A separable Bethe-Salpeter approach to deuteron structure

Adam Freese
Thomas Jefferson National Accelerator Facility

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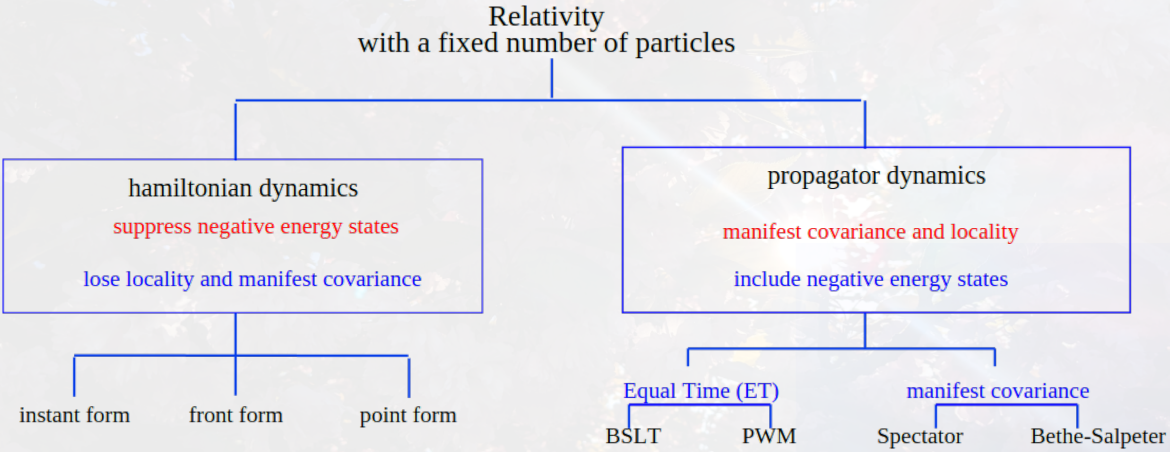
Introduction

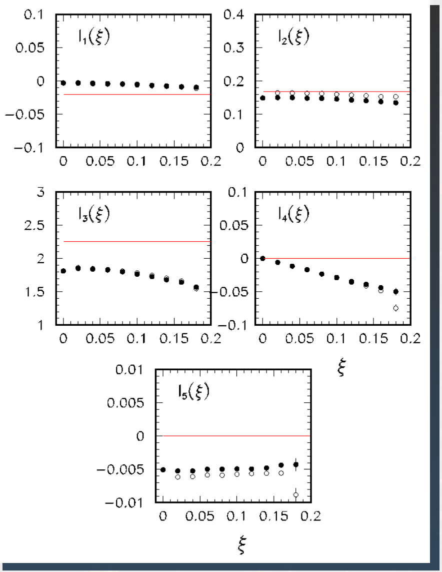
- **Main idea**: deuteron structure in a separable Bethe-Salpeter approach.
  - **Separability** means the Bethe-Salpeter equation can be solved.
- Solving a Bethe-Salpeter equation has several benefits:
  - Ensures covariance.
  - Ensures correct normalization.
  - Allows two-body currents to be derived from Lagrangian.
- The true NN interaction isn't separable—need a model.
  - This talk is about such a model.

The variety of approaches



- Figure above from Gilman & Gross, AIP Conf. Proc. 603 (2001) 55
- This talk is about a **Bethe-Salpeter** approach. (but without locality)

Why covariance matters



- Generalized parton distributions exhibit **polynomiality**.

$$\int \mathrm{d}x\, x H_1(x,\xi,t) = \mathcal{G}_1(t) + \xi^2 \mathcal{G}_3(t) \qquad \text{etc.}$$

  - Required for unambiguous extraction of energy-momentum tensor from GPDs.
- Polynomiality requires covariance.
  - X. Ji, J. Phys. G24 (1998) 1181
- Fock expansion (standard method) violates covariance.
  - Example: landmark calculation of Cano and Pire EPJA 19 (2004) 423

Non-local Lagrangian

- Adapted from **non-local NJL model**.
  - Bowler & Birse, Nucl. Phys. A582 (1995) 655
  - Modified to be a nucleon-nucleon interaction.
- V and T currents in *isosinglet* channel:

$$B^{\mu}_{Vn}(x) = \frac{1}{2}\int \mathrm{d}^4 z\, f_n(z)\psi^{\mathsf{T}}\left(x+\frac{z}{2}\right) C^{-1}\tau_2\gamma^{\mu}\psi\left(x-\frac{z}{2}\right)$$

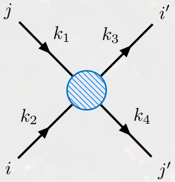
$$B^{\mu\nu}_{Tn}(x) = \frac{1}{2}\int \mathrm{d}^4 z\, f_n(z)\psi^{\mathsf{T}}\left(x+\frac{z}{2}\right) C^{-1}\tau_2\,\mathrm{i}\sigma^{\mu\nu}\psi\left(x-\frac{z}{2}\right)$$

