

# Two-particle azimuthal correlations for searching for gluon saturation

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From Quarks and Gluons to the Internal Dynamics of Hadrons  
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# Outline

- Searching for saturation with two particle correlations
- Small- $x$  TMD factorization at NLO from CGC
- Bringing back quarks at small- $x$
- Summary and Outlook

# Anatomy of nuclear matter at high-energies

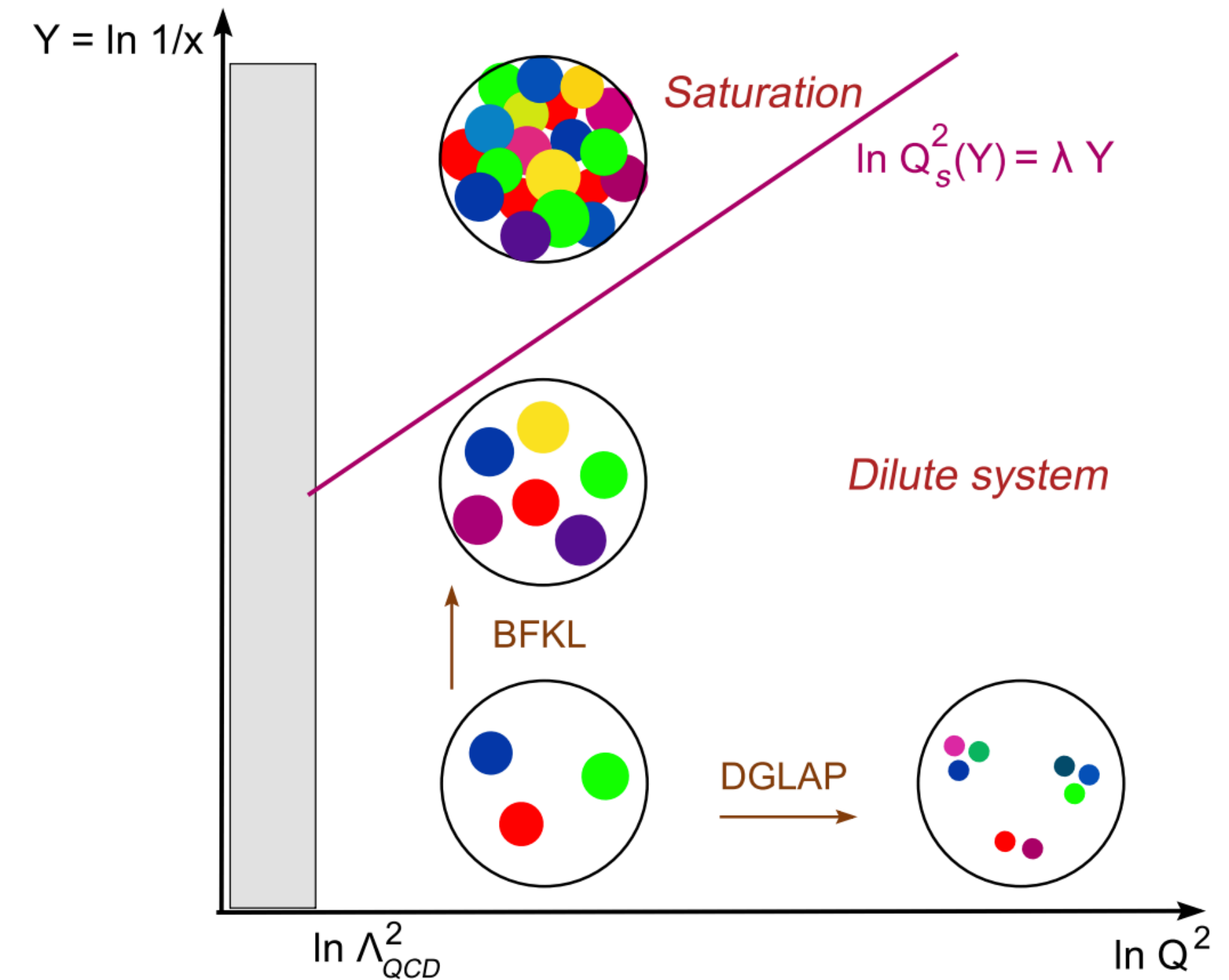
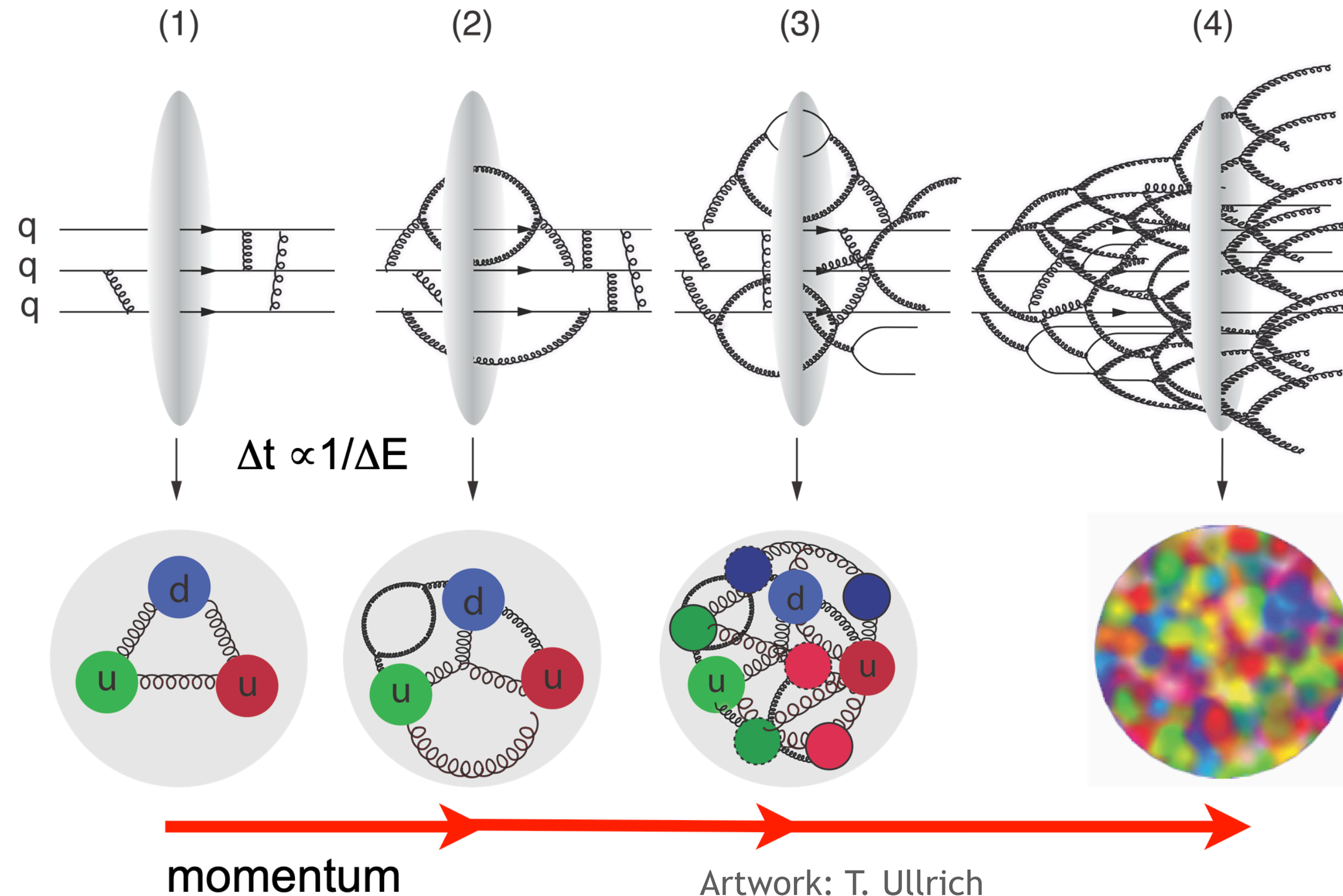


Figure from EIC White paper (2012)

Emergence of an energy and nuclear specie dependent momentum scale (saturation scale)

$$Q_s^2 \propto A^{1/3} x^{-\lambda}$$

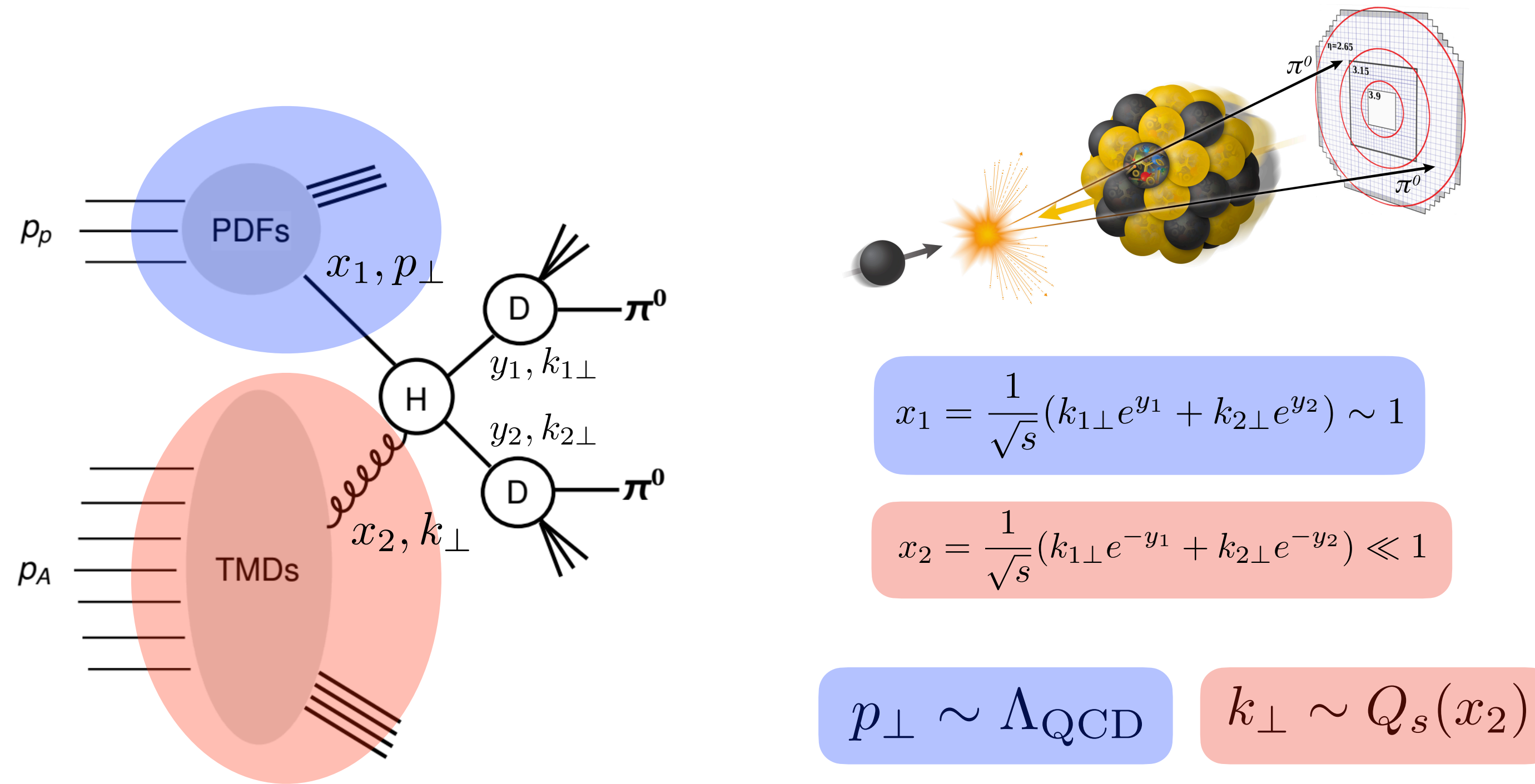
Multiple scattering (higher twist effects)

Non-linear evolution equations (BK/JIMWLK)

