Transverse Momentum Distributions from Lattice QCD without Wilson Lines

From Quarks and Gluons to the Internal Dynamics of Hadrons

May 17, 2024

YONG ZHAO

Xiang Gao, Wei-Yang Liu and Yong Zhao, Phys.Rev.D 109 (2024),
Yong Zhao, arXiv: 2311.01391.

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Outline

- **Introduction**
  - Overview of TMD physics
  - Large-Momentum Effective Theory
- **Coulomb gauge method**
  - Universality class in LaMET
  - Coulomb-gauge quasi-TMD
  - Large-momentum factorization formula
  - Soft function
  - Transverse gauge link
- **Discussions**

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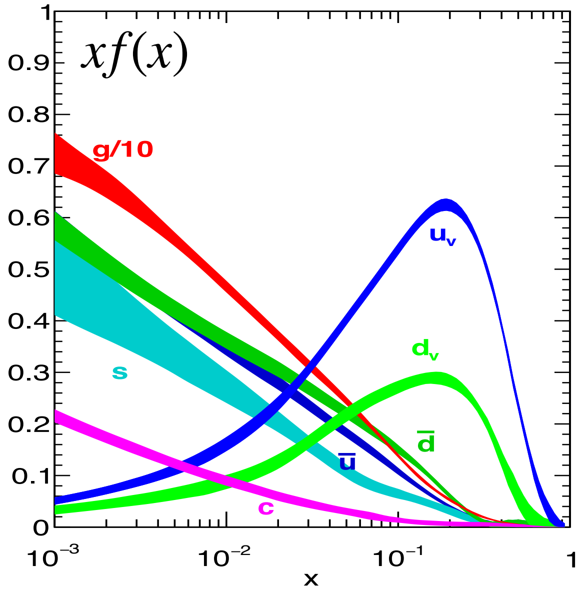
3D imaging of the proton

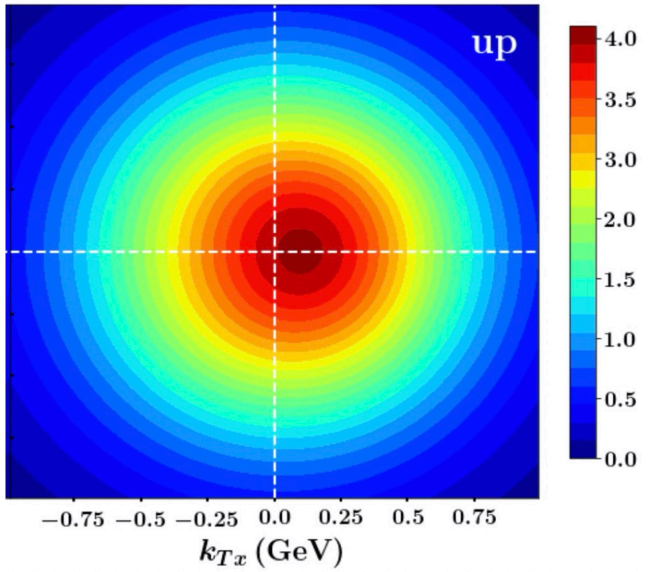
PDFs



NNPDF, EPJ C77 (2017)

$$\int d^2\vec{k}_T$$

Transvers momentum distributions (TMDs)



Cammarota, et al. (JAM), PRD **102** (2020).

$$\int d^2\vec{b}_T$$

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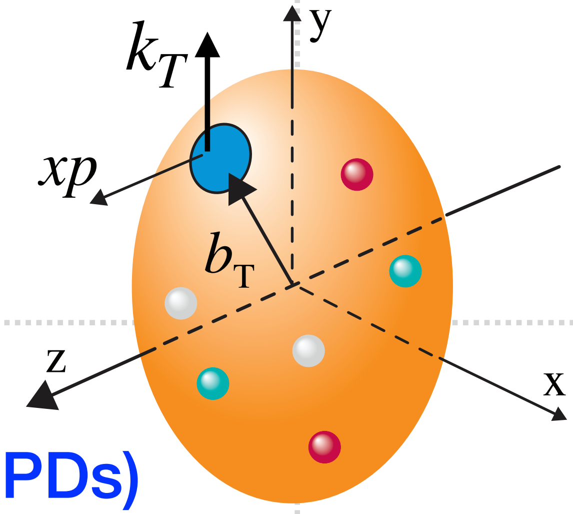
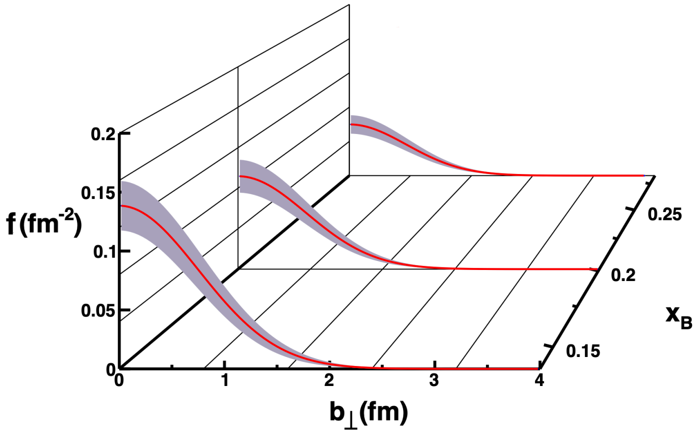


Generalized parton distributions (GPDs)



W. Armstrong et al., arXiv: 1708.00888.

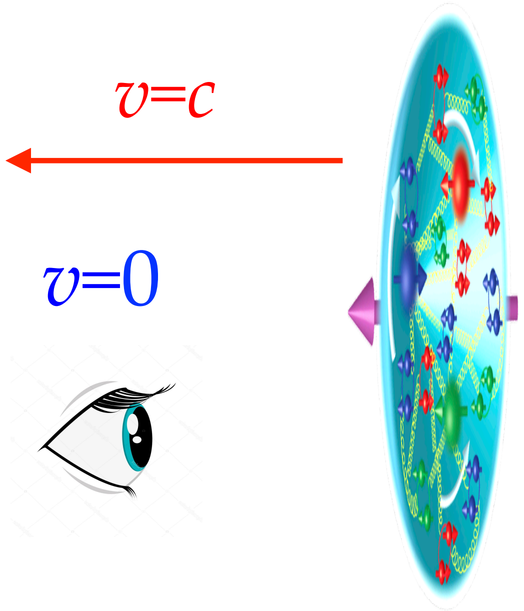
$$\int d^2\vec{k}_T$$

Wigner distributions/Generalized TMDs

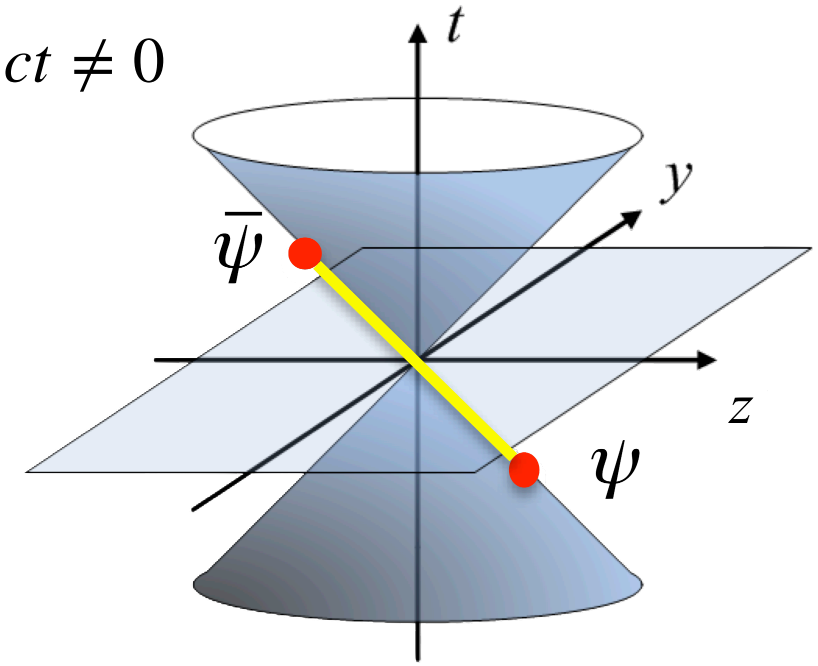
$$W(x, \vec{k}_T, \vec{b}_T)$$

Simulating partons on the Euclidean lattice?

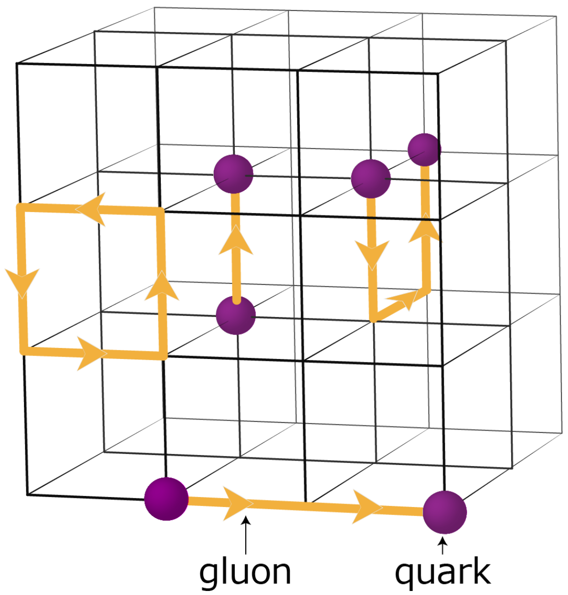
PDFs can be defined from light-cone correlations



$z + ct = 0$
$z - ct \neq 0$



$t \to i\tau$



$$f(x) = \int \frac{d\xi^-}{2\pi}\, e^{-ix\lambda} \langle P \,|\, \bar{\psi}(\xi^-)\gamma^+ W(\xi^-,0)\psi(0) \,|\, P \rangle$$

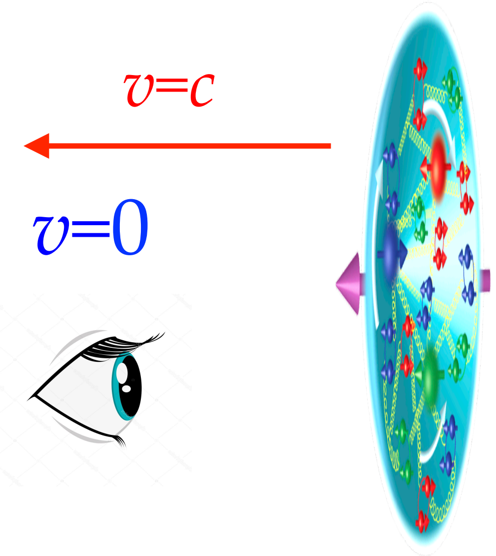
$$O(i\tau) \overset{?}{\to} O(t)$$

$\xi^- = (t - z)/\sqrt{2}$ $W(\xi^-,0)$: Wilson line that ensures SU(3) gauge invariance

Time-dependence makes it impossible to calculate the PDFs directly on the Euclidean lattice. 🙁

Large-Momentum Effective Theory (LaMET)

