Proton GPD from lattice QCD

Martha Constantinou

Temple University

From Quarks and Gluons to the Internal Dynamics of Hadrons

May 15 – 17, 2024
Center for Frontiers in Nuclear Science, Stony Brook University

Outline

★ Approaches to access information on GPDs from lattice QCD

★ Definition of light-cone GPDs vs Euclidean lattice definition (quasi GPDs)

★ New Lorentz covariant approach to access x-dependence of GPDs

★ Twist-3 GPDs

★ Future extensions - Other developments

Collaborators:

Twist-2

S. Bhattacharya, K. Cichy, J. Dodson, X. Gao, A. Metz, J. Miller, S. Mukherjee, P. Petreczky, A. Scapellato, F. Steffens, Y. Zhao

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

PHYSICAL REVIEW D **109,** 034508 (2024)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[2,†] Jack Dodson,[2] Xiang Gao,[3] Andreas Metz,[2] Joshua Miller,[2,‡] Swagato Mukherjee,[4] Peter Petreczky,[4] Fernanda Steffens,[5] and Yong Zhao[3]

Twist-3

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PHYSICAL REVIEW D **108,** 054501 (2023)

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

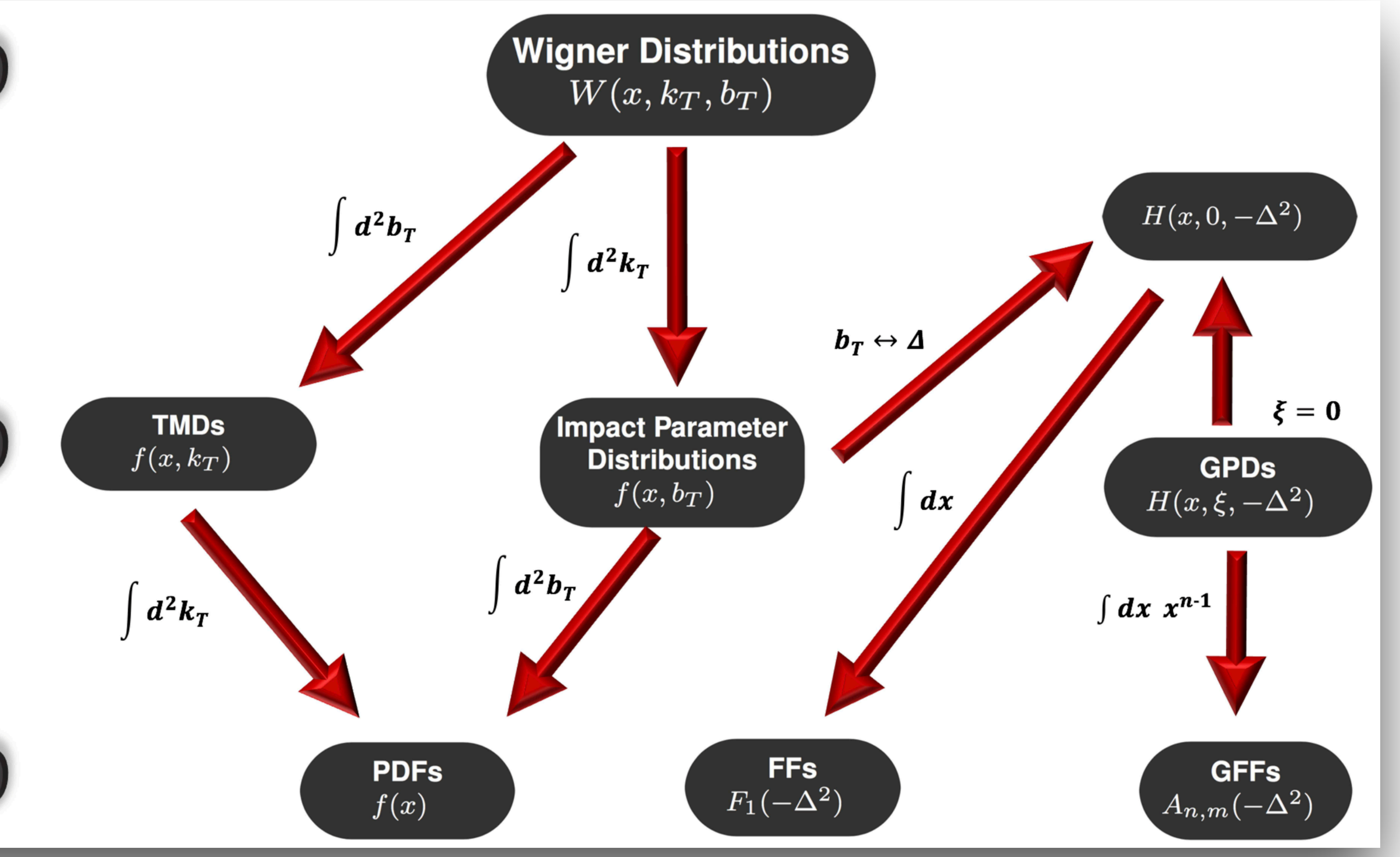
Shohini Bhattacharya,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[1] Jack Dodson,[1] Andreas Metz,[1] Aurora Scapellato,[1] and Fernanda Steffens[4]

Nucleon Characterization

Wigner distributions

★ provide multi-dim images of the parton distributions in phase space

★ encode both TMDs and GPDs in a unified picture



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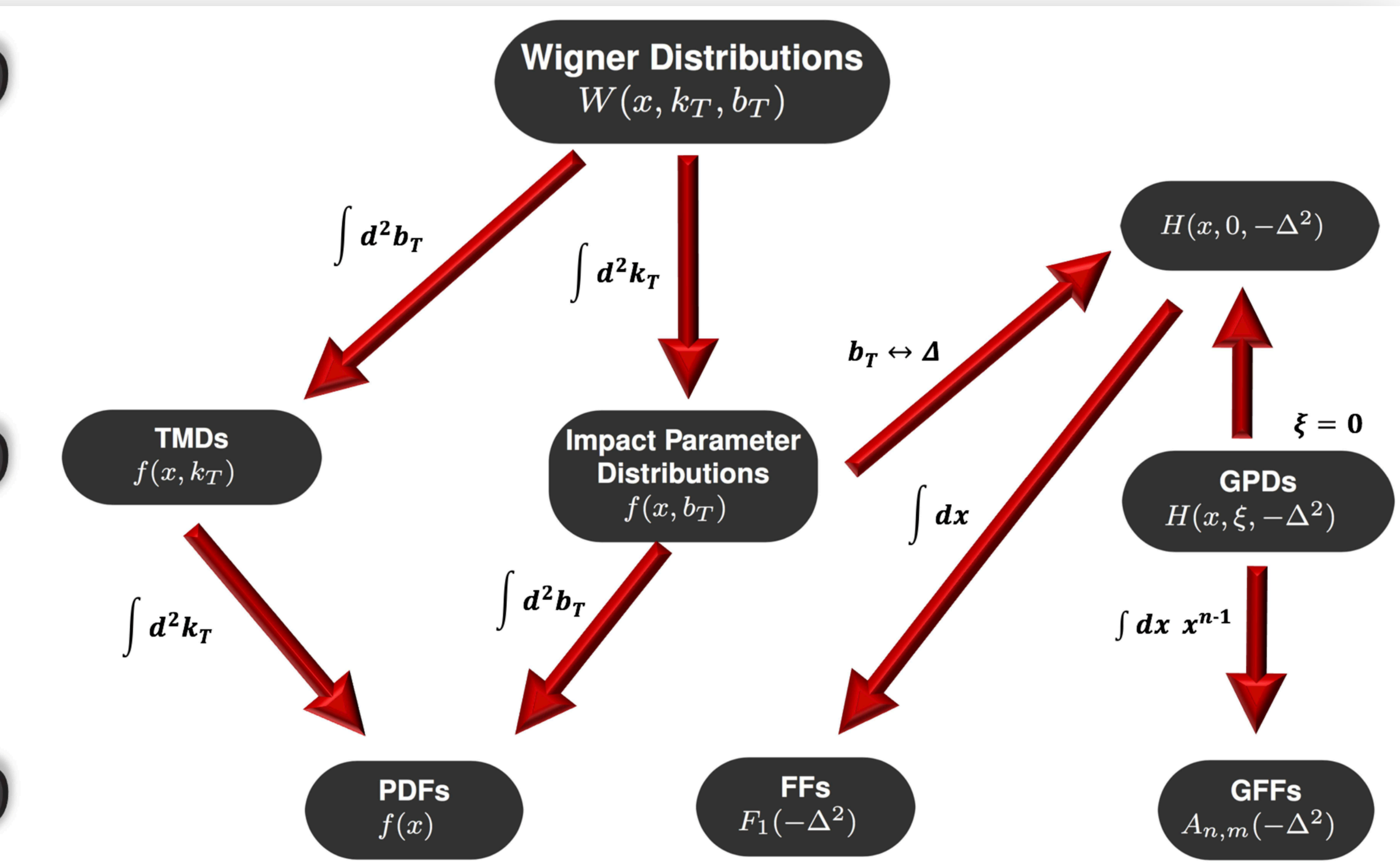
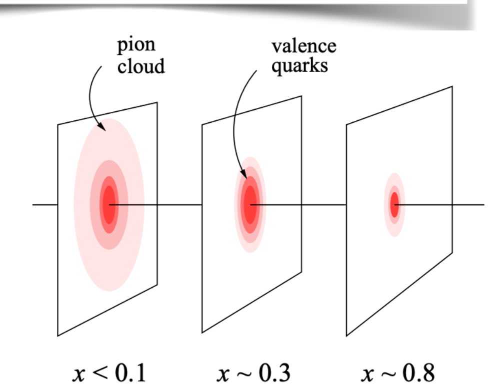


[H. Abramowicz et al., whitepaper for NSAC LRP, 2007]



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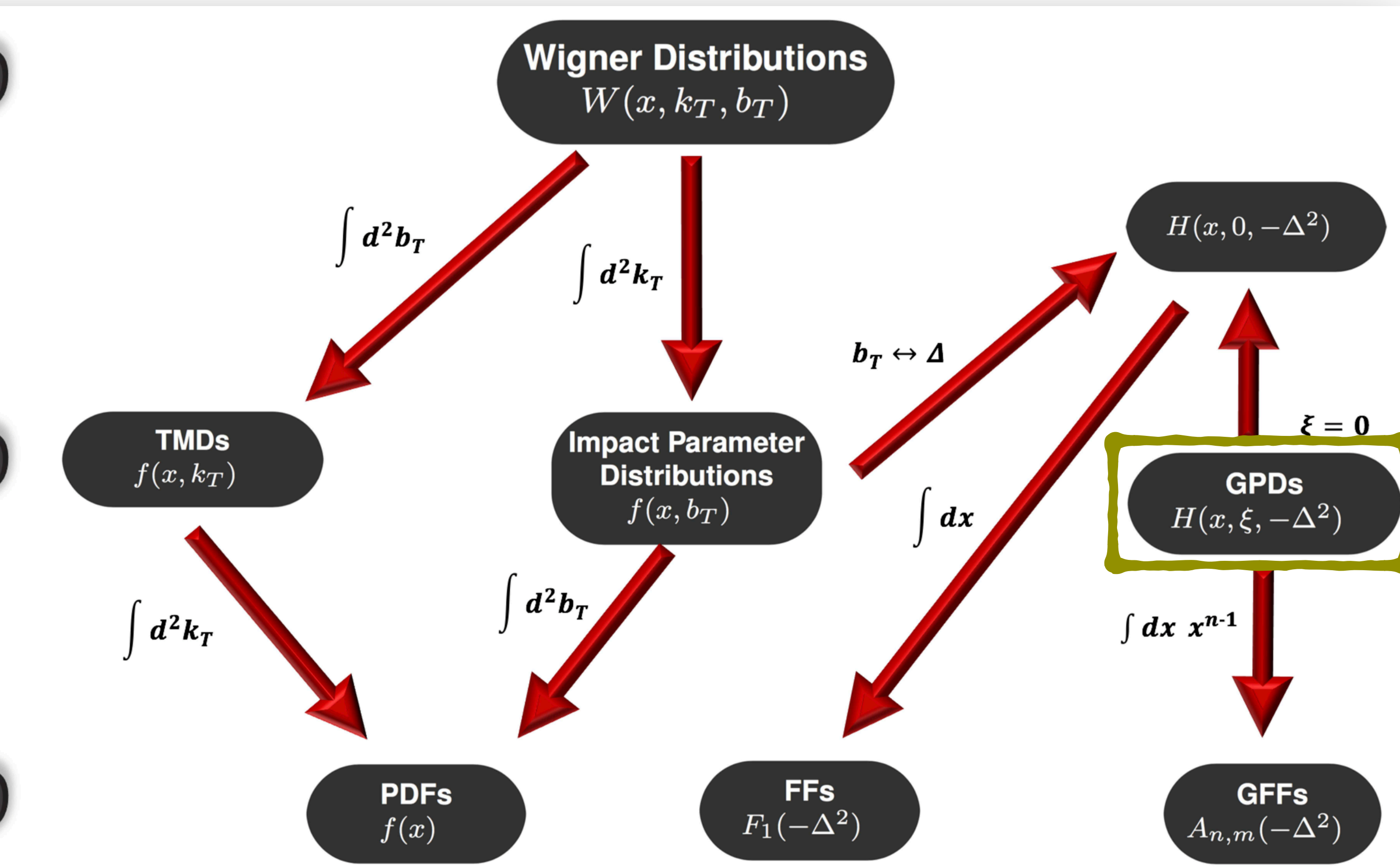

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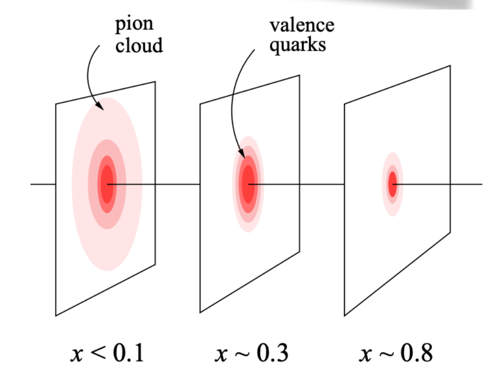

GPDs

★ "Parent" functions for PDFs, FFs, GFFs

★ Multi-dimensional objects

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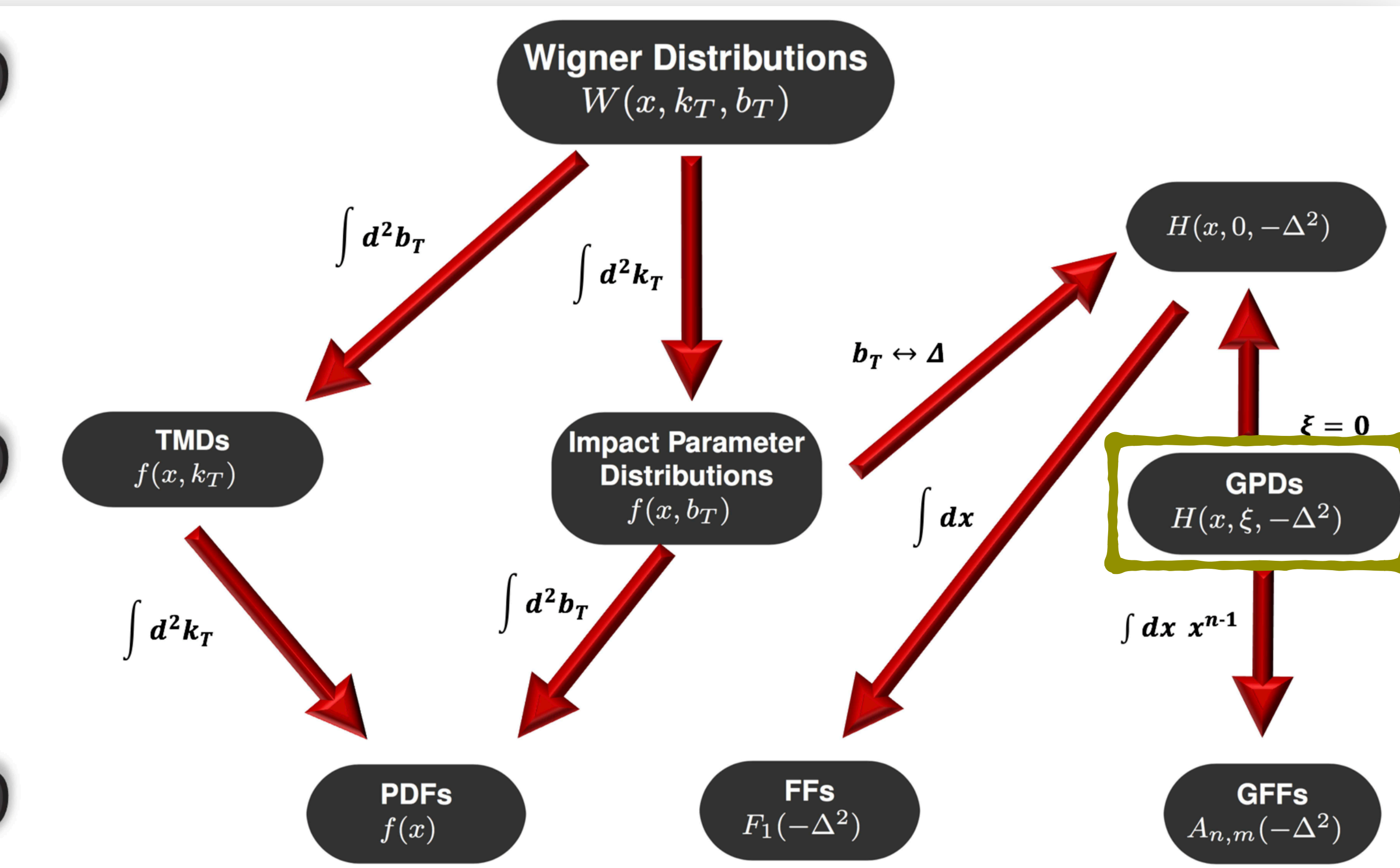

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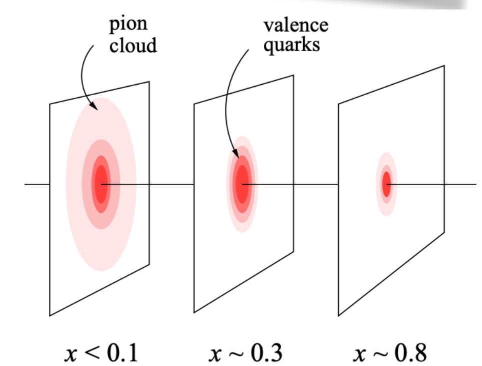


GPDs

- ★ "Parent" functions for PDFs, FFs, GFFs
- ★ Multi-dimensional objects
- ★ Provide correlation between transverse position & longitudinal momentum of the partons in the hadron

- ★ Information on the hadron's mechanical properties (OAM, pressure, etc.)

