Hadron Structure from the Lattice Compton Amplitude

G. Schierholz

Deutsches Elektronen-Synchrotron DESY

From Quarks and Gluons to the Internal Dynamics of Hadrons

May 15 - 17, 2024 – Stony Brook

Abstract

Experimentally, hadron structure functions are extracted from the forward and off-forward Compton amplitudes. On the other hand, theoretical calculations are limited to Euclidean space-time. The closest one can come to exploiting the full potential of the EIC is to compute the Compton amplitude on the lattice in full diversity. In this talk I will discuss the challenges and the potential of this approach and present some recent highlights of the calculations

With

M. Batelaan, K.U. Can, J.A. Crawford, A. Hannaford-Gunn,
R. Horsley, P.E.L. Rakow, H. Stüben, R. Young, J. Zanotti

QCDSF Collaboration

Introduction

The long-term objective is to compute hadron structure functions from first principles, overcoming the limitations of light-cone PDFs and the OPE

The structure of hadrons revealed by deep-inelastic scattering is characterized completely by the virtual Compton amplitude

$$T_{\mu\nu} = \rho_{ss'} \int d^4x \, e^{i(q+q')x} \langle p, s | T[J_\mu^{V,A}(x) J_\nu^{V,A}(0)] | p', s' \rangle$$

of vector ($J_\mu^V = \bar{q}\gamma_\mu q$) and axial vector currents ($J_\mu^A = \bar{q}\gamma_5\gamma_\mu q$), with incoming momenta q, q' sandwiched between states of momenta p, p' and polarization s, s'

In this talk I will briefly discuss the theoretical foundation and challenges of the approach and highlight present results across a range of kinematics

Forward Compton amplitude

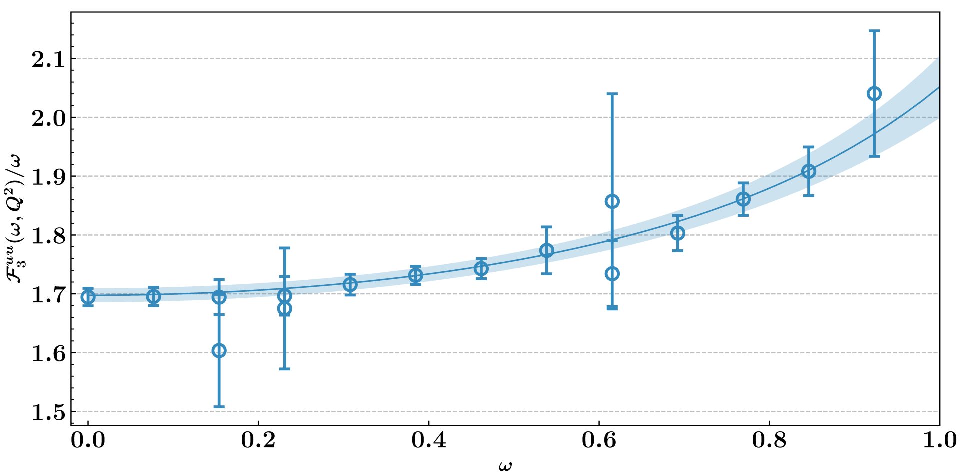
$$T_{\mu\nu}(p,q) = \delta_{\mu\nu}\mathcal{F}_1(\omega,q^2) + \frac{p_\mu p_\nu}{pq}\mathcal{F}_2(\omega,q^2) + i\epsilon_{\mu\nu\rho\sigma}\frac{p_\rho p_\sigma}{2pq}\mathcal{F}_3(\omega,q^2)$$
$$+\,\epsilon_{\mu\nu\lambda\sigma}q_\lambda s_\sigma\frac{1}{pq}\mathcal{G}_1(\omega,q^2) + \epsilon_{\mu\nu\lambda\sigma}q_\lambda\left(pq\,s_\sigma - sq\,p_\sigma\right)\frac{1}{(pq)^2}\mathcal{G}_2(\omega,q^2)$$

$$\mathcal{F}_2(\omega,q^2) = 4\omega\int_0^1 dx\,\frac{F_2(x,q^2)}{1-(\omega x)^2} = 4\sum_{n=1,2,\cdots}\omega^{2n-1}\int_0^1 dx\,x^{2n-2}F_2(x,q^2)$$
$$= 4\sum_{n=1,2,\cdots}\omega^{2n-1}M_{2n}^{(2)}(q^2)$$

$$\mathcal{F}_3(\omega,q^2) = 4\omega\int_0^1 dx\,\frac{F_3(x,q^2)}{1-(\omega x)^2} = 4\sum_{n=1,3,\cdots}\omega^{2n-1}\int_0^1 dx\,x^{2n-2}F_3(x,q^2)$$
$$= 4\sum_{n=1,3,\cdots}\omega^{2n-2}M_{2n-1}^{(3)}(q^2)$$

$$\mathcal{G}_1(\omega, q^2) = 4\omega \int_0^1 dx \, \frac{g_1(x, q^2)}{1 - (\omega x)^2} = 4 \sum_{n=1,3,\cdots} \omega^n \int_0^1 dx x^{n-1} g_1(x, q^2)$$

$$\mathcal{G}_2(\omega, q^2) = 4\omega \int_0^1 dx \, \frac{g_2(x, q^2)}{1 - (\omega x)^2} = 4 \sum_{n=1,3,\cdots}^{\infty} \omega^n \int_0^1 dx x^{n-1} g_2(x, q^2)$$



