

Structure of multi-particle states



RAÚL BRICEÑO

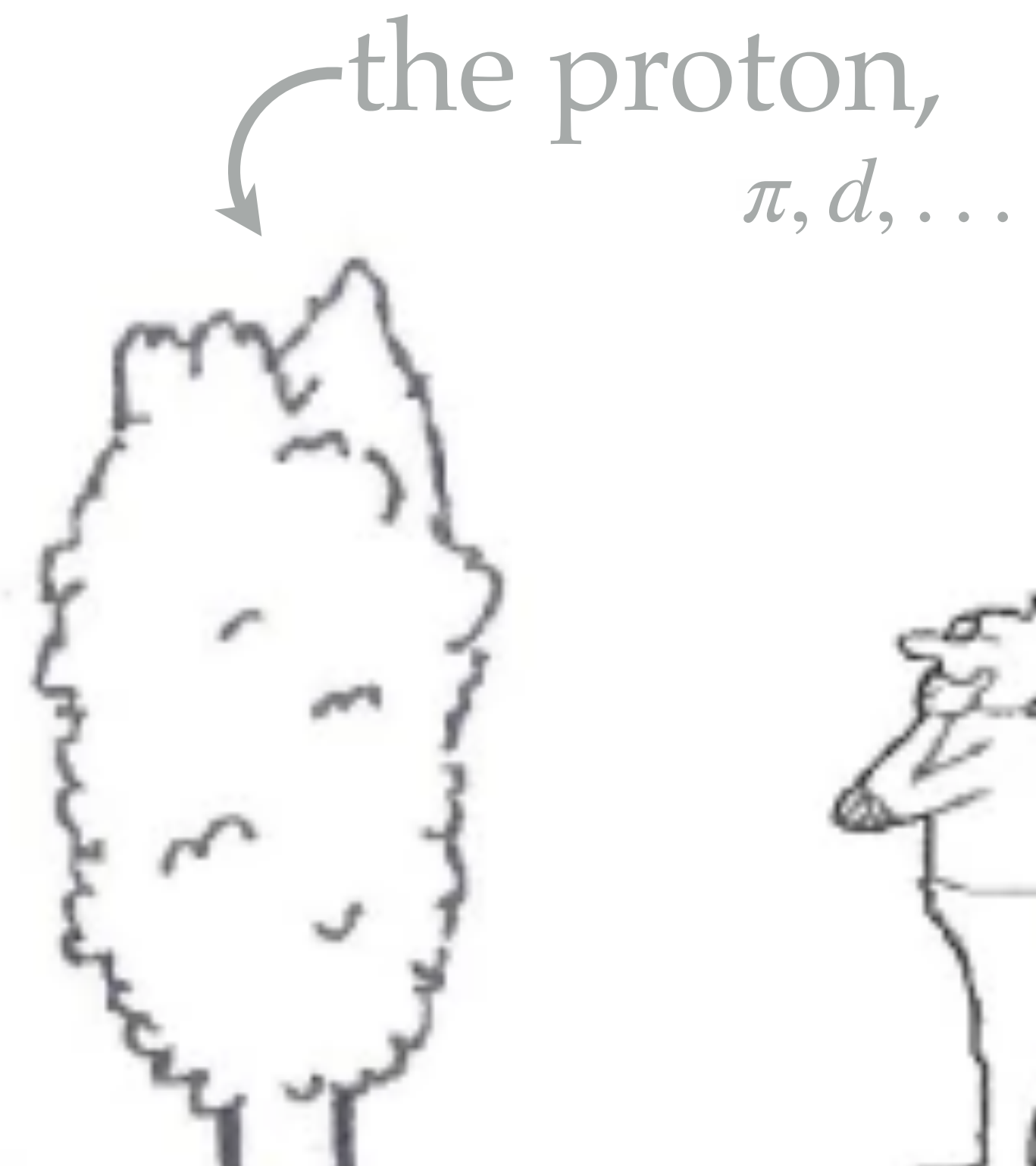
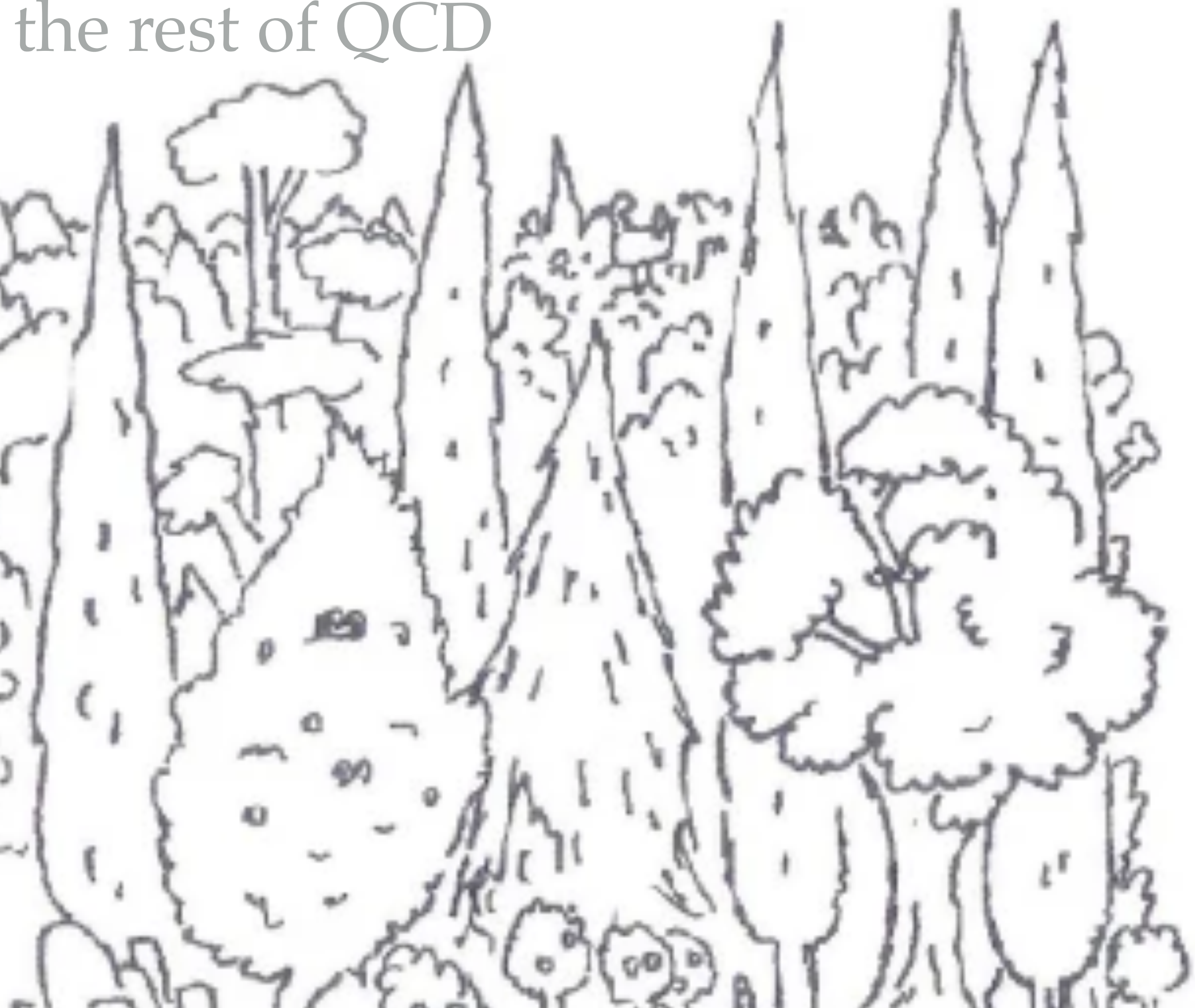
✉ rbriceno@berkeley.edu

🌐 <http://bit.ly/rbricenoPhD>

🐦 @RaulBriceno12

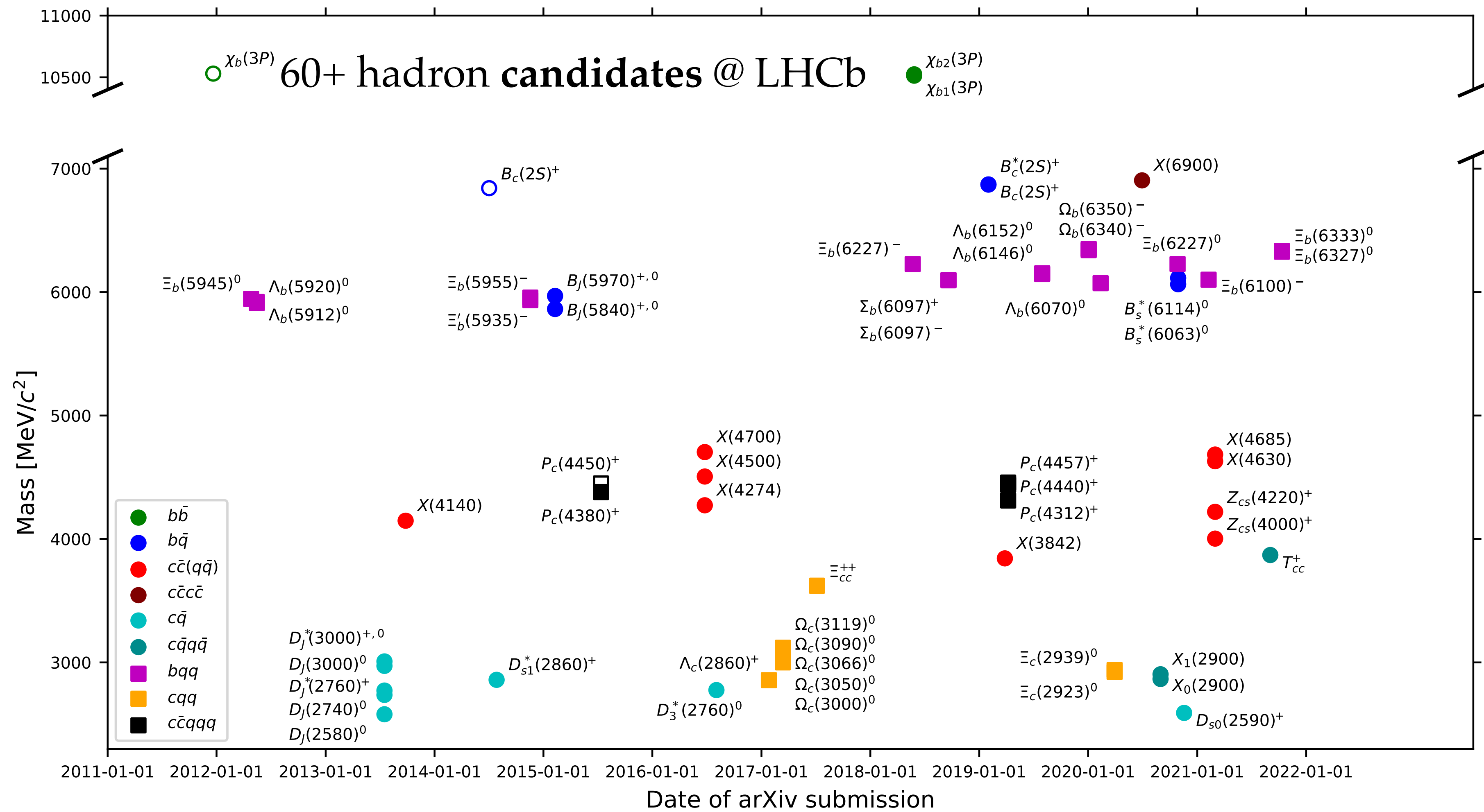
QCD is a rich forest with lots of trees deserving our attention

the rest of QCD



The particle zoo

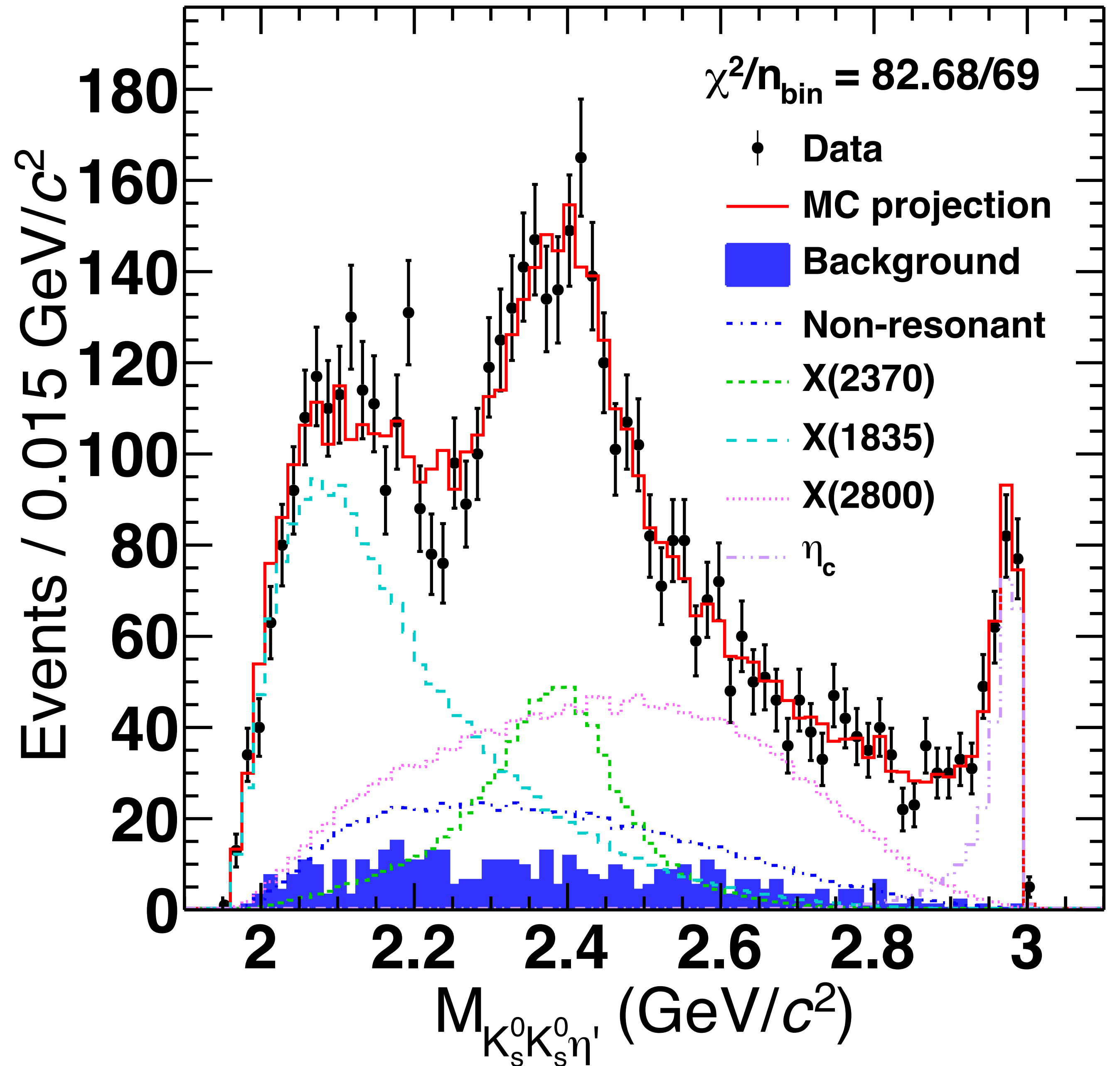
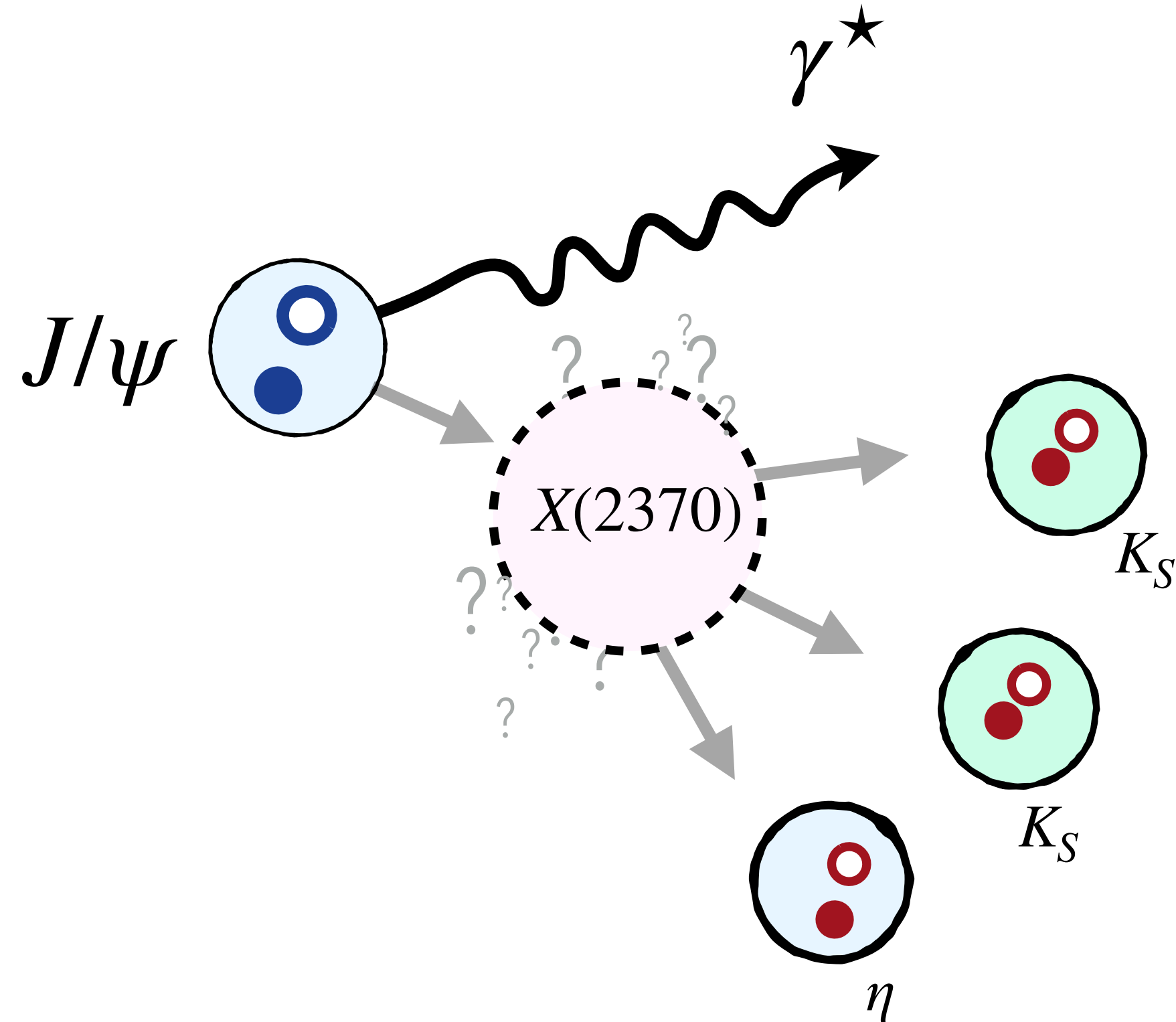
the remake