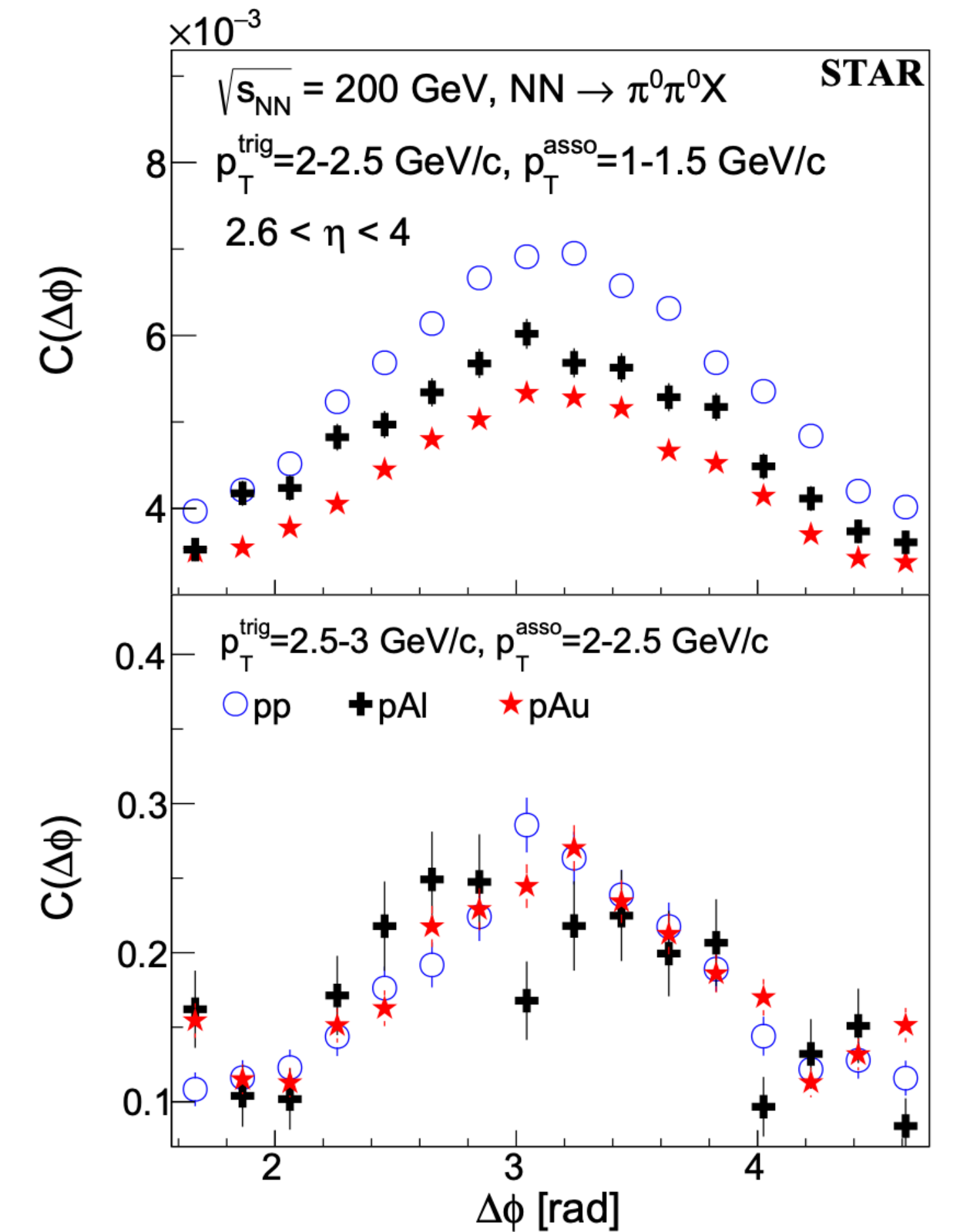
For a review see Mining gluon saturation at colliders. FS, A. Morreale (Universe 2021)

# Searching for saturation with two particle correlations

Azimuthal correlations as a probe for gluon saturation proposed  
by Kharzeev, Levin, McLerran (NPA 2005)  
Marquet (NPA 2007)



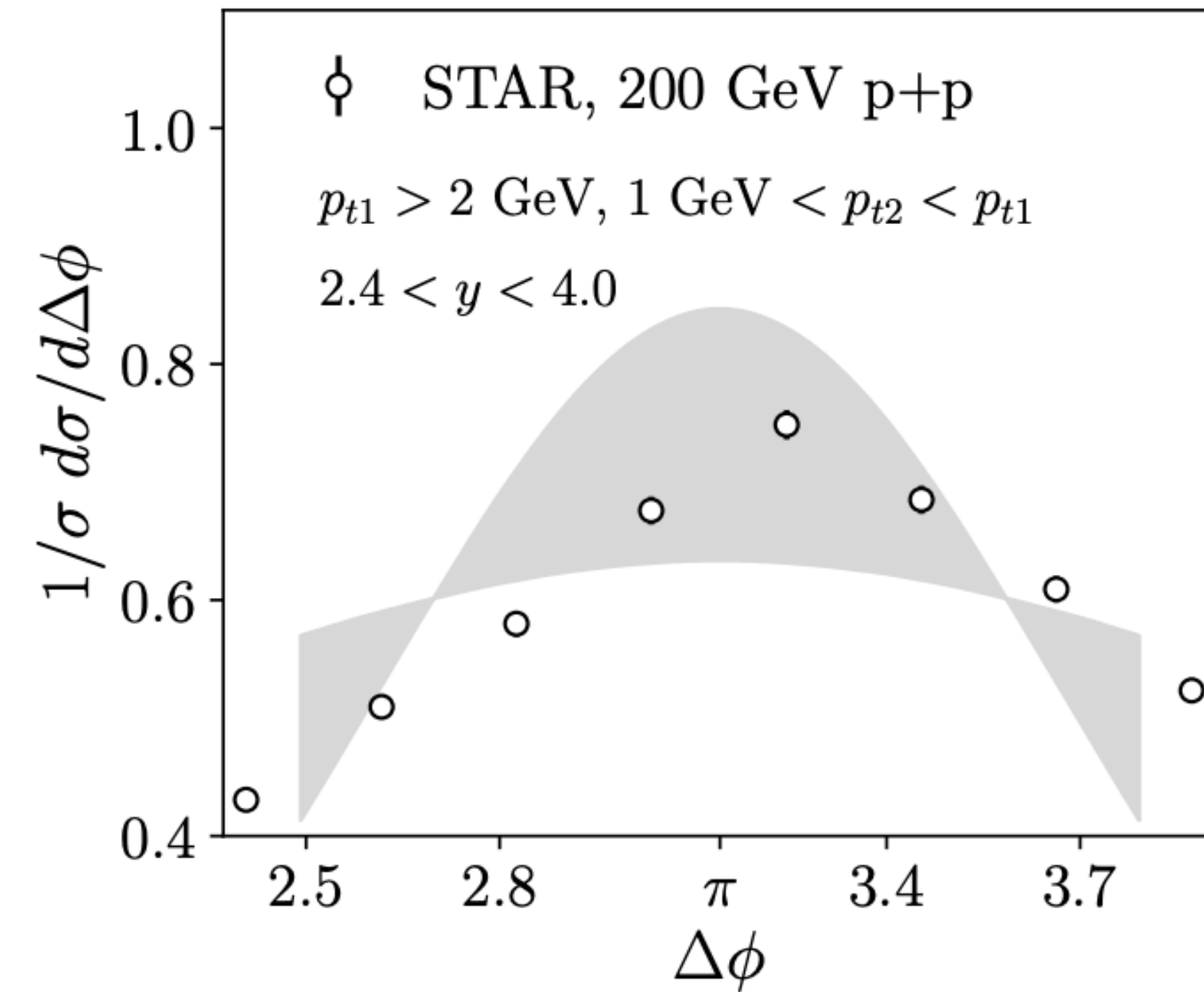
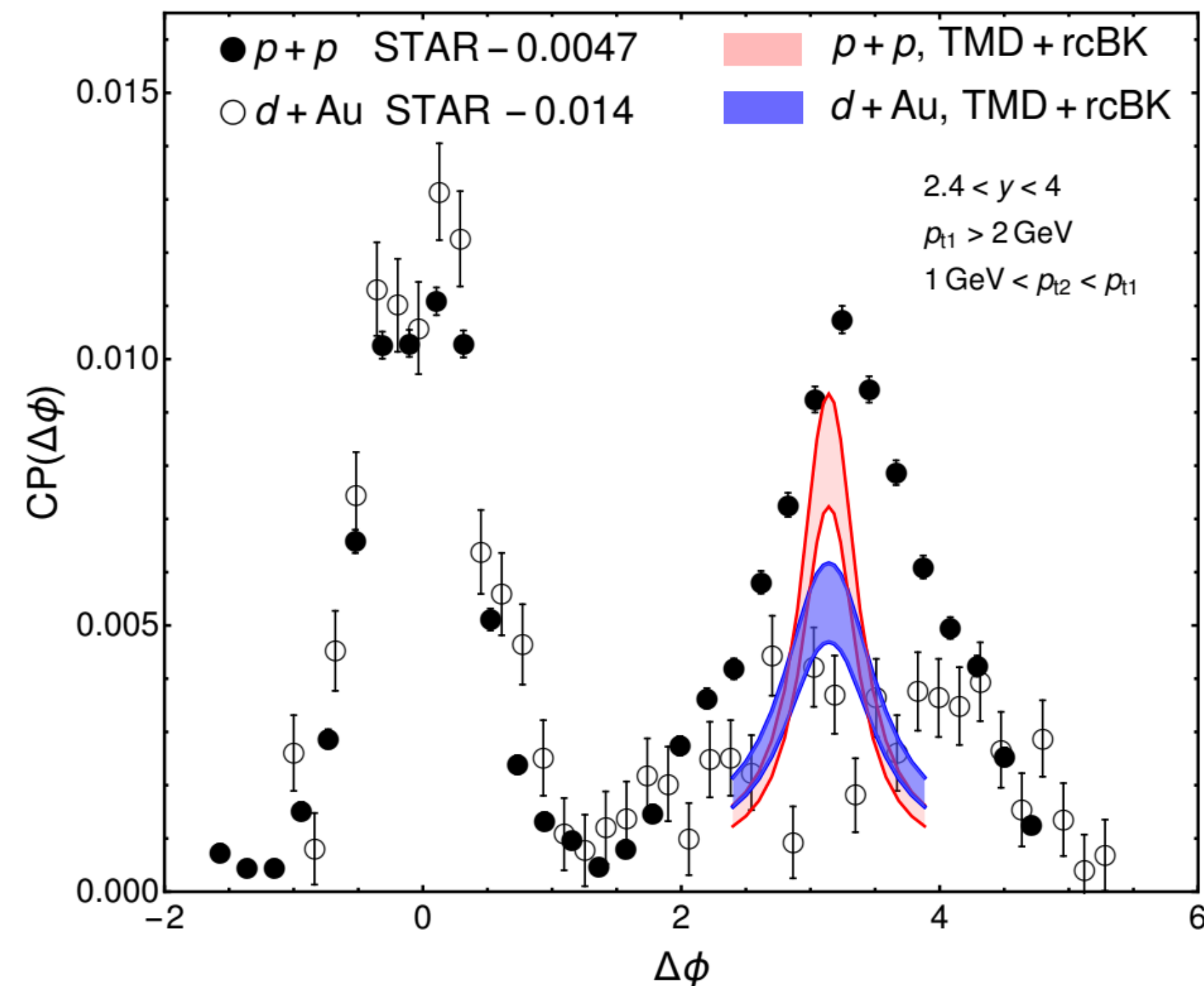
$$Q_s(x_2) \gg \Lambda_{\text{QCD}}$$



See more at Xiaoxuan's talk!



