Gauge invariant spectral analysis of hadronization dynamics

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Based on: Phys. Rev. D 108 (2023) 11, 114011

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Jefferson Lab

Outline

Introduction

Gauge invariant quark propagator

Quark propagator spectral representation

Conclusions

- **Confinement:** Quarks and gluons are not asymptotic states of QCD: are confined inside hadrons

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- How color neutral and massive hadron emerge out of colored and massless quarks and gluons?

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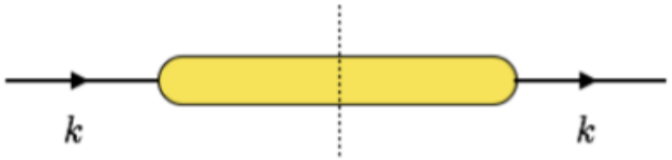
- **DCSB:** Mass generation

Nonperturbative: Gauge invariant quark propagator/jet correlator



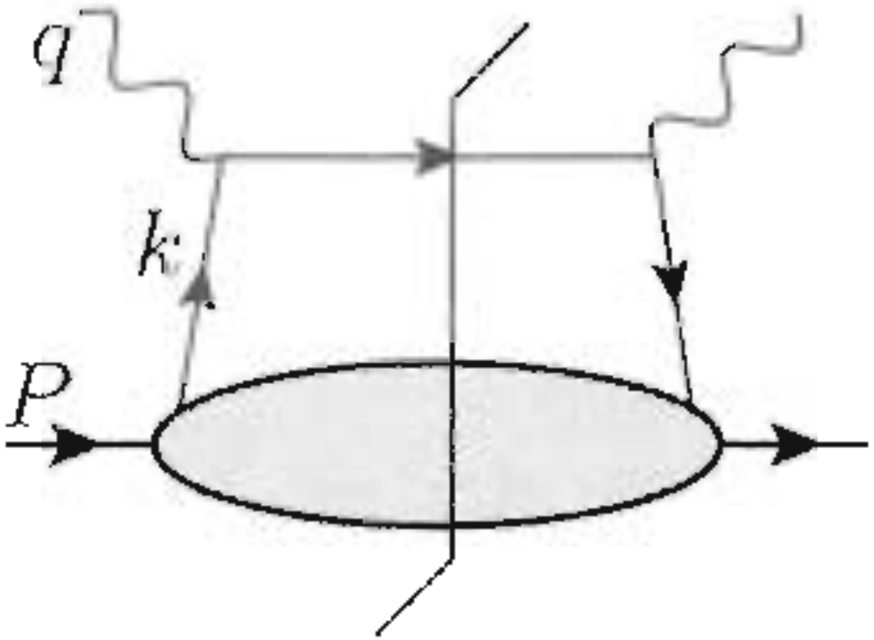
- These QCD features are intimately related to *hadronization*
- How color neutral and massive hadron emerge out of colored and massless quarks and gluons?

Chiral symmetry breaking

- Chiral symmetry: approximate symmetry of the light quark sector of QCD

$$m_u \approx 2.16\ \mathrm{MeV}, \quad m_d \approx 4.67\ \mathrm{MeV}: \quad m_u \approx m_d$$

- Chiral symmetry is broken dynamically and gives rise to:
  - the mass splitting observed in hadron spectrum
  - dressed quarks
- Gauge invariant propagator can be used to probe dynamical mass generation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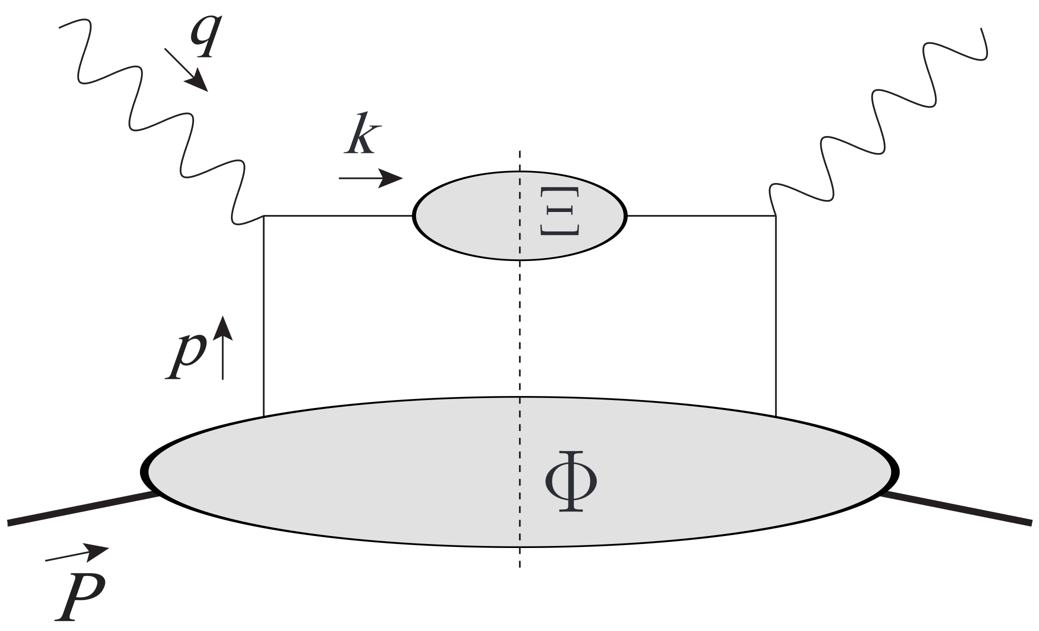

