Unraveling the internal structure of hadrons: from form factors to generalized parton distributions and future directions

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From Quarks and Gluons to the Internal Dynamics of Hadrons

Stony Brook University

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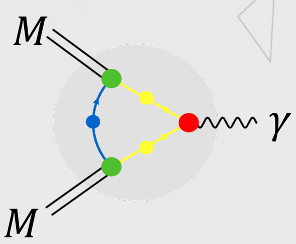
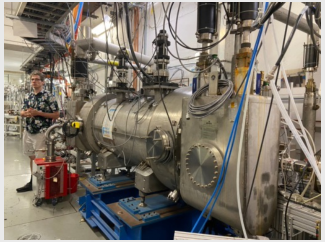
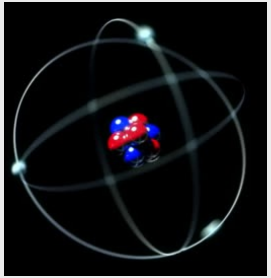
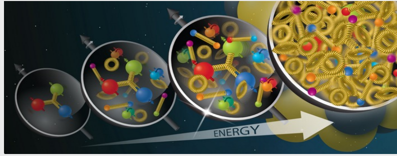
5. Final remarks.
6. Future perspectives.



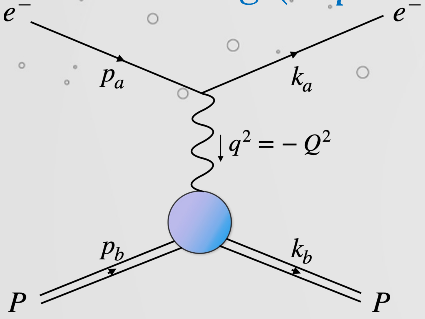




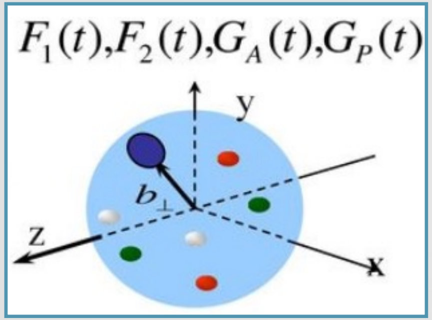
Distribution Functions



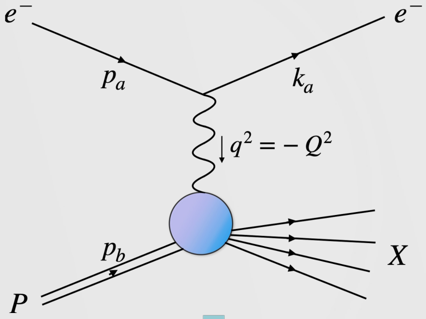
Elastic Scattering ($e^- p \to e^- p$)



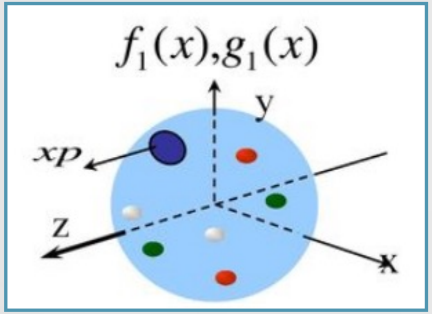
Form Factors (FFs)



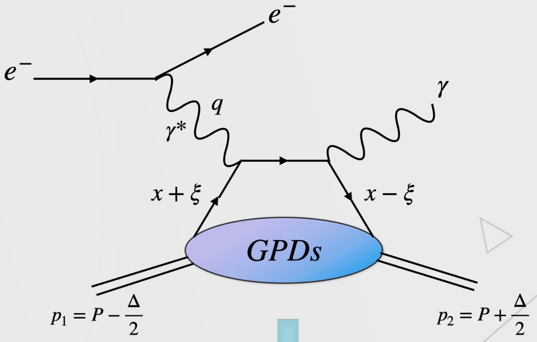
DIS ($e^- p \to e^- X$)



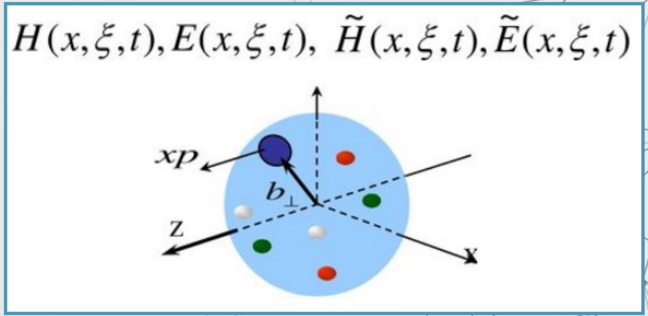
Parton Distribution Functions (PDFs)



DVCS ($e^- p \to e^- p\gamma$)



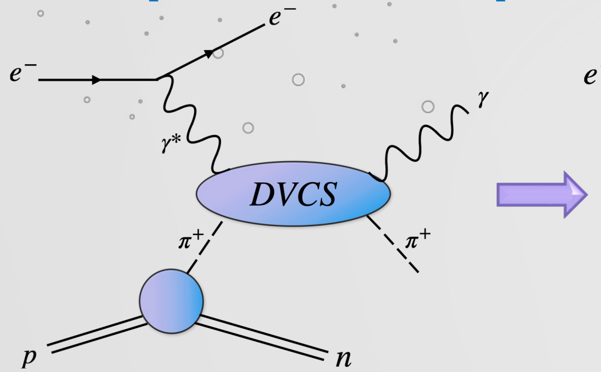
Generalized Parton Distribution (GPDs)

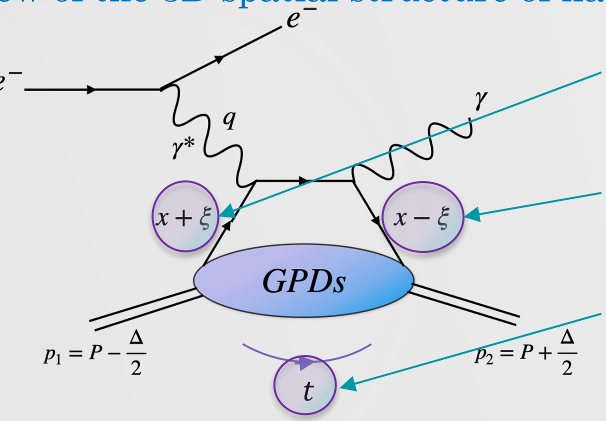


Distribution Functions

C. Mezrag, et. al *Few Body Syst. 57 (2016) 9, 729-772*
C. Mezrag, *Few Body Syst. 63 (2022) 3, 62*

- GPDs provide a kaleidoscopic view of the 3D spatial structure of hadrons.





$x + \xi$: Is the relative longitudinal momentum of the initial quark.

$x - \xi$: Is the relative longitudinal momentum of the final quark.

$t = -\Delta^2$: Total square momentum transferred to the hadron.

$\Delta^2 = (p - p')^2$

- Experimental measurements of GPDs are achievable through processes like Deeply Virtual Compton Scattering (DVCS).
- Kinematics: ➤ DGLAP region: $(|x| > |\xi|)$ ➤ ERBL region: $(|x| < |\xi|)$
- Properties:
  - ➤ Positivity:

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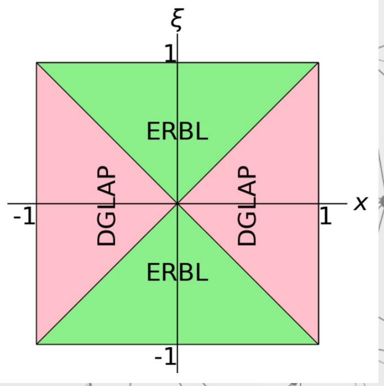
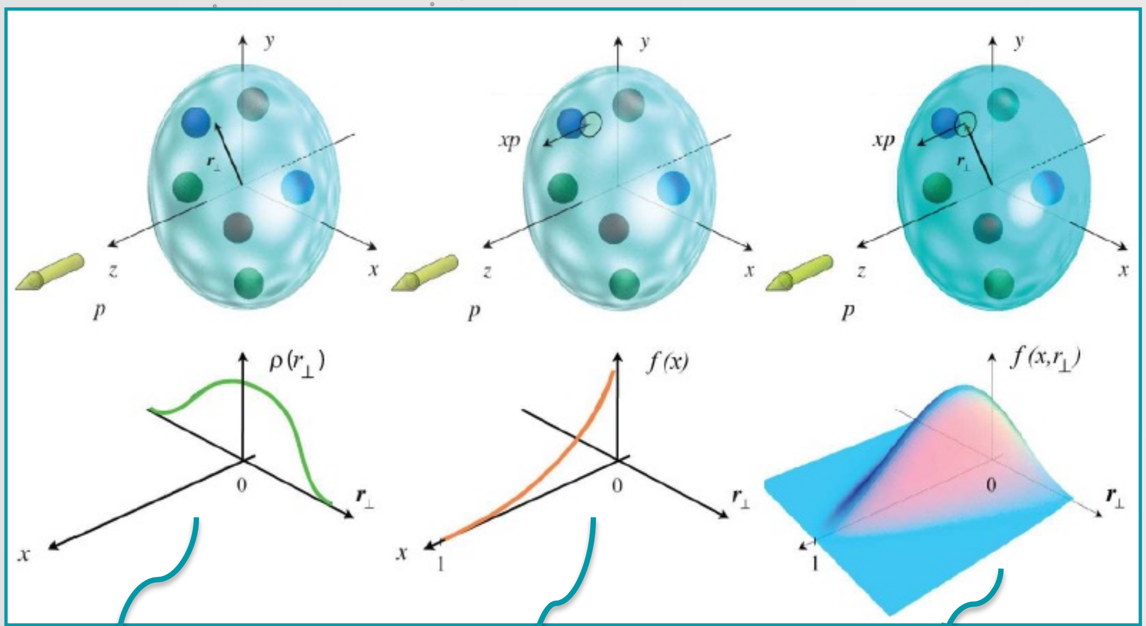
$$|H^q(x,\xi)|_{x\geq\xi}| \leq \sqrt{q\left(\frac{x-\xi}{1-\xi}\right)q\left(\frac{x+\xi}{1+\xi}\right)},$$

  - ➤ Polynomiality: Mellin moments of GPD can be rewritten as

$$\mathcal{M}_m(\xi,t) = \sum_{i=0}^{\left[\frac{m}{2}\right]}(2\xi)^{2i}A^q_{i,m}(t) + \mathrm{mod}(m,2)(2\xi)^{m+1}C_{m+1}(t),$$



Distribution Functions: Strategies



Charge density
Form Factors

Structure functions
Parton distribution functions

Generalized parton distribution functions

- Overlap representation of the LFWF:

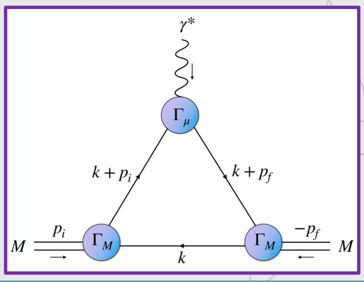
$$H_{\mathrm{M}}^{q}(x,\xi,t) = \int \frac{d^2k_\perp}{16\pi^3} \psi_{\mathrm{M}}^{q*}\left(x^-, (\mathbf{k}_\perp^-)^2\right) \psi_{\mathrm{M}}^{q}\left(x^+, (\mathbf{k}_\perp^+)^2\right)$$

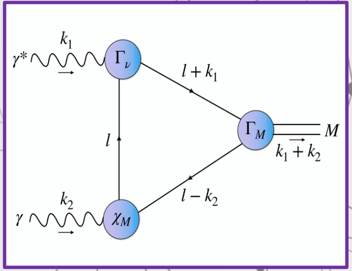
✓ Positivity condition fulfilled by construction

Restricted to the DGLAP domain ($|x| > |\xi|$)

✓ Many distribution functions can be available within this domain.

- Impulse approximation to calculate EFFs and TFFs



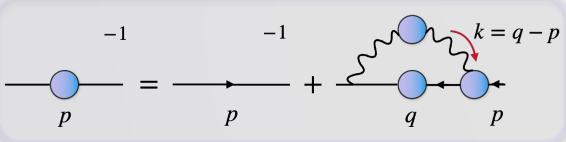


Schwinger-Dyson equations: Algebraic Model (AM)

Schwinger-Dyson equations

The SDEs form an infinite set of coupled integral equations that establish the connection between Green's functions in a quantum field theory

- The SDE for the quark propagator is:



Explicitly expressed as: $\boxed{S^{-1}(p) = Z_2(i\gamma \cdot p + m_0) + \Sigma(p)\,,}$

- The Bethe-Salpeter (BS) equation provides a fully relativistic description of two-particle bound states.

$$[\Gamma^{ab}_{\mathrm{M}}(p,P)]_{\alpha\beta} = \int \frac{d^4q}{(2\pi)^4}[\mathbf{K}]_{\alpha\gamma,\delta\beta}[\chi^{ab}_{\mathrm{M}}(q,P)]_{\gamma\delta}\,,$$

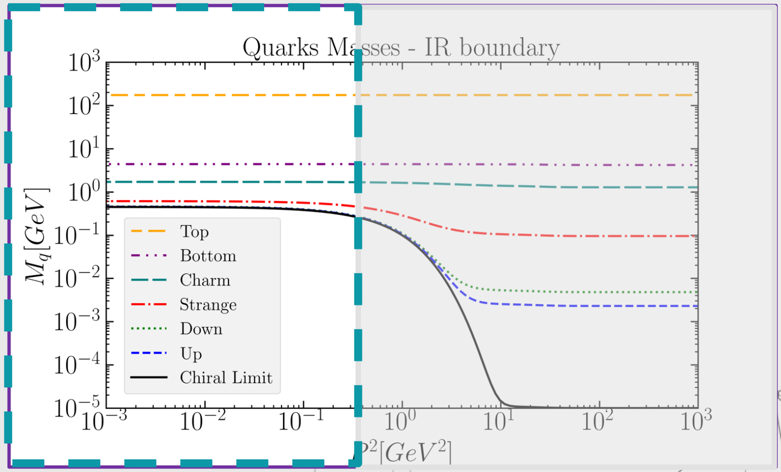
BS wave function (BSWF)

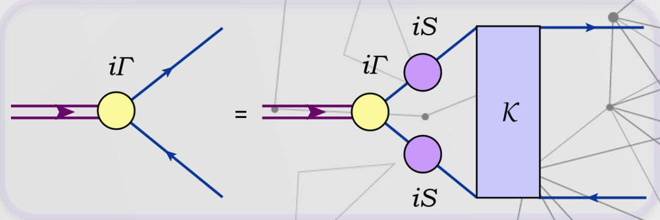
$$\chi^{ab}_{\mathrm{M}}(q,P) = S^a(q_+)\Gamma^{ab}_{\mathrm{M}}(q,P)S^b(q_-)\,,$$

The most general form of the quark propagator is:

$$S(p) \equiv \frac{F(p^2,\mu_R^2)}{i\gamma \cdot p + M(p^2,\mu_R^2)}$$

Mass function





Schwinger-Dyson equations

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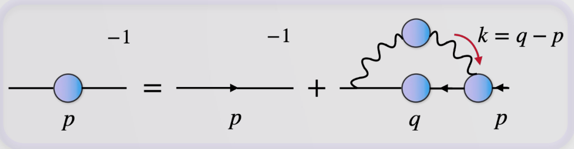

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BS wave function (BSWF)

$$\chi_{\mathrm{M}}^{ab}(q,P) = S^a(q_+)\Gamma_{\mathrm{M}}^{ab}(q,P)S^b(q_-)\,,$$

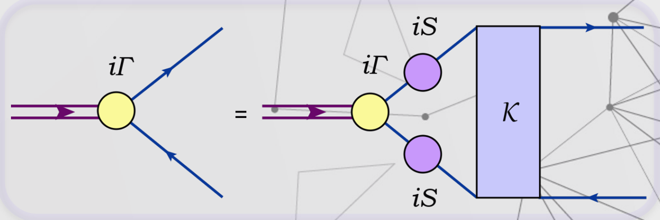
Need truncation

For example: The Rainbow-Ladder truncation

This may not be suitable for future non-perturbative calculations of hadronic functions

Parameterizations and models

Algebraic Model (AM)



Algebraic Model

Phys. Rev. D106 (2022) 3, 034003

We propose a new algebraic model for the quark propagator and the BSA grounded in NIR.

- Quark propagator
- BS amplitude

$$S_{q(\bar{h})}(k) = \left[-i\gamma \cdot k + M_{q(\bar{h})}\right] \Delta\left(k^2; M^2_{q(\bar{h})}\right) ,$$

$$n_{\mathrm{M}}\Gamma_{\mathrm{M}}(k,p) = i\gamma_5 \int_{-1}^{1} dw\, \rho_{\mathrm{M}}(w) \left[\hat{\Delta}\left(k^2_w; \Lambda^2_w\right)\right]^{\nu} .$$

$\nu = 1 + \delta$

where, $\Delta(a,b) = (a+b)^{-1}$, $\hat{\Delta}(a,b) = b\Delta(a,b)$, $k_\omega = k + (\omega/2)P$ and $P^2 = -m_M^2$.