# Searching for saturation with two particle correlations



**Experimental data:** Braidot et al [STAR Collaboration] *arXiv:1005.2378*

**Theory curves:** Albacete, Giacalone, Marquet, Matas (*PRD* 2019)

For recent phenomenology of Sudakov + Gluon Saturation see talks at DIS2022 by Marquet

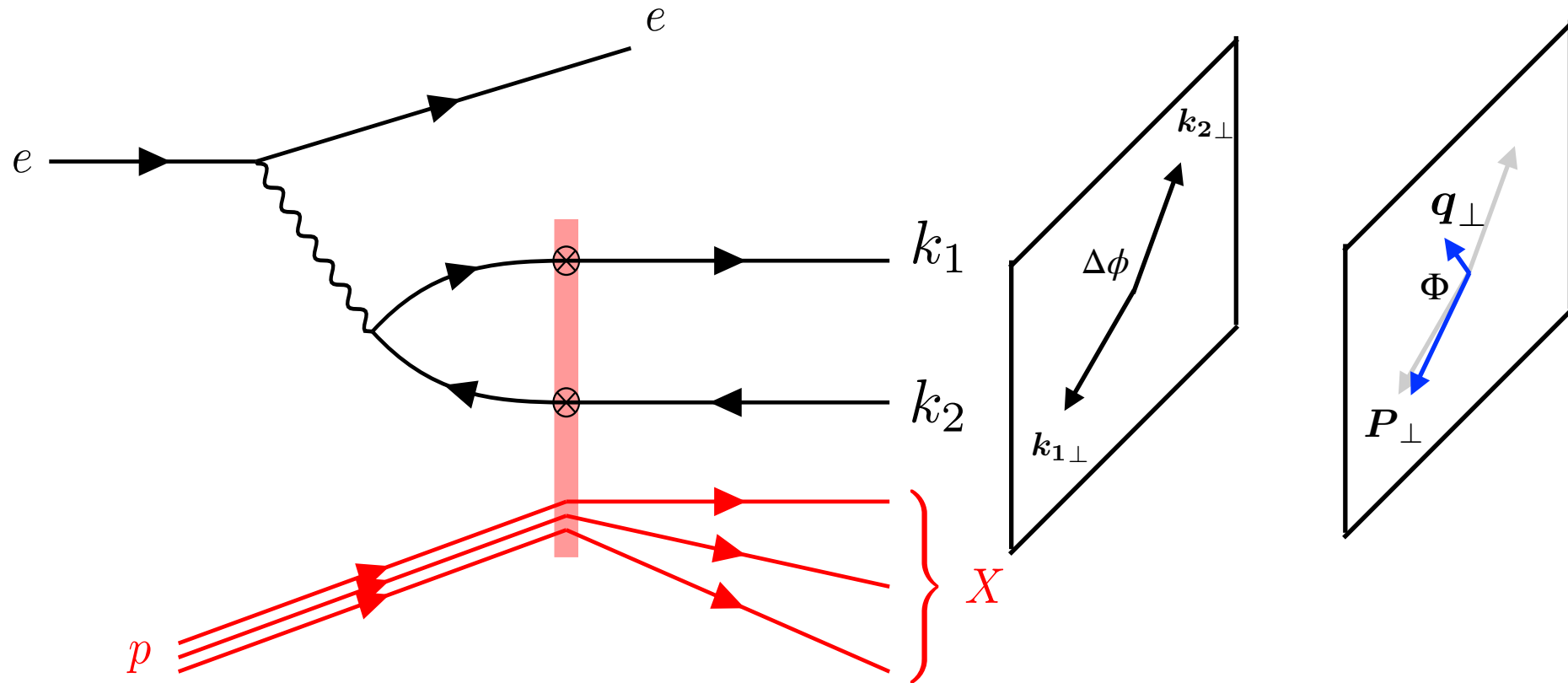
See also A. Stasto, S.Y. Wei, B.W. Xiao, F. Yuan (*PLB* 2018)

**Gluon saturation alone cannot describe data**

# Searching for saturation with two particle correlations

## Dijet production in DIS at the Electron-Ion Collider

Dominguez, Marquet, Xiao, Yuan (PRD 2011)



Jalilian-Marian, Gelis (PRD 2003)

Unpolarized differential cross-section:

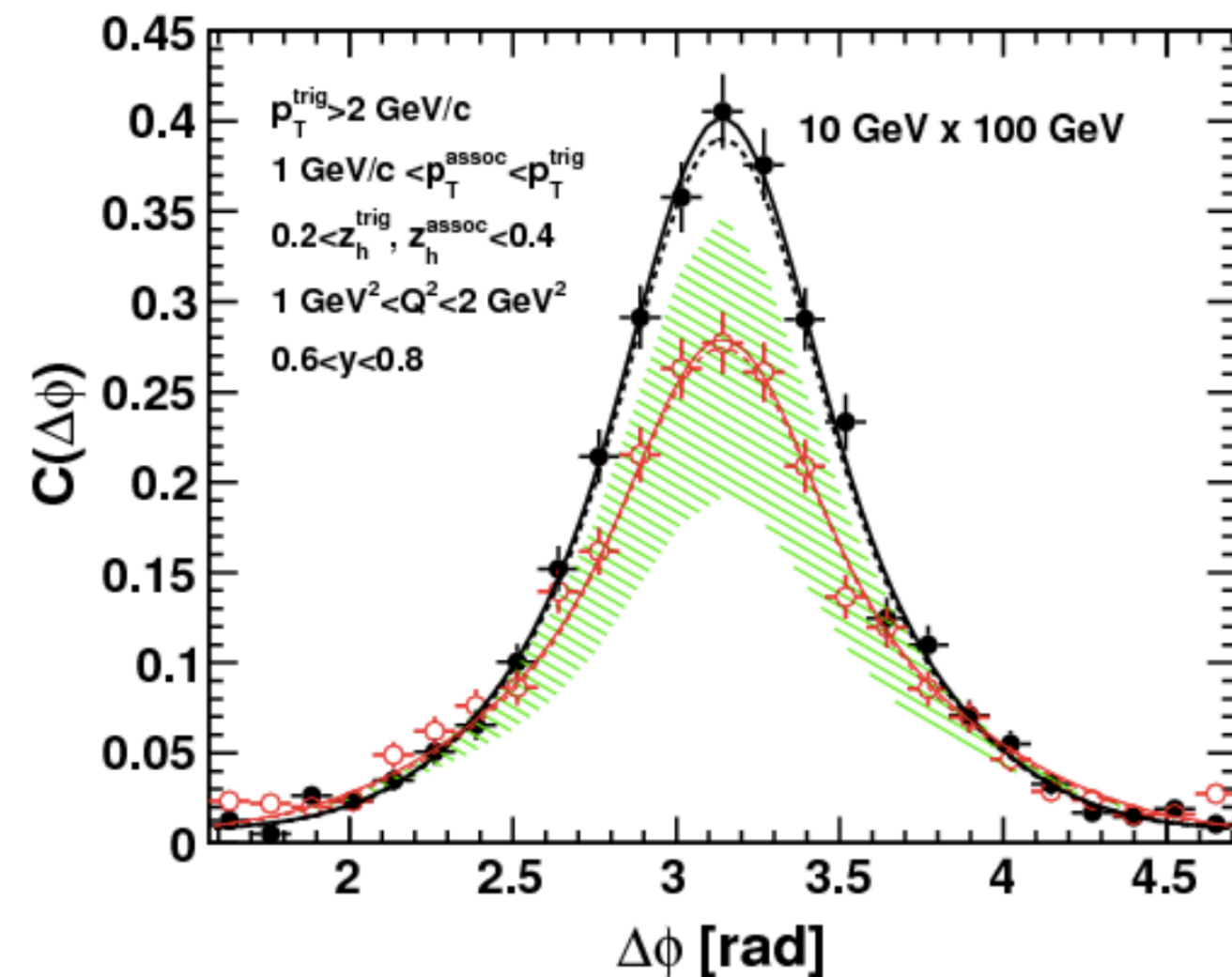
$q\bar{q}$  interaction with nucleus

$$d\sigma_{\text{CGC,LO}}^{\gamma+A \rightarrow q\bar{q}+X} = \mathcal{H}^\lambda(z, Q, \mathbf{P}_\perp; \mathbf{l}_\perp, \mathbf{l}'_\perp) \otimes_{\mathbf{l}_\perp, \mathbf{l}'_\perp} \mathcal{G}(\mathbf{q}_\perp; \mathbf{l}_\perp, \mathbf{l}'_\perp)$$

Perturbatively calculable

Two and four-point light-like Wilson correlator (contains implicitly  $Q_s$ )

### Phenomenology at the EIC Gluon saturation imprint in particle correlations



Back-to-back limit  $Q_s, q_\perp \ll P_\perp$

$$d\sigma_{\text{TMD,LO}}^{\gamma+A \rightarrow q\bar{q}+X} = H_{ij}^\lambda(z, Q, \mathbf{P}_\perp) G^{ij}(\mathbf{q}_\perp)$$

Quantitative difference between CGC and TMD approximation studied in

Mantysaari, Mueller, Salazar, Schenke (PRL 2020)

Does the correspondence between CGC and TMD hold beyond the leading order picture?

Previous studies focused on small- $x$  evolution and the Sudakov double log:

Mueller, Xiao, Yuan (PRD 2013)  
Xiao, Yuan, Zhou (Nucl Phys B 2017)

# Small- $x$ TMD factorization at NLO from CGC: Dijet and dihadron production in deep inelastic scattering

Dijet: Caucal, Salazar, Schenke, Venugopalan, Stebel (PRL 2024)

Dihadron: Salazar, Caucal (work in progress)

# Dijet production in DIS at NLO

Caucal, Salazar, Venugopalan (JHEP 2021)

Kinematic domain of validity:  $\Lambda_{QCD}^2 \ll q_{\perp}^2, P_{\perp}^2 \ll s$  High-energy limit (Forward production in lepton direction)

- Covariant Feynman rules with CGC effective vertices, dim-reg + longitudinal momentum cut-off  $z_0$