Revisit Feynman's parton picture in the infinite momentum frame



Simulating $\langle P=\infty\,|\,O(t=0)\,|\,P=\infty\rangle$? ✘

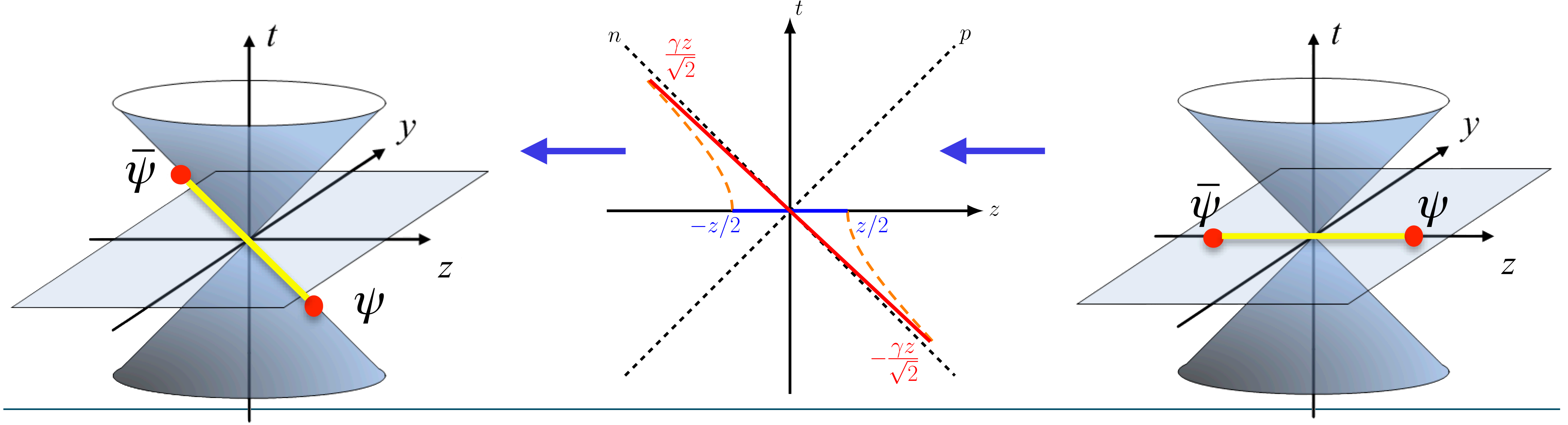
$$P \ll \frac{2\pi}{a}!$$

Nevertheless, it is possible to simulate a proton at large *P*:

$z+ct=0,\;\; z-ct\neq 0$

X. Ji, PRL **110** (2013)

$t=0,\;\; z\neq 0$



Large-Momentum Effective Theory (LaMET)

Systematic calculation of *x*-dependence:

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C^{-1}\left(\frac{x}{y}, \frac{\mu}{2xP^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) \; + \; \mathcal{O}\cancel{\left(\frac{\Lambda_{\mathrm{QCD}}}{xP^z}\right)} + \; \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{(xP^z)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{((1-x)P^z)^2}\right)$$

- **Hybrid renormalization, subtraction of linear divergence & renormalon;**
  - X. Ji, YZ, et al., NPB **964** (2021).
  - Holligan, Ji, Lin, Su and Zhang, NPB **993** (2023);
  - Zhang, Ji, Holligan and Su, PLB **844** (2023).
- **Next-to-next-to-leading order (NNLO) accuracy**
  - Chen, Zhu and Wang, PRL **126** (2021);
  - Li, Ma and Qiu, PRL **126** (2021).
- **Resummation of small-*x* (DGLAP) logarithms** $\alpha_s \ln \mu/(2xP^z)$
  - X. Gao, K. Lee, and YZ et al., PRD **103** (2021);
  - Y. Su, J. Holligan et al., NPB **991** (2023).
- **Resummation of large-*x* (threshold) logarithms** $\alpha_s \ln(1-x/y) \Big/ (1-x/y)$
  - X. Gao, K. Lee, and YZ et al., PRD **103** (2021);
  - X. Ji, Y. Liu and Y. Su, arXiv:2305.04416;
  - Y. Su and X. Ji, in preparation.

Large-Momentum Effective Theory (LaMET)

Systematic calculation of *x*-dependence:

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C^{-1}\left(\frac{x}{y}, \frac{\mu}{2xP^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) \; + \; \mathcal{O}\left(\cancel{\frac{\Lambda_{\rm QCD}}{xP^z}}\right) + \; \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

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- **Next-to-next-to-leading order**

- Chen, Zhu and Wang, PRL **126** (2021);

See Qi Shi and Rui Zhang's talks on improved lattice calculations of the pion valence GPD and distribution amplitude.

logarithms $\alpha_s \ln \mu/(2xP^z)$

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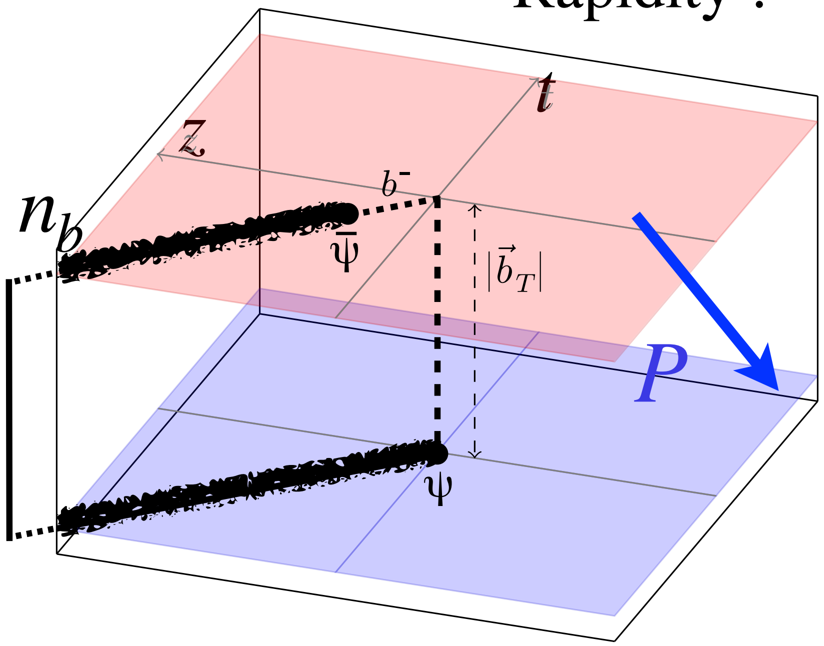
Transverse Momentum Distributions (TMDs)

- Beam function:

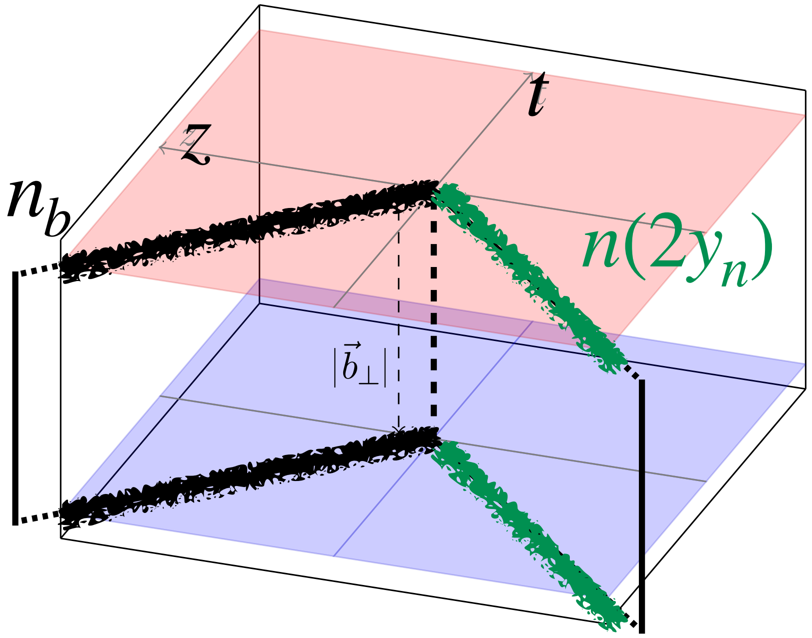
$$n_b^2 = 0$$

$$\text{Rapidity :} \quad y_B = \frac{1}{2} \ln \left| \frac{n_b^+}{n_b^-} \right| = -\infty$$



Hadronic matrix element

- Soft function :



Vacuum matrix element

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{\rm UV} \lim_{\tau \to 0} \frac{B_i}{\sqrt{S^q}}$$

Collins-Soper scale: $\zeta = 2(xP^+ e^{-y_n})^2$

Rapidity divergence regulator

Transverse Momentum Distributions (TMDs)

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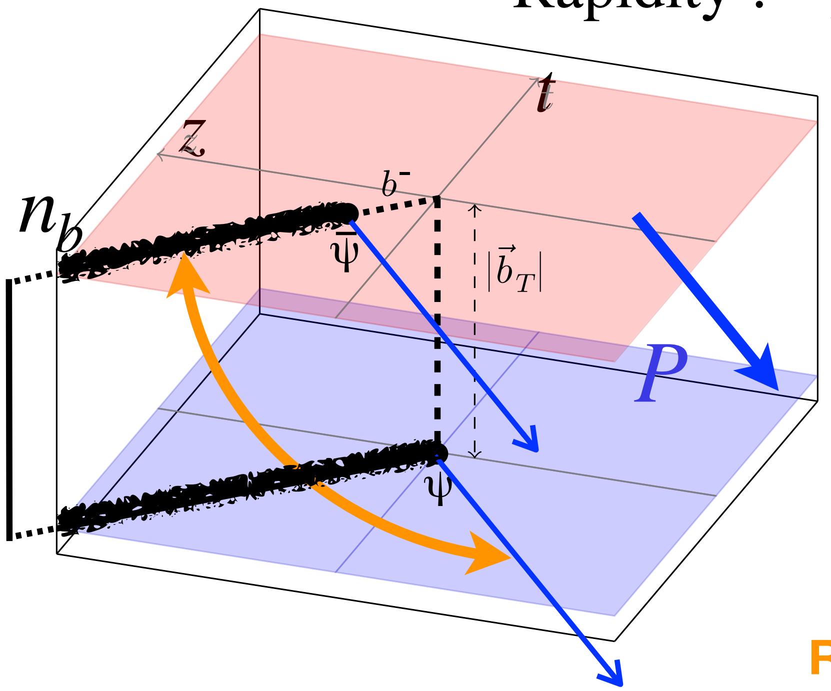

