Lattice QCD Calculation of the Pion Distribution Amplitude with Domain Wall Fermions at Physical Pion Mass

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From Quarks and Gluons to the Internal Dynamics of Hadrons,

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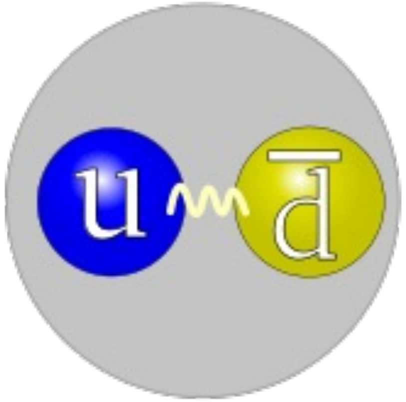
In collaboration with Ethan Baker, Dennis Bollweg, Peter Boyle, Ian Cloet, Xiang Gao, Swagato Mukherjee, Peter Petreczky, and Yong Zhao

Outline

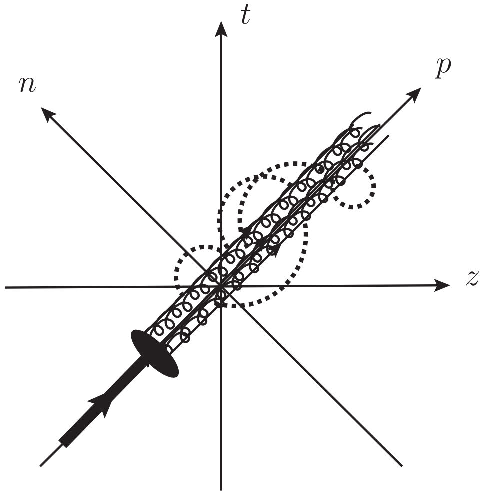
- Introduction to pion distribution amplitude
- Lattice calculation of pion quasi-DA
- Extracting x-dependence from lattice data
- Conclusion and Outlook

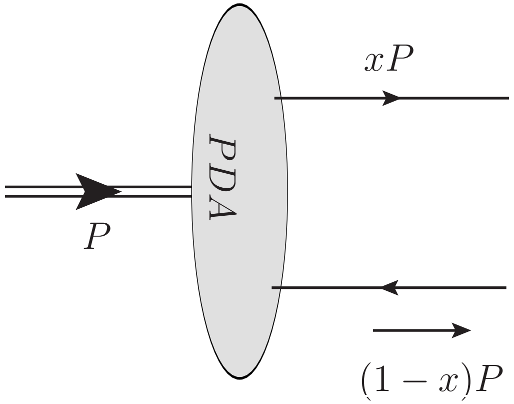
Pion Distribution Amplitude (DA)



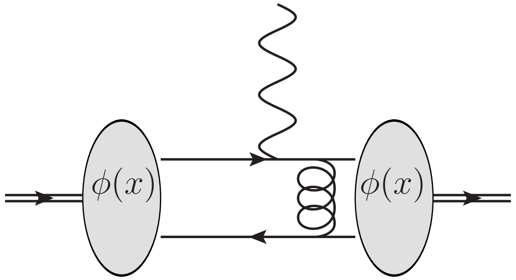
Pion lightfront DA $\phi(x)$: probability amplitude of pion in the bound state's minimal fock component $|q\bar{q}\rangle$ with collinear momentum fraction x and $1-x$

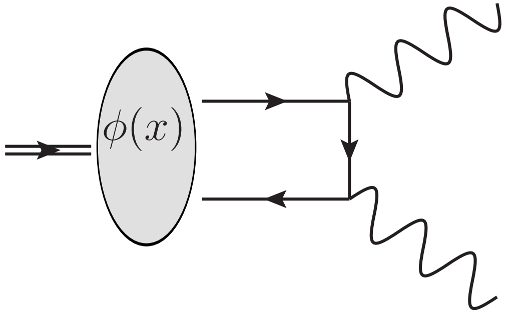
$$\phi(x,\mu) = \frac{1}{i f_\pi}\int \frac{d\xi^-}{2\pi} e^{i\left(\frac{1}{2}-x\right)\xi^- p^+} \langle 0|\bar{q}\left(\frac{\xi^-}{2}\right)\gamma^-\gamma_5 U\left(\frac{\xi^-}{2},-\frac{\xi^-}{2}\right) q\left(-\frac{\xi^-}{2}\right)|\pi(p)\rangle$$





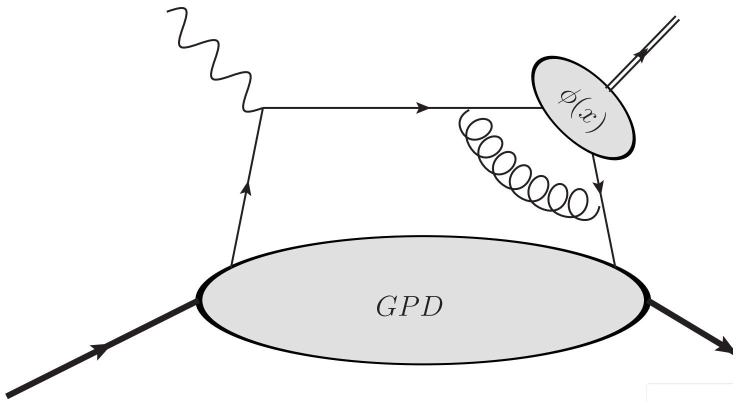
Phenomenology of pion DA

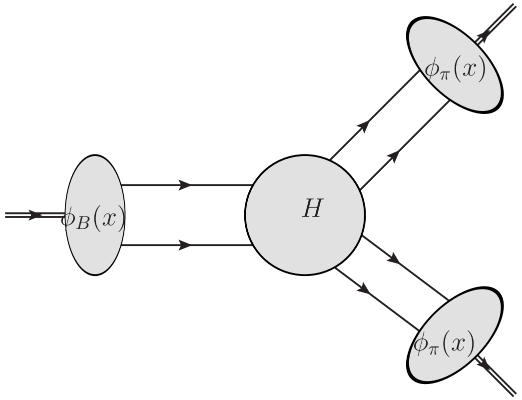


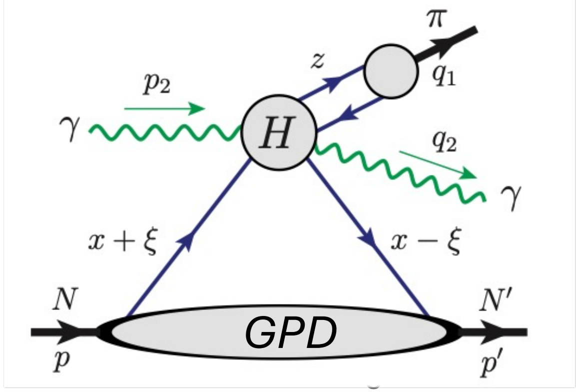


Universal inputs to various hard exclusive processes at large momentum transfer Q^2

- $\pi \to \gamma\gamma^*$ transition form factor
- Pion electromagnetic form factor
- Deeply virtual meson production Brodsky, et.al, PRD (1994)
- Heavy meson decay Beneke, et.al, PRL (1999)
- Exclusive Photoproduction Z.Yu & J.Qiu, PRL (2024)
- …







Weakly constrained by experiments! (See Zhite Yu's talk)

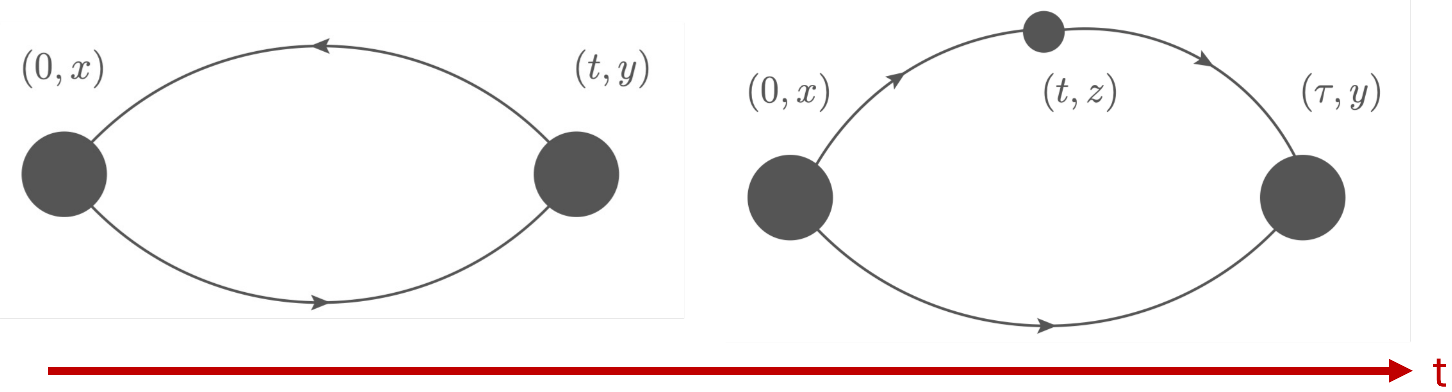
What about a direct calculation from first principle?

Lattice QCD

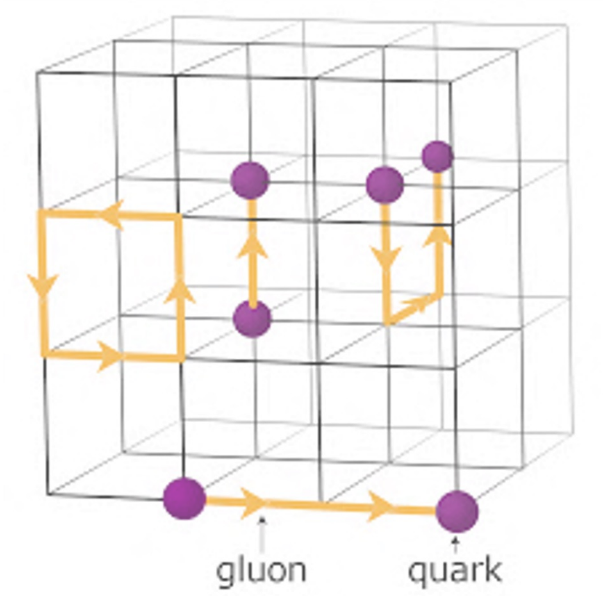
K.G. Wilson, Nobel Prize Winner (1982)

- Discretization of QCD action:
- Construction of correlators:

$C_2(t) = \langle \chi_{src}(0) | \chi_{snk}(t) \rangle$ $\quad$ $C_3(t) = \langle \chi_{src}(0) | O(t) | \chi_{snk}(\tau) \rangle$



Euclidean 4D spacetime



Savage, NNPSS (2015)

- Extraction of matrix elements:

$$C_2(t) = \sum |c_n|^2 e^{-E_n t}$$

$$C_3(t,\tau) = \sum c_m^* c_n \langle m | O | n \rangle e^{-E_m(\tau - t)} e^{-E_n t}$$