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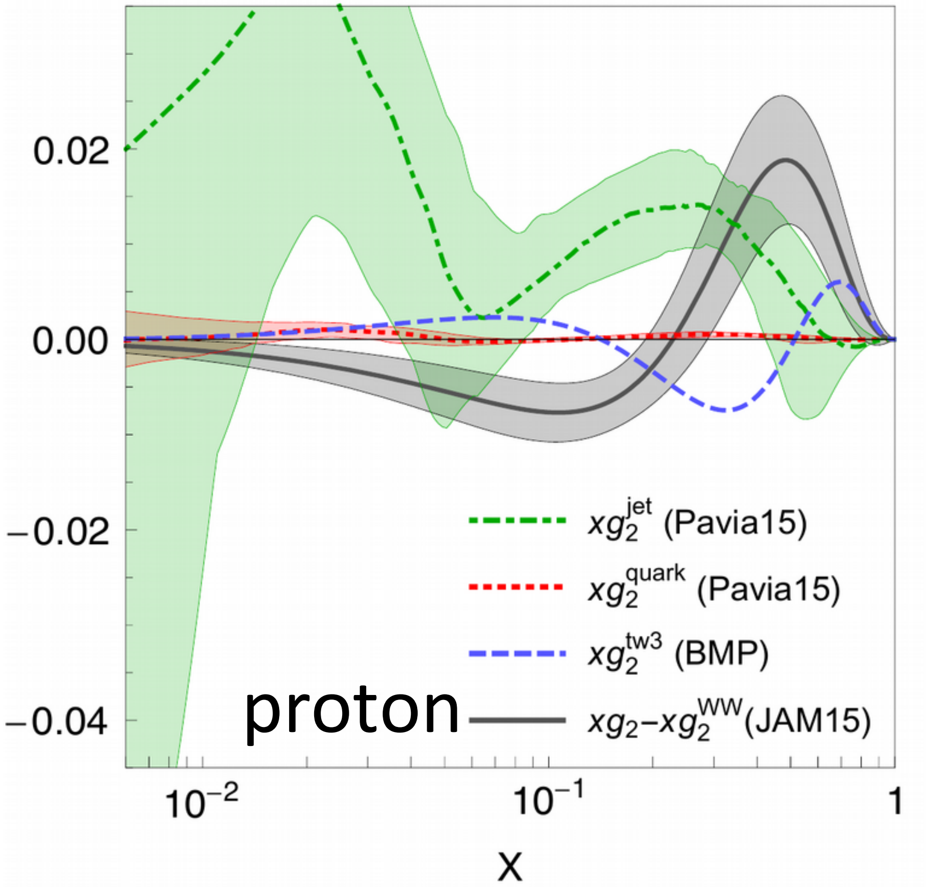
