

Nucleon Transversity and Tensor Charge from Di-hadron Production

(Andreas Metz, Temple University)

Part 1 Definition, interpretation and evolution of di-hadron fragmentation functions (DiFFs)

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Part 2 Simultaneous global analysis of DiFFs, transversity PDFs, and tensor charges

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supported by the 

Motivation

- In spin physics, DiFFs relevant for extraction of transversity h_1^q
- Access to transversity using chiral-odd spin-dependent FFs
 - Single-hadron fragmentation (Collins effect) (Collins, 1992)

$$h_1^q \otimes H_1^{\perp h/q}$$

correlation btw transverse quark spin and transverse momentum of hadron,
TMD factorization

- Single-hadron fragmentation using collinear twist-3 factorization
(Kang, Yuan, Zhou, 2010 / Metz, Pitonyak, 2012)
- Di-hadron fragmentation (Collins, Heppelmann, Ladinsky, 1993)

$$h_1^q \otimes H_1^{\triangleleft h_1 h_2/q}$$

correlation btw transverse quark spin and relative transverse momentum of (h_1, h_2) ,
collinear twist-2 factorization

- Previous work on di-hadron production related to spin physics almost exclusively by Pavia Group
- Extraction of h_1^q from global analysis of di-hadron data (Radici, Bacchetta, 2018)
 - tensor charge

$$\delta q = \int_0^1 dx (h_1^q(x) - h_1^{\bar{q}}(x)) \quad g_T = \delta u - \delta d$$

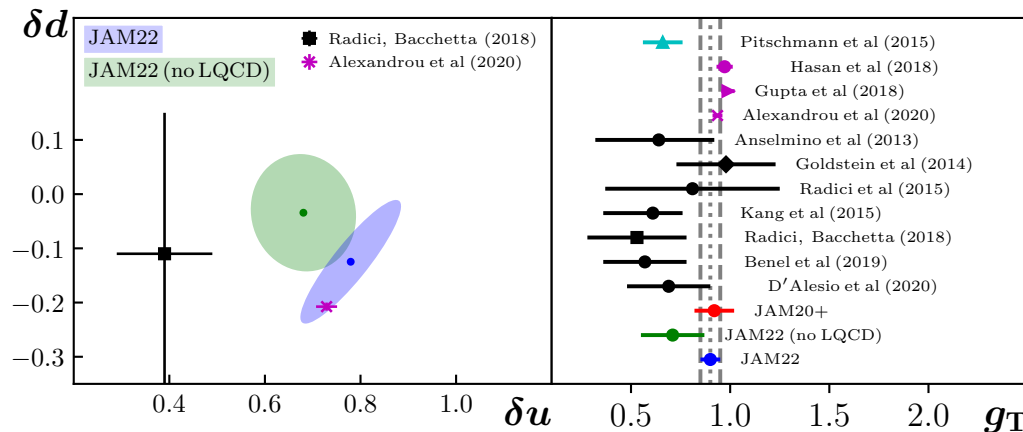


figure modified from
arXiv:2205.00999
(JAM-3D)

for δu , some tension between di-hadron channel on the one hand, and single-hadron channel and lattice QCD on the other

- Independent numerical analysis of di-hadron channel well motivated
- We also revisited the definition, interpretation and evolution of DiFFs

Lessons from Single-Hadron Fragmentation Functions

- Process and frames

$$q(k) \rightarrow h(P_h) + X \qquad P_h^- = z k^- \text{ large}$$

$$\text{hadron frame: } \vec{P}_{hT} = 0 \quad \vec{k}_T \neq 0$$

$$\text{parton frame: } \vec{P}_{h\perp} \neq 0 \quad \vec{k}_\perp = 0 \qquad (\vec{P}_{h\perp} = -z \vec{k}_T)$$

- Definition and interpretation

$$\begin{aligned} D_1^{h/q}(z, z^2 \vec{k}_T^2) &= \frac{1}{4z} \int \frac{d\xi^+ d^2 \vec{\xi}_T}{(2\pi)^3} e^{ik \cdot \xi} \text{Tr} \left[\langle 0 | \psi_q(\xi) | h, X \rangle \langle h, X | \bar{\psi}_q(0) | 0 \rangle \gamma^- \right]_{\xi^- = 0} \\ &= D_1^{h/q}(z, \vec{P}_{h\perp}^2) \end{aligned}$$

- $D_1^{h/q}(z, \vec{P}_{h\perp}^2)$ is number density (see, e.g., Collins, *Foundations of Perturbative QCD*)
- $D_1^{h/q}(z, \vec{P}_{h\perp}^2) dz d^2 \vec{P}_{h\perp}$ is number of hadrons h in $[z, z+dz]$, $[\vec{P}_{h\perp}, \vec{P}_{h\perp} + d^2 \vec{P}_{h\perp}]$
- factor $1/4z$ is crucial for interpretation as number density

- Collinear FF

$$D_1^{h/q}(z) = \int d^2 \vec{P}_{h\perp} D_1^{h/q}(z, \vec{P}_{h\perp}^2)$$

- Number of hadrons in quark q

$$\sum_h \int dz D_1^{h/q}(z) = \langle N^q \rangle$$

- Momentum sum rule (Collins, Soper, 1981 / Meissner, Metz, Pitonyak, 2010)

$$\sum_h \int dz z D_1^{h/q}(z) = 1$$

- Leading-order cross section for $e^-e^+ \rightarrow hX$

$$\frac{d\sigma}{dz} = \sum_{q,\bar{q}} \hat{\sigma}^q D_1^{h/q}(z) \quad \text{with} \quad \hat{\sigma}^q = \hat{\sigma}^{\bar{q}} = \hat{\sigma}(e^-e^+ \rightarrow \gamma^{(*)} \rightarrow q\bar{q}) = \frac{4\pi e_q^2 \alpha_{\text{em}}^2 N_c}{3Q^2}$$

- Recent work on sum rules for FFs (Collins, Rogers, 2023)
 - does not put into question the definition of FFs

Definition and Interpretation of DiFFs

- Process and frames

$$q(k) \rightarrow h_1(P_1) + h_2(P_2) + X \quad P_h^- = z k^- \text{ large}$$

$$P_h = P_1 + P_2 \quad R = \frac{1}{2} (P_1 - P_2) \quad P_{1,2}^- = z_{1,2} k^- \quad z = z_1 + z_2$$

$$\text{hadron frame: } \vec{P}_{hT} = 0 \quad \vec{k}_T \neq 0$$

$$\text{parton frame: } \vec{P}_{h\perp} \neq 0 \quad \vec{k}_\perp = 0 \quad (\vec{P}_{h\perp} = -z \vec{k}_T)$$

- Definition and interpretation

$$\frac{1}{64\pi^3 z_1 z_2} \int \frac{d\xi^+ d^2 \vec{\xi}_T}{(2\pi)^3} e^{ik \cdot \xi} \text{Tr} \left[\langle 0 | \psi_q(\xi) | h_1, h_2, X \rangle \langle h_1, h_2, X | \bar{\psi}_q(0) | 0 \rangle \gamma^- \right]_{\xi^- = 0}$$

$$= D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp}) \equiv D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp} \cdot \vec{P}_{2\perp})$$

- $D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})$ is number density for hadron pairs (h_1, h_2)
- factor $1/64\pi^3 z_1 z_2$ is crucial for interpretation as number density
- previously defined/used DiFFs in spin physics have no number density interpretation (starting from pioneering work by Bianconi, Boffi, Jakob, Radici, 1999)

- Collinear DiFFs (see also, Majumder, Wang, 2004)

$$D_1^{h_1 h_2/q}(z_1, z_2) = \int d^2 \vec{P}_{1\perp} d^2 \vec{P}_{2\perp} D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})$$

- Number of hadron pairs in quark q

$$\sum_{h_1, h_2} \int dz_1 dz_2 D_1^{h_1 h_2/q}(z_1, z_2) = \langle N^q (N^q - 1) \rangle$$

- Momentum sum rule

$$\sum_{h_1} \int_0^{1-z_2} dz_1 \int d^2 \vec{P}_{1\perp} z_1 D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp}) = (1 - z_2) D_1^{h_2/q}(z_2, \vec{P}_{2\perp}^2)$$

- sum rule after $\int d^2 \vec{P}_{2\perp}$ already in previous literature (de Florian, Vanni, 2003 / ...)
- similar (integrated) sum rule for double-parton distributions (Gaunt, Stirling, 2009 / ...)