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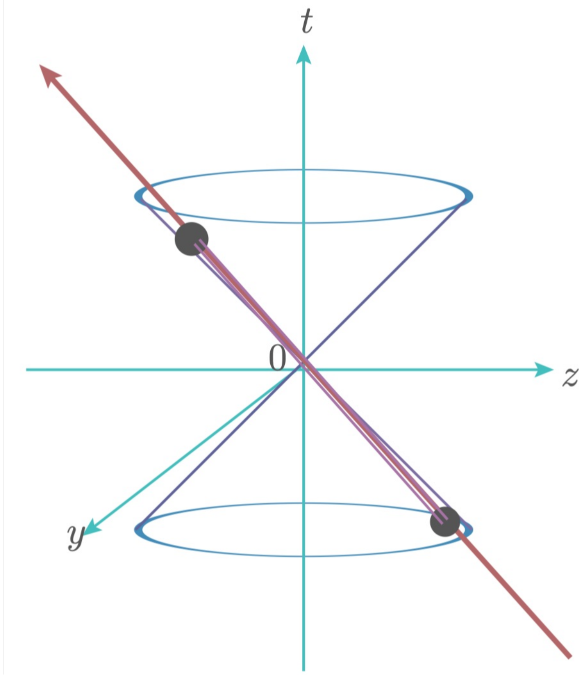
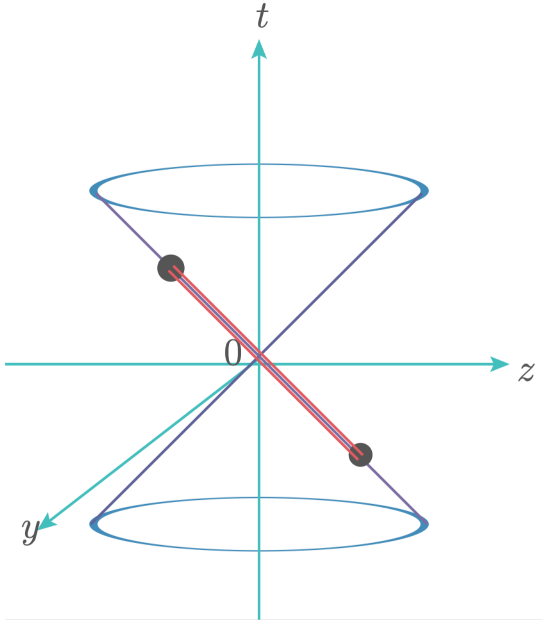
Large Momentum Effective Theory (LaMET)

Ji, PRL (2013)
Ji, SCPMA(2014)

Momentum distribution



Large P_z Expansion



$$+\mathcal{O}\left(\frac{1}{P_z^n}\right)$$

Quasi-DA: $\tilde{\phi}(x, P_z) =$

$$\int \frac{dzP_z}{2\pi} e^{i\left(\frac{1}{2}-x\right)zP_z} \langle 0|\bar{q}\left(-\frac{z}{2}\right)\gamma_t U(0,z) q\left(\frac{z}{2}\right)|\pi\rangle$$

$$C\,(x, y, \mu, P_z) \otimes \phi(y, \mu)$$

$$+\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{\bar{x}^2 P_z^2}\right)$$

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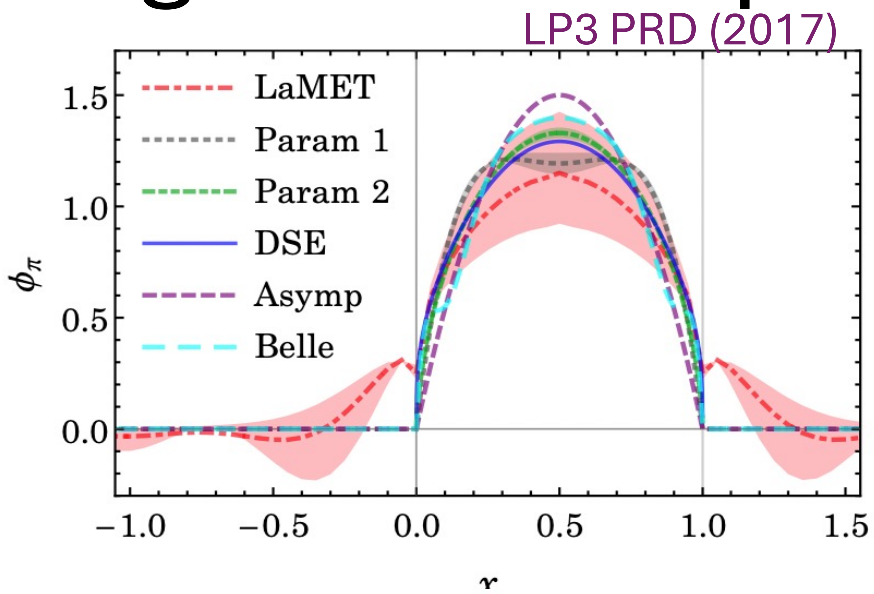
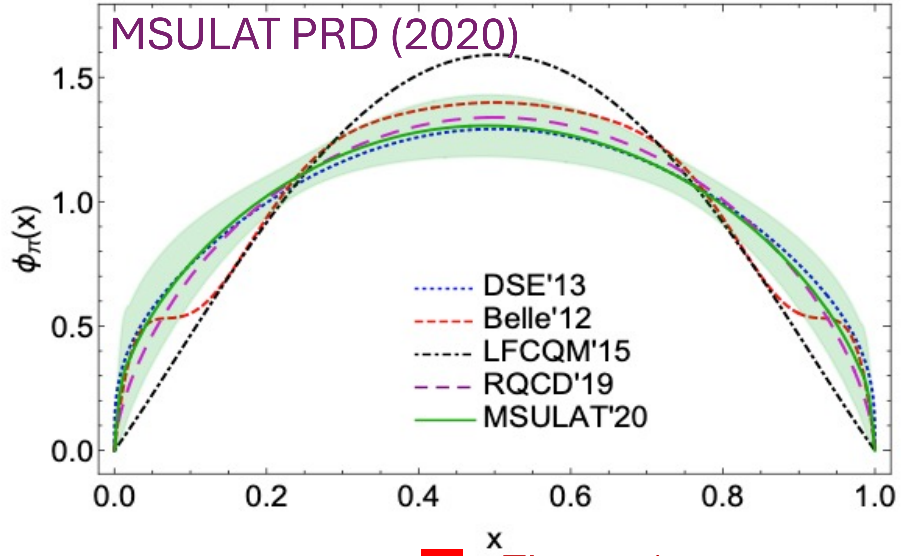
Progress in x-dependent DA calculations

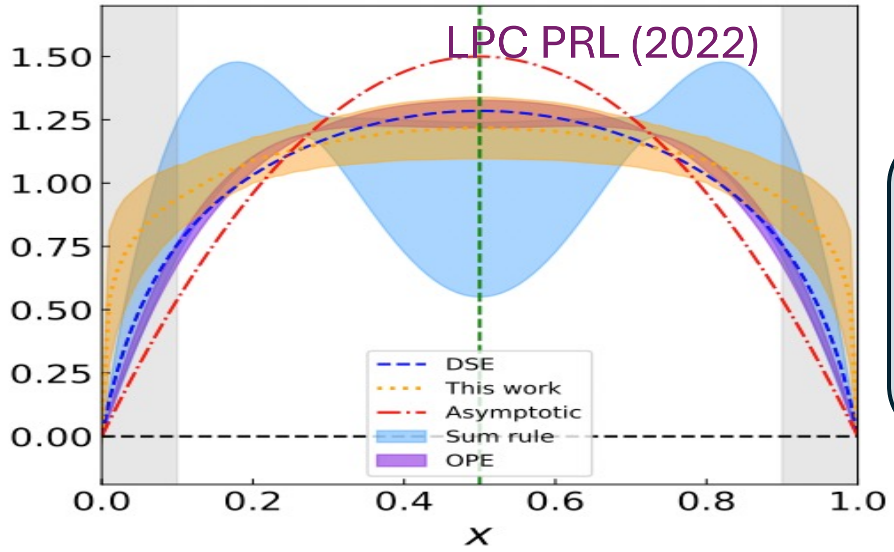


$a \to 0$

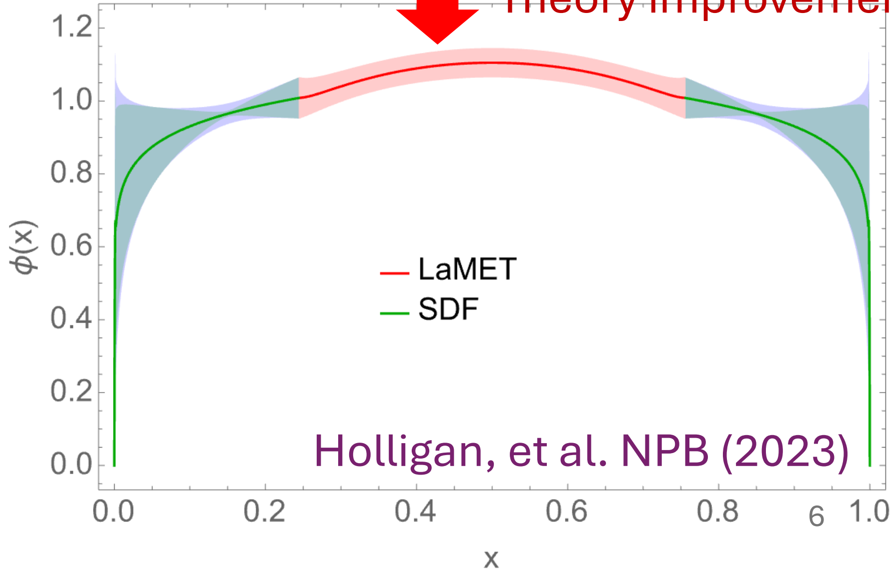


$m_\pi \to 130\ MeV$

Theory improvements



This work:
Chiral Symmetry
Large Logarithm



Lattice Setup

- Physical pion mass
- Chiral symmetric Fermion action – domain wall fermions
- Momentum smeared quark source

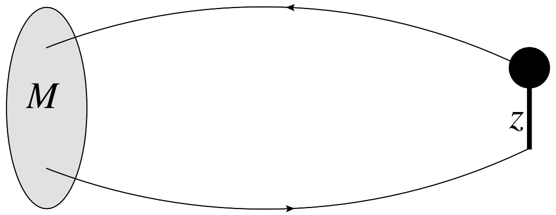
Lattice Spacing-a	Pion Mass	Lattice Volume	$m_\pi L$	Fermion Action
0.0836 fm	137 MeV	$64^3 \times 128 \times 12$	3.73	2+1f DW

Momentum Smearing	Pion Momentum	Samples	Sources	Effective Statistics
$k = \{0, 1.4\}$ GeV	$P_z = [0, 1.85]$ GeV	55	{32, 128}	Up to 28,160

Recipe

Lattice correlator

$C_{2pt} =$



Fitting matrix elements

$$C_{2pt} = i f_\pi |c_0|^2 \langle 0|O|0\rangle e^{-E_0\tau} + \cdots$$

Renormalization

$$H^R(z) = \langle 0|O|0\rangle / Z^R(z,a)$$

Extract x-dependence

$$\tilde{\phi}(x, P_z) = \int \frac{dz P_z}{2\pi} H^R(z) e^{i\left(\frac{1}{2}-x\right) z P_z}$$