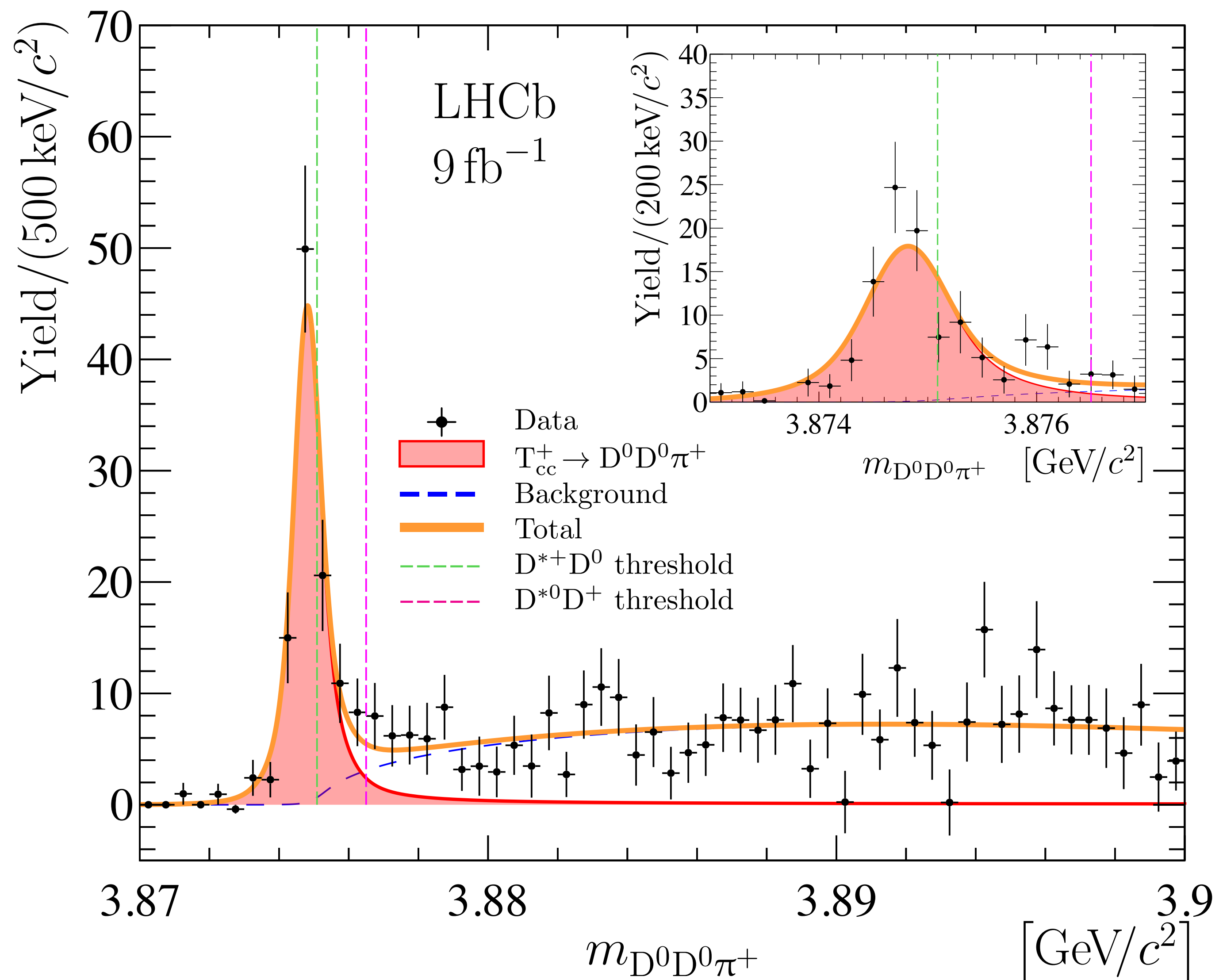
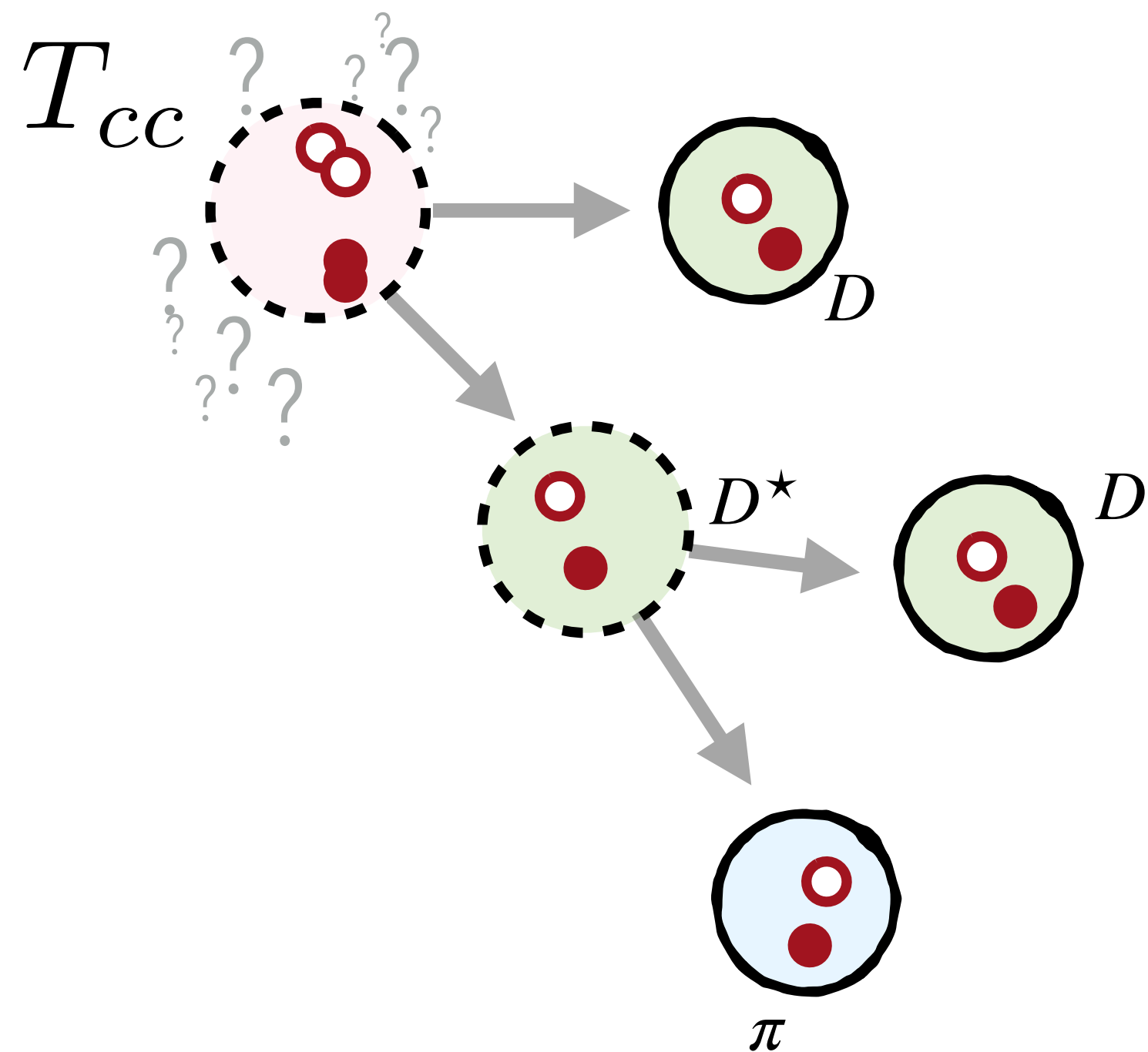
Numerous other experimental searches...



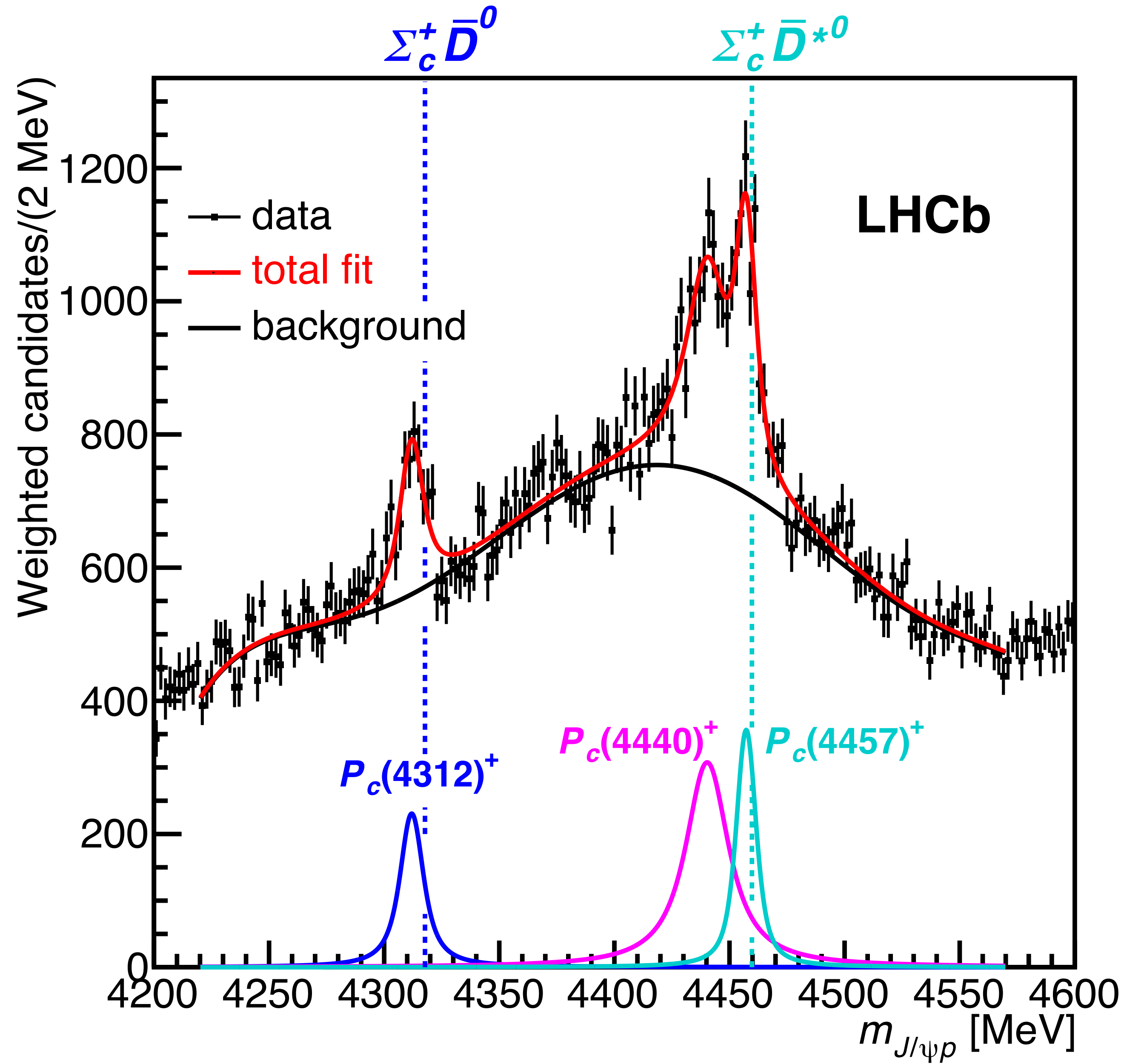
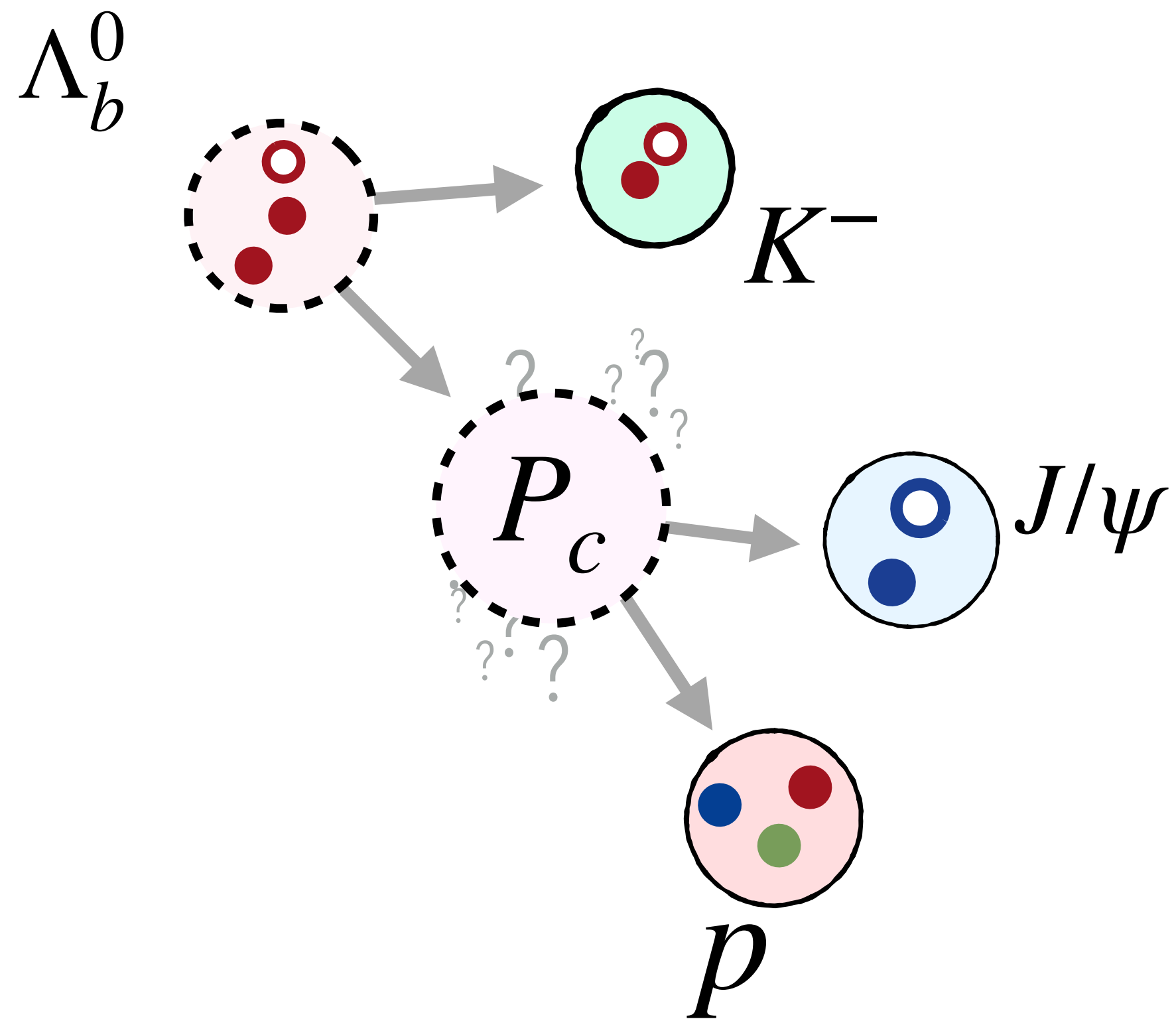
Glueballs?



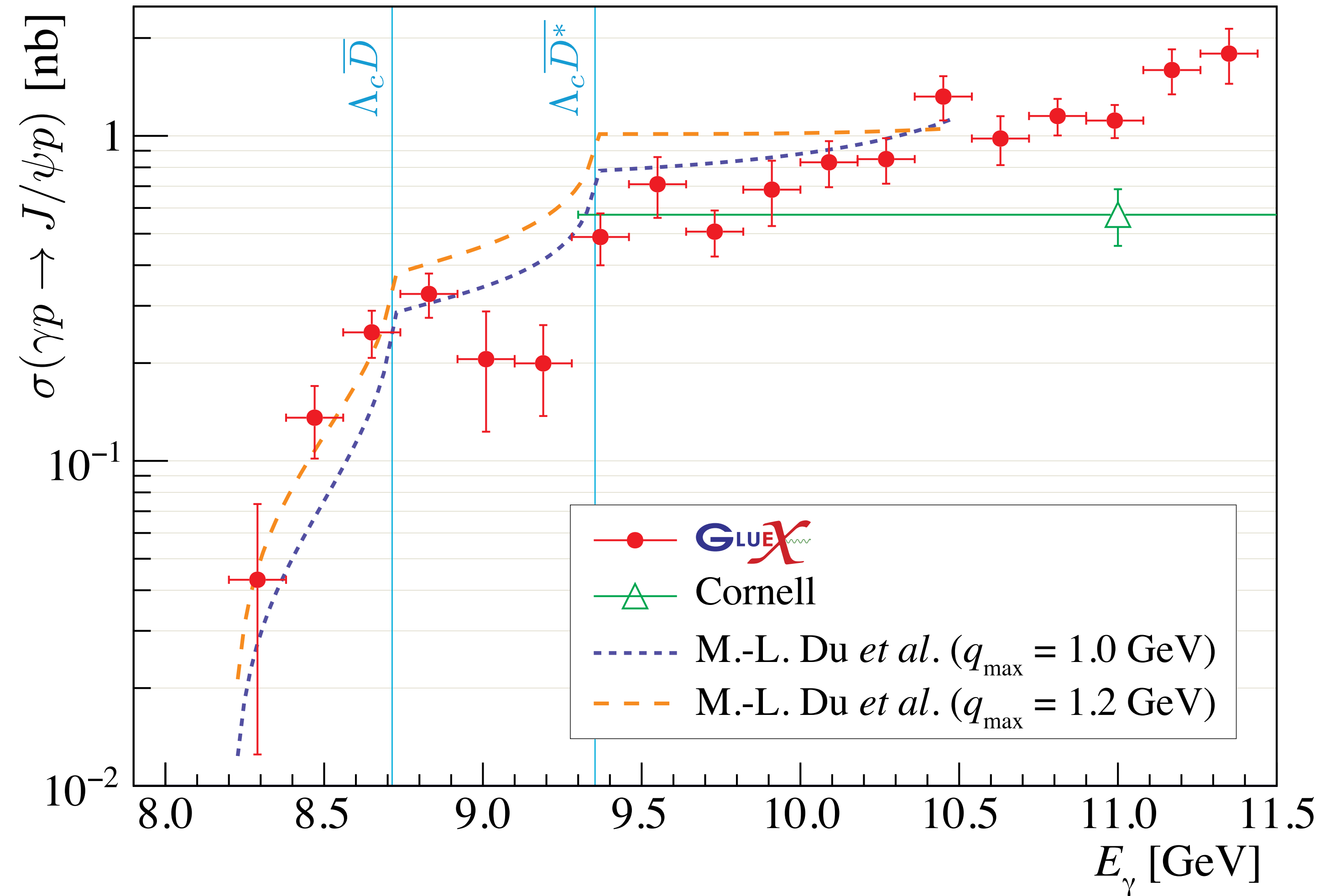
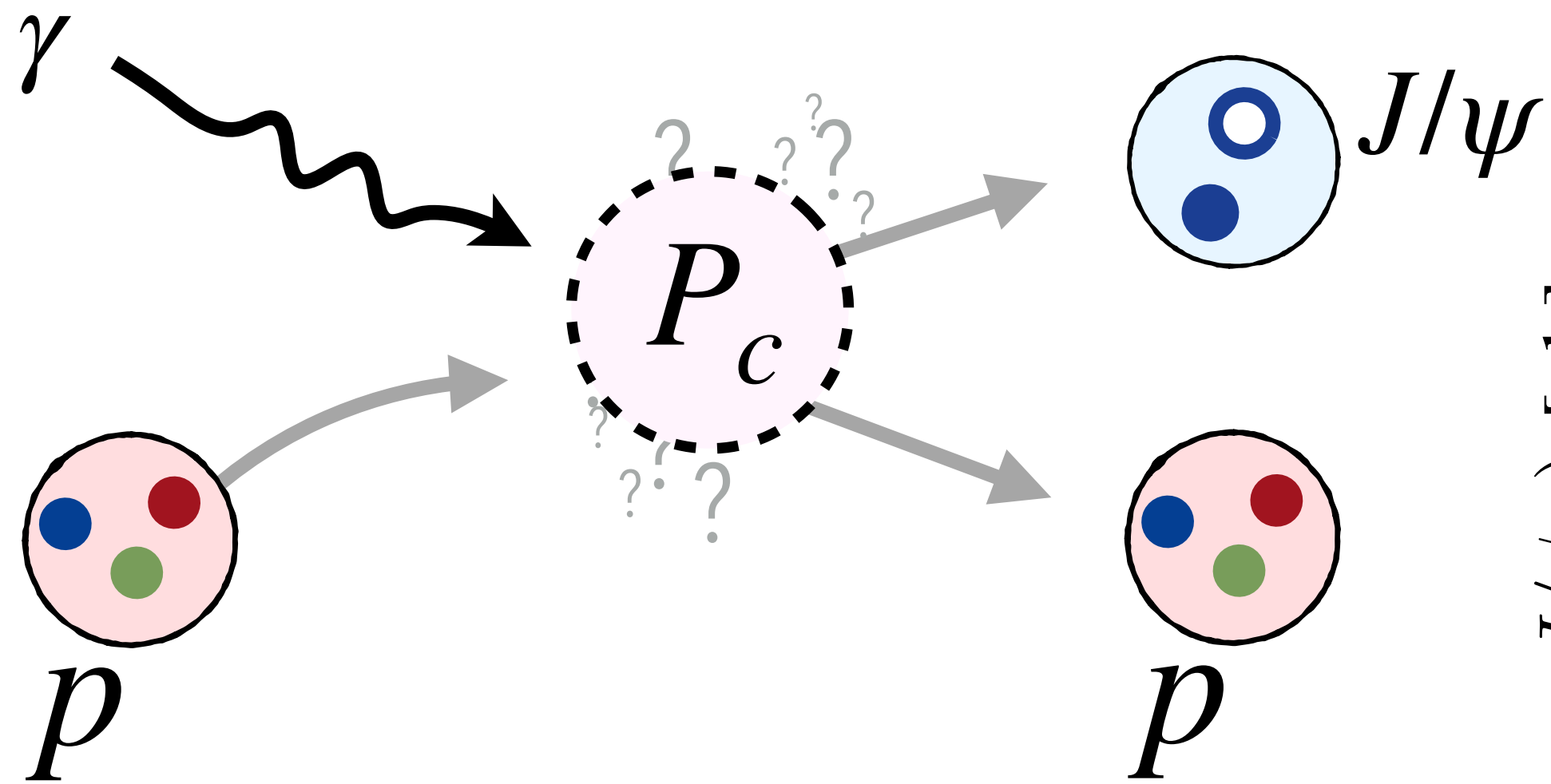
Tetraquarks?