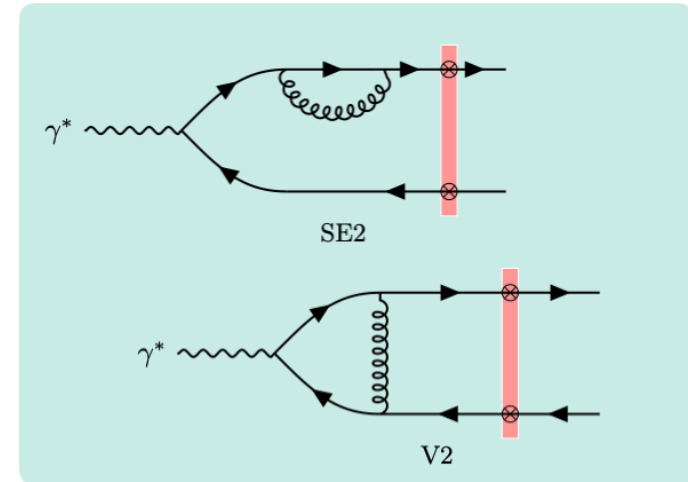
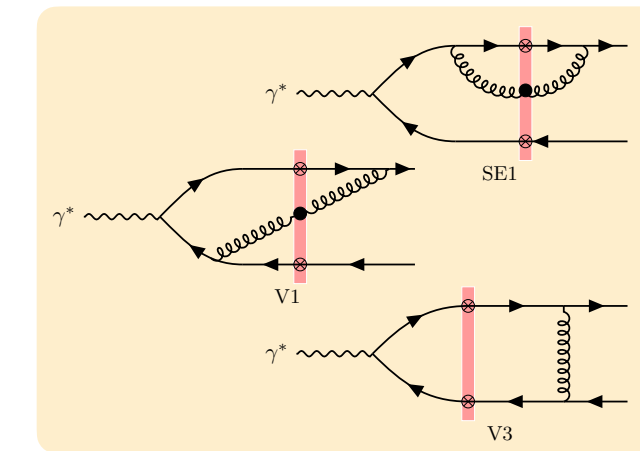
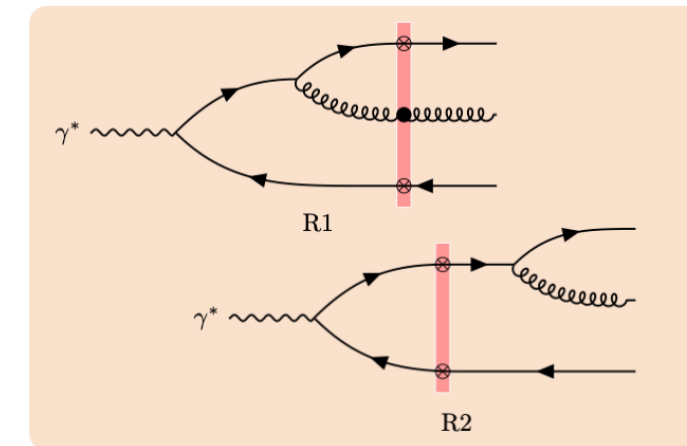
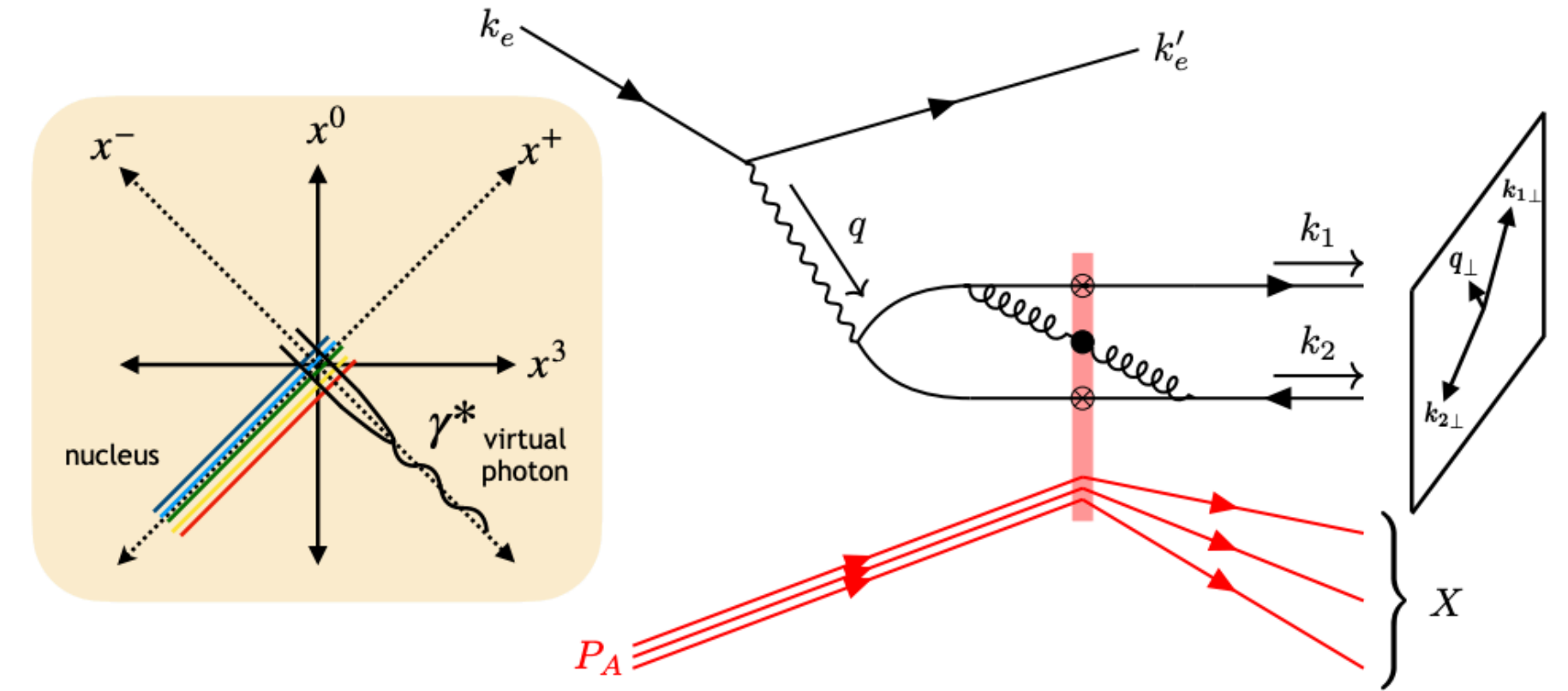
Modes with  $z_g \leq z_0$  belong to the target (background field)

- Show cancellations of UV divergences and infrared and collinear safety of this observable
- Proof of small-x factorization, large logs can be absorbed via non-linear RG

$$\alpha_s d\sigma_{1L}^{\lambda} = \ln\left(\frac{z_f}{z_0}\right) K_{JIMWLK} d\sigma_{LO}^{\lambda} \sim \ln(s/P_{\perp}^2)$$

$$+ \alpha_s \int_0^1 \frac{dz_g}{z_g} \int d^2 z_{\perp} \left[ d\tilde{\sigma}_{1L}^{\lambda}(z_g, z_{\perp}) - d\tilde{\sigma}_{1L}^{\lambda}(0, z_{\perp}) \Theta(z_f - z_g) \right]$$

- Computed genuine  $\alpha_s$  correction to the impact factor





# Dijet production in DIS at NLO

Kinematic domain of validity:

$$\Lambda_{QCD}^2 \ll q_\perp^2 \ll P_\perp^2 \ll s \text{ and } Q_s^2 \ll P_\perp^2$$

High-energy limit (Forward production in virtual photon direction) but back-to-back in the transverse plane

Previous studies focused on small-x evolution and the Sudakov double log:

Mueller, Xiao, Yuan (PRD 2013)  
Xiao, Yuan, Zhou (Nucl Phys B 2017)

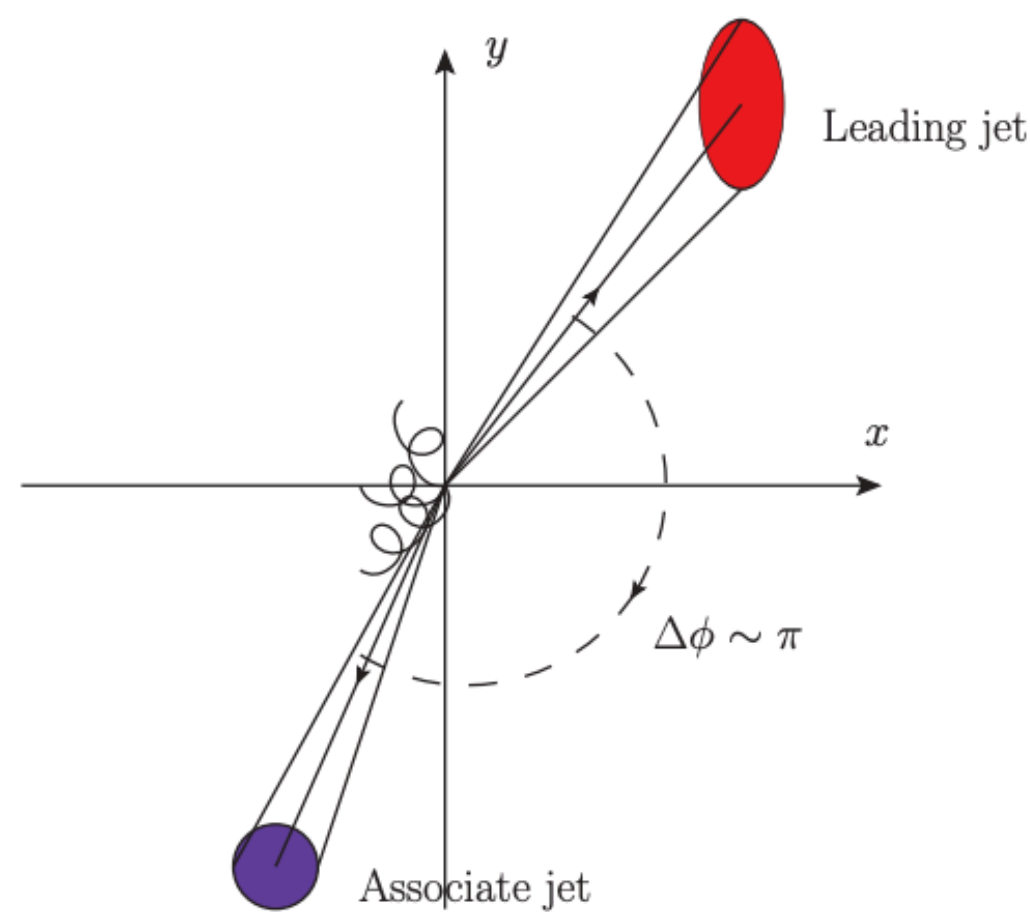
Caucal, Salazar, Schenke, Venugopalan (JHEP 2022) +  
Stebel (JHEP 2023, PRL 2024)

Both unpolarized and linearly polarized

Metz, Zhou (PRD 2011)

$$d\sigma_{\text{TMD,NLO}}^{\gamma+A \rightarrow j_1 j_2 + X} = H_{\text{NLO},ij}^\lambda(z, Q, \mathbf{P}_\perp) \int \frac{d^2 \mathbf{b}_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \tilde{G}_\eta^{ij}(\mathbf{b}_\perp) e^{-S_{\text{Sud}}(\mathbf{P}_\perp, \mu_b)}$$

$$\mu_b = c_0/b_\perp$$



Perturbatively calculable hard factor

Weizsäcker-Williams  
Small-x distribution  
obeys a modified  
non-linear RG

Sudakov factor

Analogous to the non-local equation derived by  
Ducloué, Iancu, Mueller,  
Soyez, Triantafyllopoulos  
(JHEP 2019)

$$\ln(s/P_\perp^2)$$

$$S_{\text{Sud}}(\mathbf{P}_\perp, \mu_b) = \frac{\alpha_s N_c}{4\pi} \ln^2 \left( \frac{P_\perp^2}{\mu_b^2} \right) + \frac{\alpha_s}{\pi} \left[ C_F \ln \left( \frac{1}{z_1 z_2 R^2} \right) + N_c \ln \left( 1 + \frac{z_1 z_2 Q^2}{P_\perp^2} \right) - \beta_0 \right] \ln \left( \frac{P_\perp^2}{\mu_b^2} \right)$$

Figure from  
B.W. Xiao (QCD master class 2021)