Matching to lightcone

$$\phi(x,\mu) = C^{-1}(x, y, \mu, P_z) \otimes \tilde{\phi}(y, P_z)$$

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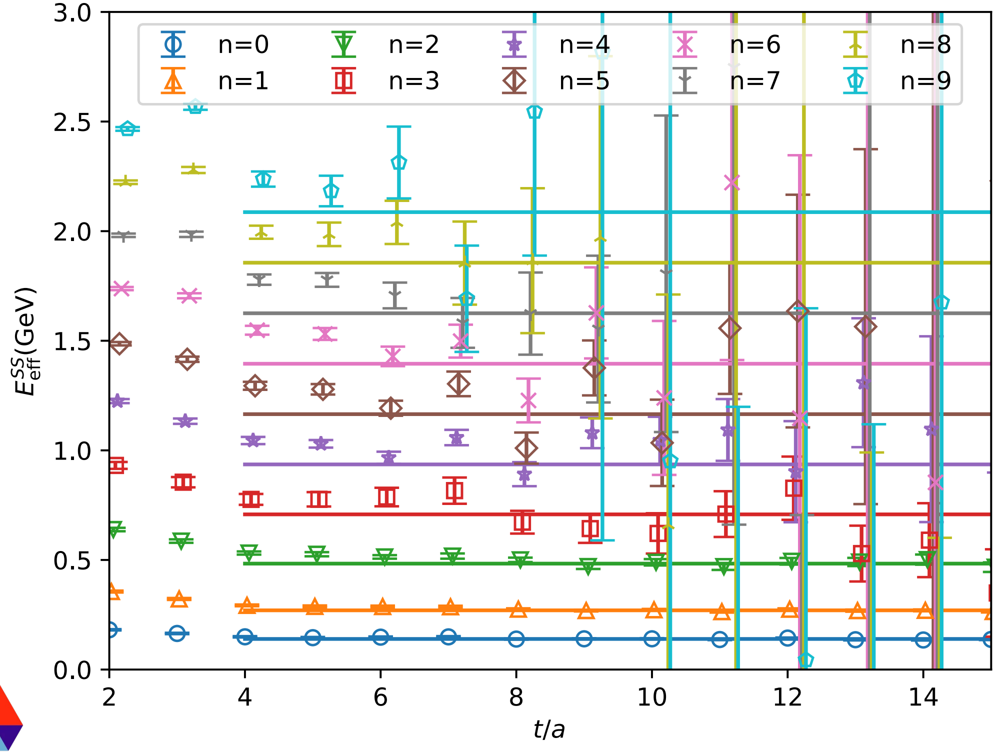
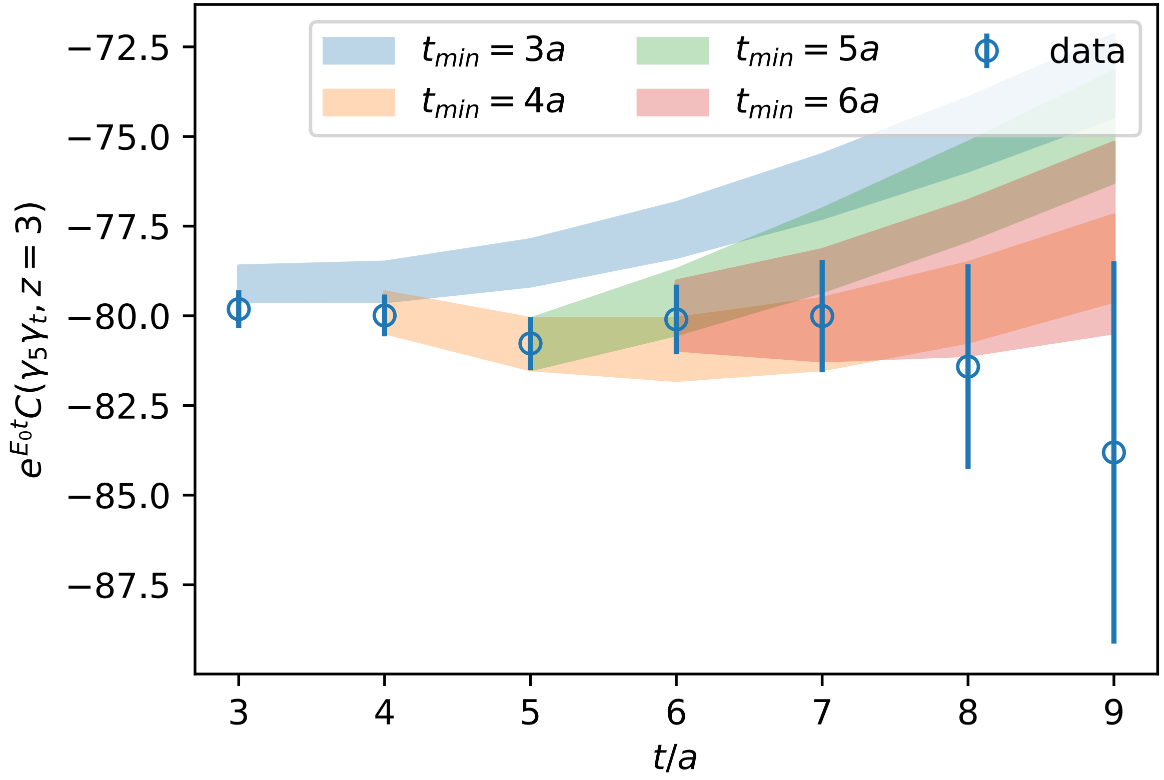
Lattice raw data and fitting

$$C_{\pi\pi}(t) = \langle O_\pi(0)|O_\pi(t)\rangle,$$
$$C_{\pi O_0}(t,z) = \langle O_\pi(0)|\bar{\psi}(-\frac{z}{2},t)\gamma_t\gamma_5 W(-\frac{z}{2},\frac{z}{2})\psi(\frac{z}{2},t)|\Omega\rangle,$$
$$C_{\pi O_3}(t,z) = \langle O_\pi(0)|\bar{\psi}(-\frac{z}{2},t)\gamma_z\gamma_5 W(-\frac{z}{2},\frac{z}{2})\psi(\frac{z}{2},t)|\Omega\rangle$$

$$C_{\pi\pi}(t) = \sum A_i^\pi(e^{-E_i t} + e^{-E_i(N_t-t)}),$$
$$C_{\pi O_0}(t,z) = \sum A_i^{O_0}(z)(e^{-E_i t} + e^{-E_i(N_t-t)}),$$
$$C_{\pi O_3}(t,z) = \sum A_i^{O_3}(z)(e^{-E_i t} + e^{-E_i(N_t-t)}),$$





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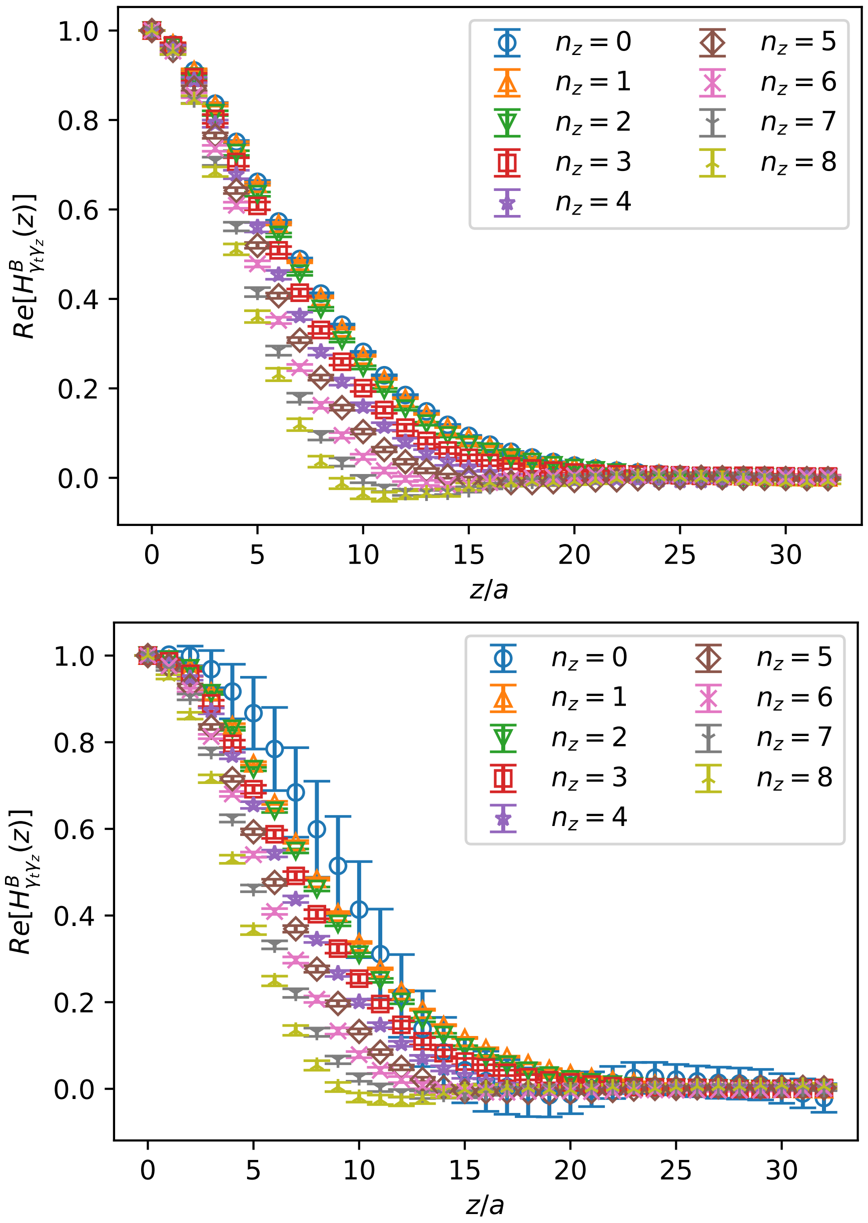
Bare matrix elements

$$A_0^\pi = \frac{|\langle O_\pi|\pi\rangle|^2}{2E_0},$$
$$A_0^{O_0}(z) = \frac{\langle O_\pi|\pi\rangle}{2E_0} f_\pi H_{\gamma_t\gamma_5}(z)E_0,$$
$$A_0^{O_3}(z) = \frac{\langle O_\pi|\pi\rangle}{2E_0} i f_\pi H_{\gamma_z\gamma_5}(z)P_z,$$

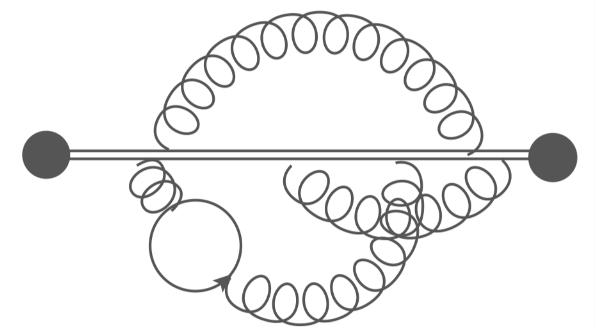
Pion DA is symmetric (vanishing imaginary part)

The lattice data decays exponentially with the Wilson link length.
The bare results contains both logarithmic and linear divergence in lattice spacing a



Renormalizing linear divergence

- Non-local operator: $\bar{q}(0)\Gamma U(0,z)q(z)$
- Linearly divergent self-energy $\delta m(a) \sim \frac{1}{a}$
  - $h^B(\mathrm{z}) \sim e^{-\delta m(a)\cdot z}$ Ji, et.al, PRL (2017)



- Renormalon ambiguity in $\Delta\big(\delta m(a)\big) \sim \Lambda_{QCD}$ Beneke, PLB (1995)
  - Renormalon also in the matching kernel Braun, et al., PRD (2018)
- $h^R(\mathrm{z}) \sim h^B(\mathrm{z}) e^{\delta m\cdot z}$ uncertain up to $e^{\mathcal{O}(\mathrm{z}\Lambda_{\mathrm{QCD}})} \to \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{\mathrm{x}\, P_z}\right)$ in $\tilde{q}$

How to remove linear ambiguity?

