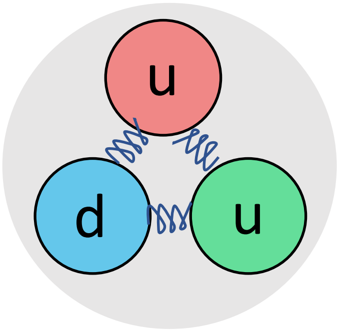
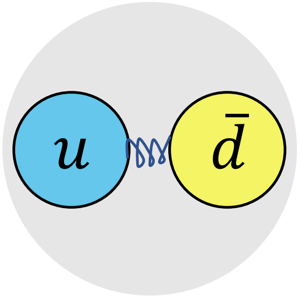
Gravitational form factors from lattice QCD



Dimitra Anastasia Pefkou

CFNS 2024
Stony Brook University



Berkeley UNIVERSITY OF CALIFORNIA

BERKELEY LAB

In collaboration with:

1.[**DAP** Hackett Shanahan PRD (2022) arXiv:2107.10368]
2.[Hackett Oare **DAP** Shanahan PRD (2023) arXiv:2307.11707]
3.[Hackett **DAP** Shanahan arXiv:2310.08484]
4.[Abbott et al, in preparation]

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4

Contents of this talk

- Introduction
- Bare gravitational form factors (GFFs) from lattice QCD
- Non-perturbative renormalization
- GFFs of the proton, pion, and other hadrons: selected results

  [Hackett Oare **DAP** Shanahan PRD (2023) arXiv:2307.11707**]**
  [Hackett **DAP** Shanahan arXiv:2310.08484]
  [**DAP** Hackett Shanahan PRD (2022) arXiv:2107.10368]

- Bonus: GFFs of scalar glueball at pure Yang-Mills

  [Abbott Hackett DAP Romero-Lopez Shanahan Urban, in preparation]

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Gravitational form factors

Gravitational form factors are the form factors of the energy-momentum tensor

$$T^{\mu\nu} = \underbrace{-F_a^{\mu\alpha}F^{\nu}_{a,\alpha} + \frac{1}{4}g^{\mu\nu}F_a^{\alpha\beta}F_{a,\alpha\beta}}_{T_g^{\mu\nu}} + \underbrace{\sum\nolimits_f i\bar{\psi}_f\gamma^\mu D^\nu\psi_f}_{T_q^{\mu\nu}}$$

increases with spin

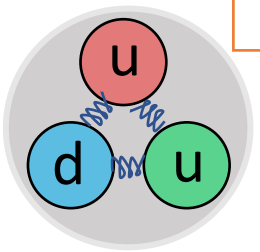
- $\langle p', s' | T_i^{\mu\nu} | p, s\rangle \sim$ Kinematic coefficients (Lorentz structure) $\times$ Gravitational form factors $\mathcal{G}_i(t)$ (scalar functions of $t = -(p'-p)^2$)
- $\partial_\mu T_i^{\mu\nu} \neq 0$ → $\mathcal{G}_q(t), \mathcal{G}_g(t)$ renormalization scheme and scale dependent
- $\partial_\mu T^{\mu\nu} = 0$ → $\mathcal{G}(t) \equiv \mathcal{G}_{q+g}(t)$ scheme and scale independent
- Poincaré symmetry constraints, e.g., $\int d^3x\, T^{00}|p,s\rangle = m|p,s\rangle$, encoded in $\mathcal{G}(t)$
- Proton: $\langle p', s' | T_i^{\mu\nu} | p, s\rangle \sim A_i(t), J_i(t), D_i(t), \bar{c}_i(t)$

totals: $A(0) = 1$, $J(0) = \frac{1}{2}$, $\bar{c}(t) = 0$, $D(0) = ?$

totals			
$A(0) = 1$	$J(0) = \frac{1}{2}$	$\bar{c}(t) = 0$	$D(0) = ?$
momentum	angular momentum	$T^{\mu\nu}$ conserved	``The last global unknown'' Polyakov Schweitzer 2018

T^{00}	T^{01}	T^{02}	T^{03}	energy
T^{10}	T^{11}	T^{12}	T^{13}	momentum density
T^{20}	T^{21}	T^{22}	T^{23}	energy flux
T^{30}	T^{23}	T^{32}	T^{33}	pressure
				shear stress
				momentum flux