Pentaquarks?

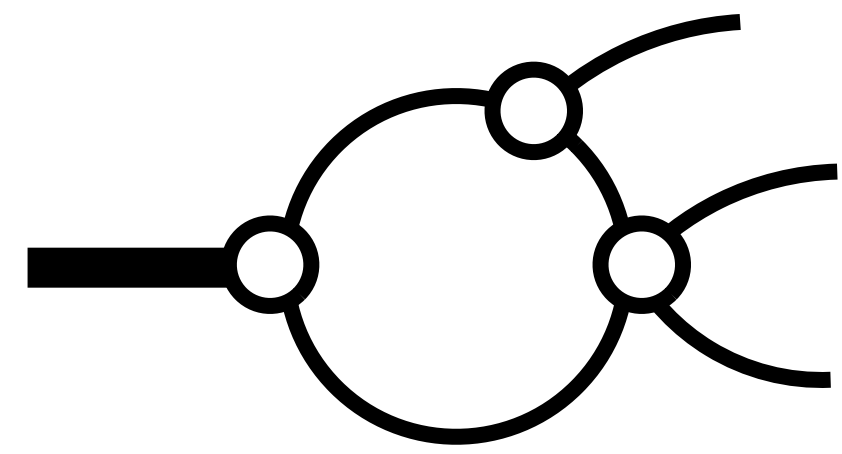


Pentaquarks?



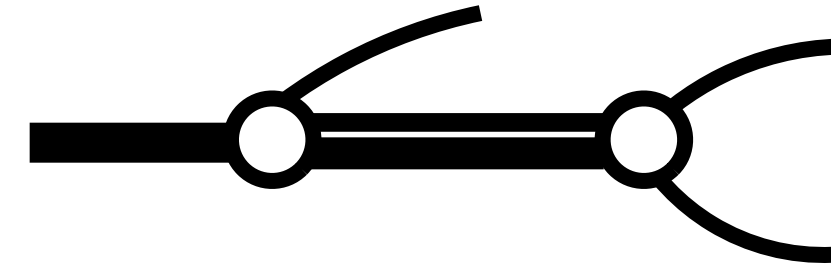
Key questions to answer

□ Which enhancements in cross sections are actual resonances?



$$\sim \log(s - s_t)$$

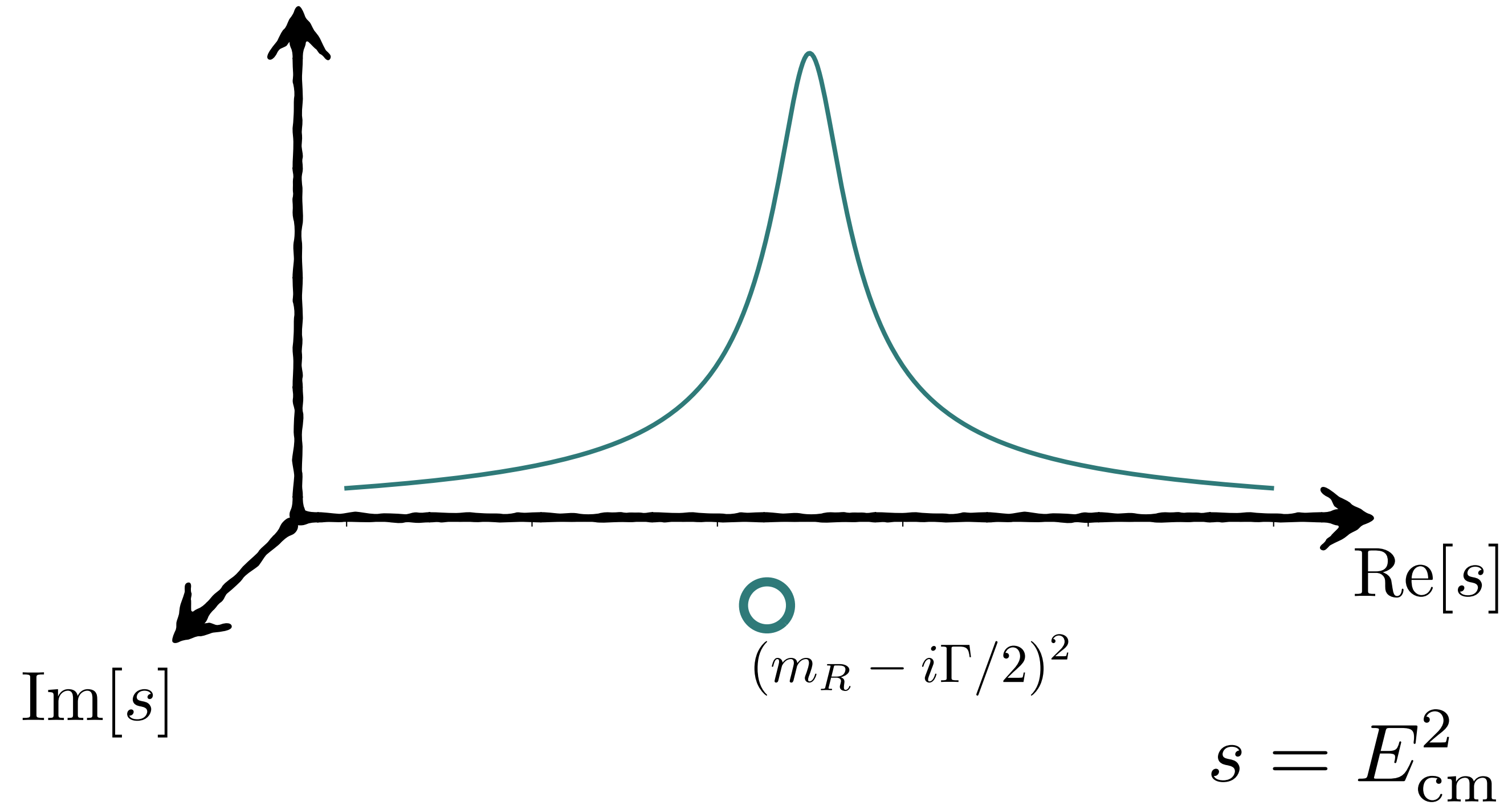
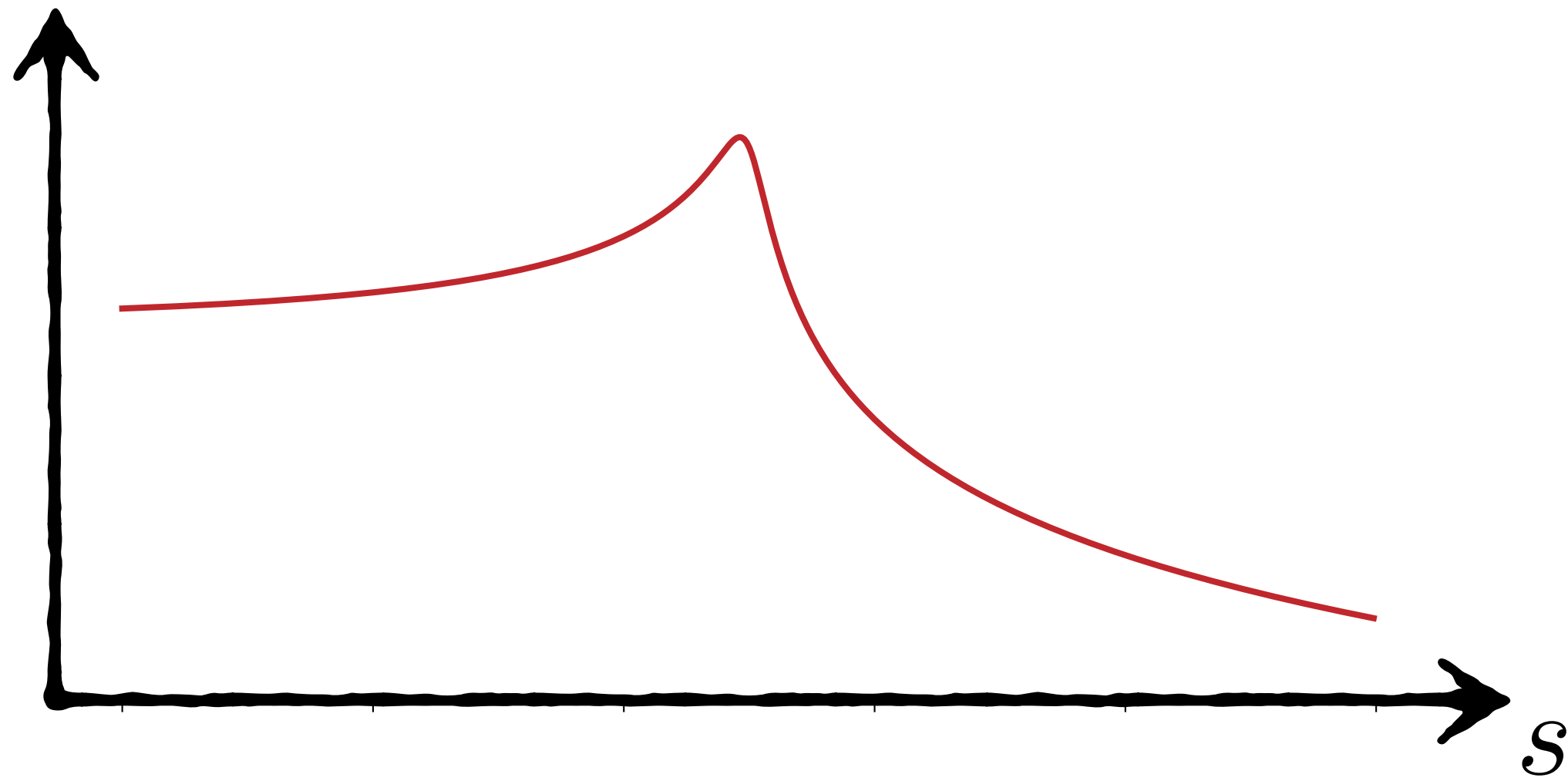
vs.



$$\sim \frac{1}{s - (m_R - \frac{i}{2}\Gamma)^2}$$

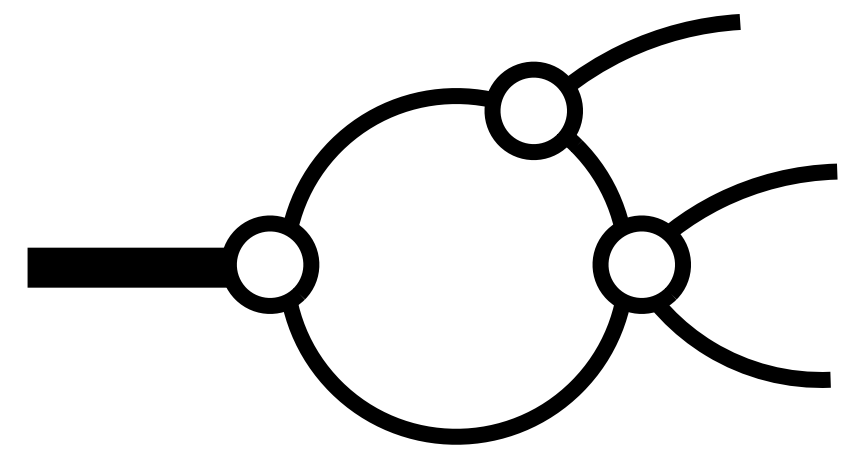
“just a poser”

“the real deal”

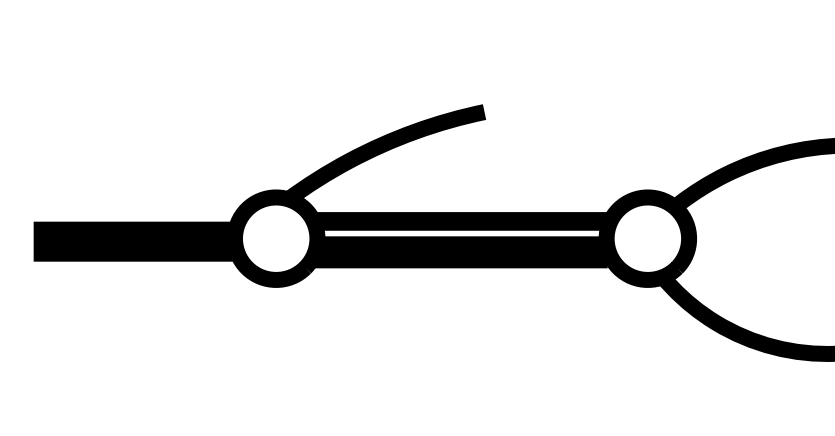


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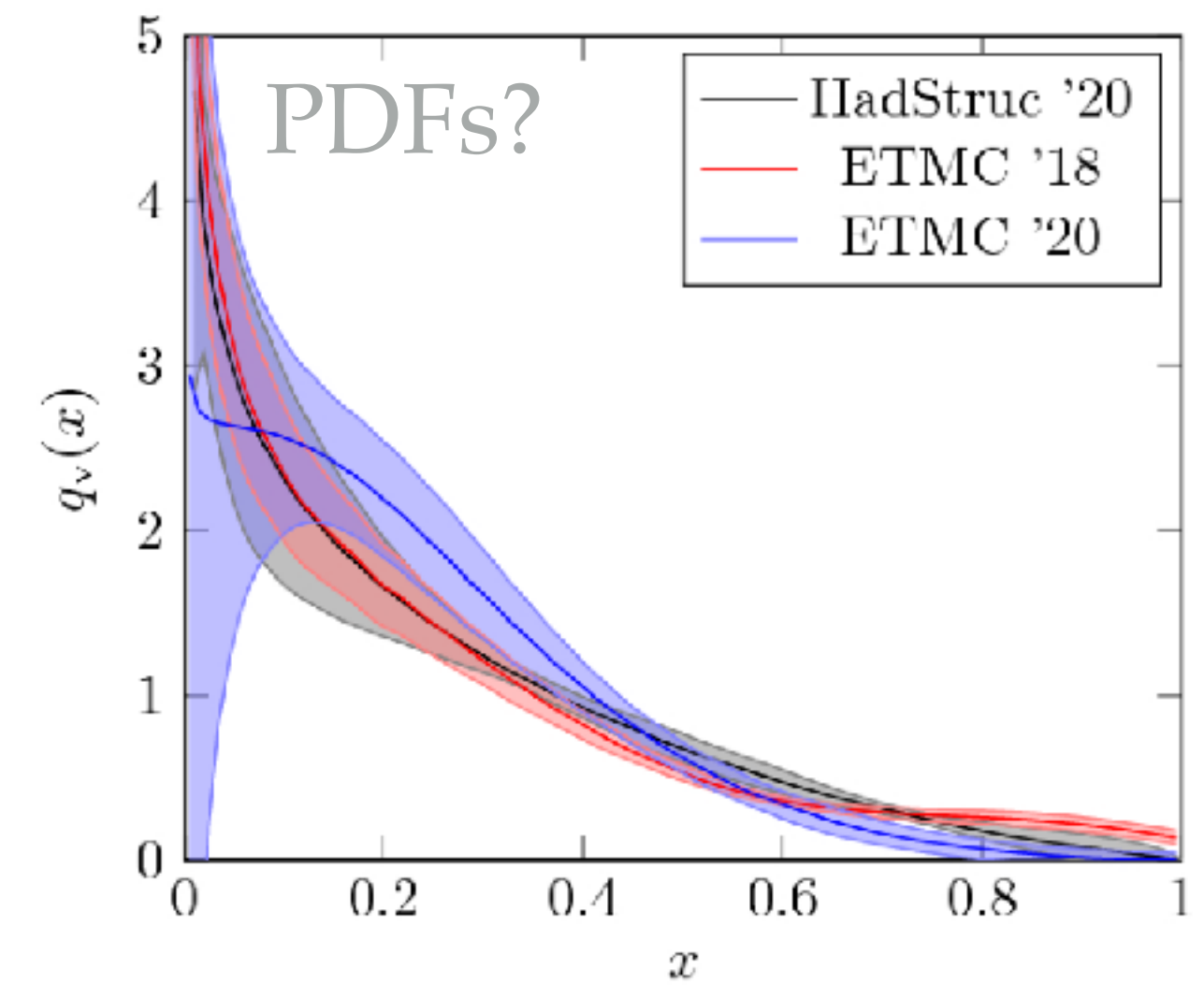
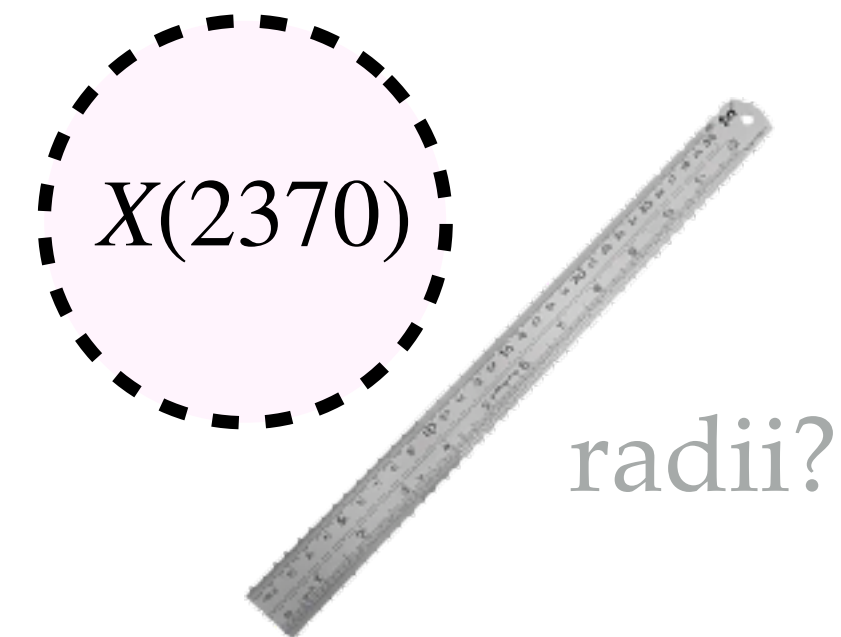

$$\sim \log(s - s_t) \quad \text{vs.}$$

“just a poser”


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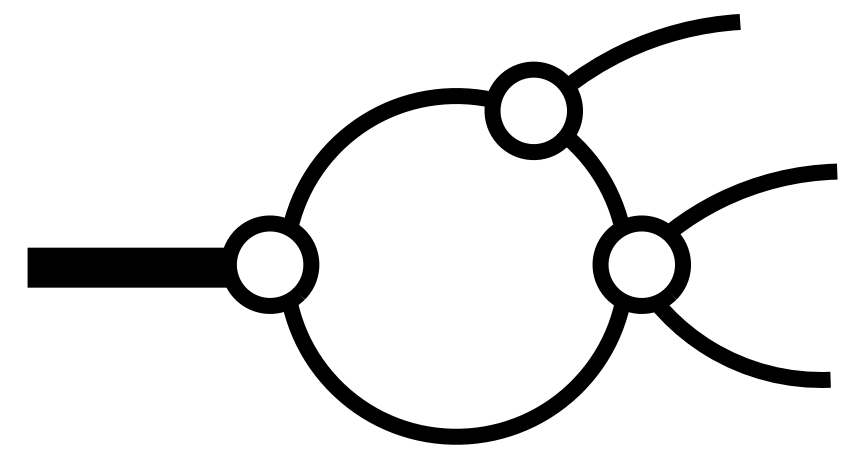
“the real deal”

- If a real resonance, what is its inner structure?