$$\ln\left(\frac{C_0(z,z^{-1})\exp(-I(z))}{h^B(z,0)}\right) = \delta m\,|z| + b$$

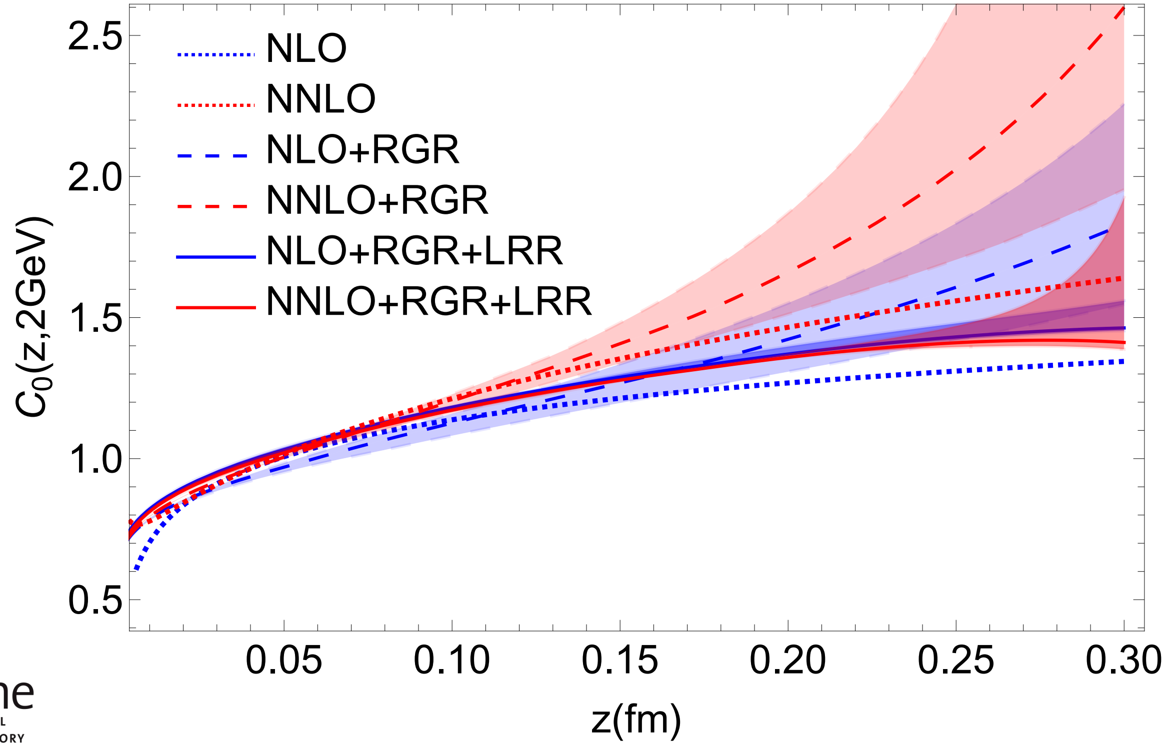
- Determine $\delta m(a)$ from matching $P=0$ lattice data to pQCD, with a consistently defined regularization of renormalon as the matching.
  - Leading renormalon resummation

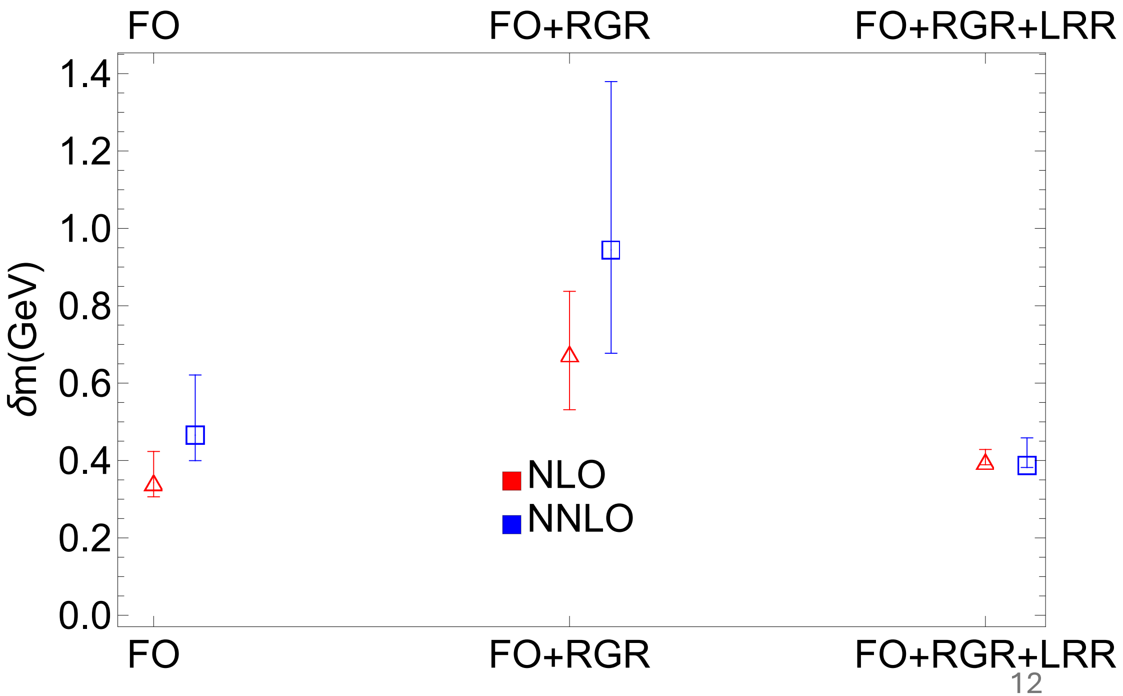
Zhang, et al., PLB (2023)

δm with leading renormalon resummation

Zhang, et al., PLB (2023)

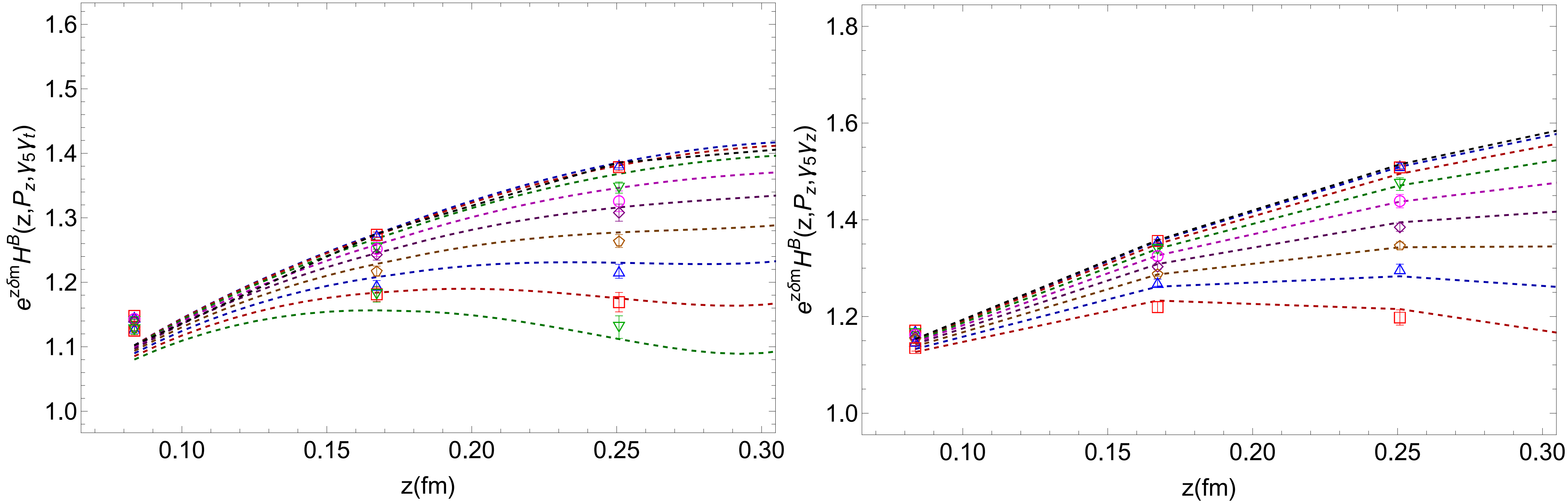
$$\ln\left(\frac{C_0(z,z^{-1})\exp(-I(z))}{h^B(z,0)}\right) = \delta m\,|z| + b$$





Consistency with OPE

$$H^R(z, P_z, \mu) = \sum_{m=0}^{\infty} \sum_{n=0}^{m} \frac{1}{m!} \left(\frac{i\, zP_z}{2} \right)^m C_{mn}(z, \mu) \langle \xi^n \rangle (\mu)$$

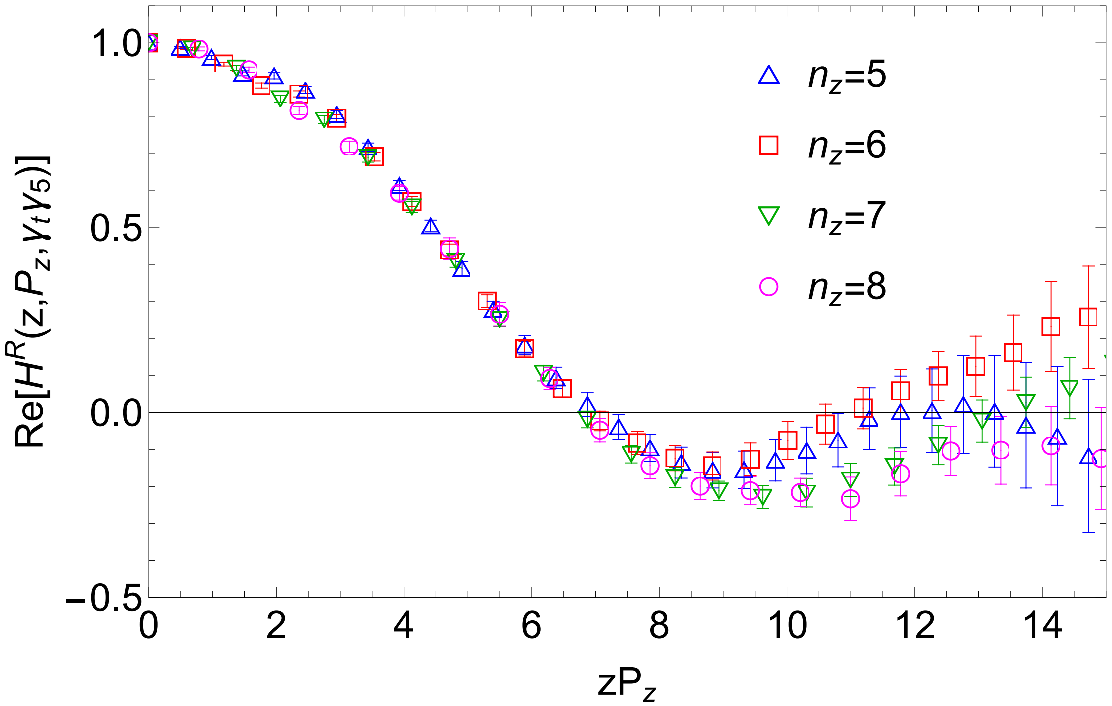


Renormalization in hybrid scheme

Ji, et al., NPB (2020)

$$h^R(z, P_z) = \frac{h^B(z, P_z)}{Z_h(z)}$$

$$Z_h(z) = \begin{cases} h^B(z, 0), & |z| < z_s \\ e^{\delta m|z-z_s|} h^B(z_s, 0), & |z| > z_s \end{cases}$$