  - $f_n(z)$ a spacetime form-factor; regulates UV divergences.
  - $f_n(z) \to \delta^{(4)}(z)$ gives (local) four-point contact interaction.
- Interaction Lagrangian:

$$\mathscr{L}_I = \sum_{n=1}^{N}\left\{g_{Vn}B^{\mu}_{Vn}(B_{Vn\mu})^* + \frac{1}{2}g_{Tn}B^{\mu\nu}_{Tn}(B_{Tn\mu\nu})^*\right\}$$

Kernel

- Momentum-space Feynman rule for interactions:



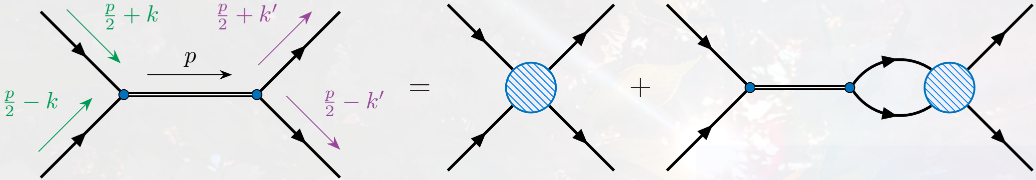
$$= \sum_{n=1}^{N} \left\{ g_{Vn} \gamma^{\mu} \otimes \gamma_{\mu} + \frac{g_{Tn}}{2} \sigma^{\mu\nu} \otimes \sigma_{\mu\nu} \right\} \widetilde{f}_n(k_1 - k_2) \widetilde{f}_n(k_3 - k_4)$$

  - **Separable interaction**: initial & final momentum dependence factorize.
  - (isospin and charge conjugation matrices suppressed for compactness)
- $\widetilde{f}_n(k)$ is Fourier transform of $f_n(z)$; I choose **Yukawa form**:

$$\widetilde{f}_n(k) \equiv \frac{\Lambda_n}{k^2 - \Lambda_n^2 + \mathrm{i}0}$$

  - Λ_n is the regulator scale (non-locality scale).
  - Each Λ_n can be different!

- Bethe-Salpeter equation (BSE) for T-matrix given by:



- **Separability** of interaction entails **separability** of T-matrix:

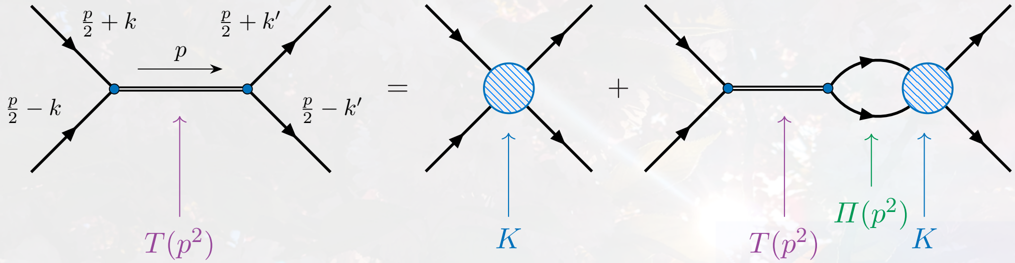
$$\mathcal{T}(p,k,k') = \sum_{\substack{n=1\\m=1}}^{N} \sum_{\substack{X=V,T\\Y=V,T}} \frac{\Lambda_n}{k'^2-\Lambda_n^2}\frac{\Lambda_m}{k^2-\Lambda_m^2}\gamma_X^{\mu}\otimes\gamma_{Y\,\mu}T_{XY}^{nm}(p^2)$$

- Helpful to arrange $T_{XY}^{nm}(p^2)$ into a matrix.

- Helpful to arrange $T^{nm}_{XY}(p^2)$ into a matrix.

$$T(p^2) = \begin{bmatrix} T^{11}_{VV}(p^2) & T^{11}_{VT}(p^2) & T^{12}_{VV}(p^2) & T^{12}_{VT}(p^2) & \cdots \\ T^{11}_{TV}(p^2) & T^{11}_{TT}(p^2) & T^{12}_{TV}(p^2) & T^{12}_{TT}(p^2) & \cdots \\ T^{21}_{VV}(p^2) & T^{21}_{VT}(p^2) & T^{22}_{VV}(p^2) & T^{22}_{VT}(p^2) & \cdots \\ T^{21}_{TV}(p^2) & T^{21}_{TT}(p^2) & T^{22}_{TV}(p^2) & T^{22}_{TT}(p^2) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

- $N \times N$ grid of 2×2 block matrices.
- With this, the Bethe-Salpeter equation will become an algebraic matrix equation!