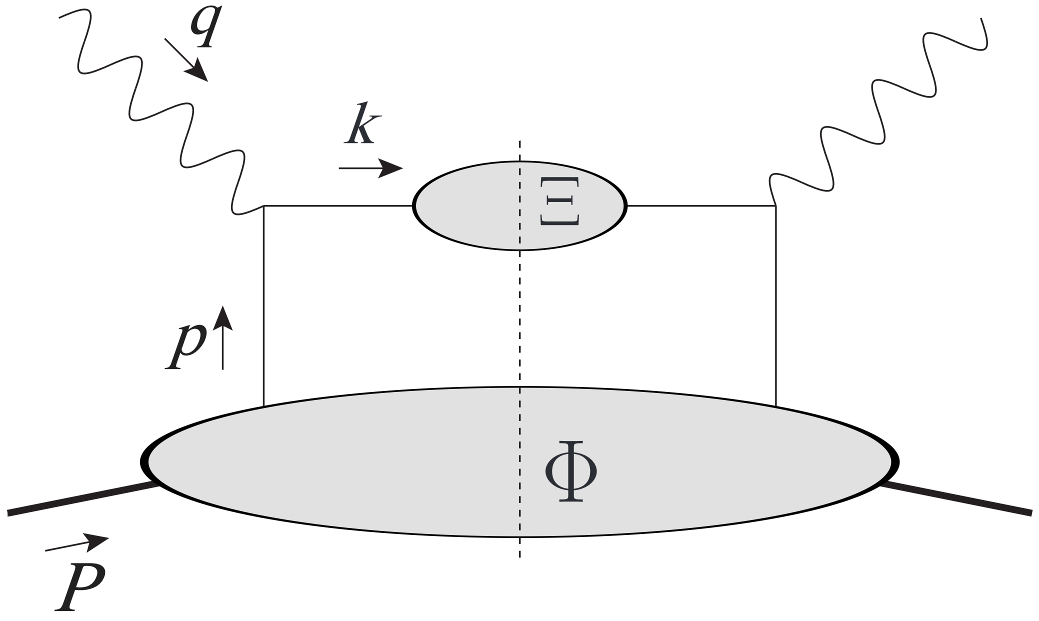
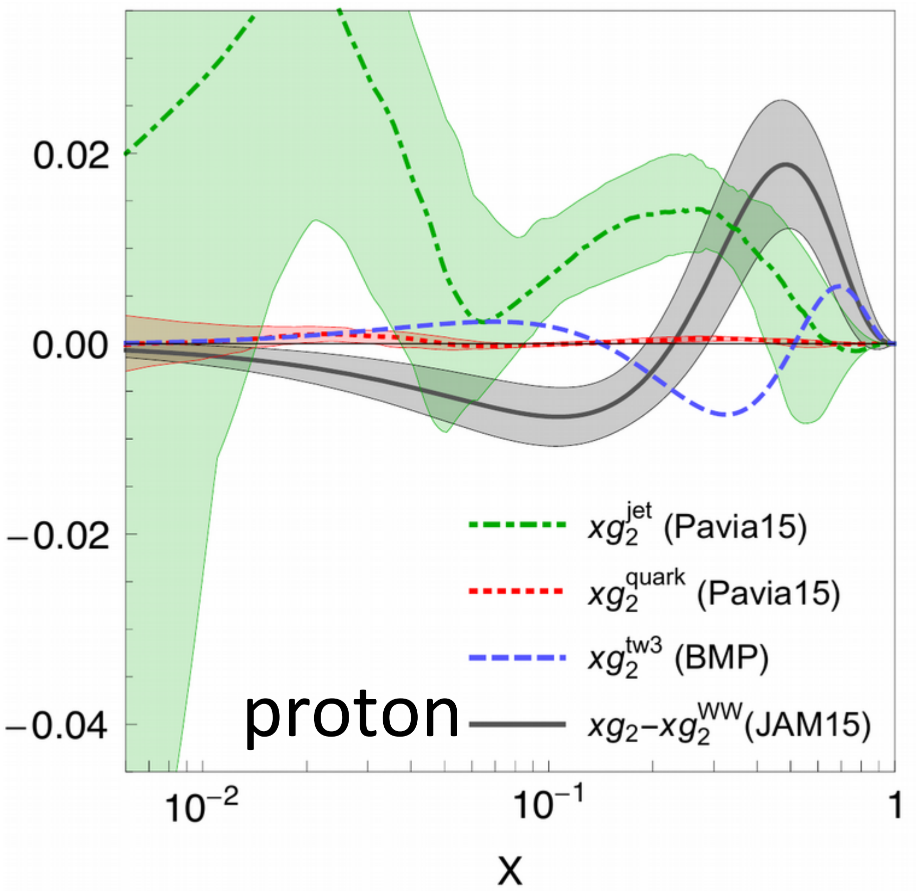
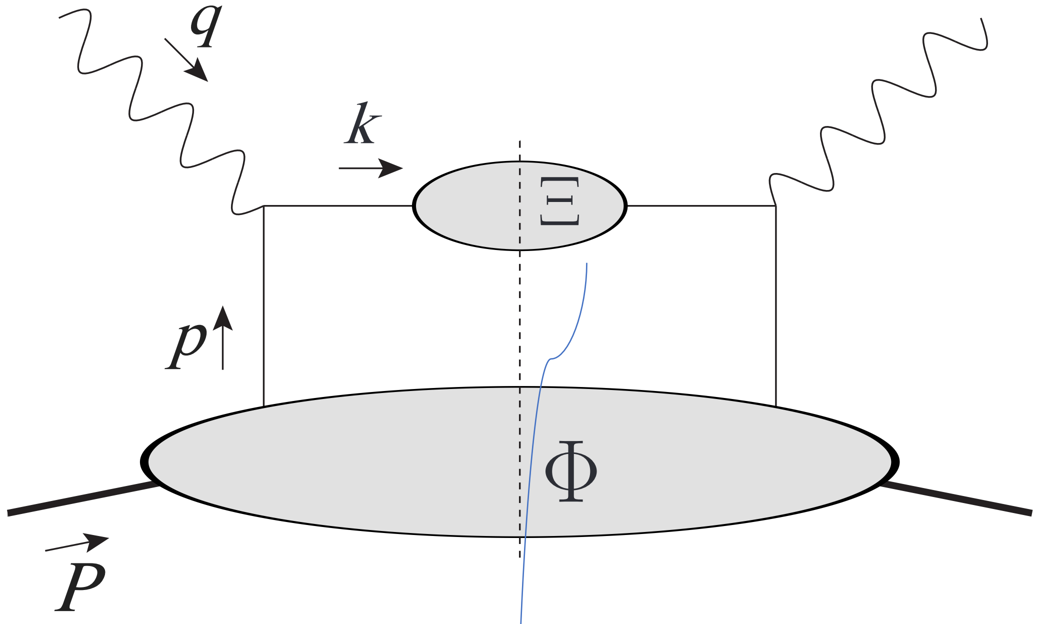