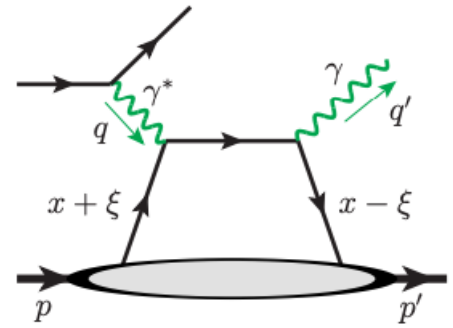
See talk by Y. Hatta

Experimental processes for GPDs

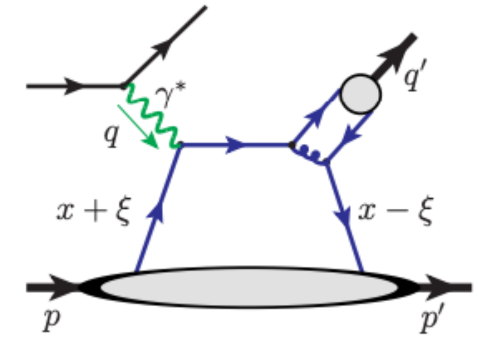
★ GPDs may be accessed via exclusive reactions (DVCS, DVMP)

[X.-D. Ji, PRD 55, 7114 (1997)]

DVCS

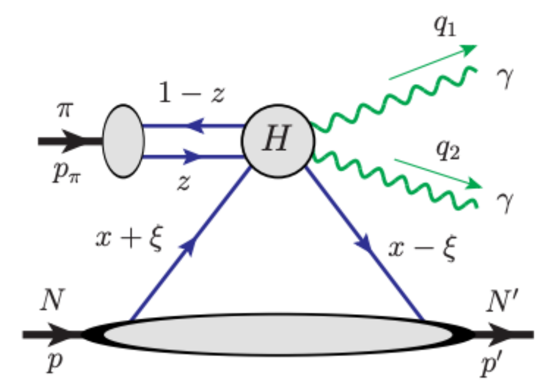


DVMP



★ exclusive pion-nucleon diffractive production of a γ pair of high $p_\perp$

[J. Qiu et al, arXiv:2205.07846]



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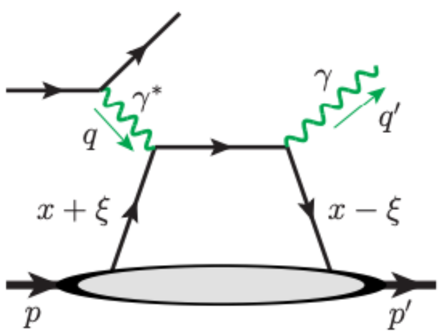

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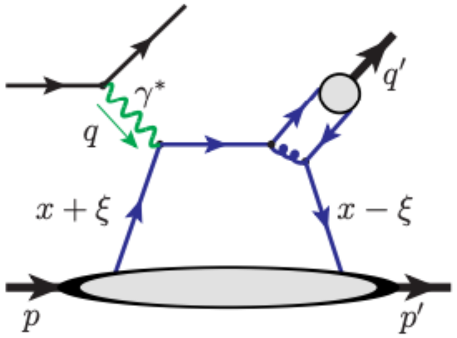

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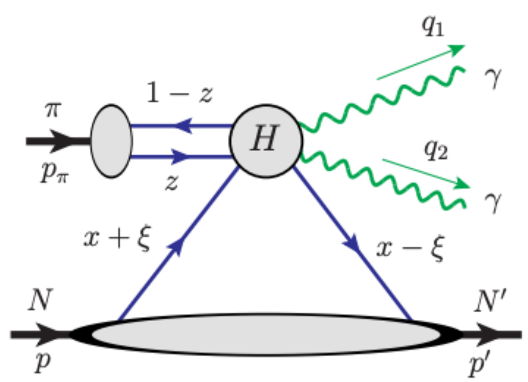

★ GPDs are not well-constrained experimentally:

- x-dependence extraction is not direct. DVCS amplitude: $\mathscr{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x-\xi+i\epsilon} dx$

(SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

- independent measurements to disentangle GPDs
- GPDs phenomenology more complicated than PDFs (multi-dimensionality)
- and more challenges …

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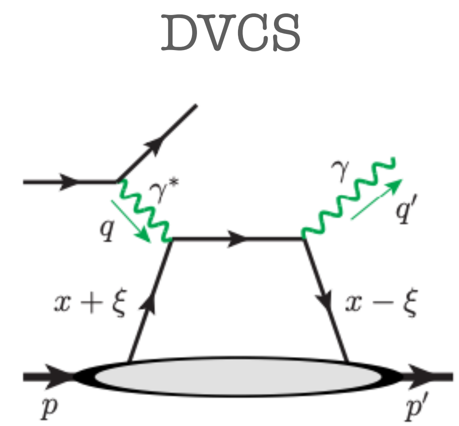

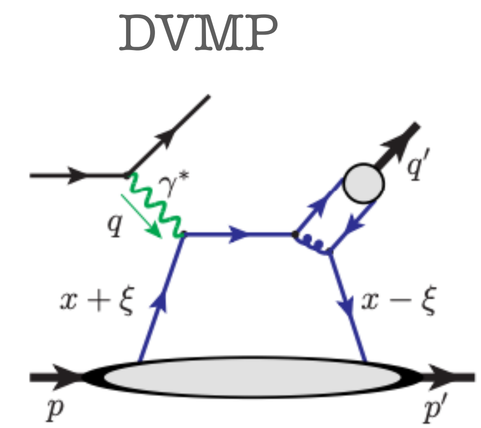

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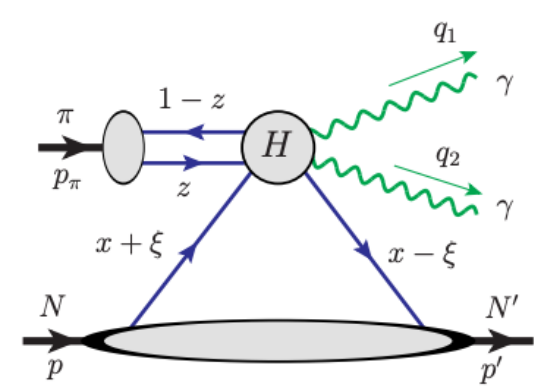

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Essential to complement the knowledge on GPD from lattice QCD

Hadron structure at core of nuclear physics

★ Tomographic imaging of proton has central role in the science program of EIC

GPDs, FFs, GFFs, TMDs, …

[R. Abdul Khalek et al.,

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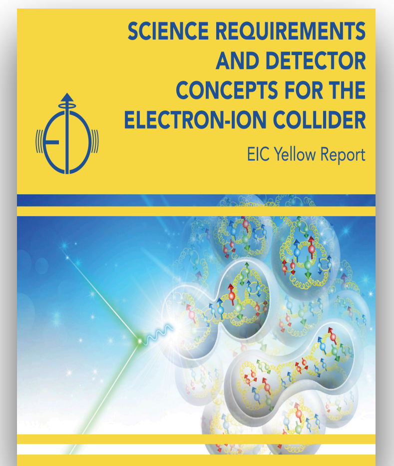

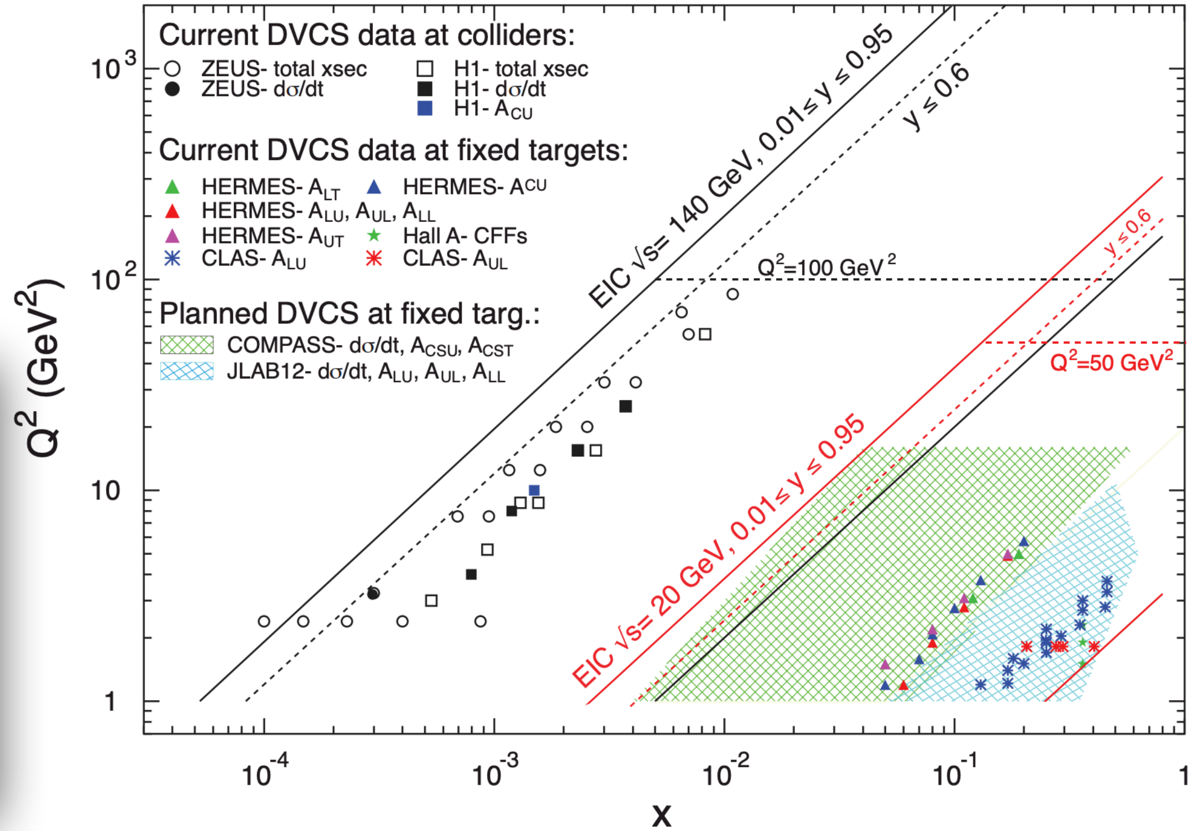

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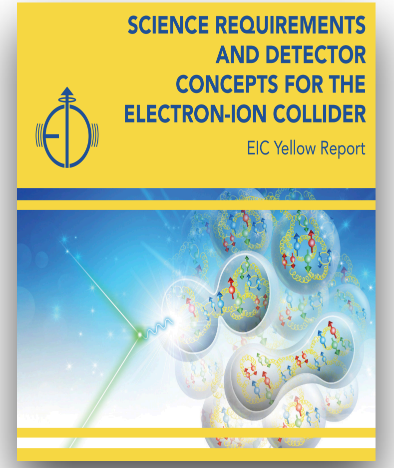

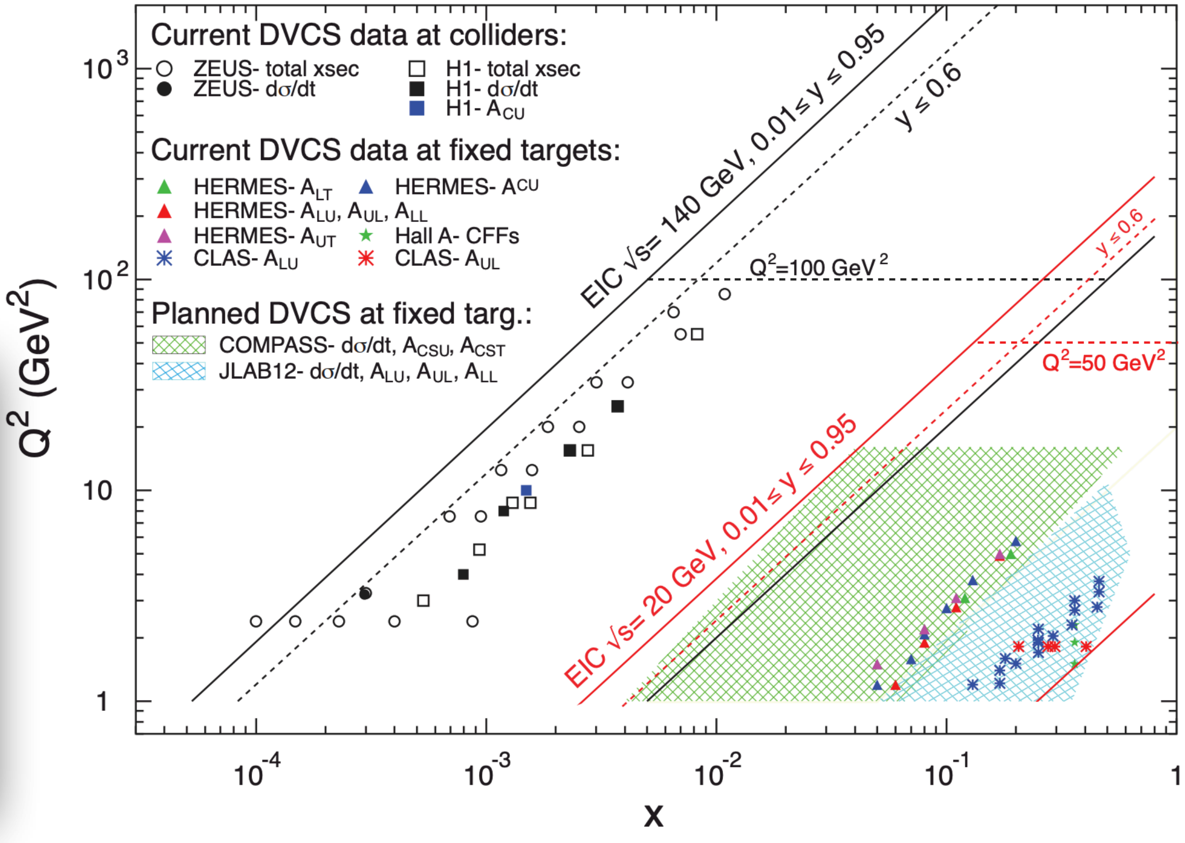

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QUARK-GLUON TOMOGRAPHY COLLABORATION

Award Number: DE-SC0023646

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

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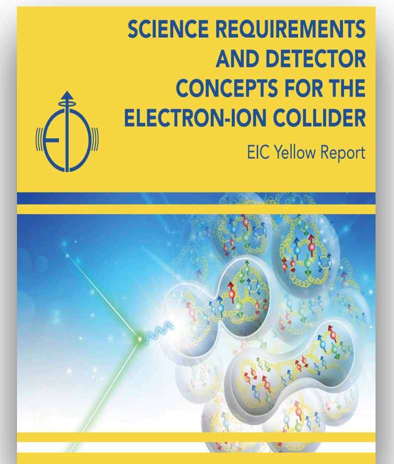

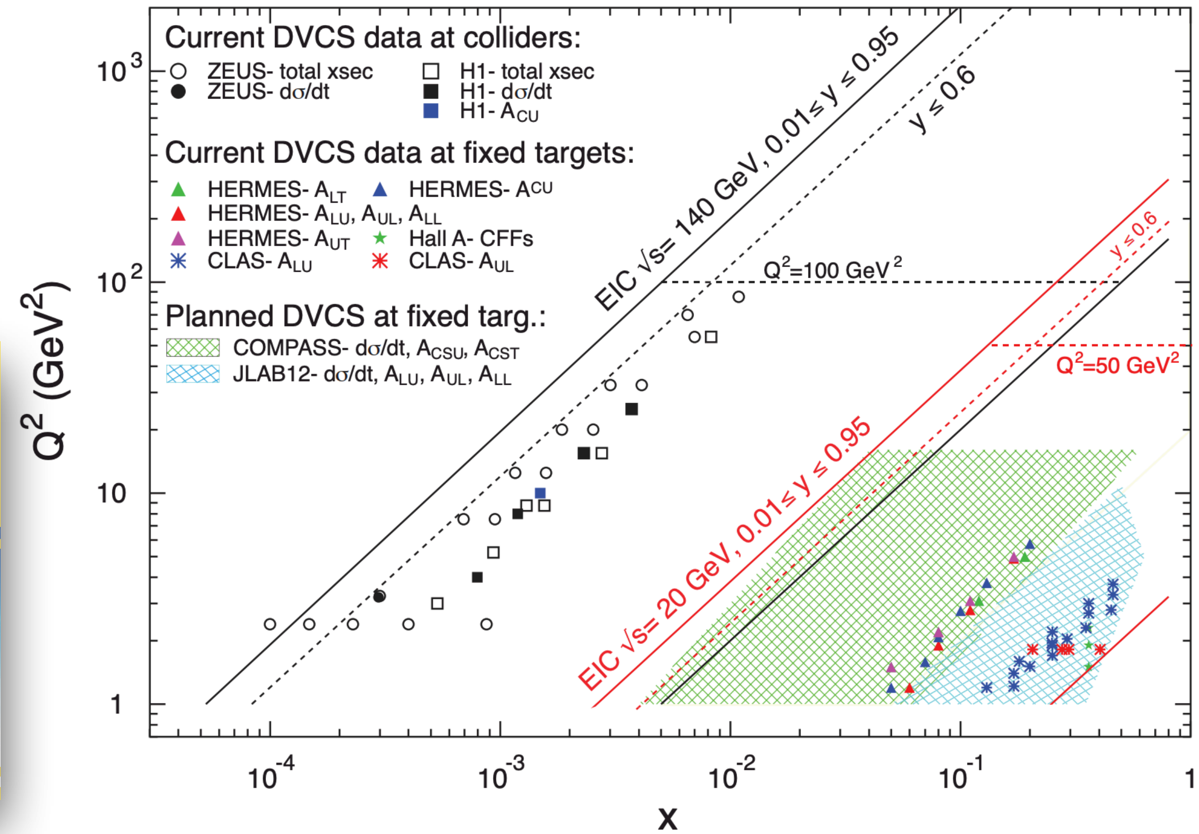

PHENOMENOLOGY
LATTICE QCD
THEORY

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Advances of lattice QCD are timely