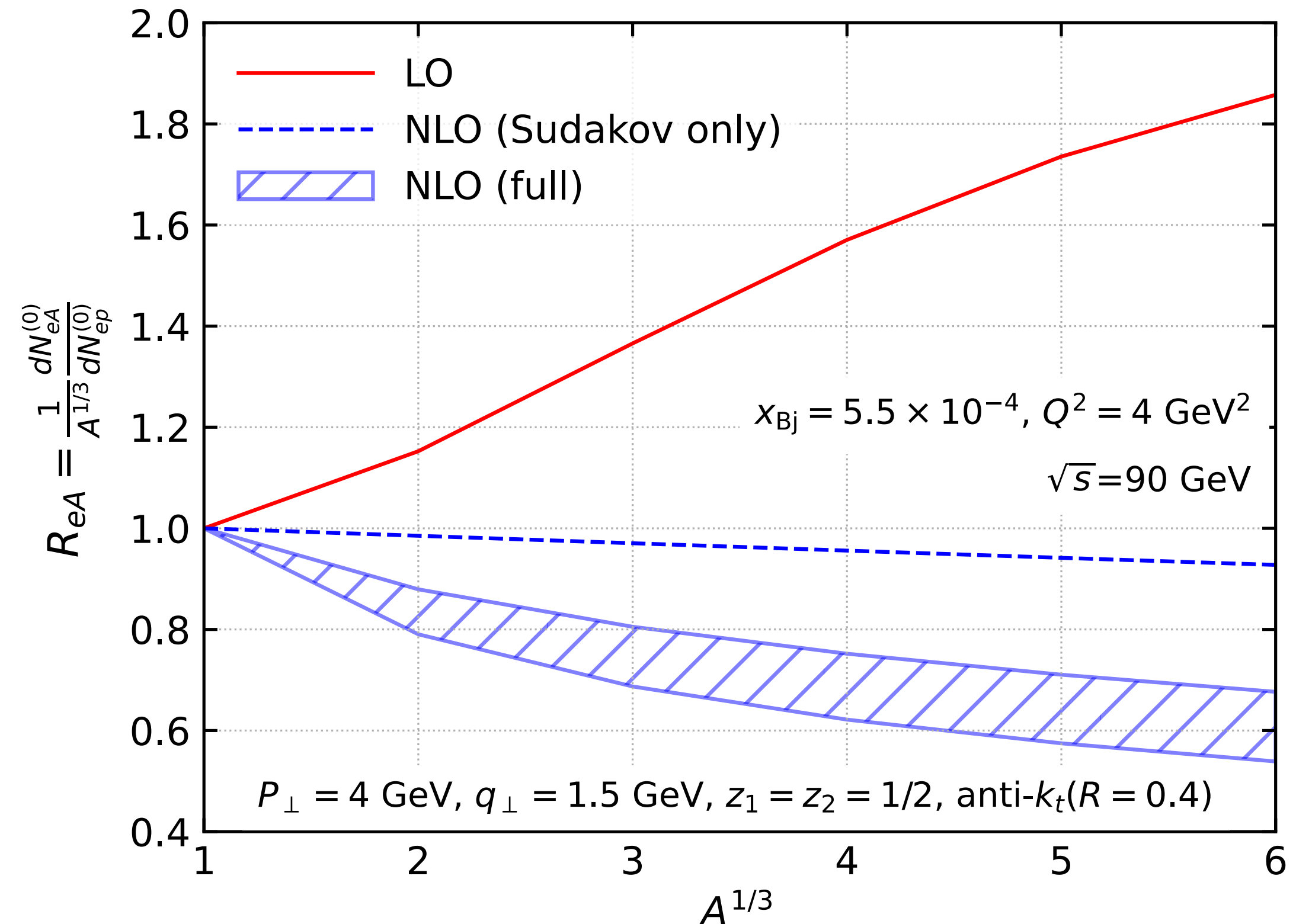
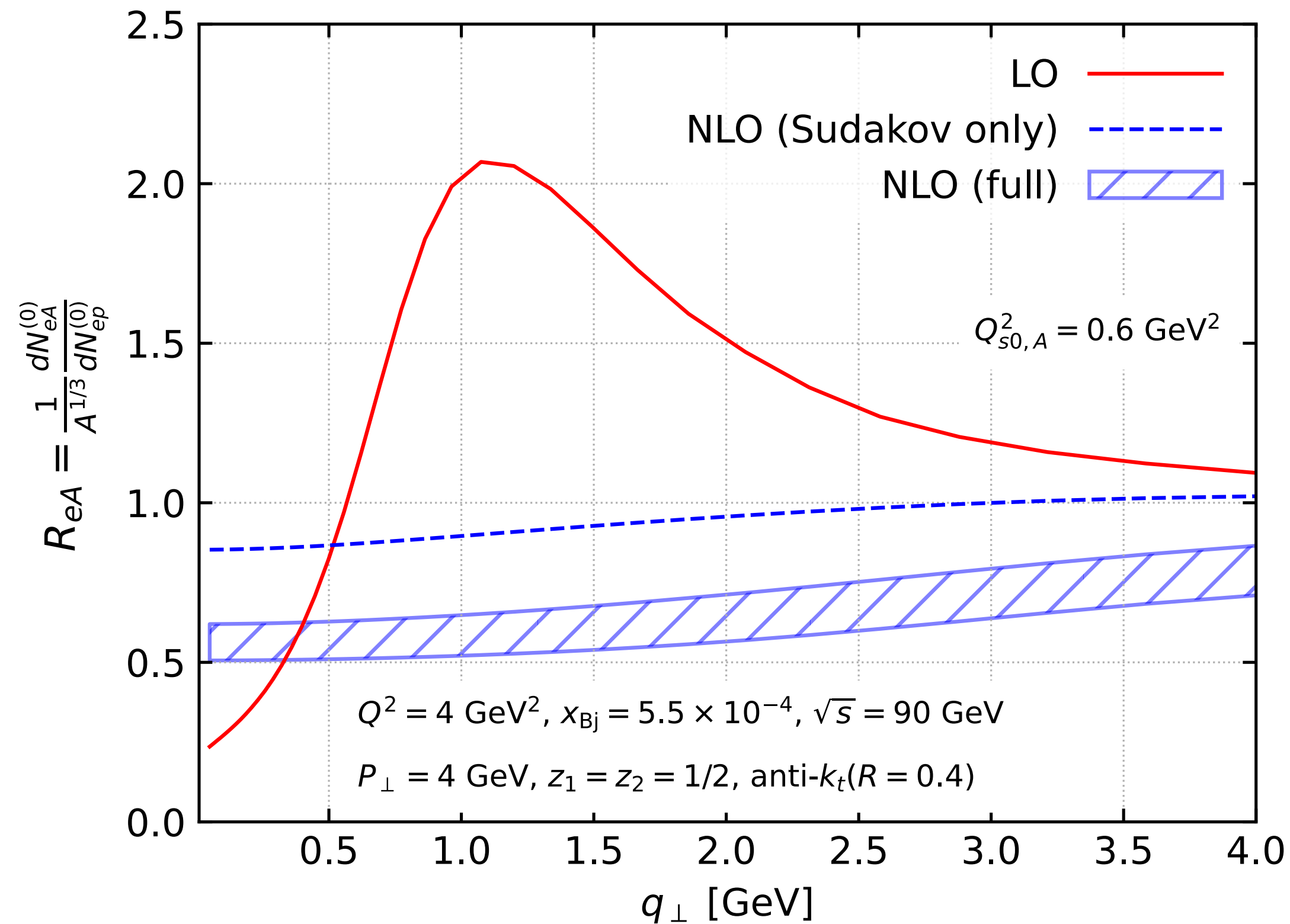
See also Tael, Altinoluk,  
Marquet, Beuf (JHEP 2022)

Sudakov factor consistent with Hatta, Xiao, Yuan, Zhou (PRD 2021)

# Dijet production in DIS at NLO

## Predictions for nuclear modification at the Electron-Ion Collider

$dN$  min-bias differential yield  $R_{eA} = \frac{1}{A^{1/3}} \frac{dN_{eA}}{dN_{ep}}$  Caucal, Salazar, Schenke, Venugopalan, Stebel (PRL 2024)



Full NLO (Sudakov + small-x evolution) shows a significant A-dependent suppression

# Dihadron production in DIS at NLO

Our techniques can also be applied to other two-particle correlations jet + photon, dijets in pA (RHIC and LHC)

extending the work in Mueller, Xiao, Yuan (PRD 2013) to complete one-loop calculation

Also we can study back-to-back dihadron production in DIS within the CGC. It can also be factorized

Caucal, Salazar (in progress)

$$\begin{aligned} d\sigma_{\text{TMD,NLO}}^{\gamma+A \rightarrow h_1 h_2 + X} &= \int_0^1 \frac{dx_1}{x_1^2} \int_0^1 \frac{dx_2}{x_2^2} \delta(1 - z_1 - z_2) \mathcal{H}_{\text{NLO}}^{\lambda,ij}(\mathbf{P}_\perp, Q, z_1, z_2) \\ &\times \int \frac{d^2 \mathbf{b}_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} G_{\eta_f}^{ij}(\mathbf{b}_\perp) e^{-S_{\text{Sud}}(\mathbf{P}_\perp, \mu_b)} \bar{D}_{h_2/\bar{q}}(x_2, \mathbf{b}_\perp; \mu_F, P_\perp) \bar{D}_{h_1/q}(x_1, \mathbf{b}_\perp; \mu_F, P_\perp) \end{aligned}$$

$$S_{\text{Sud}}(\mathbf{P}_\perp, \mu_b) = \frac{\alpha_s N_c}{4\pi} \ln^2 \left( \frac{\mathbf{P}_\perp^2}{\mu_b^2} \right) + \frac{\alpha_s}{\pi} \left[ C_F \ln \left( \frac{1}{z_1 z_2} \right) + N_c \ln \left( 1 + \frac{z_1 z_2 Q^2}{\mathbf{P}_\perp^2} \right) - \beta_0 \right] \ln \left( \frac{\mathbf{P}_\perp^2}{\mu_b^2} \right)$$

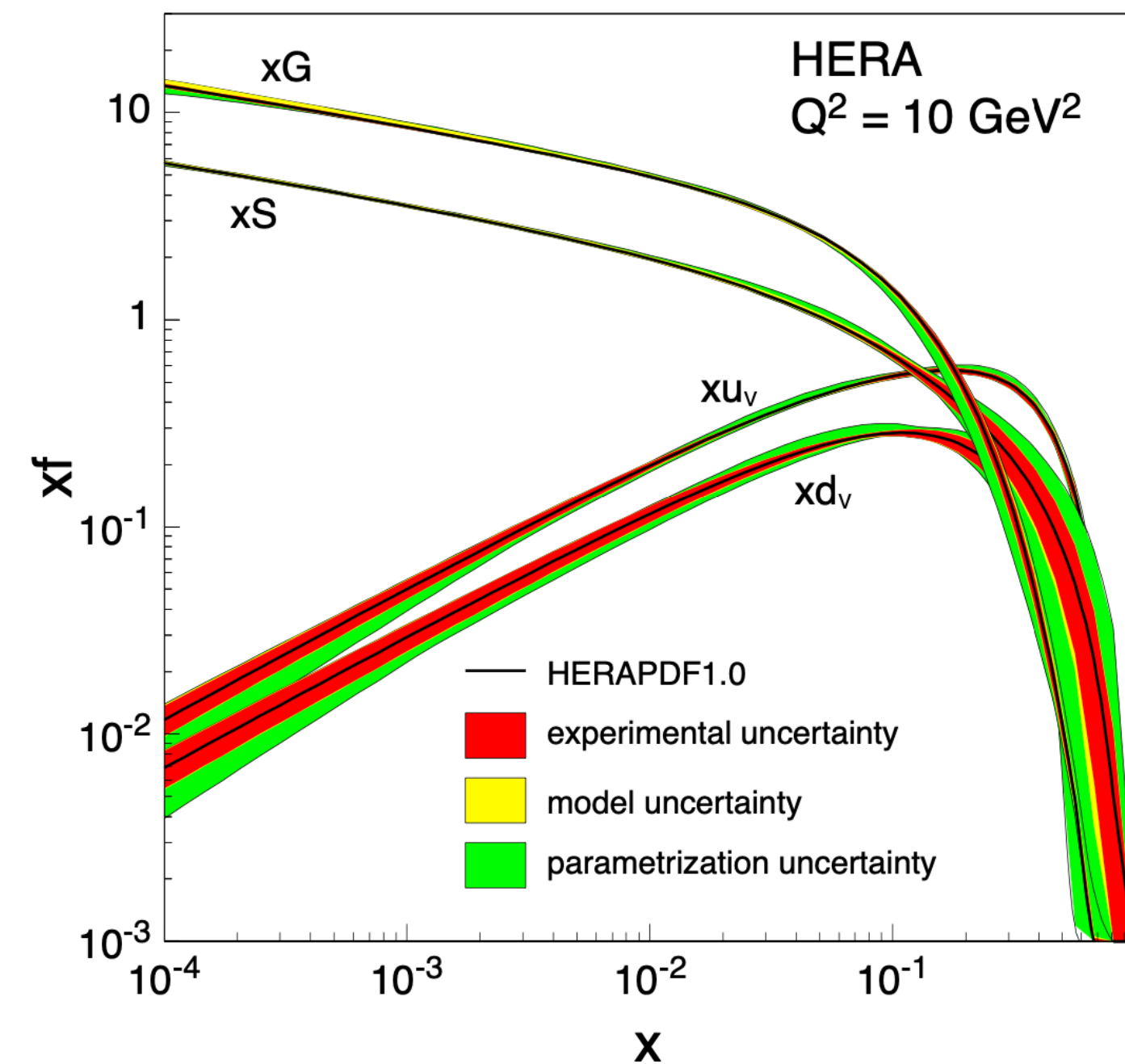
$$\bar{D}_{h_i/q}(x_1, \mathbf{r}_{bb'}; \mu_F, P_\perp) \equiv D_{h_i/q}(x_1, \mu_F) \exp \left( -\frac{\alpha_s C_F}{2\pi} \left[ \frac{1}{2} \ln^2 \left( \frac{\mathbf{P}_\perp^2}{\mu_b^2} \right) - \frac{3}{2} \ln \left( \frac{\mathbf{P}_\perp^2}{\mu_b^2} \right) \right] \right)$$

Sudakov resummation in TMD fragmentation function

State-of-art phenomenology under a unified framework coming soon...