$J_\mu^V \; J_\nu^A \qquad Q^2 \approx 4\mathrm{GeV}^2$

Moments

– Polynomial fit

Structure functions

– Singular value decomposition

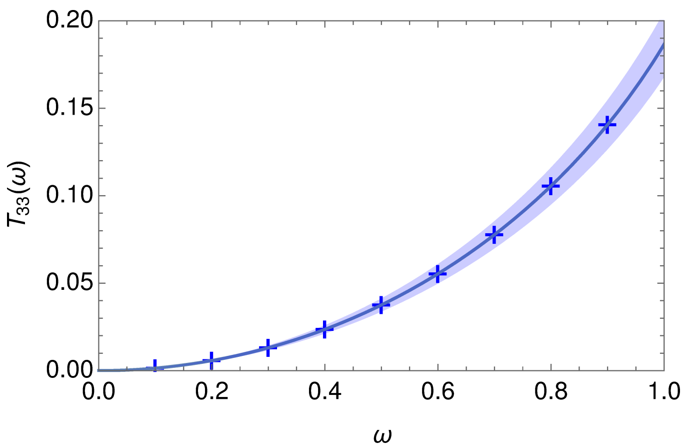
– 2^{nd} kind Fredholm equation

– Bayesian techniques

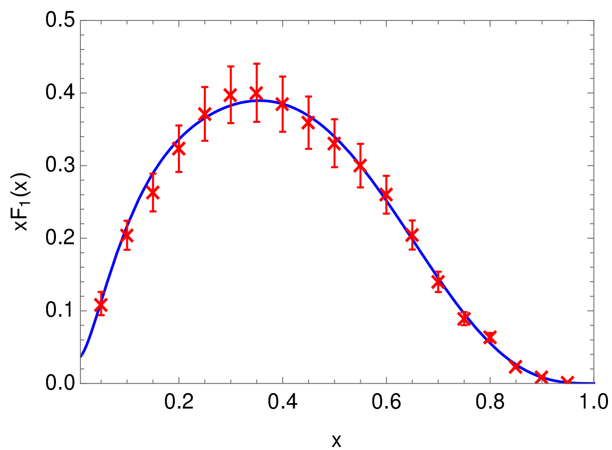
– Analytical parameterization

My personal favorite: SVD

Input



Out



With fingertip sensitivity

Feynman-Hellmann

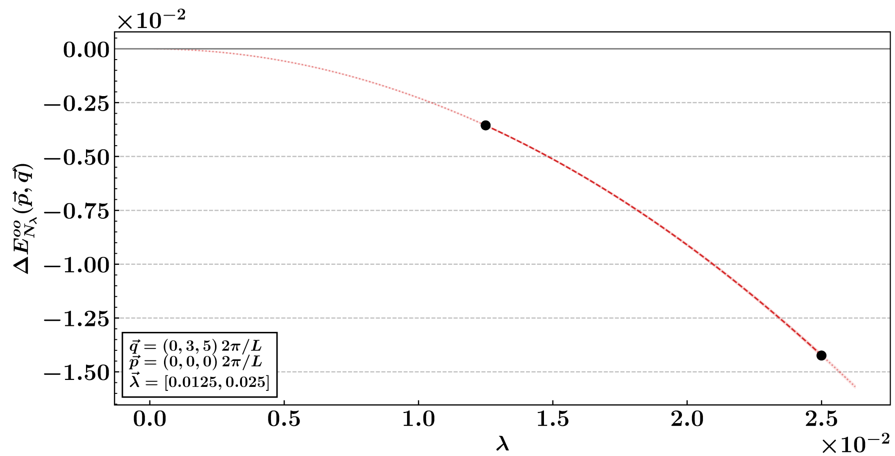
The Compton four-point amplitude can be computed most efficiently from the Feynman-Hellmann relation by introducing the perturbation to the action, e.g.

$$S \to S + \lambda_1 \int d^4x \cos qx \, J_\mu^V(x) + \lambda_2 \int d^4x \sin qx \, J_\nu^A(x)$$

Taking the derivative of the hadron two-point function with respect to λ_1 and λ_2 we obtain

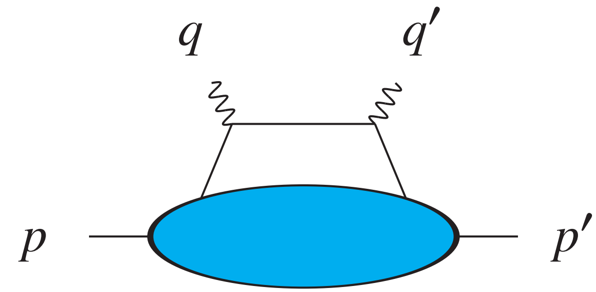
$$\left.\frac{\partial^2 E(p,q)}{\partial\lambda_1\partial\lambda_2}\right|_{\lambda_{1,2}=0} = i\,\frac{T_{[\mu\nu]}(p,q)}{E(p,q)}$$

Three-point function $$\left.\frac{\partial E(p,q)}{\partial\lambda}\right|_{\lambda=0}$$



