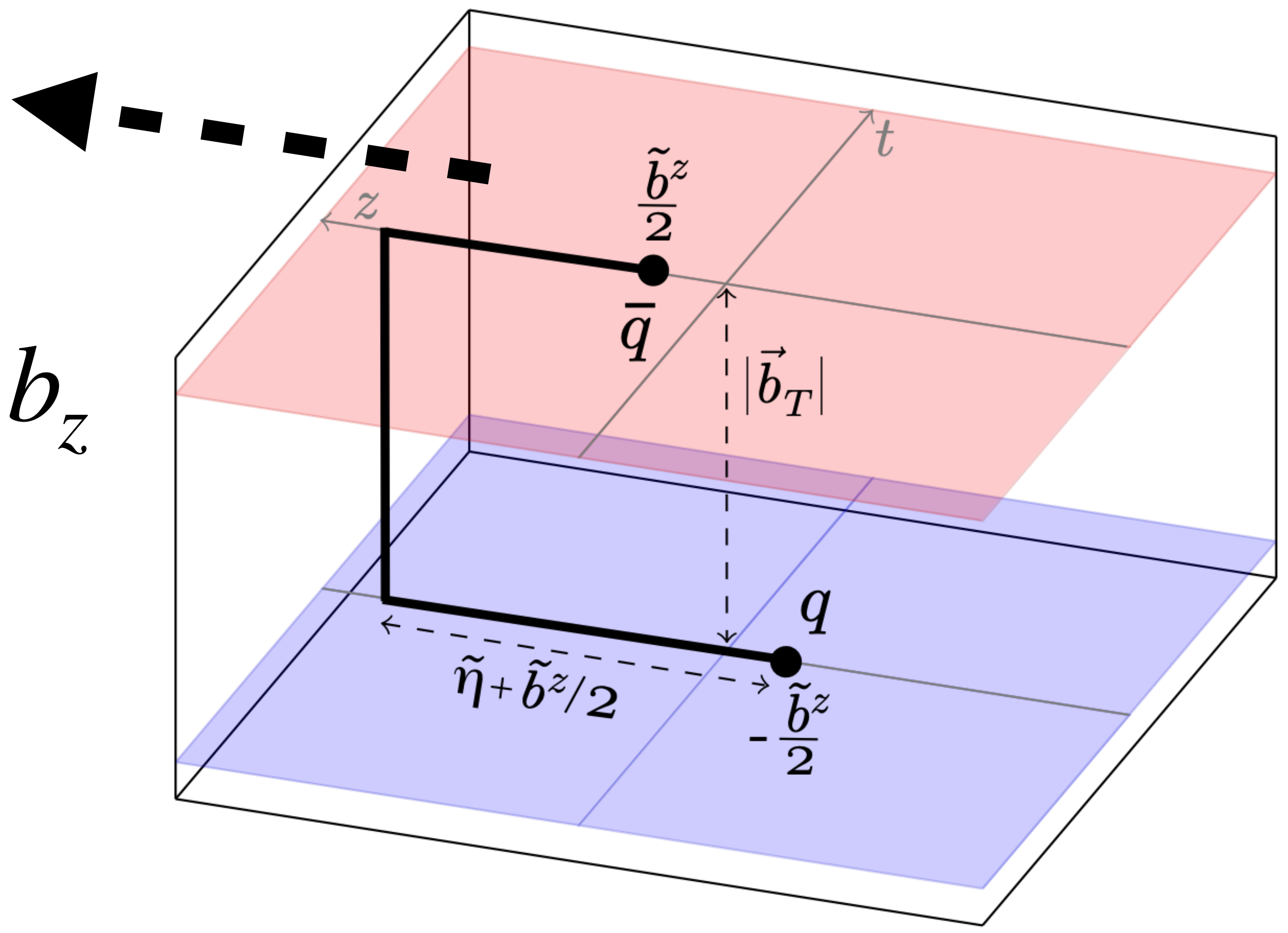
$$\langle N, S | \overline{\psi}(\frac{b^+}{2}, b_\perp) \Gamma W_{\sqsupset^+} \psi(-\frac{b^+}{2}, 0) | N, S \rangle$$

Vacuum matrix elements S_q

$$\langle \Omega | W_{\sqsubset^+}(b_\perp, 0) W_{\sqsupset^-}(b_\perp, 0) | \Omega \rangle$$

8 TMDs from lattice: quasi TMDs

Spacial direction b_z



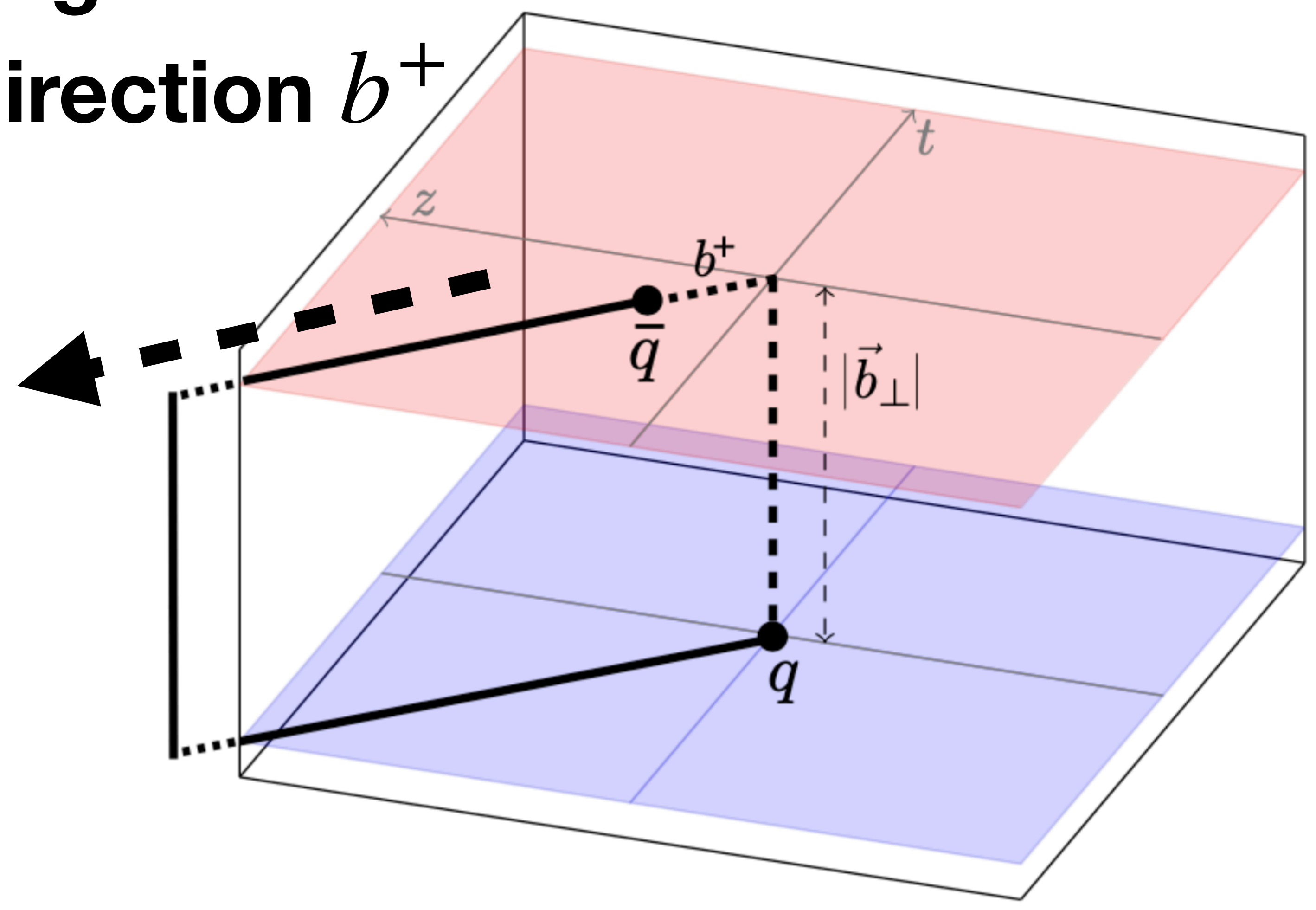
Quasi beam function

$$\langle N,S\,|\,\overline{\psi}(\frac{b_z}{2},b_\perp)\Gamma W_{\sqsupset^z}\psi(-\frac{b_z}{2},0)\,|\,N,S\rangle$$

$P_z, \eta \to \infty$

LaMET

Light-cone direction b^+



Light-cone Beam function

$$\langle N,S\,|\,\overline{\psi}(\frac{b^+}{2},b_\perp)\Gamma W_{\sqsupset^+}\psi(-\frac{b^+}{2},0)\,|\,N,S\rangle$$

Quasi-TMDs from equal-time correlators:

- Computable from Lattice QCD.
- Have same IR physics as light-cone TMDs.

- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- A. Vladimirov, A. Schäfer Phys.Rev.D 101 (2020), 074517
- I. Stewart, Y. Zhao et al., JHEP 09 (2020) 099
- X. Ji et al., Phys.Rev.D 103 (2021) 7, 074005
- I. Stewart, Y. Zhao et al., JHEP 08 (2022) 084

9 TMDs from lattice: quasi TMDs

- Quasi TMDs: first regularize QCD on a lattice (a or $\epsilon \to 0$), then take the $P_z \to \infty$ limit.
- Differ from the Collins scheme by order of $y_B \to -\infty$ (rapidity) and $\epsilon \to 0$ limit, inducing a perturbative matching.