- Leading-order cross section for $e^- e^+ \rightarrow (h_1 h_2) X$

$$\frac{d\sigma}{dz_1 dz_2} = \sum_{q, \bar{q}} \hat{\sigma}^q D_1^{h_1 h_2/q}(z_1, z_2) \quad \text{with} \quad \hat{\sigma}^q = \frac{4\pi e_q^2 \alpha_{\text{em}}^2 N_c}{3Q^2}$$

Definition and Interpretation of Extended DiFFs

- More on kinematics
 - invariant mass of di-hadron pair, and alternative variable for longitudinal momentum

$$M_h^2 = P_h^2 = (P_1 + P_2)^2 \quad \zeta = \frac{z_1 - z_2}{z}$$

- momenta P_1 and P_2 in hadron frame ($\vec{P}_{hT} = 0$)

$$P_1 = \left(\frac{M_1^2 + \vec{R}_T^2}{(1 + \zeta)P_h^-}, \frac{1 + \zeta}{2}P_h^-, \vec{R}_T \right) \quad P_2 = \left(\frac{M_2^2 + \vec{R}_T^2}{(1 - \zeta)P_h^-}, \frac{1 - \zeta}{2}P_h^-, -\vec{R}_T \right)$$

- important relation

$$\vec{R}_T^2 = \frac{1 - \zeta^2}{4}M_h^2 - \frac{1 - \zeta}{2}M_1^2 - \frac{1 + \zeta}{2}M_2^2$$

- Extended DiFFs (extDiFFs)
 - in contrast to $D_1^{h_1 h_2/q}(z_1, z_2)$, extDiFFs (also) depend on M_h (or $\vec{R}_T$)
 - extDiFFs appear in transversity-related observables

- Number density interpretation for (properly defined) extDiFFs
 - when changing variables, include **Jacobian** of transformation in definition of DiFFs
 - example

$$D_1^{h_1 h_2/q}(z, \zeta, \vec{k}_T, \vec{R}_T) = \frac{z^3}{2} D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})$$

- further extDiFFs

$$\begin{aligned} D_1^{h_1 h_2/q}(z, M_h) &= \int d\zeta D_1^{h_1 h_2/q}(z, \zeta, M_h) \\ &= \int d\zeta \frac{\pi}{2} M_h (1 - \zeta^2) D_1^{h_1 h_2/q}(z, \zeta, \vec{R}_T^2) \end{aligned}$$

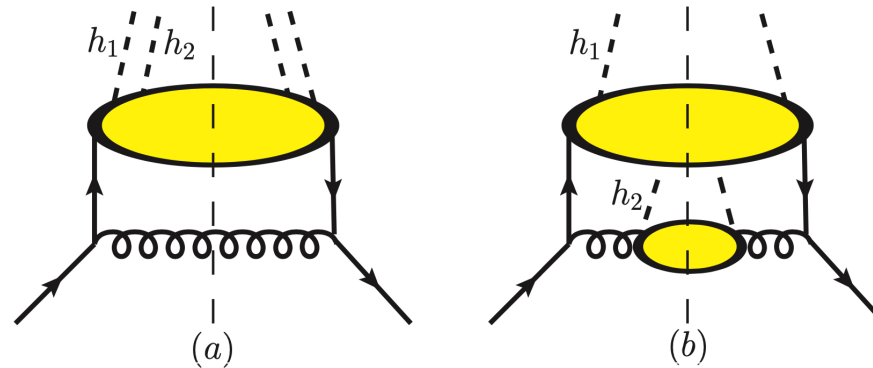
- experimental information on $D_1^{h_1 h_2/q}(z, M_h)$ from Belle (Seidl et al, 2017)

- Leading-order cross section for $e^- e^+ \rightarrow (h_1 h_2) X$ (example)

$$\frac{d\sigma}{dz dM_h} = \sum_{q, \bar{q}} \hat{\sigma}^q D_1^{h_1 h_2/q}(z, M_h) \quad \text{with} \quad \hat{\sigma}^q = \frac{4\pi e_q^2 \alpha_{\text{em}}^2 N_c}{3Q^2}$$

Evolution of DiFFs

- Homogeneous and in-homogeneous contributions to evolution (sample diagrams)



- Evolution of extDiFFs (quark non-singlet)

$$\frac{\partial}{\partial \ln \mu^2} D_1^{h_1 h_2/q}(z, \zeta, \vec{R}_T^2; \mu) = \int_z^1 \frac{dw}{w} D_1^{h_1 h_2/q}\left(\frac{z}{w}, \zeta, \vec{R}_T^2; \mu\right) P_{q \rightarrow q}(w)$$

- evolution of extDiFFs only contains homogeneous term (standard DGLAP)

(see also Ceccopieri, Bacchetta, Radici, 2007)

- corresponding evolution equation for $H_1^{\leq h_1 h_2/q}(z, \zeta, \vec{R}_T^2; \mu)$

- Upon $\int d^2 \vec{R}_T$, we recover evolution of $D_1^{h_1 h_2/q}(z_1, z_2; \mu)$ where in-homogeneous term contributes as well (de Florian, Vanni, 2003 / ...)

Simultaneous Extraction of DiFFs and Transversity PDFs

- Main observables/input for DiFFs

- unpolarized cross section in $e^-e^+ \rightarrow (h_1 h_2) X$ (data from Belle)

$$\frac{d\sigma}{dz dM_h} = \frac{4\pi\alpha_{\text{em}}^2 N_c}{3Q^2} \sum_{q,\bar{q}} e_q^2 D_1^q(z, M_h)$$

- PYTHIA event generator

- Artru-Collins asymmetry in $e^-e^+ \rightarrow (h_1 h_2)(\bar{h}_1 \bar{h}_2) X$ (data from Belle)

$$A^{e^-e^+}(z, M_h, \bar{z}, \bar{M}_h) = \frac{\sin^2 \theta \sum_{q,\bar{q}} e_q^2 H_1^{\triangleleft,q}(z, M_h) H_1^{\triangleleft,\bar{q}}(\bar{z}, \bar{M}_h)}{(1 + \cos^2 \theta) \sum_{q,\bar{q}} e_q^2 D_1^q(z, M_h) D_1^{\bar{q}}(\bar{z}, \bar{M}_h)}$$

- Further constraints on DiFFs from transverse single-spin asymmetries in semi-inclusive DIS and proton-proton collisions (simultaneous analysis)