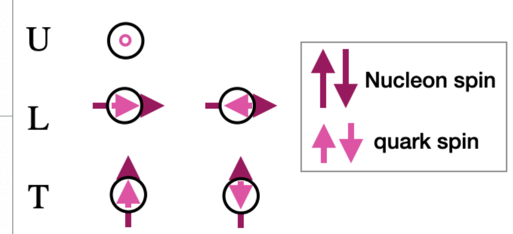
Twist-classification of PDFs, GPDs, TMDs

★ Twist: specifies the order in $1/Q$ at which the function enters factorization formula for a given observable

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L ($\gamma^+\gamma^5$)	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity



(Selected) Twist-3 $(f_i^{(1)})$

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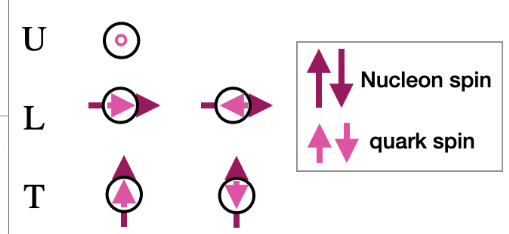

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★ Twist-2: probabilistic densities - a wealth of information exists (mostly on PDFs)

★ Twist-3: poorly known, but very important:
- as sizable as twist-2
- contain information about quark-gluon correlations inside hadrons
- appear in QCD factorization theorems for various observables (e.g. g_2)
- certain twist-3 PDFs are related to the TMDs
- physical interpretation (e.g. average force on partons inside hadron)

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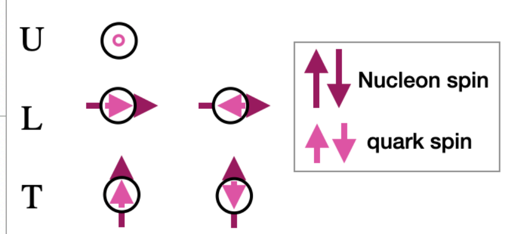

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While twist-3 $f_i^{(1)}$ share some similarities with twist-2 $f_i^{(0)}$ in their extraction, there are several challenges both experimentally and theoretically

Accessing information on GPDs

★ **Mellin moments**
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n}q\right]$$

local operators

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t) - i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\} + \frac{\Delta^{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\ \text{even}}\Big)\Big\}$$

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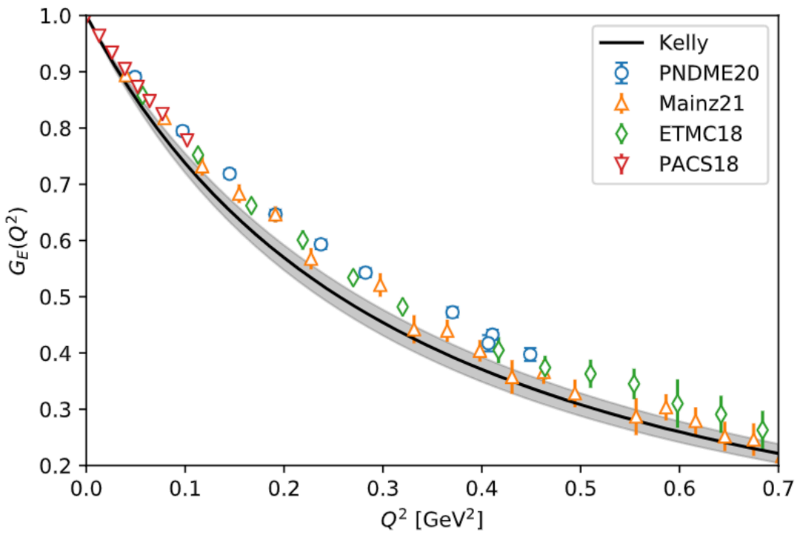

Wide -t range that comes at the cost of 1 (in the majority of cases)

Accessing information on GPDs

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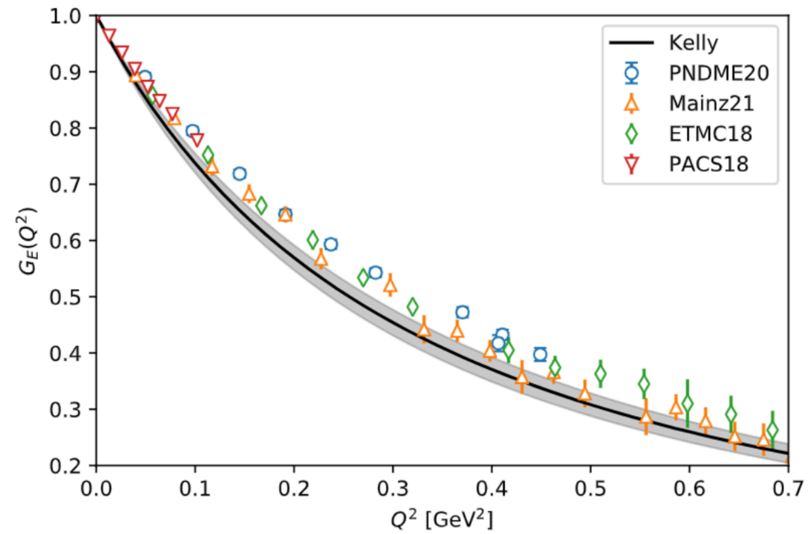

Wide -t range that comes at the cost of 1 (in the majority of cases)

★ **Matrix elements of non-local operators**
(quasi-GPDs, pseudo-GPDs, …)

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathcal{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Wilson line

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_A^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P) + \text{ht}$$

GPDs

Through non-local matrix elements

of fast-moving hadrons

Access of PDFs/GPDs on a Euclidean Lattice

- ★ Matrix elements of momentum-boosted hadrons coupled to nonlocal (equal-time) operators
- ★ Connection to light-cone GPDs through LaMET [X. Ji, PRL 110 (2013) 262002], SDF [A. Radyushkin, PRD 96, 034025 (2017)]

$$\tilde{q}_{\Gamma}^{\rm GPD}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathcal{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -\,Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

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Accessing -t dependence: Computationally intensive

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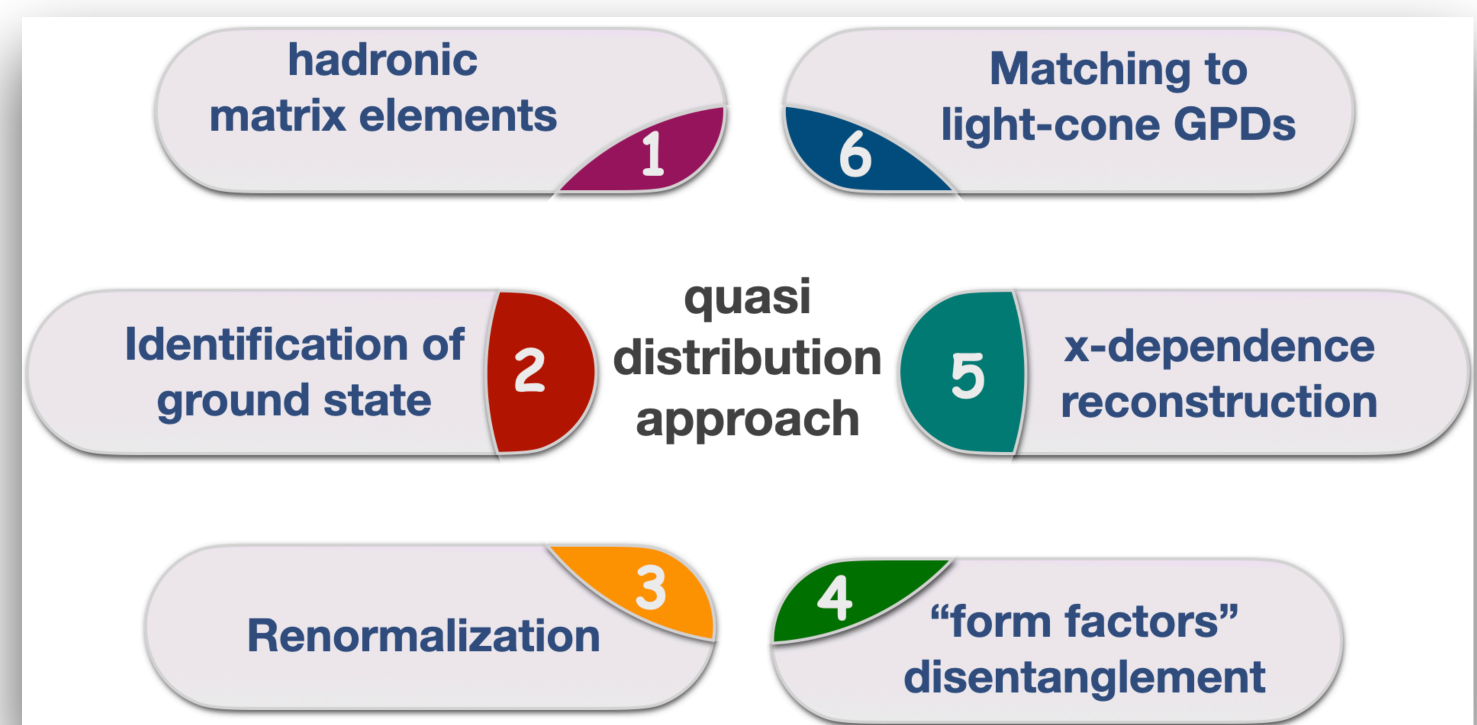
$$t = \Delta^2 = -\,Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$



Access of PDFs/GPDs on a Euclidean Lattice

- ★ Matrix elements of momentum-boosted hadrons coupled to nonlocal (equal-time) operators
- ★ Connection to light-cone GPDs through LaMET [X. Ji, PRL 110 (2013) 262002], SDF [A. Radyushkin, PRD 96, 034025 (2017)]

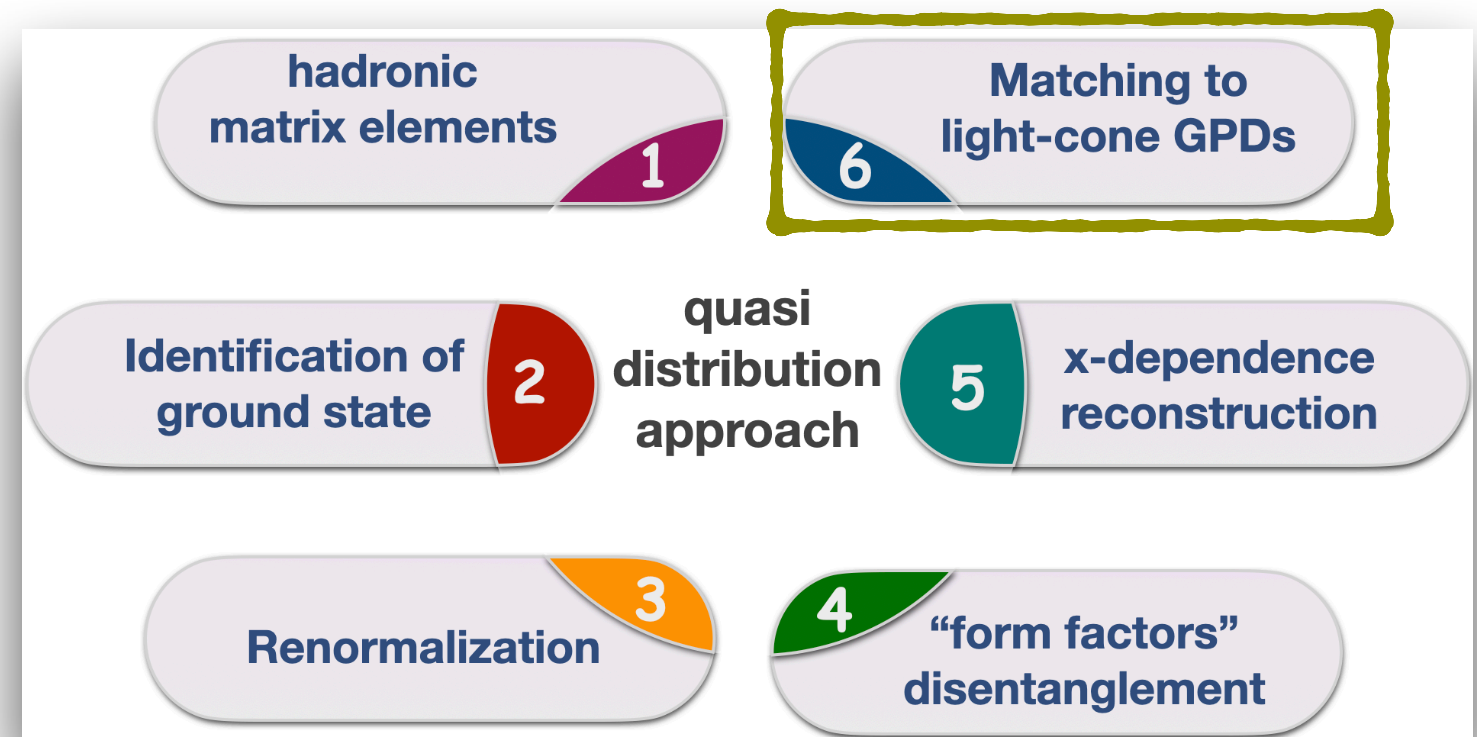
$$\tilde{q}^{\rm GPD}_{\Gamma}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Accessing -t dependence: Computationally intensive

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$



New parametrization of GPDs

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

PHYSICAL REVIEW D **109,** 034508 (2024)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[2,†] Jack Dodson,[2] Xiang Gao,[3] Andreas Metz,[2] Joshua Miller,[2,‡] Swagato Mukherjee,[4] Peter Petreczky,[4] Fernanda Steffens,[5] and Yong Zhao[3]

Theoretical setup

★ γ^+ inspired parametrization is prohibitively expensive

$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{\rm Q(0)}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{\rm Q(0)}(x,\xi,t;P^3)\right]u(p,\lambda)$$

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★ Lorentz-invariant parametrization

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu MA_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z}MA_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Goals

★ Extraction of standard GPDs using A_i obtained from any frame

★ quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone

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➡ **Proof-of-concept calculation ($\xi = 0$):**

- **symmetric frame:** $\vec{p}^{\,s}_f = \vec{P} + \frac{\vec{Q}}{2}, \quad \vec{p}^{\,s}_i = \vec{P} - \frac{\vec{Q}}{2} \qquad -t^s = \vec{Q}^2 = 0.69\,GeV^2$

- **asymmetric frame:** $\vec{p}^{\,a}_f = \vec{P}, \qquad \vec{p}^{\,a}_i = \vec{P} - \vec{Q} \qquad t^a = -\vec{Q}^2 + (E_f - E_i)^2 = 0.65\,GeV^2$

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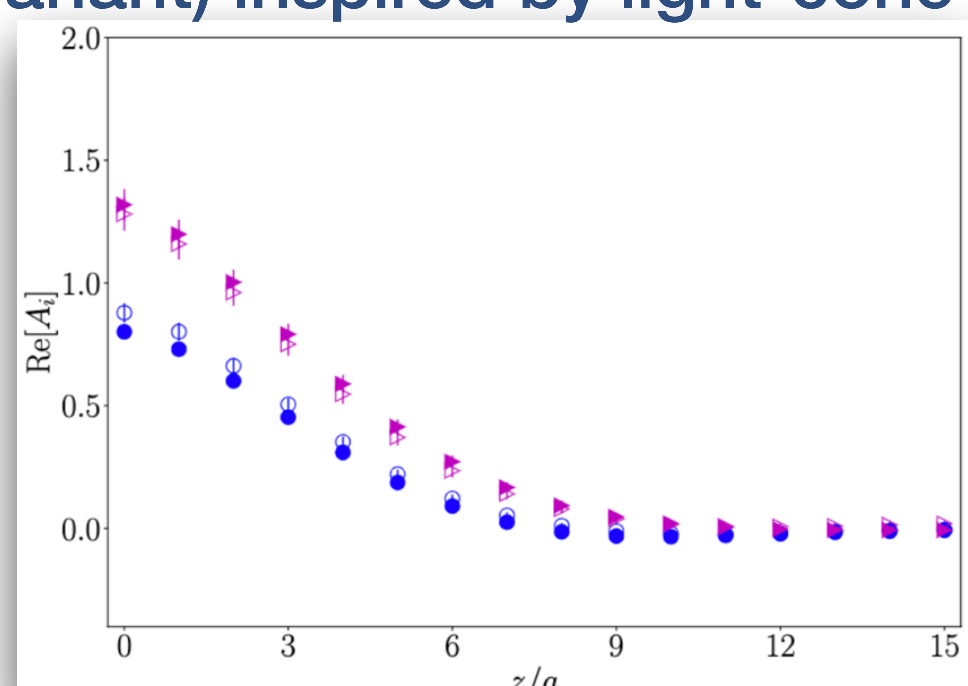

eV^2

Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	429	8	27456
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asymm	±1.25	(±2,0,0), (0,±2,0)	0.64	0	8	429	8	27456
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	429	8	27456
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	429	8	27456