Quasi beam function

Collins-Soper kernel

$$\frac{\tilde{\phi}(x, \vec{b}_T, \mu, P_z)}{\sqrt{S_r(\vec{b}_T, \mu)}} = C(\mu, xP_z)e^{\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2xP_z)^2}{\zeta}} f(x, \vec{b}_T, \mu, \zeta)\{1 + \mathcal{O}[\frac{1}{(xP_z b_T)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{(xP_z)^2}]\}$$

Physical TMD

Reduced soft factor

The Collins-Soper kernel from quasi-TMDs

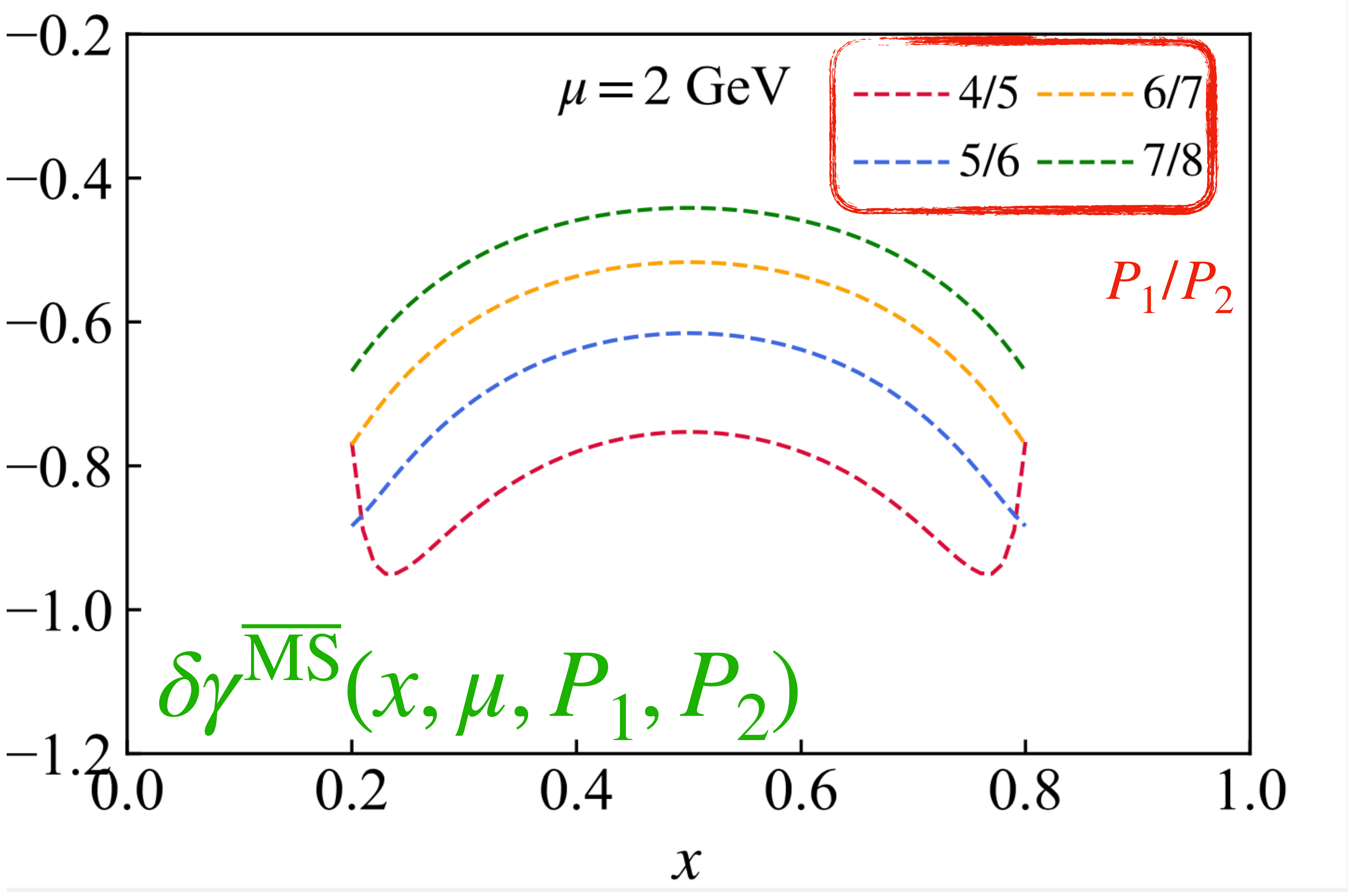
Collins-Soper kernel

Perturbative correction

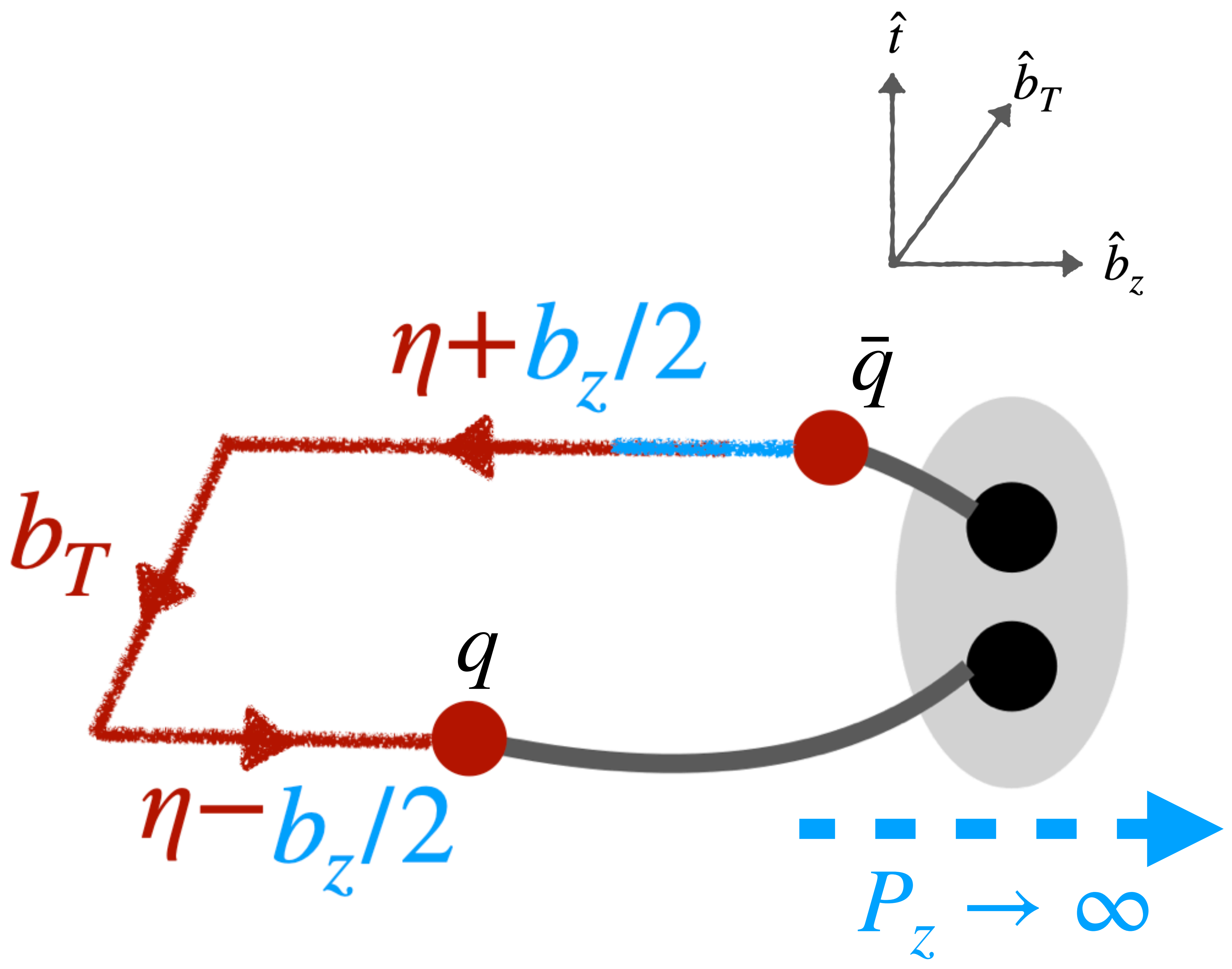
$$\gamma^{\overline{\rm MS}}(b_\perp,\mu) = \frac{1}{\ln(P_2/P_1)} \ln\left[\frac{\tilde{\phi}(x,b_\perp,P_2,\mu)}{\tilde{\phi}(x,b_\perp,P_1,\mu)}\right] + \delta\gamma^{\overline{\rm MS}}(x,\mu,P_1,P_2) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP_z)^2}, \frac{1}{(b_\perp(xP_z))^2}\right)$$

Ratio of quasi-TMDs

- The soft factor cancels in the ratio of quasi-TMDs.
- Independent (universal) of P_z and x, up to higher-order and power corrections.