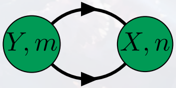


Kernel

$$K = \begin{bmatrix} g_{V1} & 0 & 0 & \dots \\ 0 & g_{T1} & 0 & \dots \\ 0 & 0 & g_{V2} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Bubble matrix

$\Pi(p^2) =$



$$T(p^2) = K - K\,\Pi(p^2)\,T(p^2)$$

Deuteron bound state pole

- T-matrix solution given by:

$$T(p^2) = (1 + K\varPi(p^2))^{-1} K$$

- Deuteron bound state pole exists where:

$$\det\big(1 + K\varPi(p^2 = M_D^2)\big) = 0$$

- Deuteron vertex from residues at pole:

$$T(p^2 \approx M_D^2) \approx -\frac{1}{p^2 - M_D^2} \begin{bmatrix} \alpha_1^2 & \alpha_1\beta_1 & \alpha_1\alpha_2 & \dots \\ \alpha_1\beta_1 & \beta_1^2 & \alpha_2\beta_1 & \dots \\ \alpha_1\alpha_2 & \alpha_2\beta_1 & \alpha_2^2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

  - α and β are coefficients in deuteron Bethe-Salpeter vertex.
  - Correct normalization automatically from solving Bethe-Salpeter equation.

- The result of all this is a deuteron Bethe-Salpeter vertex:

$$\Gamma_D^\mu(p,k) = \sum_{n=1}^{N} \frac{\Lambda_n}{k^2 - \Lambda_n^2 + \mathrm{i}0} \left\{ \alpha_n \left(\gamma^\mu - \frac{\not{p} p^\mu}{p^2} \right) + \beta_n \frac{\mathrm{i}\sigma^{\mu p}}{\sqrt{p^2}} \right\} C\tau_2$$

  - Simple k dependence fixed by separable interaction.
  - Can be used to **covariantly** calculate all sorts of observables.
- Reduces to a standard Yukawa parametrization in the non-relativistic limit!

$$\psi_{\mathrm{NR}}(\boldsymbol{k},\lambda) \sim \frac{-1}{\sqrt{8M_D}} \frac{\bar{u}(\boldsymbol{k},s_1)(\Gamma_D\cdot\varepsilon_\lambda)\bar{u}^{\mathsf{T}}(-\boldsymbol{k},s_2)}{\boldsymbol{k}^2 + m\epsilon_D} \to 4\pi\left\{ u(k)Y_{101}^\lambda(\hat{k}) + w(k)Y_{121}^\lambda(\hat{k}) \right\}$$

$$u(k) = \sum_{j=0}^{N} \frac{C_j}{\boldsymbol{k}^2 + B_j^2}$$

$$C_j = C_j(\alpha_n, \beta_n, \Lambda_n)$$

$$B_0 = \sqrt{m\epsilon_D}$$

$$w(k) = \sum_{j=0}^{N} \frac{D_j}{\boldsymbol{k}^2 + B_j^2}$$

$$D_j = D_j(\alpha_n, \beta_n, \Lambda_n)$$

$$B_n = \Lambda_n$$

Good behavior constraints

- Standard Yukawa paramatrization:

$$u(k) = \sum_{j=0}^{N} \frac{C_j}{\boldsymbol{k}^2 + B_j^2} \qquad w(k) = \sum_{j=0}^{N} \frac{D_j}{\boldsymbol{k}^2 + B_j^2}$$

- Require $B_0 = \sqrt{\epsilon_D m}$ for right large-distance behavior (**automatically satisfied!**)
- Require for correct short-distance behavior:

$$\sum_{j=0}^{N} C_j = \sum_{j=0}^{N} D_j = \sum_{j=0}^{N} D_j B_j^{-2} = \sum_{j=0}^{N} D_j B_j^2 = 0$$

  - These impose *extra constraints* on separable kernel.
  - Need $N \geq 3$ for non-zero D-wave.

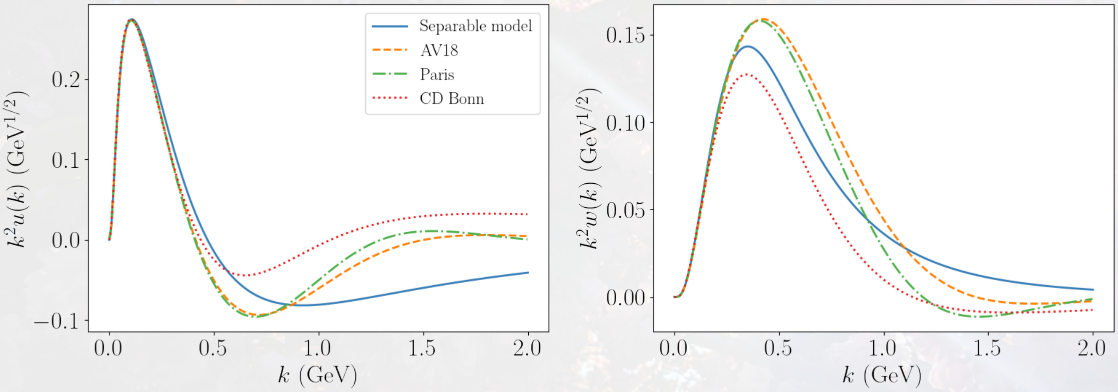
Minimal separable model

- Define **minimal separable model**:
- $N = \mathbf{3}$ to have non-zero D-wave.
- Pole in T-matrix at empirical M_D.
- Enforce good behavior constraints.
- Empirical D/S asymptotic ratio:

$$\lim_{r\to\infty} \frac{w(r)}{u(r)} = \frac{D_0}{C_0} = \eta_{D/S} = 0.0256(4)$$

- Fit unconstrained parameters to static electromagnetic properties.
  - Charge radius (r_d), magnetic moment (μ_d), quadrupole moment (Q_d)
  - In principle should fit to $^3S_1$-$^3D_1$ phase shifts, but I'm still working to get those.