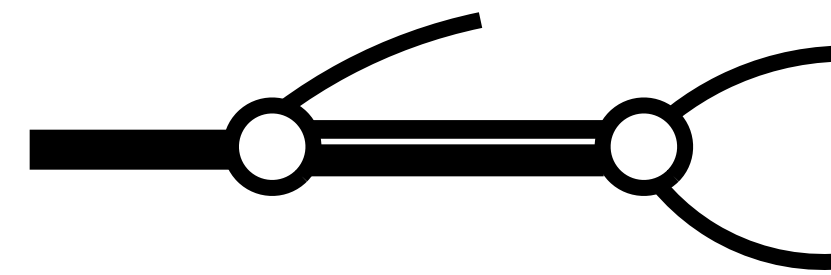


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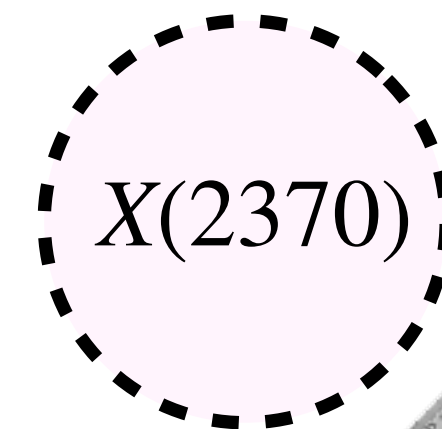


$$\sim \frac{1}{s - (m_R - \frac{i}{2}\Gamma)^2}$$

“just a poser”

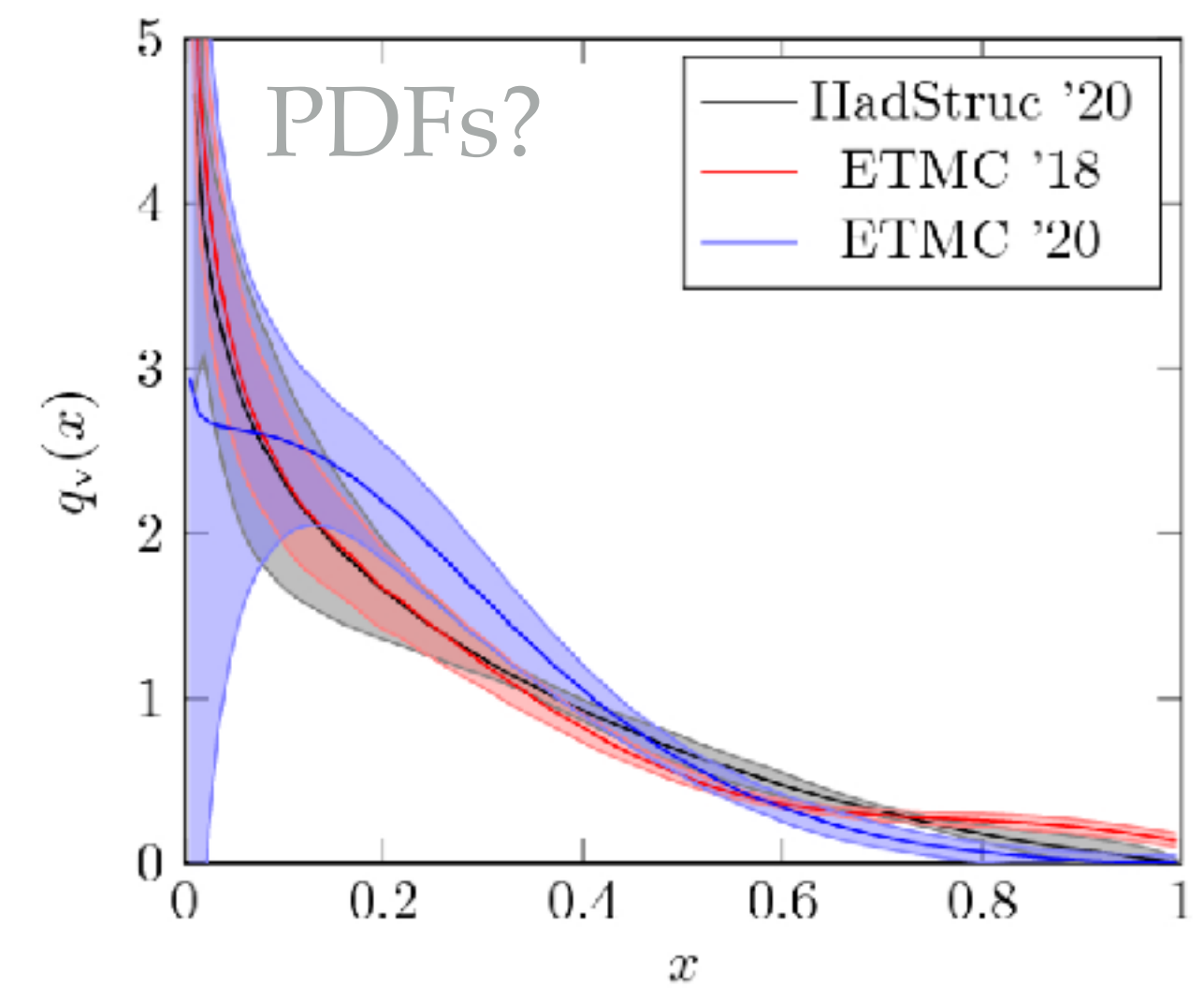
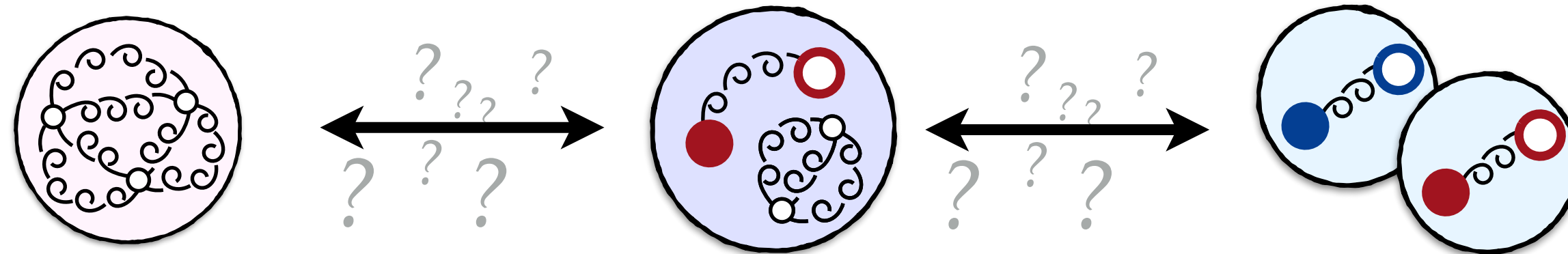
“the real deal”

- If a real resonance, what is its inner structure?



radii?

- Given structural information, can we say anything about the nature?



- Can we deduce general principles from the QCD spectrum?

Need for a Lattice QCD spectroscopy program

A Lattice QCD program running in parallel with experiments is critical.

- ☑ Guide experimental searches,
- ☑ Confirm existence [e.g. tetraquarks, pentaquarks],
- ☑ Understand their nature [*observations are not enough!*].

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Most QCD states are

- ☐ resonances or multi-body **bound states**,
- ☐ dynamical enhancement in scattering amplitudes.

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Challenges to overcome:

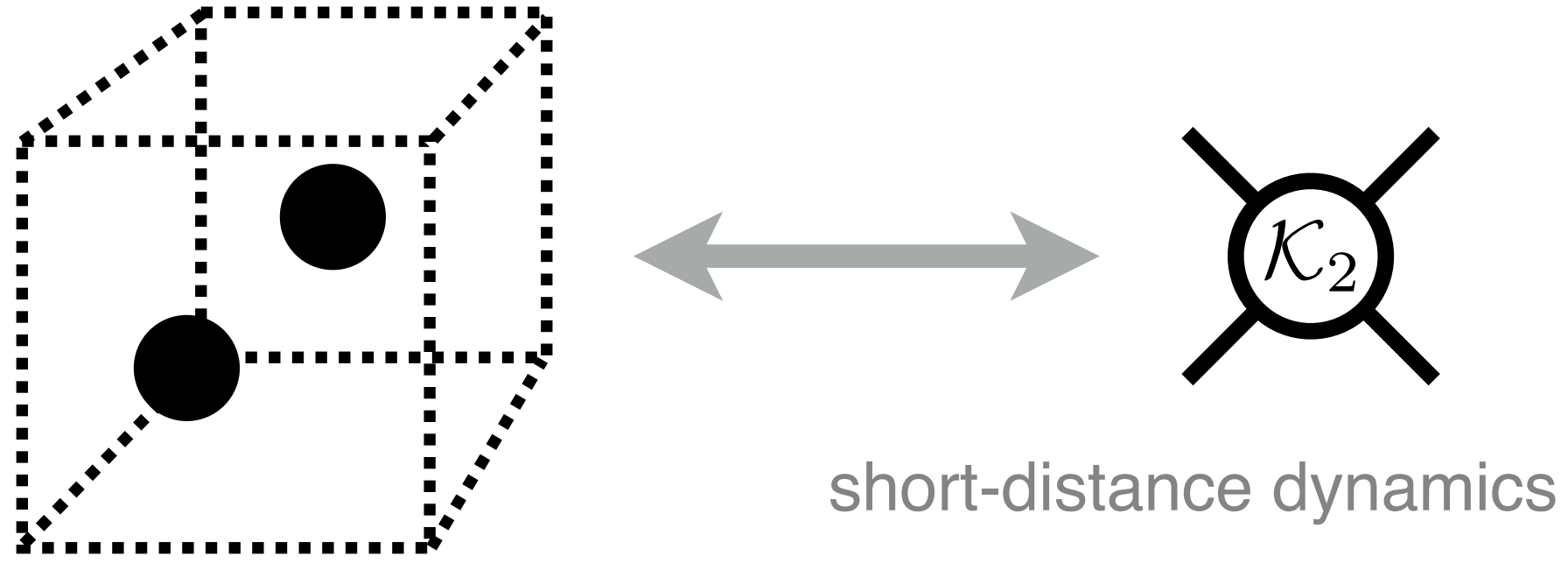
- scattering amplitudes are direct inaccessible via lattice QCD,
- formalism needed to access amplitudes,
- increasingly complicated correlators to evaluate,
- increasingly complicated amplitude analysis, }
- warm bodies,
- ...



*cross-pollination with
experimental searches*

LQCD is finally up to the challenge!

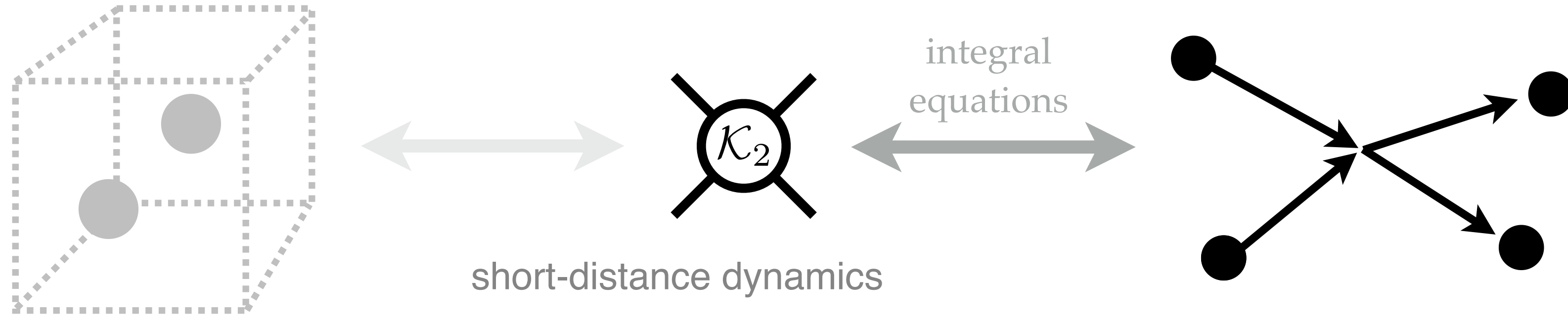
Tremendous formal progress:



$$\det [F^{-1}(P, L) + \mathcal{K}_2] = 0$$

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Tremendous formal progress:

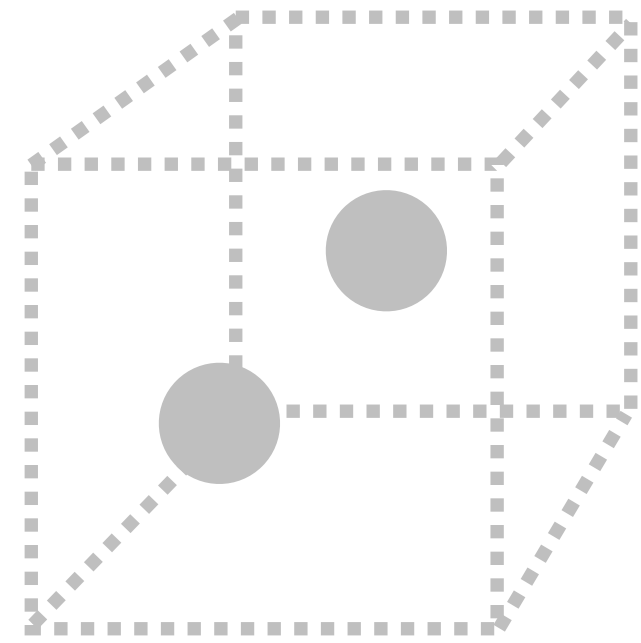


$$\det [F^{-1}(P, L) + \mathcal{K}_2] = 0$$

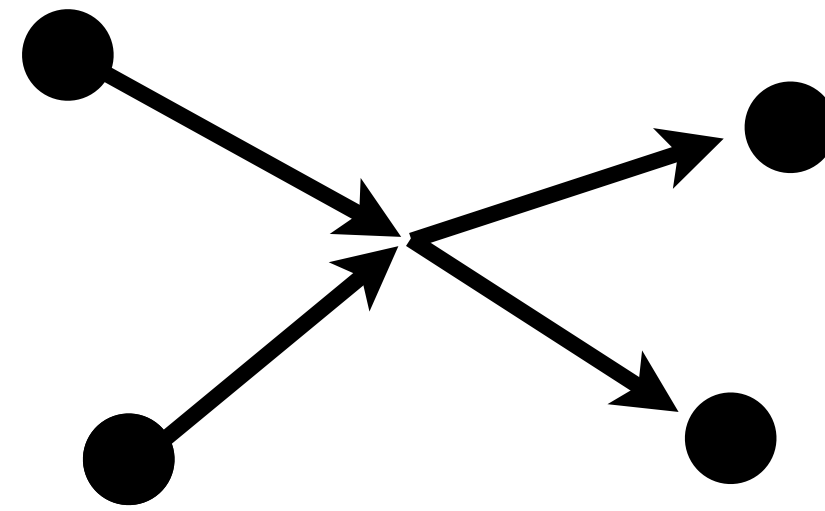
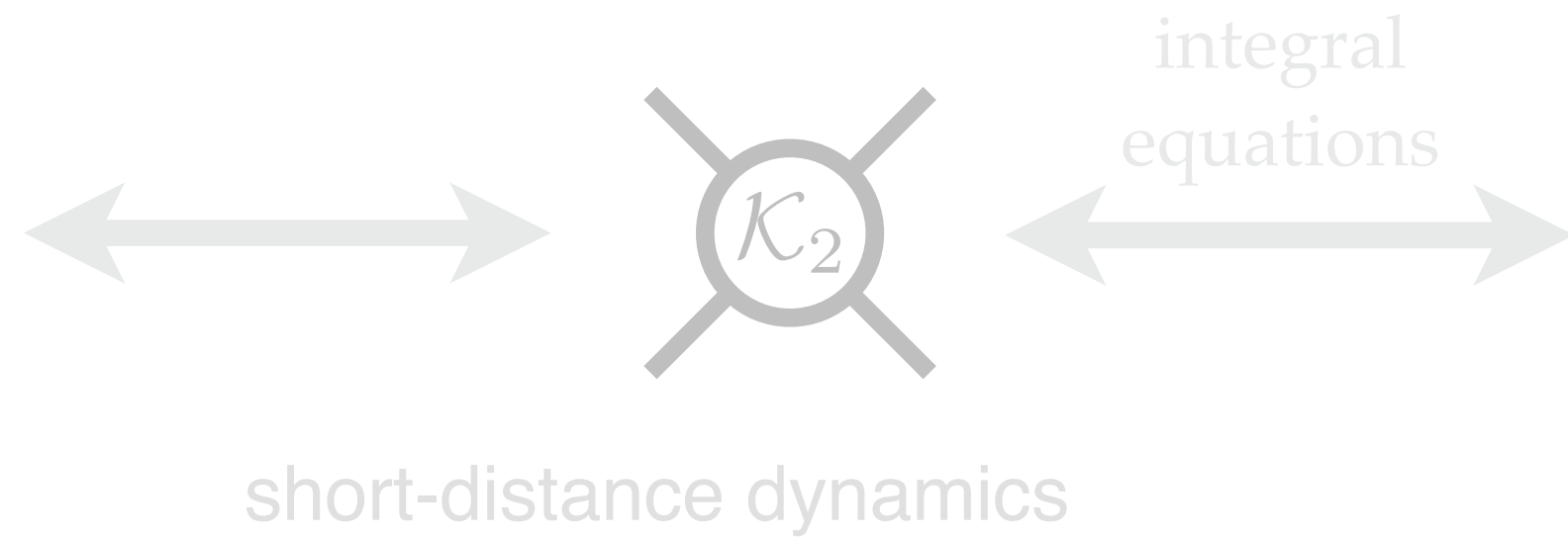
$$\mathcal{M}_2 = [\mathcal{K}_2^{-1} - i\rho]^{-1}$$

LQCD is finally up to the challenge!

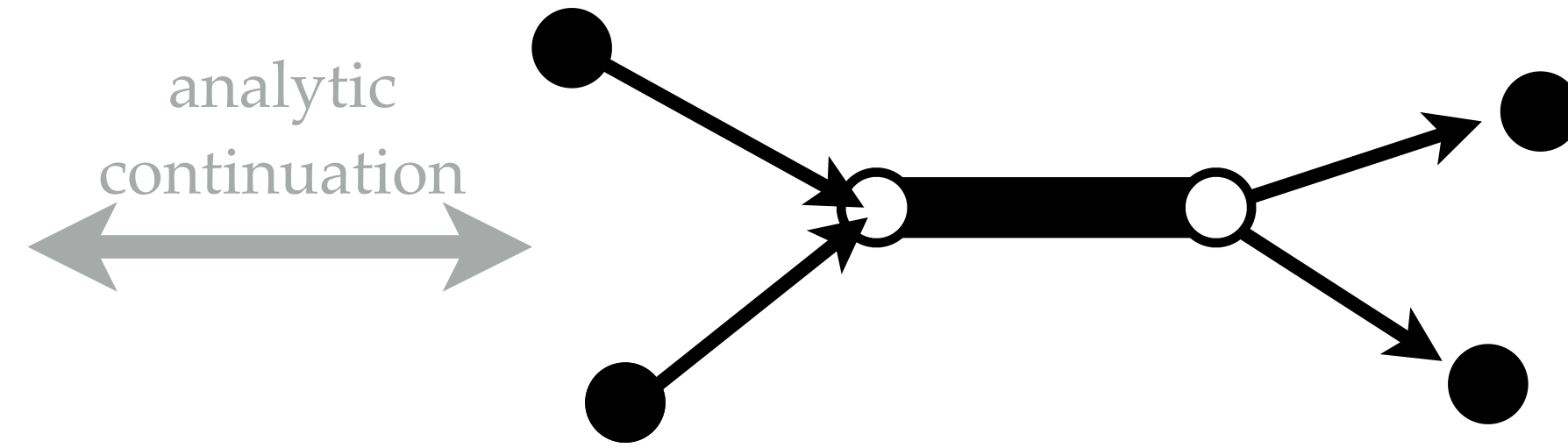
Tremendous formal progress:



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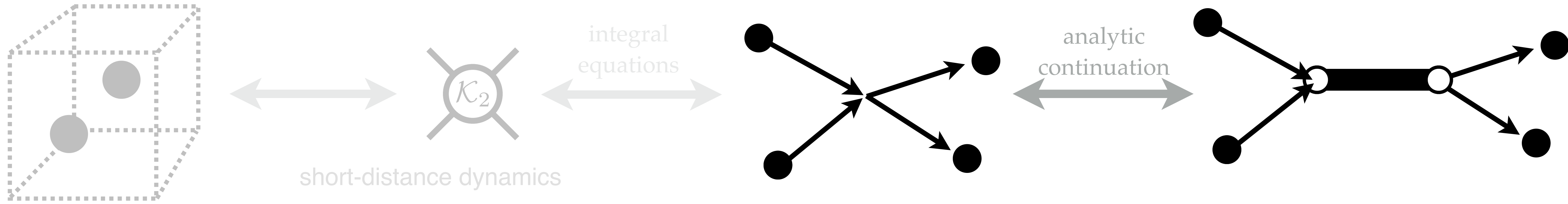
$$\mathcal{M}_2 = [\mathcal{K}_2^{-1} - i\rho]^{-1}$$



$$\mathcal{M}_2 = \frac{-g^2}{s - \left(m - i\frac{\Gamma}{2}\right)^2}$$

LQCD is finally up to the challenge!