$$g_2(x_B) - g_2^{WW}(x_B) = \frac{1}{2}\sum_a e_a^2 \left(g_2^{tw-3}(x_B) + \underbrace{\frac{m_q}{M}\left(\frac{h_1^q}{x}\right)^{\star}}_{g_2^{quark}}(x_B) + \frac{M_q - m_q}{M}\frac{h_1^q(x_B)}{x_B} \right)$$

AA, Bacchetta (2017)





dynamically generated mass: nonvanishing even when $m_q = 0$

g_2^{quark}

$$g_2(x_B) - g_2^{WW}(x_B) = \frac{1}{2}\sum_a e_a^2 \left(g_2^{tw-3}(x_B) + \frac{m_q}{M}\left(\frac{h_1^q}{x}\right)^{\star}(x_B) + \frac{M_q - m_q}{M}\frac{h_1^q(x_B)}{x_B} \right)$$

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Gauge invariant quark propagator

$$\Xi_{ij}(k;w) = \mathrm{Disc} \int \frac{d^4\xi}{(2\pi)^4} e^{\mathrm{i}k\cdot\xi} \, \frac{\mathrm{Tr}_c}{N_c} \, \langle\Omega| \big[\, \mathcal{T} \, W_1(\infty,\xi;w)\psi_i(\xi) \big] \, \big[\, \overline{\mathcal{T}} \, \overline{\psi}_j(0) W_2(0,\infty;w) \big] |\Omega\rangle$$

Accardi, Signori, 2020

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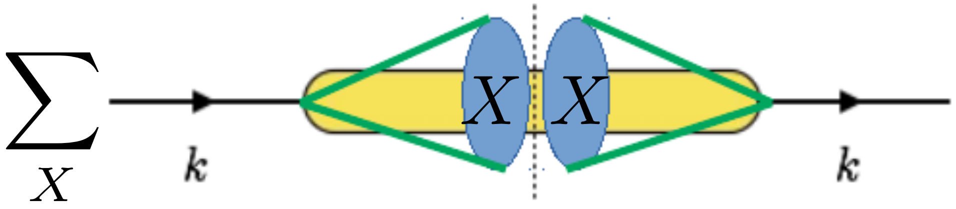
- Hadronization of a quark into an unobserved jet of particles (fully inclusive)



q → X amplitude

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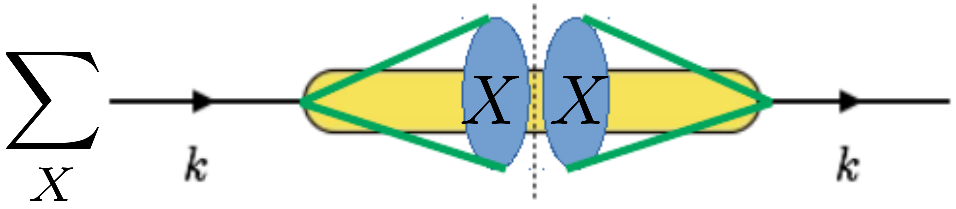

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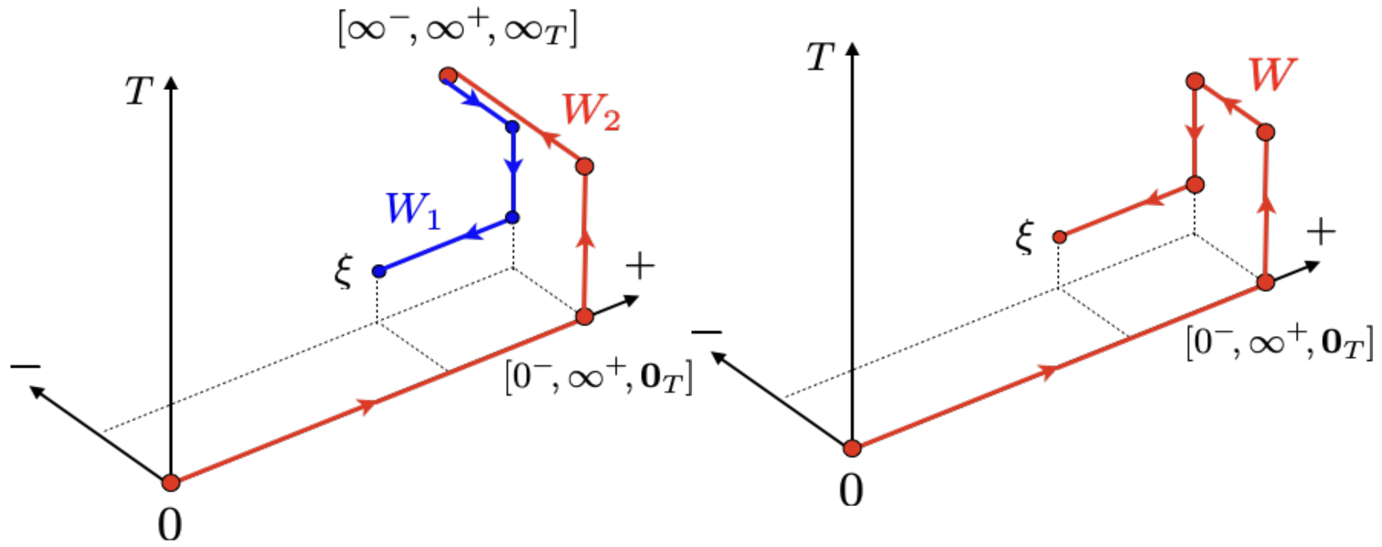

