Exploring Meson Structures from Lattice QCD

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Outline

- Electromagnetic Form Factor (EMFF) of Pion and Kaon
  — based on arXiv: 2404.04412
- Pion Light-cone Generalized Parton Distribution (GPD)
  — in preperation

Motivation

EPJA 48 (2012) 187 JPG 48 (2021) 075106 arXiv: 2102.09222

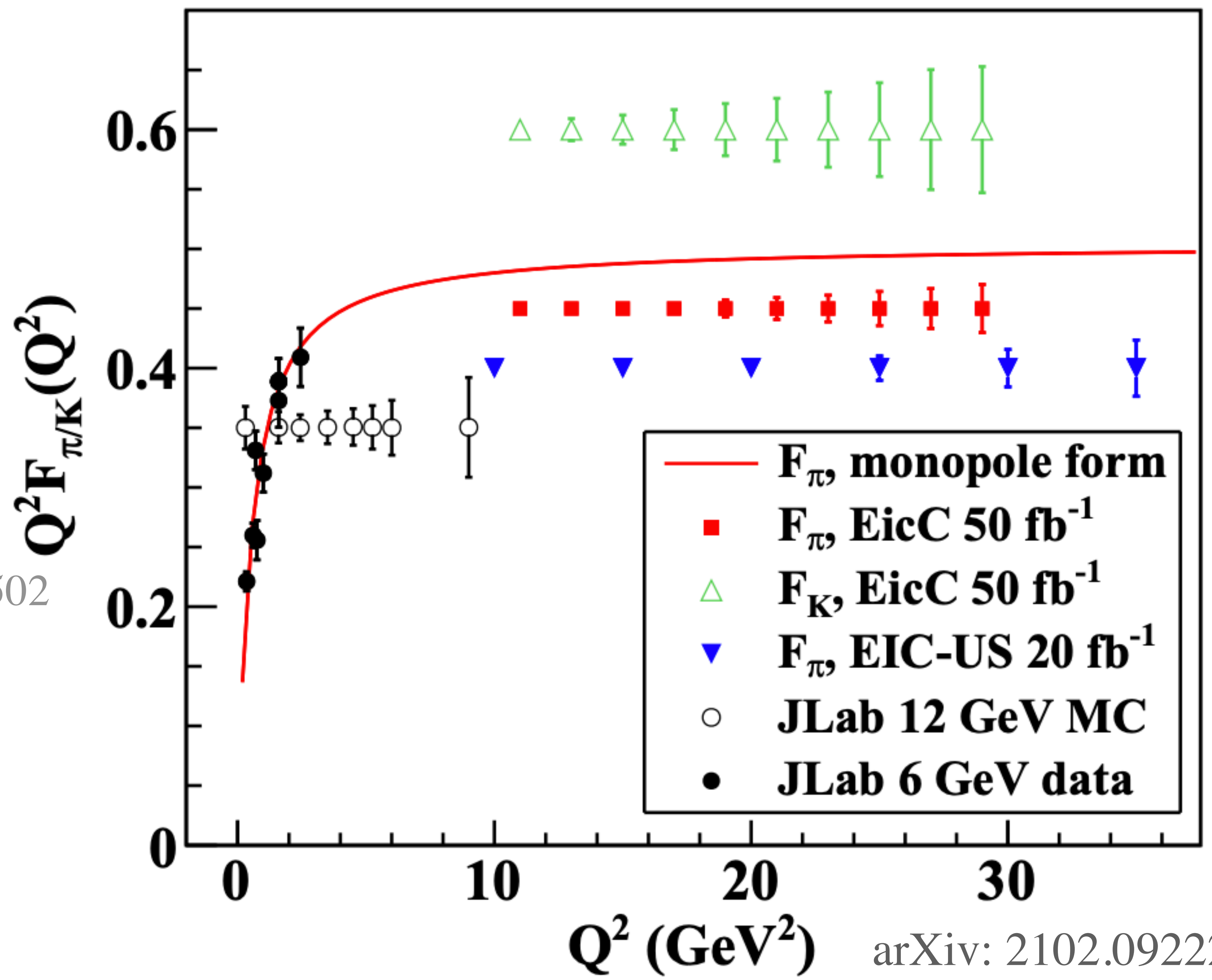
- Experiment: JLab, EIC, EicC …

Gao et al., PRD 96 (2017) 034024

- Effective theory: QCD sum rules, DSE …
- Lattice QCD: first principle

PRD 96 (2017) 114509 ETMC, PRD 105 (2022) 054502

  - State-of-the-art: $Q^2 \leq 6,\ 3\ \text{GeV}^2$
  - This work: Q^2 up to 10, 28 GeV^2



arXiv: 2102.09222

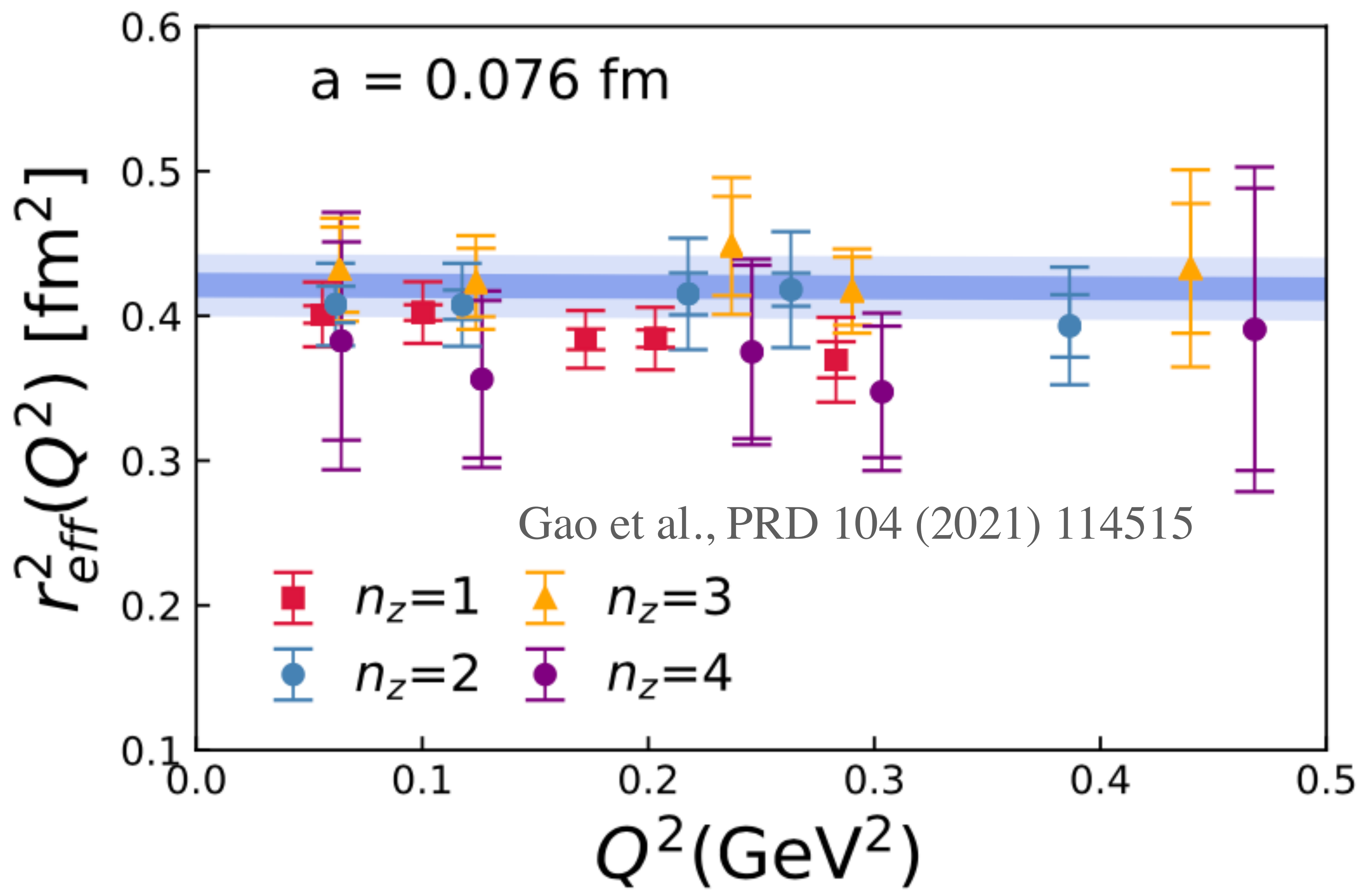
Momentum transfer $-t = Q^2$

Motivation

Low Q^2: **V**ector **M**eson **D**ominance

$$r^2_{\text{eff}}(Q^2) = 6[1/F_\pi(Q^2) - 1]/Q^2$$

$$\langle r^2_\pi \rangle = 0.42(2)\ \text{fm}^2, \quad \langle r^2_\pi \rangle_{PDG} = 0.434(5)\ \text{fm}^2$$