- Observables for transversity PDFs
 - transverse SSA in SIDIS (data from HERMES and COMPASS)

$$A_{UT}^{\text{SIDIS}} = c(y) \frac{\sum_{q,\bar{q}} e_q^2 \textcolor{red}{h}_1^q(\textcolor{red}{x}) H_1^{\triangleleft,q}(z, M_h)}{\sum_{q,\bar{q}} e_q^2 f_1^q(x) D_1^q(z, M_h)}$$

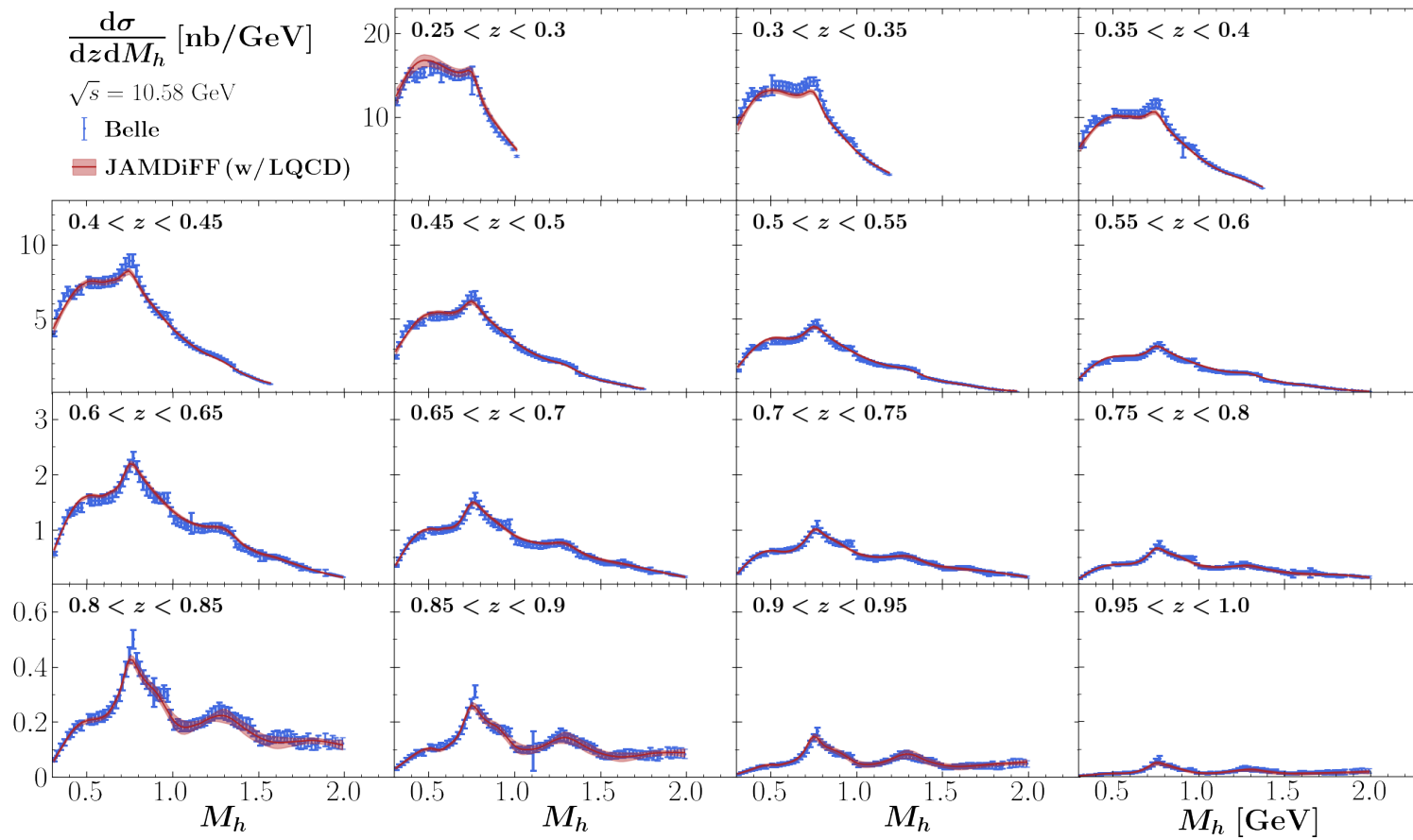
- transverse SSA in pp collisions (data from STAR)

$$A_{UT}^{pp} = \frac{\mathcal{H}(M_h, P_{hT}, \eta)}{\mathcal{D}(M_h, P_{hT}, \eta)}$$

$$\mathcal{H} = 2P_{hT} \sum_i \sum_{a,b,c} \int_{x_a^{\min}}^1 dx_a \int_{x_b^{\min}}^1 \frac{dx_b}{z} \textcolor{red}{h}_1^a(\textcolor{red}{x}_a) f_1^b(x_b) \frac{d\Delta\hat{\sigma}_{a\uparrow b\rightarrow c\uparrow}}{d\hat{t}} H_1^{\triangleleft,c}(z, M_h)$$

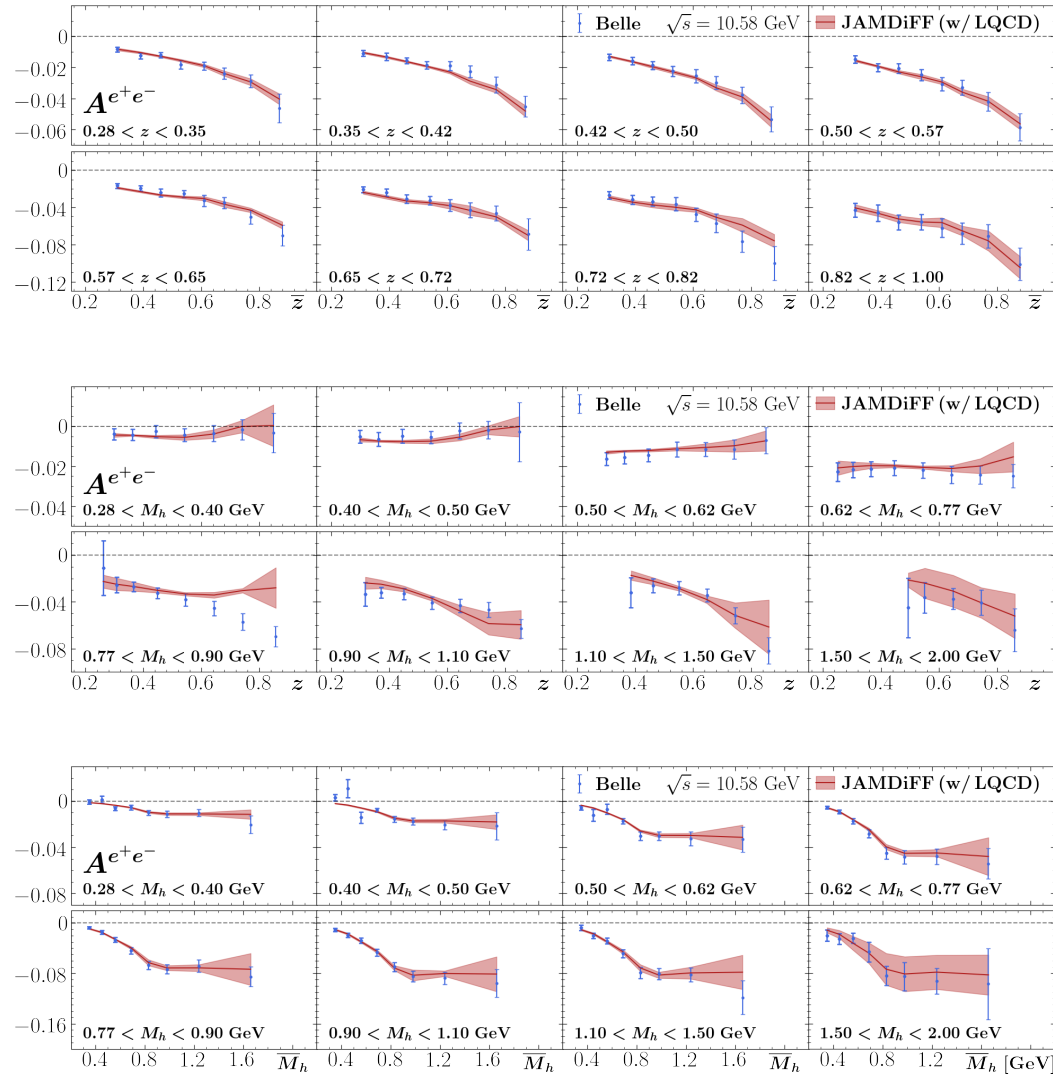
$$\mathcal{D} = 2P_{hT} \sum_i \sum_{a,b,c} \int_{x_a^{\min}}^1 dx_a \int_{x_b^{\min}}^1 \frac{dx_b}{z} f_1^a(x_a) f_1^b(x_b) \frac{d\hat{\sigma}_{a\uparrow b\rightarrow c\uparrow}}{d\hat{t}} D_1^c(z, M_h)$$