Off-forward Compton amplitude

Focus on zero skewness



inherently non-perturbative

$$T_{\mu\nu}(\bar{p},\bar{q},\Delta) = \delta_{\mu\nu}\left(\frac{\bar{u}(p)\bar{\not{q}}u(p')}{2\bar{p}\bar{q}}\,\mathcal{H}(\bar{\omega},t,\bar{q}^2) + \frac{\bar{u}(p)\,\bar{q}_\rho i\sigma_{\rho\sigma}\Delta_\sigma u(p')}{2m_N}\,\mathcal{E}(\bar{\omega},t,\bar{q}^2)\right) + \cdots$$

$$\bar{\mathcal{H}}(\bar{\omega},t,\bar{q}^2) = 2\bar{\omega}^2\int_0^1 dx\frac{H(x,t,\bar{q}^2)}{1-(\bar{\omega}x)^2} = 2\sum_{1,2,\cdots}\bar{\omega}^{2n}A_{2n}(t,\bar{q}^2)$$

GFF

$H(x,0,q^2) = 2xF_1(x,q^2)$ $\qquad$ $A_{2n}(0,q^2) \simeq \langle x^{2n-1}\rangle$

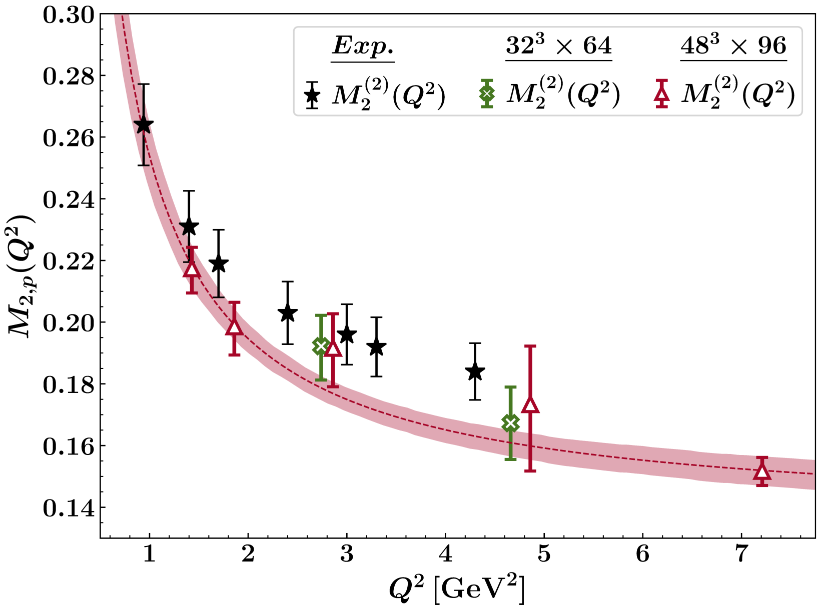
Impact parameter space

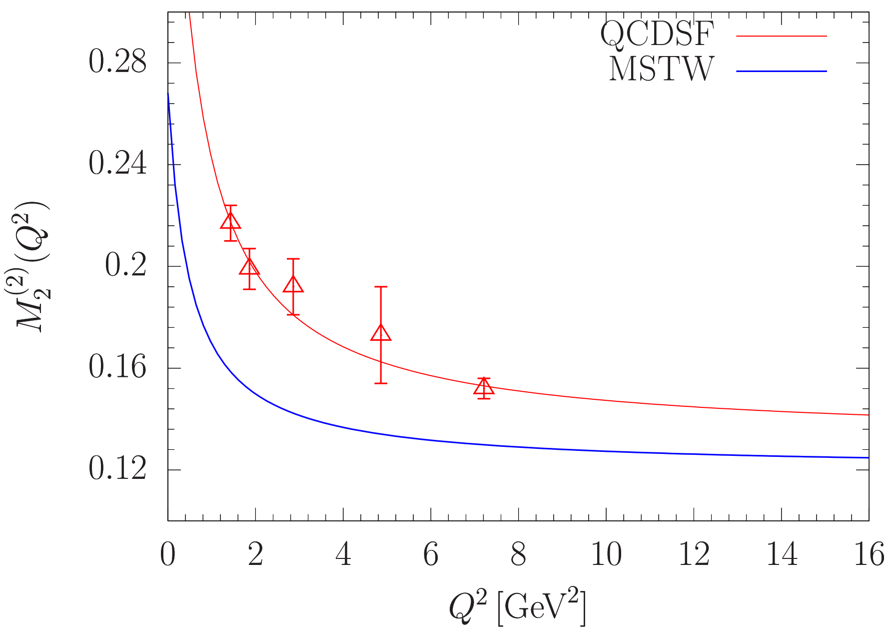
$H(x, t, \bar{q}^2) \;\Rightarrow\;$ 3D structure

$A_n(t, \bar{q}^2) \;\Rightarrow\;$ 2D structure

2^{nd} vs 1^{st} order FH: higher twist

Moment $\mathbf{M}_2^{(2)}(q^2)$





Gluon distribution function is derived from radiative decay of quark PDFs at very low Q^2 and evolved perturbatively to collider energies

GRV

Higher twist effects

Higher twist effects become more and more visible as the accuracy of the data increases. While operators of leading twist probe the light cone, higher twist effects reflect the internal motion of quarks and gluons

- Have no partonic interpretation
- Generated by transverse motion
- Window to long-distance physics
- Test of renormalon approach

$$M(Q^2) = c_2(Q^2/\mu^2)\,\langle p|O_2(\mu)|p\rangle + \frac{c_4(Q^2/\mu^2)}{Q^2}\,\langle p|O_4(\mu)|p\rangle$$