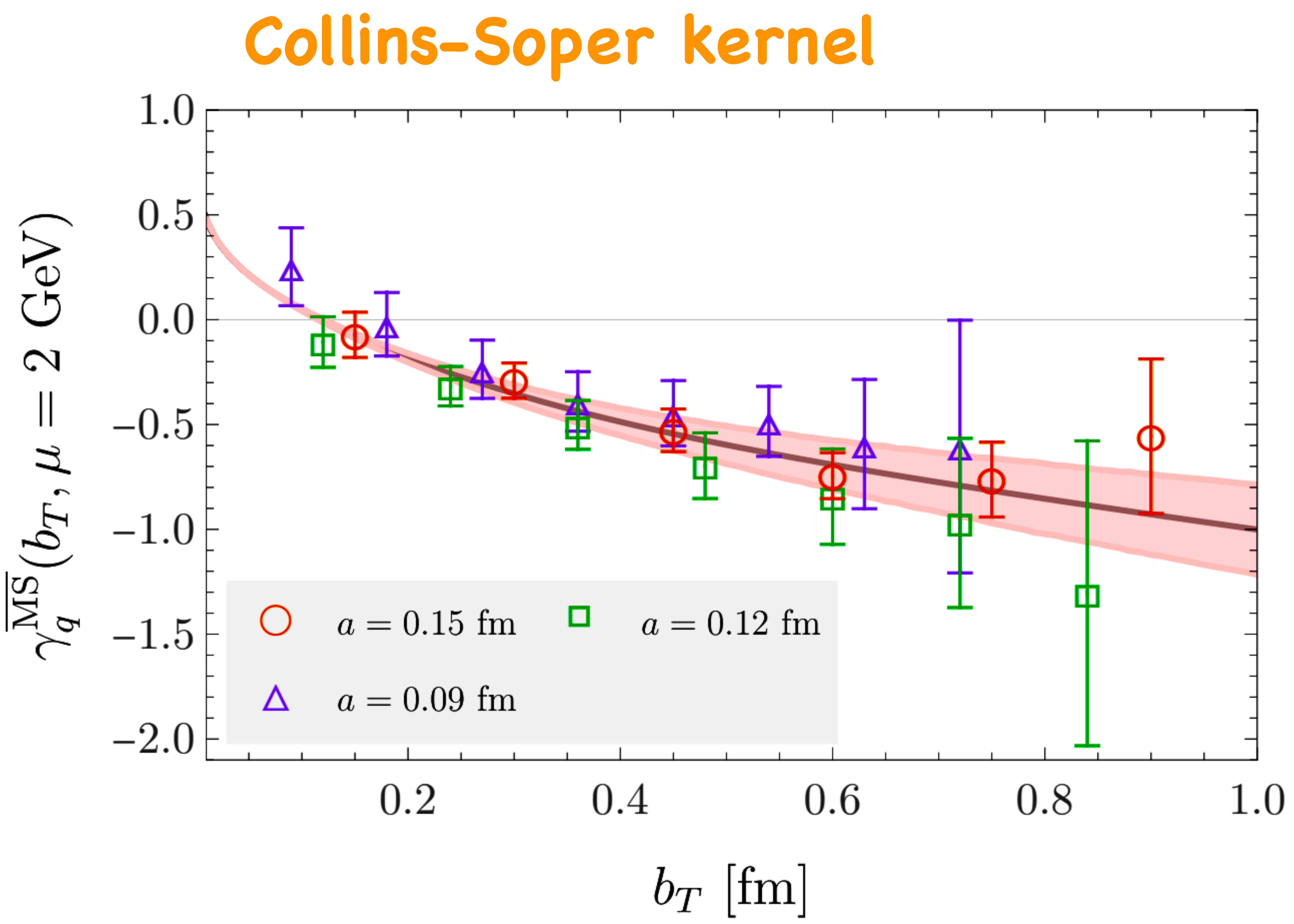


The Collins-Soper kernel from quasi-TMDs



Pion quasi TMD wave function

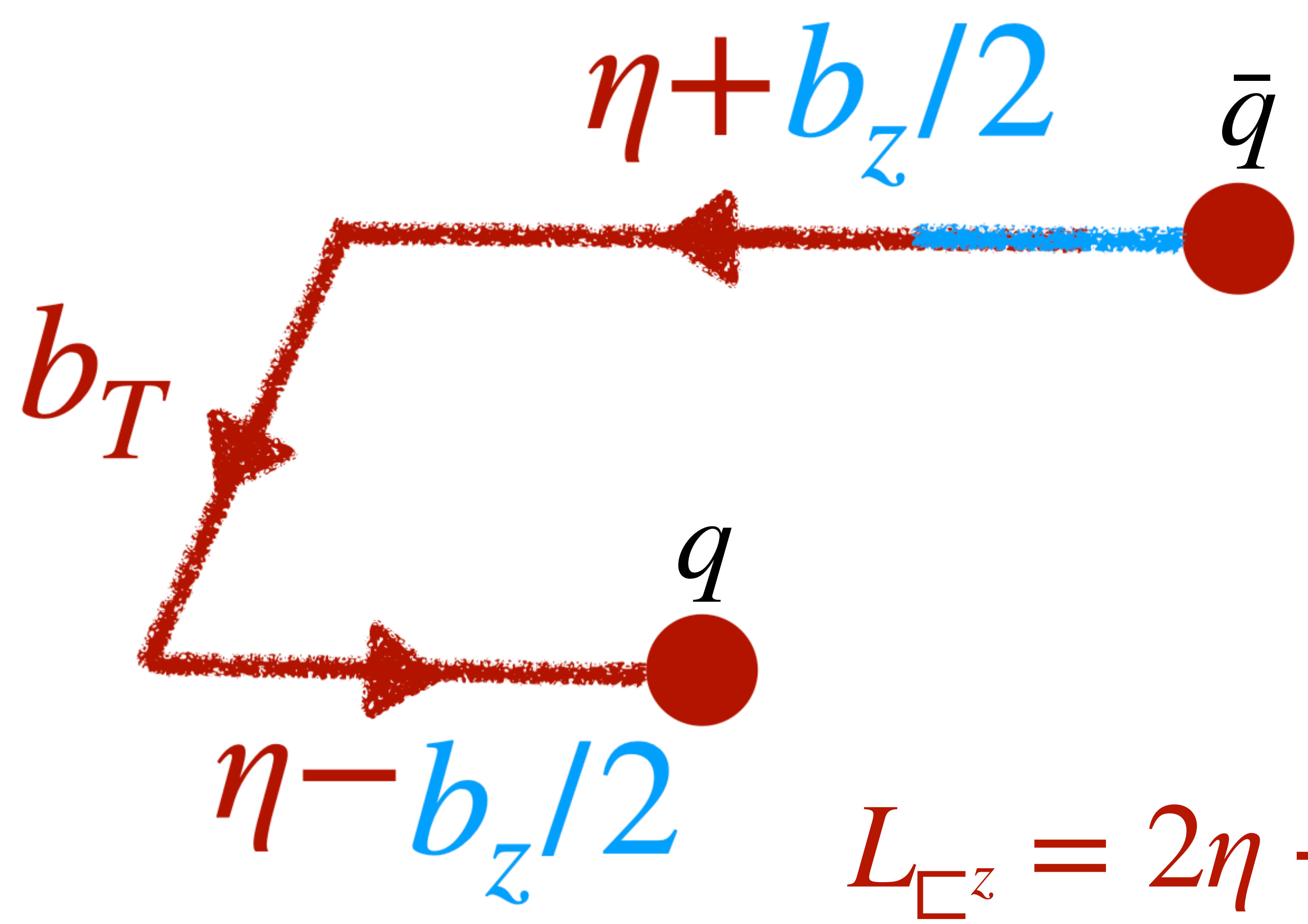
$$\langle \Omega | \overline{\psi}(\frac{b_z}{2}, b_\perp) \Gamma W_{\sqsupset z} \psi(-\frac{b_z}{2}, 0) | \pi^+, P_z \rangle$$



- Wilson-Clover fermion discretization.
- Physical pion mass.
- Three different lattice spacing.

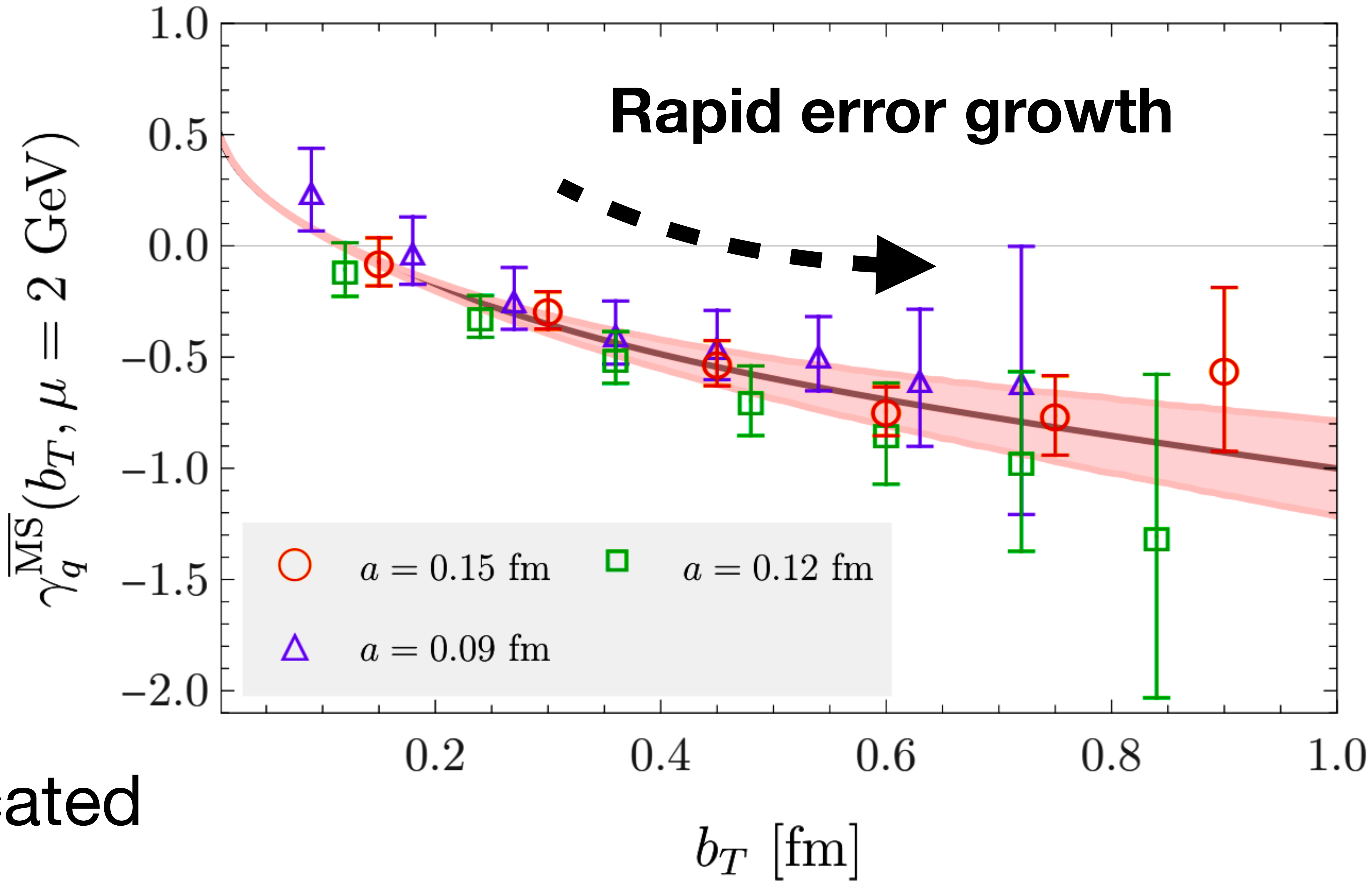
• A. Avkhadiev, et al., arXiv:2402.06725