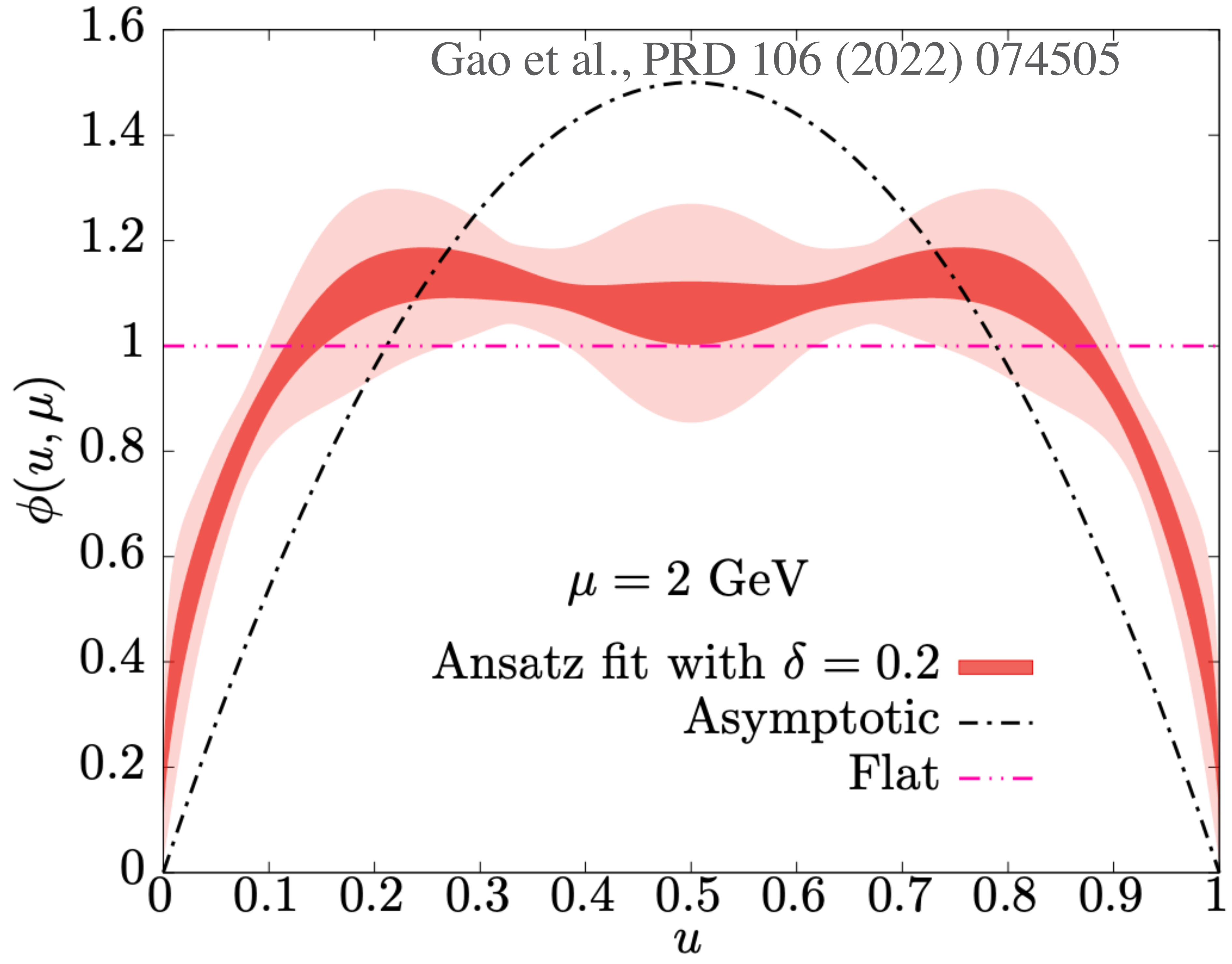


High Q^2: Factoriation framework

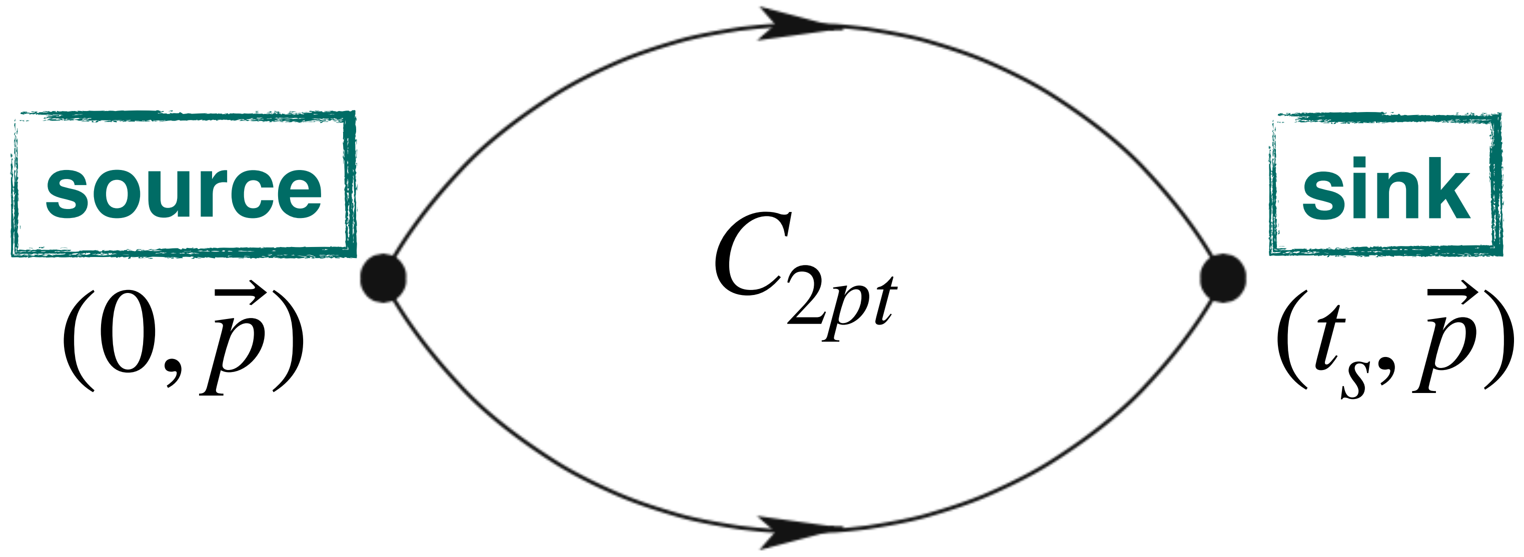
Hard-process kernel

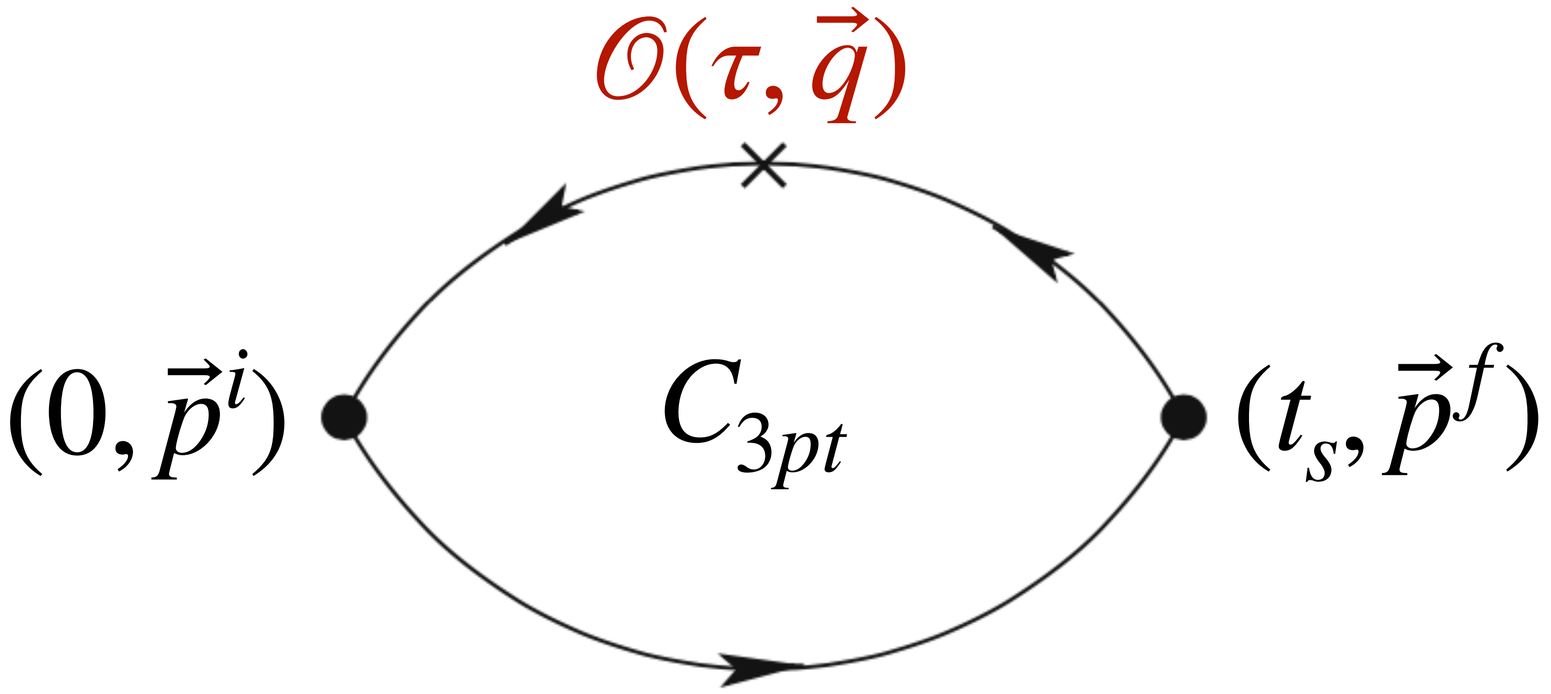
$$F(Q^2) = \int\int dxdy \Phi^*(x, \mu_F^2) T_H(x, y, Q^2, \mu_R^2, \mu_F^2) \phi(y, \mu_F^2)$$

Distribution amplitude



How to get the form factor on the lattice





$$C_{2pt}(t, \vec{p}) = \left\langle H(t_s, \vec{p}) H^\dagger(0, \vec{p}) \right\rangle$$

$$C_{3pt}(\tau, t_s; \vec{p}^i, \vec{p}^f) = \left\langle H(t_s, \vec{p}^f) \hat{\mathcal{O}}_{\gamma_\mu}(\tau, \vec{q}) H^\dagger(0, \vec{p}^i) \right\rangle$$

$$R^{fi} \sim C_{3pt} / C_{2pt} \xrightarrow{t_s \to \infty}$$

$$F^B = \left\langle E_0, \vec{p}^f \,|\, \hat{\mathcal{O}}_{\gamma^\mu}(\tau, \vec{q}) \,|\, E_0, \vec{p}^i \right\rangle$$