- Quality of fit: unpolarized cross section for $e^-e^+ \rightarrow (h_1 h_2) X$



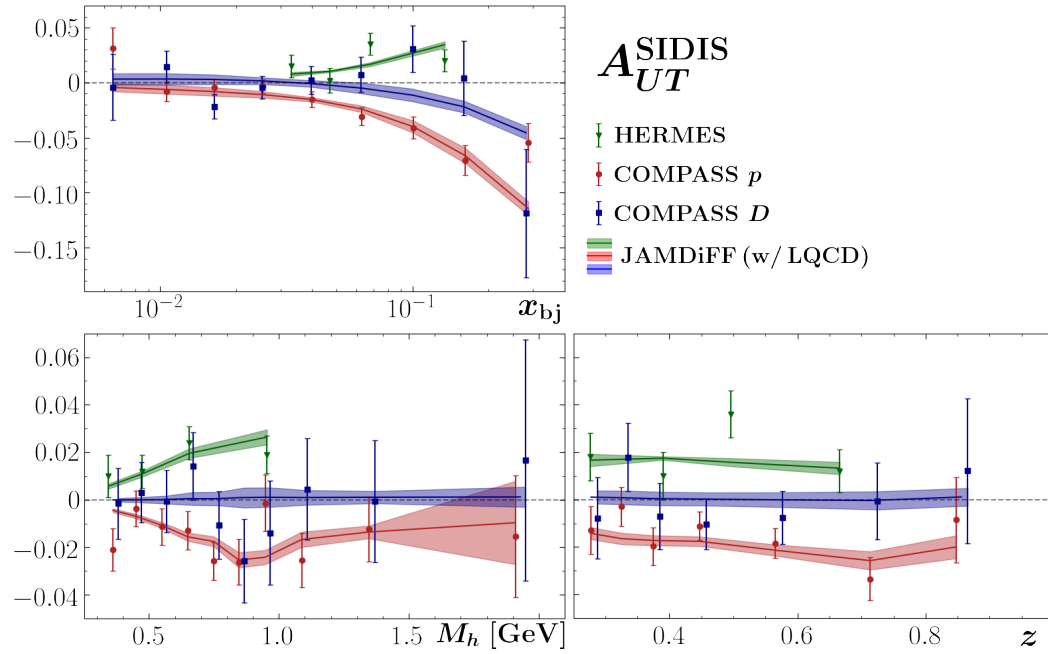
(data from Belle, 2017)

- Quality of fit: Artru-Collins asymmetry

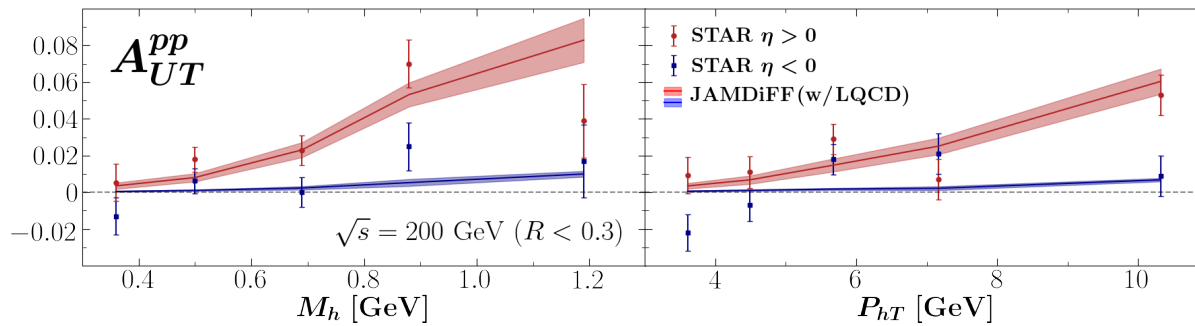


(data from Belle, 2011)

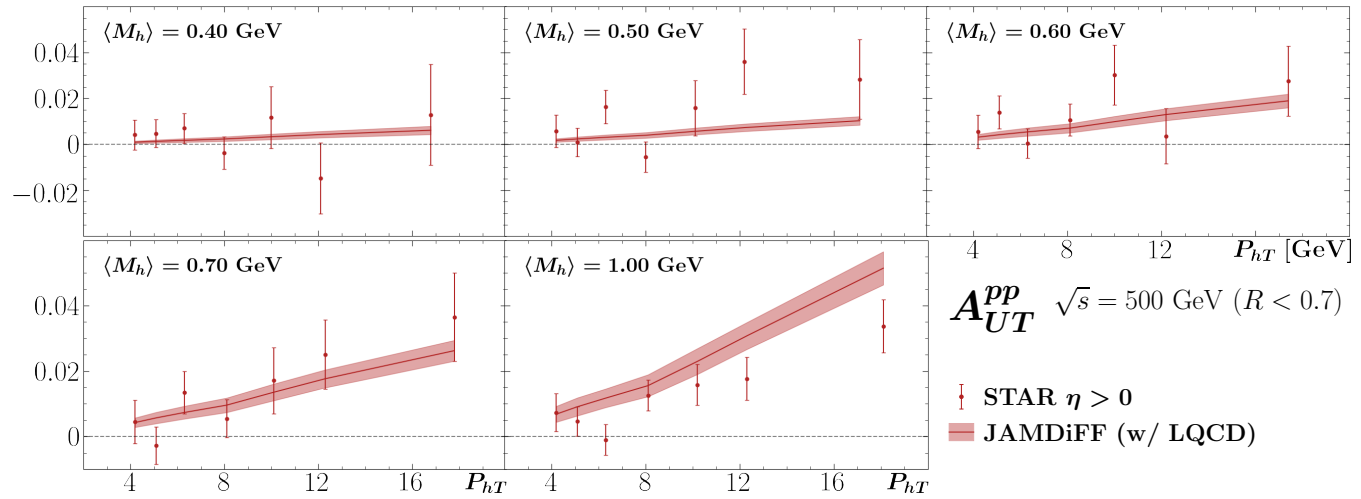
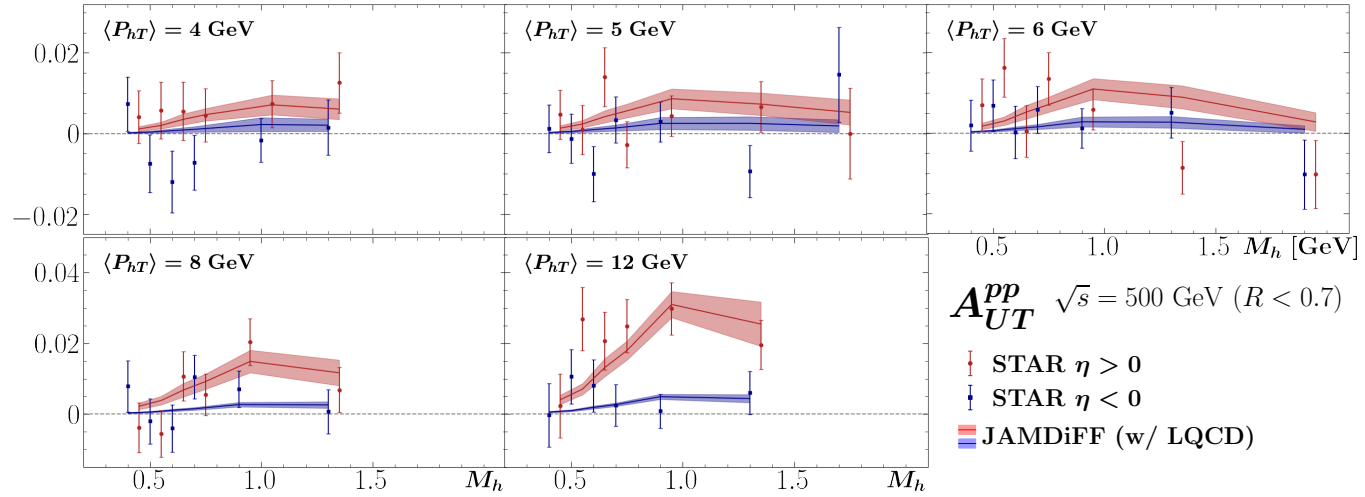
- Quality of fit: A_{UT}^{SIDIS} (data from HERMES, 2008 / COMPASS, 2023)