The shape of the spectral density, $\rho(\omega)$ determines the behavior of the associated BSA.

For our algebraic model: $\Lambda^2_w \equiv M_q^2 - \frac{1}{4}\left(1-w^2\right) m^2_{\mathrm{M}} + \frac{1}{2}(1-w)\left(M^2_{\bar{h}} - M^2_q\right)$

Remembering

$$\chi_{\mathrm{M}}\left(k_-, P\right) = S_q(k)\Gamma_{\mathrm{M}}\left(k_-, P\right) S_{\bar{h}}(k-P)$$

which can be rewritten as:

$$n_{\mathrm{M}}\chi_{\mathrm{M}}(k_-, P) = \mathcal{M}_{q,\bar{h}}(k,P) \int_0^1 d\alpha \mathcal{F}_{\mathrm{M}}(\alpha, \sigma^{\nu+2}) ,$$

$$\mathcal{F}_{\mathrm{M}}(\alpha, \sigma^{\nu+2}) = \nu(\nu+1)\left[\int_{-1}^{1-2\alpha} dw \int_{\frac{2\alpha}{w-1}+1}^{1} d\beta + \int_{1-2\alpha}^{1} dw \int_{\frac{2\alpha+(w-1)}{w+1}}^{1} d\beta\right] \frac{(1-\beta)^{\nu-1}\tilde{\rho}^{\nu}_{\mathrm{M}}(w)}{\sigma^{\nu+2}}$$

$$\sigma = [k - \alpha P]^2 + \Lambda^2_{1-2\alpha}$$

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We adopt the following Ansatz for the QPV:

$$\chi_\mu^q(k_f, k_i) = \frac{\sum_{j=1}^{3} T_\mu^{(j)} X_j}{[k_f^2 + m_q^2][k_i^2 + m_q^2]} ,$$

$$X_1 = m_q^2 ,\ \ X_2 = -1 ,\ \ X_3 = -m_q$$

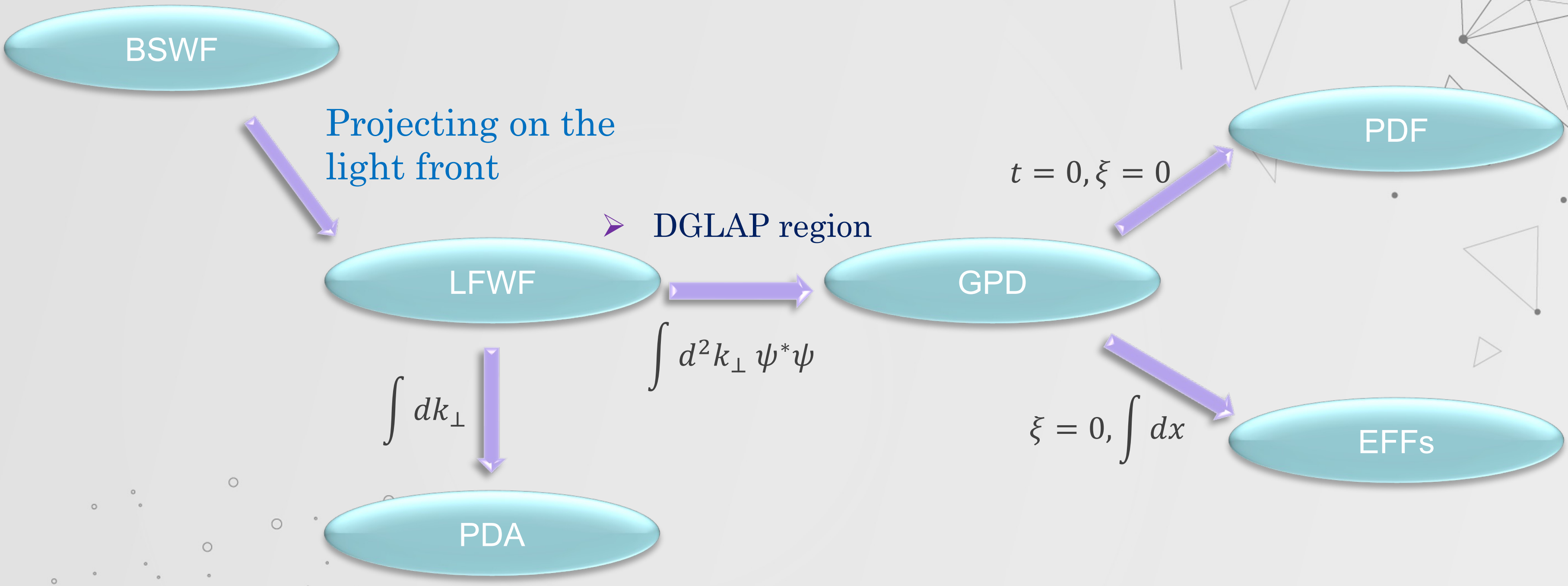
$$\begin{cases} T_\mu^{(1)} = \gamma_\mu , \\ T_\mu^{(2)} = \not{k}_f \gamma_\mu \not{k}_i + \alpha_q(\not{k}_f \gamma_\mu \not{k}_i - \not{k}_i \gamma_\mu \not{k}_f) , \\ T_\mu^{(3)} = i(\not{k}_f \gamma_\mu + \gamma_\mu \not{k}_i) \\ \qquad + i\alpha_q(\not{k}_f \gamma_\mu + \gamma_\mu \not{k}_i - \not{k}_i \gamma_\mu - \gamma_\mu \not{k}_f) \end{cases}$$

$$\alpha_q = \alpha_q^{(0)} \exp[-Q^2/(2m_q^2)]$$

Distribution functions through an AM on the LC formalism

"Pseudo-scalar mesons: light front wave functions, GPDs and PDFs", L. Albino, I.M. Higuera-Angulo, K. Raya, A. Bashir. Phys. Rev. D106 (2022) 3, 034003

Projection on the light-front



- The lowest Mellin moment of GPD → EFFs
- The forward limit of GPDs → PDFs

LFWF and PDA: Through the AM

The LFWF for pseudoscalar mesons can be obtained in the light cone formalism by projecting the meson BSWF:

$$\psi_{\mathrm{M}}^{q}\left(x, k_{\perp}^{2}\right)=\operatorname{tr} \int_{d k_{\|}} \delta_{n}^{x}\left(k_{\mathrm{M}}\right) \gamma_{5} \gamma \cdot n\, \chi_{\mathrm{M}}\left(k_{-}, P\right)$$

where $\delta_n^x(k_{\mathrm{M}}) = \delta(n\cdot k - x\, n\cdot P)$.

The Mellin moments of the distribution: $\langle x^m\rangle_{\psi_{\mathrm{M}}^q} = \int_0^1 dx\, x^m\, \psi_{\mathrm{M}}^q\left(x, k_\perp^2\right)$

The LFWF is rewritten as $\psi_{\mathrm{M}}^q(x,k_\perp^2) = \left[\frac{12}{n_{\mathrm{M}}}\frac{\mathcal{Y}_{\mathrm{M}}(x,\sigma_\perp^{\nu+1})}{\nu+1}\right]$, where $\mathcal{Y}_{\mathrm{M}}(\alpha,\sigma_\perp^{\nu+1}) = \mathcal{F}_{\mathrm{M}}(\alpha,\sigma_\perp^{\nu+1})(\alpha M_{\bar{h}} + (1-\alpha)M_q)$

On the other hand, in the light-cone formalism the PDA can be expressed as:

$$f_{\mathrm{M}}\phi_{\mathrm{M}}^q(x) = \frac{1}{16\pi^3}\int d^2k_\perp \psi_{\mathrm{M}}^q\left(x,k_\perp^2\right)$$

Integrating over $k_\perp$ we obtain a new relationship between the LFWF and the PDA:

$$\psi_{\mathrm{M}}^q(x,k_\perp^2) = 16\pi^2 f_{\mathrm{M}}\frac{\nu\Lambda_{1-2x}^{2\nu}}{(k_\perp^2+\Lambda_{1-2x}^2)^{\nu+1}}\phi_{\mathrm{M}}^q(x) \Rightarrow$$

No need to construct $\rho_{\mathrm{M}}(w)$

LFWF and PDA: Through the AM

The LFWF for pseudoscalar mesons can be obtained in the light cone formalism by projecting the meson BSWF:

$$\psi^q_{\mathrm{M}}\left(x, k^2_\perp\right) = \mathrm{tr}\int_{dk_\parallel} \delta^x_n(k_{\mathrm{M}})\gamma_5\gamma\cdot n\,\chi_{\mathrm{M}}(k_-, P)$$

$$\psi^q_{\mathrm{M}}(x, k^2_\perp) = 16\pi^2 f_{\mathrm{M}}\frac{\nu\Lambda^{2\nu}_{1-2x}}{(k^2_\perp + \Lambda^2_{1-2x})^{\nu+1}}\phi^q_{\mathrm{M}}(x)$$