Gravitational form factors encode the distribution of energy, angular momentum, and mechanical properties within hadrons



Gravitational FFs ∋ Generalized FFs

$x = \frac{-q^2}{2p^\mu q_\mu}$

$q = p' - p$

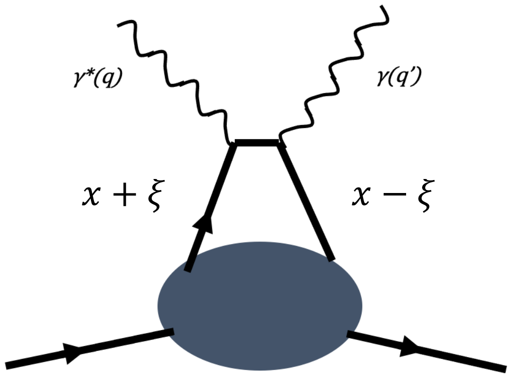
$\xi = \frac{p^+ - p'^+}{p^+ + p'^+}$

The second Mellin moment of generalized parton distributions yields the gravitational form factors (generalized form factors)

- Quark and gluon generalized parton distributions (GPDs)
  ~ matrix elements of $\bar{\psi}(-n/2)\gamma^\mu \mathcal{U}\,\psi(n/2), F^{\mu\alpha}(-n/2)\mathcal{U}F_\alpha^\nu(n/2)$

  path-ordered gauge link

  light-like vector

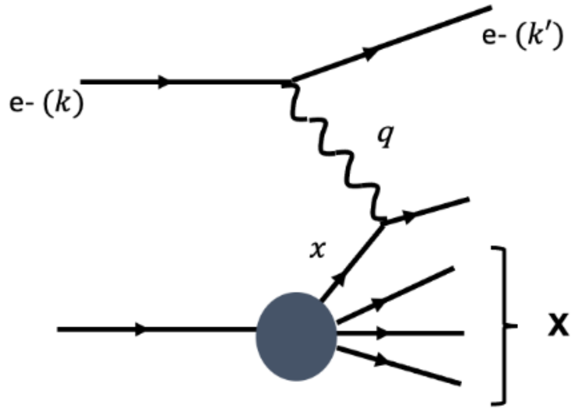


- Operator product expansion → tower of local operators
  lowest order: **traceless** $\hat{T}_q^{\mu\nu}, \hat{T}_g^{\mu\nu}$ (twist-2)

$B(t) = 2J(t) - A(t)$

- Proton: $\int_{-1}^{1} dx\, x\, H_i(x,\xi,t) = A_i(t) + \xi^2 D_i(t)$, $\int_{-1}^{1} dx\, x\, E_i(x,\xi,t) = B_i(t) - \xi^2 D_i(t)$

- Forward limit $t = 0$: 2nd Mellin moment of parton distribution functions (PDFs)
  e.g. $\int_0^1 dx\, x\, f_i(x) = A_i(0)$



Constraints on GFFs: examples

* see e.g. Burkert et al Rev.Mod.Phys. 2023 for review

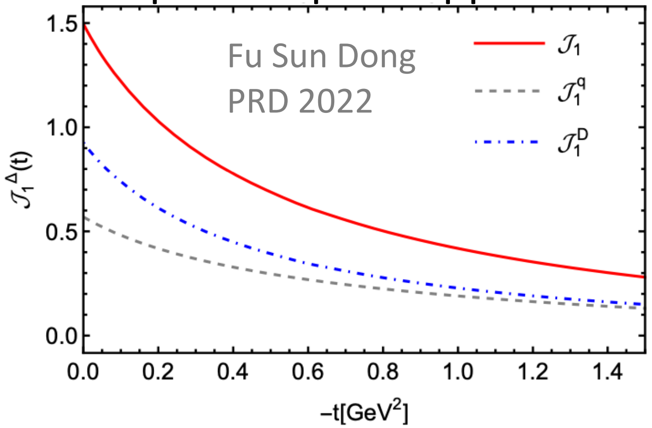
The GFFs have gained increasing interest in recent years, after their first phenomenological extractions