Zero-skewness calculation

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Zero-skewness calculation

Symmetric frame: each momentum requires <u>separate</u> computational resources

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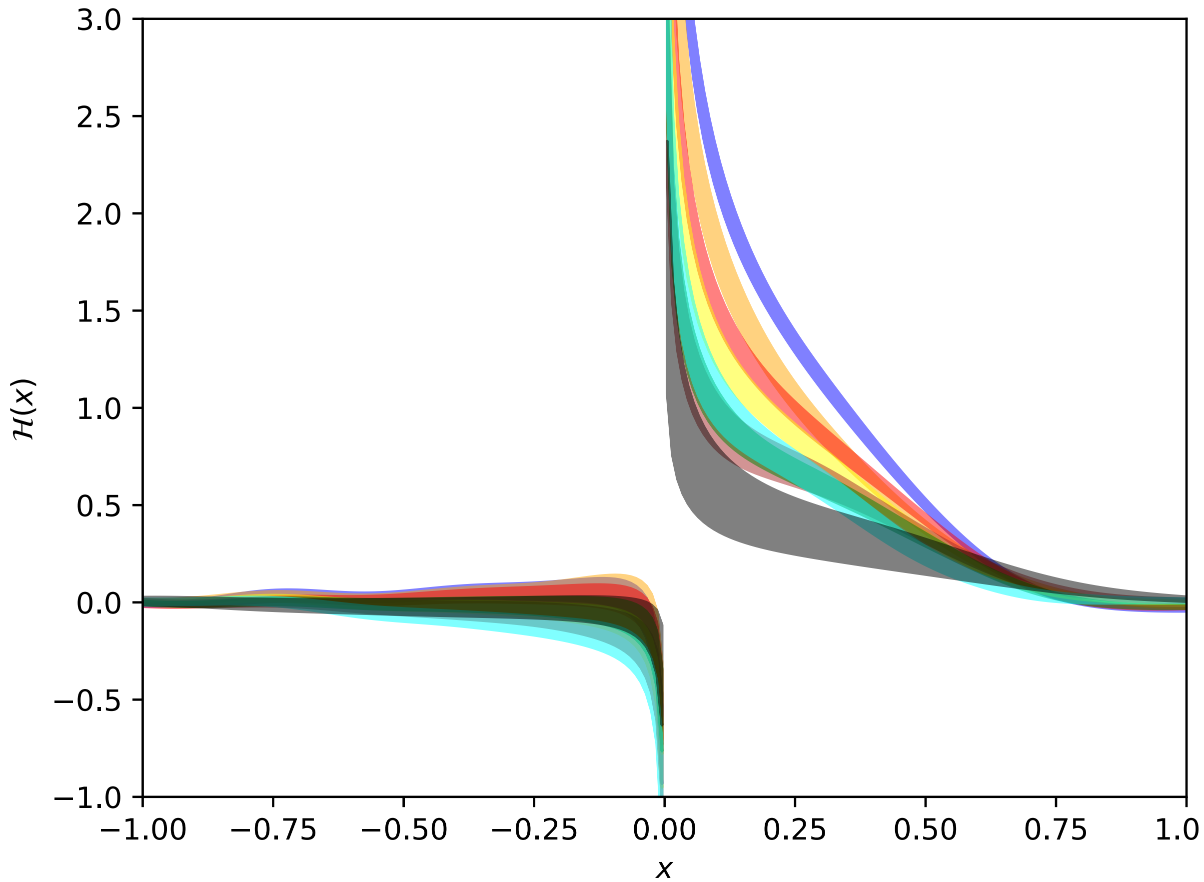
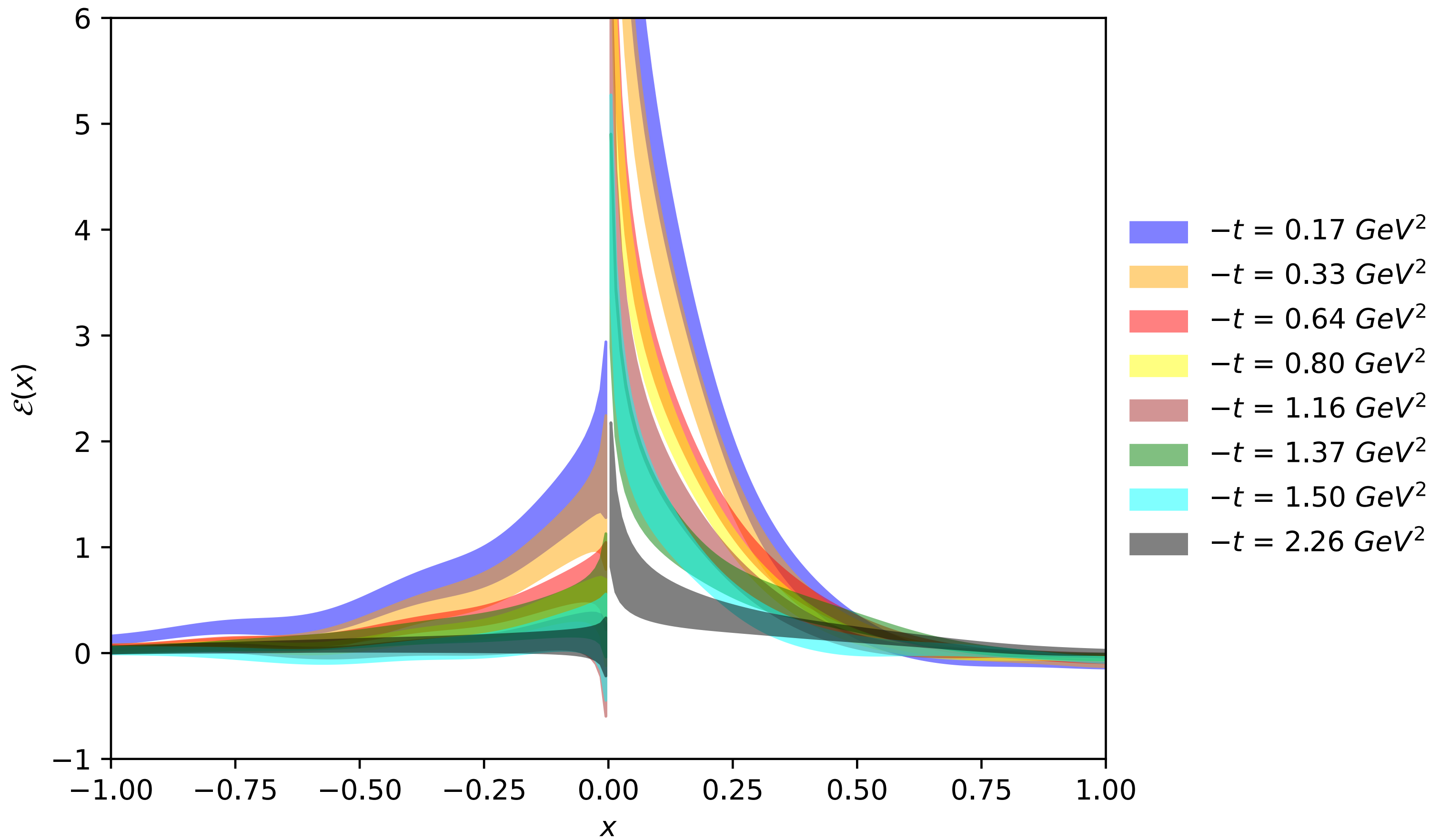
Zero-skewness calculation

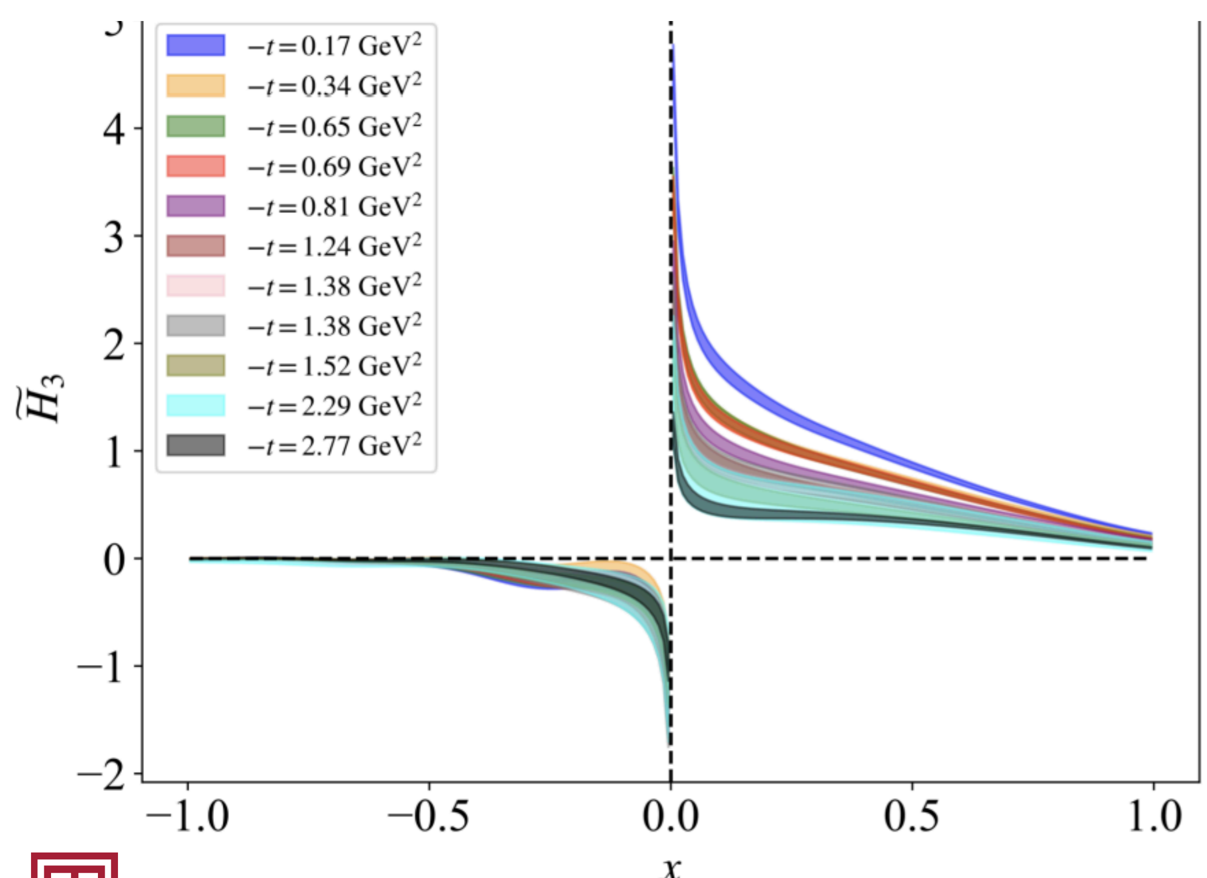
Symmetric frame: each momentum requires separate computational resources

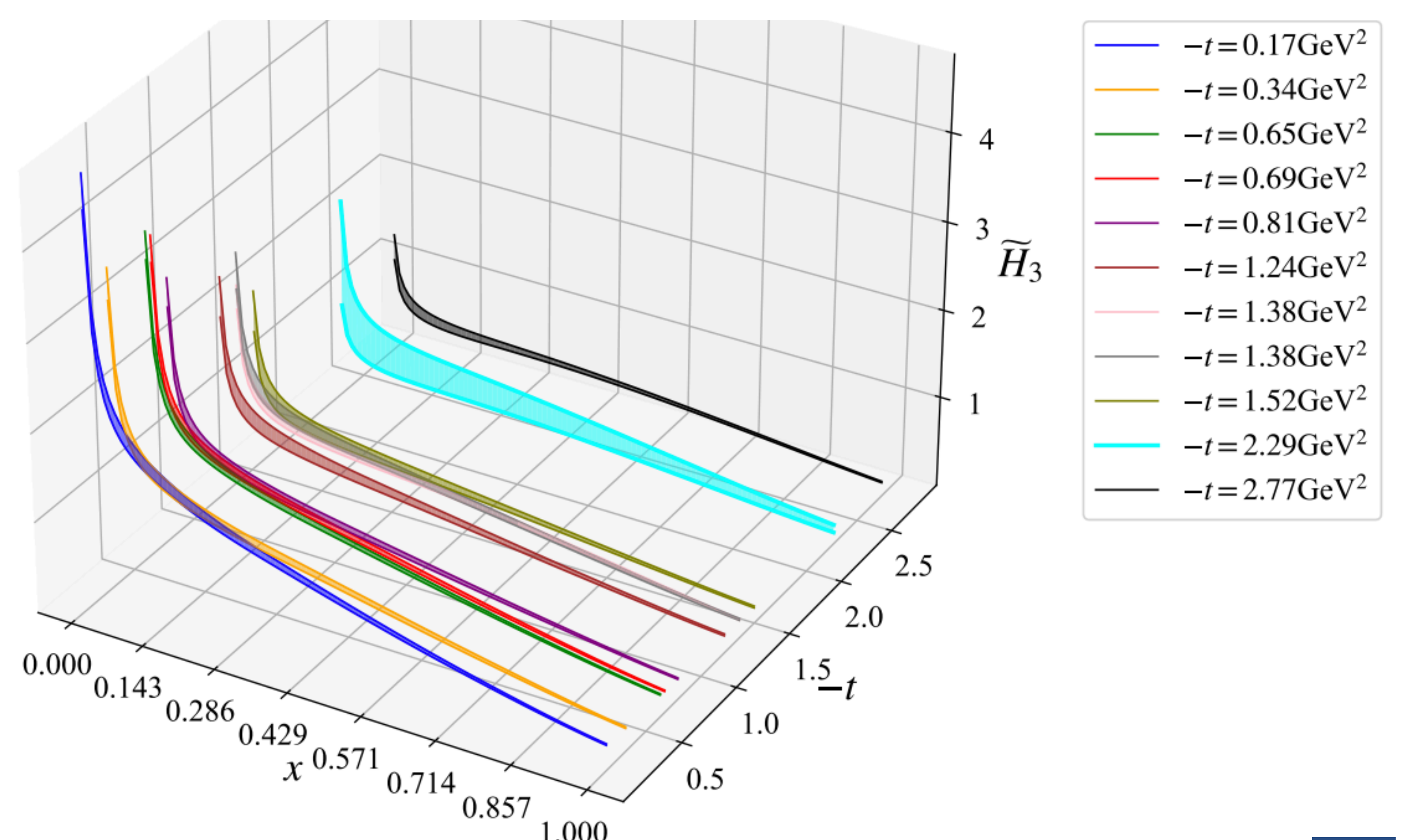
Asymmetric frame: momenta grouped in 2 sets of runs [(Q,0,0), (Qx,Qy,0)]

Light-cone GPDs







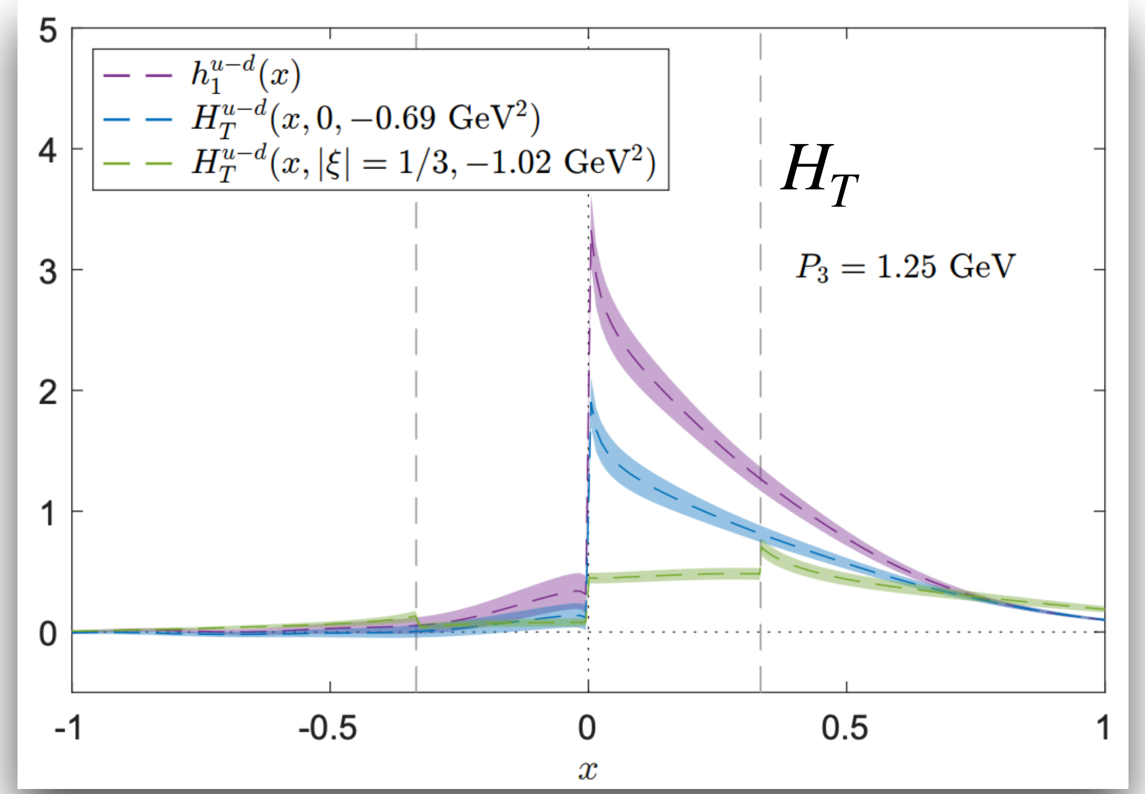


Transversity GPDs

Standard parametrization

$$h_T^j(\Gamma_\nu, z, P_f, P_i) = \langle\langle \sigma^{3j} \rangle\rangle F_{H_T}(z,\xi,t,P_3) + \frac{i}{2m}\langle\langle \gamma^3 \Delta_j - \gamma^j \Delta_3 \rangle\rangle F_{E_T}(z,\xi,t,P_3) + \frac{P_3\Delta_j - P_j\Delta_3}{m^2}\langle\langle \hat{1} \rangle\rangle F_{\widetilde{H}_T}(z,\xi,t,P_3) + \frac{1}{m}\langle\langle \gamma^3 P_j - \gamma^j P_3 \rangle\rangle F_{\widetilde{E}_T}(z,\xi,t,P_3)$$

[C. Alexandrou et al., PRD 105, 034501 (2022)]