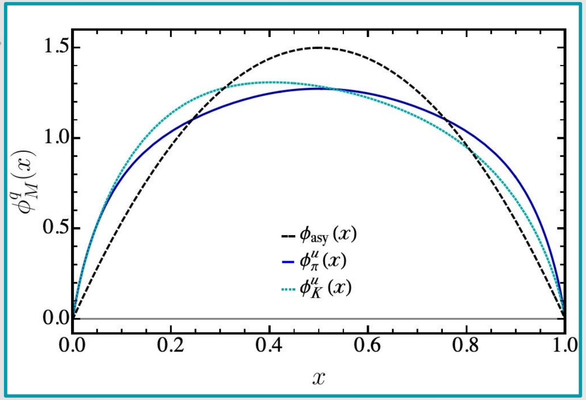
We can observe that in the chiral limit ($m_M = 0,\ M_q = M_{\bar{h}}$)

$$\psi^q_{\mathrm{M}}(x, k^2_\perp) = \left[16\pi^2 f_{\mathrm{M}}\frac{\nu M^{2\nu}_q}{(k^2_\perp + M^2_q)^{\nu+1}}\right]\phi^q_{\mathrm{M}}(x)$$

- The x and $k_\perp$ dependence of the LFWF has been completely factorized.
- So one should expect an increasingly dominant role of x and $k^2_\perp$ correlations in heavy-quarkonia and heavy-light systems.

LFWF and PDA: Through the AM

From:



Parameterized PDAs taken from:

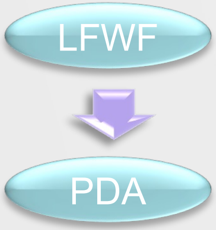
Z.-F. Cui, et. al., Eur. Phys. J. C 80, 1064 (2020).

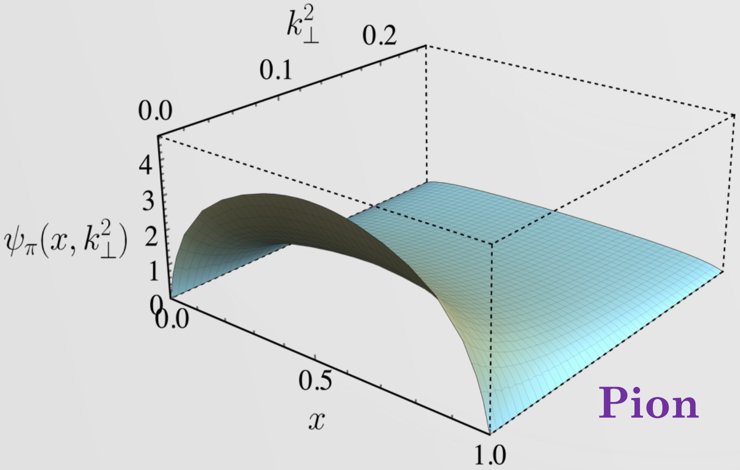
M. Ding, et. al., Phys. Lett. B 753, 330 (2016).

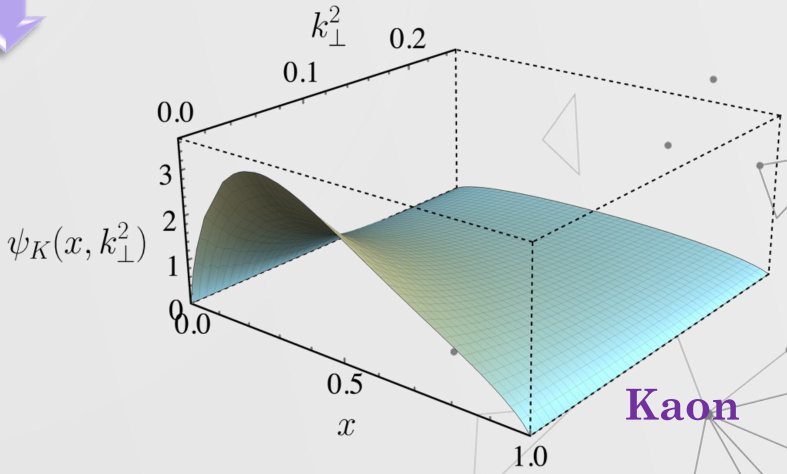
- Asymptotic limit: $6x(1-x)$.

➢ Patterns supported by Lattice:

R. Zhang, et. al., Phys.Rev.D 102 (2020) 9, 094519

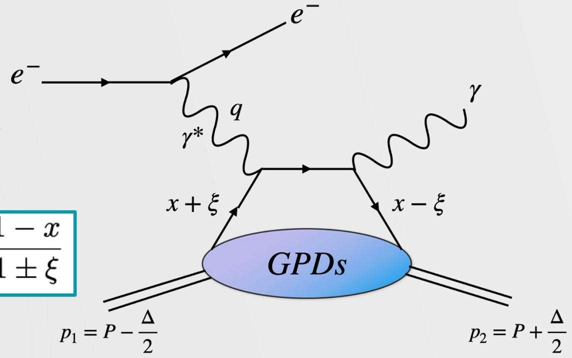






Distribution Functions: GPDs

The GPD corresponding to the valence quarks can be obtained at a hadronic scale through the overlap configuration of the LFWF:



$$H_{\mathrm{M}}^{q}(x,\xi,t) = \int \frac{d^2k_\perp}{16\pi^3} \psi_{\mathrm{M}}^{q*}\left(x^-, (\mathbf{k}_\perp^-)^2\right) \psi_{\mathrm{M}}^{q}\left(x^+, (\mathbf{k}_\perp^+)^2\right)$$

$$x^\pm = \frac{x\pm\xi}{1\pm\xi} \quad , \quad \mathbf{k}_\perp^\pm = k_\perp \mp \frac{\Delta}{2}\frac{1-x}{1\pm\xi}$$

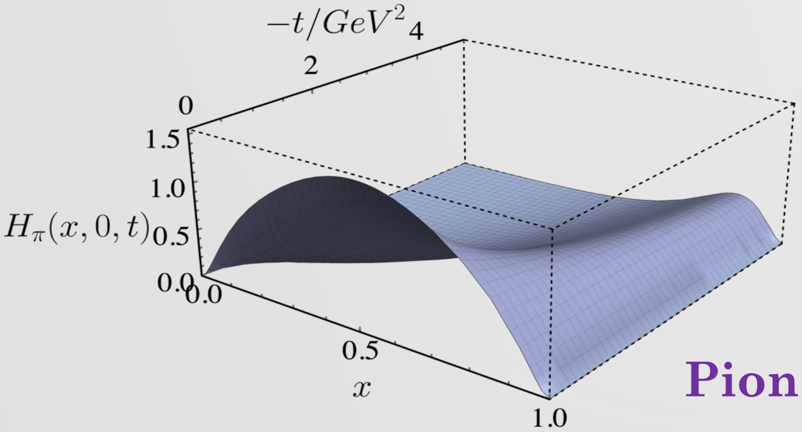
$$|x| > \xi$$

Taking advantage of the LFWF relation in terms of the PDA, the GPD take the form

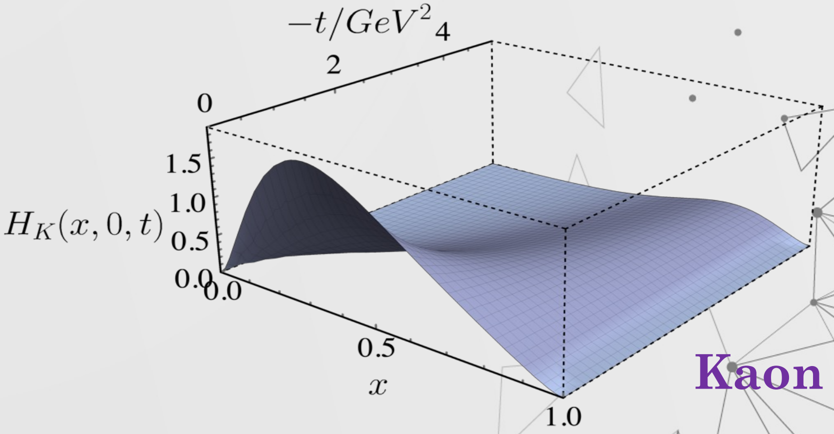
$$H_{\mathrm{M}}^{q}(x,0,t) = \frac{\Gamma(2\nu+2)}{\Gamma^2(\nu+1)} \mathcal{N} \phi_{\mathrm{M}}^{q2}(x) \Lambda_{1-2x}^{4\nu} \int_0^1 du \frac{u^\nu (1-u)^\nu}{[\mathbb{M}^2(u)]^{2\nu+1}}$$

with

$$\mathbb{M}^2(u) = t(1-x)^2 u(1-u)^2 + \Lambda_{1-2x^-}^2$$

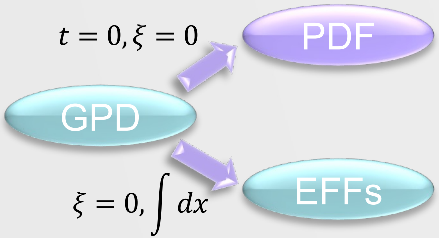


Pion



Kaon

Distribution Functions: PDFs



The PDF is defined as follows:

$$q_{\mathrm{M}}(x) \equiv H^q_{\mathrm{M}}(x,0,0) = \mathcal{N}\frac{\phi^2_{\mathrm{M}}(x)}{\Lambda^2_{1-2x}}$$

where the normalization factor is:

$$\mathcal{N} = \left[\int_0^1 dx\, \frac{\phi^2_{\mathrm{M}}(x)}{\Lambda^2_{1-2x}}\right]^{-1}$$