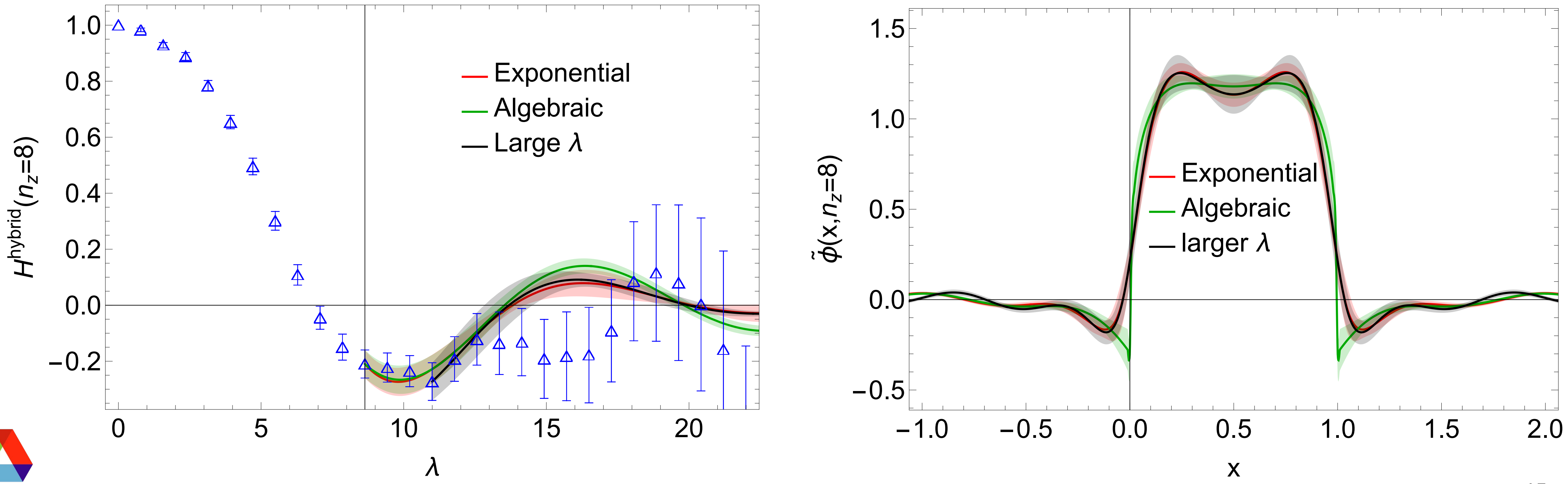
Longtail extrapolation $(\lambda = zP_z \to \infty)$

Ji, et al., NPB (2020)

Quasi-DA matrix elements have finite correlation length:

Inferred from Regge behavior

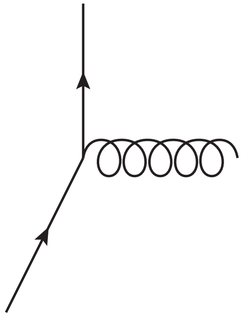
$$h^R(\lambda \to \infty) = e^{-\frac{\lambda}{\lambda_0}}\left(e^{-\frac{i\lambda}{2}}\frac{c_1}{(-i\lambda)^{d_1}} + e^{\frac{i\lambda}{2}}\frac{c_1}{(i\lambda)^{d_1}}\right)$$



Logarithms in the Matching Kernel

$$C^{\gamma_t\gamma_5}(x,y,\mu,P_z)=\delta(x-y)+\frac{\alpha_s(\mu)C_F}{2\pi}\left[\begin{cases}\frac{1+x-y}{y-x}\frac{\bar{x}}{\bar{y}}\ln\frac{(y-x)}{\bar{x}}+\frac{1+y-x}{y-x}\frac{x}{y}\ln\frac{(y-x)}{-x} & x<0\\ \frac{1+y-x}{y-x}\frac{x}{y}\ln\frac{4x(y-x)P_z^2}{\mu^2}+\frac{1+x-y}{y-x}\left(\frac{\bar{x}}{\bar{y}}\ln\frac{y-x}{\bar{x}}-\frac{x}{y}\right) & 0<x<y<1\\ \frac{1+x-y}{x-y}\frac{\bar{x}}{\bar{y}}\ln\frac{4\bar{x}(x-y)P_z^2}{\mu^2}+\frac{1+y-x}{x-y}\left(\frac{x}{y}\ln\frac{x-y}{x}-\frac{\bar{x}}{\bar{y}}\right) & 0<y<x<1\\ \frac{1+y-x}{x-y}\frac{x}{y}\ln\frac{(x-y)}{x}+\frac{1+x-y}{x-y}\frac{\bar{x}}{\bar{y}}\ln\frac{(x-y)}{-\bar{x}} & 1<x\end{cases}\right.$$

- Efremov-Radyushkin-Brodsky-Lepage logarithm
  - Physical scale of the system
    - Quark momentum logarithm $L=\ln x$
    - Anti-quark momentum logarithm $L=\ln\bar{x}$
- Threshold logarithm
  - Gluon momentum $L=\ln|x-y|$
- Only one RG equation (ERBL evolution): How to resum?



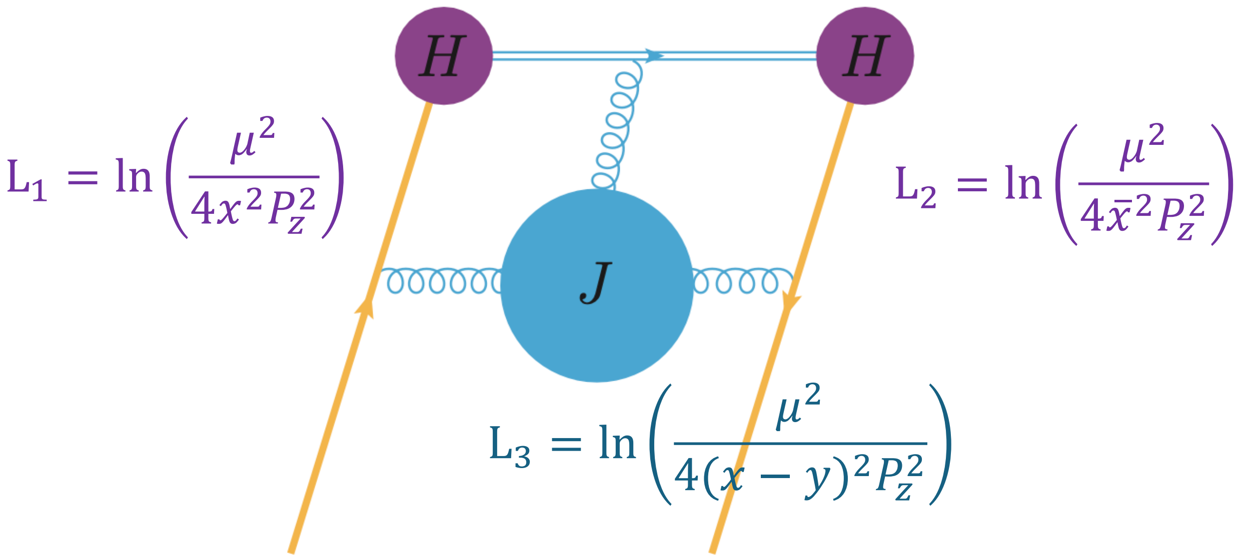
Both become important in the threshold limit $x\to y$

Factorizing Hard and “Soft” scales

- All three logsrithms are important only in the threshold limit
  - $x - y \to 0$, soft gluon emission
- Integrate out hard modes
  - Sudakov factor H
    - Quark component
    - Anti-quark component
- Integrate out hard collinear modes
  - Jet function J

Becher, Neubert & Pecjak JHEP(2007)

$$\mathcal{C}(x, y, \mu, P_z) \xrightarrow{x \to y} H(xP_z, \bar{x}P_z, \mu) \otimes J(|x-y|P_z, \mu)$$



Threshold log: soft gluon

Ji, Liu & Su JHEP (2023)

Separating all three scales

- $C(x \to y, \mu, P) \approx H(xP, \mu) H(\bar{x}P, \mu) J(|x-y|P, \mu)$
- $H\left(L_z^{\pm} = \ln\left(\frac{2xP}{\mu}\right)^2 + i\pi\,\mathrm{sgn}(zx), \mu\right) = 1 + \frac{C_F \alpha_s(\mu)}{4\pi}\left[-\frac{1}{2}(L_z^{\pm})^2 + L_z^{\pm} - 2 - \frac{5\pi^2}{12}\right]$
- $J\left(l_z = \ln\frac{z^2\mu^2 e^{2\gamma_E}}{4}, \mu\right) = 1 + \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{2}l_z^2 + l_z + \frac{\pi^2}{12} + 2\right)$
- Double logarithm come from soft and colinear divergences
- Cancellation of $\ln^2 \mu^2$ between H and J happens at all orders

Correcting the matching kernel

- Resummed Sudakov factor: $H = |H|e^{i\hat{A}}$

$$|H(\mu)| = |H(\mu_1,\mu_2)|e^{S(\mu_1,\mu)+S(\mu_2,\mu)-a_c(\mu_1,\mu)-a_c(\mu_2,\mu)} \times \left(\frac{2xP_z}{\mu_1}\right)^{-a_\Gamma(\mu_1,\mu)} \left(\frac{2\bar{x}P_z}{\mu_2}\right)^{-a_\Gamma(\mu_2,\mu)}$$

$$\hat{A}^{\rm RGR}(xP_z,\bar{x}P_z,\mu_1,\mu_2) = \pi \mathrm{sign}(z)\left[\frac{\alpha_s(\mu_1)C_F}{2\pi}\left(1-\ln\frac{4x^2P_z^2}{\mu_1^2}\right) - \frac{\alpha_s(\mu_2)C_F}{2\pi}\left(1-\ln\frac{4\bar{x}^2P_z^2}{\mu_2^2}\right) + 2\int_{\mu_1}^{\mu_2}\frac{\Gamma_{\rm cusp}}{\mu}d\mu\right]$$

- Resummed Jet function:

$$J(\Delta,\mu) = e^{[-2S(\mu_i,\mu)+a_J(\mu_i,\mu)]}\tilde{J}_z(l_z=-2\partial_\eta,\alpha_s(\mu_i))\left[\frac{\sin(\eta\pi/2)}{|\Delta|}\left(\frac{2|\Delta|}{\mu_i}\right)^\eta\right]_* \frac{\Gamma(1-\eta)e^{-\eta\gamma_E}}{\pi}\Bigg|_{\eta=2a_\Gamma(\mu_i,\mu)}$$

- $C_{TR} = (H \otimes J)_{TR} \otimes (H \otimes J)^{-1}_{NLO} \otimes C_{NLO}$
- Inverse matching:

$$C^{-1}_{TR} = C^{-1}_{NLO} \otimes (H \otimes J)_{NLO} \otimes (H \otimes J)^{-1}_{TR}$$

What are the scale choices of $\mu_{1,2}$ and μ_i?

Scale choices of resummation

- Hard scale:
  - $H(xP,\mu)$: quark momentum $\mu_{h_1} = 2xP$
  - $H(\bar{x}P,\mu)$: anti-quark momentum $\mu_{h_2} = 2\bar{x}P$
- Semi-hard scale:
  - $J(|y-x|P,\mu)$: gluon momentum $\mu_i = 2|y-x|P$?
  - This scale choice is not applicable because $\mu_i \to 0$ hits the Landau Pole for any given x!