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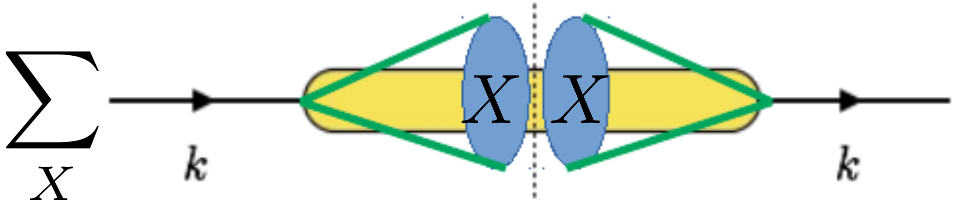
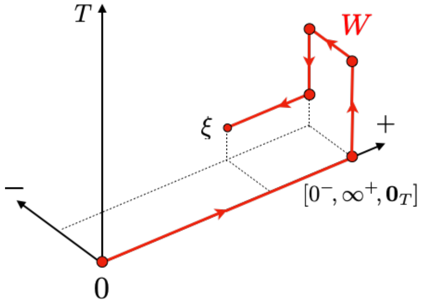


$$\Xi_{ij}(k;n_+) = \text{Disc}\int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi}\,\frac{\text{Tr}_c}{N_c}\,\langle\Omega|\psi_i(\xi)\overline{\psi}_j(0)W(0,\xi;n_+)|\Omega\rangle$$



- Gauge invariant generalization of the fully dressed quark propagator

Gauge invariant quark propagator

- Can be given a convolution representation

$$\Xi_{ij}(k;w) = \mathrm{Disc} \int d^4p \frac{\mathrm{Tr}_c}{N_c} \langle \Omega | i\widetilde{S}_{ij}(p;v) \widetilde{W}(k-p;w,v) | \Omega \rangle$$

where

$$i\widetilde{S}_{ij}(p,v) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\xi\cdot p}\, \mathcal{T}\, \psi_i(\xi) \overline{\psi}_j(0)$$

$$\widetilde{W}(k-p;\, w,v) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\xi\cdot(k-p)}\, W(0,\xi;w,v)$$

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- Decomposition of the quark bilinear operator

(axial gauges)

$$i\widetilde{S}_{ij}(p,v) = \hat{s}_3(p^2,p\cdot v)\not{p}_{ij} + \sqrt{p^2}\,\hat{s}_1(p^2,p\cdot v)\,\mathbb{I}_{ij} + \hat{s}_0(p^2,p\cdot v)\not{v}_{ij}$$

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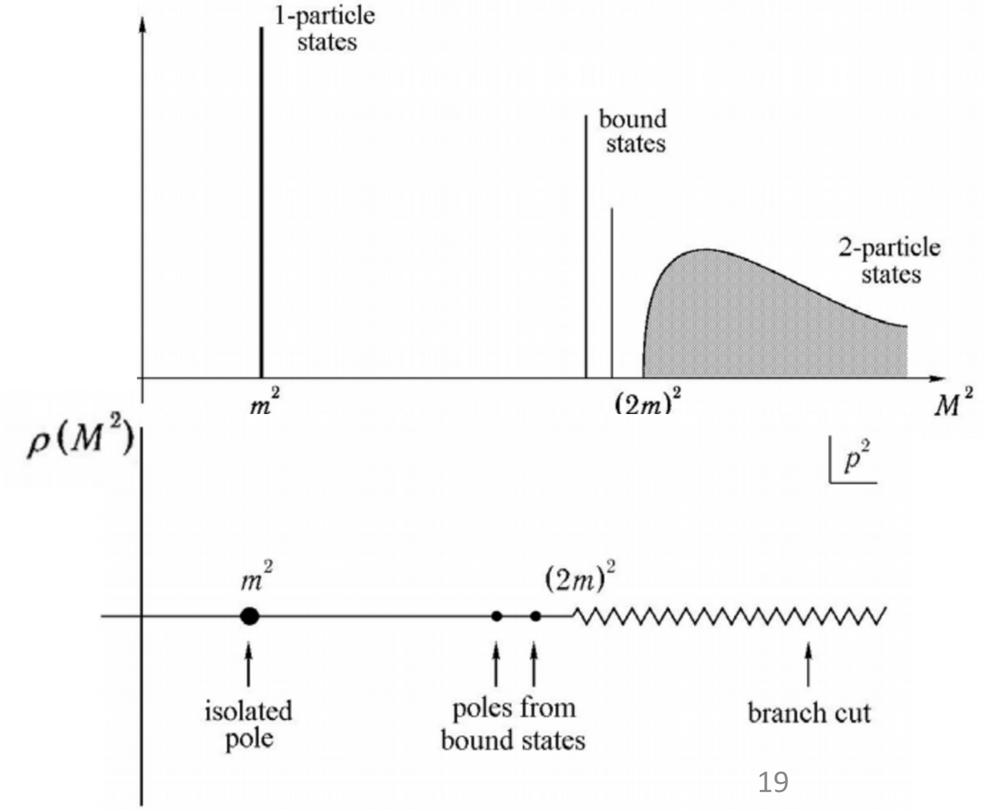
(lightlike axial gauges)

$\hat{s}_3(p^2)$ $\hat{s}_1(p^2)$ $\hat{s}_0(p^2)$: spectral operators

Spectral representation of the quark propagator

$$\frac{\mathrm{Tr_c}}{N_c}\langle\Omega|i\tilde{S}(p)|\Omega\rangle = \frac{1}{(2\pi)^4}\int_{-\infty}^{\infty} d\sigma^2 \rho(\sigma^2)\frac{i}{p^2-\sigma^2+i\epsilon}\theta(\sigma^2)$$