See Melany Higuera Angulo's talk earlier

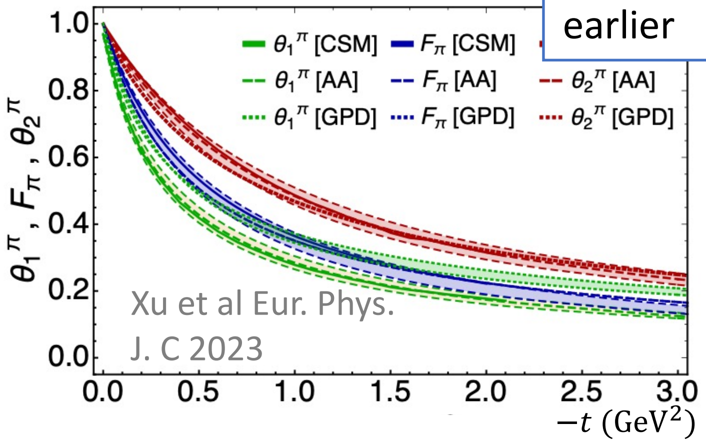
- Effective field theory and models

  - chPT: $D(0) = -1$ for the Nambu Goldstone bosons in the chiral limit (generally unknown for hadrons)

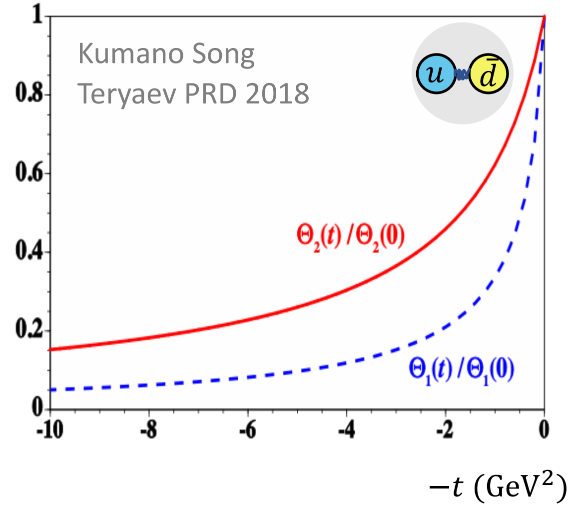
- Δ baryon in relativistic quark-diquark approach