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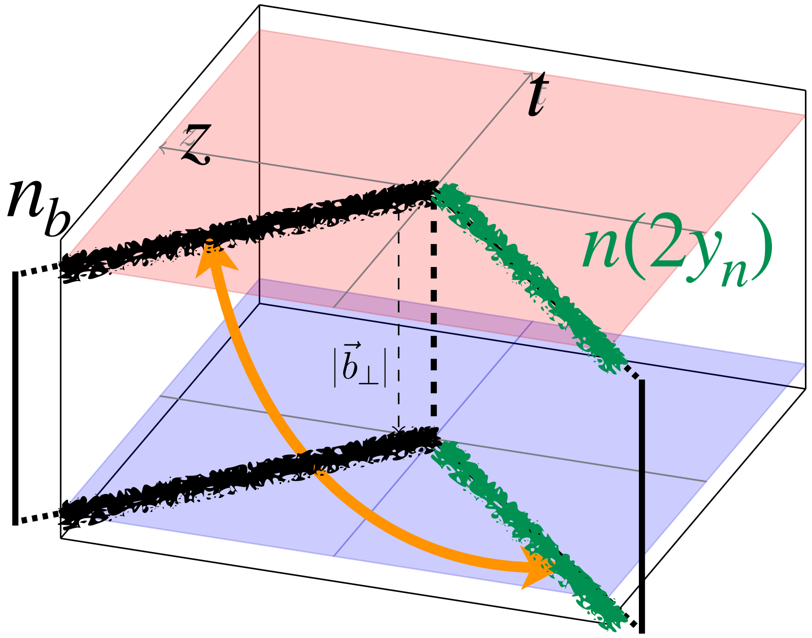

Vacuum matrix element

Rapidity divergences

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0} Z_{\mathrm{UV}} \lim_{\tau\to 0} \frac{B_i}{\sqrt{S^q}}$$

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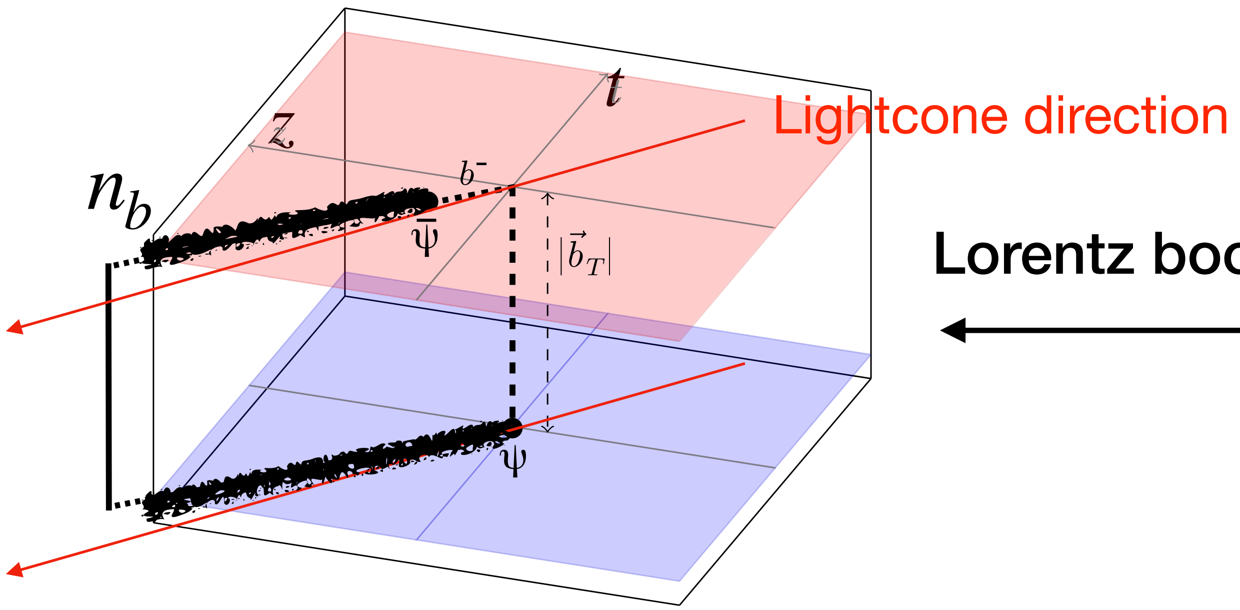
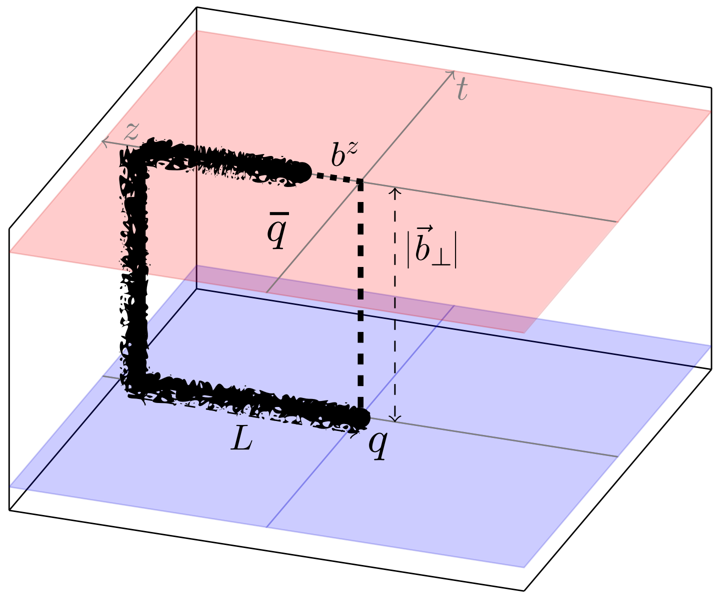
Rapidity divergence regulator

TMDs from LaMET

- Beam function (in Collins scheme):
- Quasi beam function :



Lorentz boost and $L \to \infty$



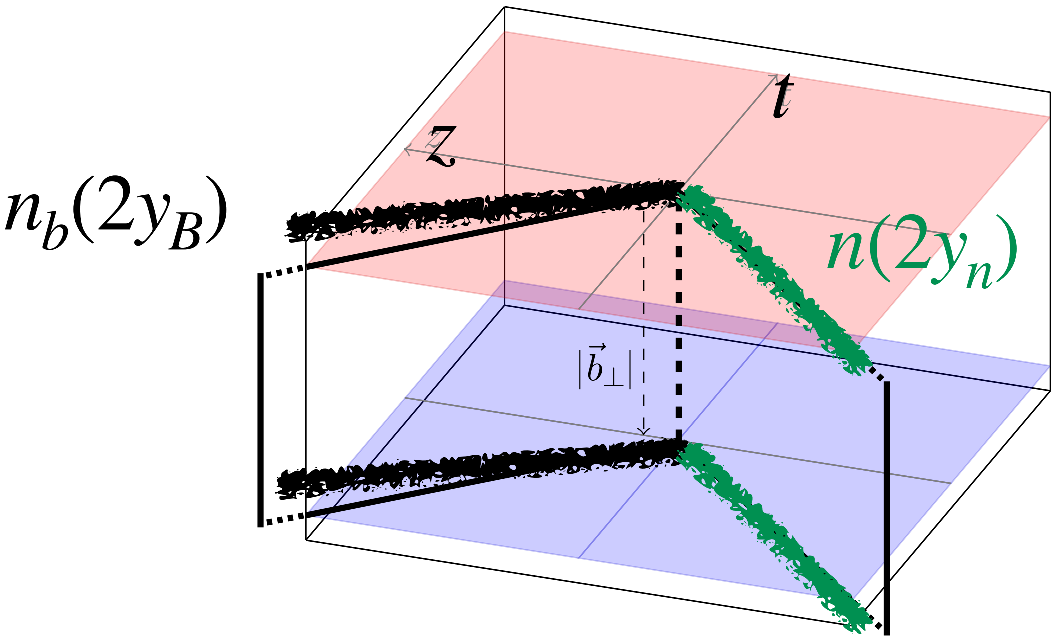
$$n_b^\mu(y_B) = (n_b^+, n_b^-, \vec{0}_\perp) = (-e^{2y_B}, 1, \vec{0}_\perp)$$

Spacelike but close-to-lightcone ($y_B \to -\infty$) Wilson lines, not calculable on the lattice ☹

Equal-time Wilson lines, directly calculable on the lattice🙂

Ebert, Schindler, Stewart and YZ, JHEP **04** (2022).

Soft factor



Collins-Soper kernel

$$\xrightarrow{y_n-y_B\to\infty} S_r(b_T,\mu)\ e^{-2(y_n-y_B)\gamma_\zeta(b_T,\mu)}$$

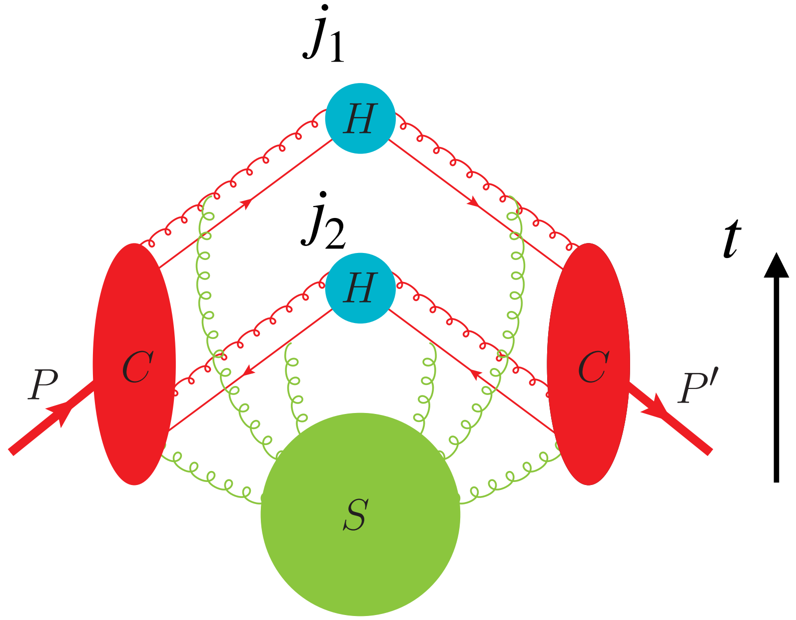
Reduced soft factor

Light-meson form factor:

$$F(b_T,P^z) = \langle \pi(-P)\,|\,j_1(b_T)j_2(0)\,|\,\pi(P)\rangle$$

$$\overset{P^z\gg m_N}{=} S_r(b_T,\mu)\int dxdx'\ H(x,x',\mu)$$

$$\times\ \Phi^\dagger(x,b_T,P^z,\mu)\Phi(x',b_T,P^z,\mu)$$



$\Phi(x,b_T,P^z,\mu)$: **quasi-TMD wave function**

- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Ji and Liu, PRD **105** (2022);
- Deng, Wang and Zeng, JHEP **09** (2022).