Difficulties in the conventional quasi-TMDs

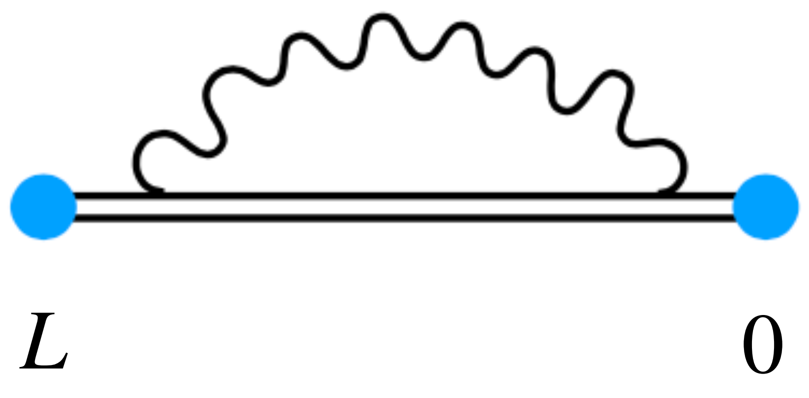


$L_{\sqsubset^z} = 2\eta + b_T$

Collins-Soper kernel



- Exponential decaying signal and complicated renormalization.



$$\sim e^{-\delta m(2\eta + b_T)}$$

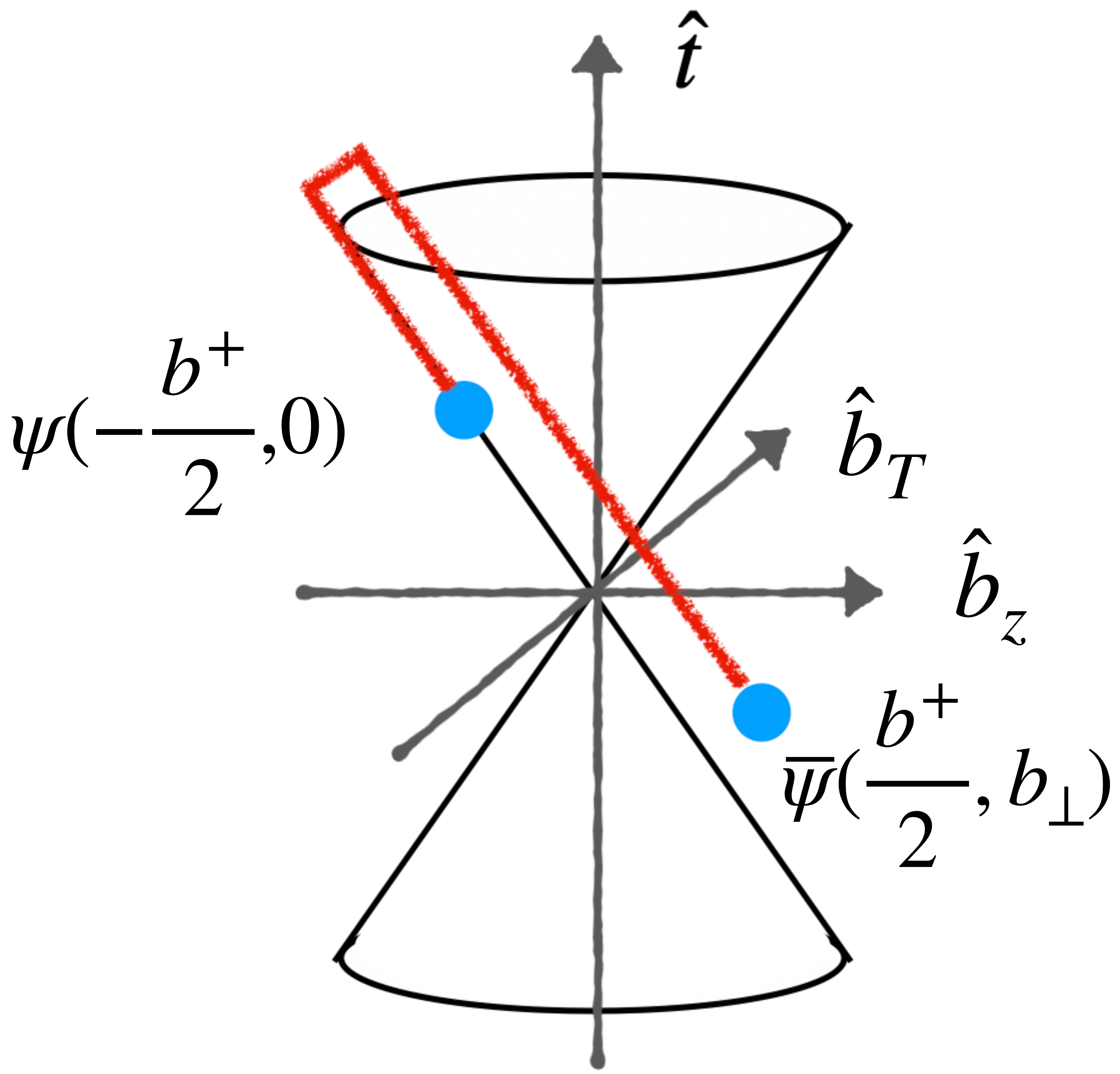
Linear divergence

- Wilson-Clover fermion discretization.
- Physical pion mass.
- Three different lattice spacing.

• A. Avkhadiev, et al., arXiv:2402.06725

Parton distributions in the light-cone gauge

Light-cone TMD

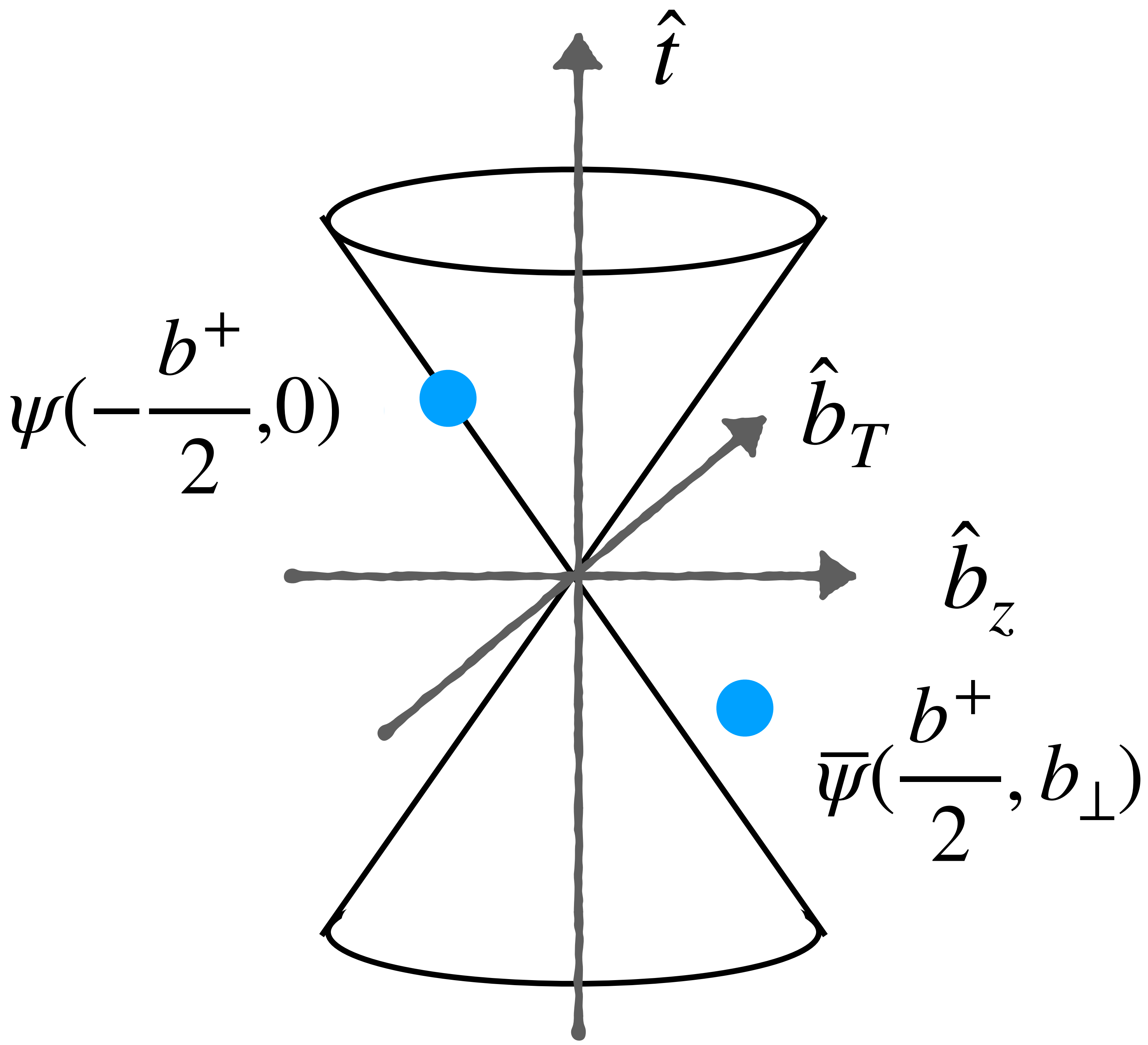


$\overline{\psi}(\frac{b^+}{2},b_\perp)\Gamma W_{\sqsupset^+}\psi(-\frac{b^+}{2},0)$