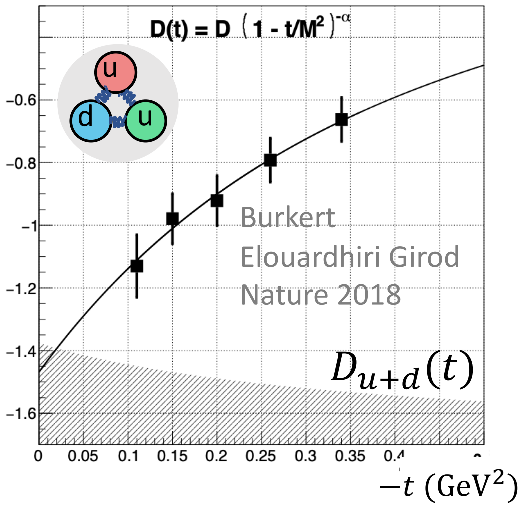


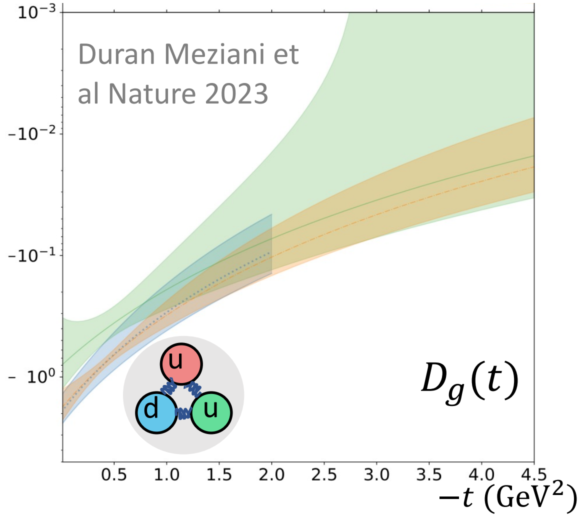
- Continuum Schwinger function methods

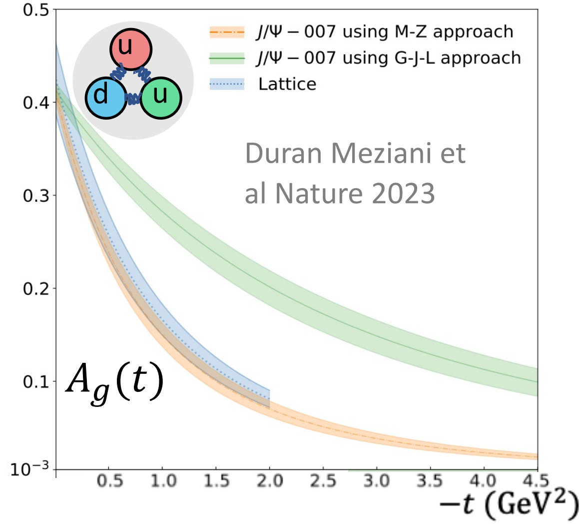


- Indirect experimental access (* Recent suggested direct access [Hatta PRD 2024])





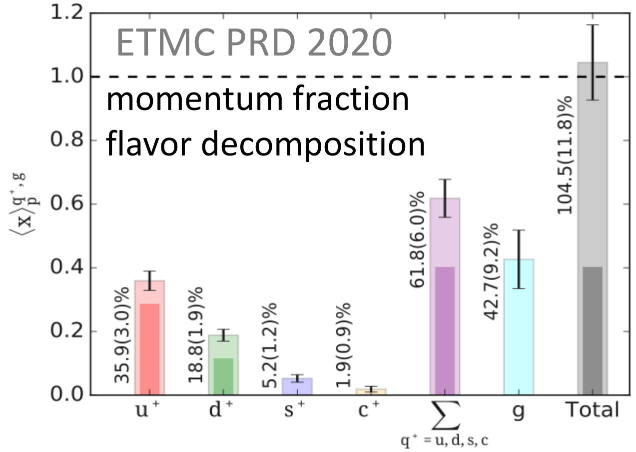


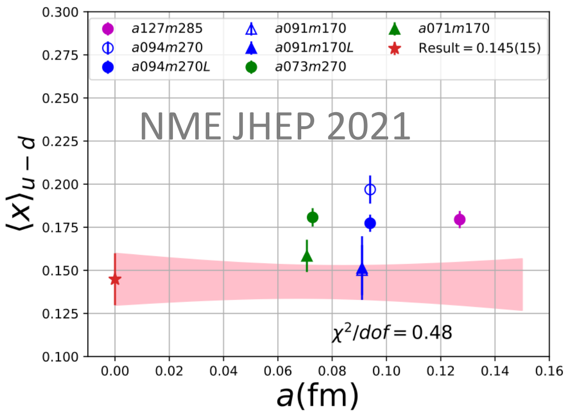


Constraints on GFFs from lattice QCD: examples

Older lattice QCD literature: connected quark momentum fraction and generalized form factors
More recently: disconnected contributions to momentum and spin fraction, gluon GFFs