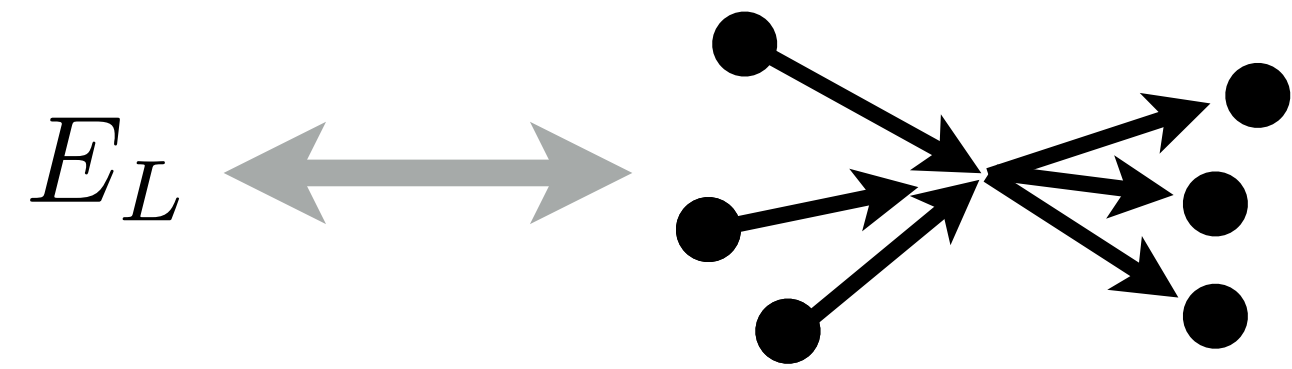
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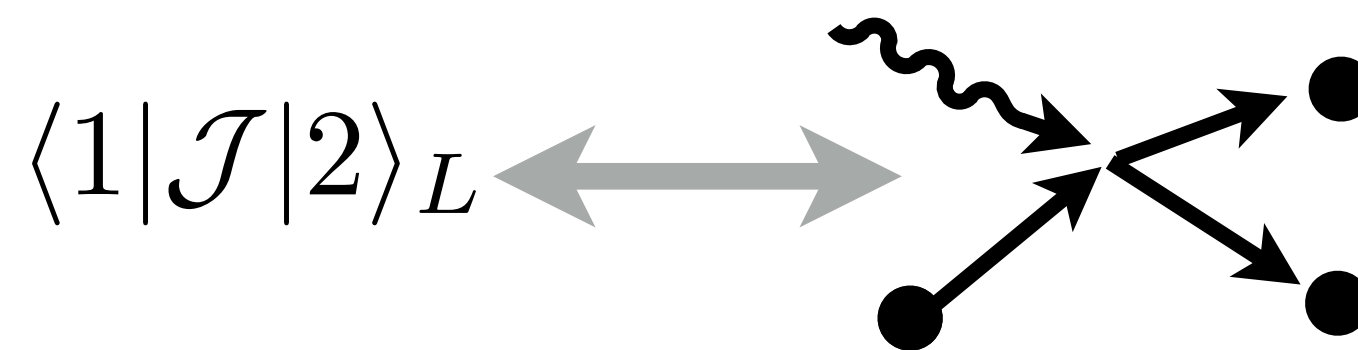
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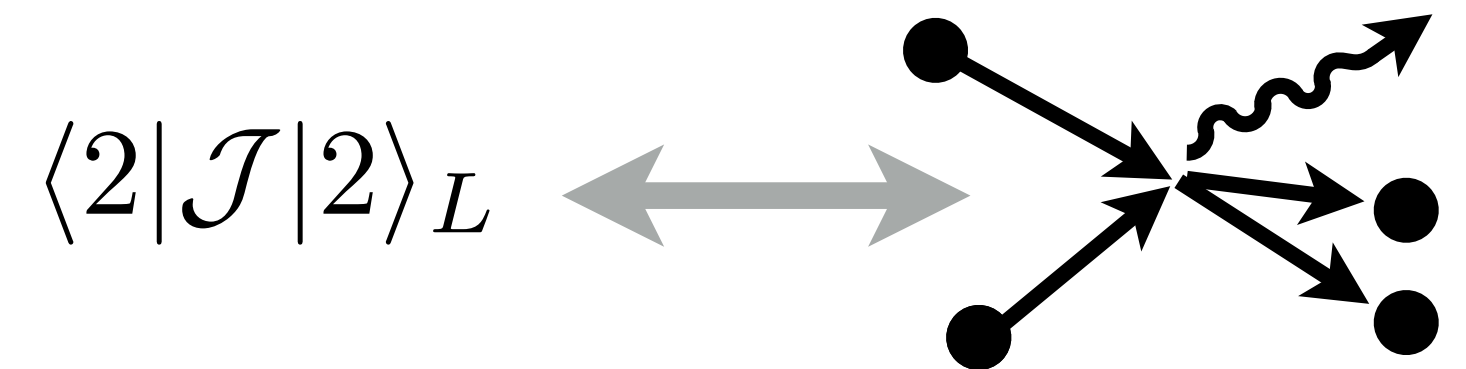
$$\mathcal{M}_2 = \frac{-g^2}{s - (m - i\frac{\Gamma}{2})^2}$$



Purely hadronic reactions



Electroweak transitions



Structural information

LQCD is finally up to the challenge!

- ☑ Generalized eigenvalue problem (GEVP),

- large basis: $\mathcal{O}_b \sim \bar{q} \Gamma_b q, \pi\pi\pi, \pi K \bar{K}, \dots,$

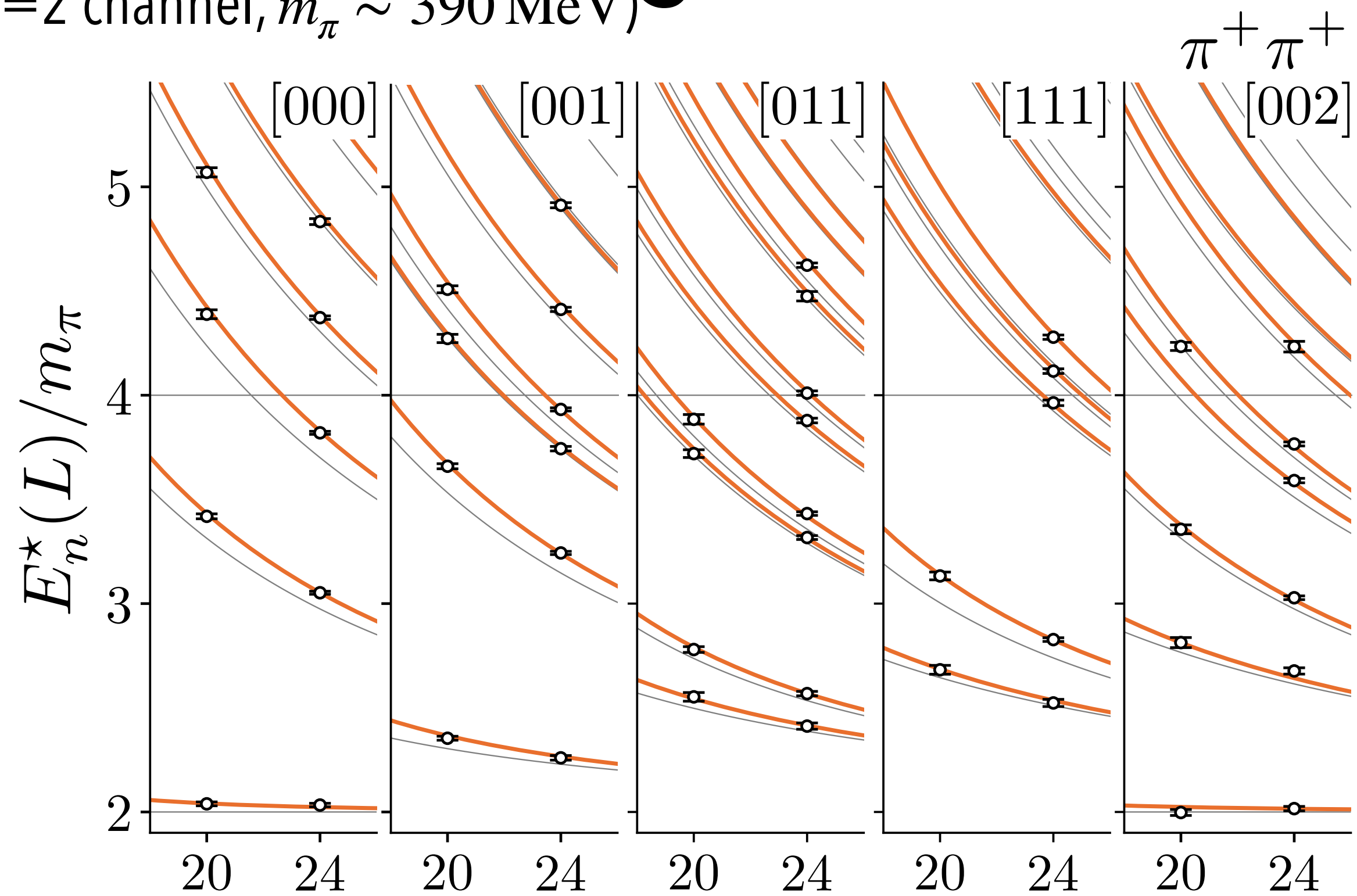
- contractions: $C_{ab}^{2pt.}(t, P) \equiv \langle 0 | \mathcal{O}_b(t, P) \mathcal{O}_a^\dagger(0, P) | 0 \rangle = \sum_n Z_{b,n} Z_{a,n}^* e^{-E_n t}$

- “diagonalization”

- It is now common to have $\mathcal{O}(100)$ kinematic points

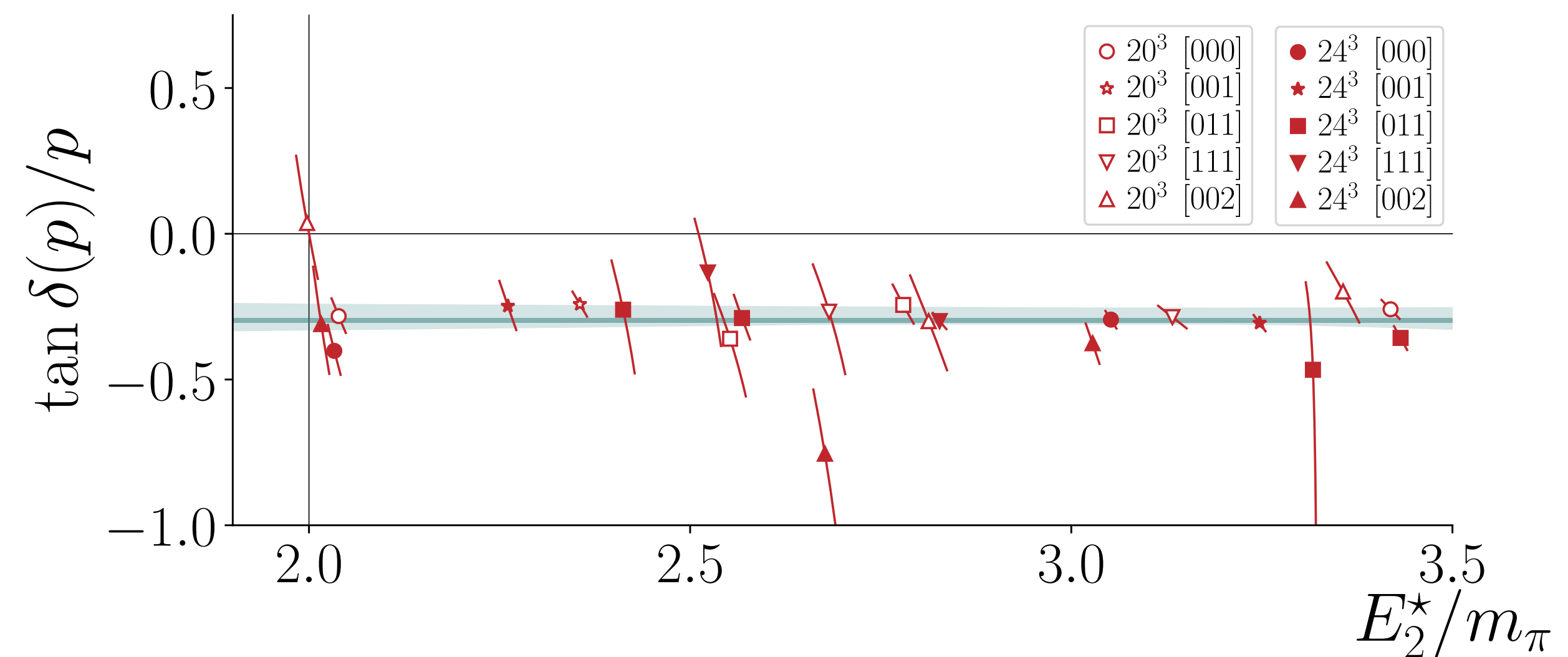
$\pi\pi$ scattering

($l=2$ channel, $m_\pi \sim 390$ MeV)



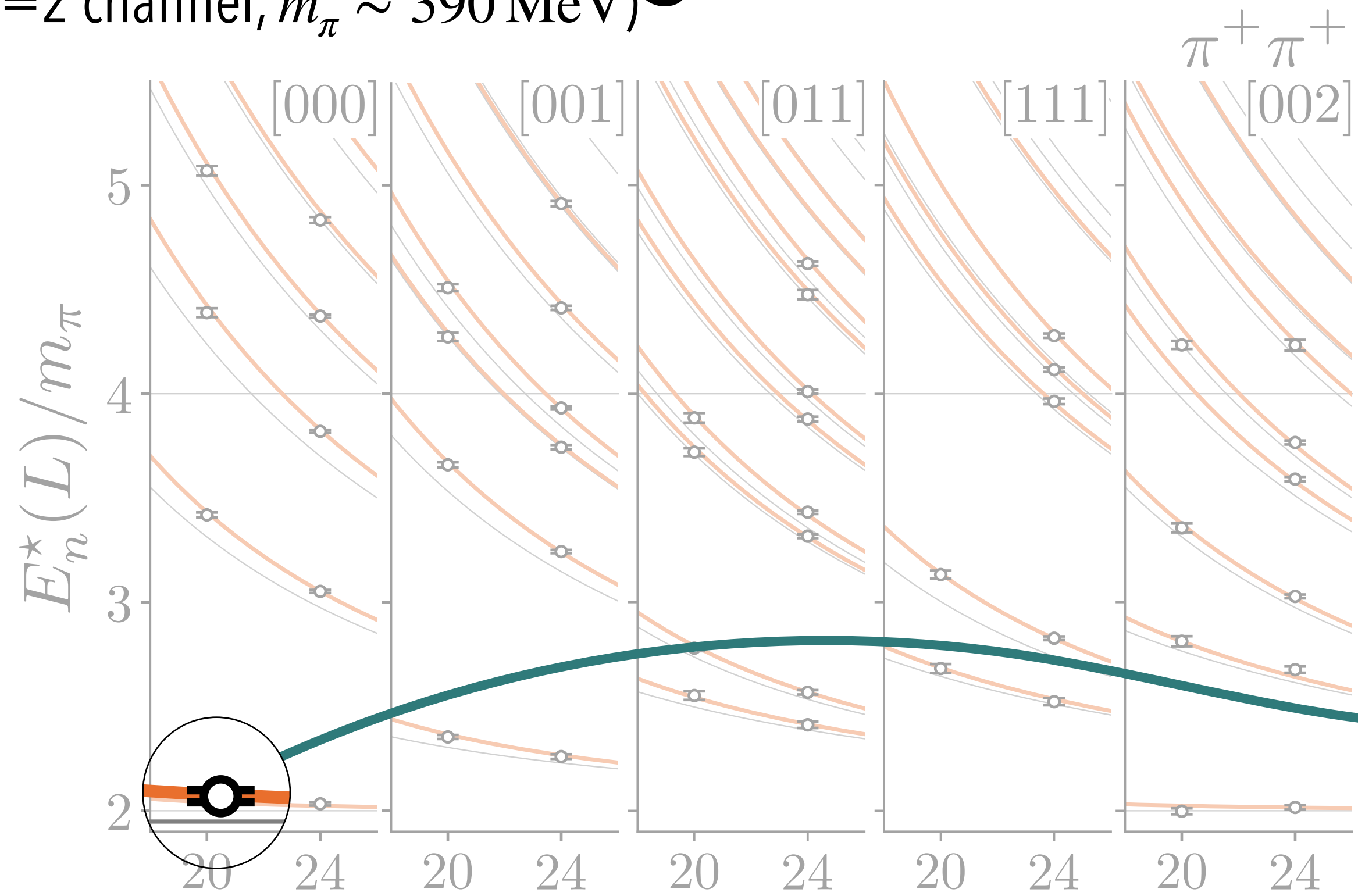
$$\mathcal{M} \sim \frac{1}{p \cot \delta - ip}$$