Factorization formula for the quasi-TMDs

$$\frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S_r(b_T, \mu)}} = C\left(\mu, x\tilde{P}^z\right) \exp\left[\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right] \times f_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\text{QCD}}^2}{(x\tilde{P}^z)^2}\right]\right\}$$

$$\gamma_\zeta(\mu, b_T) = \frac{d}{d\ln\tilde{P}^z}\ln\frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{C\left(\mu, x\tilde{P}^z\right)}$$

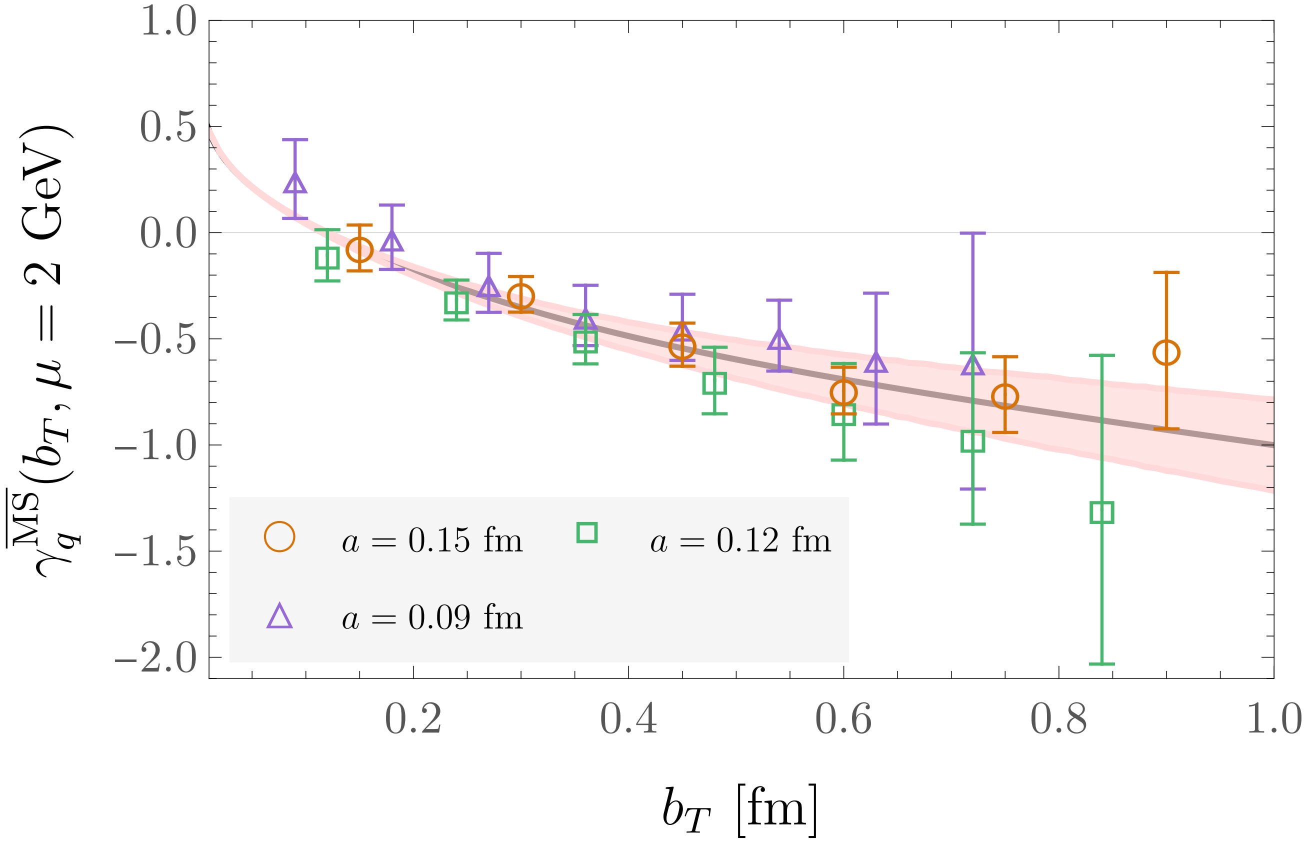
- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and **YZ**, PRD99 (2019);
- Ebert, Stewart, **YZ**, PRD99 (2019), JHEP09 (2019) 037;
- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Ebert, Schindler, Stewart and **YZ**, JHEP 09 (2020);
- Vladimirov and Schäfer, PRD 101 (2020);
- Ji, Liu, Schäfer and Yuan, PRD 103 (2021);
- Ebert, Schindler, Stewart and **YZ**, JHEP 04, 178 (2022).

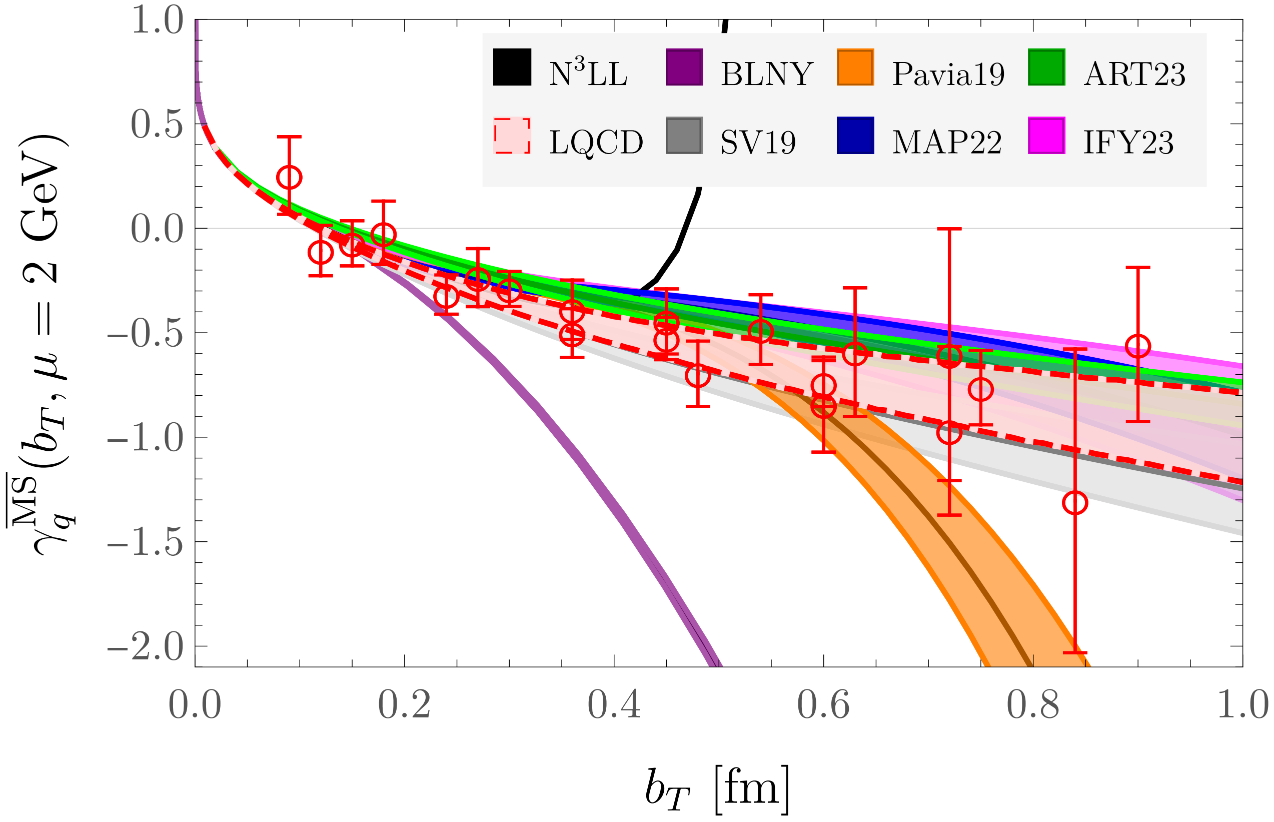
State-of-the-art determination of the Collins-Soper kernel

$$\gamma_\zeta(\mu, b_T) = \frac{d}{d\ln \tilde{P}^z}\ln\frac{\tilde{f}^{\,\text{naive}[s]}_{i/p}(x,\mathbf{b}_T,\mu,\tilde{P}^z)}{C\left(\mu, x\tilde{P}^z\right)}$$

- Physical quark masses
- Continuum limit with a = 0.15, 0.12, 0.09 fm
- Controlled renormalization and Fourier transform
- Next-to-next-to-leading logarithmic (NNLL) order