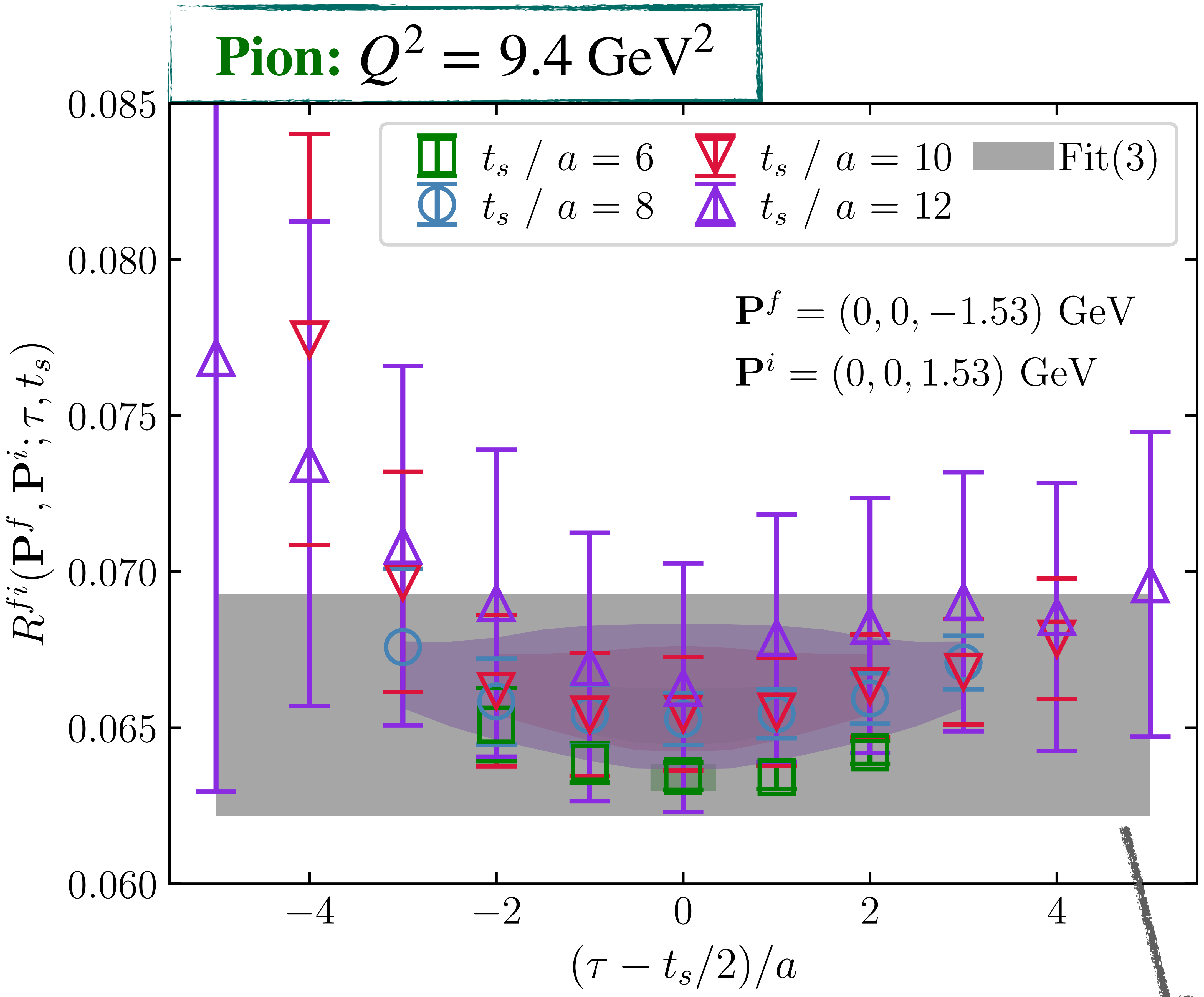
Bare Form factor

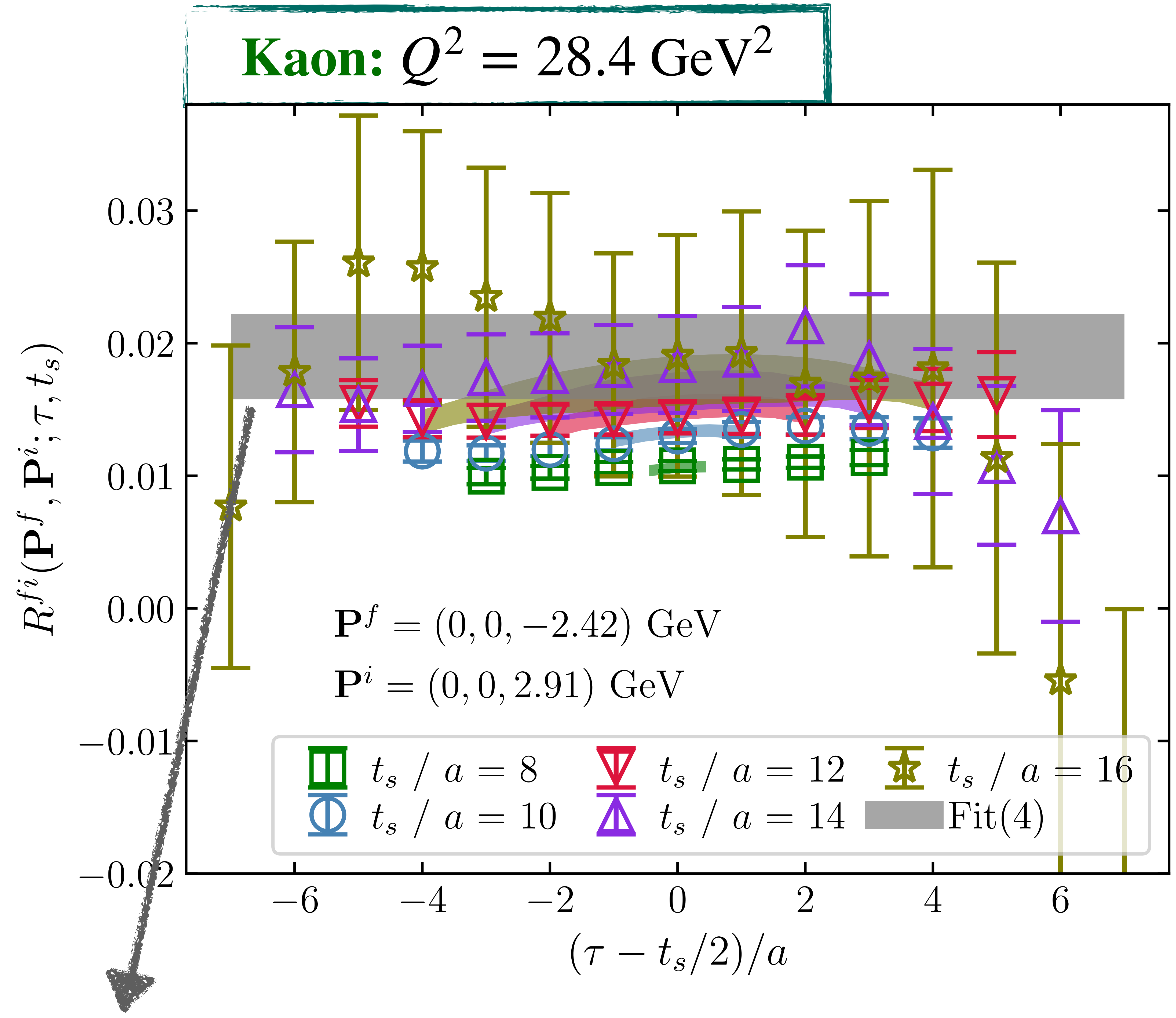
Renormalization

$$F(Q^2) = F^B \times Z_V^{-1}$$

$$(Q^2 = -t)$$

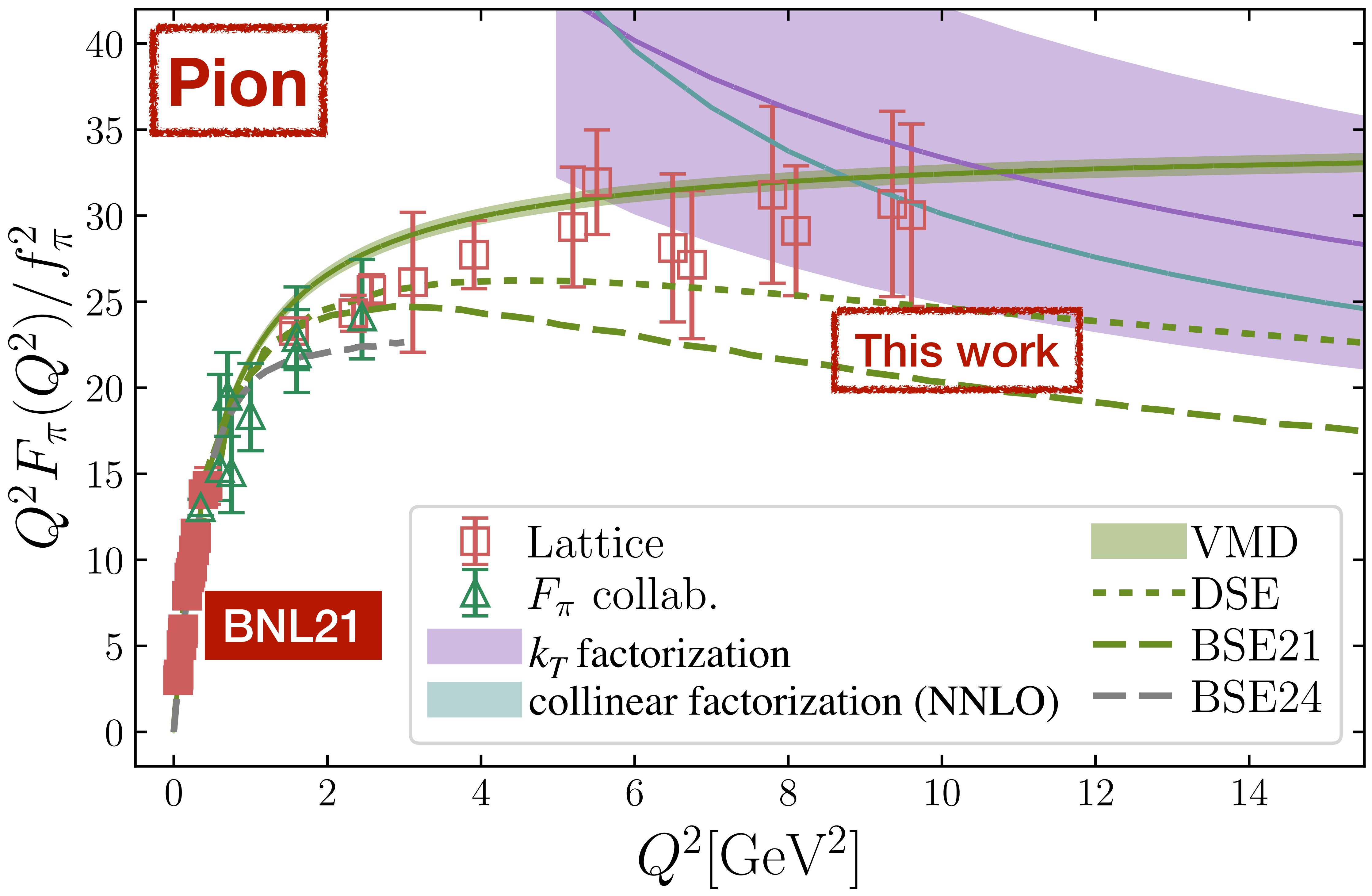
Bare Form Factor

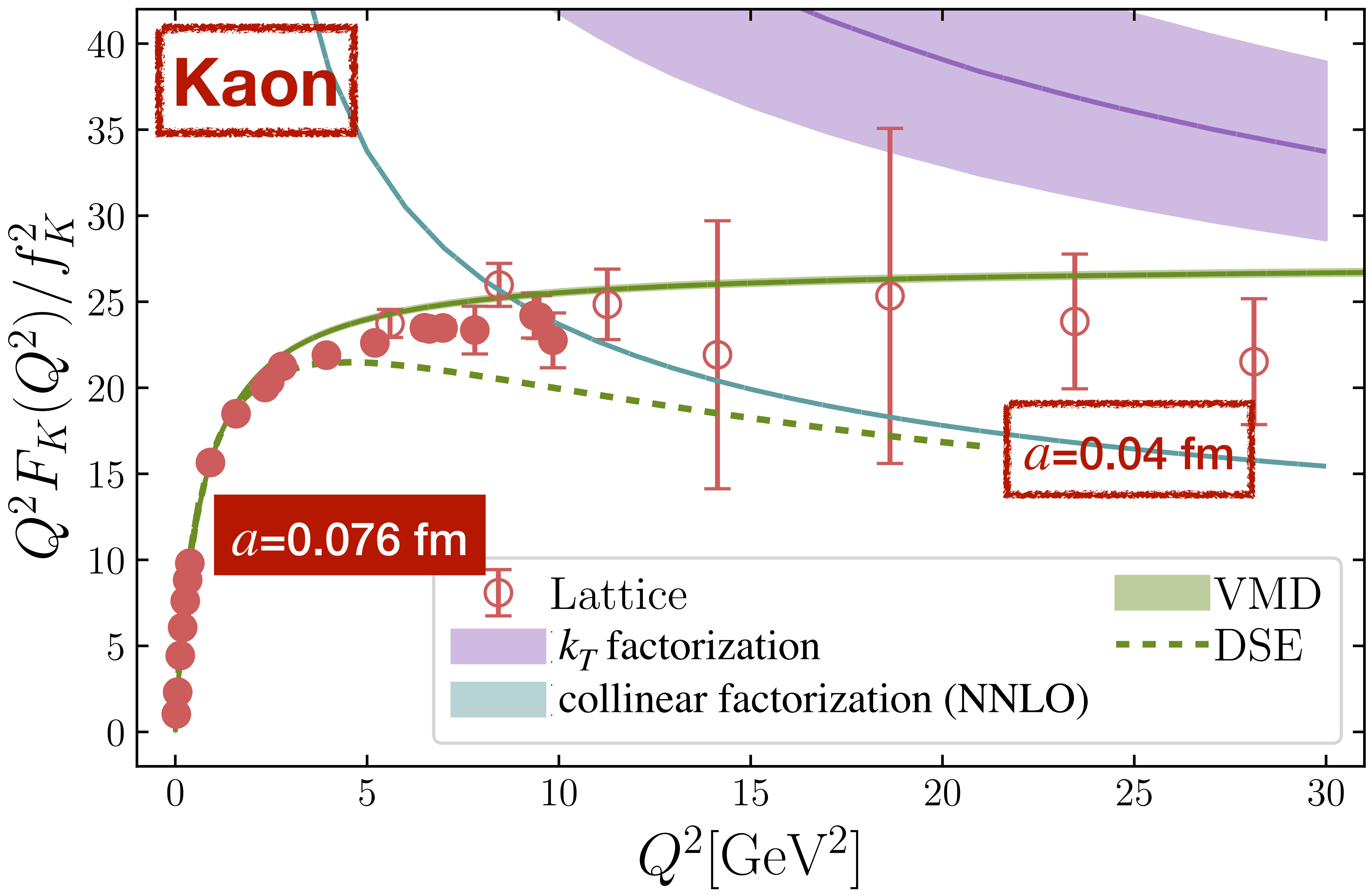




Bare form factor $F^B \times Z_V^{-1} = F(Q^2)$

Electromagnetic Form Factor





Hard-process kernel

$$F(Q^2) = \iint dxdy\,\Phi^*(x,\mu_F^2)T_H(x,y,Q^2,\mu_R^2,\mu_F^2)\phi(y,\mu_F^2)$$

Distribution amplitude

$$F(Q^2 \to \infty) = 8\pi\alpha_s(Q^2)f^2/Q^2, \quad Q^2F/f^2 \sim \text{Constant}$$

Lattice (filled symbols): Gao et al., PRD 104 (2021) 114515

F^π collaboration: Huber et al., PRC 78 (2008) 045203

DSE (Dyson-Schwinger equation): Gao et al., PRD 96 (2017) 034024

BSE21 (Bethe-Salpeter equation): Ydrefors et al., PLB 820 (2021) 136494

BSE24: Jia and Cloët, arXiv:2402.00285

k_T factorization: Cheng, PRD 100 (2019) 013007

pion: Chai et al., EPJC 83 (2023) 556

kaon: in preparation

T_H in the collinear factorization: Chen et al., arXiv:2312.17228

DA in the collinear factorization: **see Rui's talk on Friday**