Non-relativistic wave function



- Softer D-wave than AV18 & Paris
- Harder D-wave than CD-Bonn

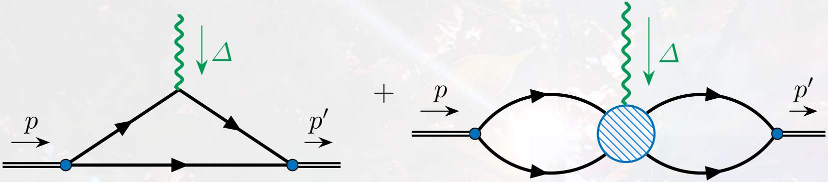
Things to do with this framework

- Phase shifts in nucleon-nucleon scattering (in progress)
- **Electromagnetic form factors** (**obtained!**)
- Collinear parton distributions (**obtained!**)
  - Includes b_1 structure function and EMC ratio.
- **Gravitational form factors** (**obtained!**)
  - Manifest covariance helpful here.
  - Previous non-covariant work (AF & Cosyn, PRD) found inconsistencies in EMT components.
- Generalized parton distributions (in progress)
  - GPDs are the **main goal** of this project.
  - Existing deuteron GPDs violate polynomiality.
  - Manifest covariance of this framework *guarantees* polynomiality.

Electromagnetic current of deuteron

- Sum of nucleon impulse and two-body currents:

$J_D^\mu(p,p') =$

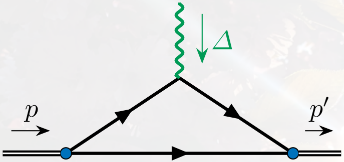


  - Two-body current uniquely determined by gauge invariance.
  - Non-local interaction requires Wilson lines.

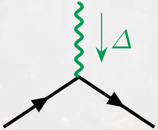
- Diagrams can be evaluated *exactly* in the separable model!
  - Symbolic algebra program needed though—results are *long* (hundreds of lines of generated Fortran code)
  - Results are covariant too.

Electromagnetic current: triangle diagram

- Nucleon impulse given by **triangle diagram**



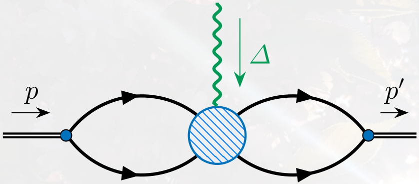
- Can use standard nucleon form factors in vertex:



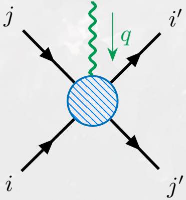
$$= \gamma^{\mu} F_{1N}(t) + \frac{i\sigma^{\mu\Delta}}{2m_N} F_{2N}(t)$$

  - Using Kelly parametrization.

- Two-body currents given by **bicycle diagram**



- Derived from Wilson lines in vertex:

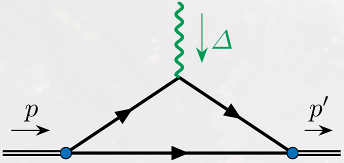


$$= e\sum_X g_X\Big\{\Big(\widetilde{h}^\mu_X(k,q) - \widetilde{h}^\mu_X(k,-q)\Big)\widetilde{f}_X(k') - \widetilde{f}_X(k)\Big(\widetilde{h}^\mu_X(k',q) - \widetilde{h}^\mu_X(k',-q)\Big)\Big\}$$

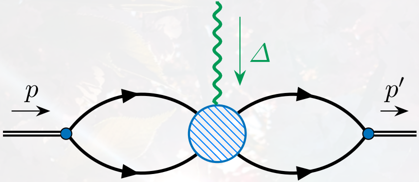
$$\times\left(\gamma^\nu_X C\tau_2\right)_{i'j'}\left(C^{-1}\tau_2\overline{\gamma}_{X\nu}\right)_{ij} F_{1N}(q^2) \quad : \quad \widetilde{h}^\mu_X(k,q) = \int_0^{+1} \mathrm{d}\tau\, \frac{\Lambda_X\left(k^\mu + \tau\frac{q^\mu}{2}\right)}{\left[\left(k+\tau\frac{q}{2}\right)^2 - \Lambda_X^2\right]^2}$$

- Standard form factor breakdown:

$$J_D^\mu(p,p') =$$



$$+$$



$$= -2P^\mu(\varepsilon\cdot\varepsilon'^*)G_1(t) + \left[\varepsilon'^{*\mu}(\varepsilon\cdot\Delta) - \varepsilon^\mu(\varepsilon'^*\cdot\Delta)\right]G_2(t) + \frac{P^\mu}{M_D^2}(\varepsilon\cdot\Delta)(\varepsilon'^*\cdot\Delta)G_3(t)$$