# Bringing back quarks at small-x: Universality of the sea-quark distributions

Caucal, Iancu, Salazar, Yuan (work in progress)





# Bringing back quarks at small-x:

A well-known example: Semi-inclusive DIS

$$\frac{d\sigma_T^{\gamma^*+A\rightarrow q+X}}{d^2\mathbf{k}_\perp} = \frac{8\pi^2\alpha_{\text{em}}e_q^2}{Q^2} \int \frac{d^2\mathbf{b}_\perp d^2\mathbf{l}_\perp}{(2\pi)^2} F(\mathbf{l}_\perp, \mathbf{b}_\perp; x) \mathcal{H}_T(\mathbf{k}_\perp, \mathbf{l}_\perp, Q)$$

$$\mathcal{H}_T(\mathbf{k}_\perp, \mathbf{l}_\perp, Q) = \frac{Q^2 N_c}{4\pi^2} \int_0^1 dz [z^2 + (1-z)^2] \left| \frac{\mathbf{k}_\perp}{\mathbf{k}_\perp^2 + z(1-z)Q^2} - \frac{(\mathbf{k}_\perp - \mathbf{l}_\perp)}{(\mathbf{k}_\perp - \mathbf{l}_\perp)^2 + z(1-z)Q^2} \right|^2$$

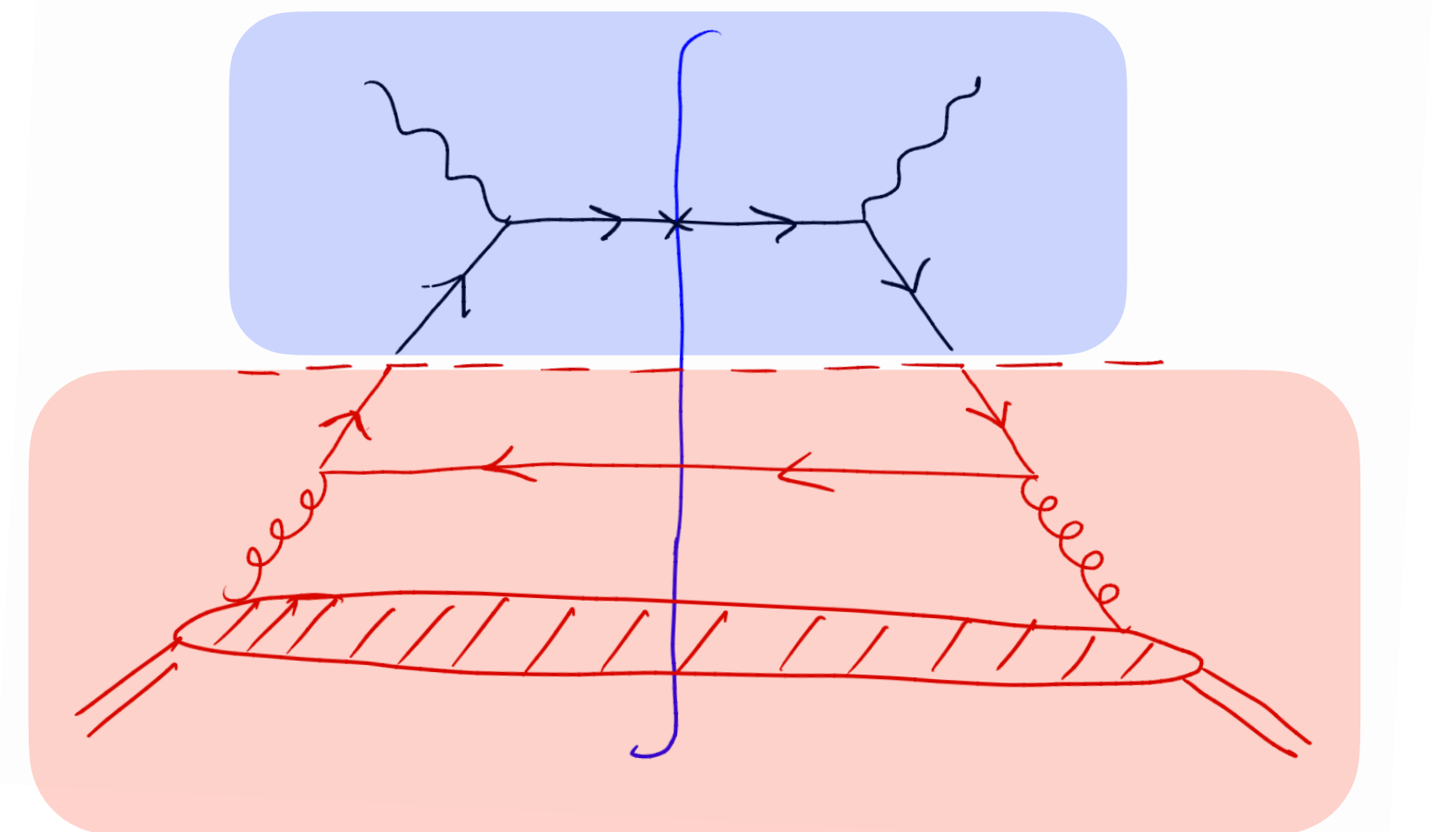
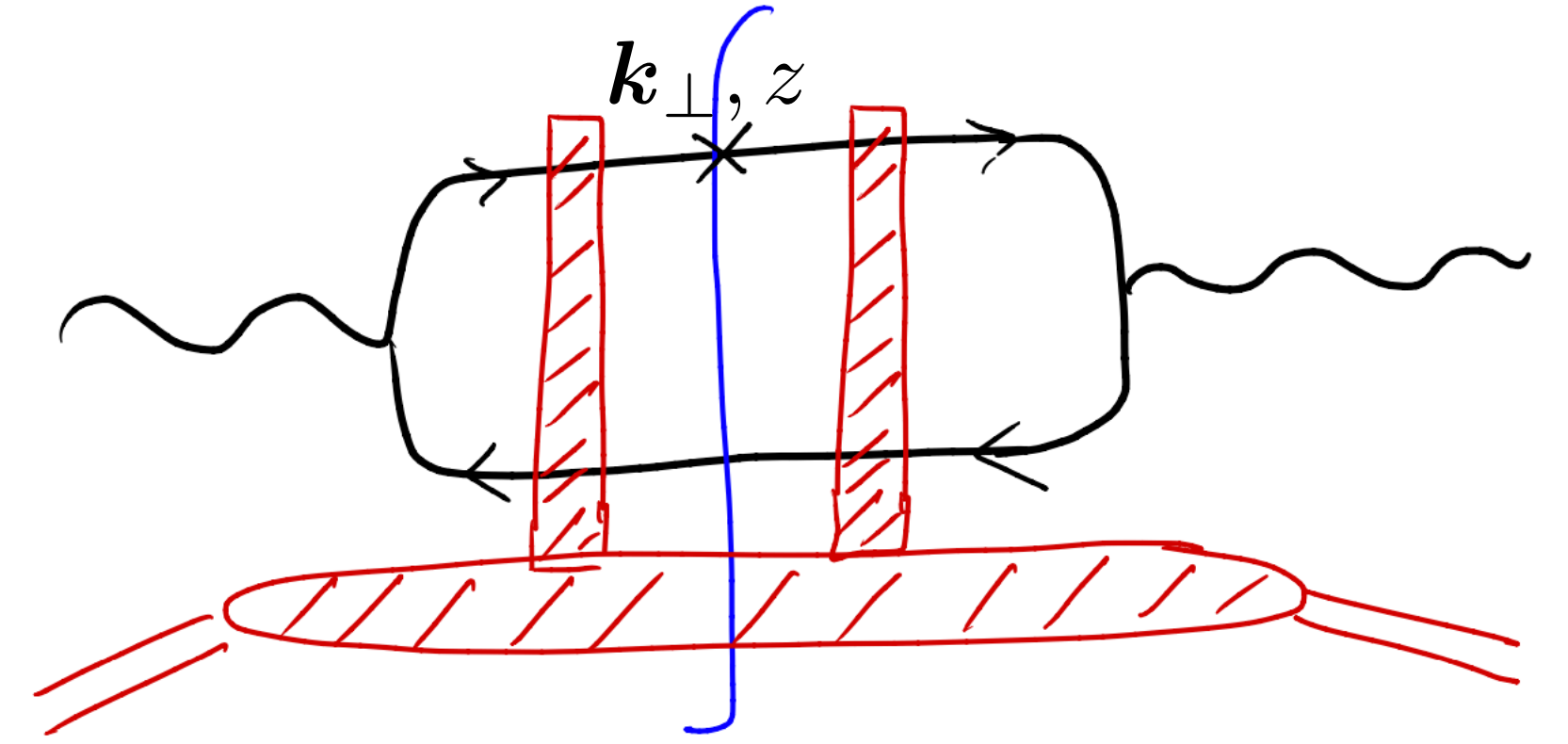
TMD regime:  $Q^2 \gg \mathbf{k}_\perp^2, \mathbf{l}_\perp^2 \sim Q_s^2$  Marquet, Yuan, Xiao (PLB 2009)

$$\frac{d\sigma_T^{\gamma^*+A\rightarrow q+X}}{d^2\mathbf{k}_\perp dz} = \frac{4\pi^2\alpha_{\text{em}}e_q^2}{Q^2} \delta(1-z) xq(x, \mathbf{k}_\perp)$$

$$xq(x, \mathbf{k}_\perp) = \frac{N_c}{4\pi^4} \int d^2\mathbf{b}_\perp d^2\mathbf{l}_\perp F(\mathbf{l}_\perp, \mathbf{b}_\perp, x) \left[ 1 - \frac{\mathbf{k}_\perp \cdot (\mathbf{k}_\perp - \mathbf{l}_\perp)}{(\mathbf{k}_\perp^2 - (\mathbf{k}_\perp - \mathbf{l}_\perp)^2)} \ln \left( \frac{\mathbf{k}_\perp^2}{(\mathbf{k}_\perp - \mathbf{l}_\perp)^2} \right) \right]$$

Sea-quark TMD built from small-x gluon by splitting

Implicit dependence on the saturation scale  
through dipole correlator  $F(\mathbf{l}_\perp, \mathbf{b}_\perp, x)$



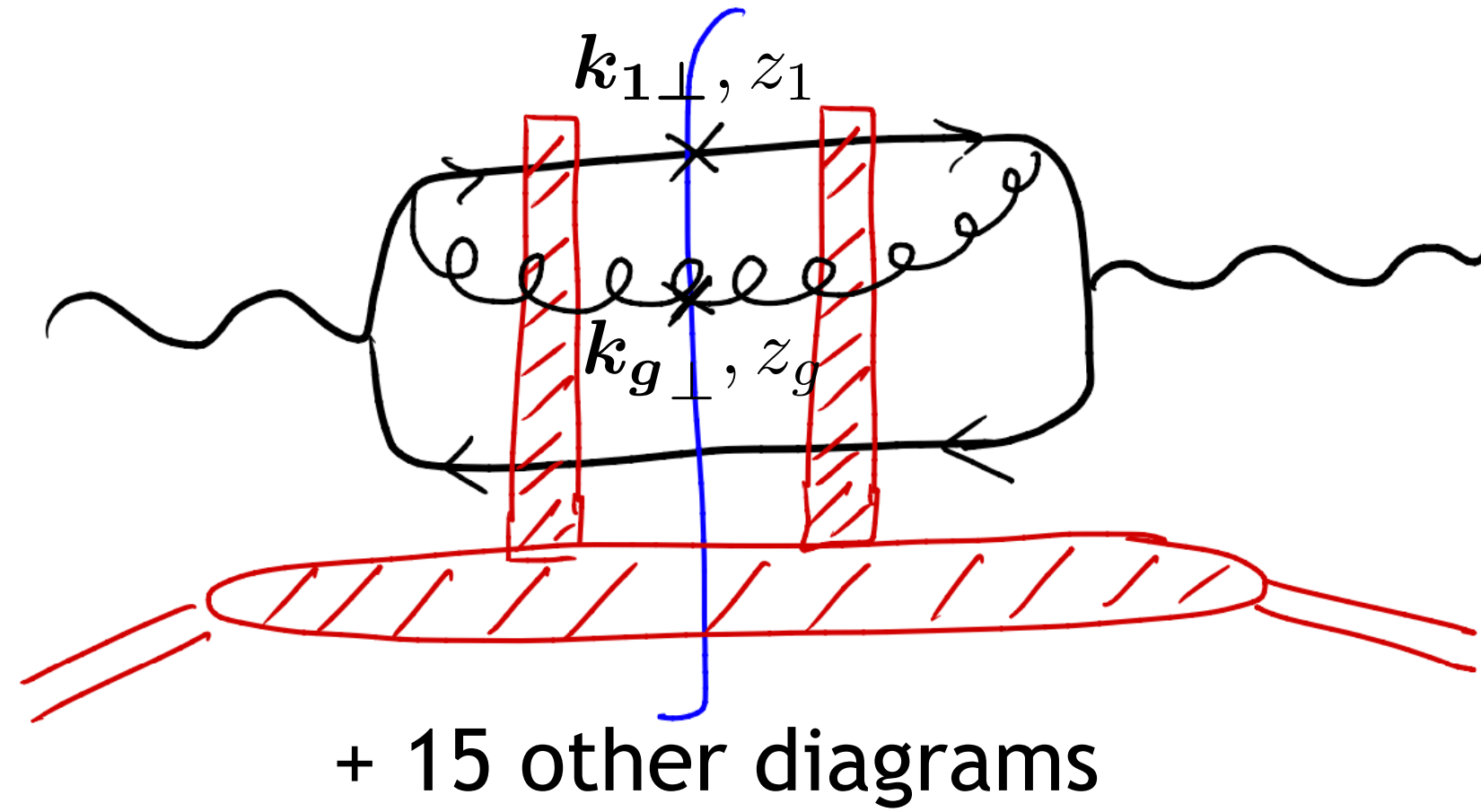
The same procedure can be carried out for Drell-Yan