$$\det[F^{-1}(P, L) + \mathcal{M}(P)] = 0$$



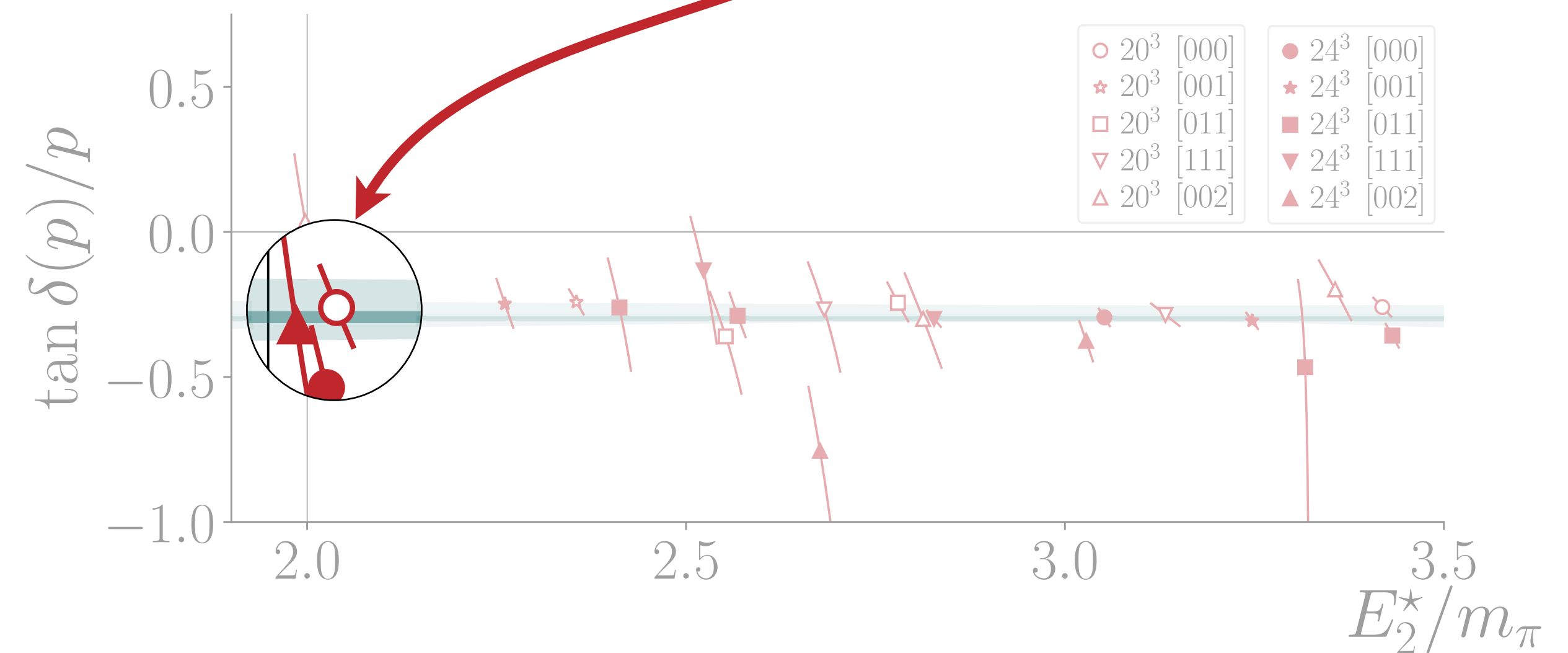
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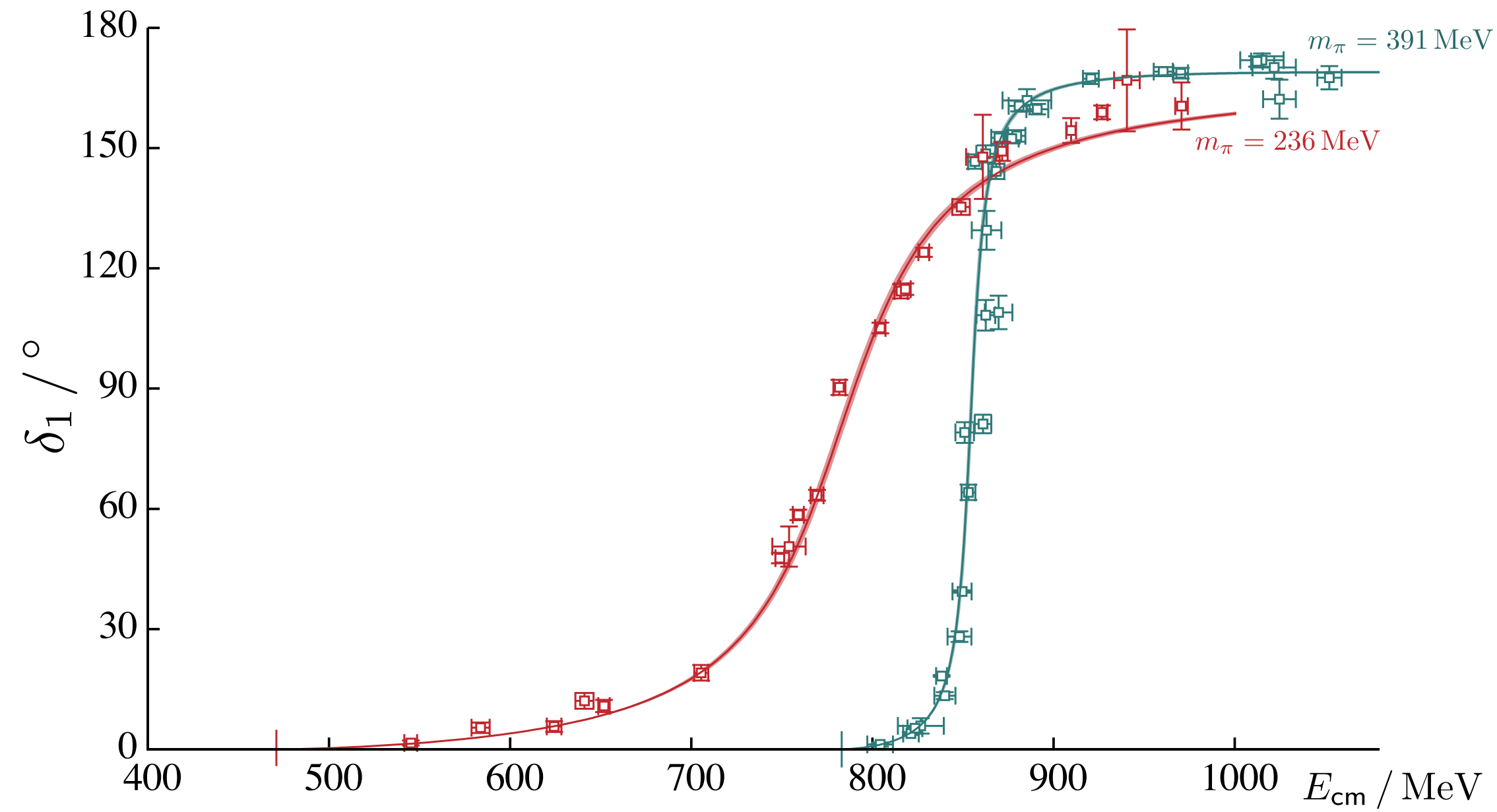
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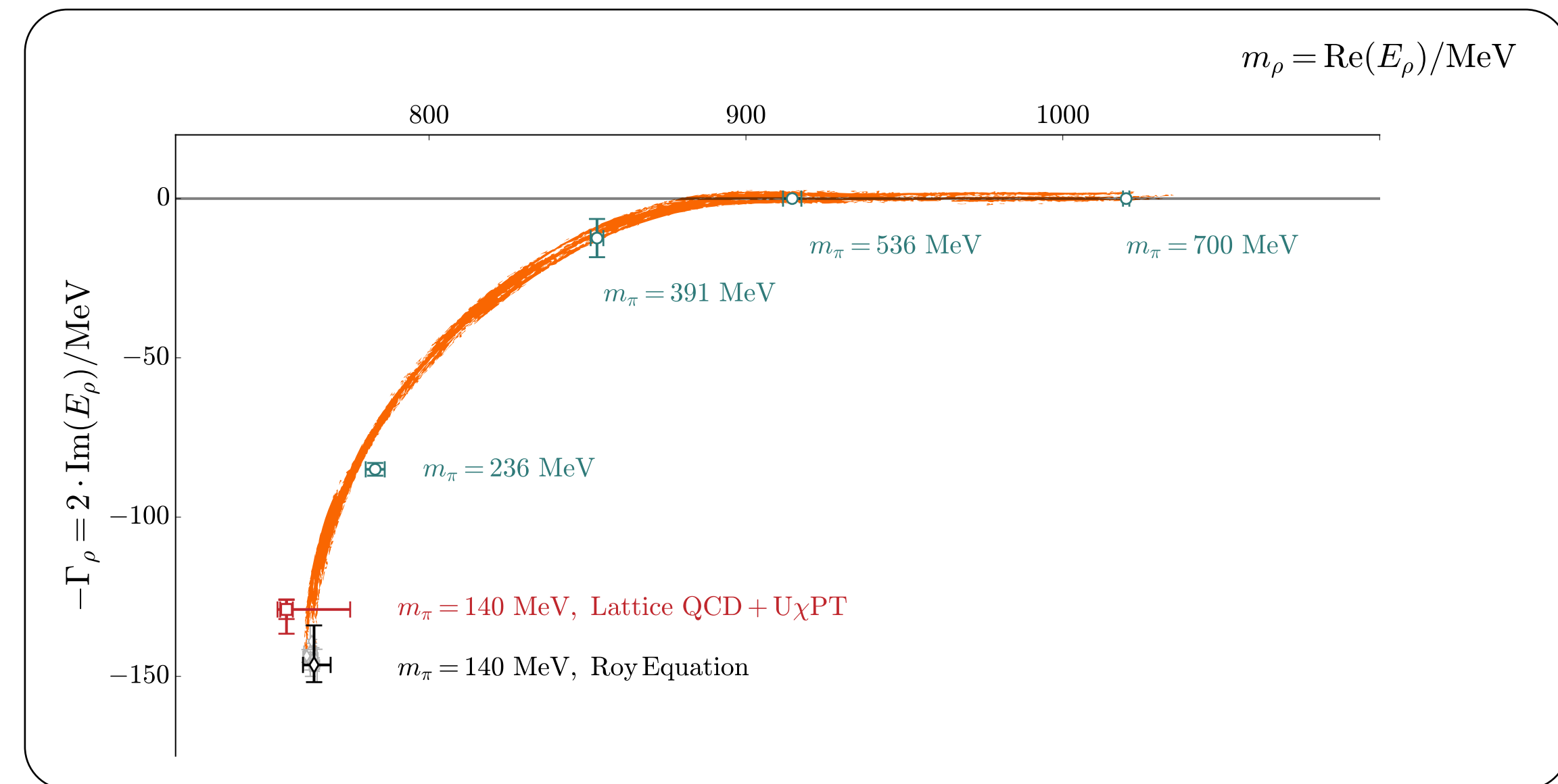
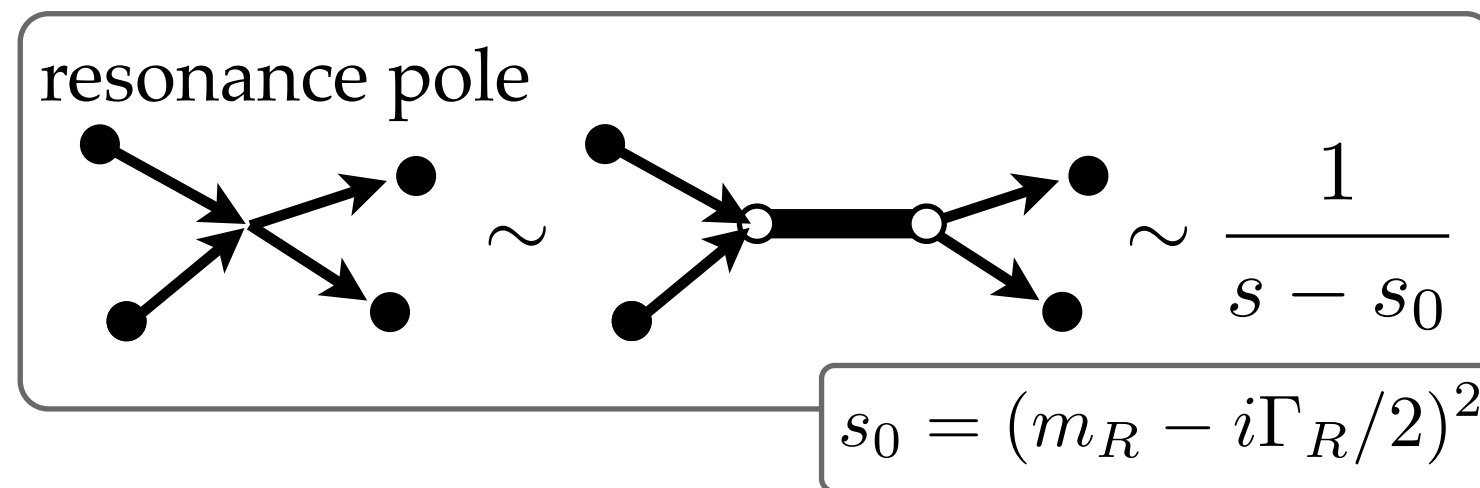


$\pi\pi$ scattering

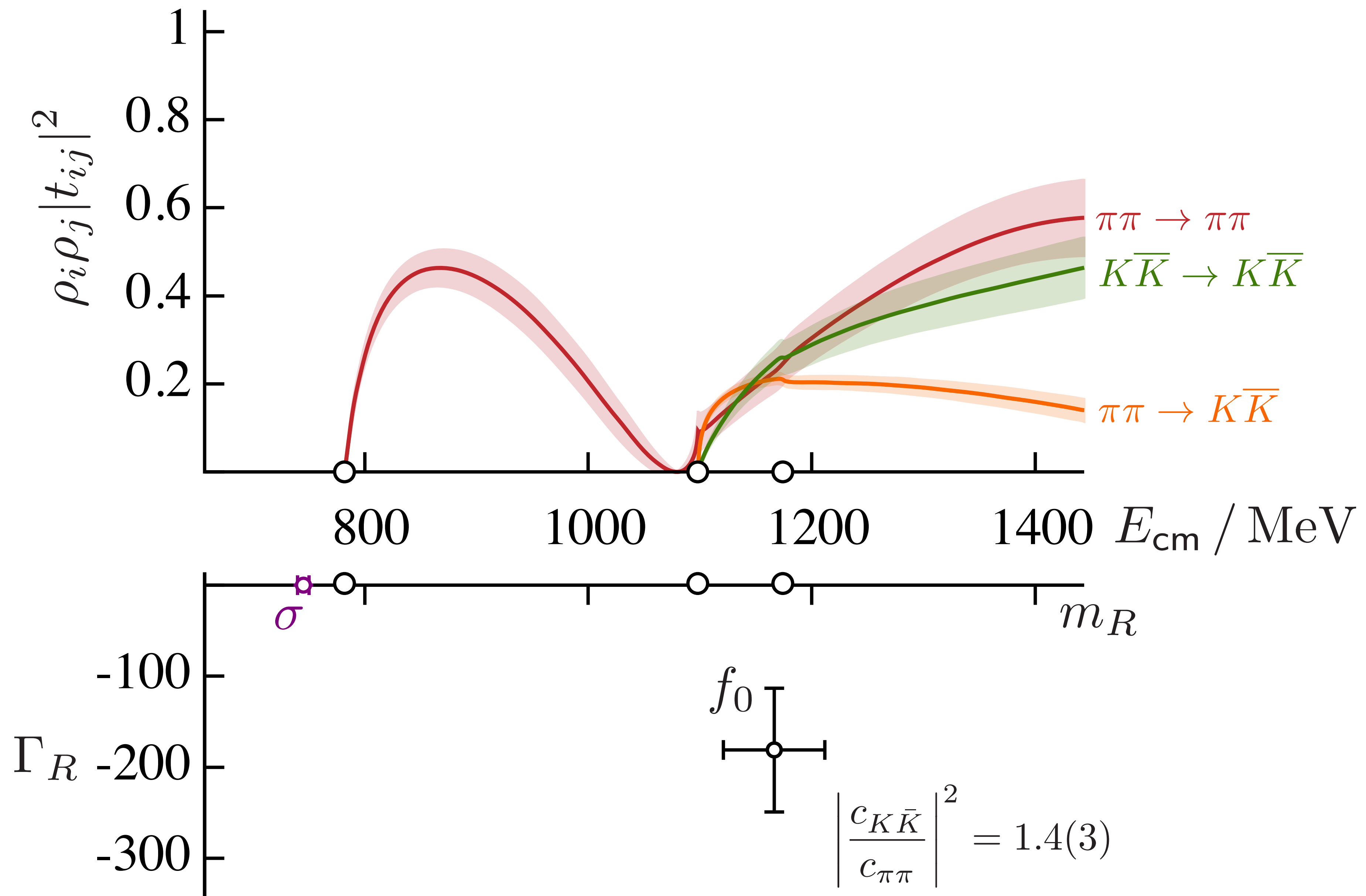
($l=1$ channel)



$$\mathcal{M} \sim \frac{1}{p \cot \delta - ip}$$

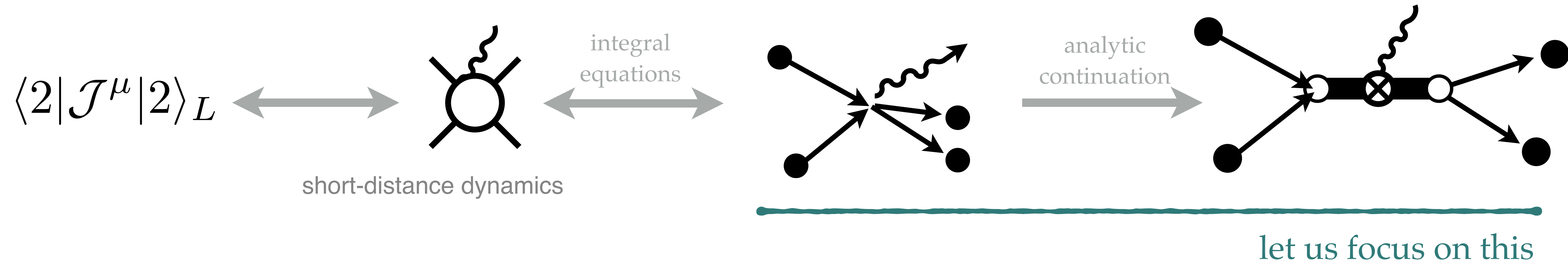


Coupled $\pi\pi$, $K\bar{K}$ and the f_0 's



Structure of multi-particles states

An idea to wet your palette



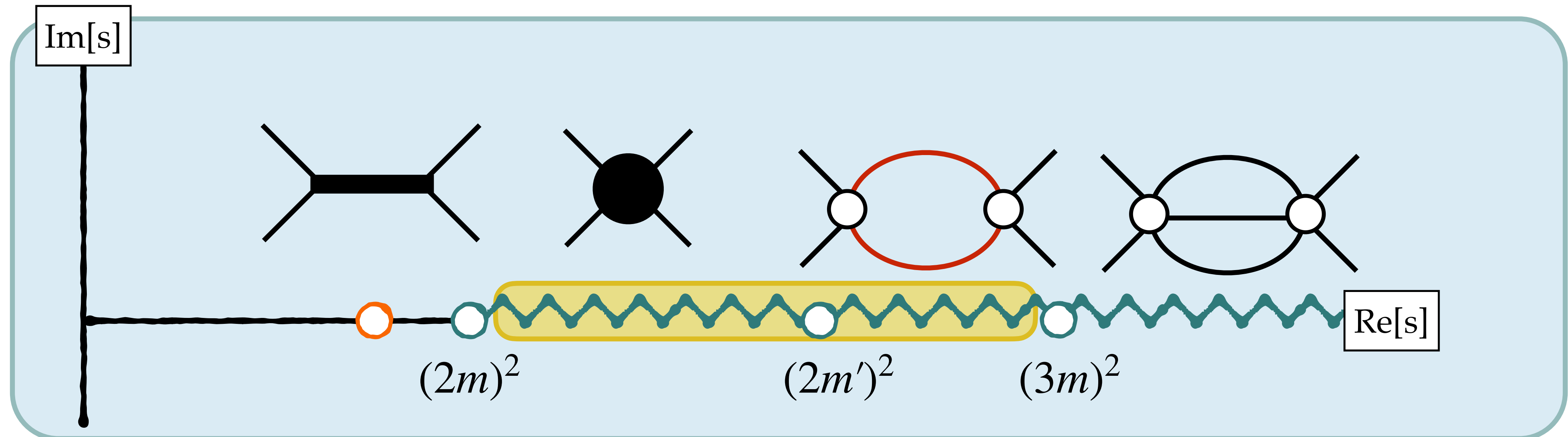
RB, Jackura, Ortega-Gama, Sherman (2021)



$$2 + \mathcal{J}^\mu \rightarrow 2$$

Goal: isolated ALL kinematic singularities of amplitudes in a kinematic region.