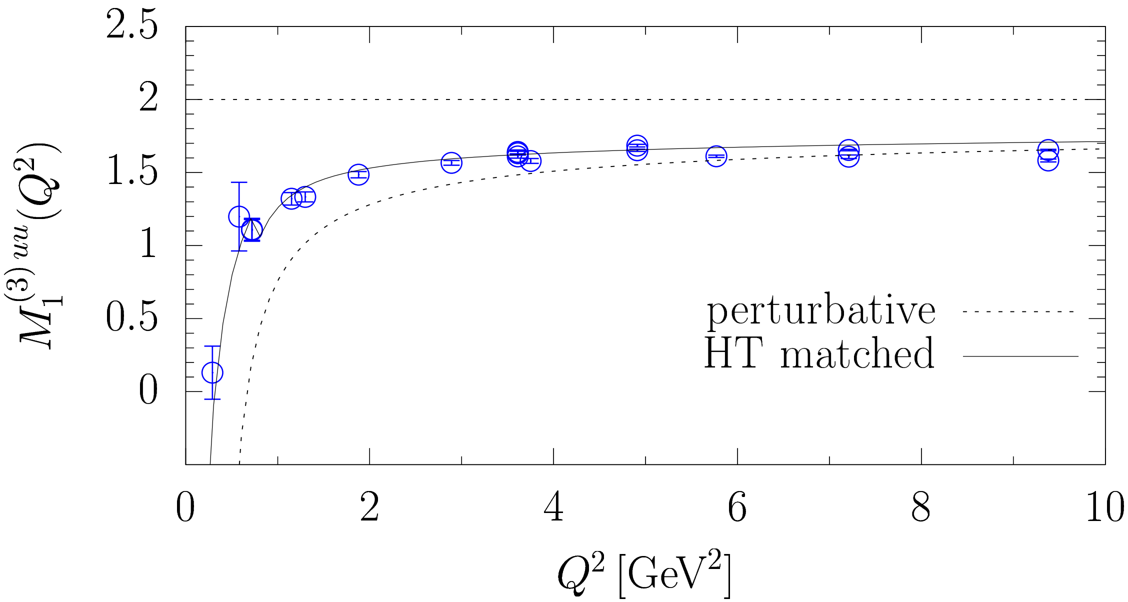
Particularly suited: Gross–Llewellyn-Smith sum rule

$$M_1^{(3)}(Q^2) = c_2(Q^2/\mu^2)\;N_q + \frac{c_4(Q^2/\mu^2)}{Q^2}\,\langle p|O_4(\mu)|p\rangle\,, \quad N_q \text{ number of quarks}$$

NNLO

Larin & Vermaseren

Consider $M_1^{(3)}$ with uu-insertion



To lowest order of c_4

$$M_1^{(3)\,uu}(Q^2) = c_2(Q^2/\mu^2)\,2 + c_4\,\frac{\mu^2}{Q^2} \qquad\qquad c_4 = 0.51 \pm 0.06\,\mathrm{GeV}^2$$

Matched to be independent of renormalization scale μ genuine HT

Twist 3

The spin-dependent structure function g_2 receives contributions from twist-2 and twist-3 operators, offering the unique possibility of directly accessing higher twist effects

The twist-3 contribution

$$d_2 = \int_0^1 dx\, x^2 \left(3\, g_2(x, q^2) + 2\, g_1(x, q^2)\right)$$

can be identified with the transverse force acting on the active quark of a transversely polarized nucleon

$$F^y(0) = \frac{1}{\sqrt{2}P^+}\langle p, s|\bar{q}(0)\gamma^+ G^{+y} q(0)|p, s\rangle = -\sqrt{2} m_N P^+ S^x\, d_2$$

OPE

Burkardt

Physical interpretation as a force

In electromagnetism the Lorentz force acting on a particle moving with $\approx$ speed of light in $-z$ direction is given by

$$F^y = [\vec{E} + \vec{v} \times \vec{B}]^y = [E^y - B^x] = -\sqrt{2}\, G^{+y}$$

Thus, d_2 can be identified with the transverse component of the color-Lorentz force acting on the struck quark in deep-inelastic scattering in the instant it has been hit by the virtual photon

In order to develop a 2D picture of the force, the generalization to off-forward kinematics is in order

$$\begin{aligned} F^i_{ss'}(\mathbf{\Delta}_\perp) &= -\frac{1}{\sqrt{2}P^+} \langle p^+, \frac{\Delta_\perp}{2}, s | \bar{q}(0)\gamma^+ G^{+i} q(0) | p^+, -\frac{\Delta_\perp}{2}, s' \rangle \\ &= \bar{u}(p,s)\left[P^+\Delta^i\gamma^+\Phi_1(t) + P^+ m_N i\sigma^{+i}\Phi_2(t) + \frac{P^+\Delta^i}{m_N} i\sigma^{+\Delta}\Phi_3(t)\right] u(p',s') \end{aligned}$$

Aslan, Burkardt & Schlegel

In impact parameter space

$$F^i_{ss'}(\mathbf{b}) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{b}\mathbf{\Delta}_\perp} F^i_{ss'}(\mathbf{\Delta}_\perp)\,,\ \Phi_i(t) = \frac{\Phi_i(0)}{(1-t/\Lambda_i^2)^2}$$