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Motivation

1D

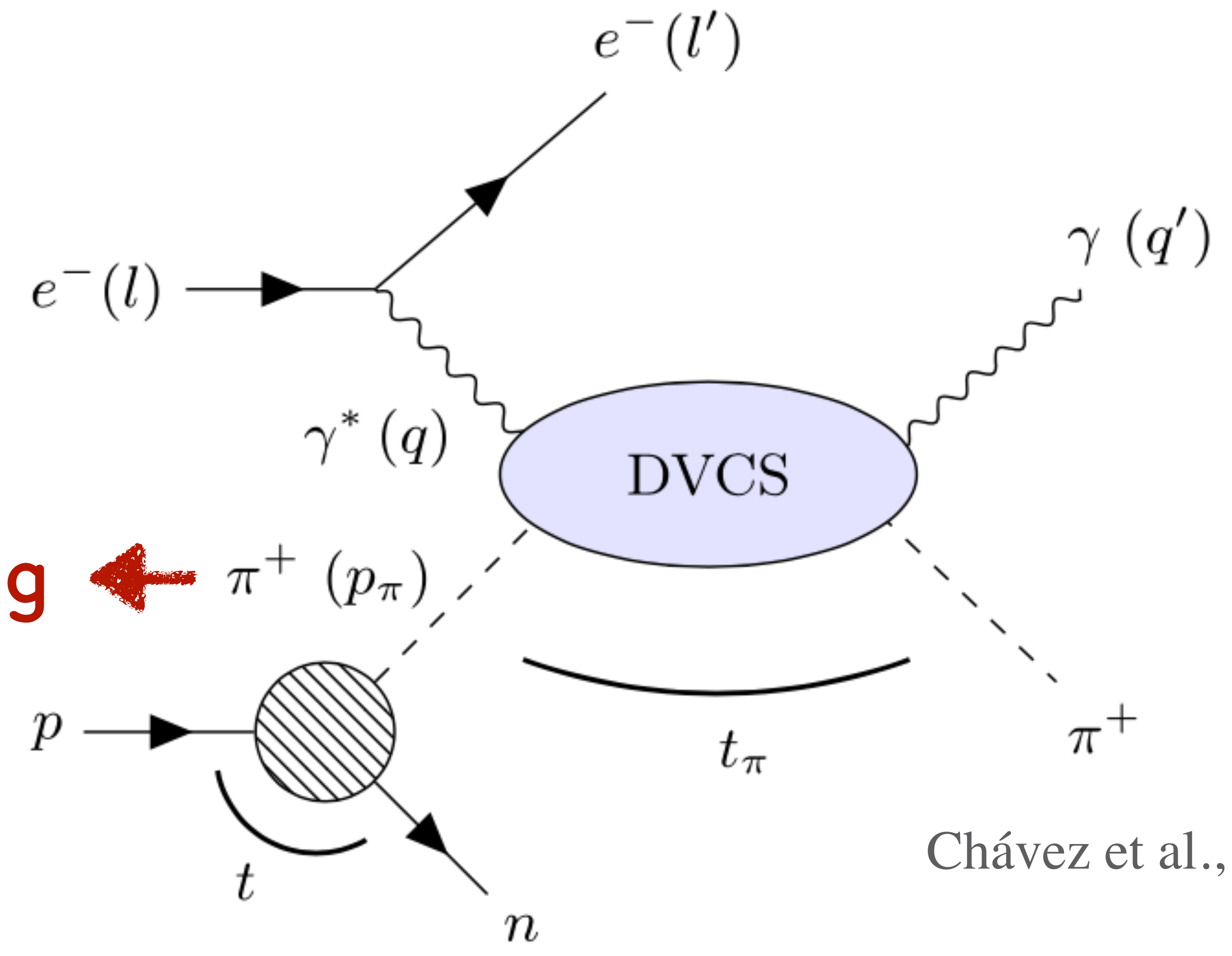
Form Factor (pion-electron scattering)

Parton Distribution Functions (Drell-Yan process)

} Generalized Parton Distributions

3D

More challenging



Chávez et al., PRL 128 (2022) 202501

\+ Models

Lattice QCD: from first principle

Frame-independent approach

Bhattacharya et al., PRD 106 (2022) 114512 See Martha's talk on Wednesday

- Calculate in the asymmetric frame
- Save computational cost

- Lorentz-invariant amplitudes A_i's

$$M^\mu(P^m u, z^\mu, \Delta^\mu) = \bar{P}^\mu A_1 + m^2 z^\mu A_2 + \Delta^\mu A_3, \quad \bar{P}^\mu = (p_f^\mu + p_i^\mu)/2, \quad \Delta^\mu = p_f^\mu - p_i^\mu .$$

$$z^\mu = (0,0,0,z),\ \Delta^\mu = (\Delta^t, \Delta^x, \Delta^y, 0) .$$

$\longrightarrow$ $A_i(\text{sym frame}) \simeq A_i(\text{asym frame})$

- Lorentz-invariant quasi-GPD $\tilde{H}_{\text{LI}}$

$$\tilde{H}_{\text{LI}}(z \cdot P, z \cdot \Delta, \Delta^2, z^2) = A_1 + \frac{z \cdot \Delta}{z \cdot \bar{P}} A_3$$

$$A_3(-z \cdot \Delta) = -A_3(z \cdot \Delta) \longrightarrow A_3(z \cdot \Delta = 0) = 0$$