Symmetric frame

Transversity GPDs

On-going work

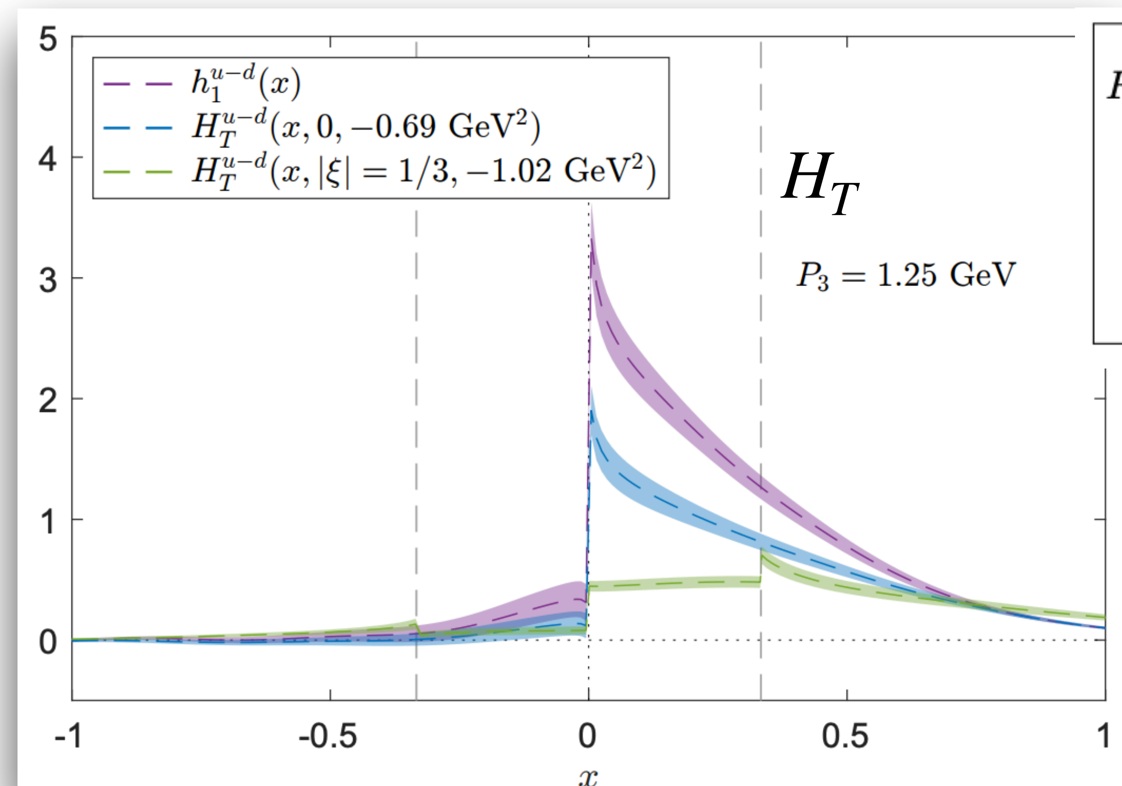
Standard parametrization

$$h_T^j(\Gamma_\nu, z, P_f, P_i) = \langle\langle \sigma^{3j} \rangle\rangle F_{H_T}(z,\xi,t,P_3) + \frac{i}{2m}\langle\langle \gamma^3\Delta_j - \gamma^j\Delta_3 \rangle\rangle F_{E_T}(z,\xi,t,P_3) + \frac{P_3\Delta_j - P_j\Delta_3}{m^2}\langle\langle \hat{1} \rangle\rangle F_{\widetilde{H}_T}(z,\xi,t,P_3) + \frac{1}{m}\langle\langle \gamma^3 P_j - \gamma^j P_3 \rangle\rangle F_{\widetilde{E}_T}(z,\xi,t,P_3)$$

[C. Alexandrou et al., PRD 105, 034501 (2022)]

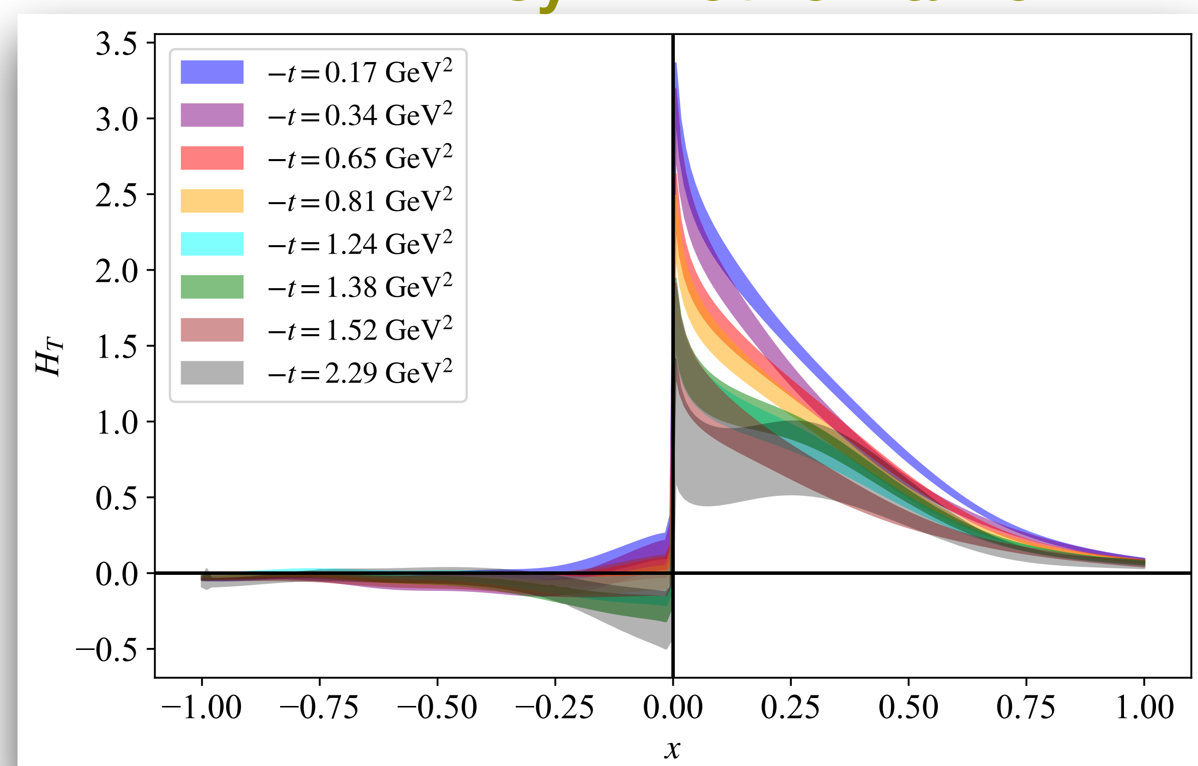
Lorentz covariant parametrization

$$F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = P^{[\mu}z^{\nu]}\gamma_5 A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + Mz^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5 + M\not{z}\gamma_5\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10} + i\epsilon^{\mu\nu P z}A_{11} + i\epsilon^{\mu\nu z\Delta}A_{12}$$



Symmetric frame

Asymmetric frame



Twist-3 GPDs

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

PHYSICAL REVIEW D **108**, 054501 (2023)

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[1] Jack Dodson,[1] Andreas Metz,[1] Aurora Scapellato,[1] and Fernanda Steffens[4]

+ Josh Miller (Temple graduate student)

Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization of coordinate-space correlation functions

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
$$+\Delta^\mu_\perp\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma^\mu_\perp\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)$$
$$+\Delta^\mu_\perp\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon^{\mu\nu}_\perp\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

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±1.25	(±4,0,0)	2.76	8	329	32	84224

Symmetric frame

Consistency Checks

★ Sum Rules (generalization of Burkhardt-Cottingham)

[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249]

$$\int_{-1}^{1} dx\, \widetilde{H}(x,\xi,t) = G_A(t)\,, \quad \int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

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[A. Efremov, O. Teryaev , E. Leader, PRD 55 (1997) 4307, hep-ph/9607217]

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_3(x,0,t) = \frac{\xi}{4} G_E \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,0,t) = \frac{1}{4} G_E(t)$$

G_E : electric FF

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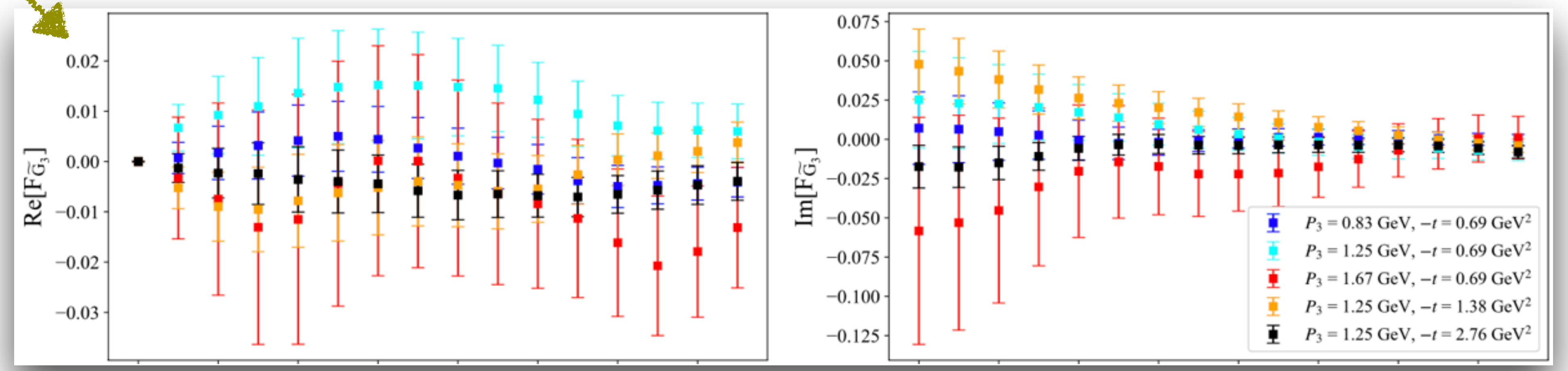
G_E : electric FF

Indeed, numerically found to be zero within uncertainties at ξ=0



Reconstruction of x-dependence & matching

★ quasi-GPDs transformed to momentum space using Backus Gilbert
[G. Backus and F. Gilbert, Geophysical Journal International 16, 169 (1968)]

★ Matching formalism to 1 loop accuracy level

$$F_X^{\overline{\rm MMS}}(x,t,P_3,\mu) = \int_{-1}^{1}\frac{dy}{|y|}\, C^{\overline{\rm MMS},\overline{\rm MS}}_{\gamma_j\gamma_5}\left(\frac{x}{y},\frac{\mu}{yP_3}\right) G_X^{\overline{\rm MS}}(y,t,\mu) \;+\; \mathcal{O}\left(\frac{m^2}{P_3^2},\frac{t}{P_3^2},\frac{\Lambda^2_{\rm QCD}}{x^2P_3^2}\right)$$

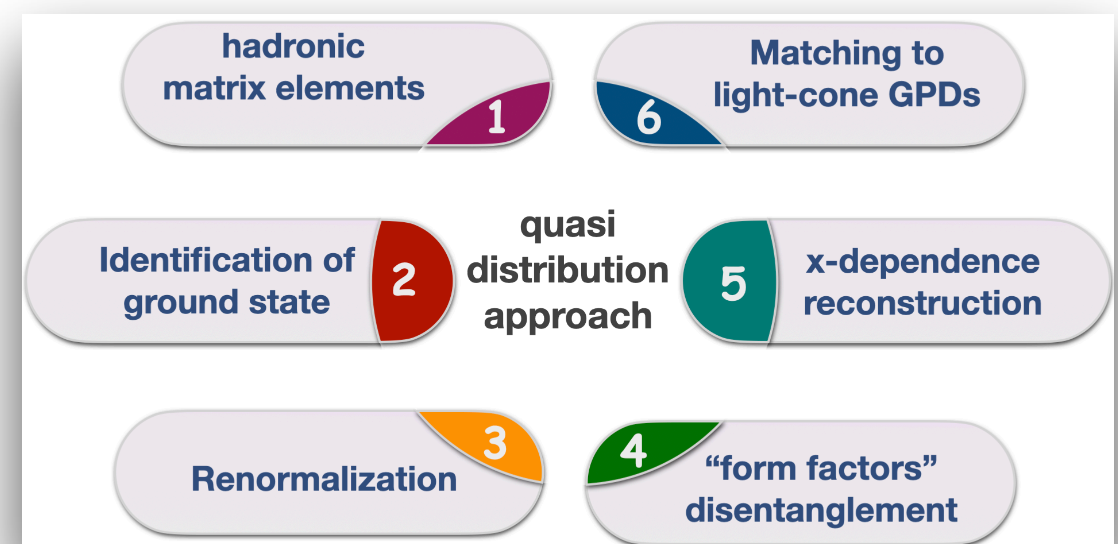
★ Operator dependent kernel

PHYSICAL REVIEW D **102,** 034005 (2020)

One-loop matching for the twist-3 parton distribution $g_T(x)$

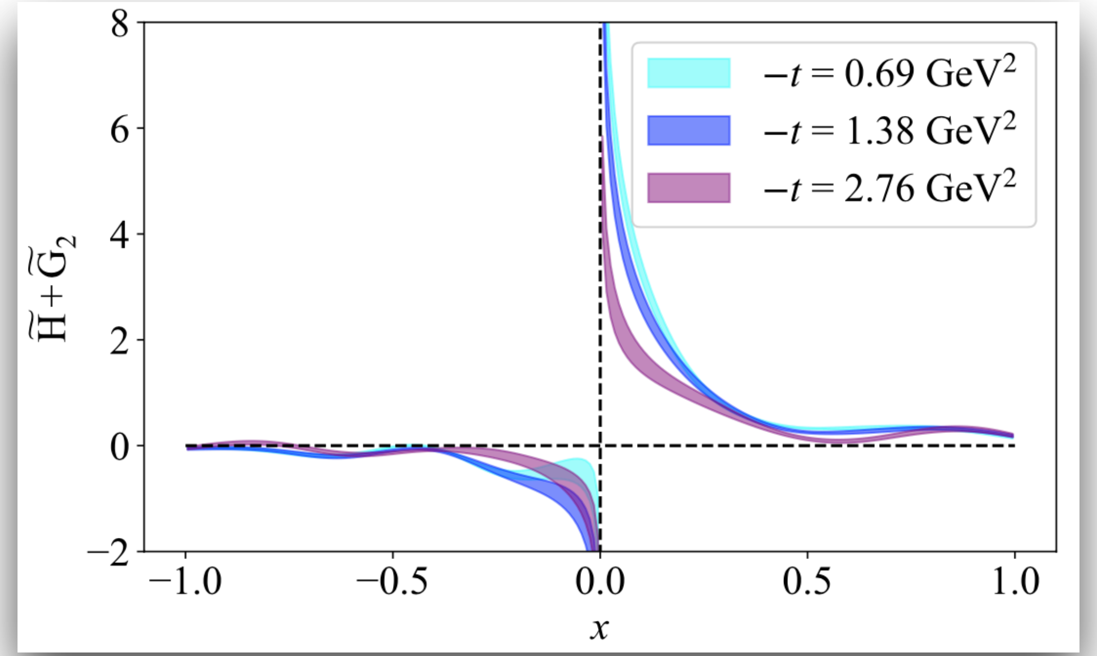
Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[2] and Fernanda Steffens[3]

$$C^{(1)}_{\overline{\rm MMS}}\left(\xi,\frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases}0\\ \delta(\xi)\\ 0\end{cases} + \frac{\alpha_s C_F}{2\pi}\begin{cases}\left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi}{\xi-1}+\frac{\xi}{1-\xi}+\frac{3}{2\xi}\right]_+ & \xi>1\\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{4\xi(1-\xi)(xP_3)^2}{\mu^2}+\frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0<\xi<1\\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi-1}{\xi}-\frac{\xi}{1-\xi}+\frac{3}{2(1-\xi)}\right]_+ & \xi<0\,,\end{cases}$$



★ Matching does not consider mixing with q-g-q correlators
[V. Braun et al., JHEP 05 (2021) 086]

Lattice Results - light-cone GPDs



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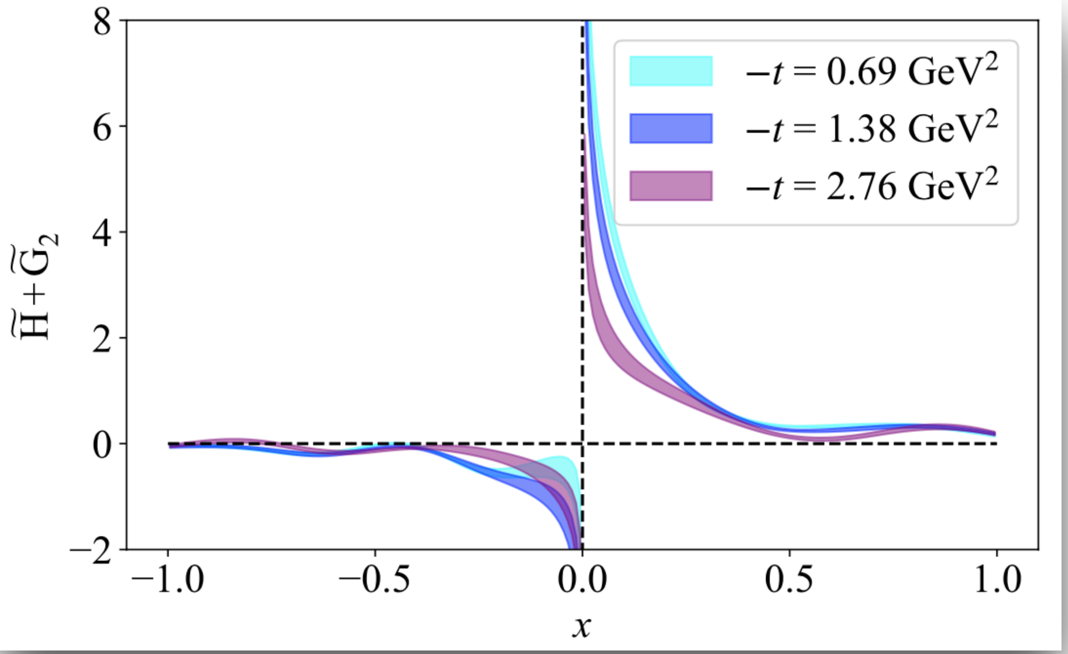
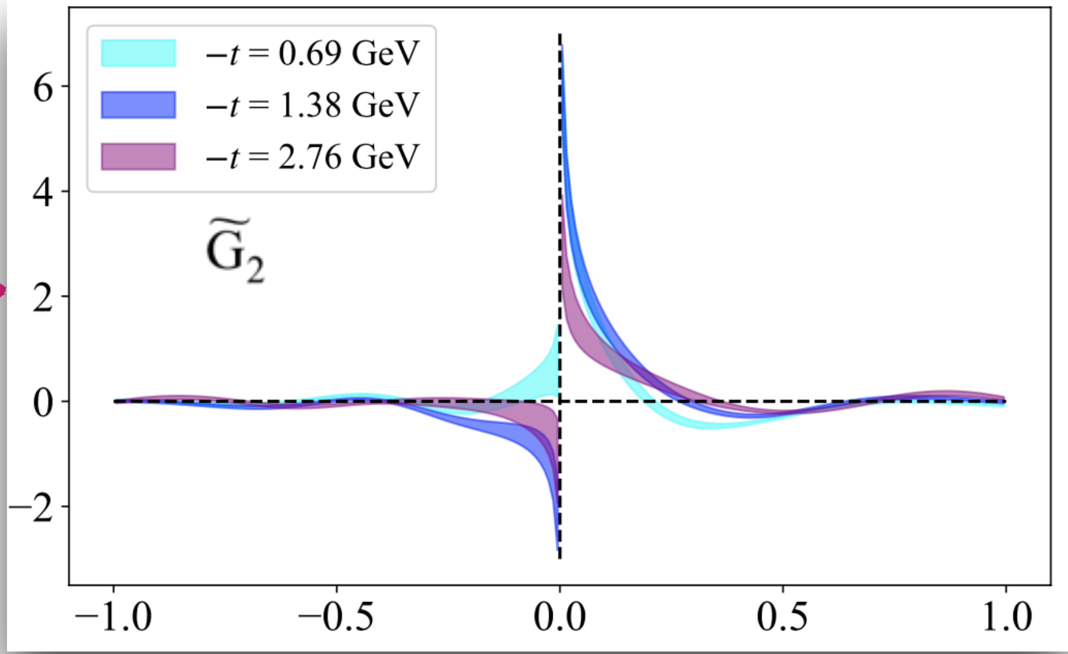