=

TMD in light gauge

$A^+=0$

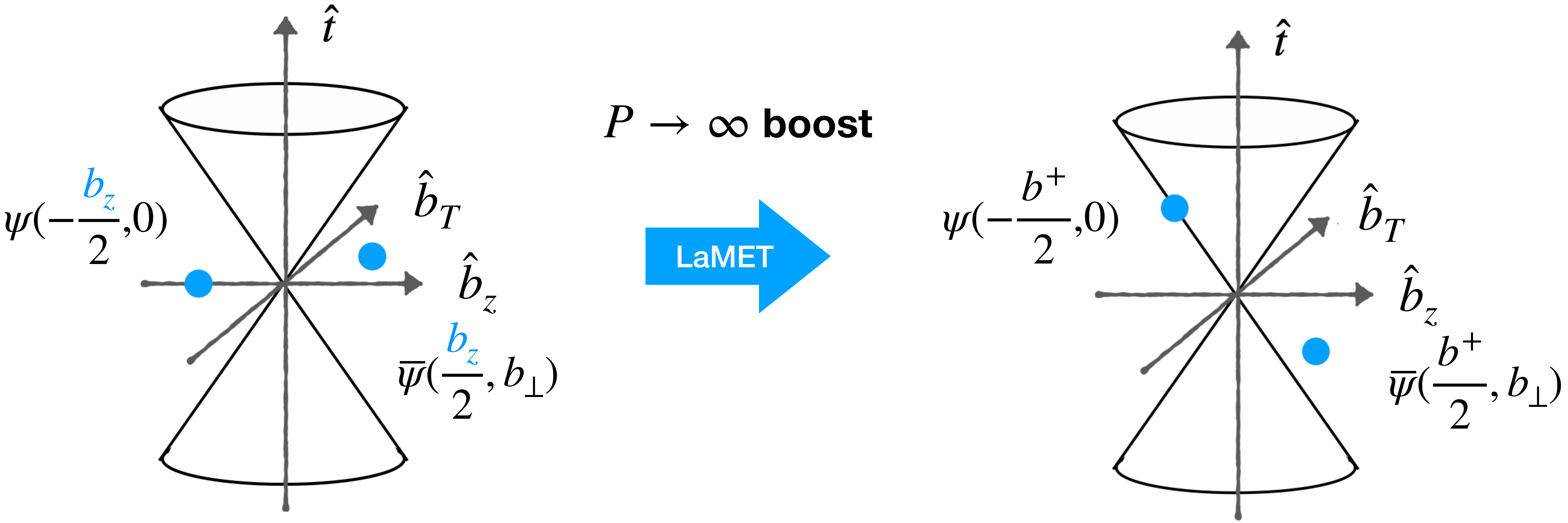


$\overline{\psi}(\frac{b^+}{2},b_\perp)\Gamma\psi(-\frac{b^+}{2},0)\,|_{A^+=0}$

A novel approach: Coulomb-gauge quasi-TMDs

quasi-TMD in physical gauge

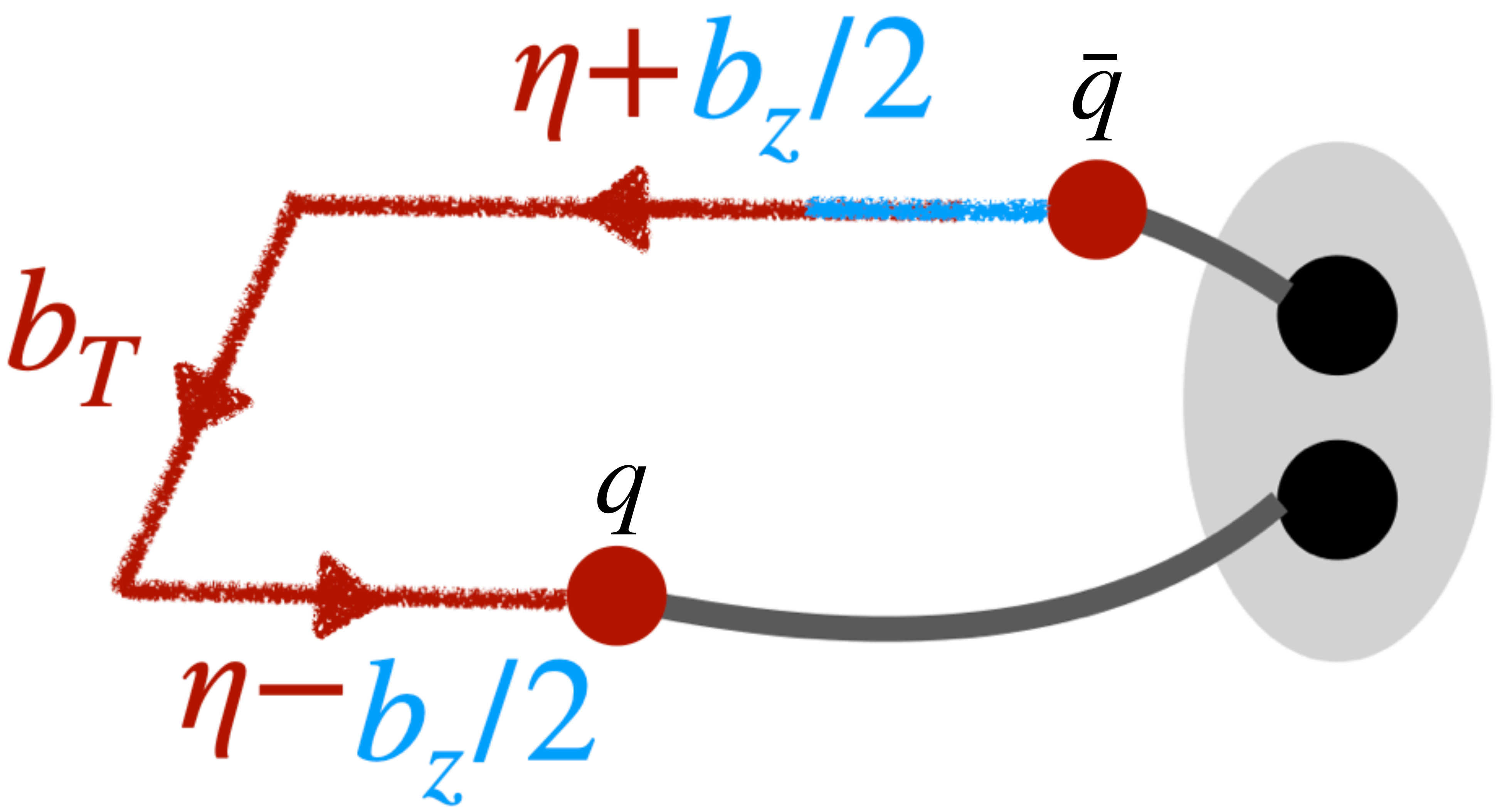
TMD in light gauge $A^+ = 0$



Boost the operator to the light-cone as well as the physical gauge, like $A^z = 0$, $A^0 = 0$, Coulomb gauge $\vec{\nabla} \cdot \vec{A} = 0$, to the light-cone gauge.

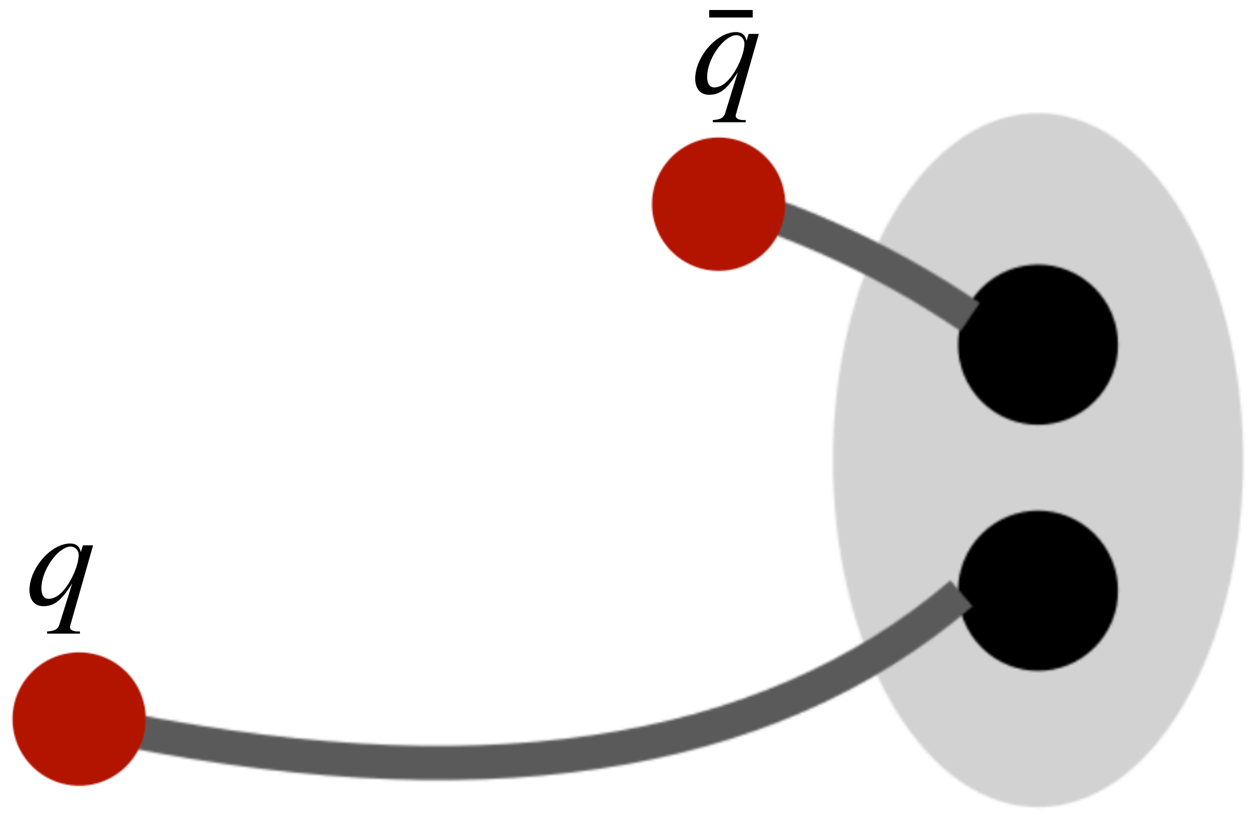
- **XG**, W.-Y. Liu, Y. Zhao, PRD 109 (2024) 9, 094506
- Y. Zhao, arXiv: 2311.01391

A novel approach: Coulomb-gauge quasi-TMDs



$$\langle \Omega | \overline{\psi}(\frac{b_z}{2}, b_\perp)\Gamma W_{\sqsupset z}\psi(-\frac{b_z}{2}, 0) | \pi^+, P_z \rangle$$

Gauge-invariant (GI)
quasi-TMDWF



$$\langle \Omega | \overline{\psi}(\frac{b_z}{2}, b_\perp)\Gamma \psi(-\frac{b_z}{2}, 0) |_{\vec{\nabla}\cdot\vec{A}=0} | \pi^+, P_z \rangle$$

Coulomb gauge (CG)
quasi-TMDWF

A novel approach: Coulomb-gauge quasi-TMDs

see Yong Zhao's talk on Friday

- Same factorization form between GI and CG.
- Both approaching to light-cone in the $P_z \to \infty$ limit but differently: different perturbative corrections and power corrections.