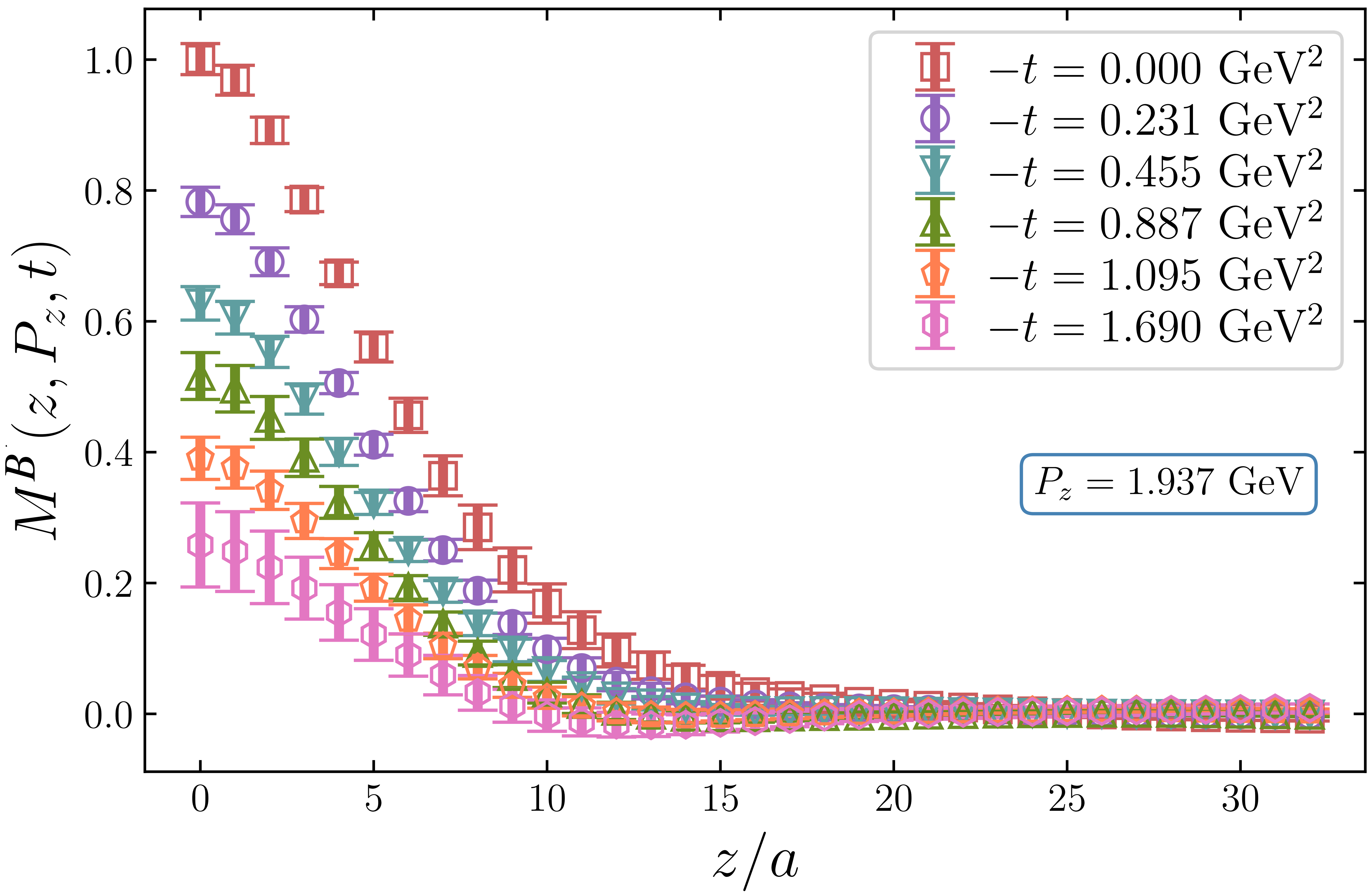
Bare matrix element

$$\tilde{H}_{\text{LI}} = A_1 = KM^t \equiv M^B$$

K: normalization factor

Bare matrix elements M^B

- Largest momentum: $P_z = 1.937$ GeV,
- Varying different momentum transfer $-t$



$z \uparrow$, $-t \uparrow$ } $M^B \downarrow$

→ **Quasi-GPDs**

Renormalization: Hybrid scheme

Ji et al., NPB 964 (2021) 115311

- RI/MOM, ratio schemes — short distance
- **Hybrid scheme** — both short and long distance

Logarithmic   Handle the renormalon ambiguity

$$M^B(z,a) = Z(a)e^{-\delta m(a)|z|}e^{-\bar{m}_0|z|}M^R(z)\,.$$

Linear

Gao et al., PRL 128 (2022) 142003

Hybrid scheme, M^R

$$\begin{cases} z \le z_S: & \dfrac{M^R(z,\vec{p},\vec{q})}{M^R(z,0,0)} = \dfrac{M^B(z,\vec{p},\vec{q})}{M^B(z,0,0)}, \quad \text{Ratio scheme} \\[2ex] z \ge z_S: & \dfrac{M^R(z,\vec{p},\vec{q})}{M^R(z_S,0,0)} = e^{(\delta m+\bar{m}_0)|z-z_S|}\dfrac{M^B(z,\vec{p},\vec{q})}{M^B(z_S,0,0)}\,. \end{cases}$$

Hybrid-scheme

Gao et al., PRL 128 (2022) 142003

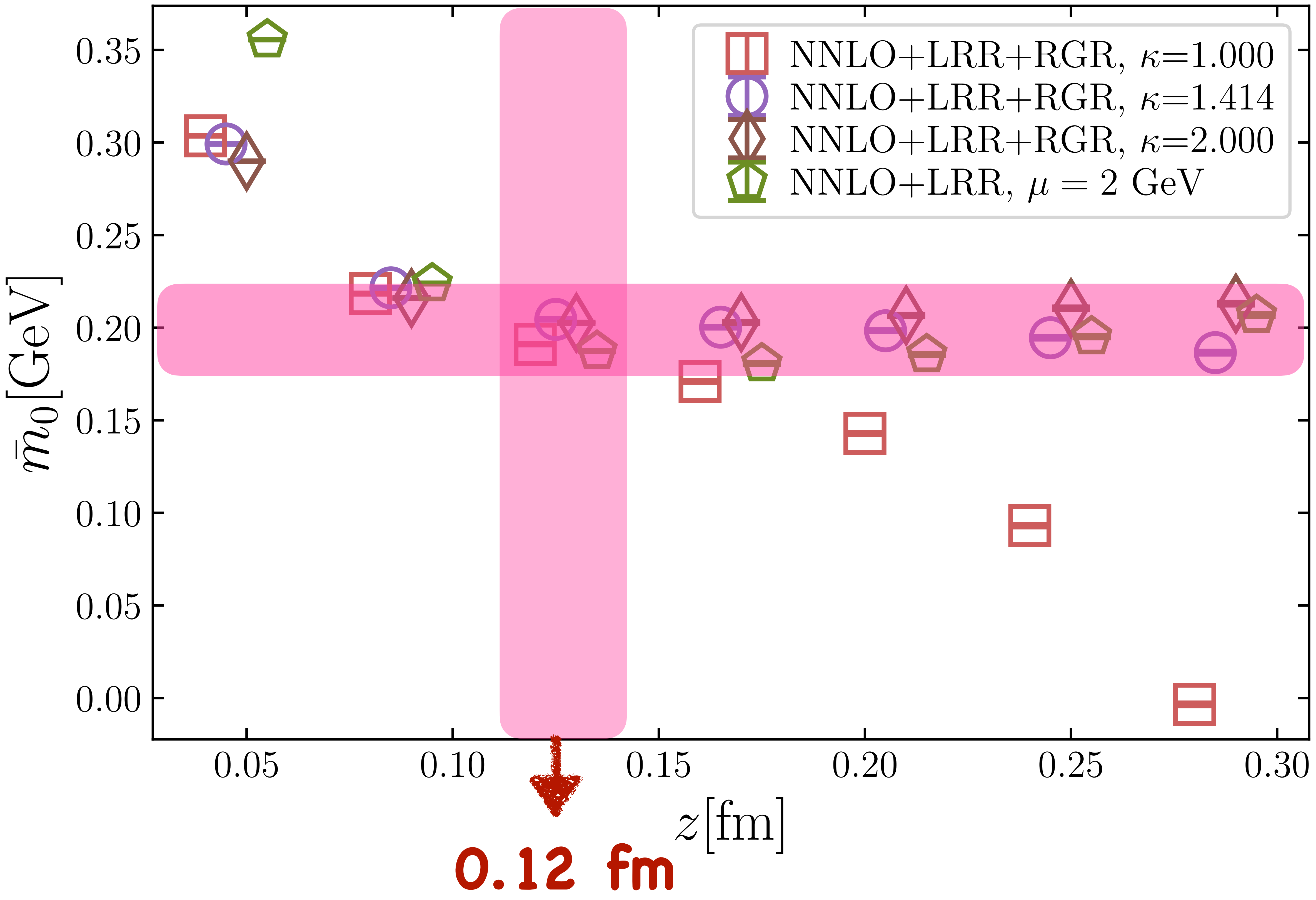
- $a\delta m(a) = 0.1508(12)$ for $a = 0.04$ fm lattice.
- $\bar{m}_0$: M^B at $P_z = 0, \vec{\Delta} = \vec{0}$; $e^{(\delta m + \bar{m}_0)\Delta z}\dfrac{M^B(z+\Delta z)}{M^B(z)} = \dfrac{C_0(\mu_0^2(z+\Delta z)^2)}{C_0(\mu_0^2 z^2)}$.