Isolating $\widetilde{G}_2$

using $\widetilde{H}$



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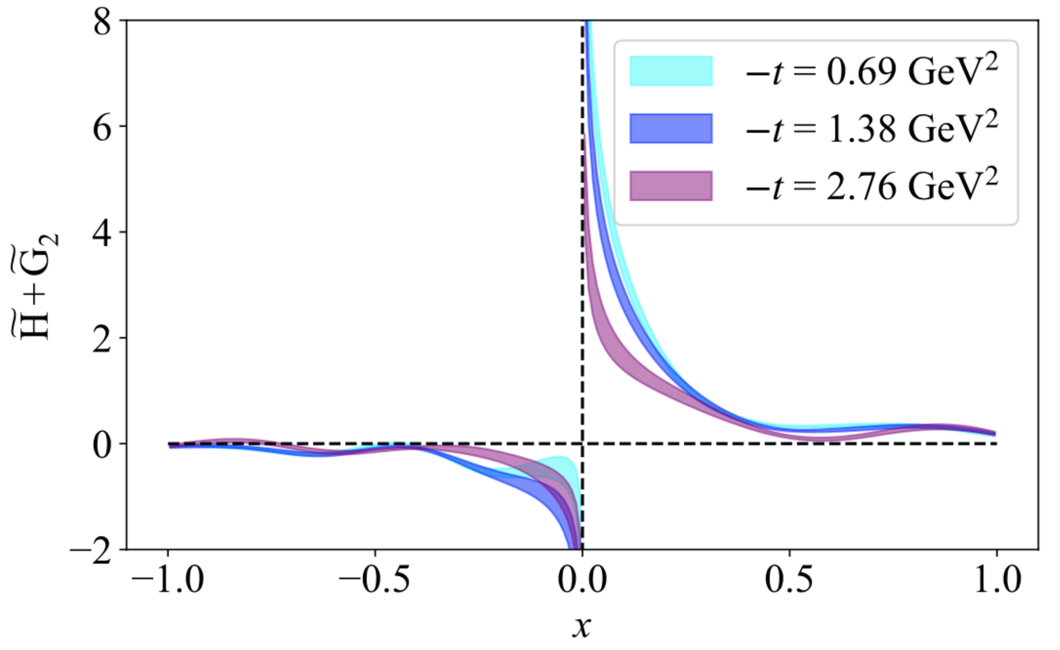

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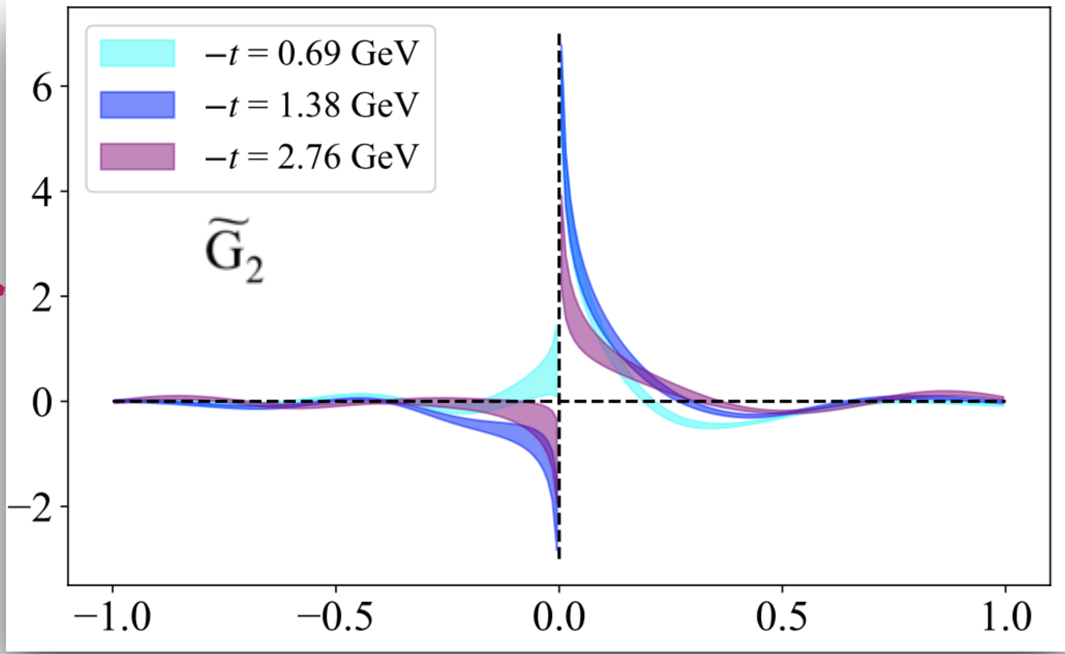

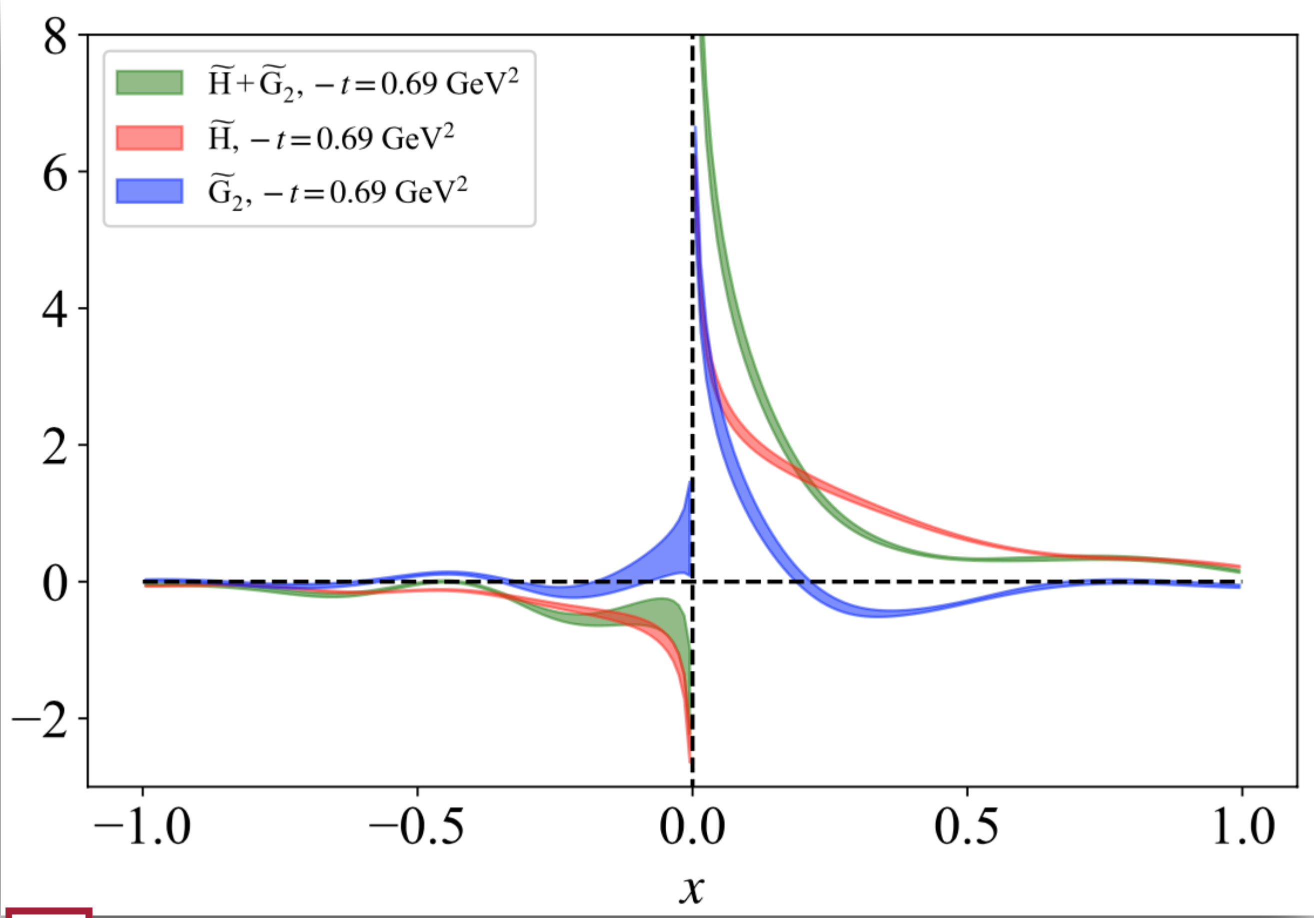

Negative areas in $\widetilde{G}_2$ theoretically anticipated:

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

Lattice Results - light-cone GPDs

★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness

$$P^\mu \frac{\Delta^3 \gamma_5}{2mP^0} F_{\widetilde{E}}(x,\xi,t;P^3)$$

★ Glimpse into $\widetilde{E}$-GPD through twist-3 :

Lattice Results - light-cone GPDs

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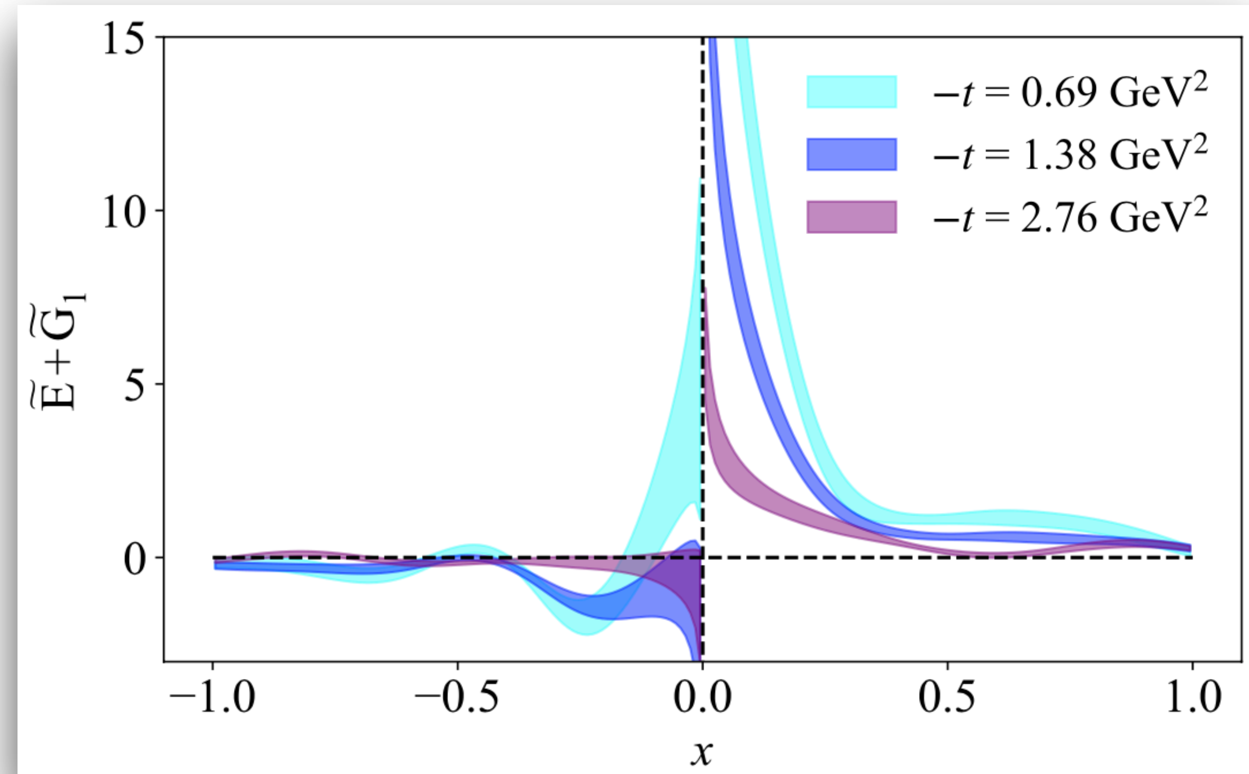

★ Sizable contributions as expected

$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

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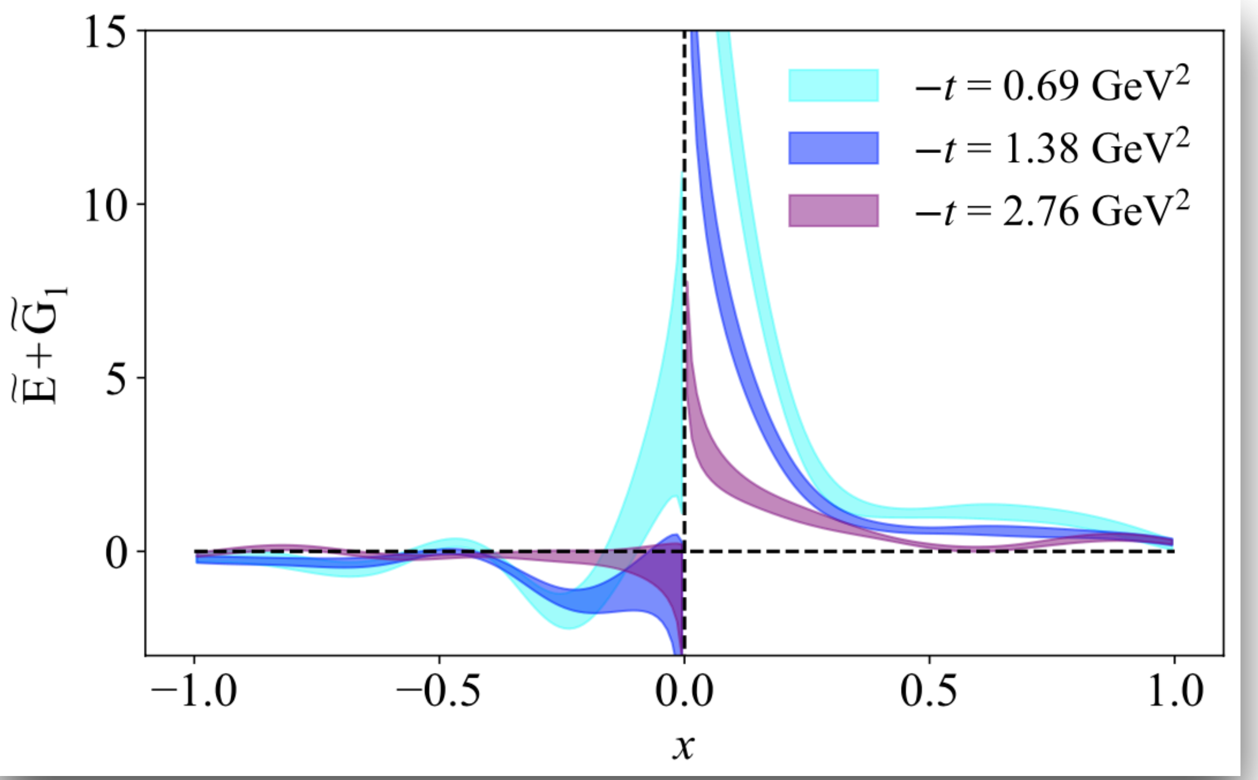

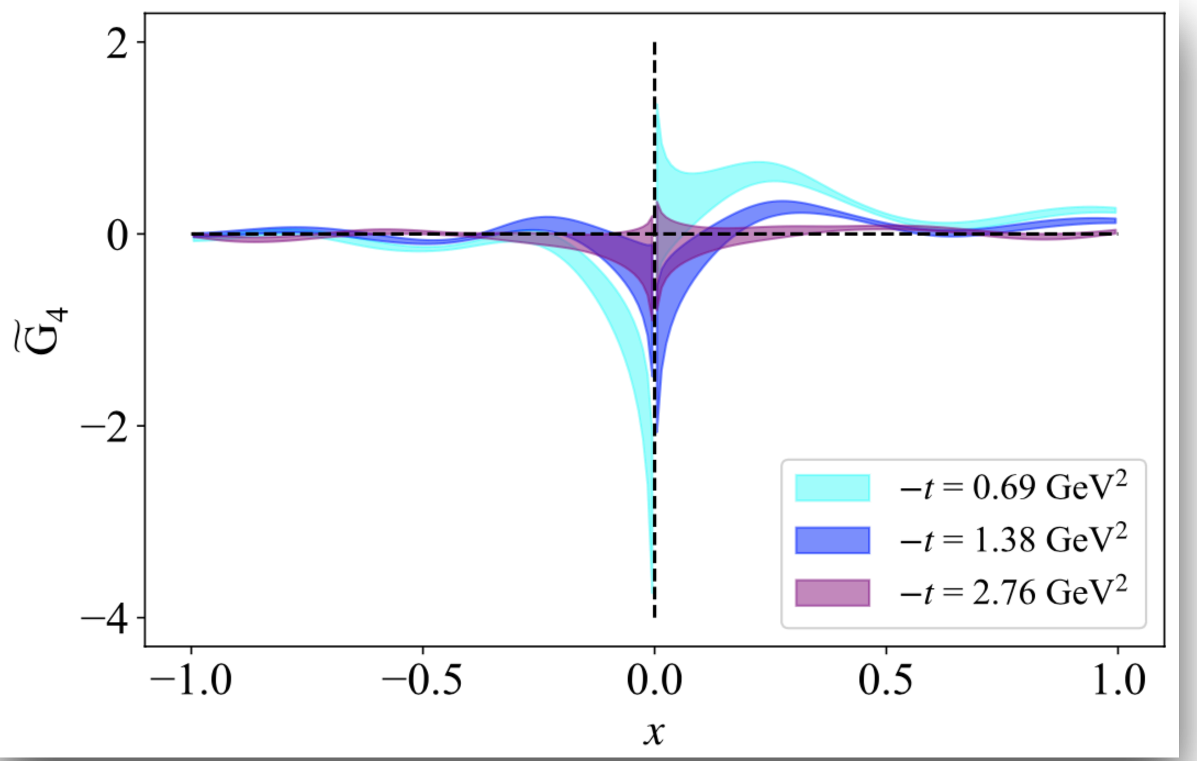

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★ $\widetilde{G}_4$ very small; no theoretical argument to be zero