Yuan, Xiao, Zhou (NPB 2017)

# Bringing back quarks at small-x:

Two-particle correlations: universality of sea quark distributions

Consider  $qg$  dijet production in DIS



Involves convolution of various multipole Wilson lines correlator with perturbative factor

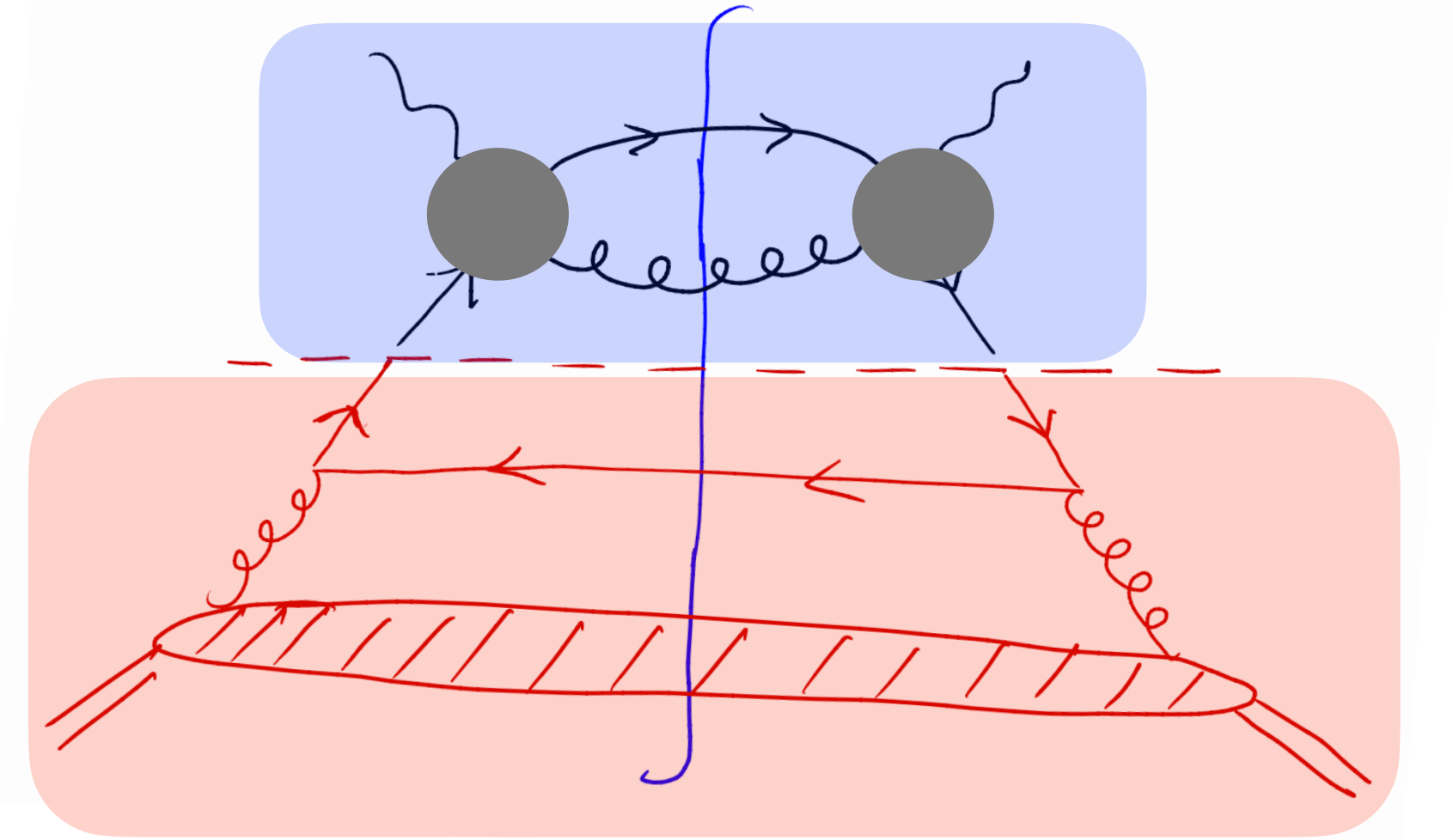
$$k_{\perp} = k_{1\perp} + k_{g\perp}$$

$$P_{\perp} = z_g k_{1\perp} - z_1 k_{g\perp}$$

$$P_{\perp}^2 \gg k_{\perp}^2, Q_s^2$$

→

TMD factorization



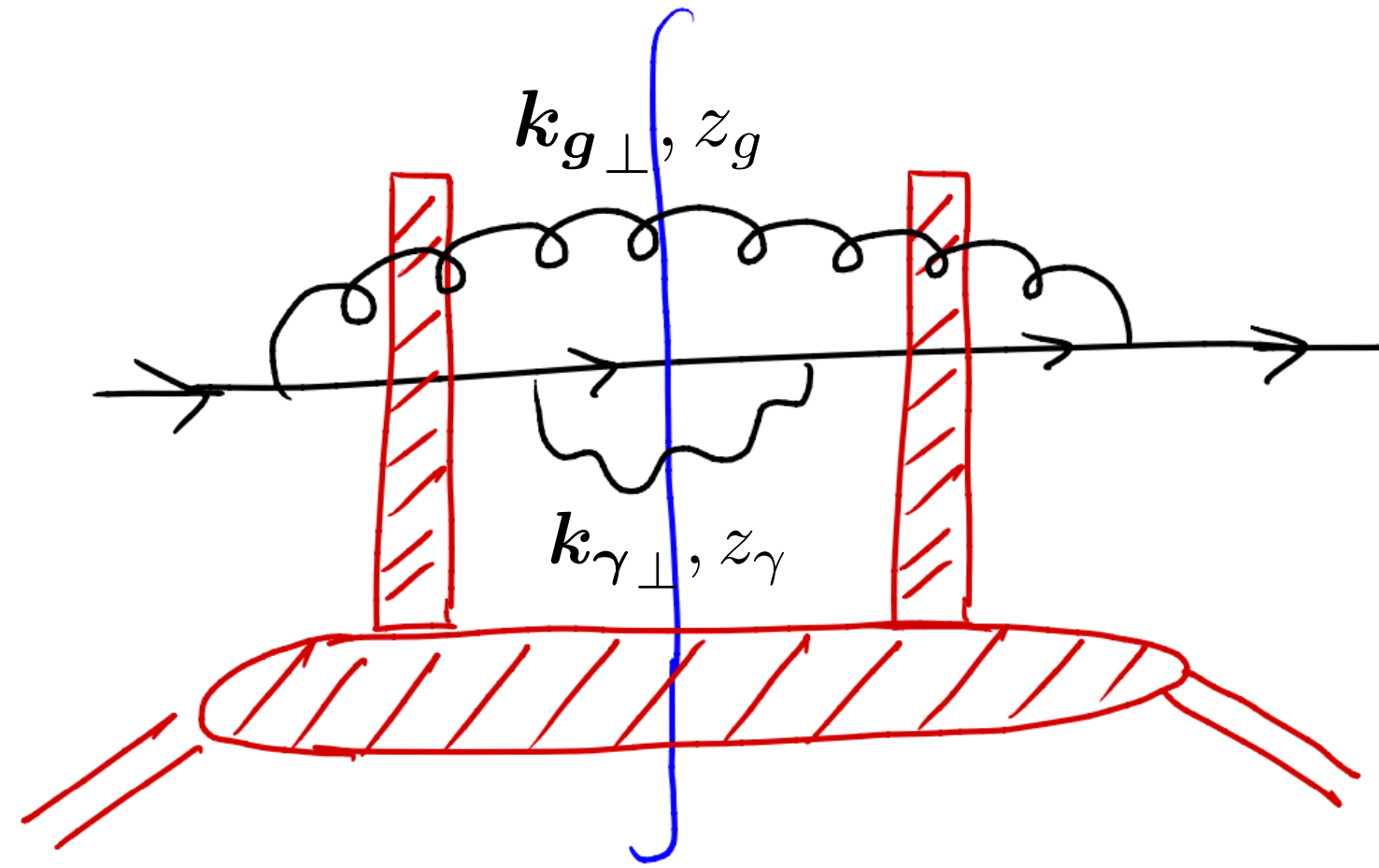
$$\frac{d\sigma^{\gamma_{\lambda=T+A}^* \rightarrow qg+X}}{d^2 P_{\perp} d^2 k_{\perp} dz_1 dz_g} = \alpha_{\text{em}} e_f^2 \alpha_s C_F \delta(1 - z_1 - z_g) \frac{2z_1 [(P_{\perp}^2 + \bar{Q}^2)^2 + z_g^2 P_{\perp}^4 + z_1^2 \bar{Q}^4]}{P_{\perp}^2 [P_{\perp}^2 + \bar{Q}^2]^3} xq(x, k_{\perp})$$

Same small-x sea quark distribution as in SIDIS and Drell Yan

# Bringing back quarks at small-x:

## Two-particle correlations: generalized universality

Consider  $g$  jet + photon production in pA



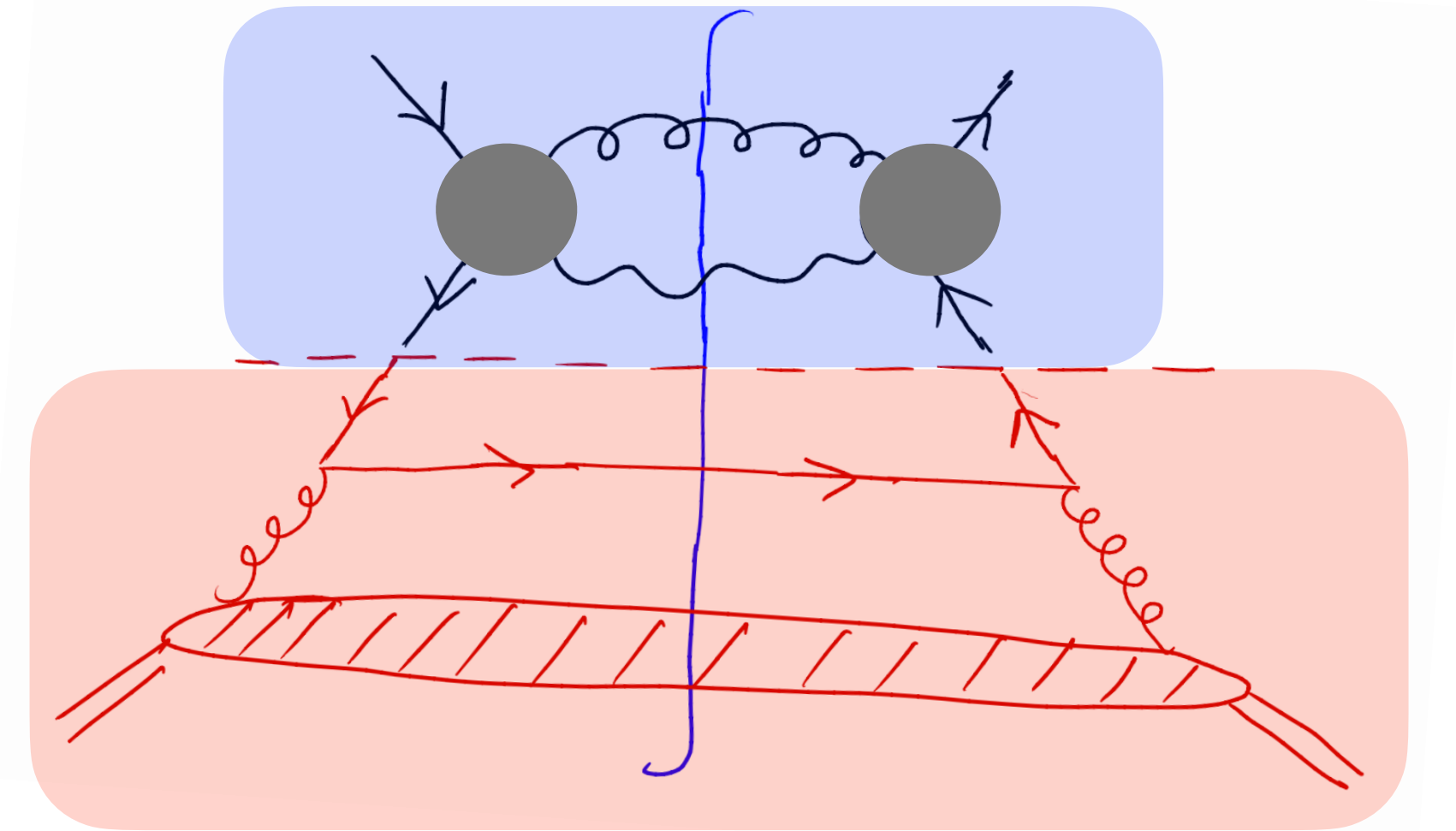
+ 35 other diagrams

$$\mathbf{k}_\perp = \mathbf{k}_{\gamma\perp} + \mathbf{k}_{g\perp}$$

$$\mathbf{P}_\perp = z_g \mathbf{k}_{\gamma\perp} - z_\gamma \mathbf{k}_{g\perp}$$