Observation: kinematic singularities are due to intermediate particles goin on-shell.



$$2 + \mathcal{J}^\mu \rightarrow 2$$

Let's isolate all possible singularities of...

$$i\mathcal{W}^\mu = \text{blob} = \text{blob with wavy line} + \dots$$

$$\text{blob with wavy line} = i\mathcal{M} \frac{i}{k^2 - m^2 + i\epsilon} i\omega^\mu + \text{“smooth”}$$

$$2 + \mathcal{J}^\mu \rightarrow 2$$

Let's isolate all possible singularities of...

$$i\mathcal{W}^\mu = \text{[blob]} = \text{[blob with wavy line]} + \text{[blob with loop]} + \dots$$

$$\text{[loop with wavy line]} = \text{[loop with dashed line]} + \text{[loop with double line]} + \text{[loop with triple line]} + \text{[loop with quadruple line]}$$

Lorentz decomposition of off-shell $1 + \mathcal{J} \rightarrow 1$ amplitude:

$$\omega^\mu(k_f, k_i) = \sum_j K_j^\mu(k_f, k_i) f_j(Q^2, k_f^2, k_i^2)$$

Off-shell form factor

$$\begin{aligned} f_j(Q^2, k_f^2, k_i^2) &= f_j(Q^2) && \text{[on-shell]} \\ &+ [f_j(Q^2, k_f^2, k_i^2) - f_j(Q^2, k_f^2, m^2)] && \text{[left-cut]} \\ &+ [f_j(Q^2, k_f^2, k_i^2) - f_j(Q^2, m^2, k_i^2)] && \text{[right-cut]} \\ &+ [f_j(Q^2, k_f^2, m^2) + f_j(Q^2, m^2, k_i^2) - f_j(Q^2, k_f^2, k_i^2) - f_j(Q^2, m^2, m^2)] && \text{[smooth]} \end{aligned}$$

$$2 + \mathcal{J}^\mu \rightarrow 2$$

Let's isolate all possible singularities of...

$$i\mathcal{W}^\mu = \text{[Diagram: a solid black circle with four external lines, one wavy line at the top]} = \text{[Diagram: a solid black circle with four external lines, one wavy line at the top connected to a small white circle]} + \text{[Diagram: a loop of two solid black circles with four external lines, one wavy line at the top connected to a small white circle]} + \underbrace{\text{[Diagram: a solid black circle with four external lines, one wavy line at the top connected to a small white circle]} + \dots}_{\left\{ \begin{array}{l} \text{[Diagram: a white circle with four external lines, one wavy line at the top]} + \text{[Diagram: a white circle with two external lines and two wavy lines]} + \text{[Diagram: a white circle with two external lines and two wavy lines]} + \dots + \text{[Diagram: a white circle with four external lines, one wavy line at the top]} + \dots \end{array} \right\}}$$

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Let's isolate all possible singularities of...

$$i\mathcal{W}^\mu = \text{[Diagram: a black circle with four external lines]} = \text{[Diagram: a black circle with four external lines and a wavy line]} + \text{[Diagram: a black circle with four external lines and a loop]} + \text{[Diagram: a white circle with four external lines and a wavy line]} + \text{[Diagram: a white circle with four external lines and a loop]} + \text{[Diagram: a black circle with four external lines and a loop]} + \text{[Diagram: a black circle with four external lines and a loop]} + \text{[Diagram: a black circle with four external lines and a loop]}$$

$$\begin{aligned} \text{[Diagram: a white circle with four external lines]} &= \int \frac{d^4k}{(2\pi)^4} [iB(k, P)]^2 \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{(P - k)^2 - m^2 + i\epsilon} \\ &= \int \frac{d^3k}{(2\pi)^3} \frac{[iB(k, P)]^2}{(2\omega_k)^2} \pi \delta(E - 2\omega_k) + \text{"PV integral"} \\ &= [iB_{on}] \rho [iB_{on}] + \text{"PV integral"} \\ &= \text{[Diagram: a white circle with four external lines and a vertical dashed line]} + \text{[Diagram: a white circle with four external lines and the label 'PV' inside]} \end{aligned}$$

$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$

square root singularity.

$$2 + \mathcal{J}^\mu \rightarrow 2$$

Let's isolate all possible singularities of...

$$\begin{aligned}
 i\mathcal{W}^\mu &= \text{[Diagram: a large black circle with four external lines]} = \text{[Diagram: black circle with 3 lines]} + \text{[Diagram: black circle with 2 lines, loop, black circle with 2 lines]} + \text{[Diagram: white circle with 3 lines]} + \text{[Diagram: white circle with 2 lines, loop, black circle with 2 lines]} + \text{[Diagram: black circle with 2 lines, loop, white circle with 2 lines]} + \text{[Diagram: black circle with 2 lines, loop, white circle with 2 lines, loop, black circle with 2 lines]} \\
 &= \text{[Diagram: black circle with 2 lines, wavy line]} + \{1 + \text{[Diagram: square in loop with vertical dashed line]} + \dots\} \{ \text{[Diagram: crossed square with wavy line]} + \text{[Diagram: square in loop with dashed line and wavy line]} \} \{1 + \text{[Diagram: square in loop with vertical dashed line]} + \dots\}
 \end{aligned}$$

$$2 + \mathcal{J}^\mu \rightarrow 2$$

Let's isolate all possible singularities of...

$$\begin{aligned}
 i\mathcal{W}^\mu &= \text{[Diagram: a black circle with four external lines and a wavy line]} = \text{[Diagram: black circle with 3 lines, wavy line to white circle]} + \text{[Diagram: black circle with 2 lines, loop, black circle with 2 lines, wavy line to white circle]} \\
 &\quad + \text{[Diagram: white circle with 3 lines, wavy line]} + \text{[Diagram: white circle with 2 lines, loop, black circle with 2 lines, wavy line]} + \text{[Diagram: black circle with 2 lines, loop, white circle with 2 lines, wavy line]} + \text{[Diagram: black circle with 1 line, loop, white circle with 1 line, loop, black circle with 2 lines, wavy line]} \\
 &= \text{[Diagram: black circle with 2 lines, wavy line to white circle]} + \left\{ 1 + \text{[Diagram: white square in a circle with a vertical dashed line]} + \dots \right\} \left\{ \text{[Diagram: crossed white square with wavy line]} + \text{[Diagram: white square in a circle with wavy line to white circle]} \right\} \left\{ 1 + \text{[Diagram: white circle with a vertical dashed line and white square]} + \dots \right\} \\
 &\quad \underbrace{\hspace{10em}}_{\text{K-matrix}} \left\{ \text{[Diagram: white circle with 4 lines]} + \text{[Diagram: white circle with 2 lines, loop, PV, white square]} \right\}
 \end{aligned}$$

$$2 + \mathcal{J}^\mu \rightarrow 2$$

Let's isolate all possible singularities of...

$$i\mathcal{W}^\mu = \text{[Diagram: a black circle with four external lines]} = \text{[Diagram: black circle with 3 lines and 1 wavy line]} + \text{[Diagram: black circle with 2 lines and 1 loop]} + \text{[Diagram: white circle with 3 lines and 1 wavy line]} + \text{[Diagram: white circle with 2 lines and 1 loop]} + \text{[Diagram: black circle with 1 line and 1 loop]} + \text{[Diagram: black circle with 1 line and 2 loops]} + \text{[Diagram: black circle with 1 line and 3 loops]}$$

$$= \text{[Diagram: black circle with 3 lines and 1 wavy line]} + \underbrace{\{1 + \text{[Diagram: square in a circle with a vertical dashed line]} + \dots\} \{ \text{[Diagram: crossed square in a circle with a wavy line]} + \text{[Diagram: square in a circle with a wavy line]} \} \{1 + \text{[Diagram: square in a circle with a vertical dashed line]} + \dots\}}$$

$$i\mathcal{W}_{\text{df}}^\mu = \mathcal{M} \cdot \left(\text{[Diagram: black circle with 3 lines and 1 wavy line]} + i \sum_j f_j(Q^2) \mathcal{G}_j^\mu \right) \cdot \mathcal{M}$$

unknown real function, to be determined via lattice QCD

$$2 + \mathcal{J}^\mu \rightarrow 2$$

Let's isolate all possible singularities of...

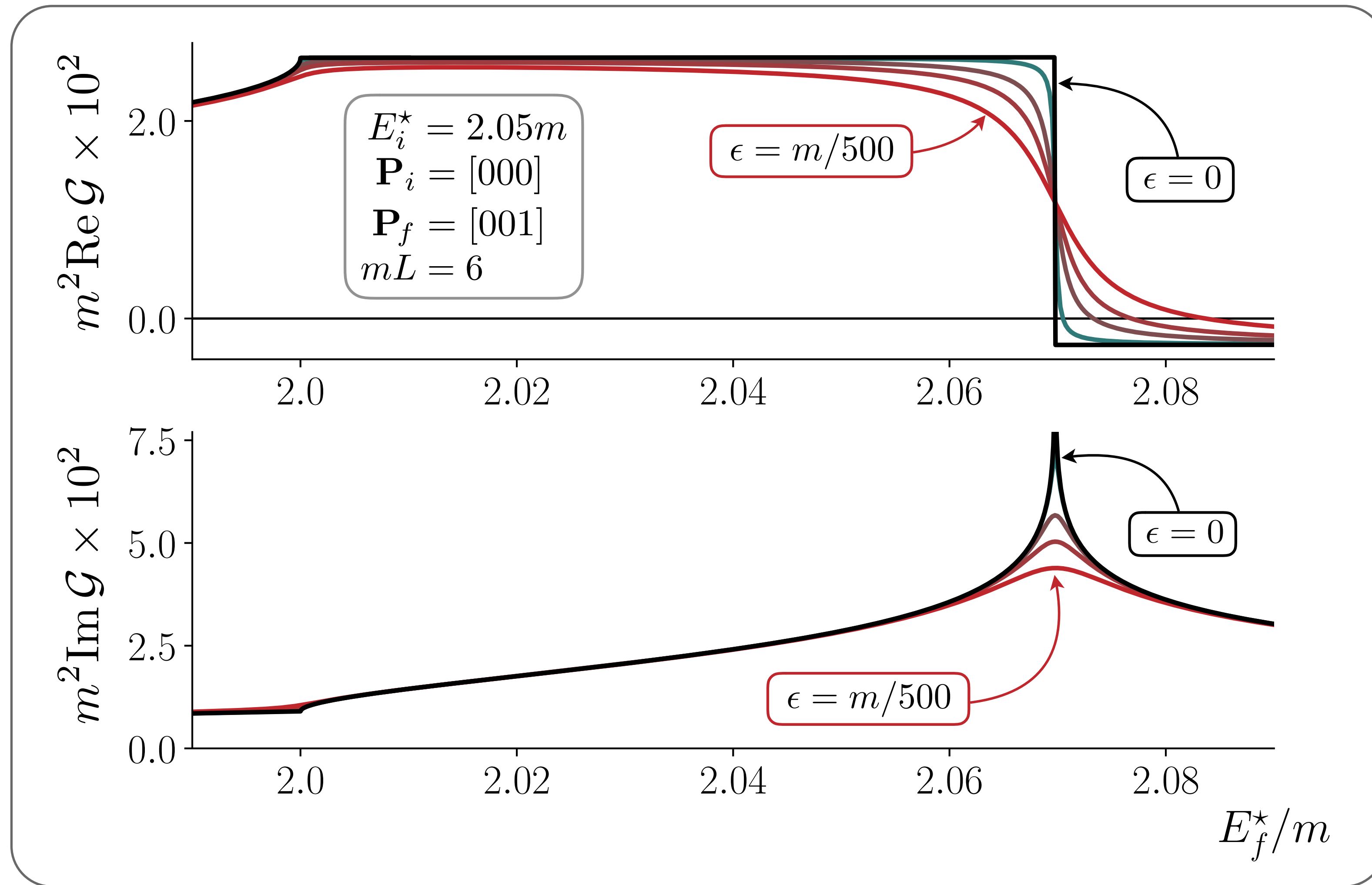
$$i\mathcal{W}^\mu = \text{[Diagram: a black circle with four external lines]} = \text{[Diagram: a black circle with four external lines and a wavy line]} + \text{[Diagram: a black circle with four external lines and a loop]} + \text{[Diagram: a white circle with four external lines and a wavy line]} + \text{[Diagram: a white circle with four external lines and a loop]} + \text{[Diagram: a black circle with four external lines and a loop]} + \text{[Diagram: a black circle with four external lines and a loop]} + \text{[Diagram: a black circle with four external lines and a loop]}$$

$$= \text{[Diagram: a black circle with four external lines and a wavy line]} + \underbrace{\{1 + \text{[Diagram: a square in a circle with a vertical dashed line]} + \dots\} \{ \text{[Diagram: a crossed square in a circle with a wavy line]} + \text{[Diagram: a square in a circle with a wavy line]} \} \{1 + \text{[Diagram: a square in a circle with a vertical dashed line]} + \dots\}}$$

$$i\mathcal{W}_{\text{df}}^\mu = \mathcal{M} \cdot \left(i\mathcal{A}^\mu + i \sum_j f_j(Q^2) \mathcal{G}_j^\mu \right) \cdot \mathcal{M}$$

$\mathcal{G}_j^\mu = \int \frac{d^4 k}{(2\pi)^4} \frac{\mathcal{Y}_f iK_j^\mu \mathcal{Y}_i}{(k^2 - m^2) ((P_f - k)^2 - m^2) ((P_i - k)^2 - m^2)}$

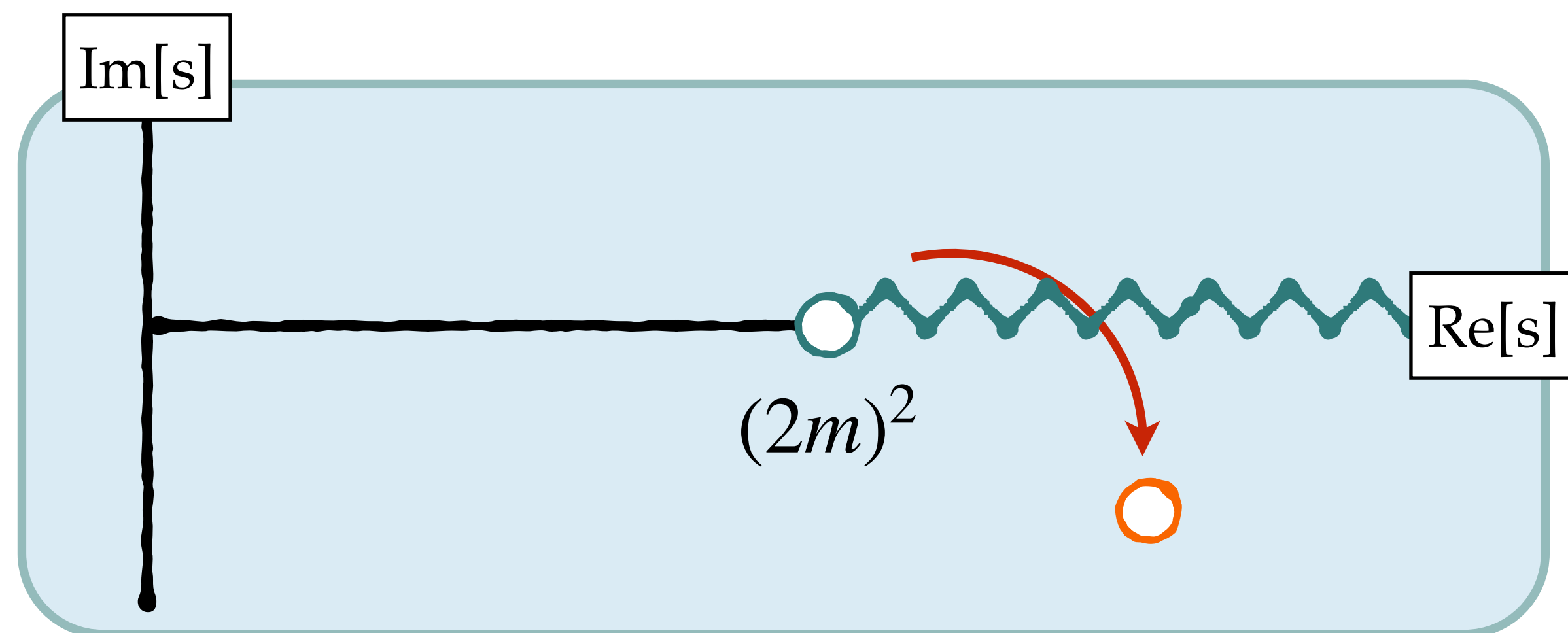
$$2 + \mathcal{J}^\mu \rightarrow 2$$



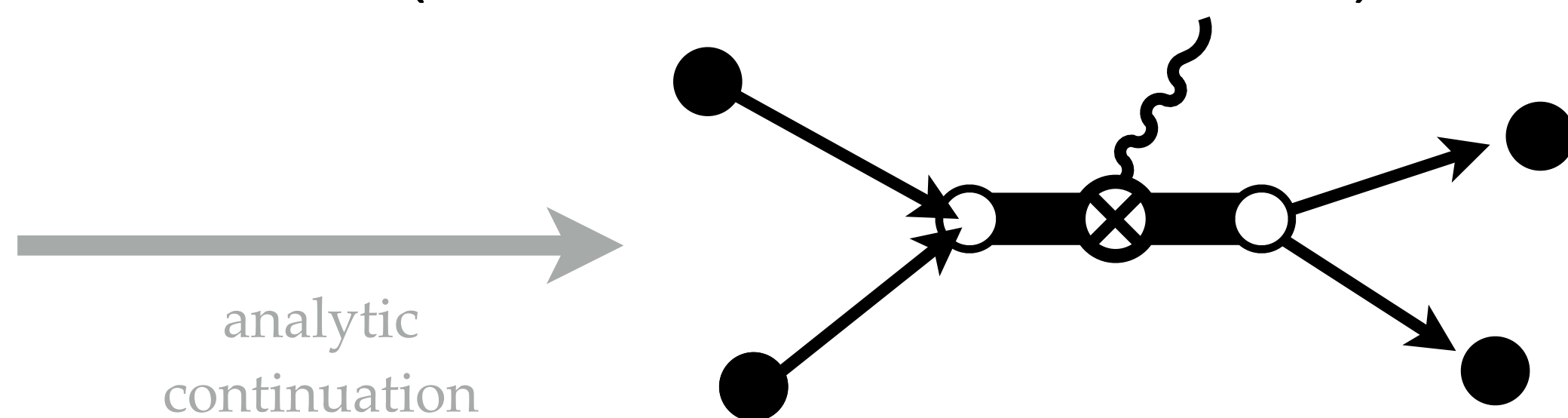
$$\mathcal{G}_j^\mu = \int \frac{d^4 k}{(2\pi)^4} \frac{\mathcal{Y}_f iK_j^\mu \mathcal{Y}_i}{(k^2 - m^2) ((P_f - k)^2 - m^2) ((P_i - k)^2 - m^2)}$$

$$R + \mathcal{J}^\mu \rightarrow R$$

Analytic continuation towards resonance pole



$$i\mathcal{W}_{\text{df}}^\mu = \mathcal{M} \cdot \left(i\mathcal{A}^\mu + i \sum_j f_j(Q^2) \mathcal{G}_j^\mu \right) \cdot \mathcal{M}$$