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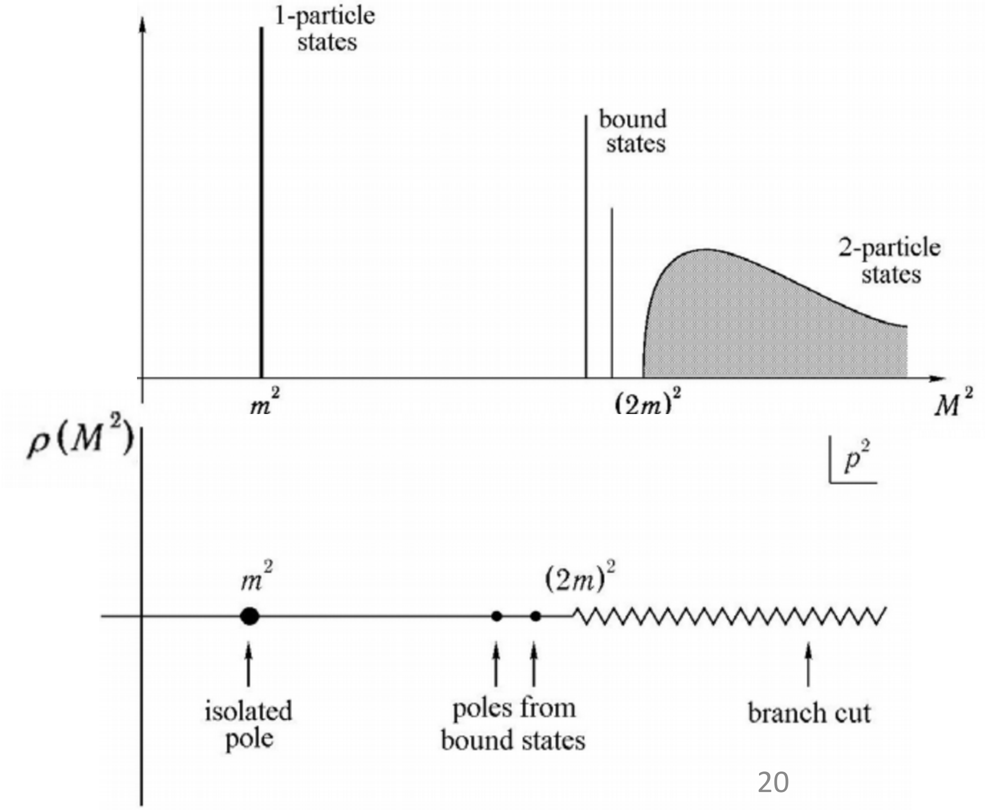

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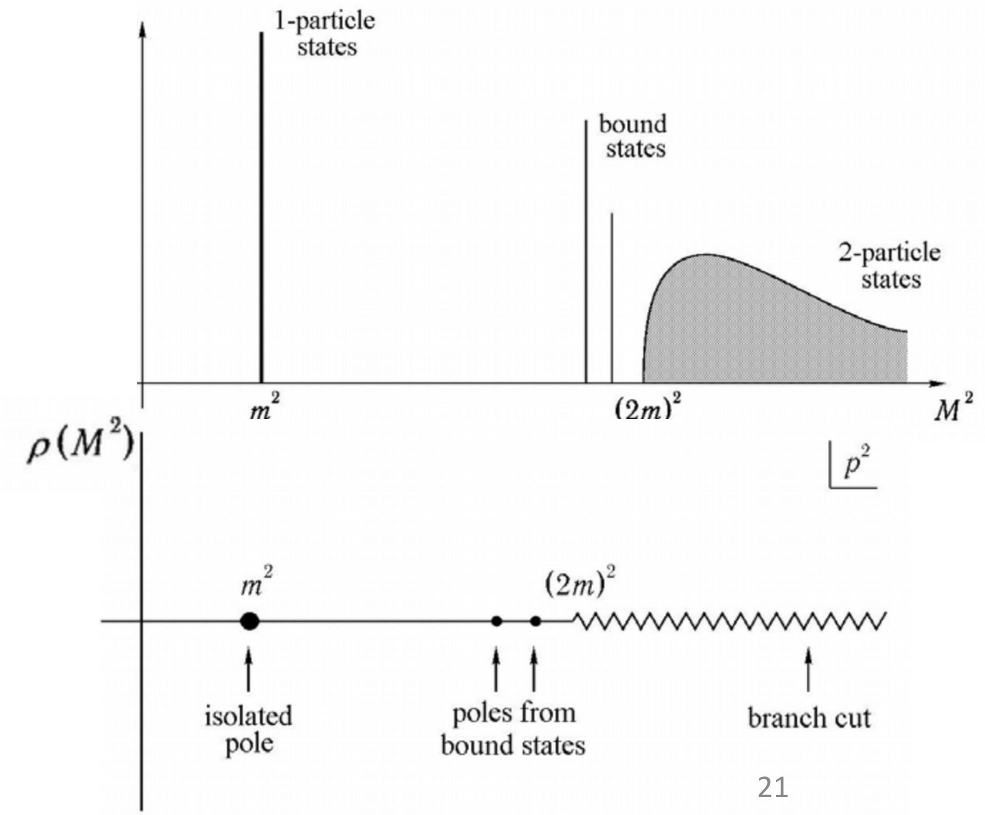