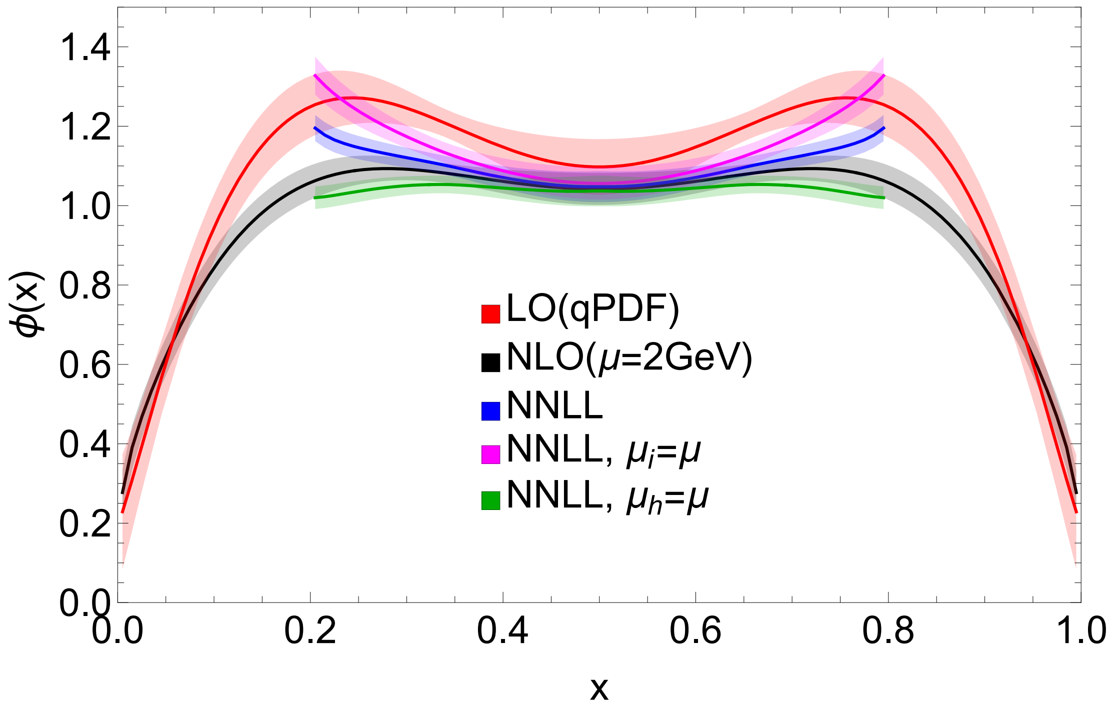
Becher, Neubert & Pecjak JHEP(2007)

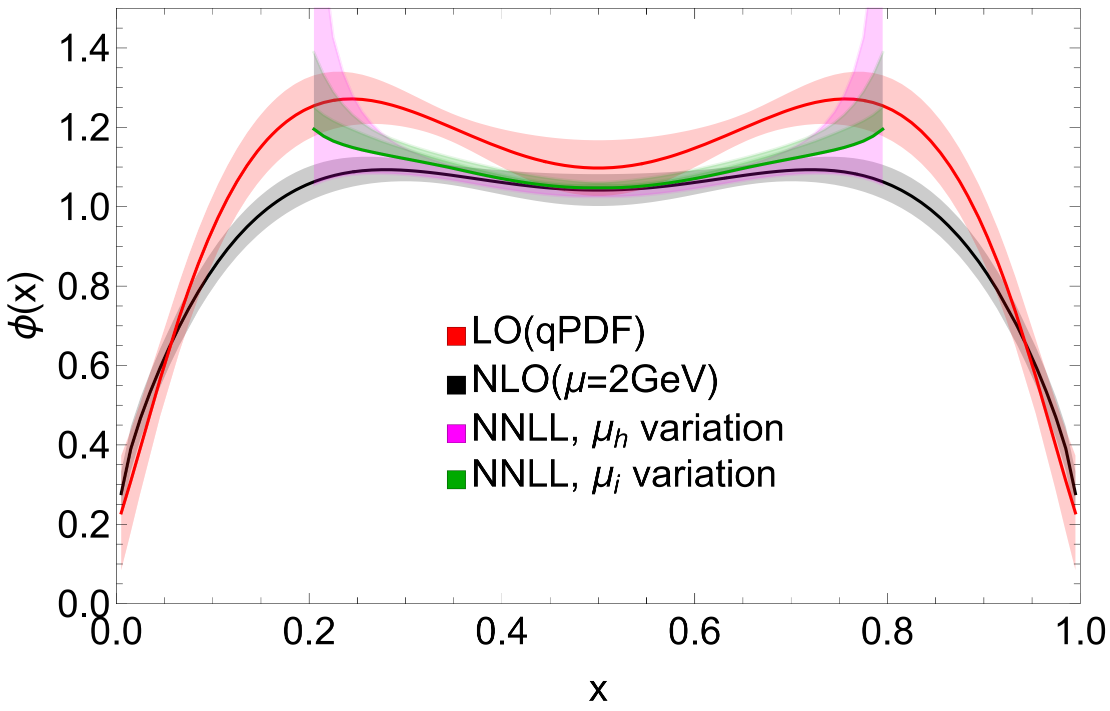
- Actual semi-hard scale choice turns out to be
  - $2xP$ when $x \to 0$
  - $2\bar{x}P$ when $x \to 1$
  - We choose $\mu_i = 2\min(x,\bar{x})\,P$