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4} G_E$$

Extension of calculation

★ Alternative kinematic setup can be utilized

$$F_{\widetilde{H}+\widetilde{G}_2} = \frac{1}{2m^2}\frac{z_3 P_0^2 (\Delta_\perp)^2}{P_3} + A_2$$

$$F_{\widetilde{E}+\widetilde{G}_1} = \frac{2 z_3 P_0^2}{P_3} + 2A_5$$

$$F_{\widetilde{G}_3} = \frac{1}{2m^2}\left(z_3 P_0^2 \Delta_3 - z_3 P_3 P_0 \Delta_0\right) A_1 - z_3 P_3 A_8$$

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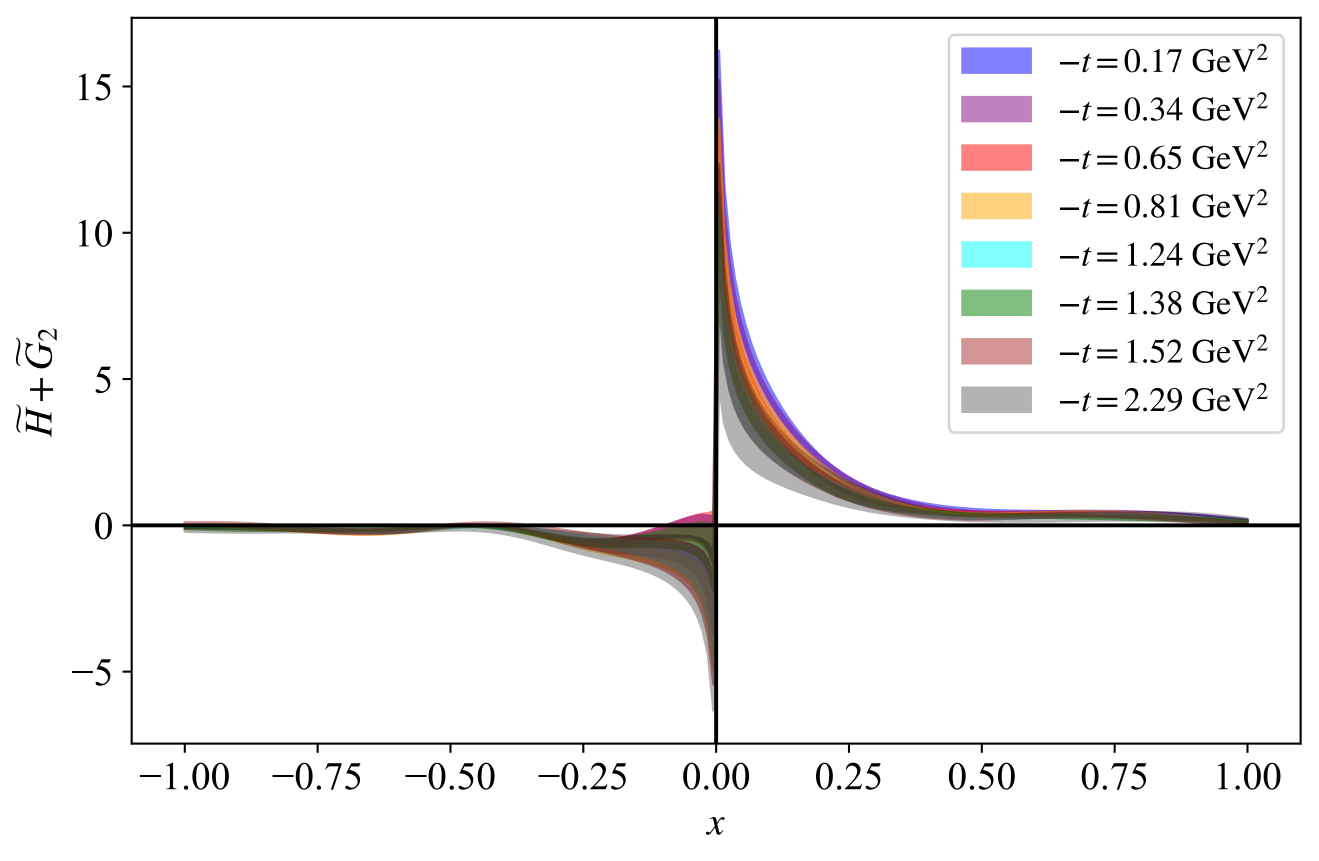

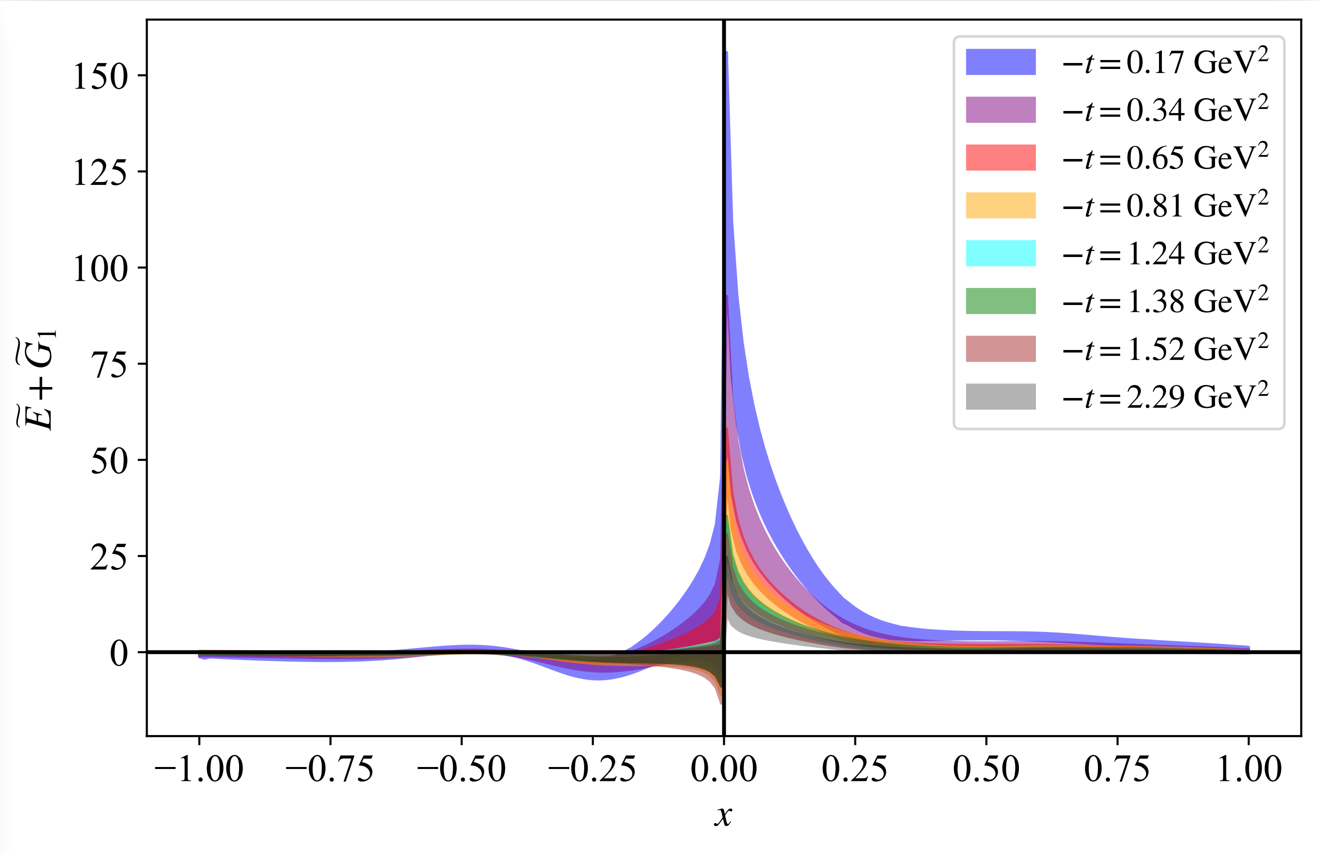

On-going work

How to lattice QCD data fit into the overall effort for hadron tomography

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★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

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QUARK-GLUON TOMOGRAPHY COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science

Award Number: DE-SC0023646

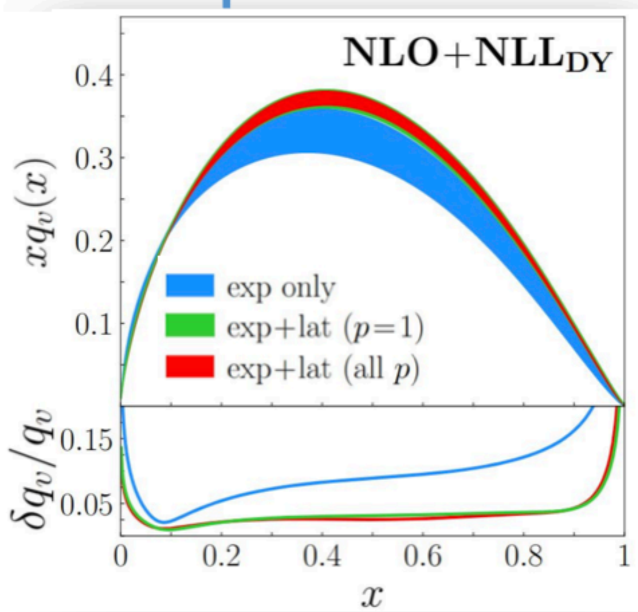
1. Theoretical studies of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties

2. Lattice QCD calculations of GPDs and related structures

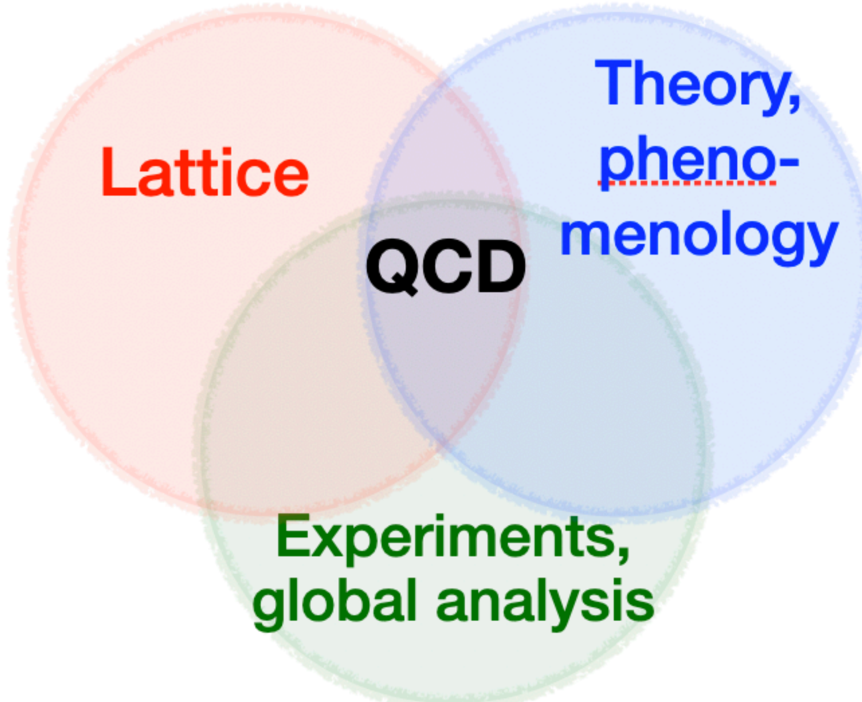
3. Global analysis of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

Synergies: constraints & predictive power of lattice QCD

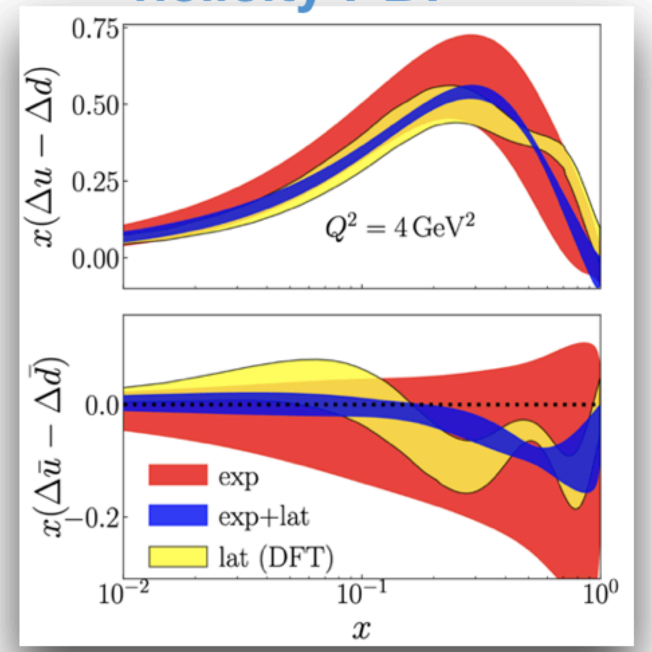
pion PDF



[JAM/HadStruc, PRD105 (2022) 114051]

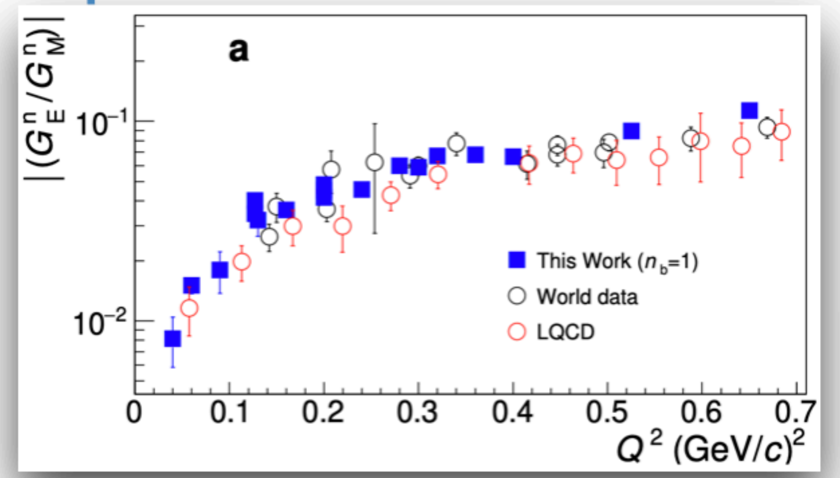


helicity PDF



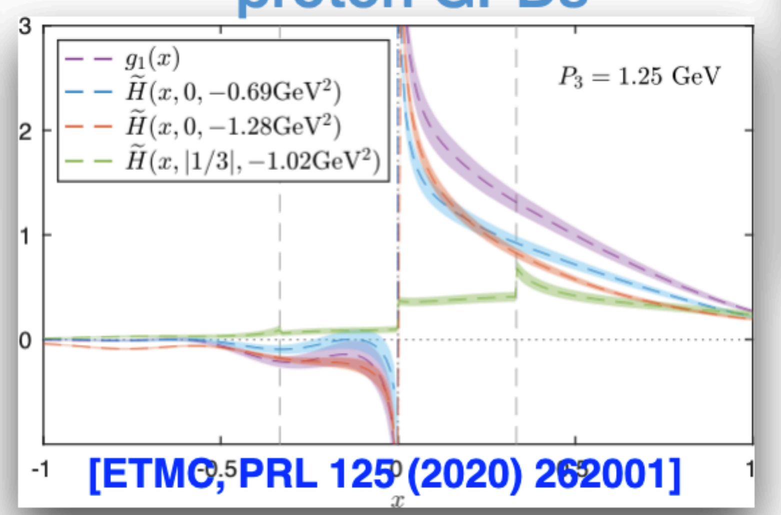
[JAM & ETMC, PRD 103 (2021) 016003]

proton & neutron radius



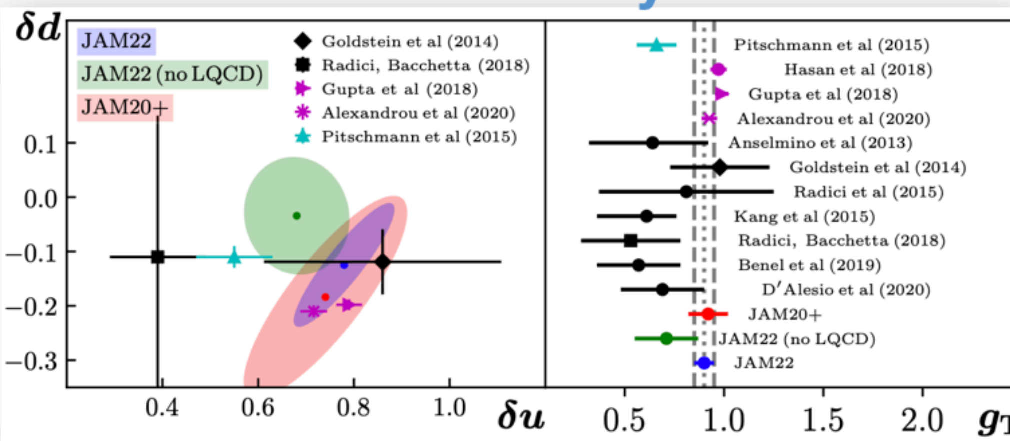
[Atac et al., Nature Comm. 12, 1759 (2021)]

proton GPDs



[ETMC, PRL 125 (2020) 262001]

transversity PDF



[JAM, PRD 106 (2022) 3, 034014]

And many more!

Summary

- ★ Definitions of quasi-GPDs on Euclidean lattice: intrinsically frame dependent. Historically used symmetric frame is computationally very expensive

- ★ Novel Lorentz covariant decomposition has great advantages:
  - access to symmetric-frame GPDs from matrix elements in any frame
  - significant reduction of computational cost
  - access to a broad range of t and ξ

- ★ Numerical results demonstrate the validity of the approach

- ★ Future calculations have the potential to transform the field of GPDs

- ★ Lattice QCD data on GPDs will play an important role in the pre-EIC era and can complement experimental efforts of JLab@12GeV.

- ★ Synergy with phenomenology is an exciting prospect!