- Quality of fit: A_{UT}^{pp} (sample data for $\sqrt{s} = 200$ GeV from STAR, 2015)

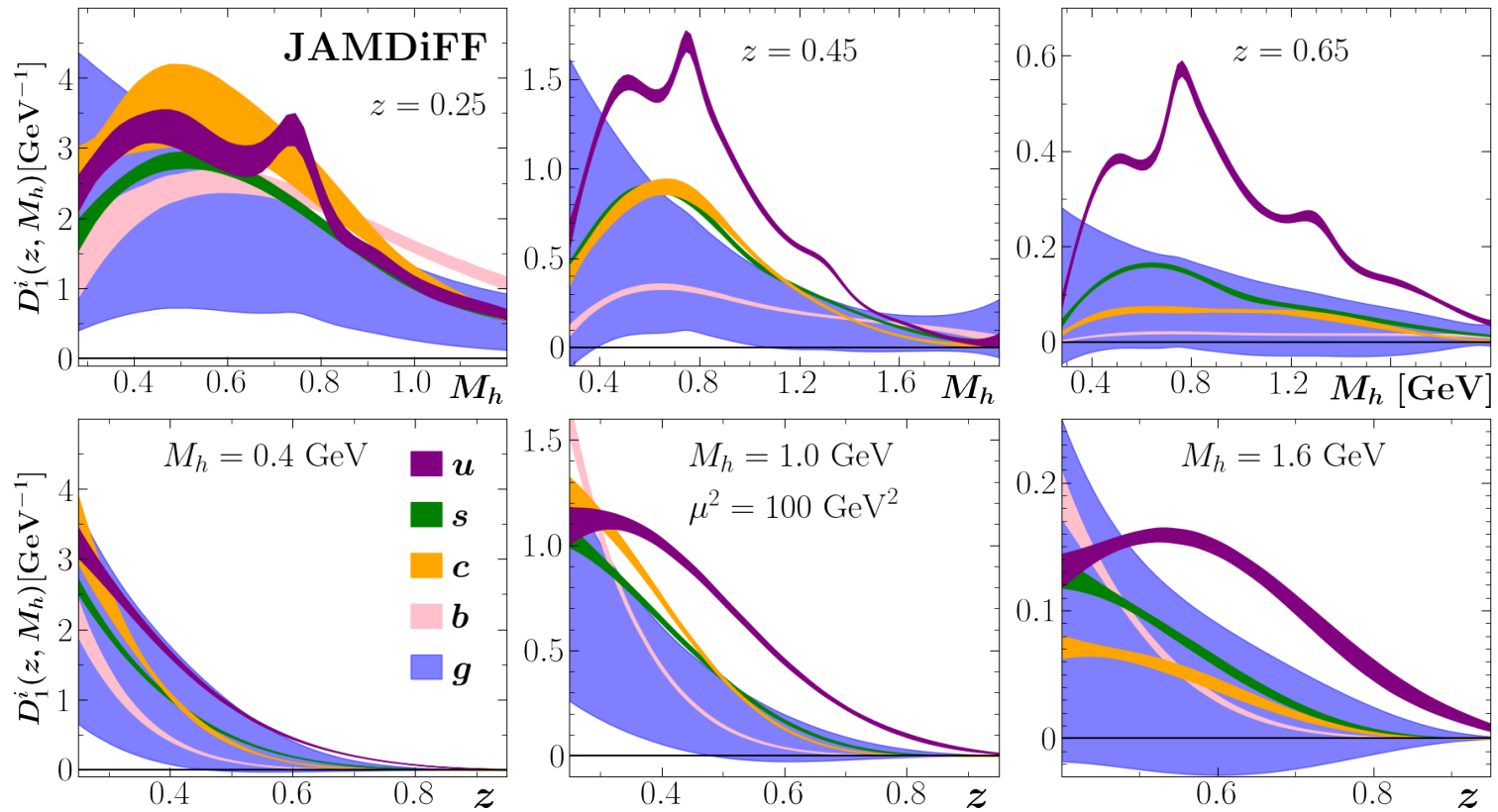


- Quality of fit: A_{UT}^{pp} (sample data for $\sqrt{s} = 500$ GeV from STAR, 2017)



- Extracted DiFFs $D_1^{\pi^+\pi^-/a}$ ($a = q, g$)