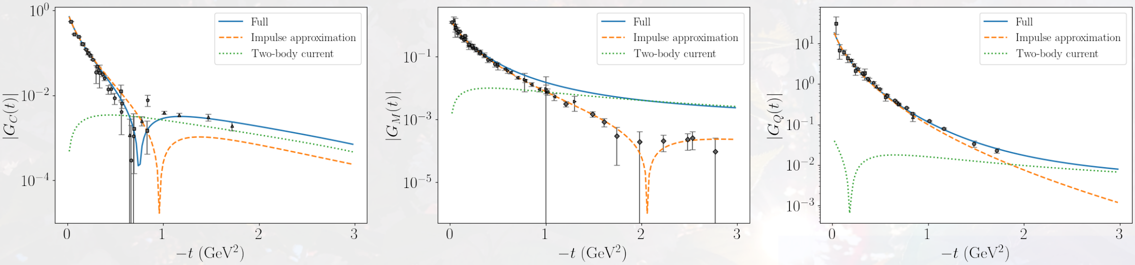
- Often use Sachs-like form factors:

$$G_C(t) = G_1(t) - \frac{t}{6M_D^2}G_Q(t)$$ **Coulomb** form factor

$$G_M(t) = G_2(t)$$ **magnetic** form factor

$$G_Q(t) = G_1(t) - G_2(t) + \left(1 - \frac{t}{4M_D^2}\right)G_3(t)$$ **quadrupole** form factor

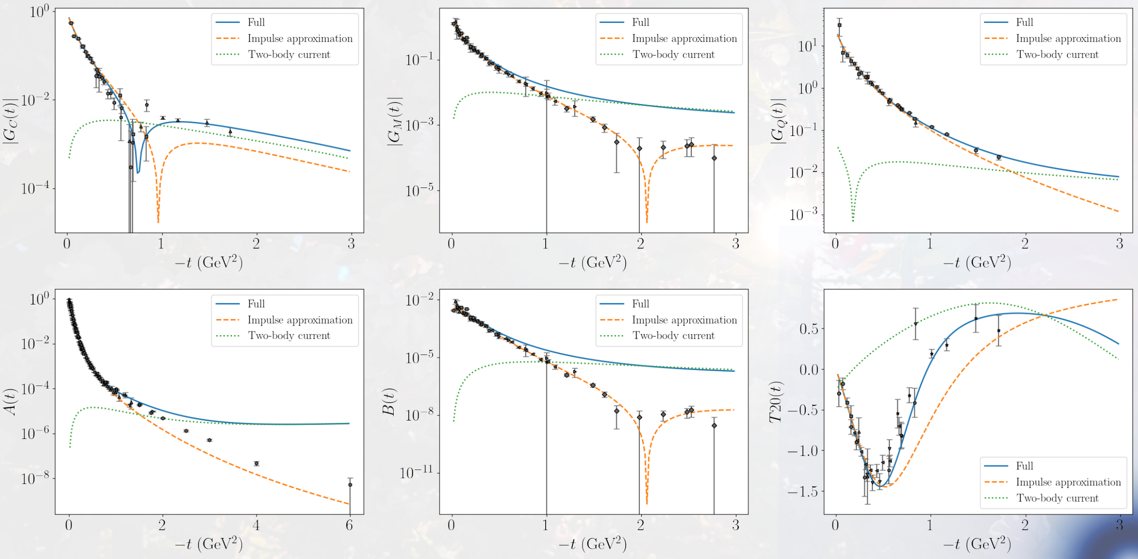
Model results for form factors



	Empirical	Model (total)	Impulse approx.	Two-body current
r_d (fm)	2.12799	2.126	2.127	-0.0776
μ_d (μ_N)	0.8574382284	0.876	0.876	0
Q_d (e-fm^2)	0.2859	0.296	0.286	0.010

- Model has mixed success.
  - Better at smaller $-t$.
- Two-body currents are significant.

Electromagnetic structure



- The energy-momentum tensor describes **density** and **flow** of energy & momentum.

Energy density

Momentum densities

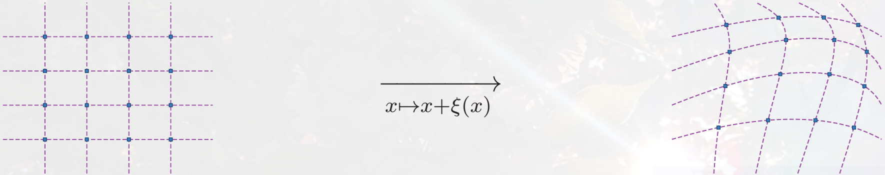
$$T^{\mu\nu}(x) = \begin{bmatrix} T^{00}(x) & T^{01}(x) & T^{02}(x) & T^{03}(x) \\ T^{10}(x) & T^{11}(x) & T^{12}(x) & T^{13}(x) \\ T^{20}(x) & T^{21}(x) & T^{22}(x) & T^{23}(x) \\ T^{30}(x) & T^{31}(x) & T^{32}(x) & T^{33}(x) \end{bmatrix}$$