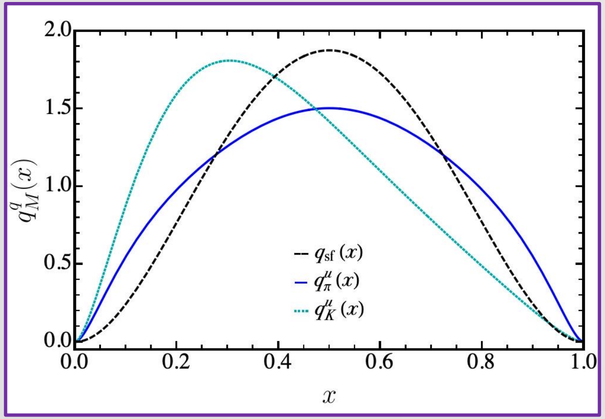
$$\frac{\partial q^{NS}}{\partial ln\mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} P_{qq} \otimes q^{NS},$$

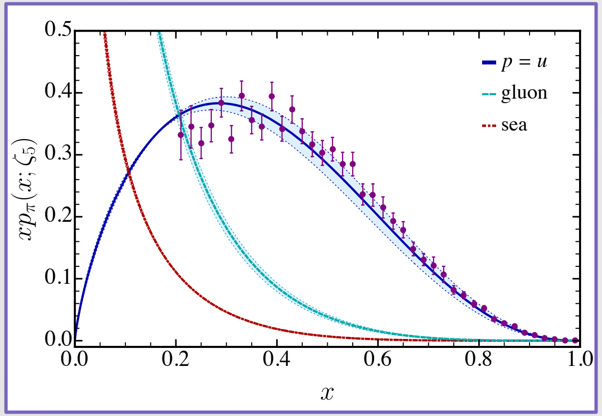
$$\frac{\partial}{\partial ln\mu^2}\begin{pmatrix} q^S \\ g \end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi}\begin{pmatrix} P_{qq} & 2N_f P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q^S \\ g \end{pmatrix}$$

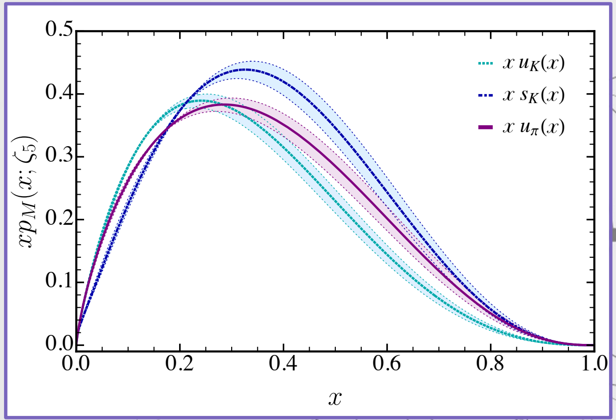
G. Altarelli and G. Parisi, Nucl. Phys.B126, 298 (1977).

Applying QCD-evolution equations to PDFs, they can be evolve up to $\zeta_5 = 5.2\ GeV$ where experimental data for pion is available.

L. Albino, I.M. Higuera-Angulo, K. Raya, A. Bashir. Phys. Rev. D106 (2022) 3, 034003



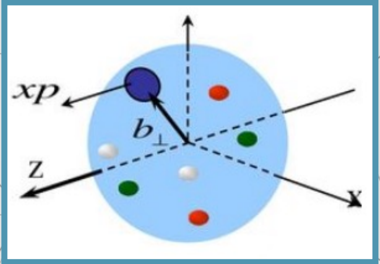




Impact Parameter Space GPD

- Likelihood of locating a parton at a transverse position b⊥ relative to the transverse momentum center of the meson.

$$u_M(x, b_\perp^2, \zeta_H) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta) H_M(x, 0, t)$$

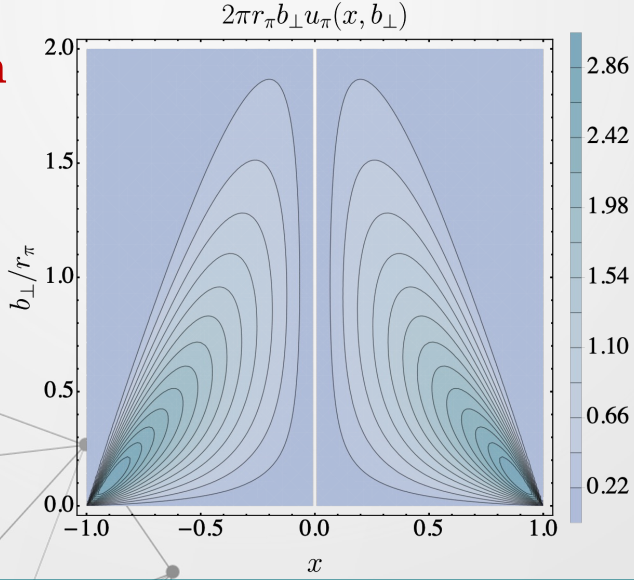


where J_0 is a cylindrical Bessel function.

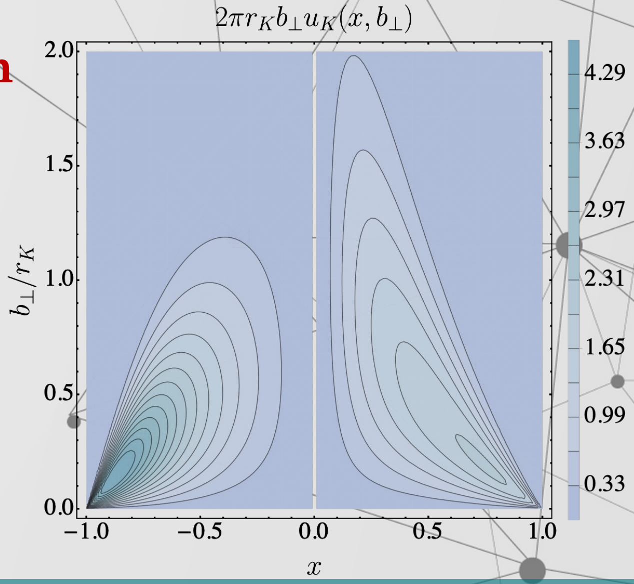
The AM allows analytical integration, leading to:

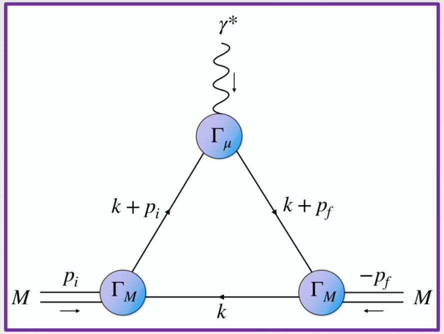
$$u_M(x, b_\perp^2, \zeta_H) = \frac{q_M(x)}{4\pi f(x)} e^{-\frac{b_\perp^2}{4f(x)}}$$

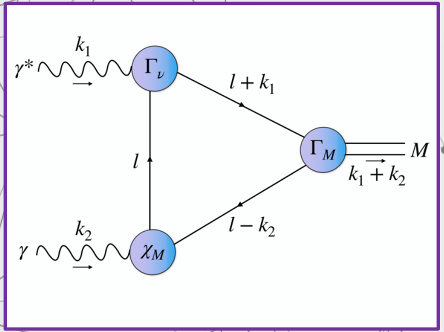
Pion



Kaon







Triangle diagram in the calculation of EFFs and TFFs

"Data-driven algebraic model computation of electromagnetic and transition form factors of pseudoscalar mesons to γγ*", I.M. Higuera-Angulo, A. Bashir, R.J. Hernandez-Pinto, K. Raya.