Λ_i from fit of $\Phi_i' s$

Probability interpretation

Results in three distinct contributions

$$F^i_1(\mathbf{b}) = -2\sqrt{2}P^+ b^i \Phi_1'(b^2)$$

$$F^i_2(\mathbf{b}) = m_n^2 \epsilon^{ij} s^j \Phi_2(b^2)$$

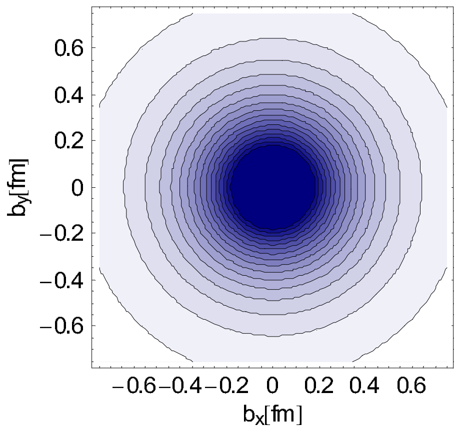
$$F^i_3(\mathbf{b}) = -\epsilon^{jk} s^k [2\delta^{ij}\Phi_3'(b^2) + 4b^i b^j \Phi_3''(b^2)]$$

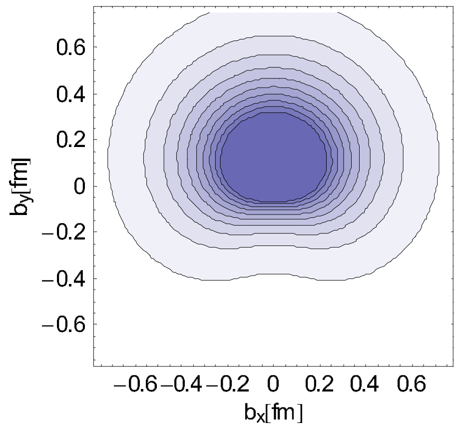
$\Phi_i(b^2)'s$ Fourier transforms of $\Phi_i(t)'s$

Work in nucleon rest frame $P^+ = \frac{m_N}{\sqrt{2}}$

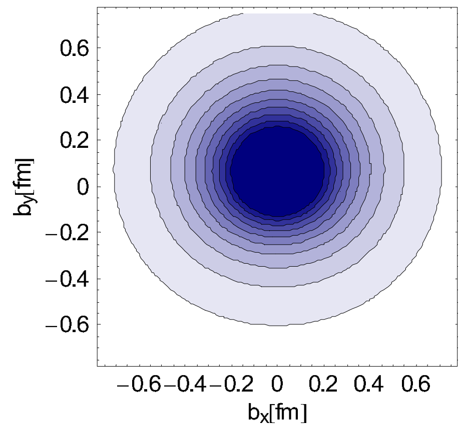
Remember $\langle p^+, s|\bar{q}(\mathbf{b})\sigma^{+i}\gamma_5 q(\mathbf{b})|p^+, s\rangle$ QSDSF