Energy fluxes

Stress tensor

Noether's theorems and spacetime distortions

- Conserved current from *local* spacetime translations (**Noether's second theorem**):



  - **Noether's theorems**: symmetries imply conservation laws
  - *Local* translation: move spacetime differently everywhere

- The **energy-momentum tensor** is a response to these deformations

$$\Delta S_{\text{QCD}} = \int \mathrm{d}^4 x \, T^{\mu\nu}_{\text{QCD}}(x) \partial_\mu \xi_\nu(x)$$

  - Conserved if the action is invariant
  - Basically, equivalent to doing a gravitational gauge transform.

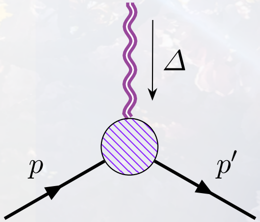
Gravitational form factors

- The energy-momentum tensor is parametrized using **gravitational form factors**
  - It's just a name.
  - The energy-momentum tensor is the source of gravitation
  - But we don't really use gravitation to measure them
- Spin-zero example:

$$\langle p' | \hat{T}^{\mu\nu}(0) | p \rangle = 2P^\mu P^\nu A(t) + \frac{1}{2}(\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}) D(t)$$

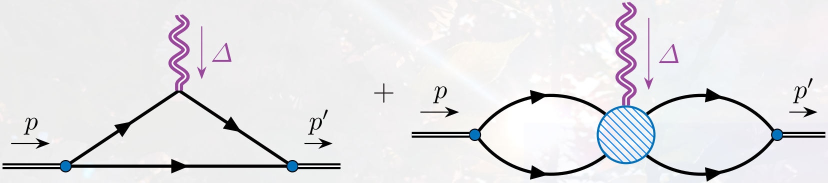
$$P^\mu = \frac{1}{2}\left(p^\mu + p'^\mu\right)$$

- $A(t)$ encodes momentum density
- $D(t)$ encodes stress distributions (anisotropic pressures)
- Mix of both encodes energy density



- Sum of nucleon impulse and two-body currents:

$$T_D^{\mu\nu}(p, p') =$$

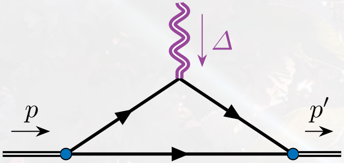


  - Two-body current uniquely determined via Noether's second theorem.

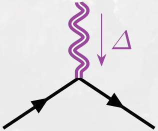
- Diagrams can be evaluated *exactly* in the separable model!
  - Symbolic algebra program needed though—results are *long* (hundreds of lines of generated Fortran code)
  - Results are covariant too.

Energy-momentum currents: triangle diagram

- Nucleon impulse given by **triangle diagram**



- Can use nucleon form factors in vertex:

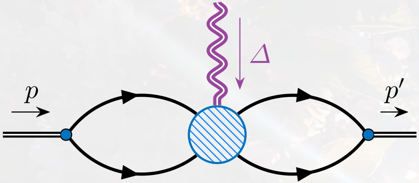


$$= \gamma^\mu P^\nu A_N(t) + \frac{\mathrm{i}\sigma^{\mu\Delta}P^\nu}{2m_N}B_N(t) + \frac{\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu}}{4m_N}D_N(t)$$

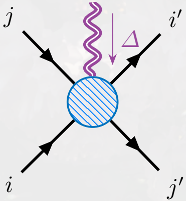
$$- \frac{\mathrm{i}\varepsilon^{\mu\nu\Delta\rho}\gamma_\rho\gamma_5}{2}L_N(t) - (\not{k} - m_N)g^{\mu\nu}A_N(t)$$

  - Using Mamo-Zahed model for nucleon form factors.

- Two-body currents given by **bicycle diagram**



- Derived from Noether's second theorem:



$$= \frac{1}{2}\sum_X g_X \left\{ \left[\left(\frac{p}{2} - k\right)^\nu \widetilde{h}^\mu_X(k,\Delta) - \left(\frac{p}{2} + k\right)^\nu \widetilde{h}^\mu_X(k,-\Delta) \right] \widetilde{f}_X(k') \right.$$

$$\left. + \left[\left(\frac{p}{2} - k'\right)^\nu \widetilde{h}^\mu_X(k',\Delta) - \left(\frac{p}{2} + k'\right)^\nu \widetilde{h}^\mu_X(k',-\Delta) \right] \widetilde{f}_X(k) \right\} \left(\gamma^\nu_X C\tau_2\right)_{i'j'} \left(C^{-1}\tau_2\overline{\gamma}_{X\nu}\right)_{ij} - g^{\mu\nu} \times \text{kernel}$$

  - Treat spinors like scalars under local translation; get asymmetric EMT.
  - Use $A_N(t)$ to fold in nucleon structure.