Matching with Resummed Kernel

Scale variation: $\mu_i \to c * \mu_i,\ c = [\frac{1}{\sqrt{2}}, \sqrt{2}]$

When scale variation becomes large, perturbation theory is no longer reliable

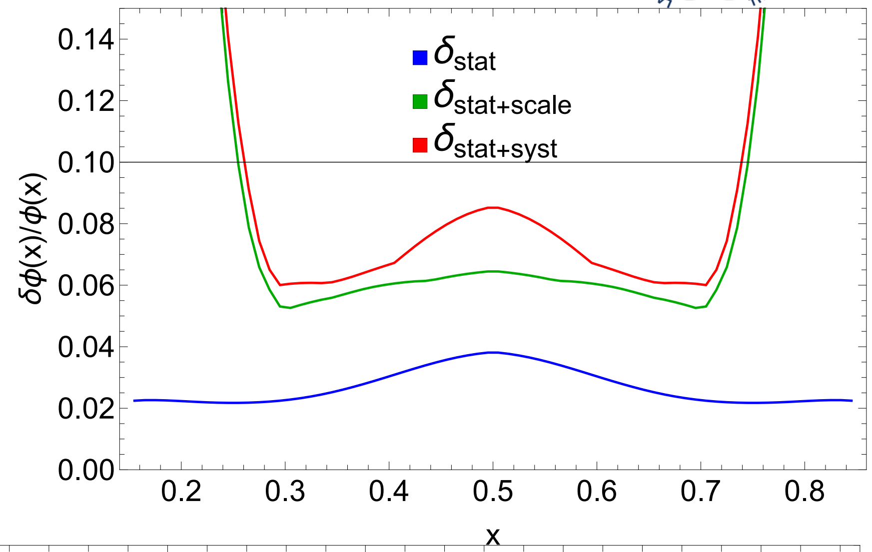


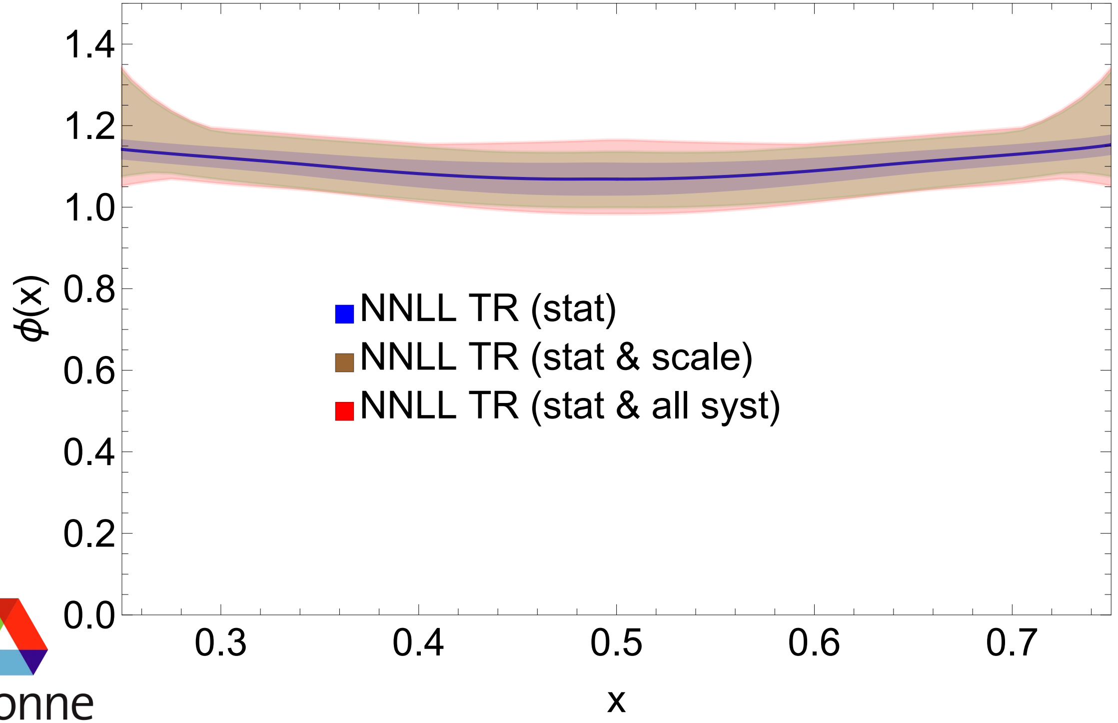


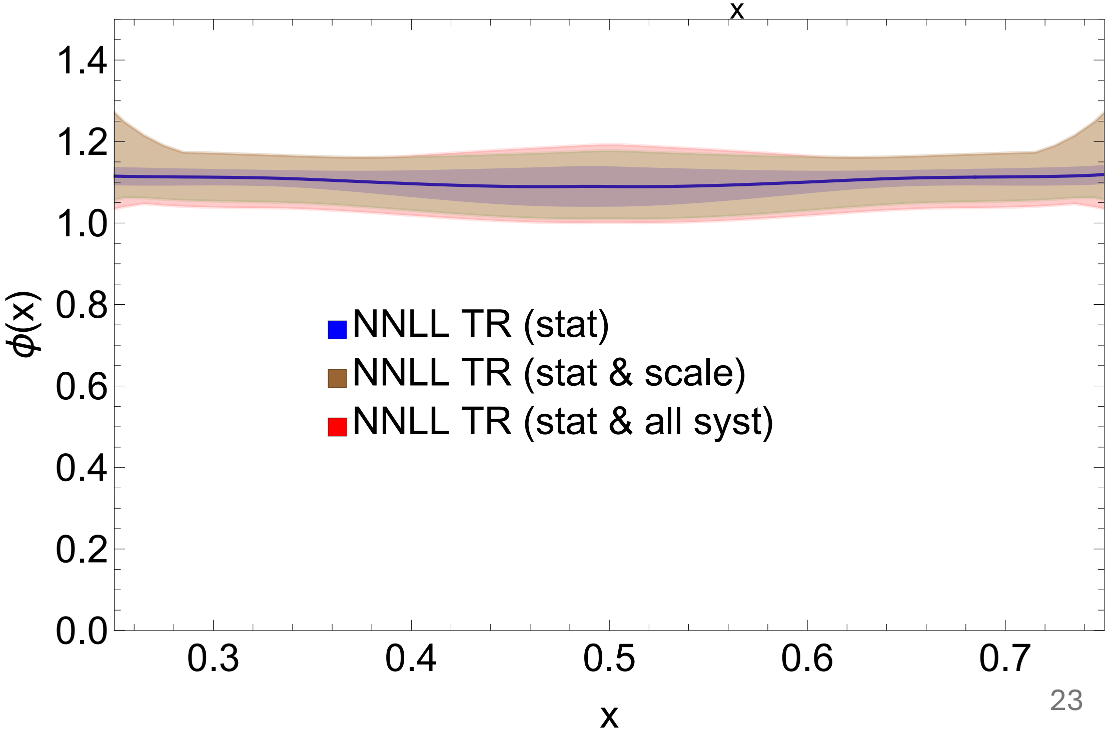
Including all systematics

- Different z_s
- Different extrapolations
- Scale Variation

$x \in [0.25, 0.75]$
$\delta_{total} \approx 10\%$

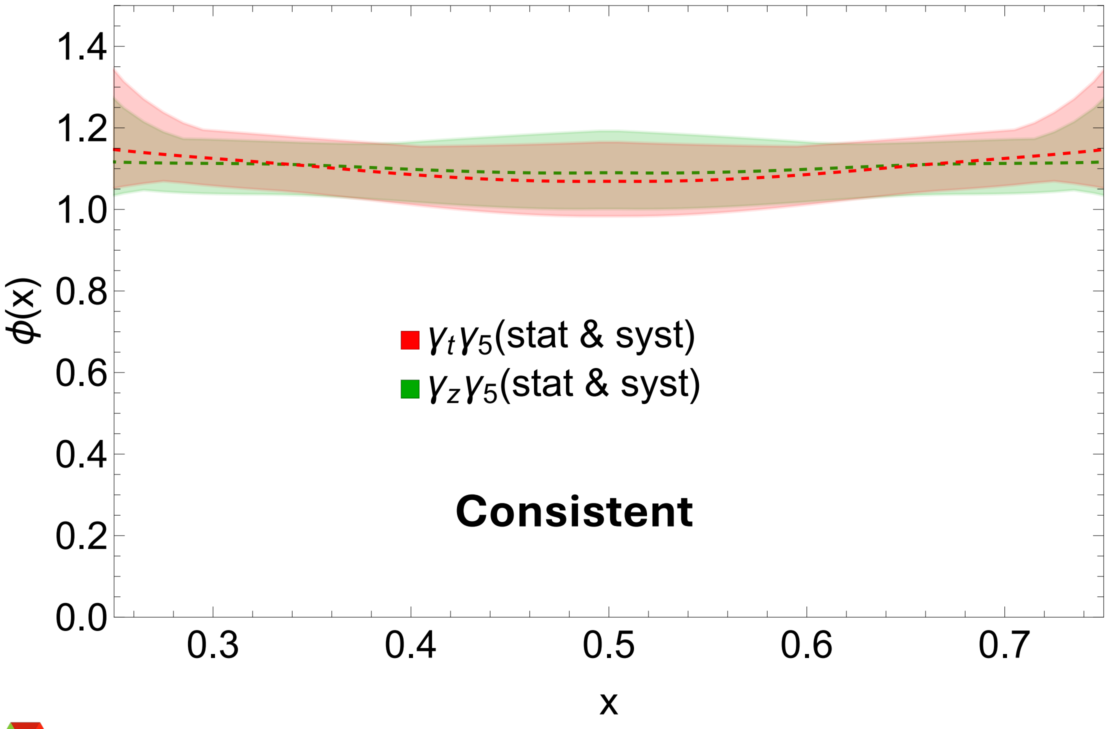




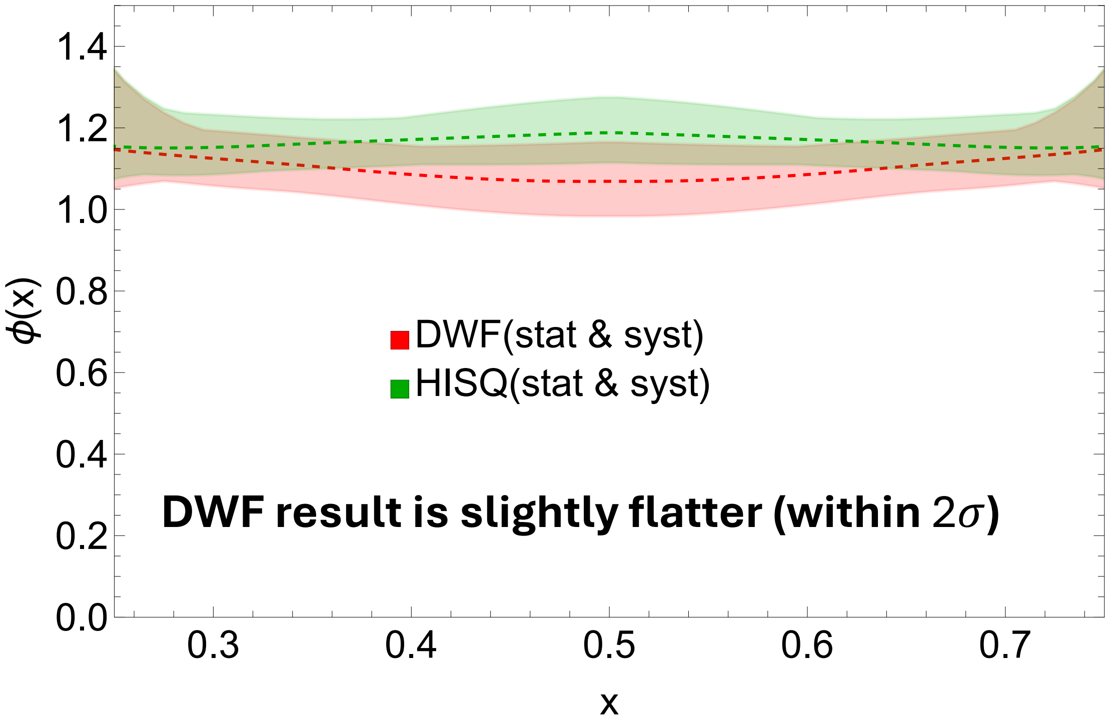


Comparison of Final Results

- Different operators



- Different fermion actions on similar lattice



Argonne NATIONAL LABORATORY

QUARK-GLUON TOMOGRAPHY COLLABORATION

Conclusion and Outlook

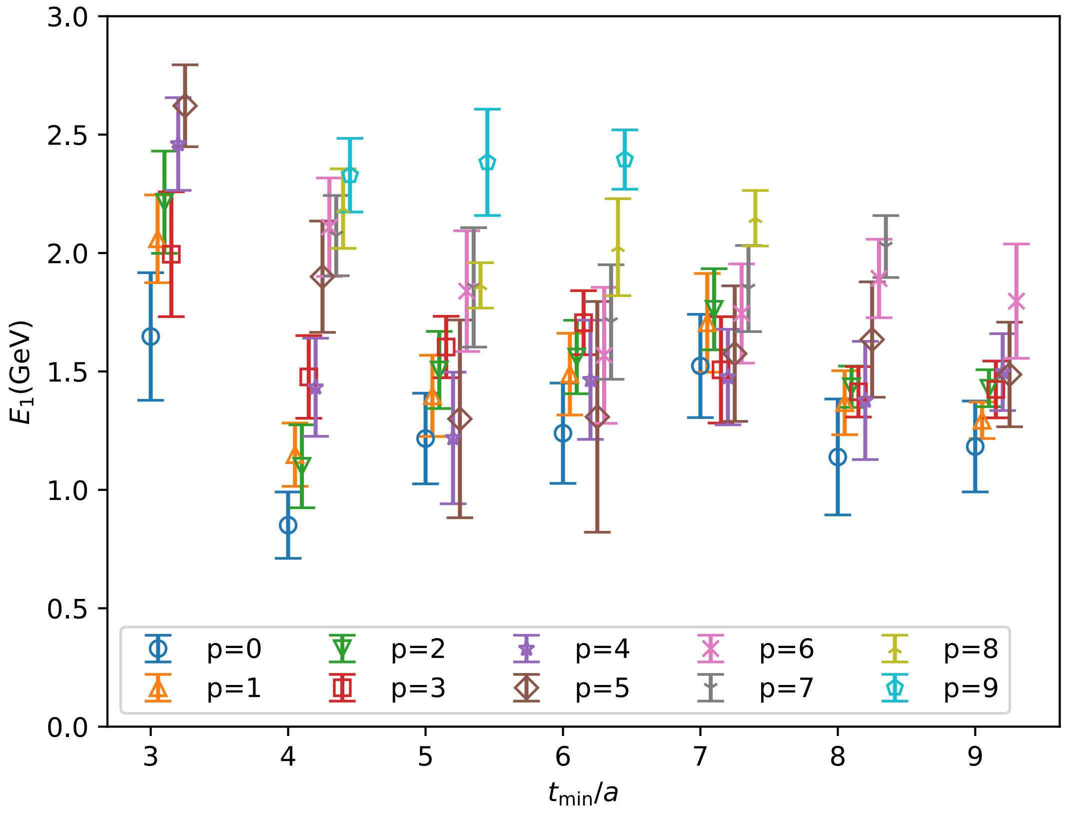
- We present a pion DA calculation using gauge ensembles with domain wall fermions;
- We propose and develop a more robust method to resum the small-momentum logarithms in the perturbative matching kernel of DA, the first implementation of threshold resummation in the LaMET DA calculation;
- We observe a slightly flatter distribution for domain wall fermions.

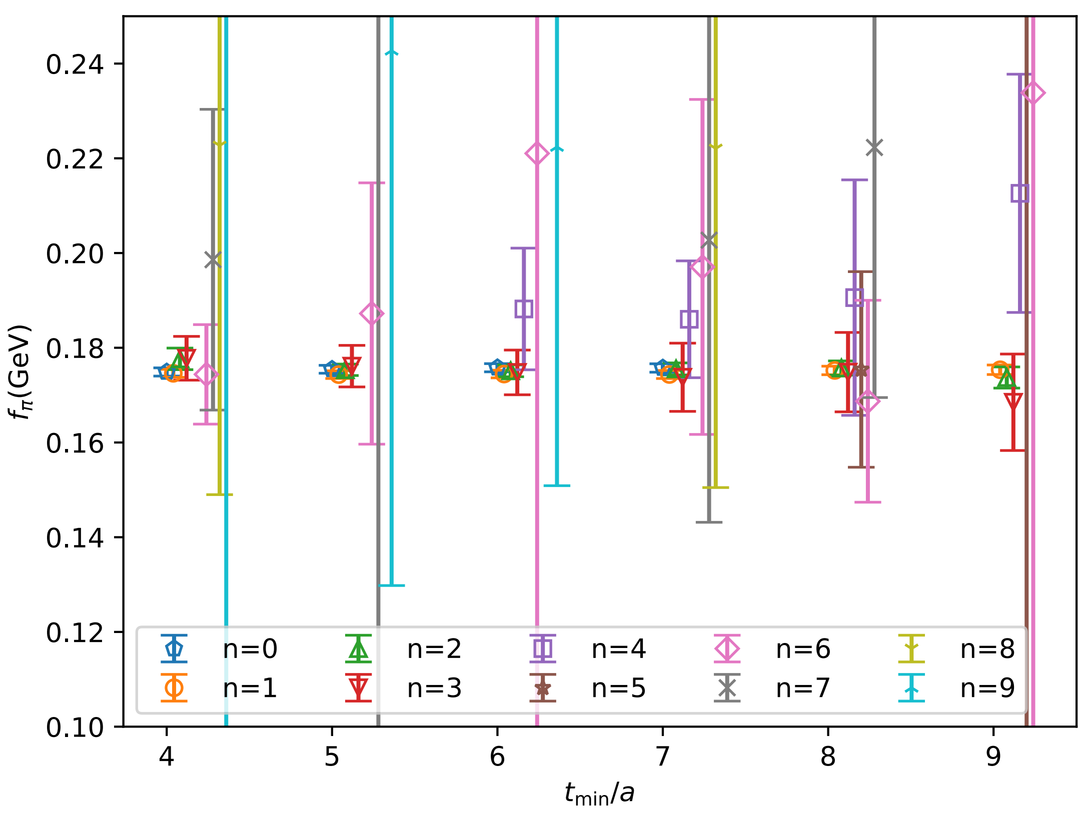
- ❑ Continuum limit is needed for a more conclusive comparison
- ❑ Larger pion momentum is needed to extend the x range of calculation
- ❑ More precise measurement of DA longtail is needed

Thank you for listening!