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Thank you

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QUARK-GLUON TOMOGRAPHY COLLABORATION
Award Number: DE-SC0023646

U.S. DEPARTMENT OF ENERGY | Office of Science
DOE Early Career Award (NP)
Grant No. DE-SC0020405

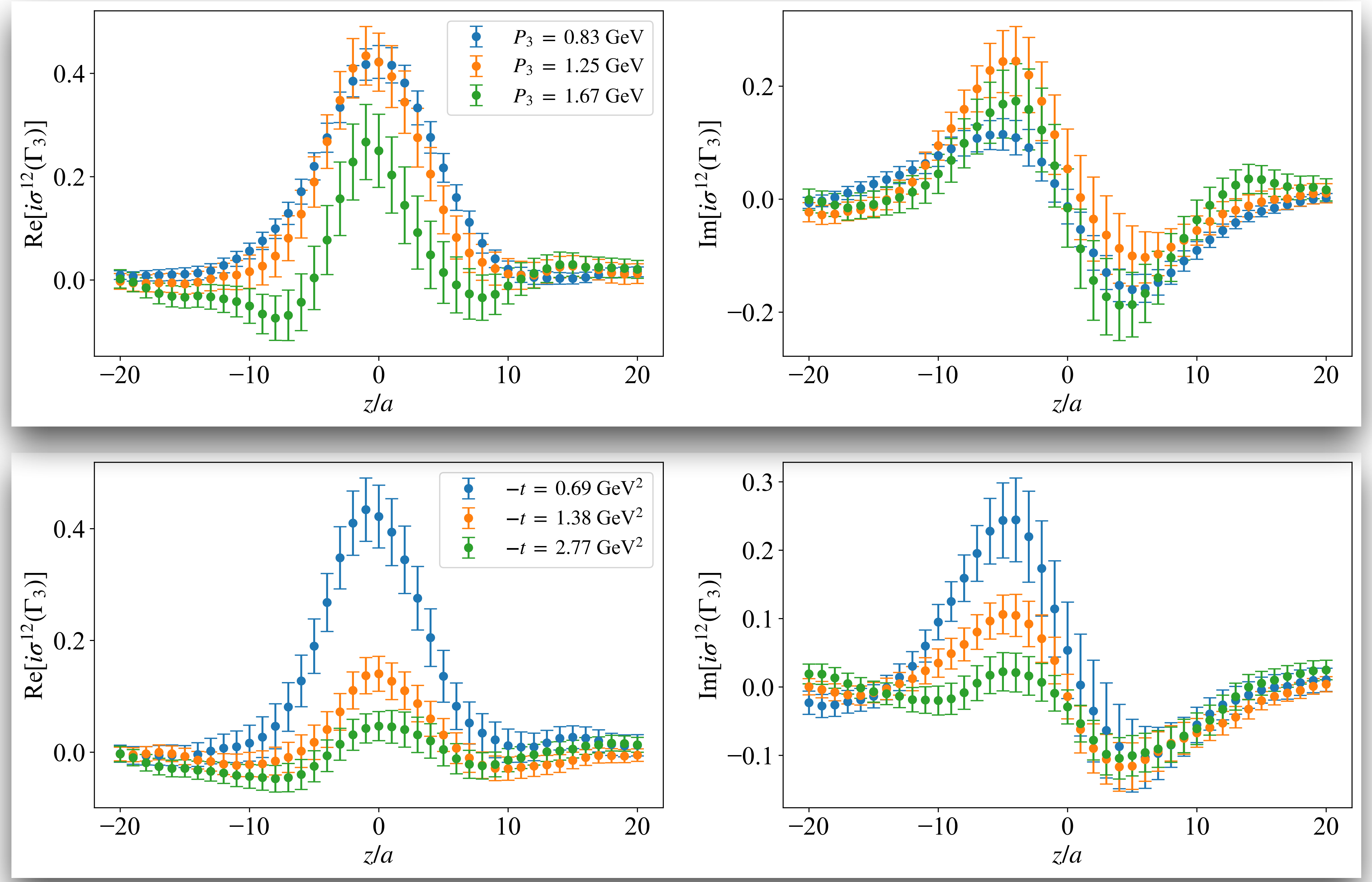
U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING
NERSC
ASCR Leadership Computing Challenge ALCC

Miscellaneous

Extension to twist-3 tensor GPDs

★ **Parametrization** [Meissner et al., *JHEP* 08 (2009) 056]

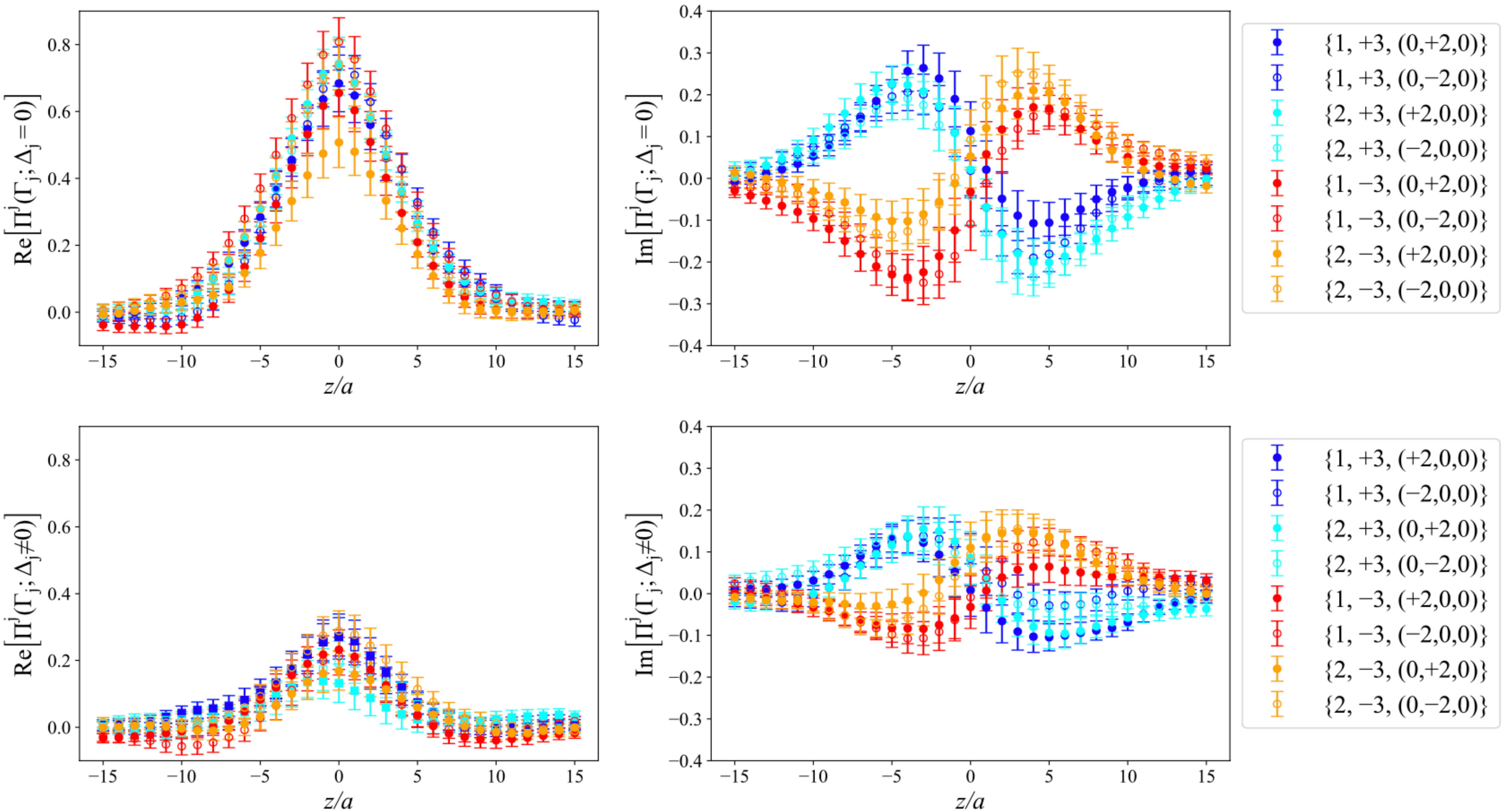
$$F^{[\sigma^{+-}\gamma_5]} = \bar{u}(p')\left(\gamma^+\gamma_5\,\widetilde{H}_2' + \frac{P^+\gamma_5}{M}\,\widetilde{E}_2'\right)u(p)$$



Lattice Results - Matrix Elements

★ Bare matrix elements

$$\Pi^1(\Gamma_1) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2}\,\frac{(4m(E+m)+\Delta_y^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\,\frac{\Delta_x^2(E+m)}{8m^3} + F_{\widetilde{G}_4}\,\frac{\mathrm{sign}[P_3]\,\Delta_y^2(E+m)}{4m^2 P_3}\right)$$



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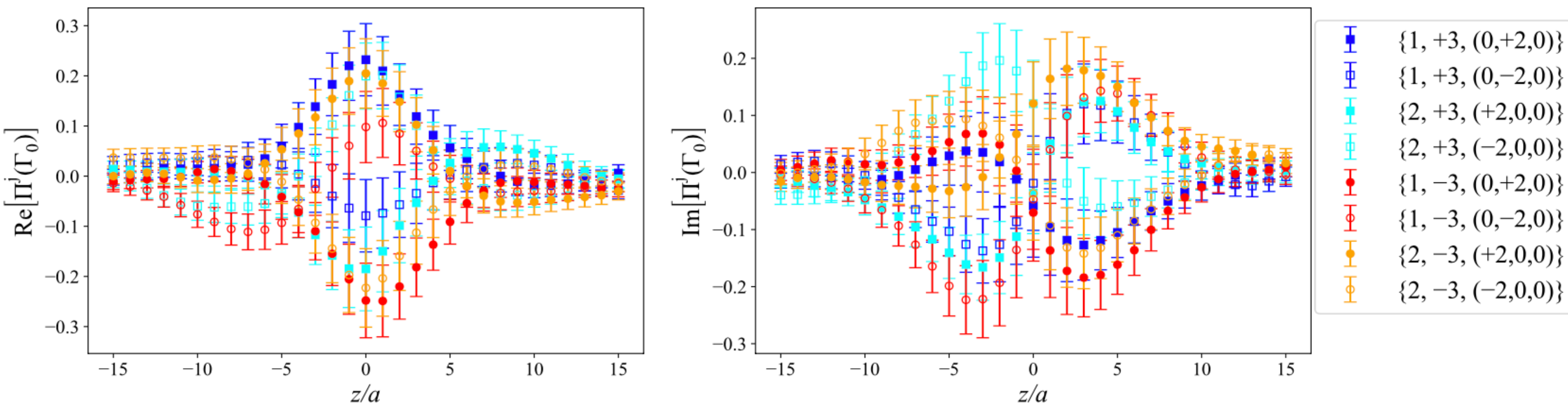

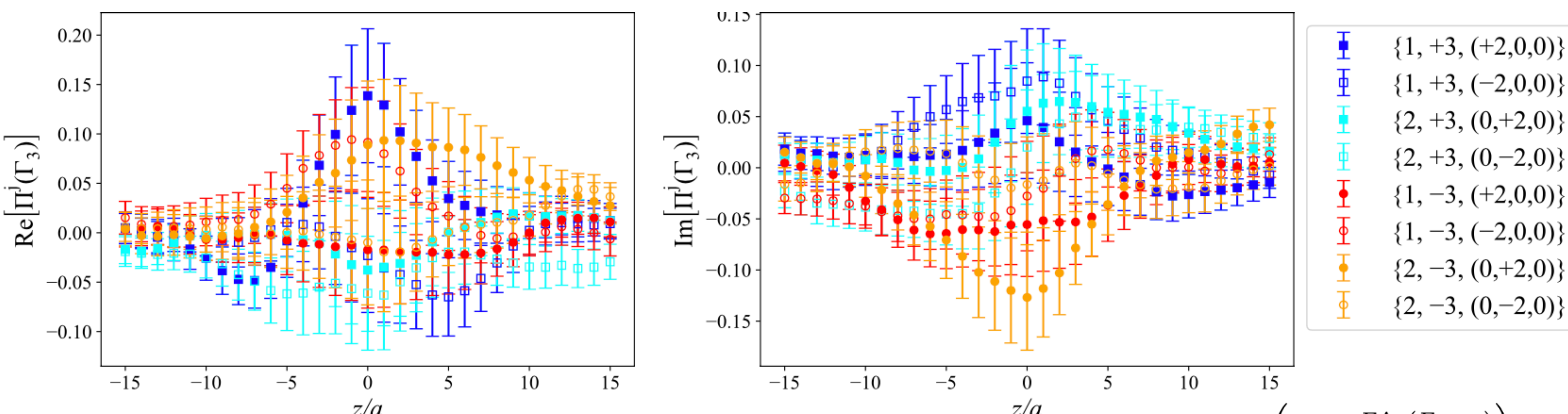

$$\Pi^1(\Gamma_3) = C\left(- F_{\widetilde{G}_3} \frac{E\Delta_x(E+m)}{2m^2 P_3} \right)$$

★ Suppressed signal compared to $\gamma_+ \gamma_5$ operators

Consistency checks

★ Norms satisfied   encouraging results

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

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★ Alternative kinematic setup can be utilized

[Fernanda Steffens]

$$F_{\widetilde{H}+\widetilde{G}_2} = \frac{1}{2m^2}\frac{z_3 P_0^2 (\Delta_\perp)^2}{P_3} + A_2$$

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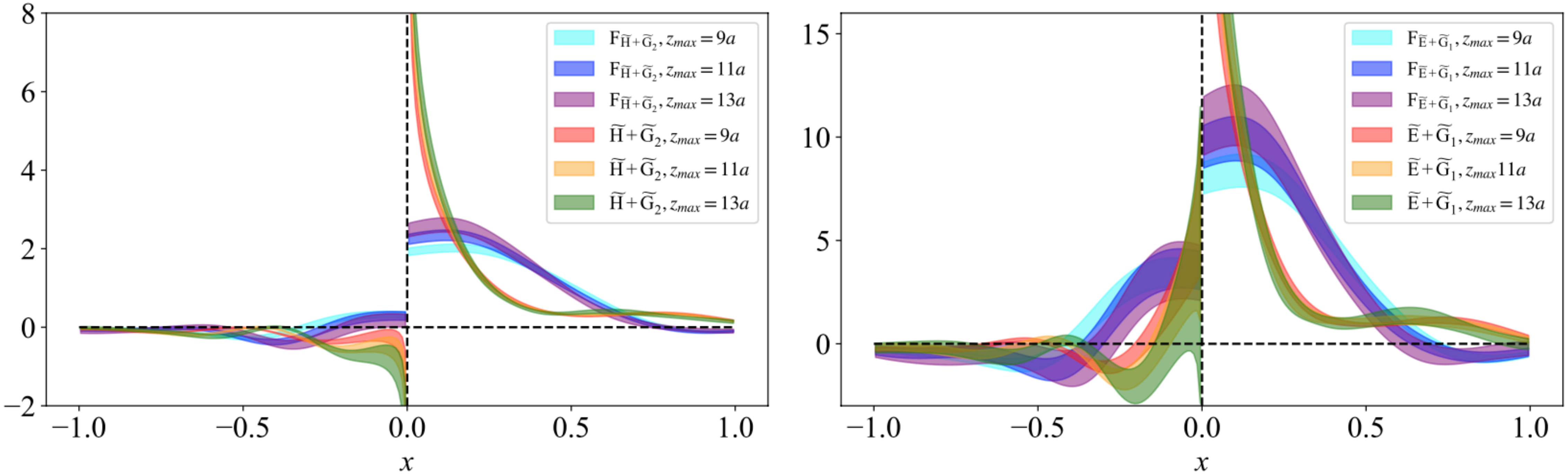

FIG. 10. z_{max} dependence of $F_{\widetilde{H}+\widetilde{G}_2}$ and $\widetilde{H}+\widetilde{G}_2$ (left), as well as $F_{\widetilde{E}+\widetilde{G}_1}$ and $\widetilde{E}+\widetilde{G}_1$ (right) at $-t = 0.69$ GeV^2 and $P_3 = 1.25$ GeV. Results are given in the $\overline{\text{MS}}$ scheme at a scale of 2 GeV.

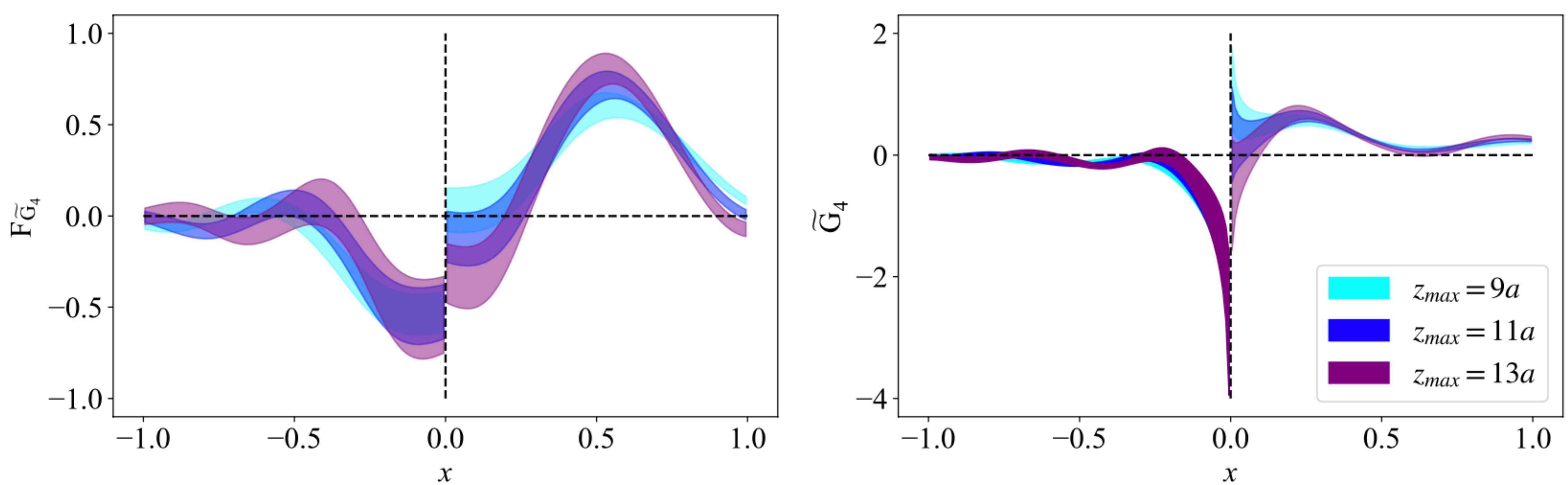

FIG. 11. z_{max} dependence of $F_{\widetilde{G}_4}$ and $\widetilde{G}_4$ at $-t = 0.69$ GeV^2 and $P_3 = 1.25$ GeV. Results are given in $\overline{\text{MS}}$ scheme at a scale of 2 GeV.