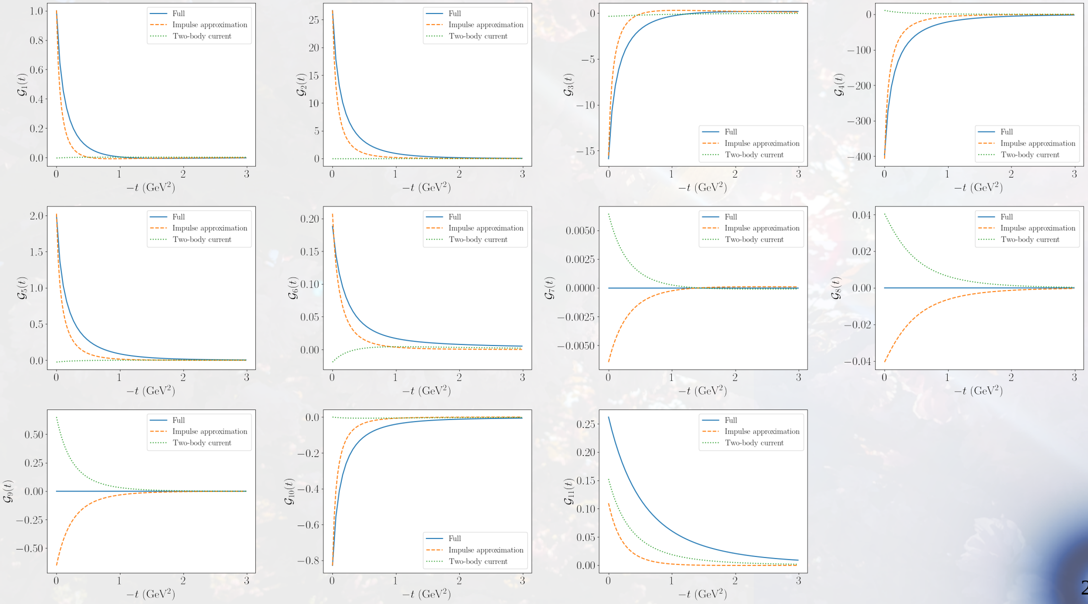
Gravitational form factors for spin-one targets

- Spin-one breakdown:

$$
\begin{aligned}
\langle p',\lambda'|T^{\mu\nu}(0)|p,\lambda\rangle = & -2P^\mu P^\nu \left[(\varepsilon'^*\cdot\varepsilon)\mathcal{G}_1(t) - \frac{(\Delta\cdot\varepsilon'^*)(\Delta\cdot\varepsilon)}{2M_D^2}\mathcal{G}_2(t)\right] \\
& - \frac{1}{2}(\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu})\left[(\varepsilon'^*\cdot\varepsilon)\mathcal{G}_3(t) - \frac{(\Delta\cdot\varepsilon'^*)(\Delta\cdot\varepsilon)}{2M_D^2}\mathcal{G}_4(t)\right] \\
& + P^{\{\mu}\left(\varepsilon'^{*\nu\}}(\Delta\cdot\varepsilon) - \varepsilon^{\nu\}}(\Delta\cdot\varepsilon'^*)\right)\mathcal{G}_5(t) \\
& + \frac{1}{2}\left[\Delta^{\{\mu}\left(\varepsilon'^{*\nu\}}(\Delta\cdot\varepsilon) + \varepsilon^{\nu\}}(\Delta\cdot\varepsilon'^*)\right) - \varepsilon'^{*\{\mu}\varepsilon^{\nu\}}\Delta^2 - g^{\mu\nu}(\Delta\cdot\varepsilon'^*)(\Delta\cdot\varepsilon)\right]\mathcal{G}_6(t) \\
& + \frac{1}{2}\varepsilon'^{*\{\mu}\varepsilon^{\nu\}}M_D^2\mathcal{G}_7(t) + g^{\mu\nu}M_D^2(\varepsilon'^*\cdot\varepsilon)\mathcal{G}_8(t) + \frac{1}{2}g^{\mu\nu}(\Delta\cdot\varepsilon'^*)(\Delta\cdot\varepsilon)\mathcal{G}_9(t) \\
& + P^{[\mu}\left[\varepsilon'^{*\nu]}(\Delta\cdot\varepsilon) - \varepsilon^{\nu]}(\Delta\cdot\varepsilon'^*)\right]\mathcal{G}_{10}(t) + \frac{1}{2}\left[\Delta^\mu\left(\varepsilon'^{*\nu}(\Delta\cdot\varepsilon) + \varepsilon^\nu(\Delta\cdot\varepsilon'^*)\right) - \Delta^2\varepsilon'^{*\{\mu}\varepsilon^{\nu\}}\right]\mathcal{G}_{11}(t)
\end{aligned}
$$

- Well, that's a lot. (Eleven form factors!)
- $\mathcal{G}_7(t) = \mathcal{G}_8(t) = \mathcal{G}_9(t) = 0$ required by energy/momentum conservation!
- **Tensor for $\mathcal{G}_{11}(t)$ differs from Cosyn *et al.*** (EPJC 2019)
  - Mixed symmetry (not symmetric or antisymmetric).
  - Because symmetric & antisymmetric parts of EMT *not separately conserved*.