Quasi beam function

Collins-Soper kernel

$$\frac{\tilde{\phi}^{\rm CG}(x, \vec{b}_T, \mu, P_z)}{\sqrt{S_r^{\rm CG}(\vec{b}_T, \mu)}} = C^{\rm CG}(\mu, xP_z) e^{\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2xP_z)^2}{\zeta}} f(x, \vec{b}_T, \mu, \zeta)\{1 + \mathcal{O}[\frac{1}{(xP_z b_T)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP_z)^2}]\}$$

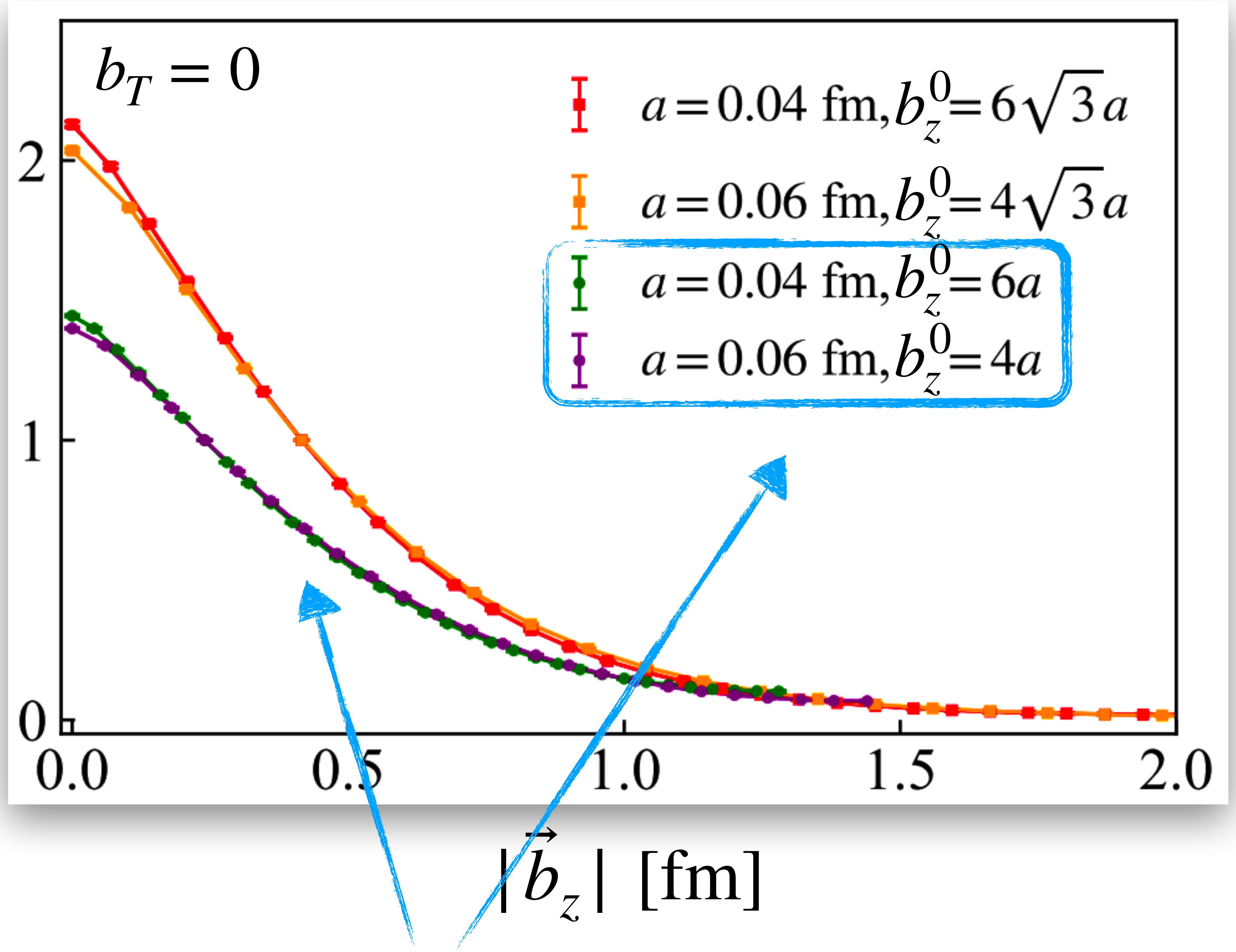
Physical TMD

Reduced soft factor

- Y. Zhao, arXiv: 2311.01391
- Y.-Z. Liu, Y.-S. Su., JHEP 02 (2024) 204

CG quasi-TMDs: simplified renormalization

Renormalized matrix elements



Two lattice spacings: excellent continuum limit!

- No linear divergence: the renormalization is an overall constant.

$$[\bar{\psi}(-\frac{\vec{b}}{2})\Gamma\psi(\frac{\vec{b}}{2})]_B = Z_\psi(a)[\bar{\psi}(-\frac{\vec{b}}{2})\Gamma\psi(\frac{\vec{b}}{2})]_R$$

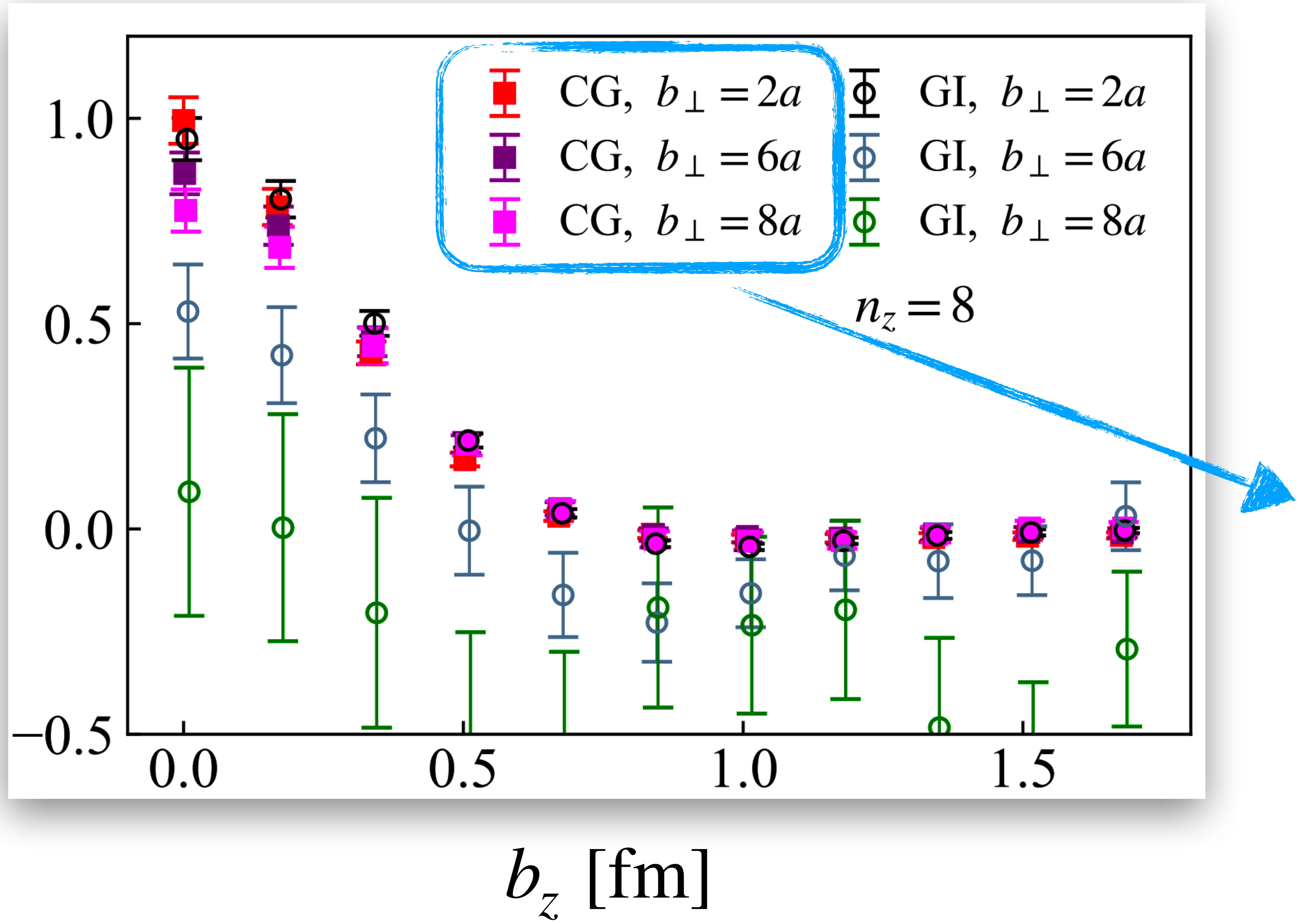
- Matrix elements with any $\vec{b}$ can be used to remove the UV divergence.