Status: Approaching the conclusion of the writing process.

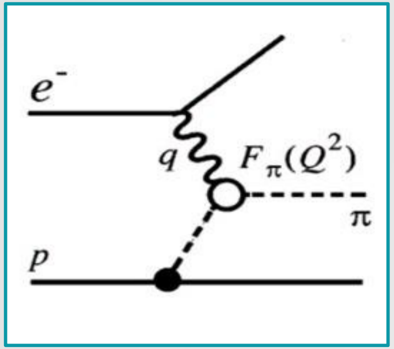
Planned for publication in Phys. Rev. D.

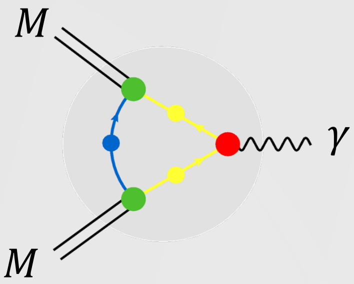
http://bibliotecavirtual.dgb.umich.mx:8083/xmlui/handle/DGB_UMICH/16592

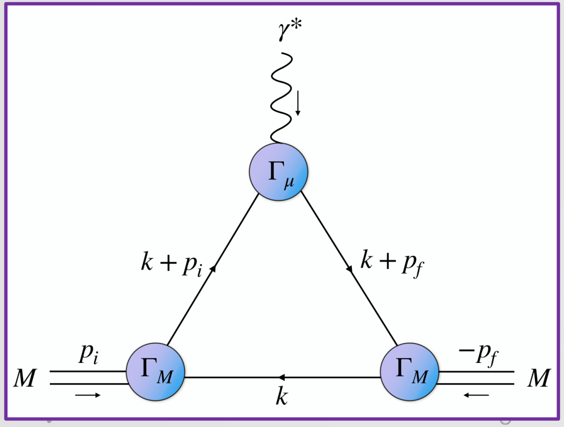
Electromagnetic Form Factors

$$F(q^2) = \int \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} d^3\mathbf{r}$$

$e^-p \to e^-p\pi$







In order to compute the **EFFs** we need to solve the **MγM−vertex**.

$$K_\mu F_M^q(Q^2) = N_c \,\mathrm{tr} \int \frac{d^4k}{(2\pi)^4} \chi_\mu^q(k+p_f, k+p_i) \Gamma_M(k_i, p_i) S_{\bar{h}}(k) \Gamma_M(k_f, p_f)\,,$$

where, $p_{f,i} = K \pm Q/2,$ $k_{f,i} = k + p_{f,i}/2,$ $K \cdot Q = 0$ $p_{f(i)}^2 = K^2 + Q^2/2 = -m_M^2.$

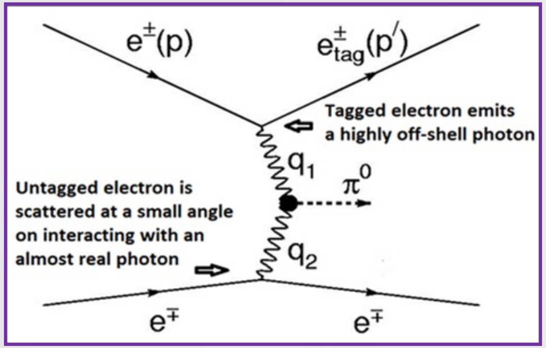
Once again, the total meson **EFF** is:

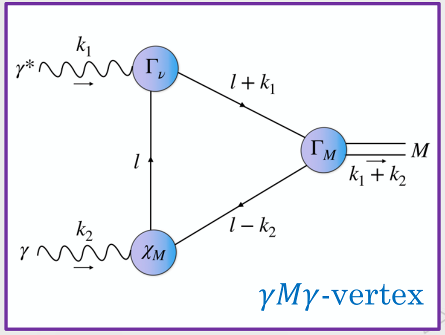
$$F_M(Q^2) = e_q F_M^q(Q^2) + e_{\bar{h}} F_M^{\bar{h}}(Q^2)$$

Transition Form Factors: AM

The $\gamma\gamma^* \to M$ transition appears in the process:

$$e^- e^+ \to e^- e^+ \pi^0$$





Then, the Transition Form Factors (TFF) of $\gamma\gamma^* \to M$ is expressed as:

$$\begin{aligned} \mathcal{T}_{\mu\nu}(k_1, k_2) &= \mathcal{T}_{\mu\nu}(k_1, k_2) + \mathcal{T}_{\nu\mu}(k_2, k_1) \\ &= \frac{e^2}{4\pi^2} \epsilon_{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} G_{\mathrm{M}}(k_1^2, k_2^2, k_1 \cdot k_2)\,, \end{aligned}$$

$$\mathcal{T}_{\mu\nu}(k_1, k_2) = \mathrm{tr} \int_l i\mathcal{Q}\chi_\mu(l, l+k_1)\Gamma_\pi(l+k_1, l-k_2)S(l-k_2)i\mathcal{Q}\Gamma_\nu(l-k_2, l)$$

where $\boxed{\mathcal{Q} = \mathrm{diag}[e_q, e_h]}$ and the kinematic conditions: $k_1^2 = Q^2$, $k_2^2 = 0$ *and* $2k_1 \cdot k_2 = -(m_M^2 Q^2)$.

Results: the π meson case

Global analysis using the standard χ^2 statistical test:

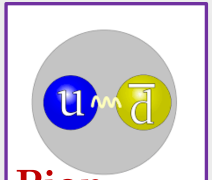
$$\chi^2 = \sum_{i=1}^{N} \frac{(T_i - E_i)^2}{\delta E_i^2},$$

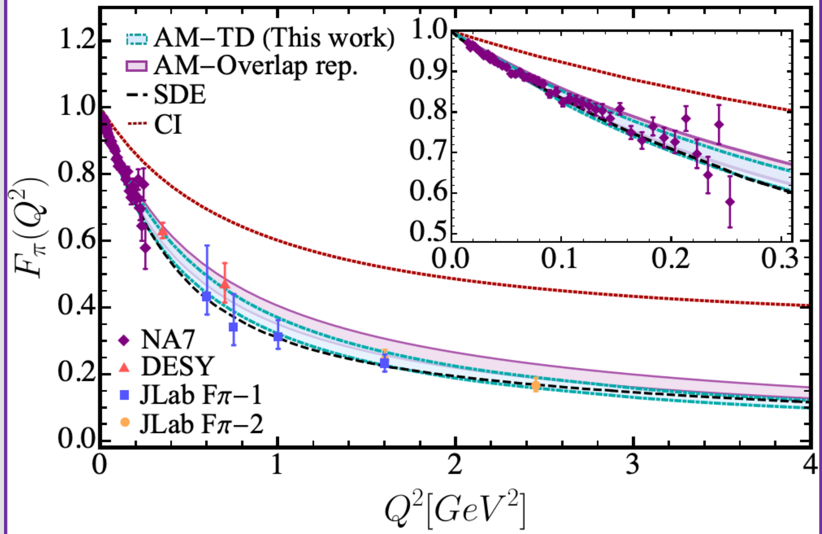
Error bands ➡ 5% variation on the charge radii