Nice agreement with phenomenology ☺

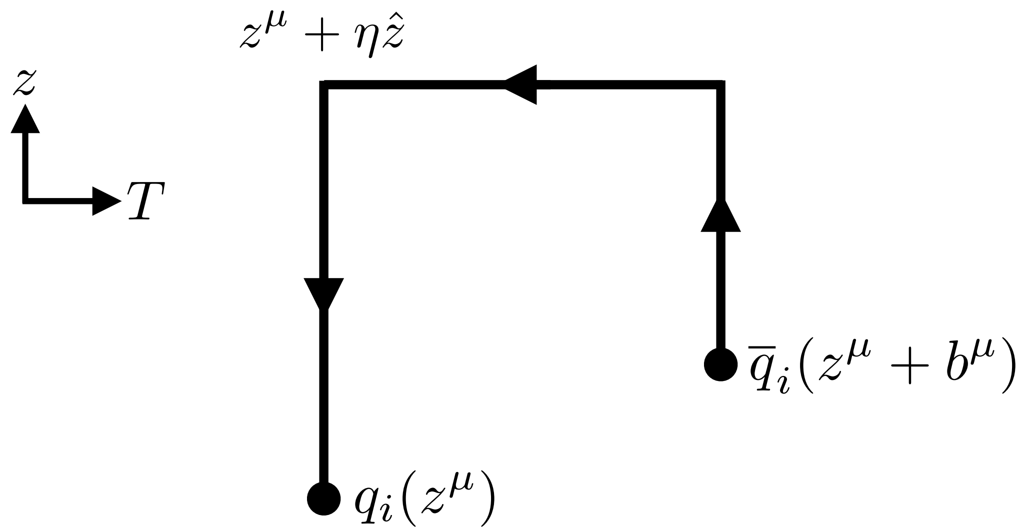




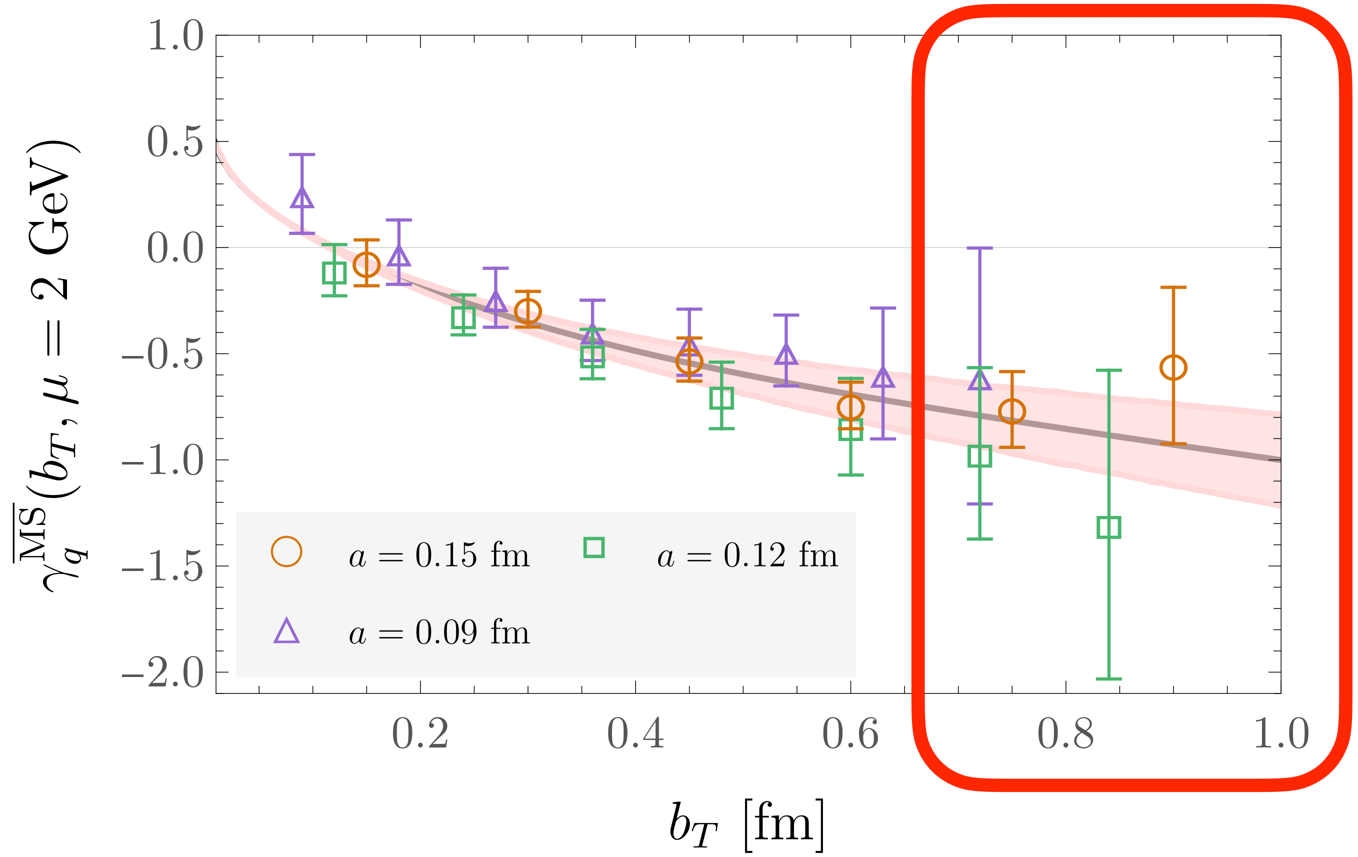
- A. Avkhadiev, P. Shanahan, M. Wagman and **YZ**, Phys.Rev.D 108 (2023);
- A. Avkhadiev, P. Shanahan, M. Wagman and **YZ**, 2402.06725, accepted by PRL.

Systematics in lattice calculation

Staple-shaped Wilson line



$$\eta \gg \{b^z, b_T\},\ xP^z \gg 1/b_T$$



- A large staple (gauge link) induces large statistical noises, which becomes worse at larger b_T. Smearing or gradient flow is required to reach reasonable precision;
- Complex operator mixings induced by the staple geometry;
- Additional power correction of order b^z/b_T, or equivalently $1/(xP^z b_T)$, from the staple self energy, which has not been handled by renormalization so far.

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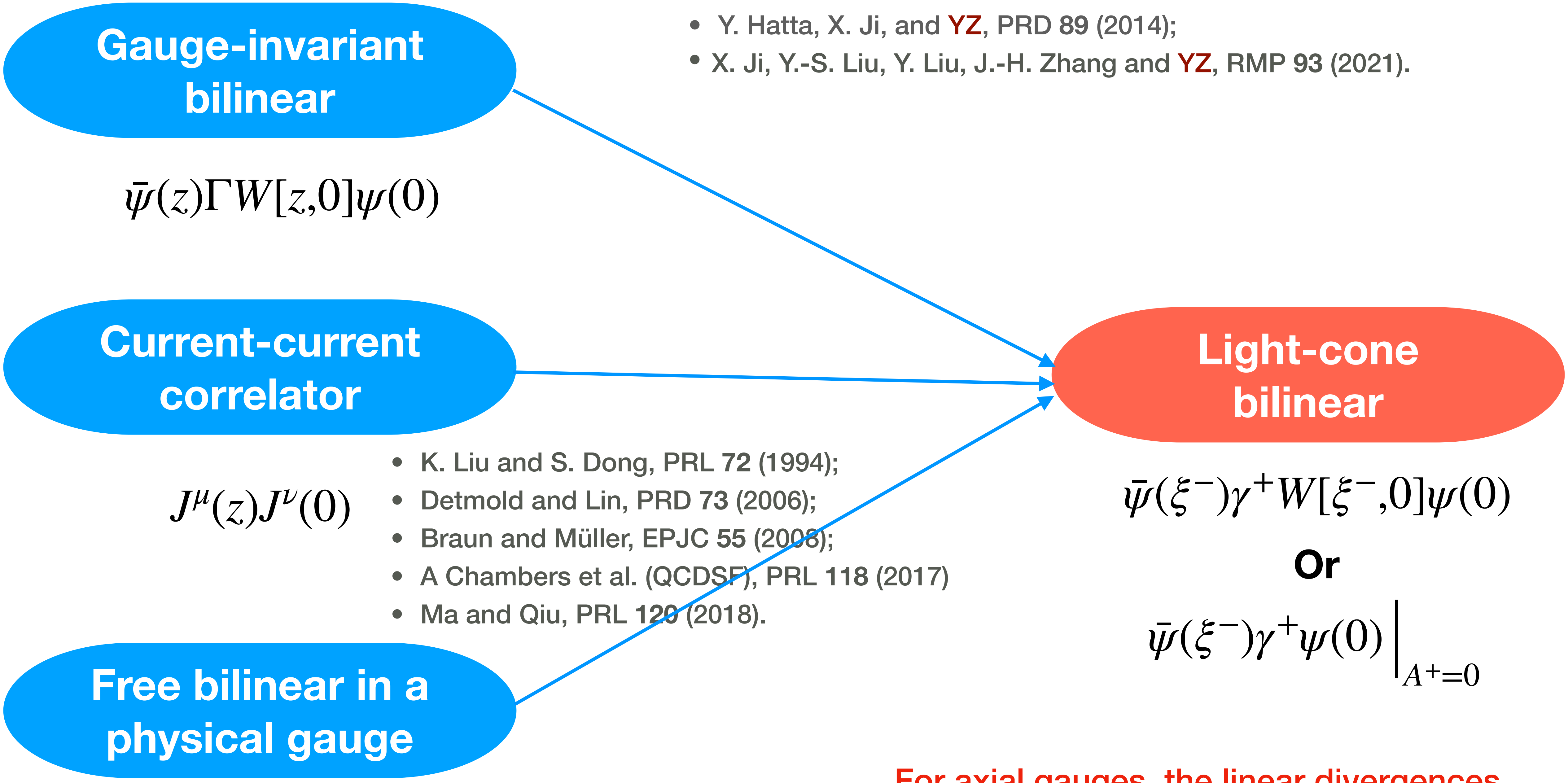
Universality in LaMET



- Y. Hatta, X. Ji, and YZ, PRD **89** (2014);
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP **93** (2021).

Gauge-invariant bilinear

$$\bar{\psi}(z)\Gamma W[z,0]\psi(0)$$

Current-current correlator

$$J^{\mu}(z)J^{\nu}(0)$$

- K. Liu and S. Dong, PRL **72** (1994);
- Detmold and Lin, PRD **73** (2006);
- Braun and Müller, EPJC **55** (2008);
- A Chambers et al. (QCDSF), PRL **118** (2017)
- Ma and Qiu, PRL **120** (2018).

Free bilinear in a physical gauge

$$\bar{\psi}(z)\Gamma\psi(0)\Big|_{G(A)=0} \qquad G(A) = A^0, A^z, \nabla\cdot\mathbf{A}$$

Light-cone bilinear

$$\bar{\psi}(\xi^-)\gamma^+W[\xi^-,0]\psi(0)$$

Or

$$\bar{\psi}(\xi^-)\gamma^+\psi(0)\Big|_{A^+=0}$$

For axial gauges, the linear divergences still appear in the quark self-energy 🙁

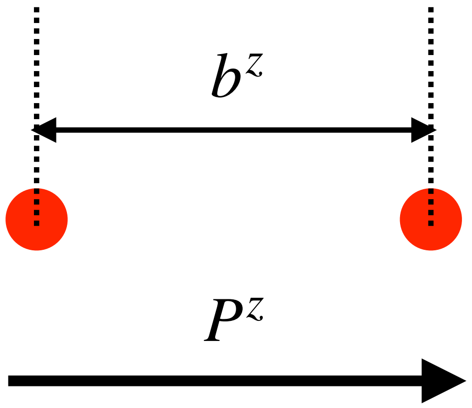
X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP **93** (2021).

Quasi-TMD in the Coulomb gauge

$$\tilde{h}(\vec{b}, \vec{P}, \mu) = \frac{1}{2P^t}\langle P | \bar{\psi}(\vec{b})\gamma^t\psi(0)\Big|_{\nabla\cdot\mathbf{A}=0} | P\rangle$$

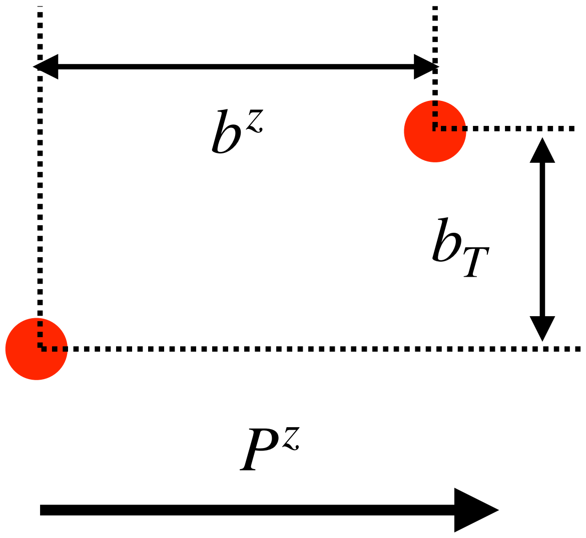
$$\tilde{f}(x, b_T, P^z, \mu) = P^z \int_{-\infty}^{\infty} \frac{db^z}{2\pi}\, e^{ixP^z b^z}\, \tilde{h}(b^z, b_T, P^z, \mu)$$

Quasi-PDF



X. Gao, W.-Y. Liu and YZ,
Phys.Rev.D 109 (2024)

Quasi-TMD



YZ, arXiv: 2311.01391.