Backup Slides

fits


3.0
2.5
2.0
1.5
1.0
0.5
0.0
E_1(GeV)
3
4
5
6
7
8
9
t_{min}/a
p=0
p=1
p=2
p=3
p=4
p=5
p=6
p=7
p=8
p=9


0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10
f_π(GeV)
4
5
6
7
8
9
t_{min}/a
n=0
n=1
n=2
n=3
n=4
n=5
n=6
n=7
n=8
n=9

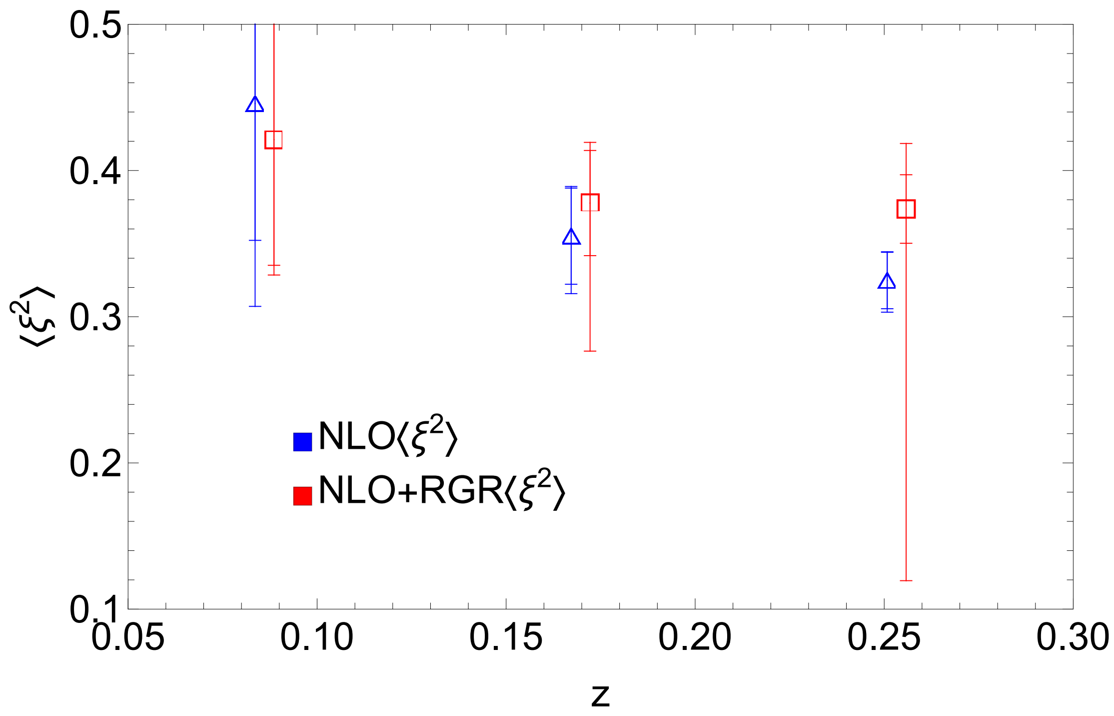
Extracting Moments

- RG-invariant ratio:

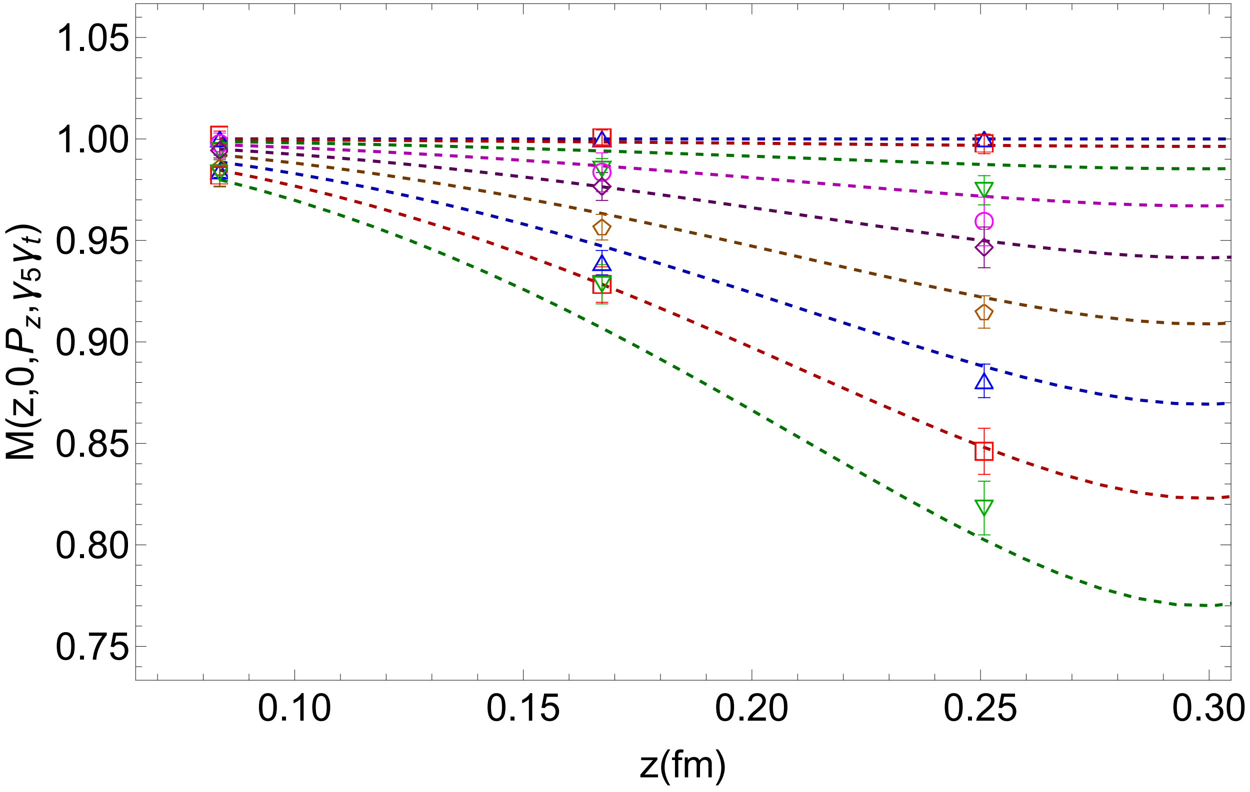
$$\mathcal{M}(z,P_1,P_2)=\lim_{a\to 0}\frac{H^B(z,P_2,a)}{H^B(z,P_1,a)}=\frac{H^R(z,P_2)}{H^R(z,P_1)}$$

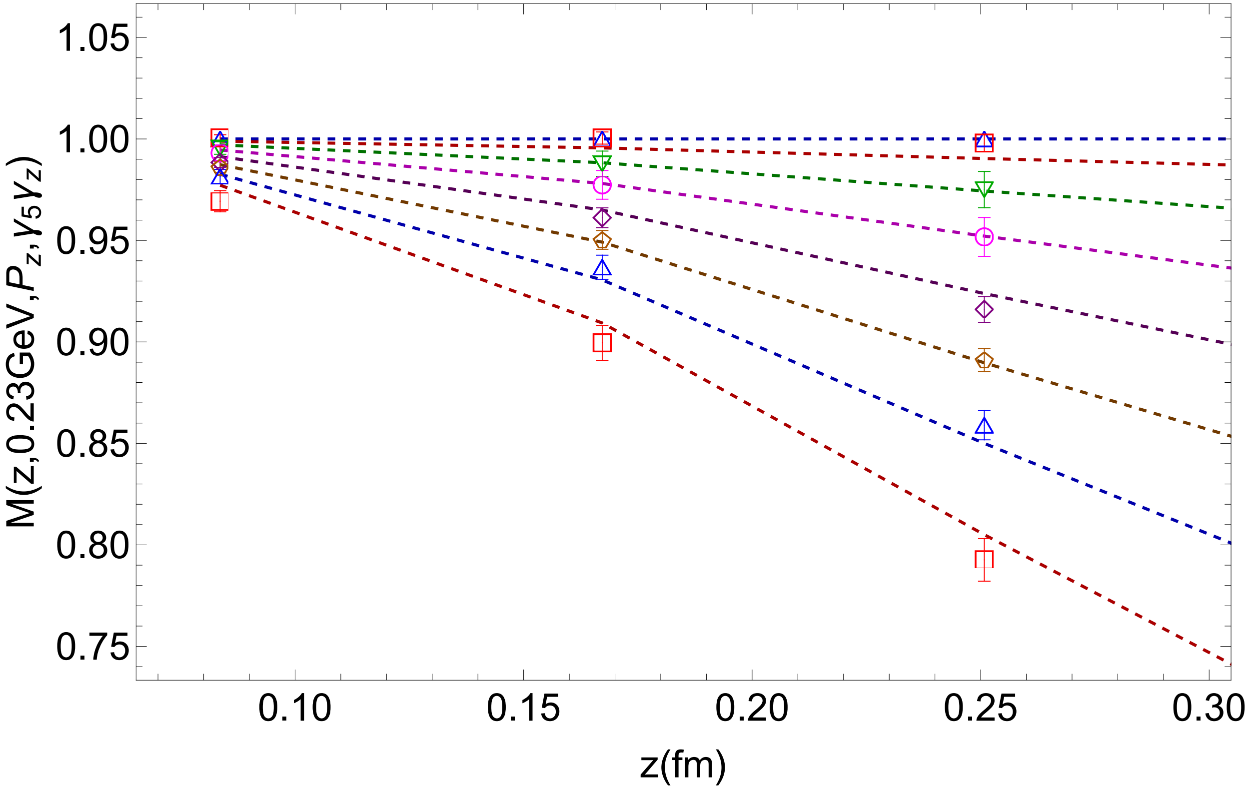
- Fit to OPE

$$\mathcal{M}(z,P_1,P_2)\approx\frac{\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{(\frac{-izP_2}{2})^n}{n!}C_{nm}(z,\mu)\langle\xi^m\rangle}{\sum_{n}^{\infty}\sum_{m=0}^{n}\frac{(\frac{-izP_1}{2})^n}{n!}C_{nm}(z,\mu)\langle\xi^m\rangle}$$



Consistency with OPE (Ratio)





Different momenta

- Compare with $P_z = 1.6GeV$
- The range of calculation increases with momentum