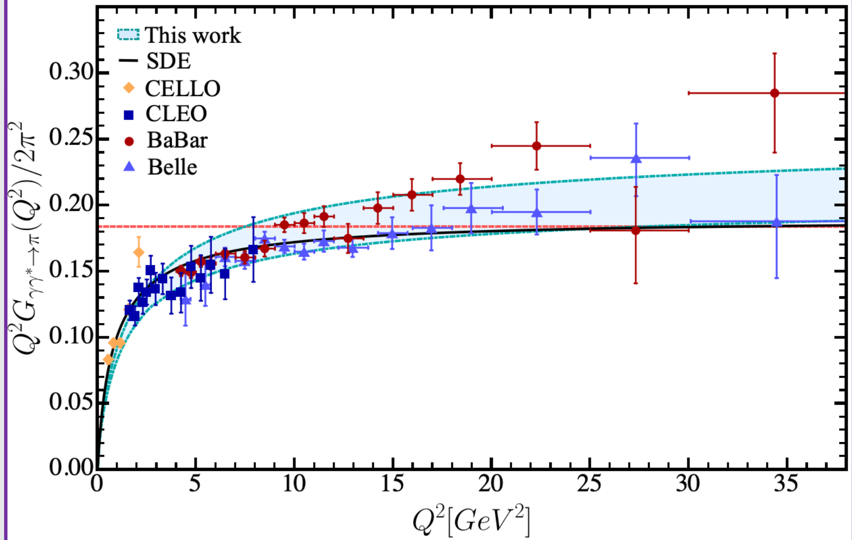
➤ Best fitted values for the pion meson in the AM:

m_u	m_π	ν_π	$\alpha_u^{(0)}$
0.3135	0.1395	0.8428	0.1964

$\chi^2/d.o.f. \sim 1.93$



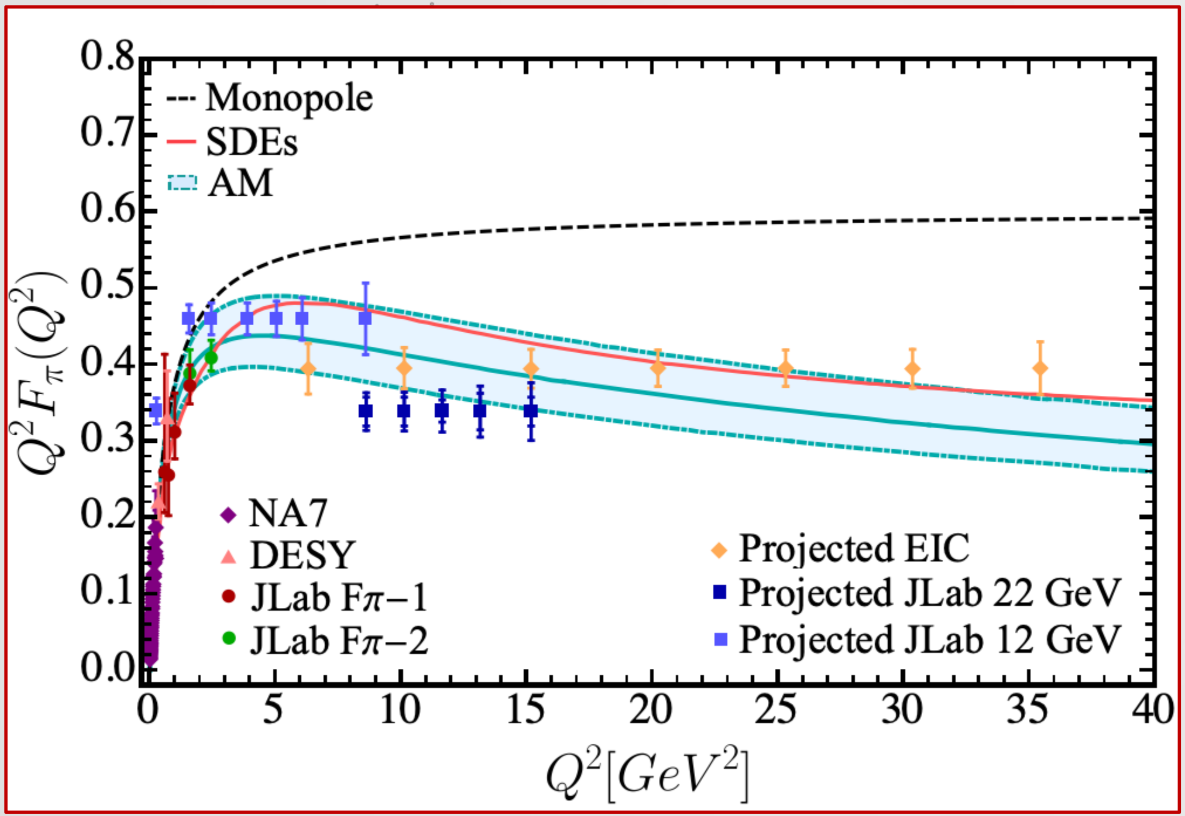


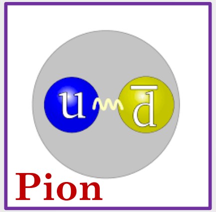


Red dotted line: *R.J. Hernandez-Pinto, et.al., Phys.Rev.D 107 (2023) 5, 054002*

Conform limit: $\exists Q_0 > \Lambda_{\mathrm{QCD}} \mid Q^2 G(Q^2) \overset{Q^2 \geq Q_0^2}{\approx} 4\pi^2 f_\pi$

Results: the π meson case





- The charge radius of pions from EEF

$r_\pi^{\text{fit}} = 0.67\ \text{fm}$

We analyze the expected theoretical predictions of the AM for EFF of pions for the expected center of mass energies of the EIC and Jlab.

- Eur. Phys. J. A 55.10 (2019)
- JLAB-PHY-23-3840 (2023)

Results: the K case

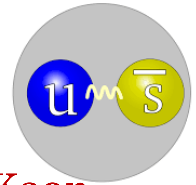
- Best fitted values for the pion meson in the AM