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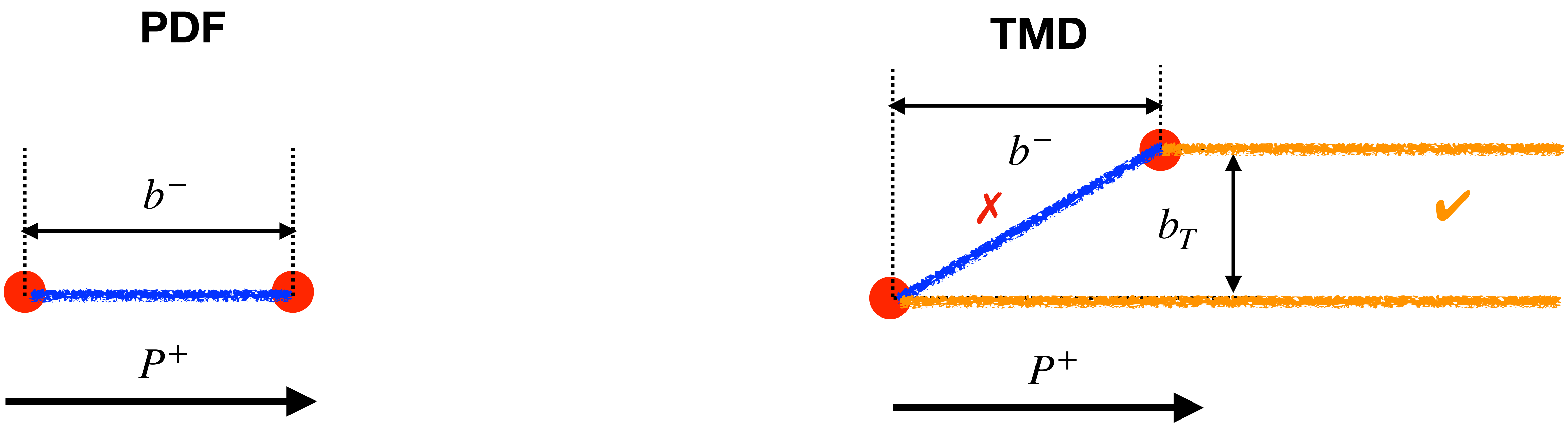

Parton distributions probe the correlation of energetic quarks and gluons dressed in the gauge background, which can be formulated by fixing a physical gauge condition.

$$G(A) = 0, \quad G(A) = A^0, A^z, \nabla\cdot\mathbf{A}, A^+$$

Quasi-TMD under the infinite boost

- Gauge-invariant extension:

$$\Psi_C(x) = U_C(x)\psi(x)$$

$$\vec{\nabla}\cdot\left[U_C\vec{A}U_C^{-1}+\frac{i}{g}U_C\vec{\nabla}U_C^{-1}\right]=0$$

Under arbitrary compact gauge transformation $U(x)$

$$\psi_C(x)\to U\psi_C(x),\quad U_C\to U_CU^{-1},\quad \Psi_C(x)\to\Psi_C(x)$$

Classical solution:

$$U_C=\sum_{n=0}^{\infty}\frac{(ig)^n}{n!}\omega_n$$

$$\omega_1=-\frac{1}{\nabla^2}\vec{\nabla}\cdot\vec{A},$$

$$\frac{\omega_2}{2!}=\frac{1}{\nabla^2}\left(\vec{\nabla}\cdot\left(\omega_1^{\dagger}\vec{\nabla}\omega_1\right)-[\vec{\nabla}\omega_1,\cdot\vec{A}]\right),$$

$$\cdots$$

$$U_C(A)\Big|_{\vec{\nabla}\cdot\vec{A}=0}=1$$

- Infinite boost limit along the *z* direction:

$$-\frac{1}{\nabla^2}\vec{\nabla}\cdot\vec{A}(x)=i\int\frac{d^4k}{(2\pi)^4}\,e^{-ik\cdot x}\frac{1}{k_z^2+k_\perp^2}\left[k^z\tilde{A}^z(k)+k_\perp\cdot\tilde{A}_\perp(k)\right]$$

$$\approx i\int\frac{d^4k}{(2\pi)^4}\,e^{-ik\cdot x}\frac{k^+}{(k^+)^2+\epsilon^2}\tilde{A}^+(k)$$

$$=\frac{1}{2}\left[\int_{-\infty^-}^{x^-}+\int_{+\infty^-}^{x^-}\right]d\eta^-\;A^+(x^+,\eta^-,x_\perp)\equiv\frac{1}{\partial_{\rm pv}^+}A^+(x)$$

Principle-value (P.V.) prescription:

$$\frac{k^+}{(k^+)^2+\epsilon^2}=\frac{1}{2}\left[\underbrace{\frac{1}{k^++i\epsilon}}_{\text{Past pointing}}+\underbrace{\frac{1}{k^+-i\epsilon}}_{\text{Future pointing}}\right]$$

Quasi-TMD under the infinite boost

- **Gauge-invariant extension:**

P. A. M. Dirac, Can. J. Phys. **33** (1955);
M. Lavelle and D. McMullan, Phys. Rept. **297** (1997).

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$$\omega_1 = -\frac{1}{\nabla^2}\vec{\nabla}\cdot\vec{A},$$

$$\frac{\omega_2}{2!} = \frac{1}{\nabla^2}\left(\vec{\nabla}\cdot\left(\omega_1^\dagger\vec{\nabla}\omega_1\right) - [\vec{\nabla}\omega_1, \cdot\vec{A}]\right) ,$$

$$\cdots$$

$$U_C(A)\Big|_{\vec{\nabla}\cdot\vec{A}=0} = 1$$

- **Infinite boost limit along the *z* direction:**

$$\frac{\omega_n}{n!} \to \frac{1}{\partial^+_{\rm pv}}\left(\ldots\left(\frac{1}{\partial^+_{\rm pv}}\left((\frac{1}{\partial^+_{\rm pv}}A^+)A^+\right)A^+\right)\ldots A^+\right)$$

Path-ordered integral for future/past pointing $1/\partial^+$

$$U_C \to \mathscr{P}\exp\left[-ig\int_{x^-}^{\mp\infty^-} dy^- A^+(y^-)\right] \equiv W_n^\dagger(x, \mp\infty^-)$$

Infinite "P.V. prescribed" light-like Wilson line

Factorization formula

- Power counting: $\lambda \ll 1, \quad P^+ \gg \Lambda_{\rm QCD}$

Collinear $(k^+, k^-, k_\perp) = (1, \lambda^2, \lambda)P^+,$
$(b^+, b^-, b_\perp) = (\lambda^{-2}, 1, \lambda^{-1})(P^+)^{-1}$

Soft $(k^+, k^-, k_\perp) = (\lambda, \lambda, \lambda)P^+,$
$(b^+, b^-, b_\perp) = (\lambda^{-1}, \lambda^{-1}, \lambda^{-1})(P^+)^{-1}$

Described by SCET$_{\rm II}$

- Bauer, Pirjol and Stewart, PRD **65** (2002), **66** (2002), **68** (2003)
- Beneke, Chapovsky, Diehl and Feldmann, NPB 643 (2002)
- Bauer et al., PRD **66** (2002)

$$\psi = \psi_n + \psi_s + \psi_{us} + \ldots$$

$$\{\psi_n, \psi_s, \psi_{us}\} \sim \{\lambda, \lambda^{3/2}, \lambda^3\}P^+$$

$$A^\mu = A_n^\mu + A_s^\mu + A_{us}^\mu + \ldots$$

$$A_n^\mu = (A_n^+, A_n^-, A_n^\perp) \sim (1, \lambda^2, \lambda)P^+$$

$$\{A_s^\mu, A_{us}^\mu\} \sim \{\lambda, \lambda^2\}P^+$$

Since $(1, \lambda^2, \lambda) + (\lambda^2, \lambda^2, \lambda^2) \sim (1, \lambda^2, \lambda)$, we do not need to separate the ultrasoft modes from the collinear modes here.

$$\Rightarrow U_C(A) = U_C(A_s)U_C(A_n) \equiv U_C^s U_C^n \approx U_C^s W_n^\dagger$$

$U_C^s(A_s)$ cannot be expanded because $-\frac{1}{\nabla^2}\vec{\nabla} \cdot \vec{A}_s(x) = i\int \frac{d^4k_s}{(2\pi)^4}\, e^{-ik_s\cdot x}\frac{\vec{k}_s \cdot \vec{\tilde{A}}_s(k_s)}{k_x^2 + k_y^2 + k_z^2}$

Factorization formula

- **QCD**
- **SCET$_{\text{II}}$**

$$\begin{tikzcd} \Psi_C(x) = U_C(x)\psi(x) \arrow[r, "\text{Collinear expansion}"] & e^{i\hat{\mathscr{P}}\cdot b}U_C^s(x)W_n^\dagger(x)\xi_n(x) + O(\lambda^2) \arrow[d, "\text{SCET gauge invariance}", "{\text{Soft gauge transformation} \quad U_C^s \to U_C^s V_s^{-1}, \quad S_n \to V_s S_n}"'] \\ & e^{i\hat{\mathscr{P}}\cdot b}U_C^s(x)S_n(x)W_n^\dagger(x)\xi_n(x) \end{tikzcd}$$

Bauer, Pirjol and Stewart, PRD **65** (2002)

Matching QCD dressed quark field to SCET

$$U_C\psi(b) = e^{i\hat{\mathscr{P}}\cdot b}C(\hat{\mathscr{P}}^+/\mu)\left[U_C^s S_n W_n^\dagger \xi_n\right](b) + O(\lambda^2)$$

$C(\hat{\mathscr{P}}^+/\mu)$

Matching coefficient

Matching quasi-TMD correlator to SCET

$$\langle P|\Psi_C^\dagger(b)\gamma^t\Psi_C(0)|P\rangle = \frac{1}{\sqrt{2}}e^{i\hat{\mathscr{P}}\cdot b}\left\langle P\left|\left[\bar{\xi}_n W_n S_n^\dagger (U_C^s)^\dagger\right](b)\ C(\hat{\mathscr{P}}^+/\mu)^\dagger\ \gamma^+ C(\hat{\mathscr{P}}^+/\mu)\left[U_C^s S_n W_n^\dagger \xi_n\right](0)\right|P\right\rangle$$

Factorization formula

$$\langle P|\Psi_C^\dagger(b)\gamma^t\Psi_C(0)|P\rangle = \frac{1}{\sqrt{2}}e^{i\hat{\mathscr{P}}\cdot b}\left\langle P\left|\left[\bar{\xi}_n W_n S_n^\dagger (U_C^s)^\dagger\right](b)\ C(\hat{\mathscr{P}}^+/\mu)^\dagger\ \gamma^+ C(\hat{\mathscr{P}}^+/\mu)\left[U_C^s S_n W_n^\dagger \xi_n\right](0)\right|P\right\rangle$$

Equal time separation: $b^\mu = (0, b_\perp, b^z)$

- Mode separation: soft modes decouple from the collinear modes and can be treated as the background field;
- Multipole expansion: $b^z \sim 1/(xP^z) \sim O(1)/P^z, \quad b_\perp \sim O(\lambda^{-1})/P^z$

$$U_C^s(b) = U_C^s(b_\perp) + b^z\partial_z U_C^s(b_\perp) + \ldots \approx U_C^s(b_\perp) + O(\lambda)$$

$$\langle P|\Psi_C^\dagger(b)\gamma^t\Psi_C(0)|P\rangle = \frac{1}{\sqrt{2}}e^{i\hat{\mathscr{P}}\cdot b}\left\langle P\left|\left[\bar{\xi}_n W_n\right](b) C\left(\hat{\mathscr{P}}^+/\mu\right)^\dagger \gamma^+ C\left(\hat{\mathscr{P}}^+/\mu\right)\left[W_n^\dagger \xi_n\right](0)\right|P\right\rangle$$