up



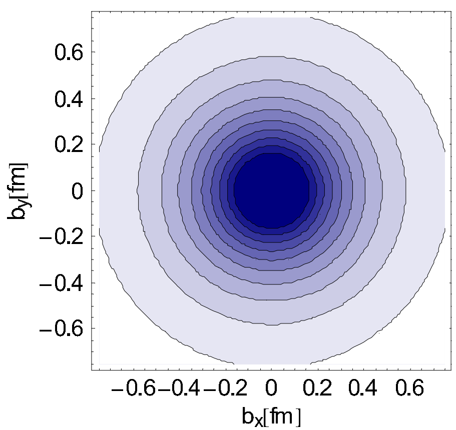


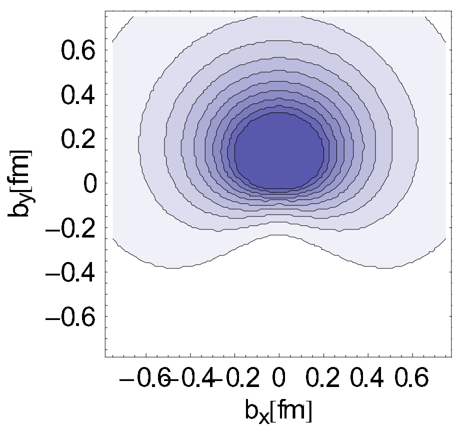
$\rightarrow$



$\Rightarrow$

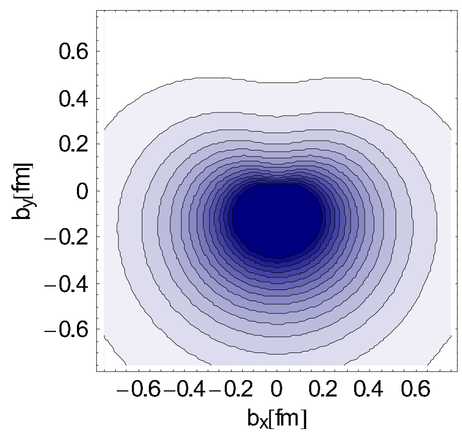
down





$\rightarrow$

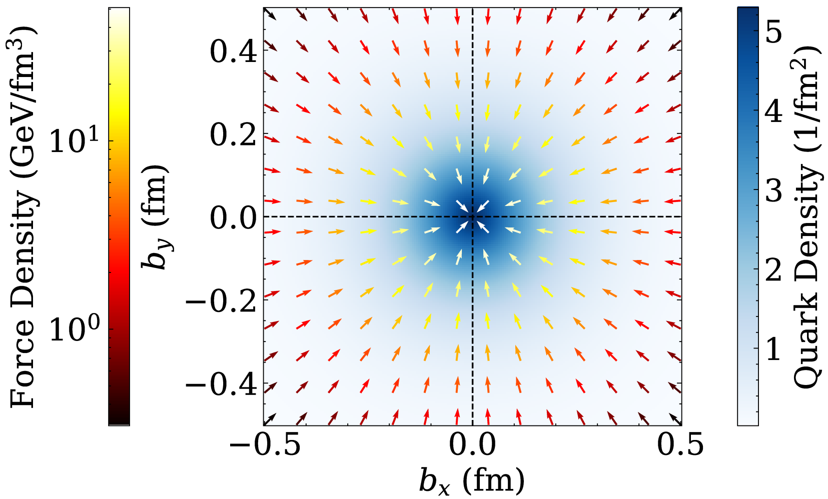
Boer–Mulders effect



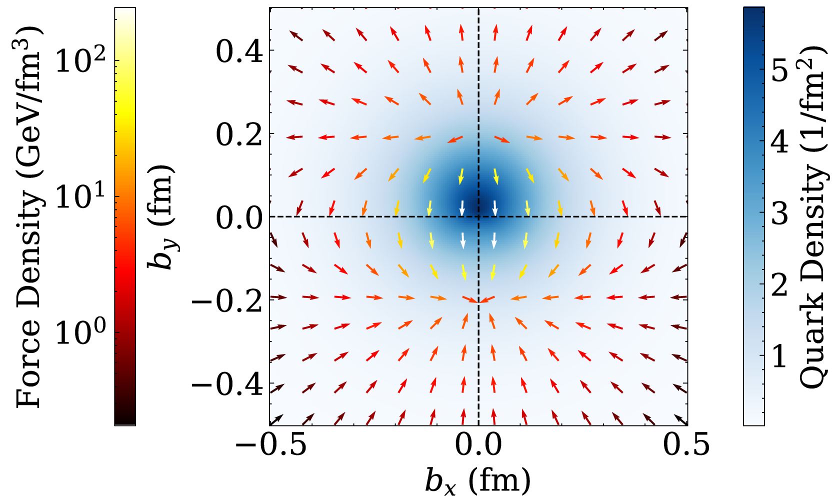
$\Rightarrow$

Sivers effect

Struck up quark



Unpolarized

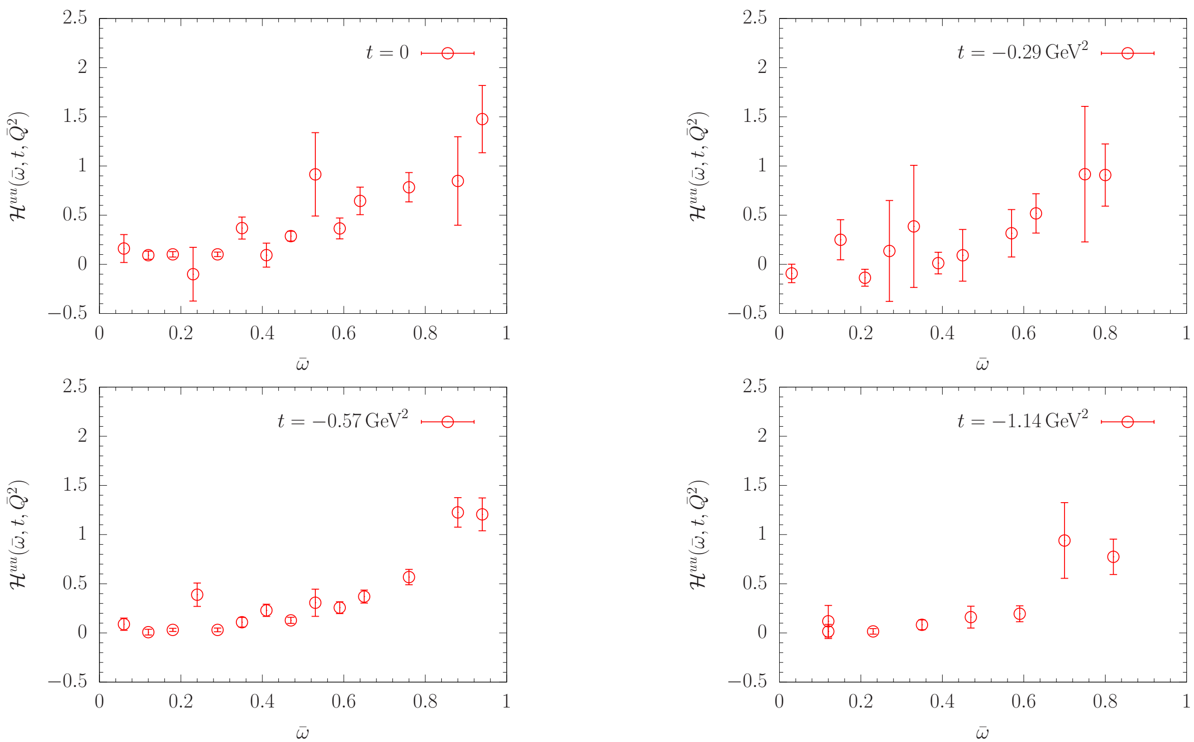


Polarized

3D Structure

$$\bar{\mathcal{H}}(\bar{\omega}, t, \bar{q}^2) = 2\bar{\omega}^2 \int_0^1 dx \frac{H(x, t, \bar{q}^2)}{1 - (\bar{\omega}x)^2}$$