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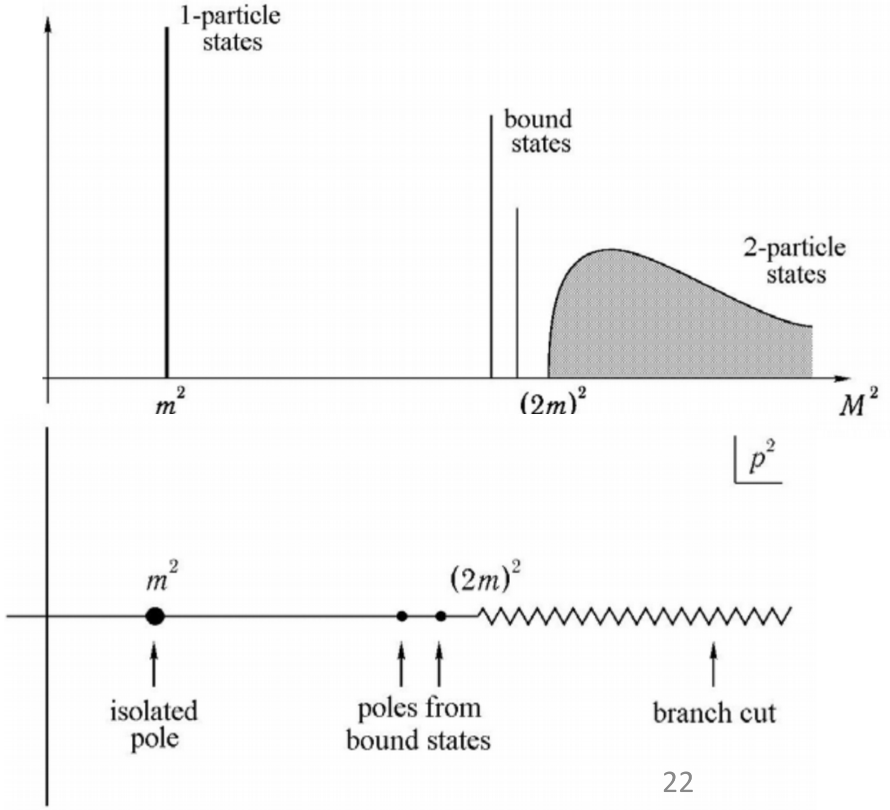
$$\mathrm{Disc}\frac{\mathrm{Tr_c}}{N_c}\langle\Omega|i\tilde{S}(p)|\Omega\rangle = \frac{1}{(2\pi)^3}\rho(p^2)\theta(p^2)\theta(p^-)$$

$$\mathrm{Disc}\frac{\mathrm{Tr}_c}{N_c}\langle\Omega|\hat{s}_{3,1,0}(p,v)|\Omega\rangle = \frac{1}{(2\pi)^3}\rho_{3,1,0}(p^2)\theta(p^2)\theta(p\quad)$$



Integrated g.i. quark propagator

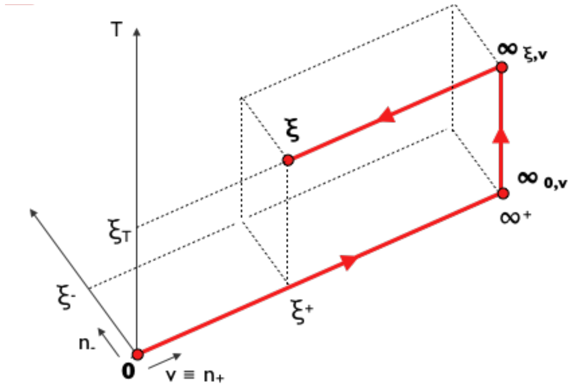
- Boost quark at large light-cone momentum:

$$k^- \gg |\mathbf{k}_\perp| \gg k^+$$

$w = n^+$

Integrate out the suppressed component of the quark momentum:

$$J_{ij}(k^-, \vec{k}_\perp; n_+) \equiv \frac{1}{2} \int dk^+ \Xi_{ij}(k; n_+)$$



- Generalizes the perturbative quark propagator that appears in inclusive and semi-inclusive DIS

$$W_{\rm TMD}(\xi^+, \boldsymbol{\xi}_\perp) = \mathcal{U}_{n_+}[0^-, 0^+, \mathbf{0}_\perp; 0^-, \infty^+, \mathbf{0}_\perp]\, \mathcal{U}_{\boldsymbol{n}_\perp}[0^-, \infty^+, \mathbf{0}_\perp; 0^-, \infty^+, \boldsymbol{\xi}_\perp]\, \mathcal{U}_{n_+}[0^-, \infty^+, \boldsymbol{\xi}_\perp; 0^-, \xi^+, \boldsymbol{\xi}_\perp]$$

$$W_{\rm coll}(\xi^+) = \mathcal{U}_{n_+}[0^-, 0^+, \mathbf{0}_\perp; 0^-, \xi^+, \mathbf{0}_\perp]$$

Integrated g.i. quark propagator

- Expand in Dirac structures, in powers of $1/k^-$

$$J(k^-, \boldsymbol{k}_\perp; n_+) = \frac{1}{2}\alpha(k^-)\gamma^+ + \frac{\Lambda}{k^-}\left[\zeta(k^-)\mathbb{I} + \alpha(k^-)\frac{\not{\boldsymbol{k}}_\perp}{\Lambda}\right] + \frac{\Lambda^2}{2(k^-)^2}\omega(k^-, \boldsymbol{k}_\perp^2)\gamma^-$$