Beam

$$\times \frac{1}{N_c}\langle 0|T\left[S_n^\dagger(b_\perp)(U_C^s)^\dagger(b_\perp)U_C^s(0)S_n(0)\right]|0\rangle$$

Soft S_C^0

Operator definition obtained for the first time!

$$\Big\downarrow \frac{P^z}{2P^t}\int\frac{db^z}{2\pi}e^{ixP^zb^z}$$

$$\tilde{B}(x, b_\perp, \mu, P^z) = |C(xP^+/\mu)|^2 B(x, b_\perp, \ldots, xP^+)S_C^0(b_\perp, \ldots) + O(\lambda^2)$$

Factorization formula

$$\tilde{B}(x, b_\perp, \mu, P^z) = |C(xP^+/\mu)|^2 B(x, b_\perp, \ldots, xP^+) S_C^0(b_\perp, \ldots) + O(\lambda^2)$$

"…": UV and rapidity regulators

Physical (or subtracted) TMD PDF:

$$f(x, b_\perp, \mu, \zeta) = B(x, b_\perp, \ldots, xP^+) S(b_\perp, \ldots, y_n)$$

Collins-Soper scale

$$\zeta = 2(xP^+)^2 e^{-2y_n}$$

$$\downarrow$$

$$\frac{\tilde{B}(x, b_\perp, \mu, P^z)}{\tilde{S}_C(b_\perp, \mu, y_n)} = |C(xP^+/\mu)|^2 f(x, b_\perp, \mu, \zeta) + O(\lambda^2)$$

$$\tilde{S}_C(b_\perp, \mu, y_n) \equiv \frac{S_C^0(b_\perp, \ldots)}{S(b_\perp, \ldots, y_n)}$$

Quasi soft factor

Or

$$\frac{\tilde{B}(x, b_\perp, \mu, P^z)}{\tilde{S}_C(b_\perp, \mu, 0)} = |C(xP^+/\mu)|^2 \exp\left[\frac{1}{2}\gamma_\zeta(b_\perp, \mu) \ln \frac{2(xP^+)^2}{\zeta}\right] \times f(x, b_\perp, \mu, \zeta) + O(\lambda^2)$$

Verified at 1-loop!

Quasi soft factor

$$\tilde{S}_C(b_\perp, \mu, y_n) \equiv \frac{S_C^0(b_\perp, \ldots)}{S(b_\perp, \ldots, y_n)}$$

$$S_C^0 = \frac{1}{N_c}\langle 0 \,|\, T\left[S_n^\dagger(b_\perp)(U_C^s)^\dagger(b_\perp)U_C^s(0)S_n(0)\right] |\, 0\rangle$$

Both S_C^0 and S involve real-time dependence, so they are not directly calculable on the lattice.

Light-meson form factor:

$$F(b_\perp, P^z) = \langle \pi(-P) \,|\, j_1(b_\perp) j_2(0) \,|\, \pi(P)\rangle$$

$$= \int dx_1 dx_2 \; H_F(x_1, x_2, P^z, \mu)$$

Hard kernel: known at 1-loop

$$\times \frac{\phi(x_1, b_\perp, \mu, P^z)}{\tilde{S}_C(b_\perp, \mu, 0)} \frac{\phi(x_2, b_\perp, \mu, P^z)]}{\tilde{S}_C(b_\perp, \mu, 0)}$$

$\phi(x, b_T, \mu, P^z)$: **Coulomb gauge quasi-TMD wave function** ✔

$\phi^* = \phi$ due to the P.V. prescription

Transverse link and *T*-odd TMDs

- *T*-odd light-cone TMD:

P.V. prescription = anti-symmetric boundary condition $A_\perp^\mu(\infty^-) = -A_\perp^\mu(-\infty^-)$

Need a transverse link to define the *T*-odd TMDs

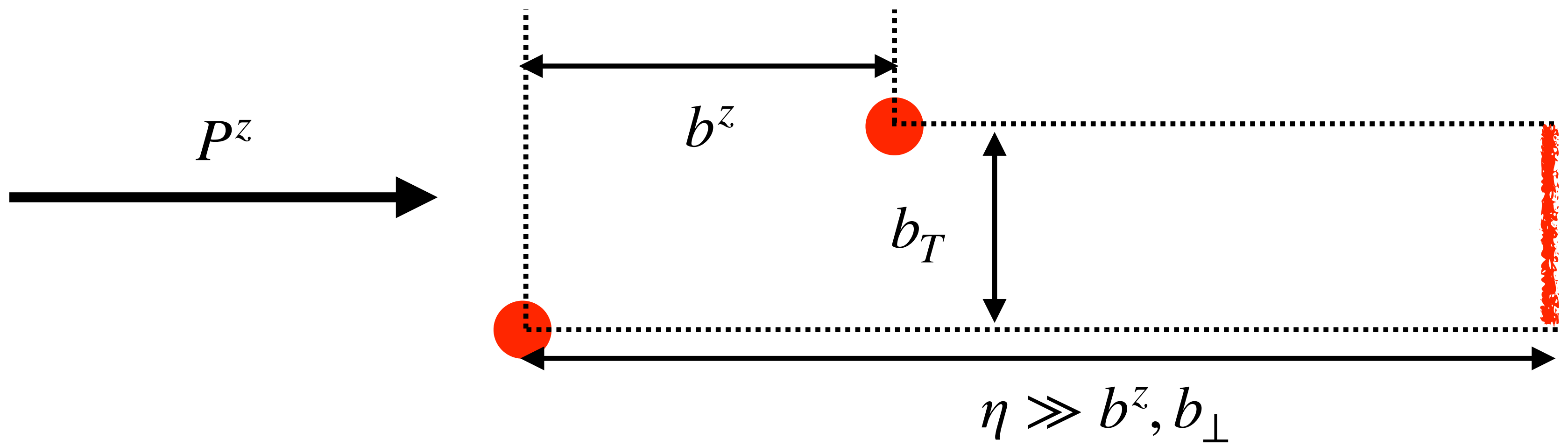
Contribution in a Feynman diagram: $\frac{e^{-i\infty^- k^+}}{[k^+]_{\rm pv}} = -i\pi\delta(k^+)$

- Ji and Yuan, PLB **543** (2002)
- Belitsky, Ji and Yuan, NPB **656** (2003)

- Coulomb-gauge quasi-TMD:

$$\tilde h(\vec b, \vec P, \mu, \pm) = \frac{1}{2P^t}\langle P|\bar\psi(\vec b)\mathscr{W}_\perp(\pm\infty\hat z; b_\perp, 0_\perp)\gamma^t\psi(0)\Big|_{\nabla\cdot\mathbf{A}=0}|P\rangle$$

Quasi-TMD with a transverse link



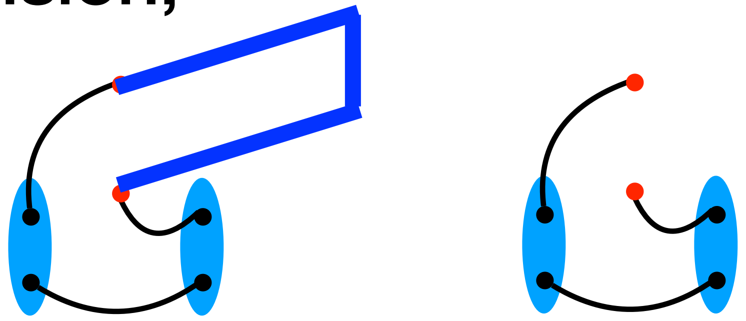
$$\frac{k^z e^{-i\infty^z k^z}}{k_z^2 + k_\perp^2} \xrightarrow{k^z \gg k_\perp} -i\pi\delta(k^z) \ ?$$

Outline

- Introduction
    - Overview of TMD physics
    - Large-Momentum Effective Theory
- Coulomb gauge method
    - Universality class in LaMET
    - Coulomb-gauge quasi-TMD
    - Large-momentum factorization formula
    - Soft function
    - Transverse gauge link
- **Discussions**

Advantages

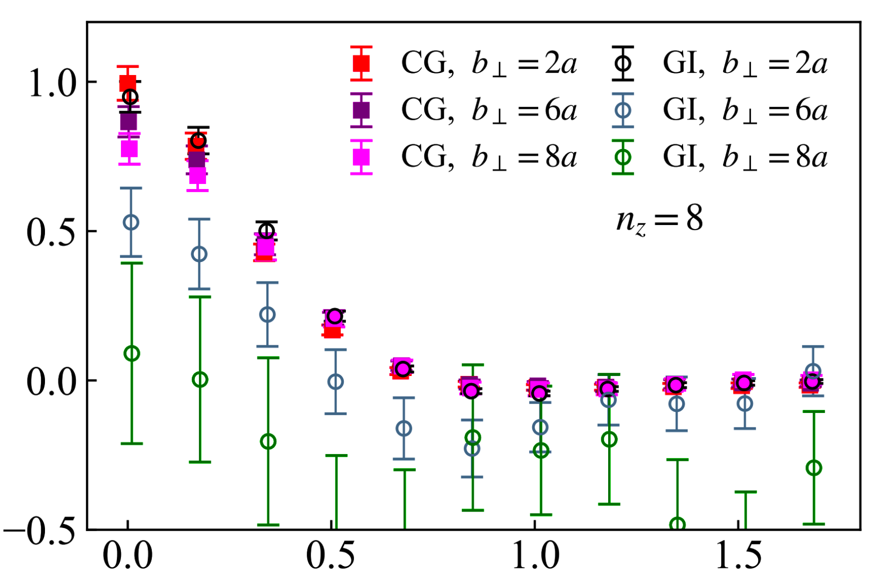
- Significantly improved statistical precision;



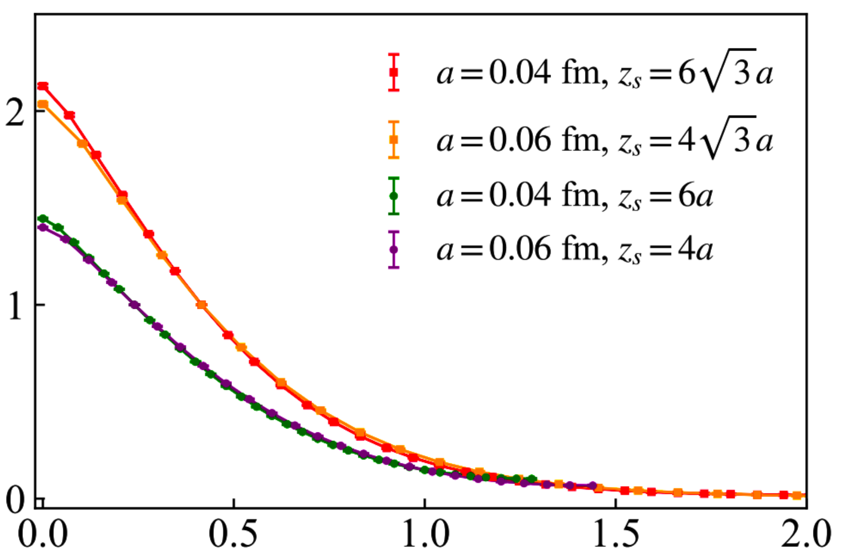
- Absence of linear power divergence and multiplicative renormalization;