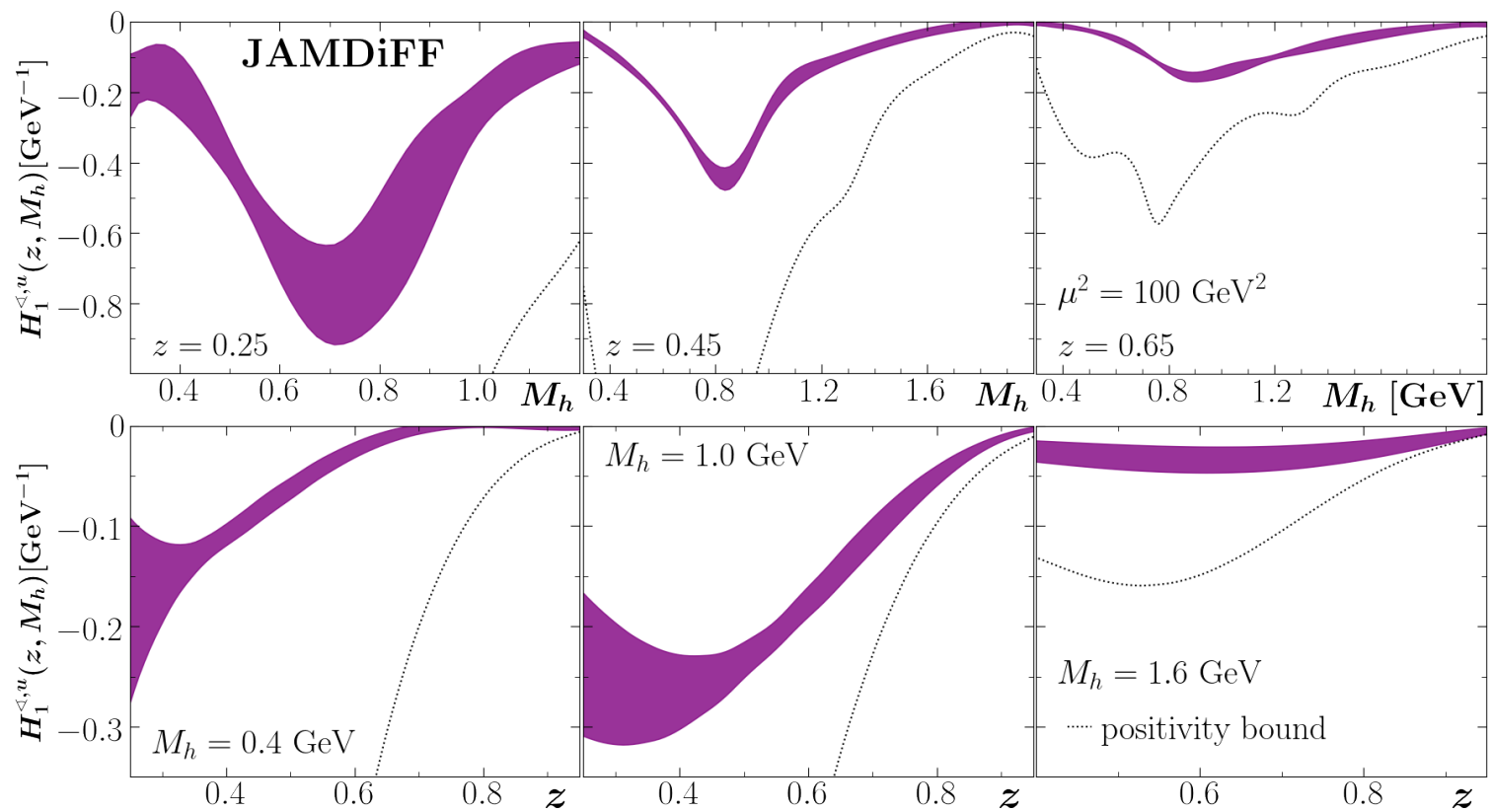
$$D_1^u = D_1^d = D_1^{\bar{u}} = D_1^{\bar{d}} \quad D_1^s = D_1^{\bar{s}} \quad D_1^c = D_1^{\bar{c}} \quad D_1^b = D_1^{\bar{b}} \quad D_1^g$$



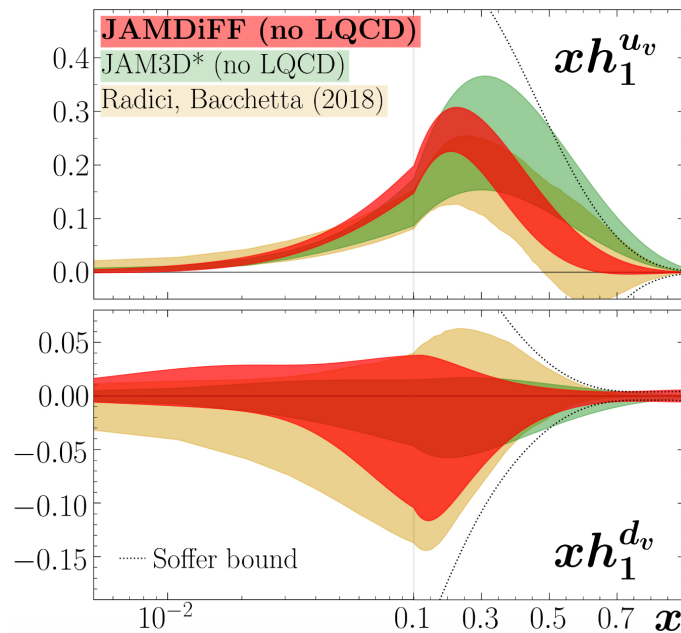
- with our new definition of DiFFs we can compute $\langle M_h \rangle$ and $\langle z \rangle$

- Extracted DiFF $H_1^{\triangleleft \pi^+ \pi^- / u}$

$$H_1^{\triangleleft u} = -H_1^{\triangleleft d} = -H_1^{\triangleleft \bar{u}} = H_1^{\triangleleft \bar{d}} \quad H_1^{\triangleleft q} = 0 \text{ for } q = s, c, b$$



- Extracted transversity PDFs



- fit of $h_1^{u_v}$, $h_1^{d_v}$, $h_1^{\bar{u}} = -h_1^{\bar{d}}$
large- N_c constraint for antiquarks (Pobylitsa, 2003)

- Soffer bound (Soffer, 1995)

$$h_1^q(x) \leq \frac{1}{2} |f_1^q(x) + g_1^q(x)|$$

- small- x constraint (Kovchegov, Sievert, 2019)

$$h_1^q \xrightarrow{x \rightarrow 0} x^{\alpha_q} \quad \alpha_q \approx 0.17 \pm 0.085$$

- JAM3D* = JAM3D-22 (no LQCD)

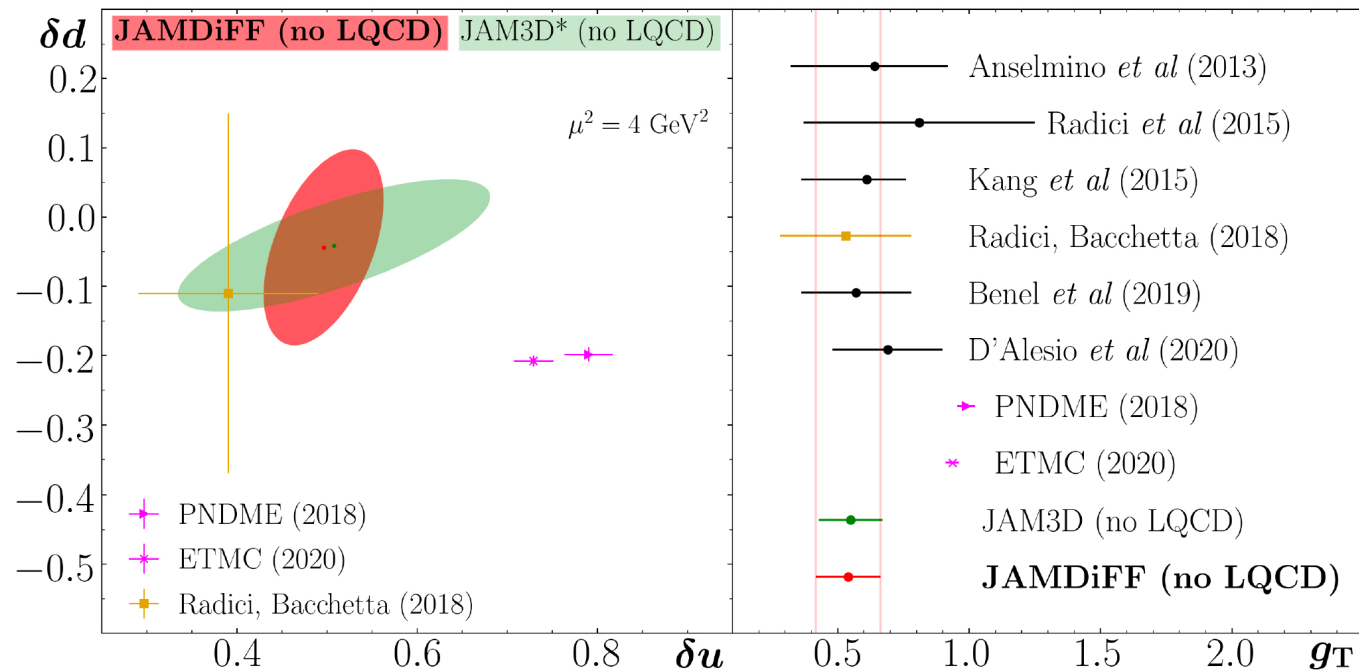
+ antiquarks with $h_1^{\bar{u}} = -h_1^{\bar{d}}$