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$$\alpha(k^-) = J^{[\gamma^-]}$$

$$\zeta(k^-) = \frac{k^-}{\Lambda}J^{[\mathbb{I}]}$$

$$\omega(k^-, \boldsymbol{k}_\perp^2) = \left(\frac{k^-}{\Lambda}\right)^2 J^{[\gamma^+]}$$

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$$J(k^-, \boldsymbol{k}_T; n_+) = \frac{\theta(k^-)}{4(2\pi)^3\, k^-}\left\{k^-\,\gamma^+ + \not{k}_T + M_j\mathbb{I} + \frac{K_j^2 + \boldsymbol{k}_T^2}{2k^-}\gamma^-\right\}$$

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Average mass of the particles produces in the hadronization process

Integrated g.i. quark propagator

- Expand in Dirac structures, in powers of $1/k^-$

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Jet viruality

Average mass of the particles produces in the hadronization process

- In ***<u>any gauge</u>***:

$$(k \quad) = J^{[\quad]} = \frac{\theta(k \quad)}{2(2\pi)^3} \int_0^\infty dp^2 \, \boxed{\rho_3(p^2)}$$

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$$\omega(k \quad, \boldsymbol{k_T}) = \left(\frac{k}{\Lambda}\right)^2 J^{[\quad]} = \frac{\theta(k \quad)}{(2\Lambda)^2(2\pi)^3} \quad \mu_j^2 + \tau_j^2 + \boldsymbol{k}_{\boldsymbol{T}}^2)$$

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Sum rules

- In <u>*any gauge*</u>:

$$1 = \int_0^\infty dp^2 \rho_3(p^2)$$

$$M_j = \int_0^\infty dp^2 \, \sqrt{p^2} \, \rho_1(p^2)$$

$$0 = \int_0^\infty dp^2 \, p^2 \, \rho_0(p^2)$$

- Can be used to verify actual calculations of the quark propagator!

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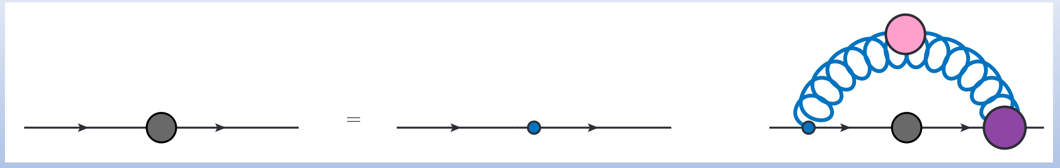

(e.g, DSE calculations)

- Can be used to verify actual calculations of the quark propagator!

- In *any gauge*:

$$M_j = \int dp^2 \, \sqrt{p^2} \, \rho_1(p^2)$$

Gauge invariant generalization of the gauge dependent dressed quark mass

- Calculable, should you you know the chiral odd quark spectral function (in progress)

- In *any gauge*:

$$M_j = \int dp^2 \, \sqrt{p^2} \, \rho_1(p^2)$$

Gauge invariant generalization of the gauge dependent dressed quark mass

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 = Unpolarized		$h_1^{\perp}$ = Boer-Mulders
	L		g_1 = Helicity	$h_{1L}^{\perp}$ = Worm-gear
	T	$f_{1T}^{\perp}$ = Sivers	$g_{1T}^{\perp}$ = Worm-gear	h_1 = Transversity; $h_{1T}^{\perp}$ = Pretzelosity

Experimentally accessible in spin assymetry measurements!

$$\propto \; (\, M_j - m \,) h_1$$

In *any gauge*:

$$M_j = \int dp^2 \; \sqrt{p^2} \; \rho_1(p^2)$$

Gauge invariant generalization of the gauge dependent dressed quark mass

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		D_1 = Unpolarized		$H_1^\perp$ = − Collins
Polarized Hadrons	L		G_1 = − Helicity	$H_{1L}^\perp$ = −
	T	$D_{1T}^\perp$ = − Polarizing FF	$G_{1T}^\perp$ = −	H_1 = − Transversity $H_{1T}^\perp$ = −

Experimentally accessible in spin assymetry measurements!

$$\propto \; (\, M_j - m \,) H_1$$

In *light-cone gauge*:

$$K_j^2 = \mu_j^2 + \tau_j^2 = \int_0^\infty dp^2 \; p^2 \; \rho_3^{\mathrm{lcg}}(p^2)$$

Final state interactions ”vanish”

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Summary

- Non-vanishing even in the chiral limit (dynamically generated mass)
- It's calculable, but moreover.. It can be measured!
- Provides a direct way to probe dynamical chiral symmetry breaking
- New perspectives to study hadronization effects involving different observables.

Snowmass 2021 White Paper
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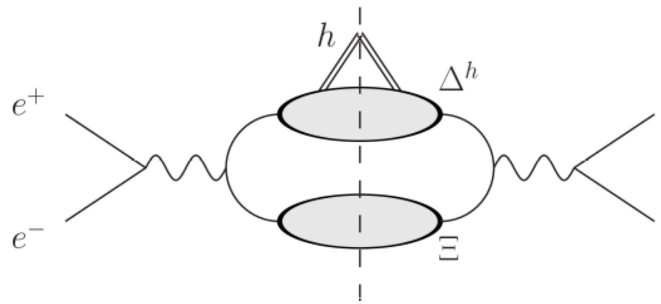

Thank you!