- Access to larger off-axis momenta.

$$\vec{P} = (0,P^z,P^z), \qquad \vec{b} = (b_\perp, b^z, b^z)$$



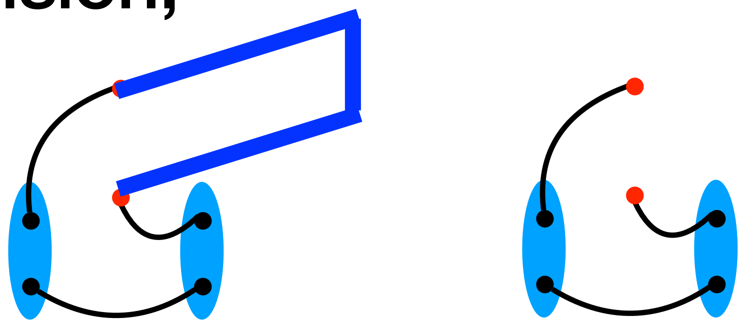
D. Bollweg, X. Gao, S. Mukherjee and YZ, Phys.Lett.B **852** (2024)



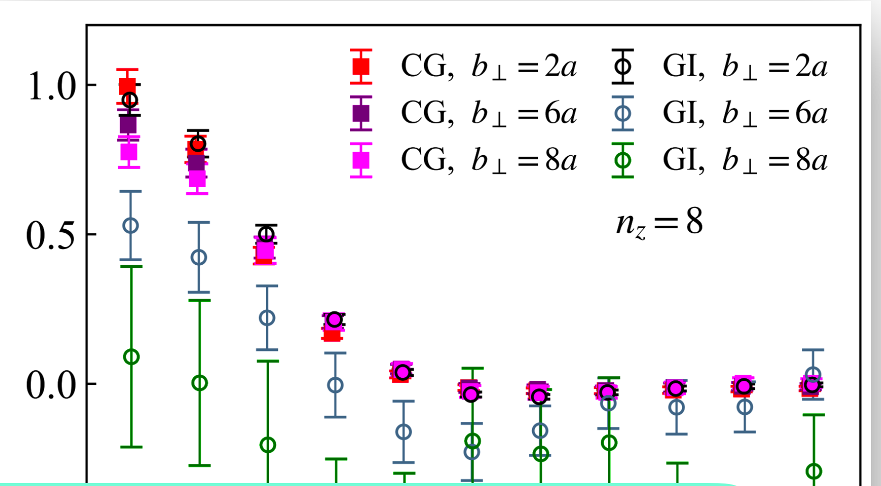
X. Gao, W.-Y. Liu and YZ, Phys.Rev.D 109 (2024)

Advantages

- Significantly improved statistical precision;



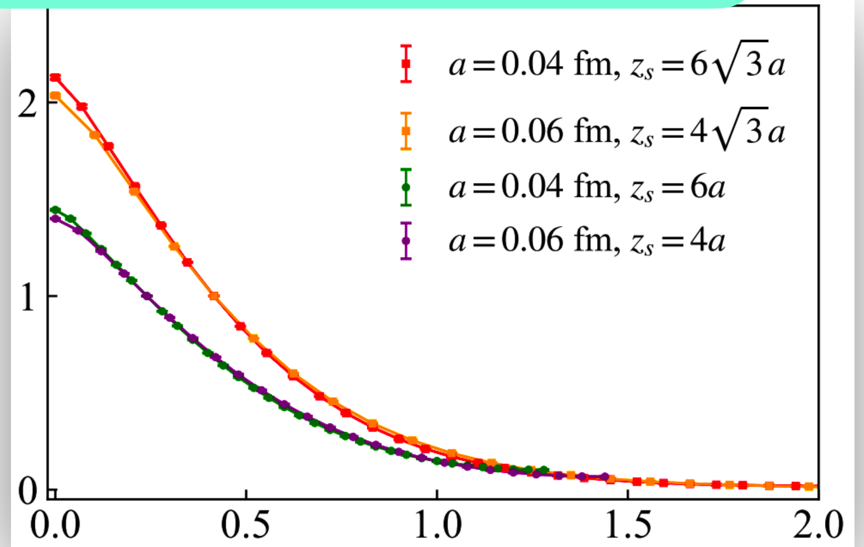
- Absence of linear power divergence a
multiplicative renormalization;



See Xiang Gao's talk.

YZ,

- Access to larger off-axis momenta.



$$\vec{P} = (0, P^z, P^z), \qquad \vec{b} = (b_\perp, b^z, b^z)$$

X. Gao, W.-Y. Liu and YZ, Phys.Rev.D 109 (2024)

Gauge fixing and Gribov copies

- Find the gauge transformation Ω of link variables $U_i(t, \vec{x})$ that minimizes:

$$F[U^\Omega] = \frac{1}{9V} \sum_{\vec{x}} \sum_{i=1,2,3} \left[-\textbf{re Tr } U_i^\Omega(t, \vec{x}) \right]$$

Precision ~ 10^{-7}

- Gribov copies correspond to different local extrema.
- Gauge-variant correlations may differ in different Gribov copies.
- However, the copies should contribute to the statistical noise if the algorithm randomly select the copies.

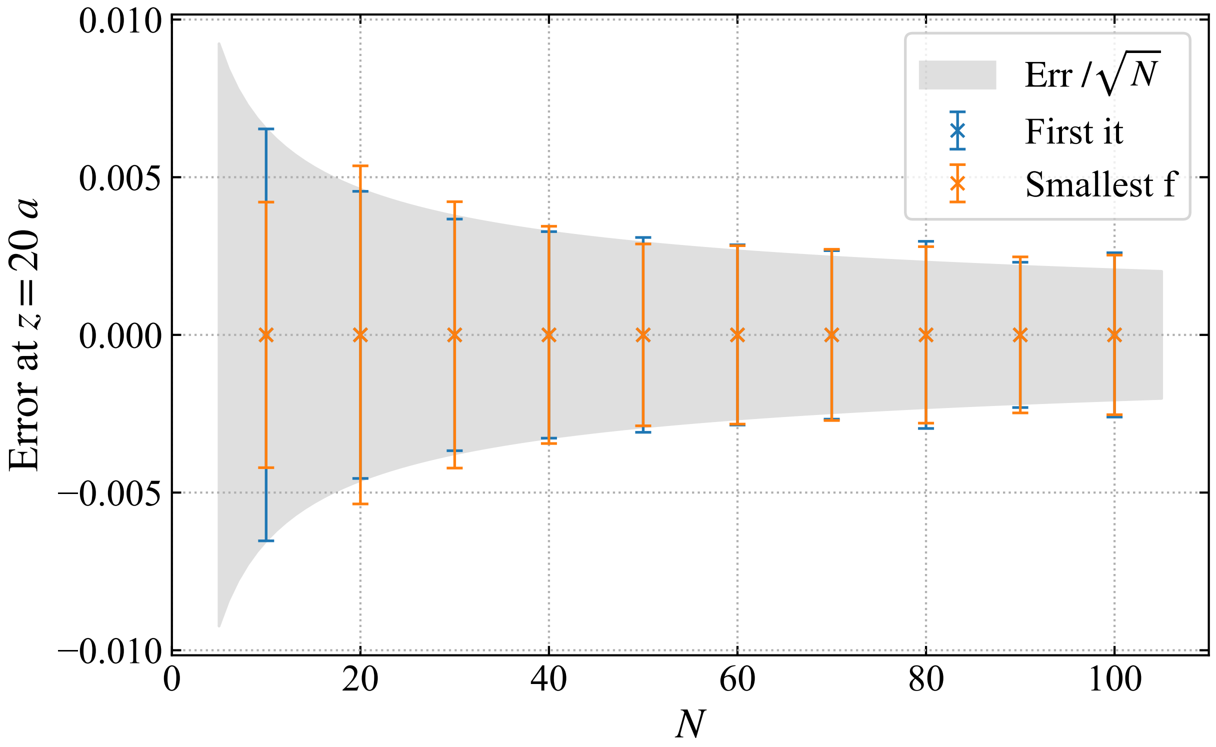
Test of the Gribov copies effect

For each configuration, obtain *n* daughter configurations that satisfy the criteria.

"First it": select the first daughter configuration.

"Smallest f": select the configuration with the lowest functional value.

Measure the pion quasi-PDF matrix element.



J. He et al., work in preparation.

Gluon TMDs

- Factorization formula can be derived using SCET as well.

- No linear power divergence.

- Mixing with gauge-variant operators. However, the number of mixings is finite under the constraint of SO(3) symmetry.

- Could be more susceptible to the Gribov noise, but it may still easily beat the statistical precision achieved with Wilson line operators.

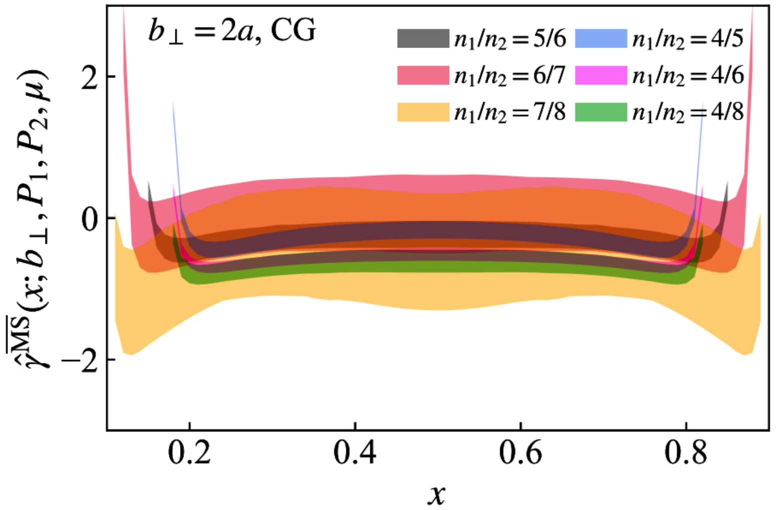
High-order corrections

- Known to be difficult due to non-covariant nature, but it is just one scale in massless integrals.

- No complete 2-loop result so far, even for the quark wave function renormalization.

- Linear renormalon in the matching coefficient.
  - Corresponds to a linear power correction of order $\Lambda_{\rm QCD}/P^z$;
  - However, strength of the renormalon is not easy to estimate. An NNLO calculation can offer a lot of insight.
  - Lattice calculation of the Collins-Soper kernel suggests that the result converges in P^z quite well.

Y. Liu and Y. Su, JHEP **2024** (2024)



D. Bollweg, X. Gao, S. Mukherjee and YZ, Phys.Lett.B **852** (2024)

Summary

- The Coulomb-gauge quasi-TMD can be factorized into the physical TMDs at large momentum;

- The factorization formula can be derived using SCET, which also leads to the operator definition of the quasi soft factor.

- It corresponds to the principle value prescription for the light-like Wilson lines, which can be used to calculate *T*-even TMDs;

- A transverse link needs to be added to access the *T*-odd TMDs;

- It can significantly reduce the statistical error, simplify the renormalization and access higher off-axis momenta, thus providing a more efficient way to calculate the TMDs.