+ small- x constraint

- agreement between all three analyses
within errors

Extraction of Tensor Charges

- Tensor charges and comparison with results from LQCD



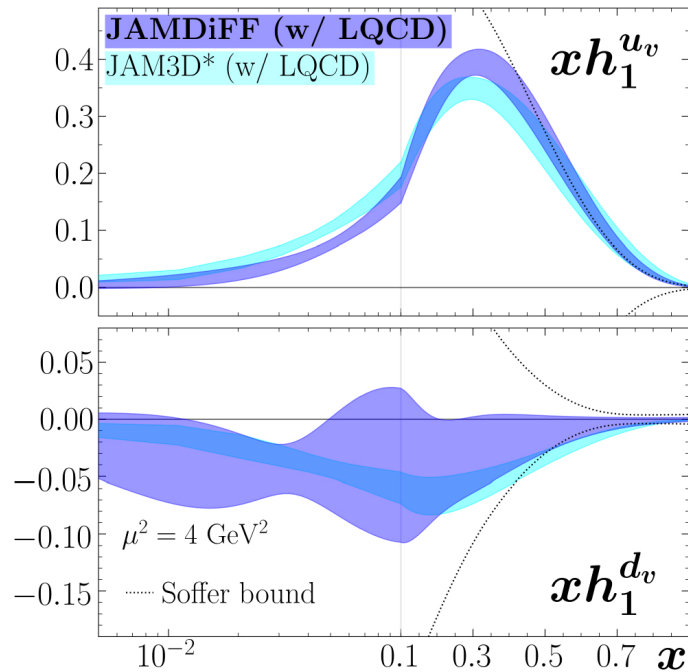
- for δu , we find discrepancy with ETMC and with PNDME
- what happens if LQCD results for δu and δd are included in the fit?

- Quality of fit: χ^2 values for various data sets

Experiment	N_{dat}	χ^2_{red}	
		w/ LQCD	no LQCD
Belle (cross section) [63]	1094	1.01	1.01
Belle (Artru-Collins) [92]	183	0.74	0.73
HERMES [72]	12	1.13	1.10
COMPASS (p) [71]	26	1.24	0.75
COMPASS (D) [71]	26	0.78	0.76
STAR (2015) [94]	24	1.47	1.67
STAR (2018) [64]	106	1.20	1.04
ETMC δu [28]	1	0.71	—
ETMC δd [28]	1	1.02	—
PNDME δu [25]	1	8.68	—
PNDME δd [25]	1	0.04	—
Total χ^2_{red} (N_{dat})		1.01 (1475)	0.98 (1471)

- successful fit after inclusion of LQCD tensor charges

- Extracted transversity PDFs (w/ LQCD)



- Soffer bound (Soffer, 1995)

$$h_1^q(x) \leq \frac{1}{2} |f_1^q(x) + g_1^q(x)|$$

- small- x constraint (Kovchegov, Sievert, 2019)

$$h_1^q \xrightarrow{x \rightarrow 0} x^{\alpha_q} \quad \alpha_q \approx 0.17 \pm 0.085$$

- after inclusion of LQCD tensor charges:

(1) increase of $h_1^{u_v}$ for $x \gtrsim 0.3$

(2) $h_1^{d_v}$ tends to become negative

- JAM3D* = JAM3D-22 (w/ LQCD)

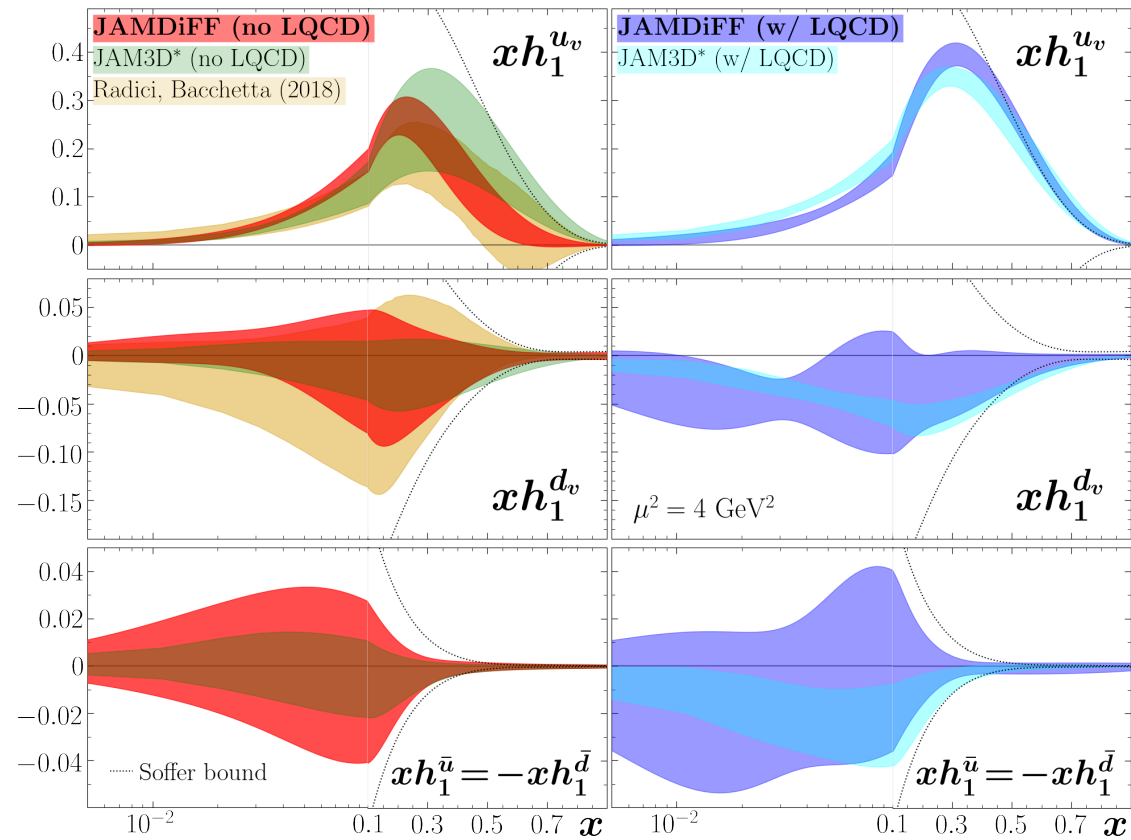
+ antiquarks with $h_1^{\bar{u}} = -h_1^{\bar{d}}$