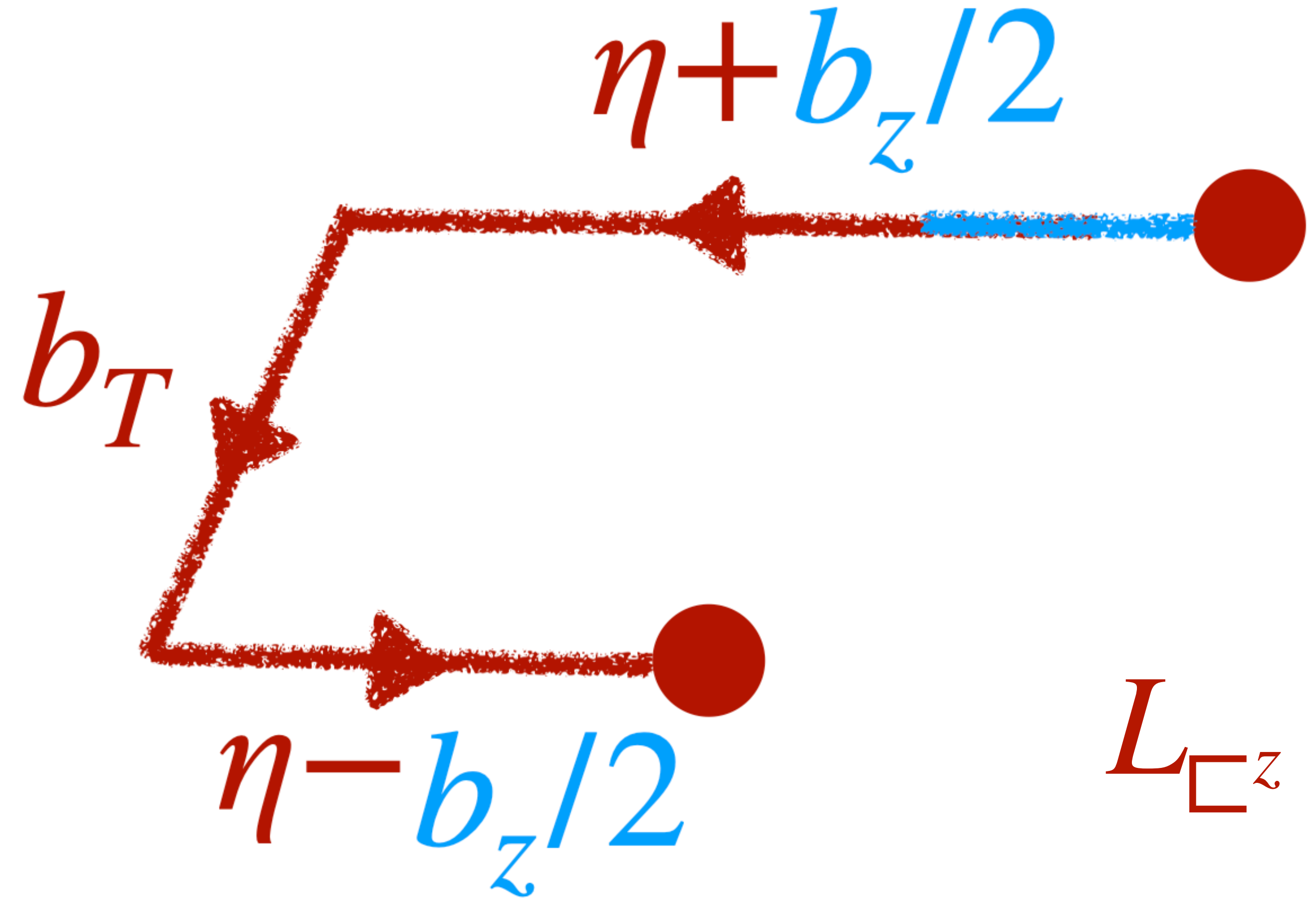
$$\frac{\tilde{h}^B(b_T, b_z, a)}{\tilde{h}^B(b_T^0, b_z^0, a)} = \frac{\tilde{h}^R(b_T, b_z, \mu)}{\tilde{h}^R(b_T^0, b_z^0, \mu)}$$

• XG, W.-Y. Liu, Y. Zhao, PRD 109 (2024) 9, 094506

CG quasi-TMDs: enhanced long-range precision

Renormalized matrix elements





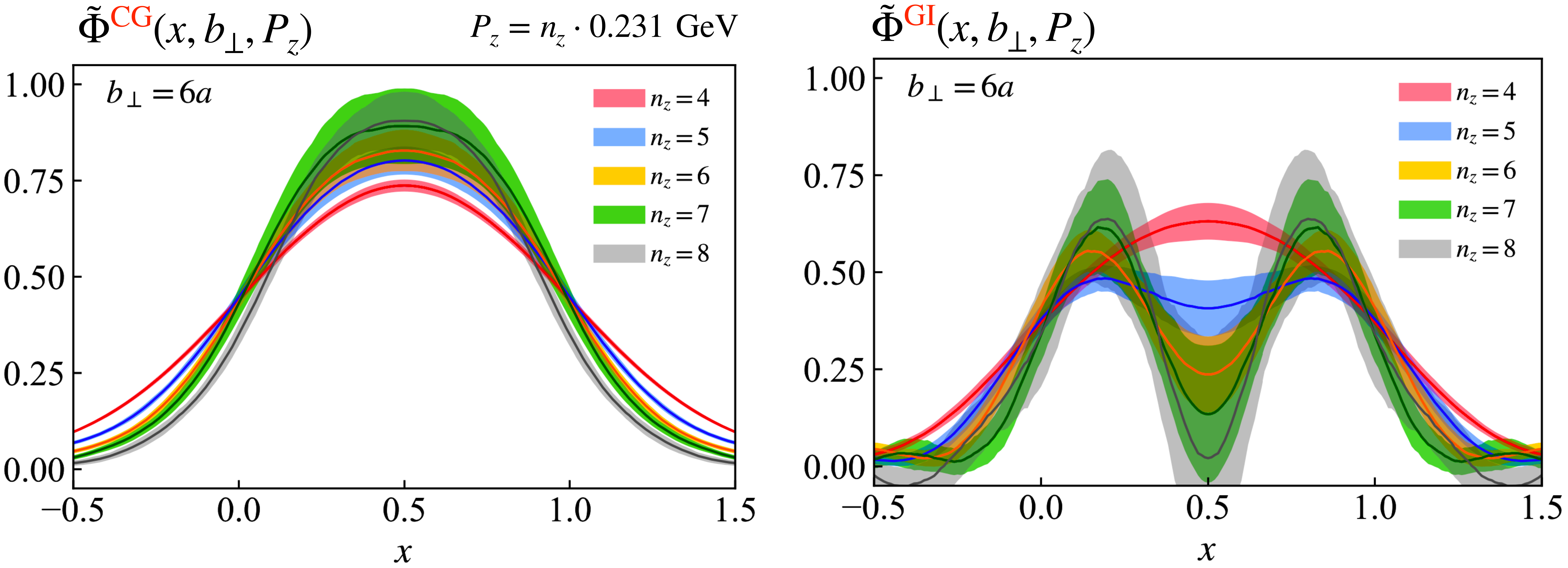
$L_{\sqsubset^z} = 2\eta + b_T$

b_T: $2a \rightarrow 6a \rightarrow 8a$

- CG shows much slower signal decay compared to the GI cases.

• D. Bollweg, **XG**, S. Mukherjee, Y. Zhao, PLB 852 (2024) 138617

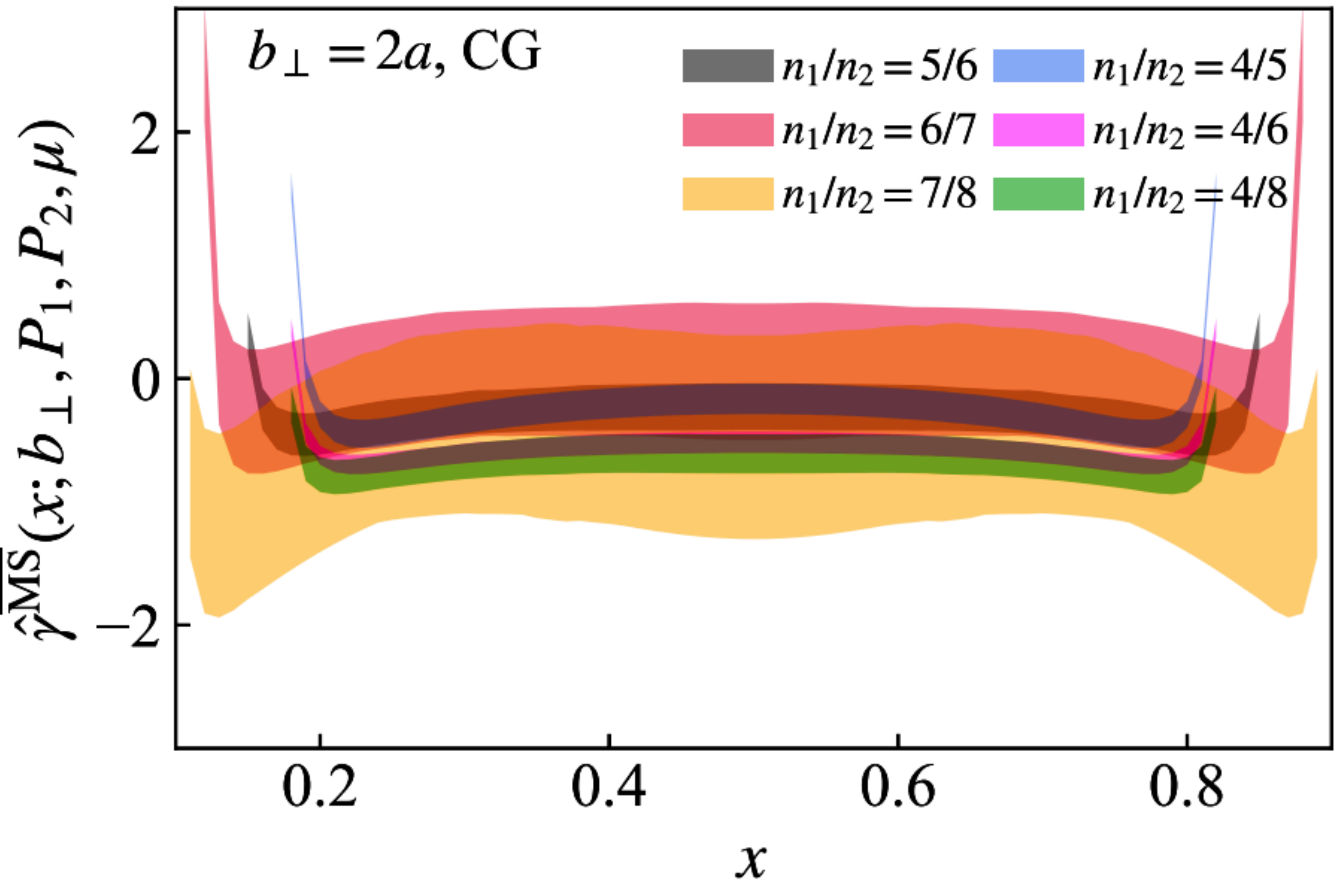
Quasi-TMD wave functions after F.T.