$$\mathbf{P}_\perp^2 \gg \mathbf{k}_\perp^2, Q_s^2$$

TMD factorization



$$\frac{d\sigma^{qA \rightarrow \gamma g + X}}{d^2 \mathbf{P}_\perp d^2 \mathbf{K}_\perp d\eta_\gamma d\eta_g} = \frac{\alpha_{\text{em}} e_f^2 \alpha_s}{N_c} \delta(1 - z_\gamma - z_g) \frac{z_\gamma z_g (z_\gamma^2 + z_g^2)}{\mathbf{P}_\perp^4} xq^{(2)}(x, \mathbf{k}_\perp)$$

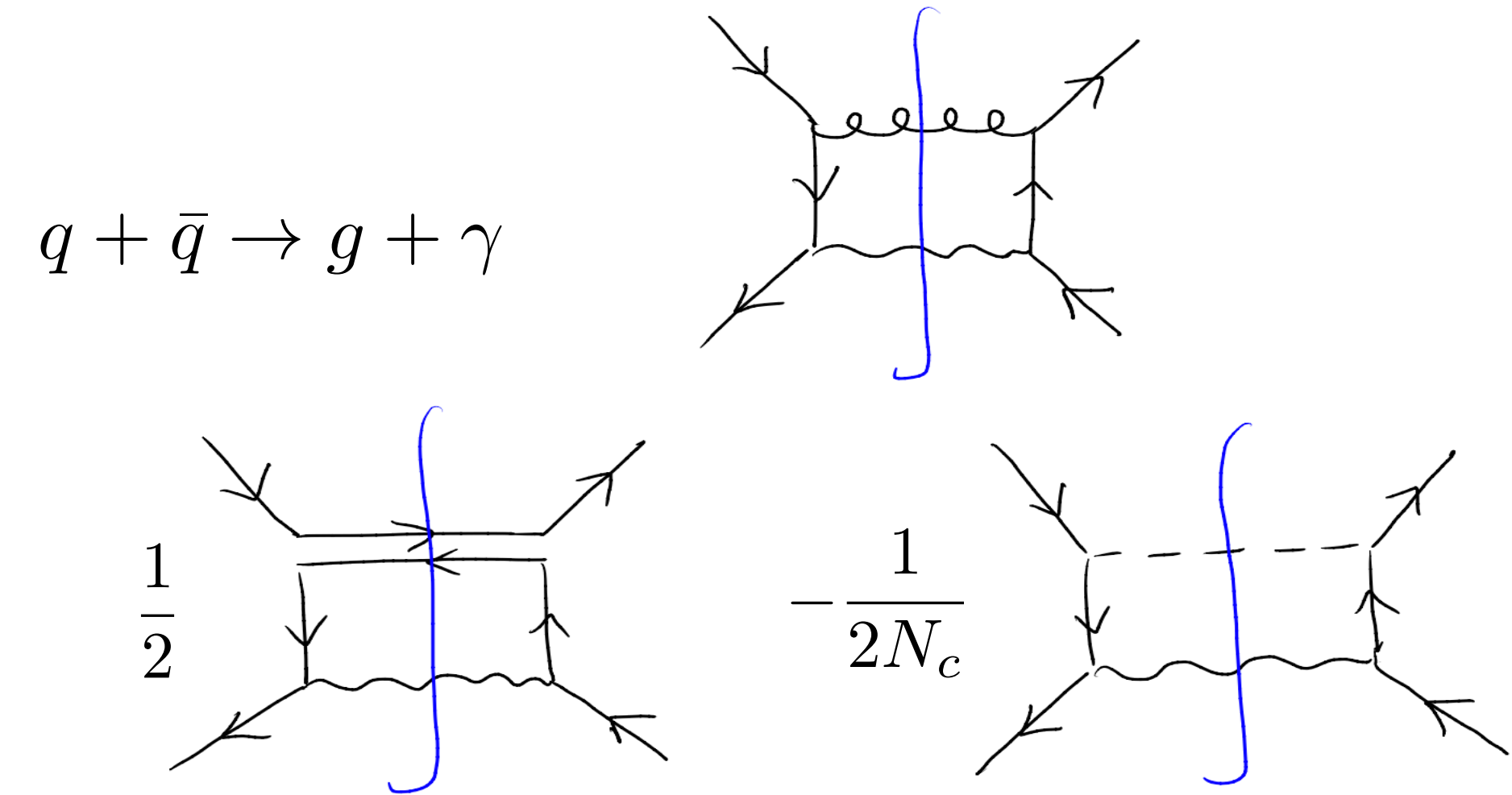
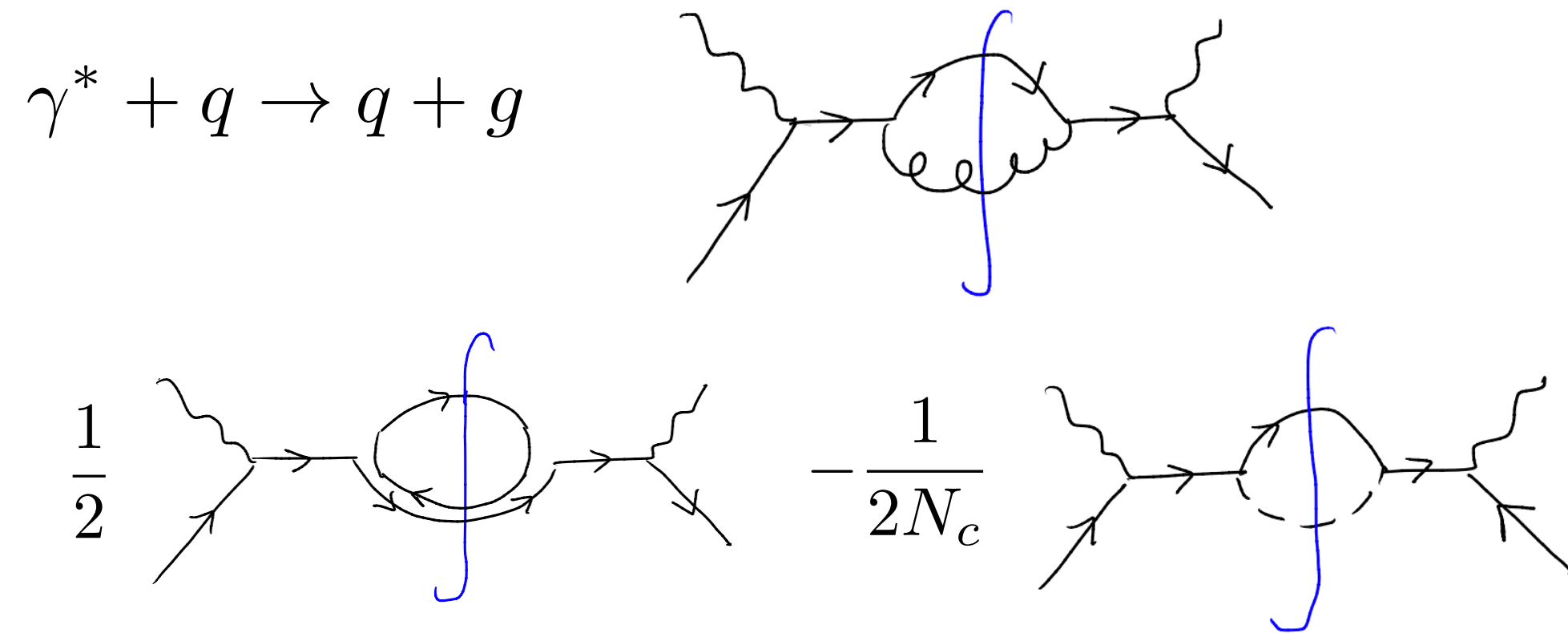
$$xq^{(2)}(x, \mathbf{k}_\perp) = \frac{1}{C_F} \left[ \frac{N_c}{2} xq(x, \mathbf{k}_\perp) \otimes F(x, \mathbf{k}_\perp) - \frac{1}{2N_c} xq(x, \mathbf{k}_\perp) \right]$$

New quark TMD built from TMD in SIDIS/DY and the dipole

# Bringing back quarks at small-x:

## Non-trivial gauge link structure of TMDs for different processes

Following Bomhof, Mulders, Pijlman (EPJC 2006)



$$\frac{1}{2} \text{Tr} [\Phi_q \mathcal{U}^{[+]}] \text{Tr} [\mathcal{U}^{[+]} (\mathcal{U}^{[+]} )^\dagger] - \frac{1}{2N_c} \text{Tr} [\Phi_q \mathcal{U}^{[+]}]$$

$$\frac{N_c}{2} \text{Tr} [\Phi_{\bar{q}} (\mathcal{U}^{[+]} )^\dagger] \frac{1}{N_c} \text{Tr} [\mathcal{U}^{[\square]}] - \frac{1}{2N_c} \text{Tr} [\Phi_{\bar{q}} (\mathcal{U}^{[-]} )^\dagger]$$

$$xq(x, \mathbf{k}_\perp) = \frac{1}{C_F} \left[ \frac{N_c}{2} xq(x, \mathbf{k}_\perp) - \frac{1}{2N_c} xq(x, \mathbf{k}_\perp) \right]$$

$$xq^{(2)}(x, \mathbf{k}_\perp) = \frac{1}{C_F} \left[ \frac{N_c}{2} xq(x, \mathbf{k}_\perp) \otimes F(x, \mathbf{k}_\perp) - \frac{1}{2N_c} xq(x, \mathbf{k}_\perp) \right]$$

Only final state interactions

Both initial and final state interactions

Same procedure can be applied to more complex processes



# Summary

- Two particle correlations a window to saturation

Gluon saturation imprints on particle correlations  
Soft gluon radiation (Sudakov) competing effect

- Small- $x$  TMD factorization at NLO from CGC

Joint resummation of small- $x$  and Sudakov logs needs a kinematic constraint  
Computed NLO finite pieces, and numerical predictions for EIC

- Bringing back quarks at small- $x$

Establish TMD factorization for small- $x$  (sea) quark-initiated channels  
(Generalized) universality consistent with non-trivial gauge link structure  
Sea quark TMDs can be computed from the CGC dipole

# Outlook

- Prove small- $x$  TMD factorization for other processes, e.g. photon+hadron(jet) production in pA
- Does the small- $x$  TMD factorization for quark initiated channels hold at NLO?
- Future phone studies of two-particle correlations at NLO: joint resummation, kinematic constraint, finite pieces, quark initiated channel contributions, etc

EIC potential for saturation signals dihadrons, dijets, etc

ALICE Focal at LHC will measure dihadrons, hadron + photon, etc at very small values of  $x$

- Polarized observables? Diffractive observables?