+ small- x constraint

+ $\delta u, \delta d$ from ETMC & PNDME

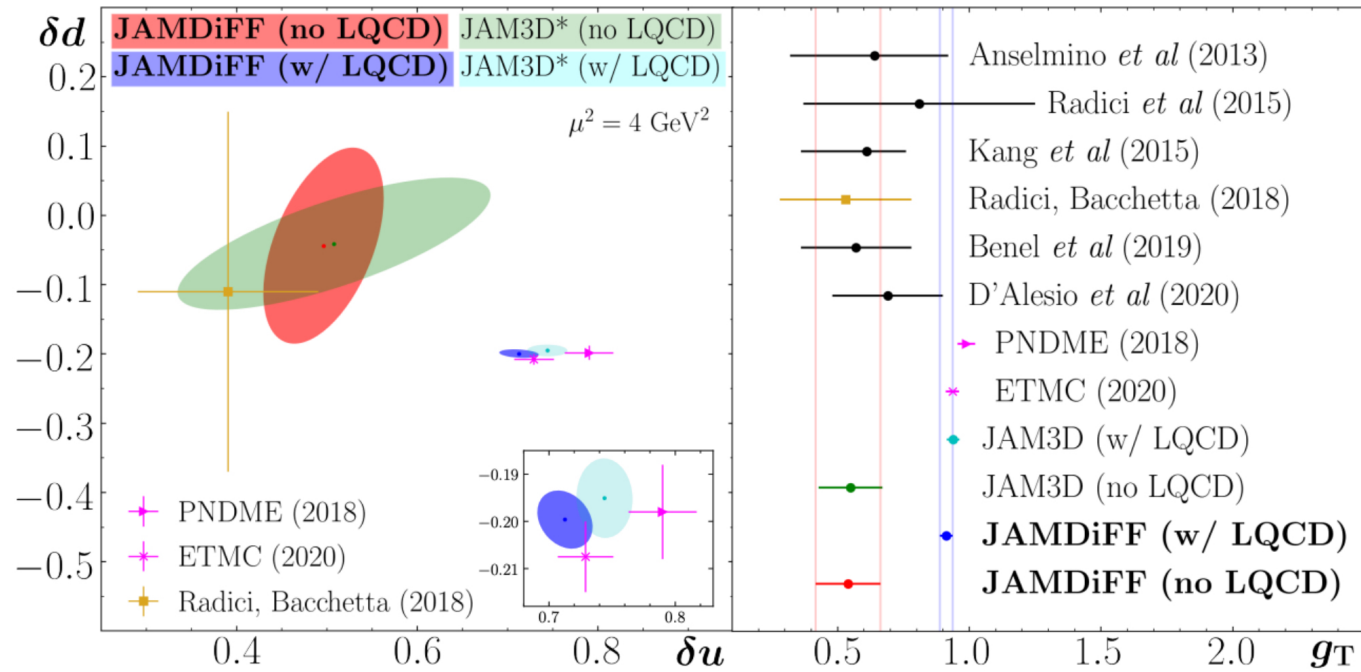
- agreement between two analyses within errors

- Extracted transversity PDFs: comp. no LQCD and w/ LQCD fits (with antiquarks)



- $h_1^{u_v}$ is by far the largest distribution
- Soffer bound implies $h_1^{\bar{q}} \approx 0$ for $x \gtrsim 0.4$

- Tensor charges (no LQCD vs w/ LQCD)

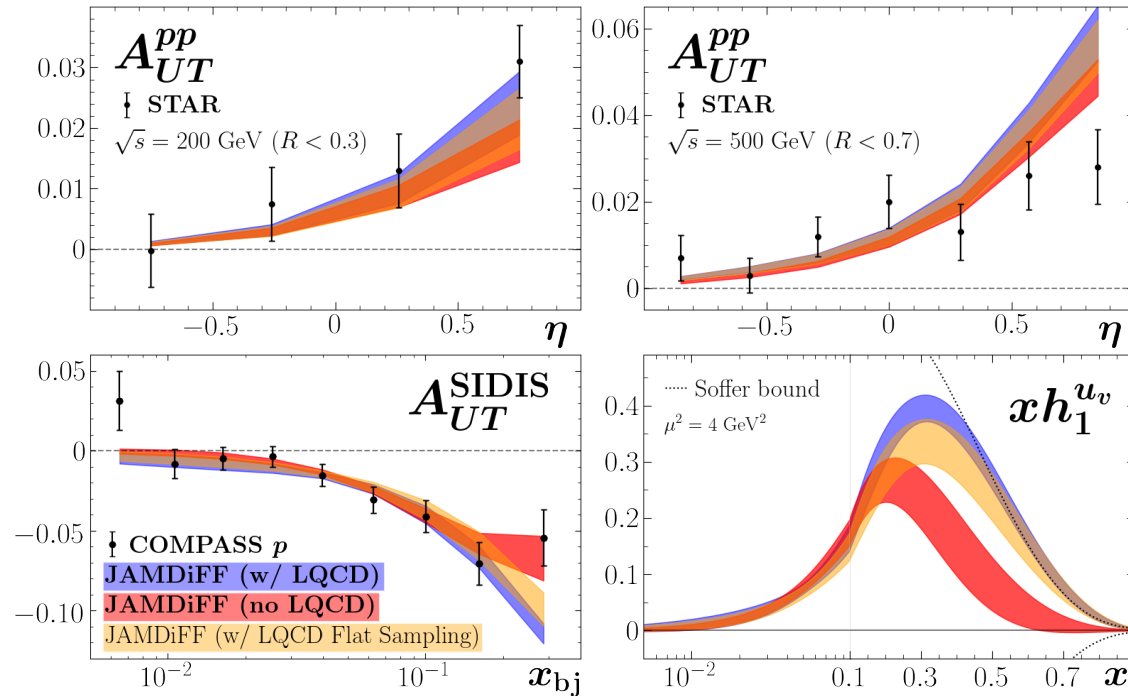


– noticeable shift for δu after including LQCD results

Overall finding: universal nature of all available information on h_1^q —
 (1) data for di-hadron production, (2) data for single-hadron production,
 (3) LQCD results for tensor charge, (4) Soffer bound, (5) small- x constraint

Need for New Data

- Unpolarized data for di-hadron production in SIDIS and pp collision would help to constrain $D_1^{\pi^+\pi^-}/a$
- (Precise) data for A_{UT} at any x would better constrain $h_1^{qv}(x)$, $h_1^{\bar{q}}(x)$
- For the tensor charge δq , (new) data for $x \gtrsim 0.2$ seems most important



Summary

- For both quarks and gluons, we propose a field-theoretic definition of DiFFs which have an interpretation as number densities
- Number density interpretation can be obtained for different variables of interest (including extDiFFs)
- Operator definition of DiFFs also allows one to “easily” obtain their evolution
- New numerical study of data on di-hadron production: Simultaneous global analysis of DiFFs, transversity PDFs, and tensor charges
- Main differences compared to previous analyses of di-hadron data: (1) inclusion of Belle cross section data, STAR 500 GeV data, all binnings for Artru-Collins and SIDIS asymmetries, (2) simultaneous fit of DiFFs and transversity PDFs, (3) small- x theory constraint, (4) improved functional form for DiFF fit functions, (5) inclusion of LQCD results for tensor charges, (6) fit of antiquark transversities
- We find compatibility of all available information on transversity PDFs