$\bar{Q}^2 \approx 5\,\mathrm{GeV}^2$



We are finally interested in

$$H(x, b^2, \bar{Q}^2) = \int \frac{d^2\boldsymbol{\Delta}}{(2\pi)^2} e^{i\mathbf{b}\boldsymbol{\Delta}} H(x, \Delta^2, \bar{Q}^2)$$

inherently nonperturbative

Need analytic expression to assess the spatial distribution of quarks and gluons

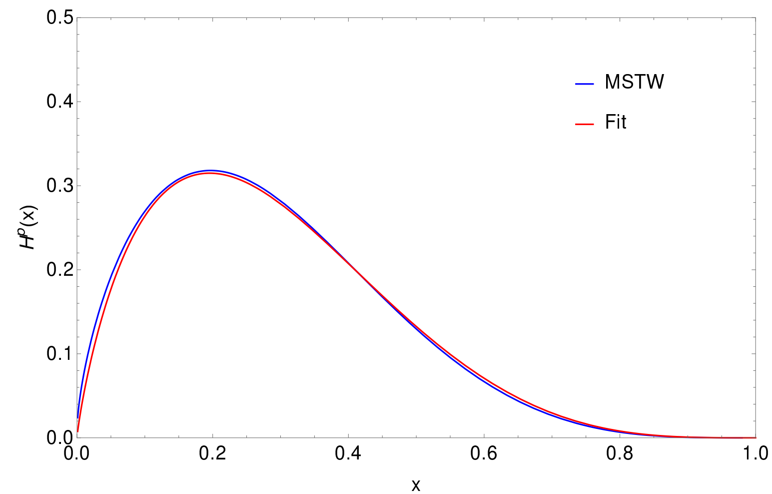
Ansatz derived from dual, Veneziano-type Compton amplitude resp. light-cone dominated current commutator

$$H(x, t, \bar{Q}^2) = A x^{1-\alpha(t)} (1-x)^{\beta}$$

Ademollo & Del Giudice
Gatto & Preparata

$\alpha(t) = \alpha(0) + \alpha'(0)\, t\,, \quad \beta$ constant trajectory

$t = 0$



$\alpha(0) = 0.15\,, \ \beta = 3.5$

effective

Matching MSTW parameterization with Regge ansatz arXiv:0901.0002

$$xq_v(x) = A\,x^{1-a}(1-x)^b(1+e\sqrt{x}+gx)$$

$$= A\,(x^{1-a} + ex^{1-(a-0.5)} + gx^{1-(a-1)}(1-x)^b$$

$\alpha_{\rho/\omega}$ α_η α_f

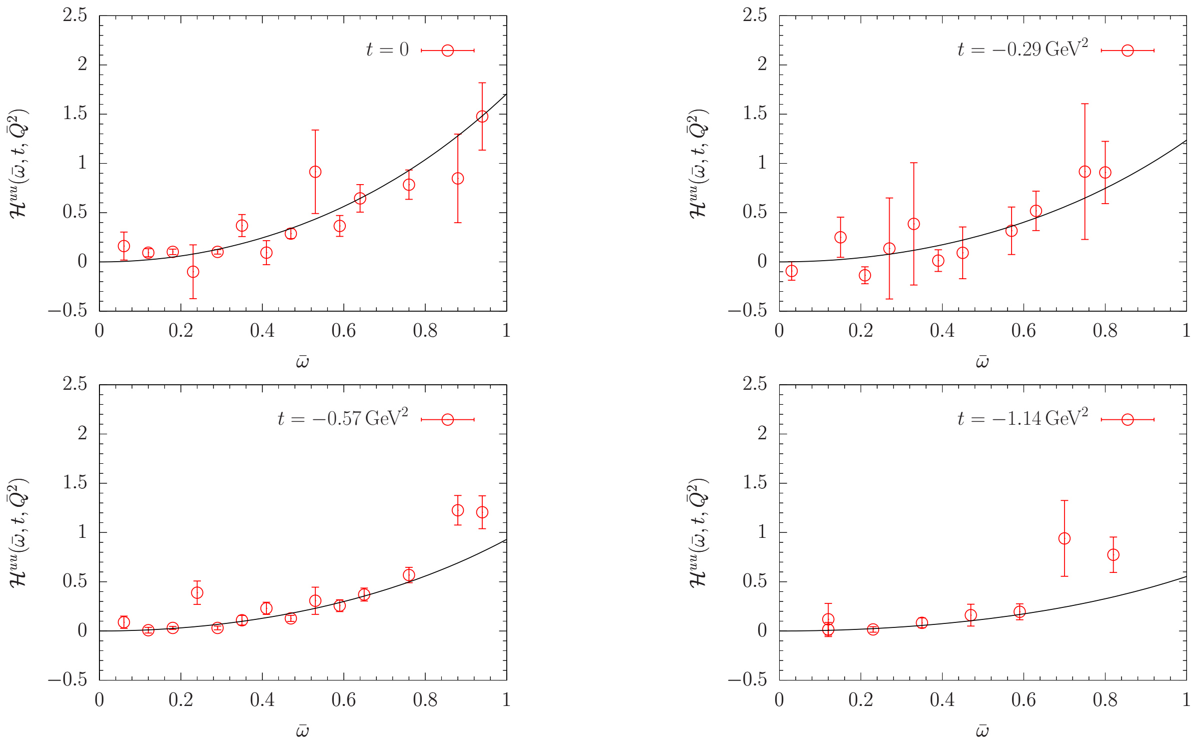
Accordingly, we should find

$$a \approx \alpha_{\rho/\omega}(0) \quad a - 0.5 \approx \alpha_\eta(0) \quad a - 1 \approx \alpha_f(0)$$