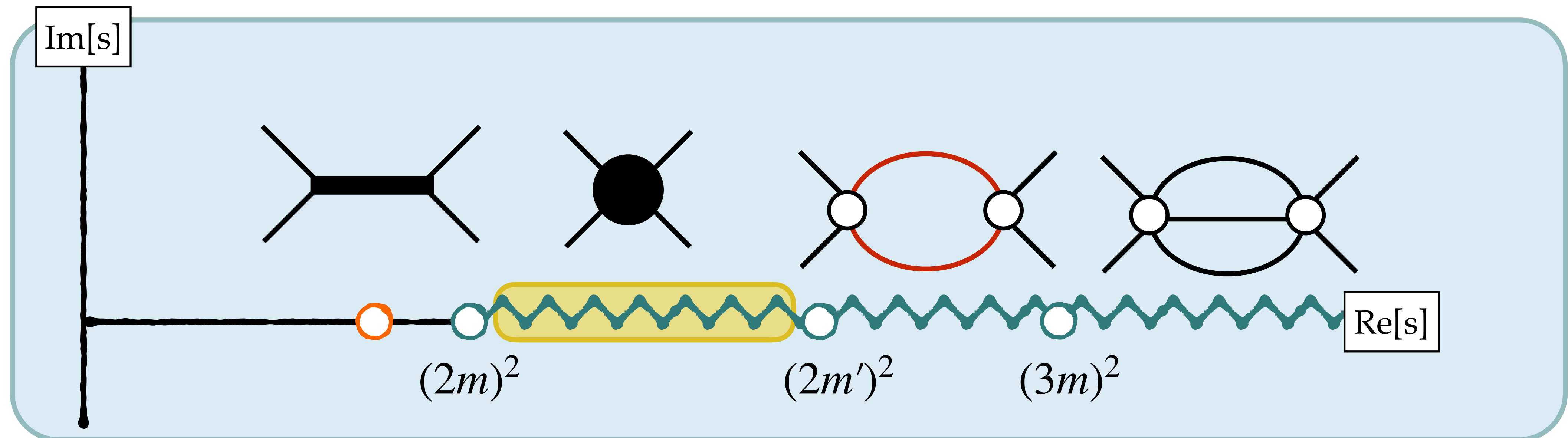


$$= g \frac{1}{s_f - s_R} iK^\mu F_R(Q^2) \frac{1}{s_i - s_R} g$$

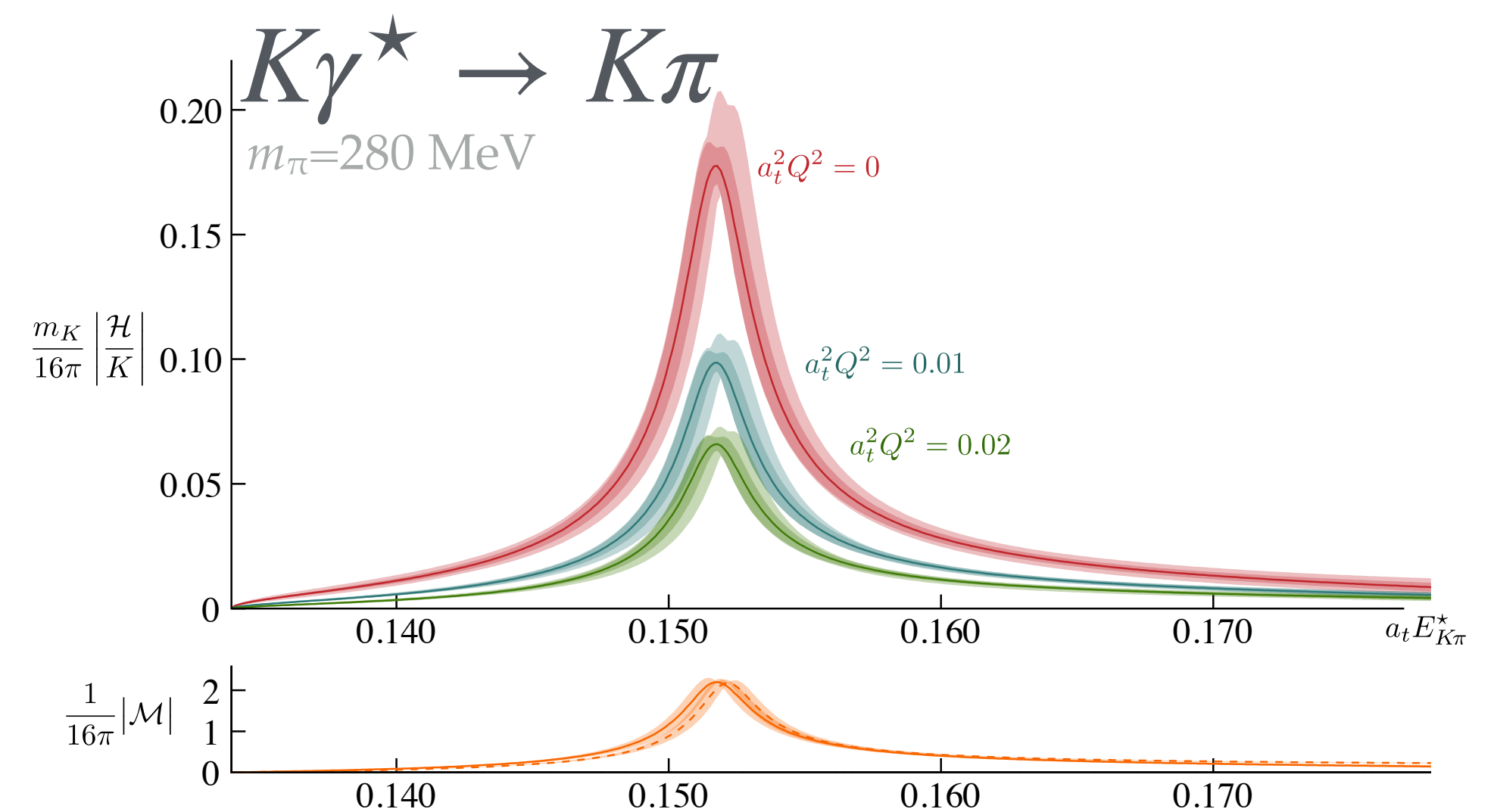
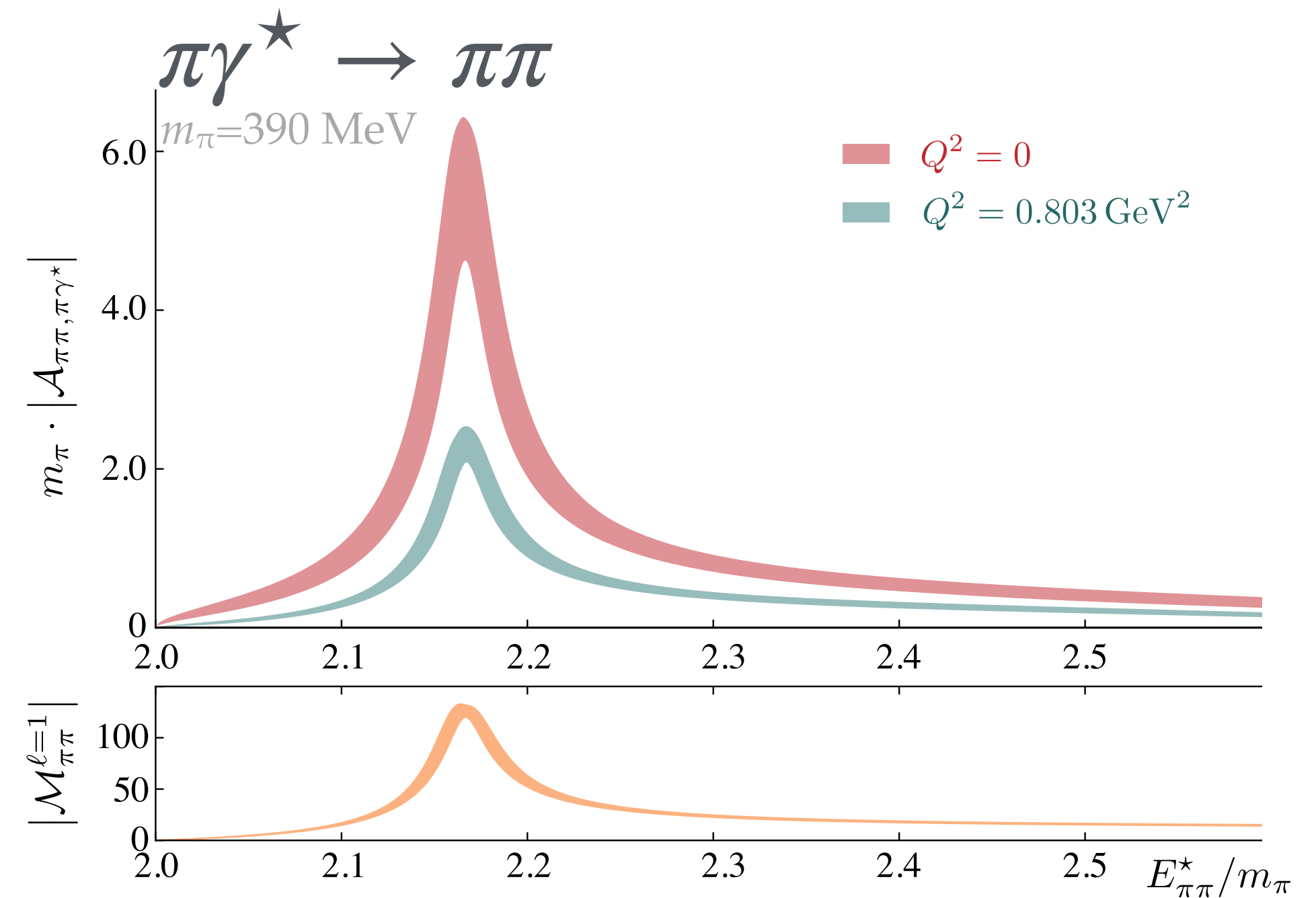
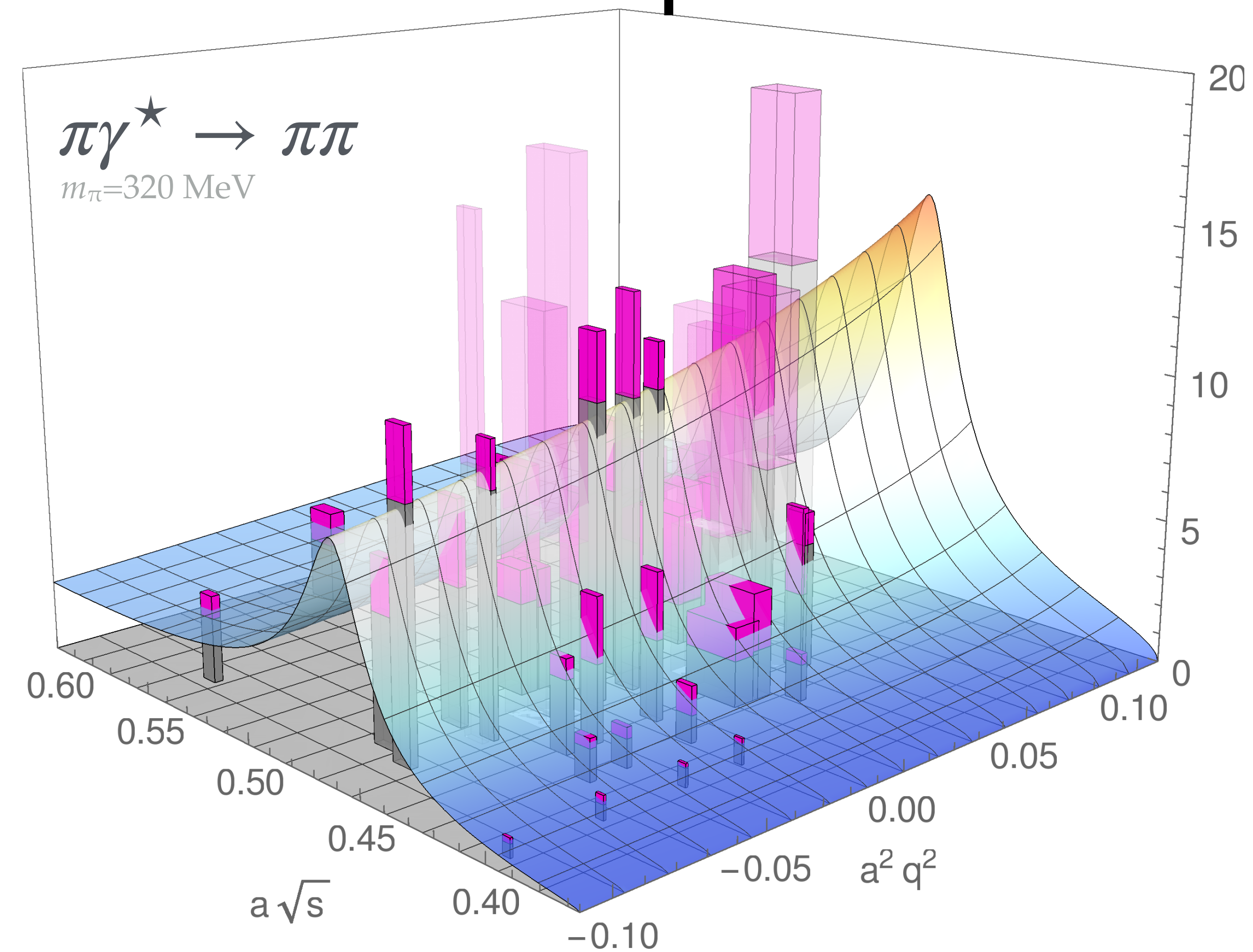
$$2 + \mathcal{J}^\mu \rightarrow 2$$

Goal: isolated ALL kinematic singularities of amplitudes in a kinematic region.

Observation: kinematic singularities are due to intermediate particles goin on-shell.



Transition amplitudes

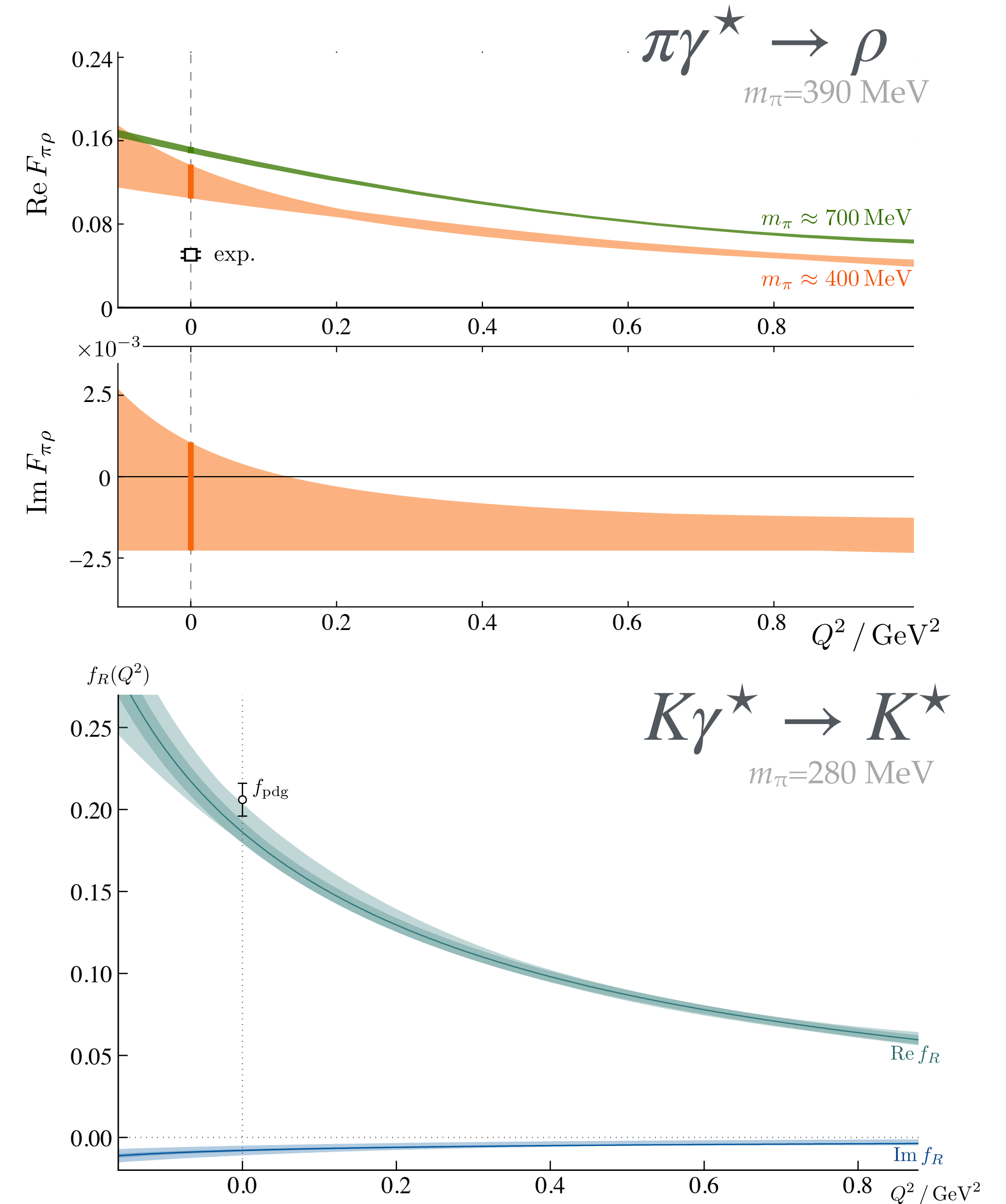
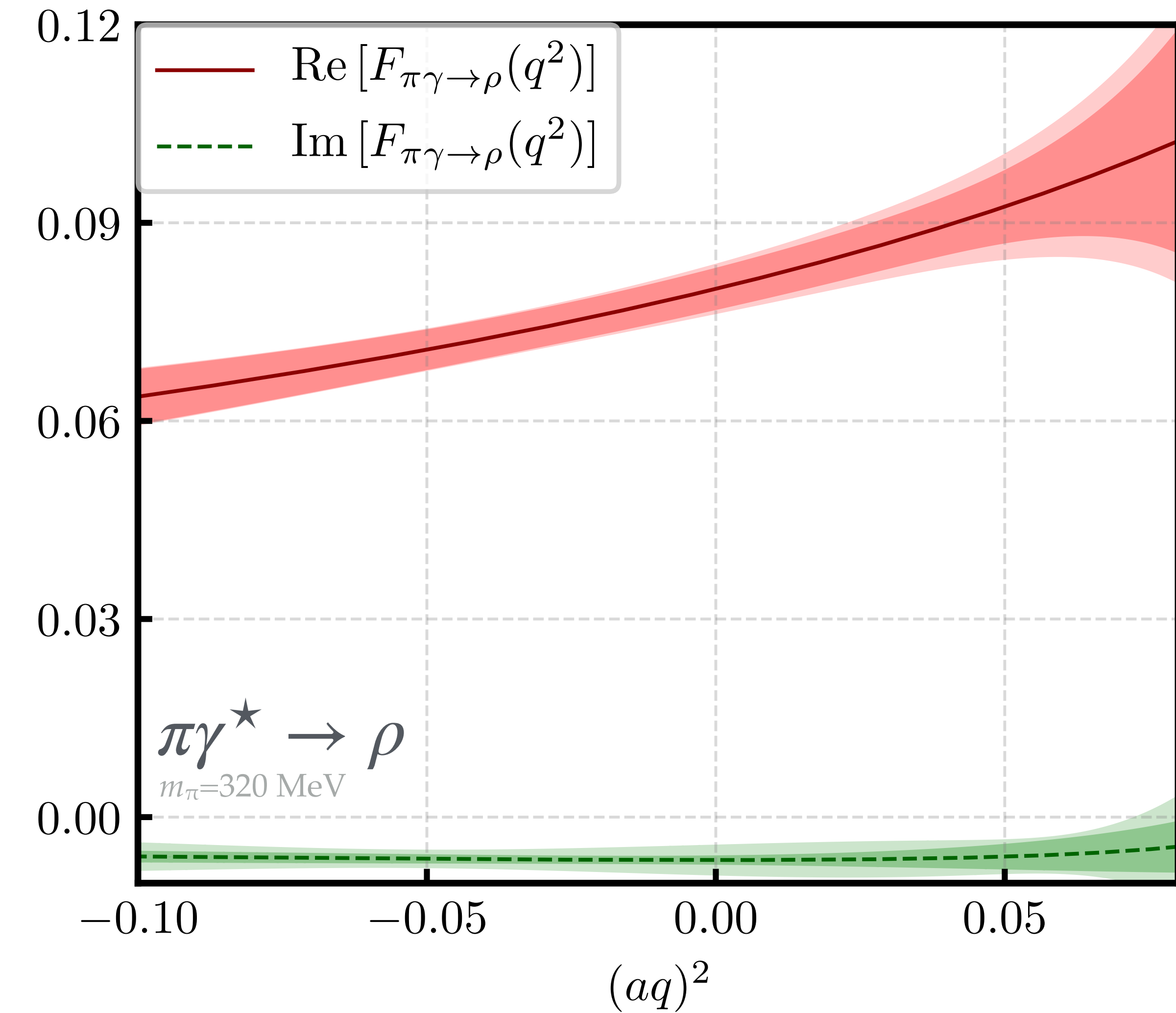


RB, Dudek, Edwards, Schultz, Thomas, Wilson (2016)

Radhakrishnan, Dudek, Edwards (2022)

Alexandrou, Leskovec, et al. (2018)

Transition form factors



RB, Dudek, Edwards, Schultz, Thomas, Wilson (2016)

Radhakrishnan, Dudek, Edwards (2022)

Alexandrou, Leskovec, Meinel, et al. (2018)

QCD is a rich forest with lots of trees deserving our attention



Outreach & mentoring

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Python4physics

```
# get objects selected in the viewport
viewport_selection = bpy.context.selected_objects

# get export objects
obj_export_list = viewport_selection
if self.use_selection_setting == False:
    obj_export_list = [i for i in bpy.context.scene.objects]

# deselect all objects
bpy.ops.object.select_all(action='DESELECT')

for item in obj_export_list:
    item.select = True
    if item.type == 'MESH':
        file_path = os.path.join(folder_path, "{}.obj".format(item.name))
        bpy.ops.export_scene.obj(filepath=file_path, use_selection=True,
                                axis_forward=self.axis_forward_setting,
                                axis_up=self.axis_up_setting,
                                animation=self.use_animation_setting,
                                mesh_modifiers=self.use_mesh_modifiers_setting,
                                use_edges=self.use_edges_setting,
                                use_smooth_groups=self.use_smooth_groups_setting,
                                use_smooth_groups_bitflags=self.use_smooth_groups_bitfl
                                use_normals=self.use_normals_setting,
                                use_unwrap=self.use_unwrap_setting,
```

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