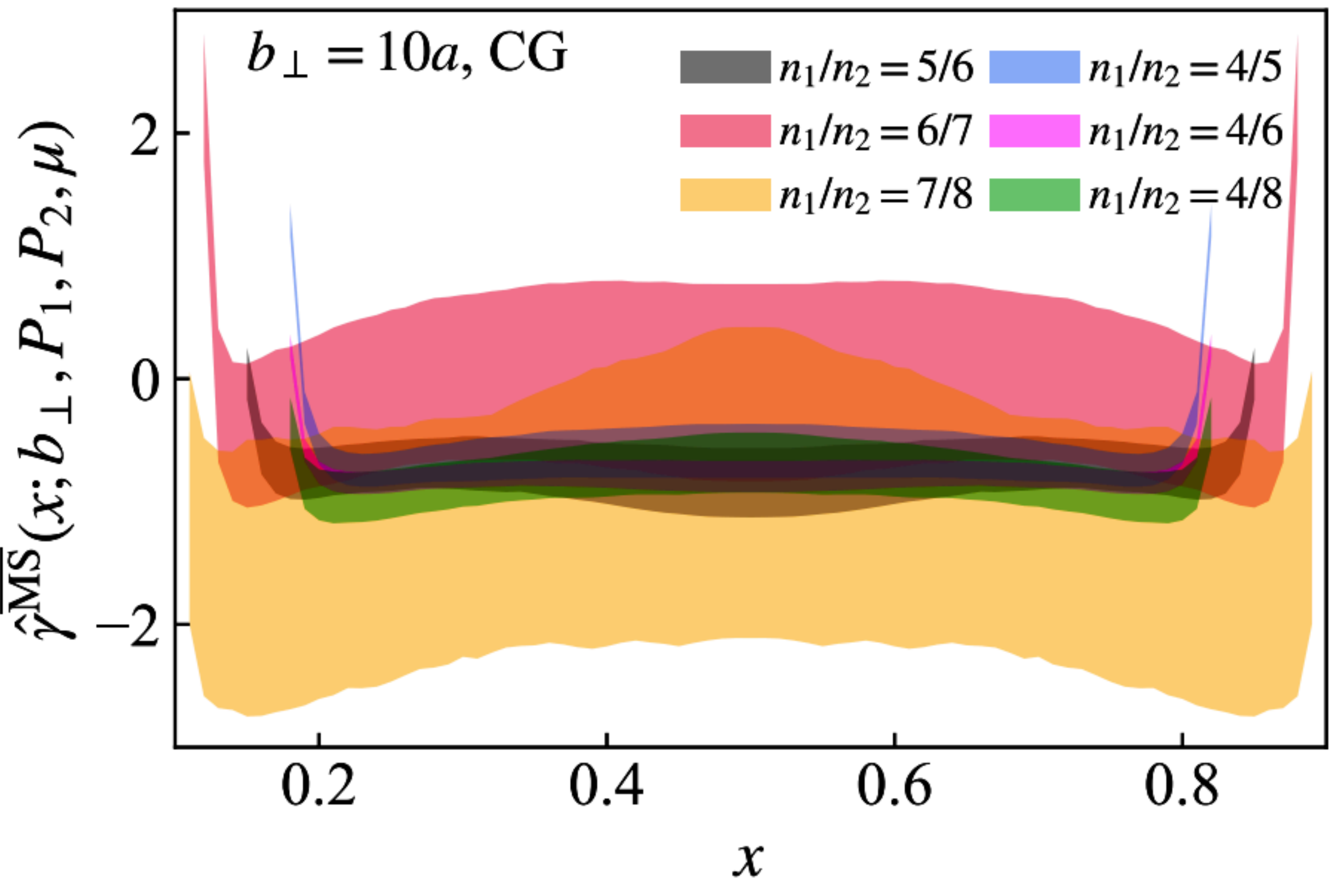


- The CG quasi-TMD wave functions are more stable and show better signal.

The CS kernel from NLL matching

$a = 0.084$ fm, $\mathrm{P_z = n_z \cdot 0.23}$ GeV

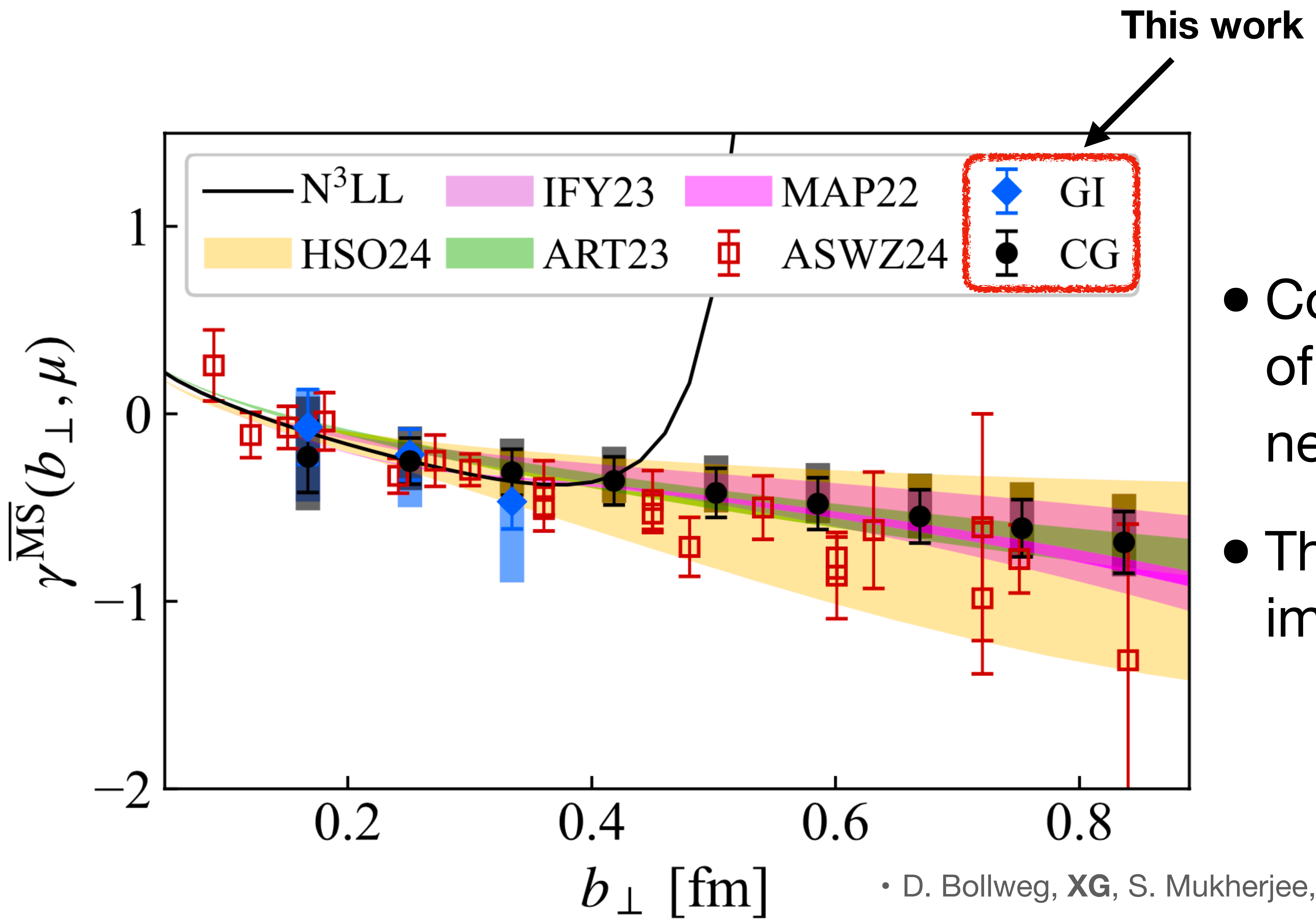




- Small $b_T \sim 0.1$ fm: visible P_z dependence.
- Non-negligible power corrections.

- Large b_T: no x and P_z dependence.
- Perturbative factorization work well!

The Collins-Soper kernel



- Consistent with recent parametrization of experimental data and favors a near-linear dependence of $b_\perp$.
- The novel CG quasi-TMDs greatly improve the efficiency of calculations.

• D. Bollweg, **XG**, S. Mukherjee, Y. Zhao, PLB 852 (2024) 138617

- Chiral symmetry preserved discretization: Domain wall fermion.
- $64^3 \times 128$, a = 0.084 fm, physical quark masses.

Summary

- The TMDs can be extracted from quasi-TMD correlators. The novel CG quasi-TMDs have the advantages of the simplified renormalization and enhanced long-range precision.
- We extracted the non-perturbative CS kernel from the quasi-TMD wave functions in the CG which appears to be consistent with recent parametrization of experimental data.
- The CG methods could have broader use in the future particularly in the non-perturbative regime of TMD physics, including the gluons and the Wigner distributions.

Thanks for your attention!