m_s	m_K	ν_K	$\alpha_s^{(0)}$
0.5274	0.4936	0.913	–

$\chi^2/d.o.f. \sim 0.705$

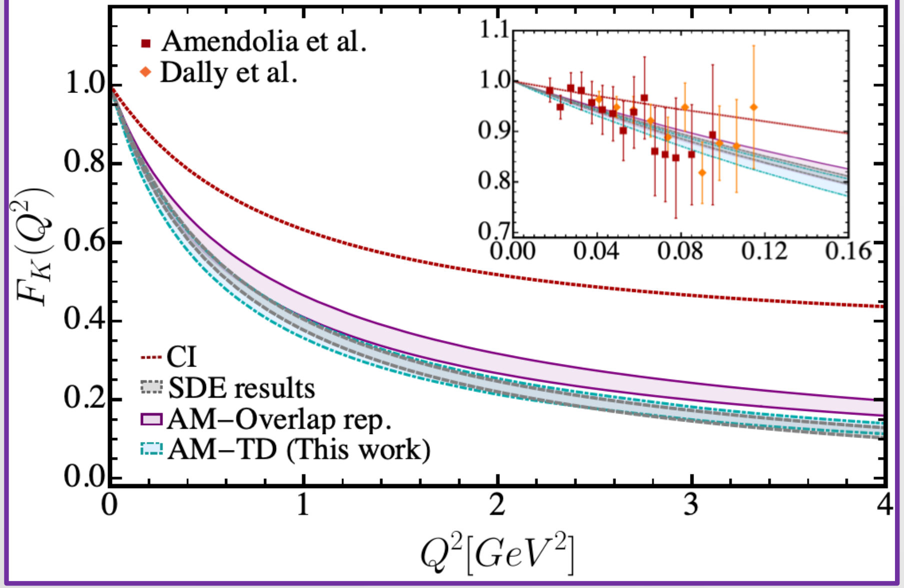
- The charge radius of pions from EEF

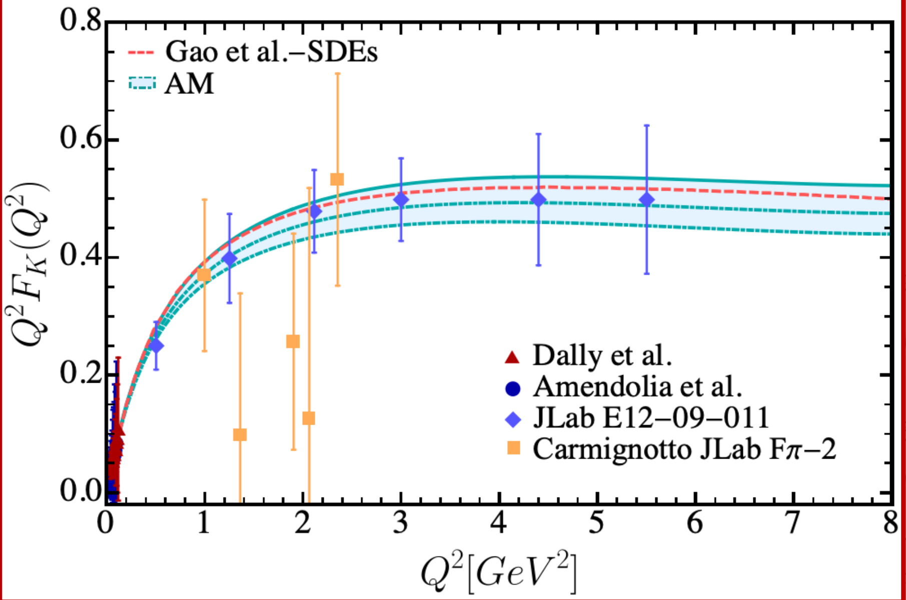
$r_K^{\text{fit}} = 0.64$ fm



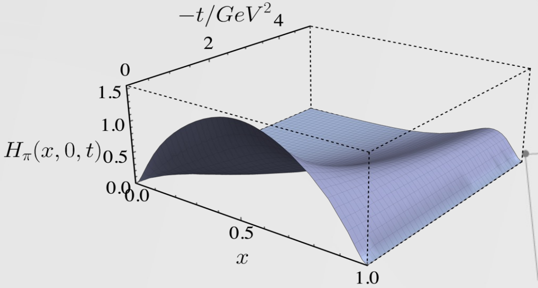
Kaon

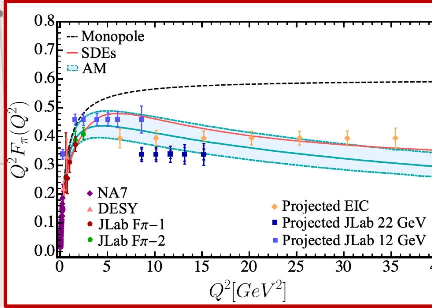
Jlab E12-09-011 Analysis in progress





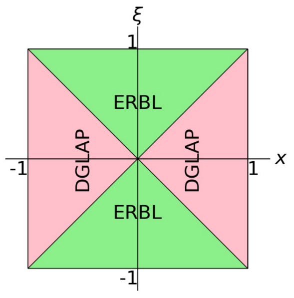
Final remarks





- $(\Lambda \to \Lambda(\omega))$ ⟶ Analytical calculations of all the distribution functions.

- Analytical relationship between all the non-perturbative functions and the PDAs



- The overlap representation only takes into account the DGLAP region
  - ⟶ There is no contribution from the ERBL region. ⟶ Covariant extension.

N. Chouika *Eur.Phys.J.C 77 (2017) 12, 906*

  - ⟶ The DGLAP domain still provides a wealth of information that also closely aligns with experimental findings.

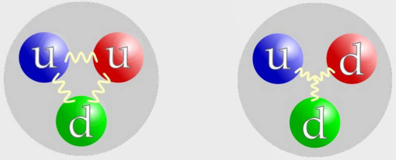
- Good agreement with experimental and SDE results ⟶ Allows new predictions.

- Using the TD approximation we can also extract EFFs by solving the interaction $M\gamma M$ vertex and the TFFs by solving the interaction $\gamma M \gamma^*$ vertex.
  - ⟶ By performing a phenomenological analysis we observed more reliable results.

- Much more to know: complete GPDs

Future perspectives

Proton Neutron



- Three-body case: Nucleons ⟶ Other models
- Realistic extractions of GPDs ⟶ From experimental measurements emanating from the DVCS, DVMP,...
- The GPDs are directly related to the CFFs :

$$\boxed{\mathcal{H}^q(\xi,Q^2)=\int_{-1}^{1}\frac{\mathrm{d}x}{2\xi}T^q\left(\frac{x}{\xi},\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)H^{q(+)}(x,\xi,\mu^2)}$$

⟶ Non-trivial

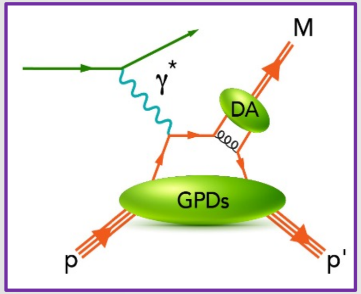
Deconvolution problem

Existence of shadow GPD.

V. Bertone, et. al, Phys.Rev.D 103 (2021) 11,114019

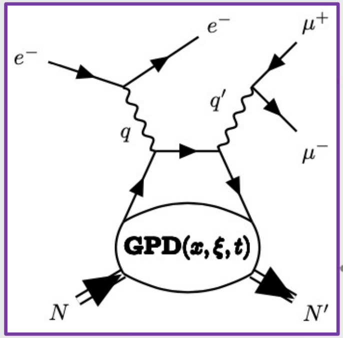
- Different exclusive processes to observe:

- DVMP:
  - ⟶ Depends on the meson DA
  - ⟶ Acces to gluon GPD at LO



Jefferson Lab Hall A Collaboration, Phys.Rev.Lett. 117 (2016) 26, 262001

- Double DVCS:
  - ⟶ Possibility of directly measuring GPDs for $x \neq \pm\xi$

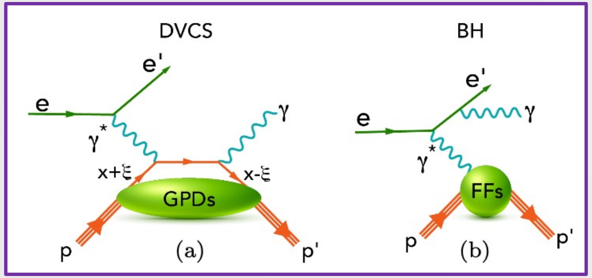


K. Deja, et. al, Acta Phys.Polon.Supp. 16 (2023) 7, 7-A24

- TCS...

Future perspectives

V.D. Burkert, et. al, *Rev.Mod.Phys.* 95 (2023) 4, 041002



- Furthermore, we can access to the gravitational form factors (GFFs) from the GPDs

Gravitational form factors

$$\int_{-1}^{1} dx\; xH^q(x,\xi,t) = \boxed{A^q(t)} + \xi^2 \boxed{D^q(t)}$$

$$\int_{-1}^{1} dx\; xE^q(x,\xi,t) = \boxed{B^q(t)} - \xi^2 \boxed{D^q(t)}$$

  - The GFFs can elucidate the proton's mass and spin decompositions.
  - "Spin puzzle": Understanding the spin composition of the nucleon using the Ji's sum rule

$$J_Q = \sum_q \frac{1}{2}\int_{-1}^{1} dx\; x(H^q(x,\xi,0) + E^q(x,\xi,0)) = \sum_q \frac{1}{2}\boxed{(A^q(t) + B^q(t))}$$

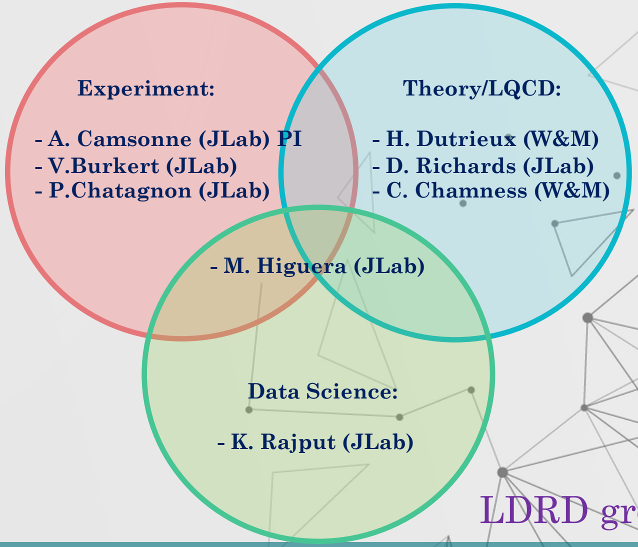
New tools! → Neural Networks

- We can fit all GPDs data available using NN.

Tools

- Phyton software framework dedicated to the study of GPDs
- K. Kumerički, D. Müller, K. Passek-Kumerički, Towards a fitting procedure for deeply virtual Compton scattering at next-to-leading order and beyond, Nuclear Physics B, Volume 794, Issues 1–2, 2008

- C++ software framework, dedicated to the phenomenology of GPDs.
- B. Berthou et al., PARTONS: PARtonic Tomography Of Nucleon Software Eur. Phys. J. C78 (2018), 478.



LDRD group

Goal:

- The goal is to fit all GPDs available data using NN.
  - Extract the GFFs
- Handle experimental and LQCD data.
- Use state-of-the-art ML libraries: Pytorch.

Thank you for your attention!

Backups

GPDs strategies

- Overlap representation of the LFWF

$$\boxed{H^q_{\mathrm{M}}(x,\xi,t) = \int \frac{d^2k_\perp}{16\pi^3}\psi^{q*}_{\mathrm{M}}\left(x^-,(\mathbf{k}^-_\perp)^2\right)\psi^q_{\mathrm{M}}\left(x^+,(\mathbf{k}^+_\perp)^2\right)}$$

  ✓ Positivity condition fulfilled by construction
  Restricted to the DGLAP domain ($|x| > |\xi|$)

  ✓ Many distribution functions can be available within this domain.

- Double distribution representation: The GPD can be written as the Radon transform of the sum of DD

$$\boxed{H(x,\xi,t) = \int_\Omega \mathrm{d}\beta\mathrm{d}\alpha\,\delta(x-\beta-\alpha\xi)[F(\beta,\alpha,t)+\underbrace{\xi\delta(\beta)D(\alpha,t)}_{\text{D-term}}].}$$

  ✓ Polynomiality can be explicitly proven
  Positivity is not guaranteed