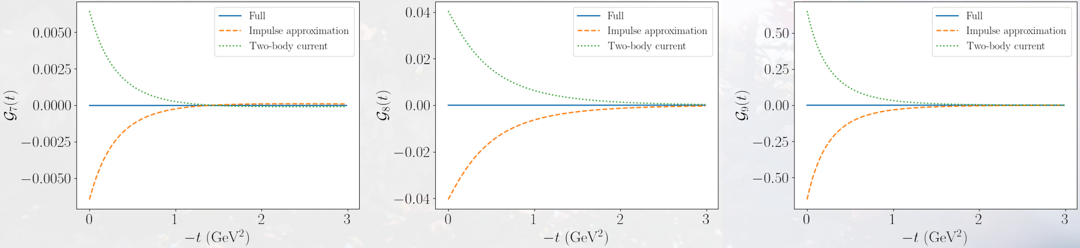
Big plot of all the form factors



- Momentum sum rule: $\mathcal{G}_1(0) = 1$
- Spin sum rule: $\mathcal{G}_5(0) = 2$

	Impulse approximation	Two-body current	Total
$\mathcal{G}_1(0)$	1.0031	-0.0031	1
$\mathcal{G}_5(0)$	2.0259	-0.0259	2

- Continuity equation: $\mathcal{G}_7(t) = \mathcal{G}_8(t) = \mathcal{G}_9(t) = 0$



- No sum rule for $\mathcal{G}_{11}(t)$ (unlike in QCD)
- **Two-body currents needed to satisfy all sum rules!**

Outlook

- Presented a covariant model of deuteron structure.
  - Separable kernel.
  - Bethe-Salpeter equation solvable in Minkowski spacetime.
  - Covariance means GPDs *will obey polynomiality*.
- Reproduced known deuteron properties in this framework.
  - Necessary sanity check.
  - **Two-body currents** (bicycle diagrams) must be accounted for!
- Much more to be done:
  - Energy-momentum densities (already have gravitational form factors!)
  - **Generalized parton distributions** (the main purpose of this project!)

Thank you for your time!

Backup slides

Quantum numbers in kernel

- Kernel encodes channels with multiple quantum numbers:

$$\gamma^\mu C \otimes C^{-1}\gamma_\mu = \left(\gamma^\mu - \frac{\not{p} p^\mu}{p^2}\right) C \otimes C^{-1} \left(\gamma_\mu - \frac{\not{p} p_\mu}{p^2}\right) + \frac{1}{p^2}\not{p} C \otimes C^{-1} \not{p}$$

spin-one spin-zero

$$\sigma^{\mu\nu} C \otimes C^{-1} \sigma_{\mu\nu} = \frac{1}{p^2}\sigma^{\mu p} C \otimes C^{-1} \sigma_{\mu p} + \left(\sigma^{\mu\nu} - \frac{\sigma^{\mu p} p^\nu - \sigma^{\nu p} p^\mu}{p^2}\right) C \otimes C^{-1} \left(\sigma^{\mu\nu} - \frac{\sigma^{\mu p} p^\nu - \sigma^{\nu p} p^\mu}{p^2}\right)$$

even parity odd parity

  - p is center-of-mass momentum (deuteron momentum)
- Need only structures with deuteron quantum numbers:

$$\gamma_V^\mu \equiv \gamma^\mu - \frac{\not{p} p^\mu}{p^2} \qquad \gamma_T^\mu \equiv \frac{i\sigma_{\mu p}}{\sqrt{p^2}}$$

  - Other structures fully decouple in the T-matrix equation!

Why F_{1N}?

- A free Lagrangian for pointlike nucleons might look like:

$$\mathscr{L} = \overline{\psi}(x)\left(\frac{\mathrm{i}}{2}\overleftrightarrow{\not\partial} - e\not{A}(x)\right)\psi(x) - \frac{e\kappa}{4m_N}F_{\mu\nu}(x)\overline{\psi}(x)\sigma^{\mu\nu}\psi(x)$$

  - Effectively, $F_{1N}(t) = 1$ and $F_{2N}(t) = \kappa$.
  - Anomalous magnetic moment in non-minimal Puali coupling.
- Smearing this over spacetime might look like:

$$\mathscr{L} = \frac{\mathrm{i}}{2}\overline{\psi}(x)\overleftrightarrow{\not\partial}\psi(x) - e\int \mathrm{d}^4y\,\widetilde{F}_{1N}(y)\overline{\psi}(x)\not{A}(x+y)\psi(x) - \frac{e}{4m_N}\int \mathrm{d}^4y\,\widetilde{F}_{2N}(y)F_{\mu\nu}(x+y)\overline{\psi}(x)\sigma^{\mu\nu}\psi(x)$$

  - Wilson lines should be smeared if minimal coupling is.
  - F_{1N} is what smears the non-minimal coupling.