The ρ/ω trajectory has been studied the most, which gave $\alpha_{\rho/\omega}(0) \approx 0.45$. The MSTW parameterization then suggests $\alpha_\eta(0) \approx -0.05\,,\ \alpha_f(0) \approx -0.55$, which is in broad agreement with phenomenology

$$\bar{\mathcal{H}}(\bar{\omega}, t, \bar{Q}^2) = 2\,A \sum_{n=2,4,\cdots} \bar{\omega}^n \frac{\Gamma(1+\beta)\Gamma(n-\alpha(t))}{\Gamma(1+\beta+n-\alpha(t))}$$

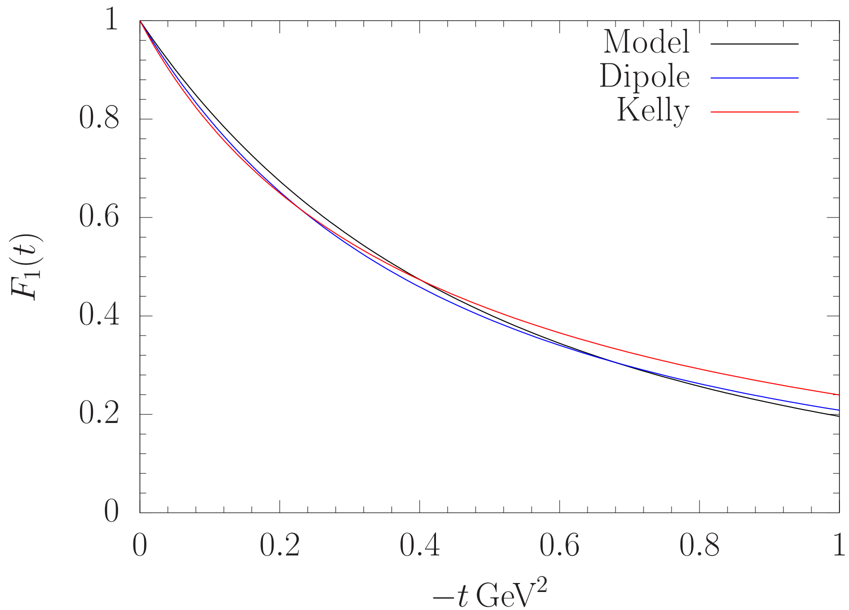
Sums up to ${}_3F_2$



$$\alpha'(0) = 0.85\,\mathrm{GeV}^{-2}$$

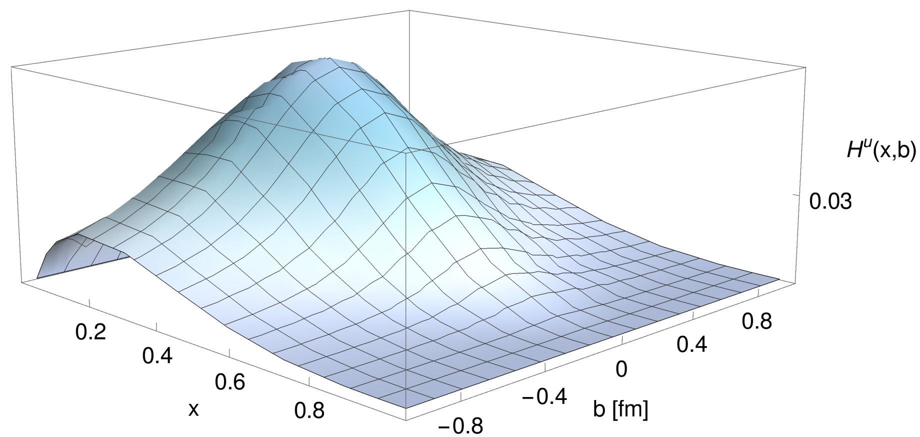
universal

$$F_1(t) = \frac{\Gamma(2+\beta-\alpha(0))}{\Gamma(1-\alpha(0))}\frac{\Gamma(1-\alpha(t))}{\Gamma(2+\beta-\alpha(t))}$$



$$H(x, b^2, \bar{Q}^2) \;=\; \frac{A}{4\sqrt{\pi \log(1/x)}}\, e^{-z}\, I_0(z)\; x^{1-\alpha(0)}\,(1-x)^{\beta}$$

$$z = \frac{b^2}{8\,\alpha'(0)\;\log(1/x)}$$



peripheral at $x \approx 0.2$

Conclusions

- Our approach is different from the mainstream. Starting point is the all-comprehensive, forward and off-forward, virtual Compton amplitude
- By working with the physical amplitude, there is no need to resort to the OPE, facing problems of factorization and renormalization, nor is the calculation bound to light-cone kinematics
- The computation of the Compton amplitude is made possible by recent advances in the application of the second-order Feynman-Hellmann theorem
- The emphasis of this talk has been on quantities that are not easily, or not at all, assessible by standard techniques. First results on higher-twist effects and the 2D and 3D structure of the nucleon look promising
- To convert experimental and theoretical information on GPDs into an image of the internal structure of the nucleon, an underlying model must be in place
- Synergies between this approach, covering a wide range of kinematics, and the Electron-Ion Collider EIC can be expected