C_0: **NNLO,**

Leading-**R**enormalon **R**esummation,

Renormalization **G**roup **R**esummation

$(\mu_0 = 2\kappa e^{-\gamma_E}/z \to \mu = 2\text{ GeV})$



Renormalization

Lattice artifacts

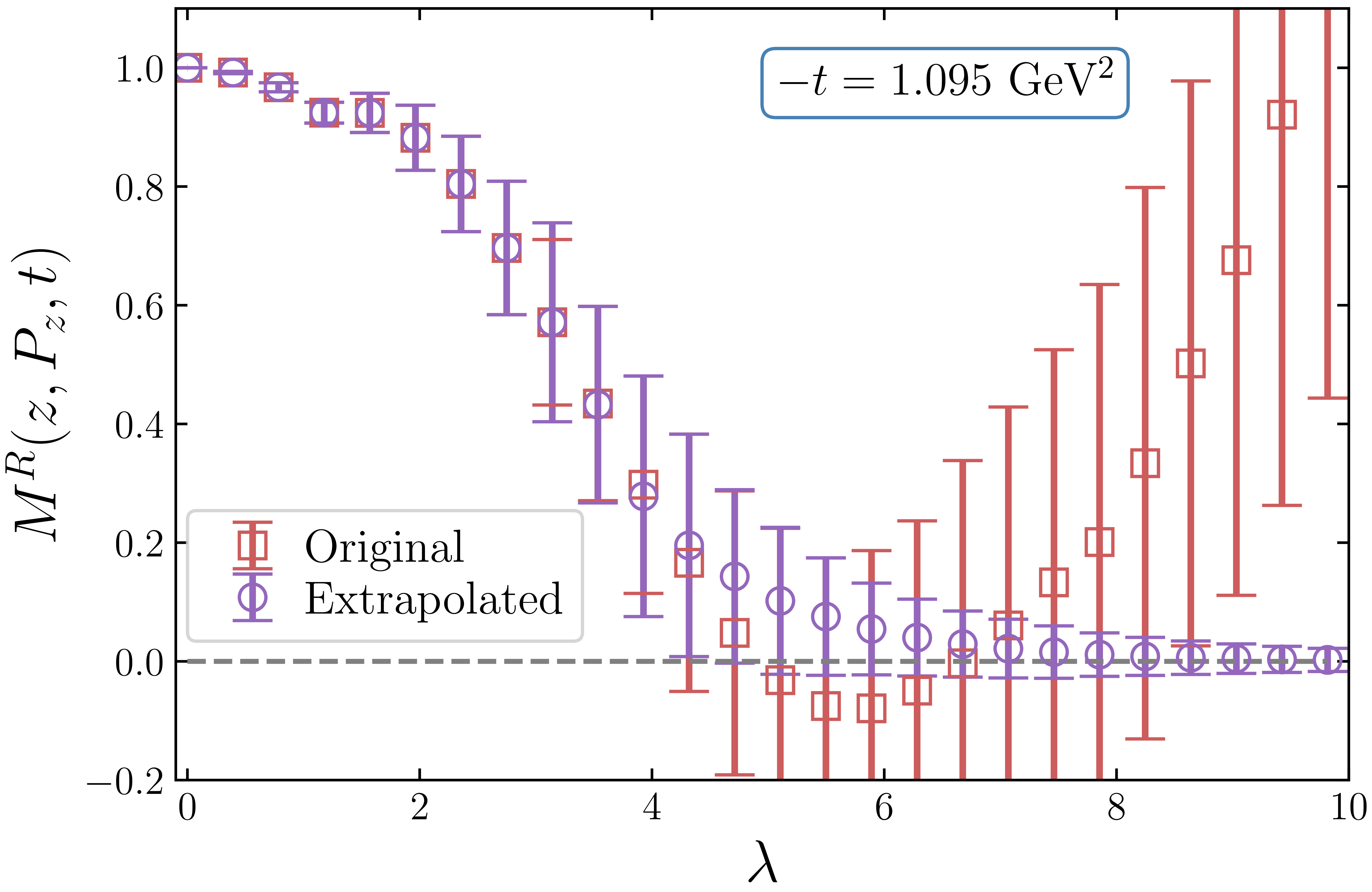
↓

Extrapolation

$$M^R = A\frac{e^{-mz}}{\lambda^d}, \lambda = zP_z$$

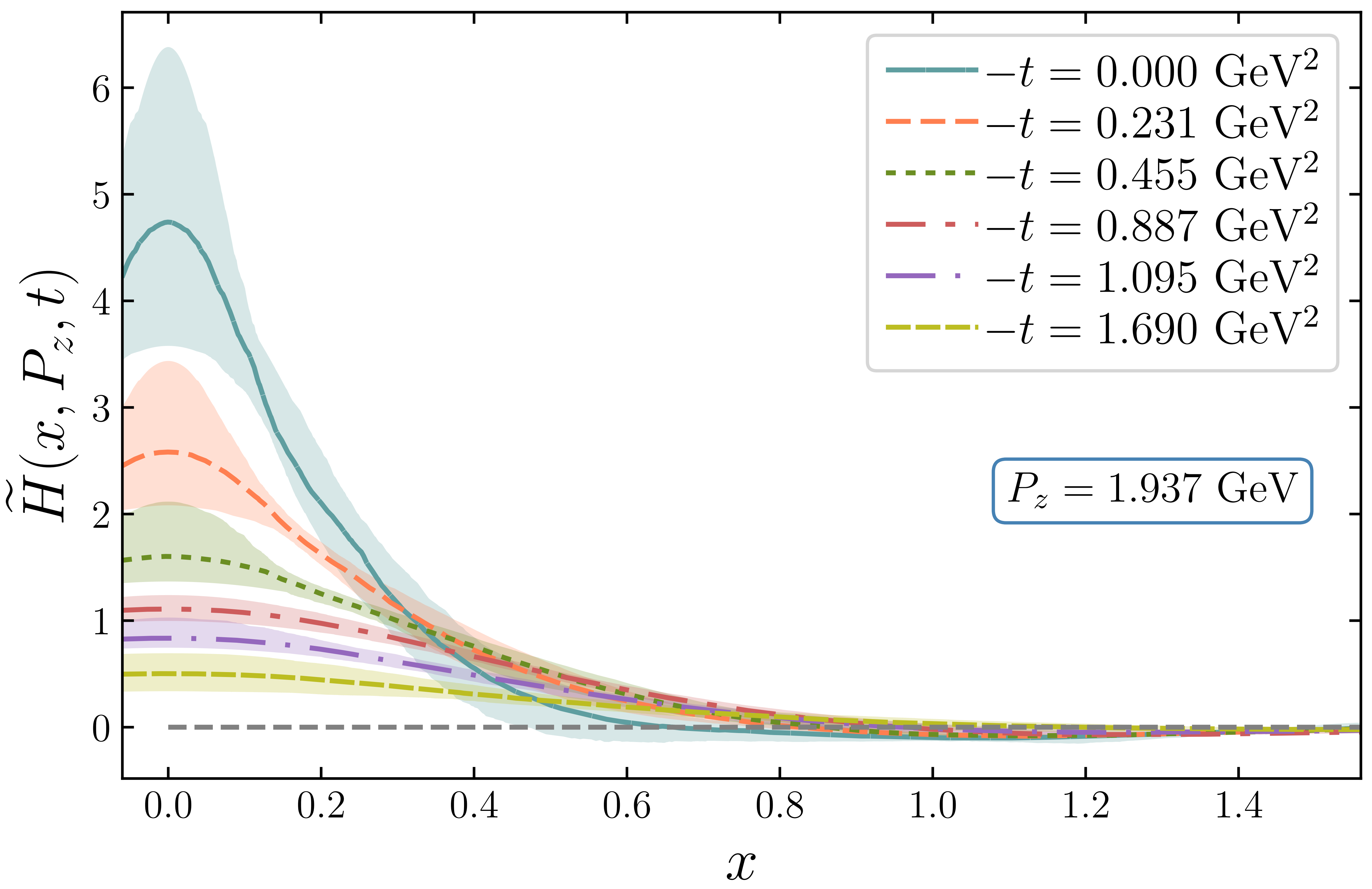
↓

Remove the unphysical oscillations in quasi-GPDs



Quasi-GPDs

$$\tilde{H}(x, P_z, t) = \int \frac{d\lambda}{2\pi} e^{ix\lambda} M^R(z, P_z, t)$$



$$H(x, t) = \int \frac{dk}{|k|} \int \frac{dy}{|y|} C_{\text{evo}}^{-1}\left(\frac{x}{k}, \frac{\mu}{\mu_0}\right) C^{-1}\left(\frac{k}{y}, \frac{\mu_0}{yP_z}, |y|\lambda_S\right) \tilde{H}(y, P_z, t, z_S, \mu_0) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{(xP_z)^2}, \frac{\Lambda_{\text{QCD}}^2}{[(1-x)P_z]^2}\right)$$

LC GPDs **quasi-GPDs**

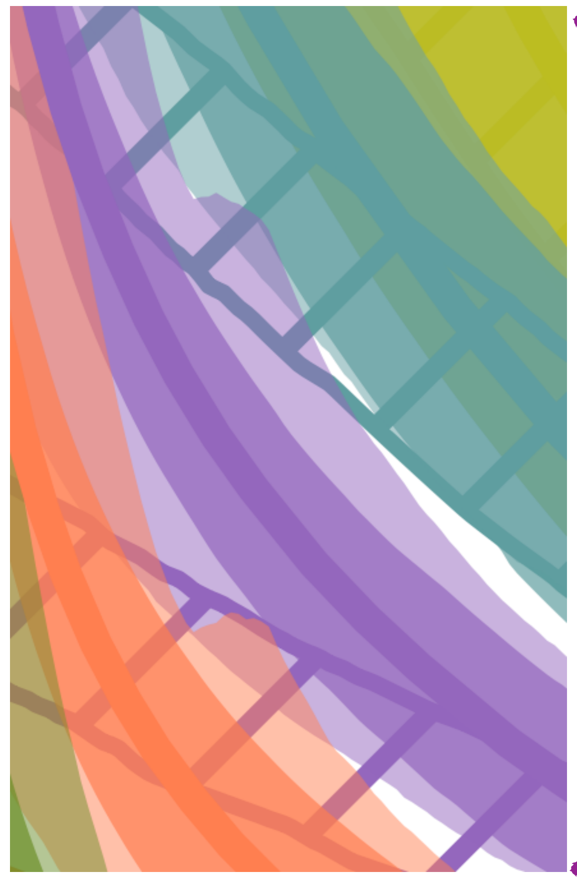
Valence LC GPDs: $-t$-dependence

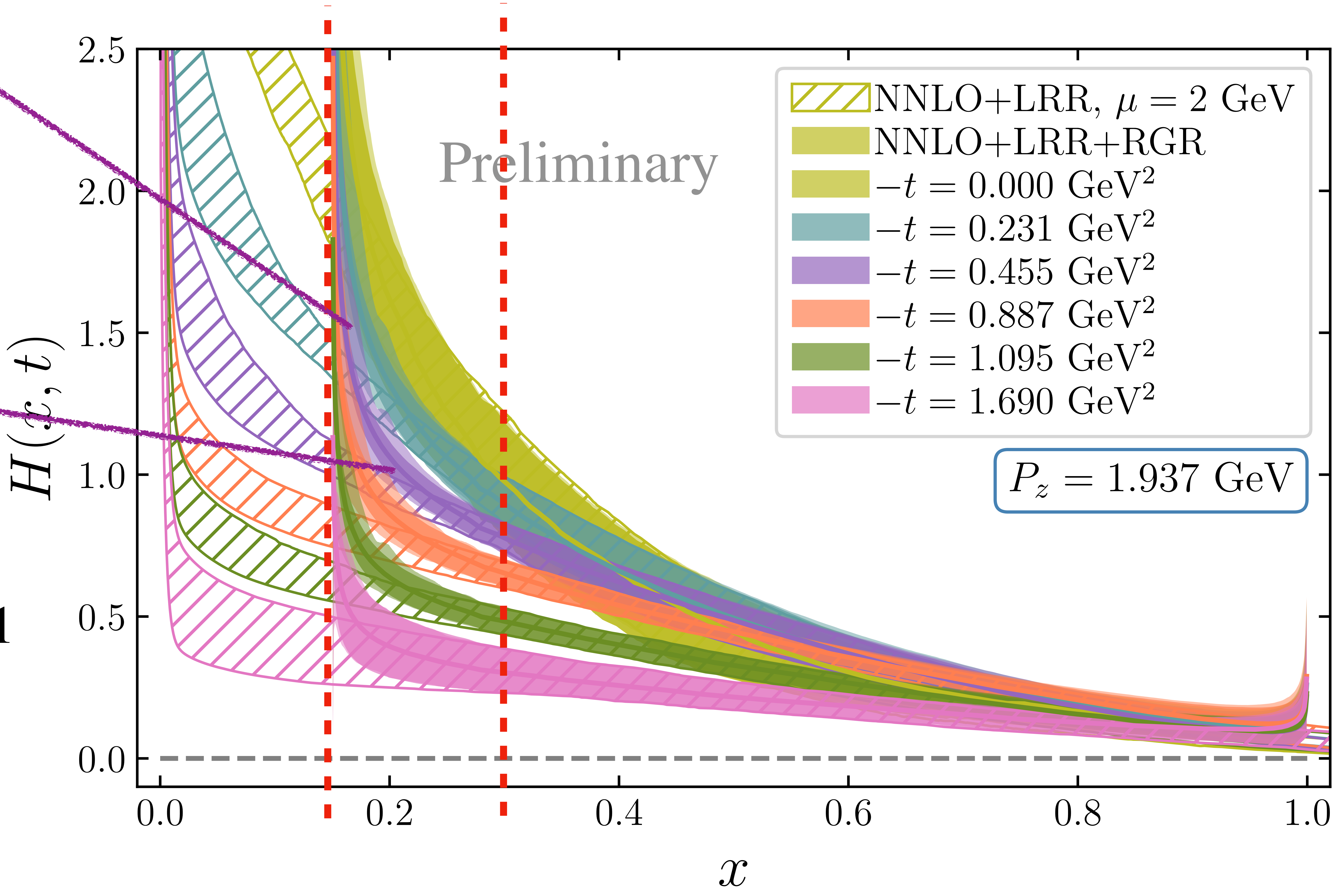
Statistical:

$\kappa = 1.414$

Systematic:

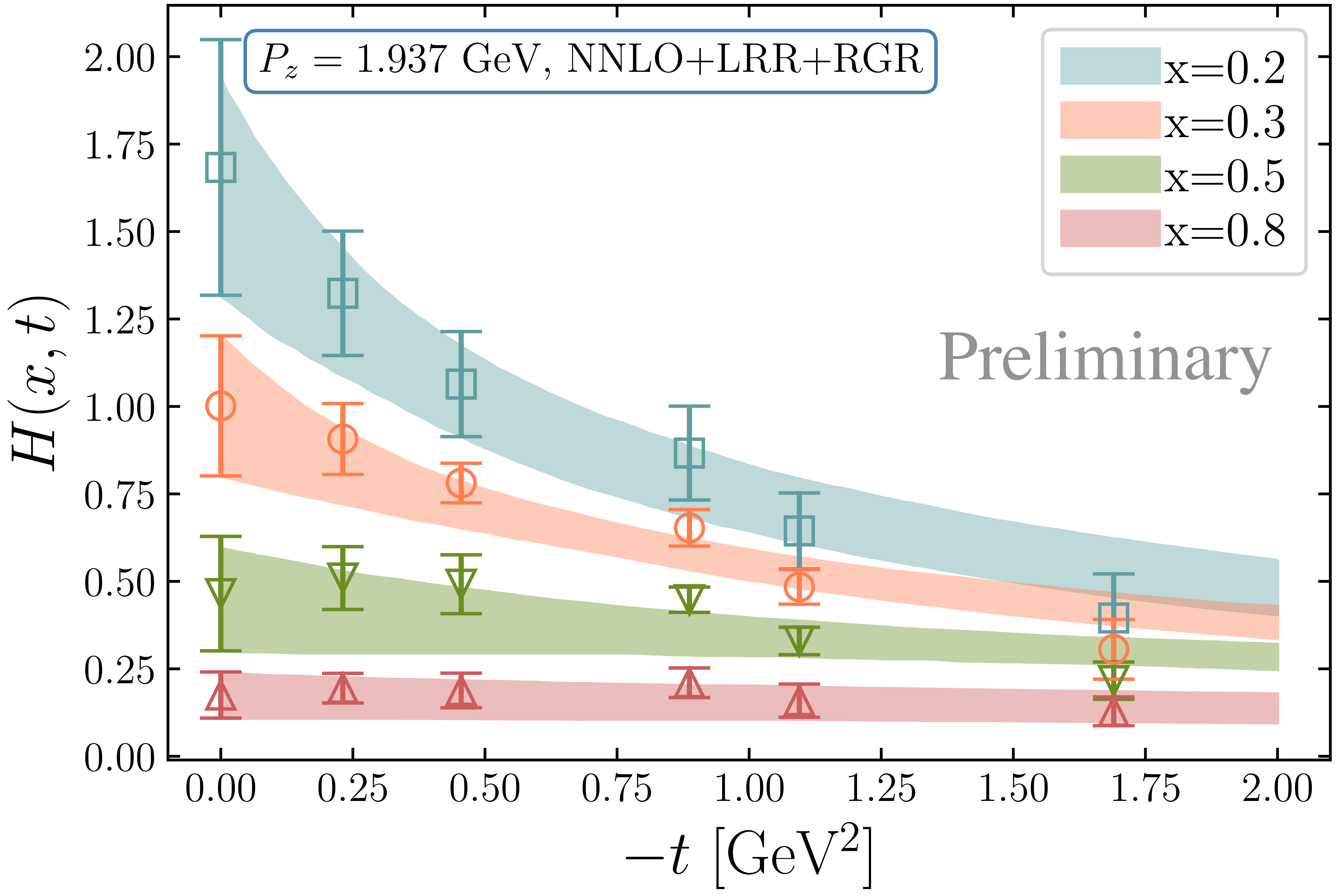
$1 < \kappa < 2$





- w/ & w/o RGR:
  - same: $x > 0.3$
  - correction: $0.15 < x < 0.3$, $x \sim 1$
  - breakdown: $x < 0.15$
- Scale variation (lighter filled bands):
  - $0.15 < x < 0.3$

Valence LC GPDs: x-dependence



x increases

↓

Declining trend -> stable plateau

Summary

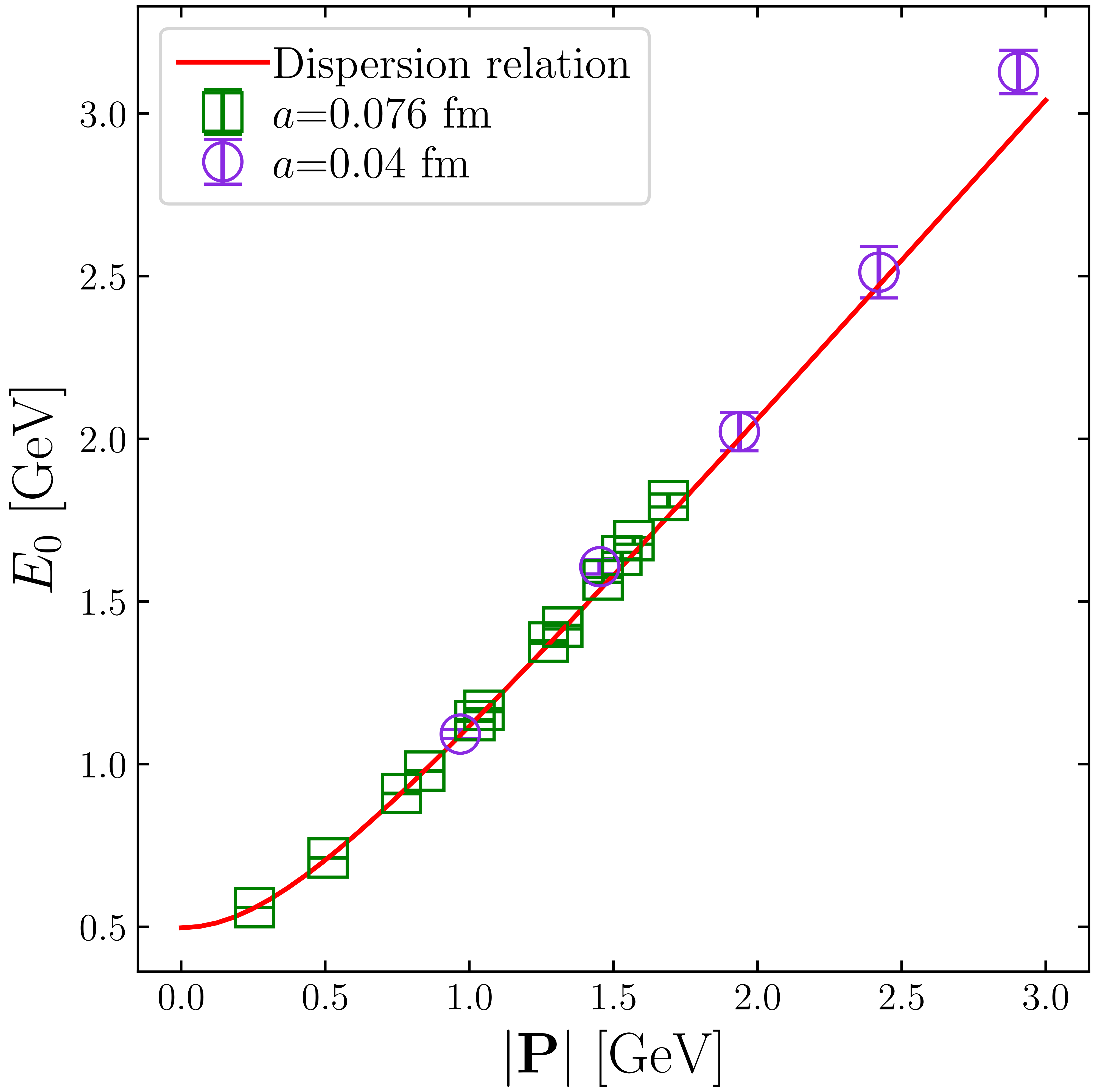
- **Pion and kaon EMFF at the physical point**
  - $-t$ up to 10 and 28 GeV^2 for the pion and kaon
  - Consistent with the existing experimental results
  - Consistent with the collinear factorizaiton results when $-t > 5$ GeV^2
- **Pion LC GPD in the asymmetric frame**
  - Hybrid-scheme renormalization
  - Matching with NNLO + LRR + RGR
  - $-t$-, x-dependence of the LC GPDs

Thanks for your attention!

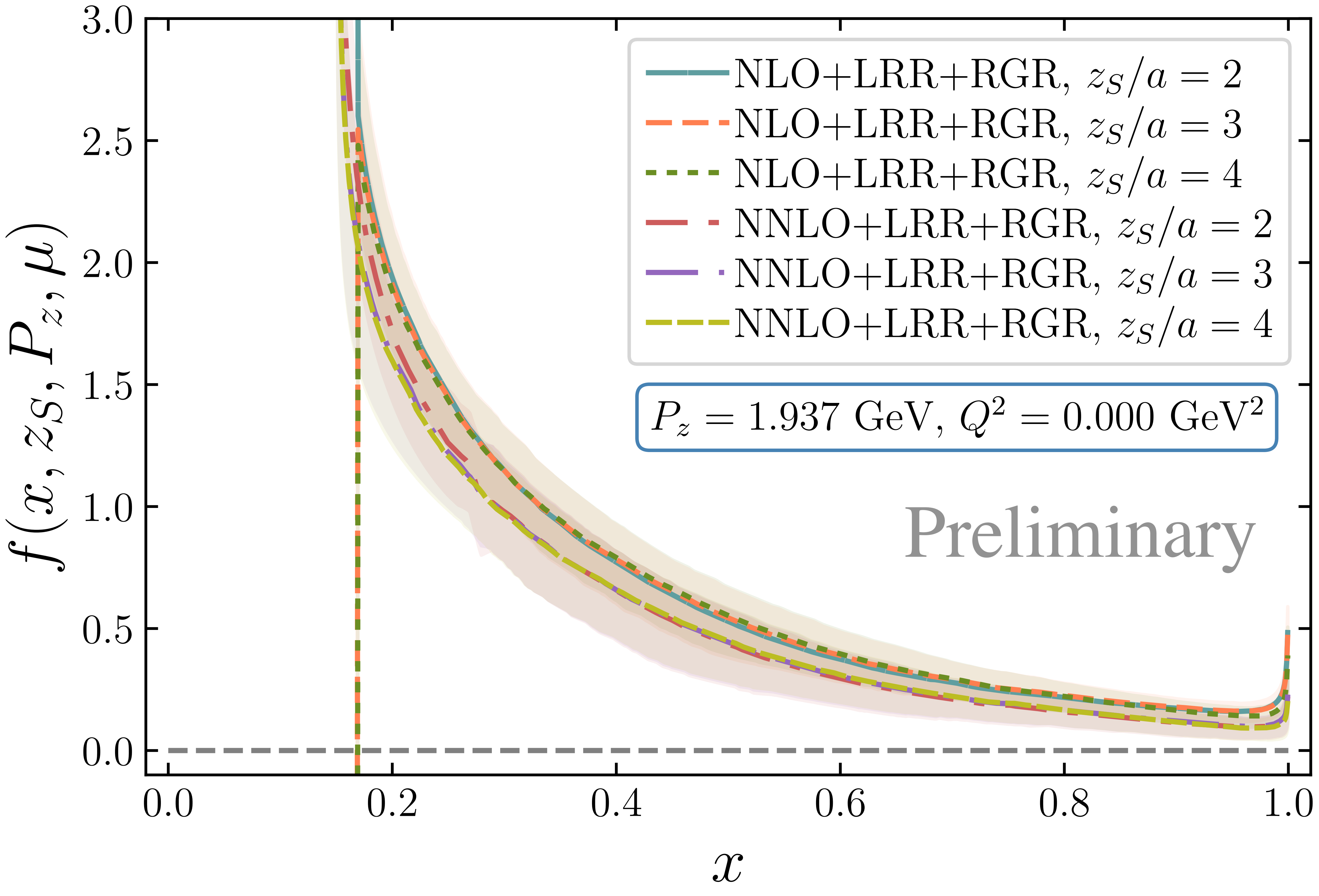
Backup

Extract Energy and Amplitude

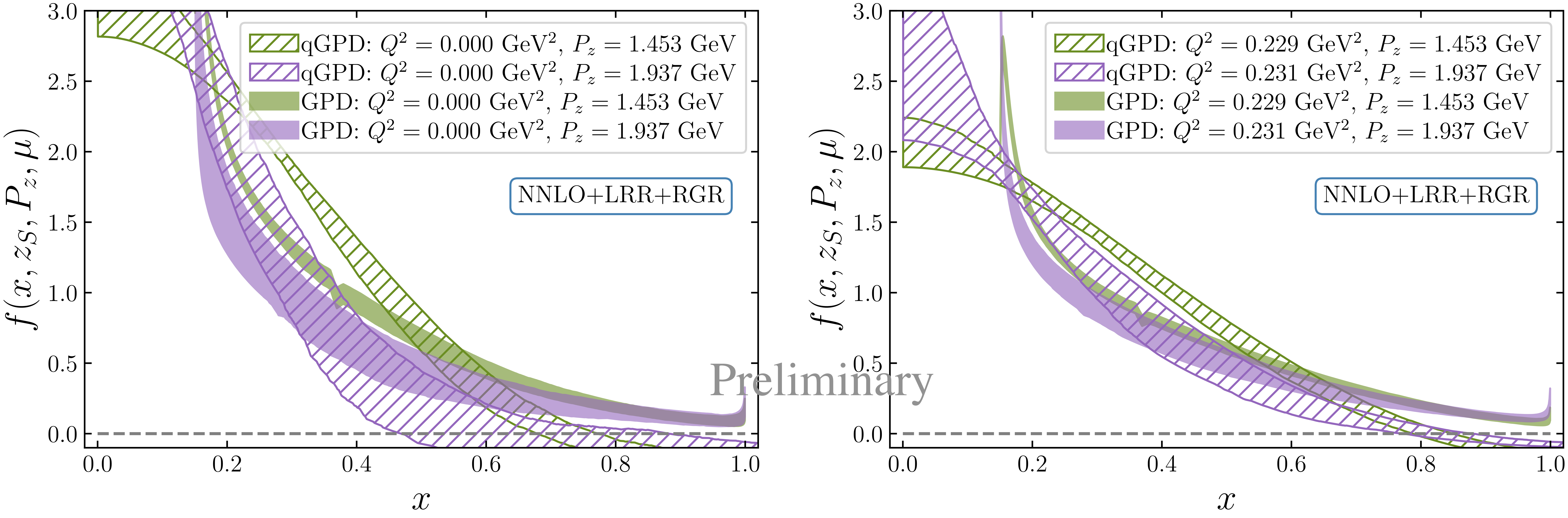
Take kaon data as an example



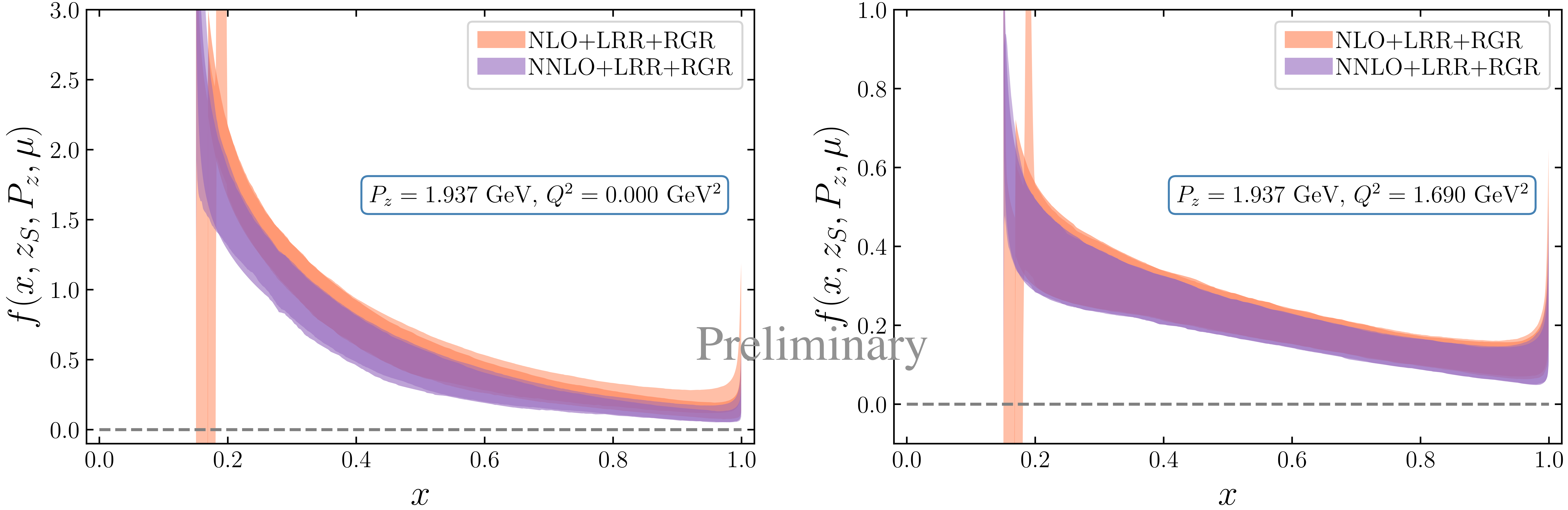
Valence LC GPDs: z_S-dependence



Valence LC GPDs: P_z-dependence



Valence LC GPDs: Order-dependence

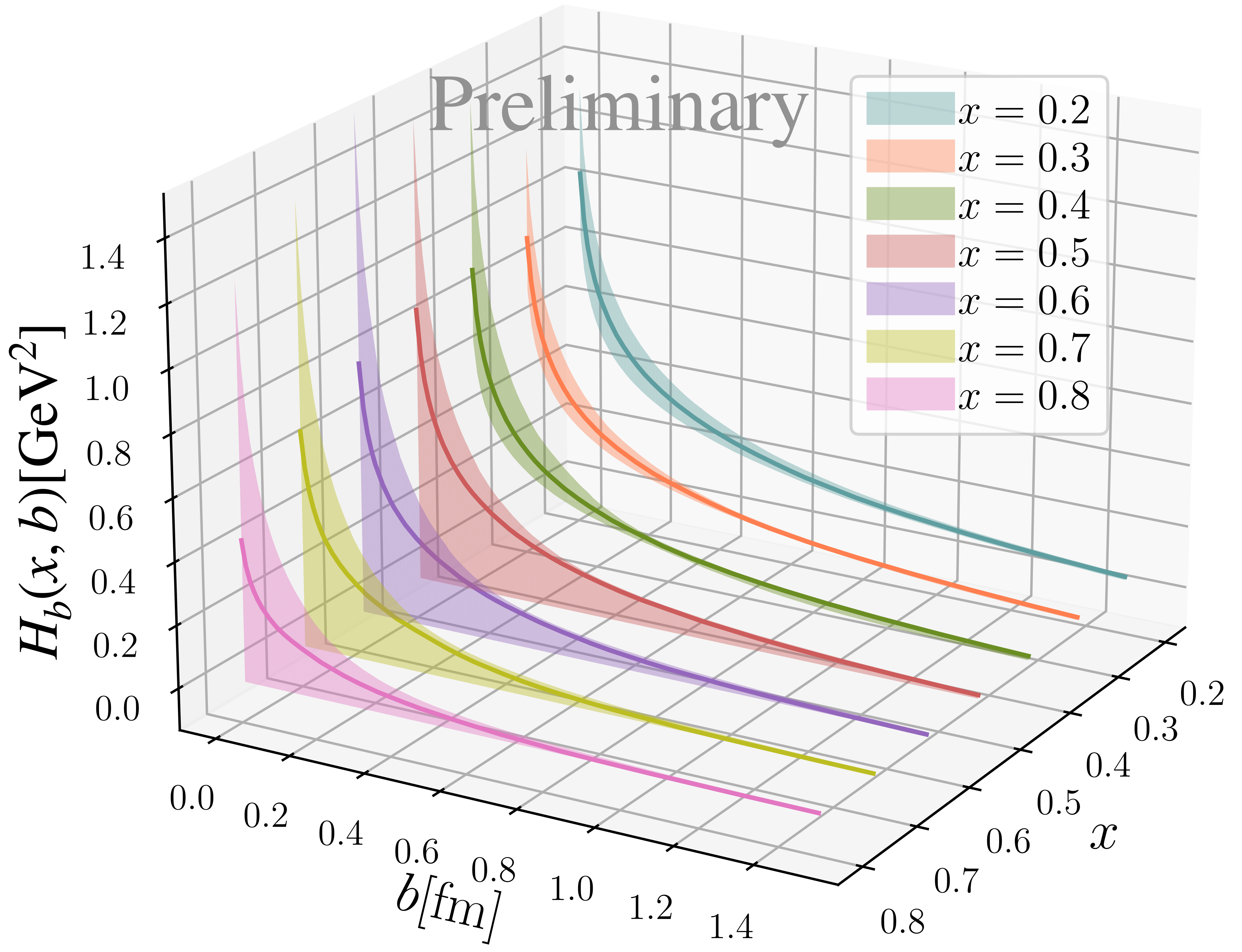


$Q^2 = -t$ increases → Order-dependence reduces

NNLO: better perturbative convergence

Impact-parameter dependent parton distributions

$$H_b(x,b) = \int \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2} H(x,\xi=0,\mathbf{q}_\perp^2) e^{i\mathbf{b}_\perp\cdot\mathbf{q}_\perp}$$



Impact-parameter dependent parton distributions