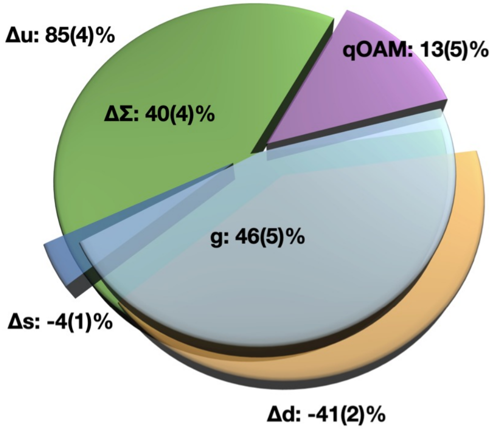
Proton

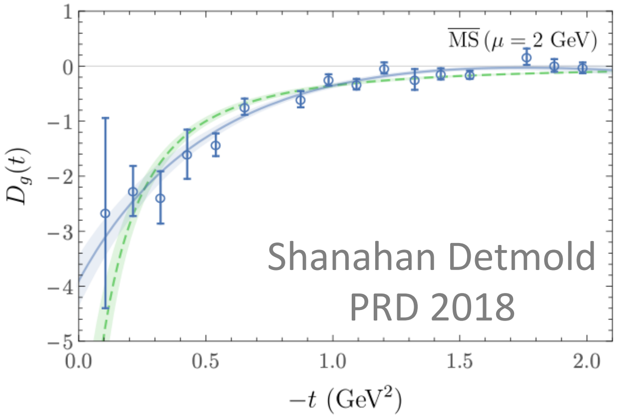




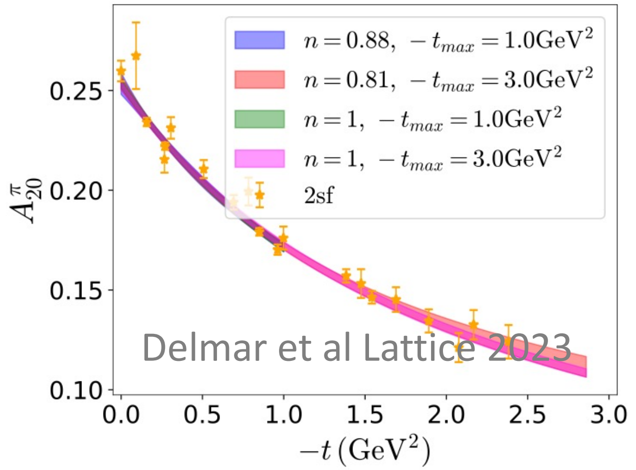
χQCD PRD 2022

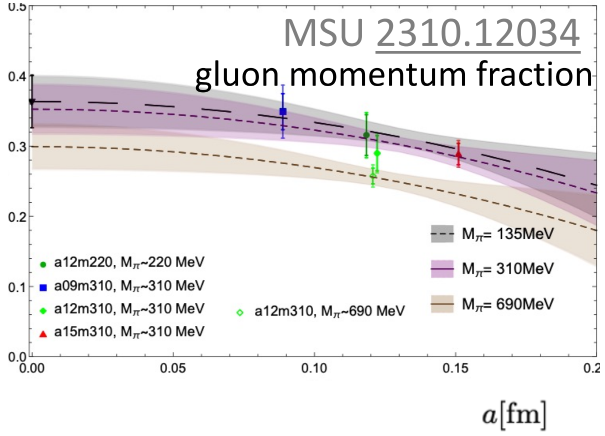
spin decomposition

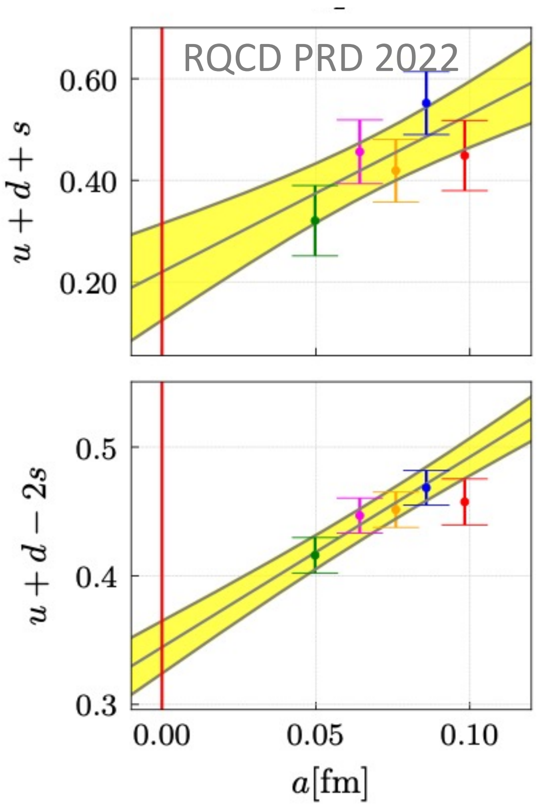




Pion







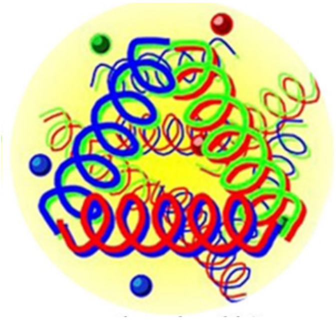
quark momentum fraction

Other hadronic states?

Understanding the structure of unstable hadrons is possible from lattice QCD

See Raúl Briceño's talk on Thursday

- States unstable under strong interactions: e.g. resonances (rho, delta, etc…)
- Information encoded in GFFs (e.g. gluon momentum fraction, radii from form factors) highly interesting, inaccessible experimentally
- GFFs from Lattice QCD: possible (e.g. Baroni et al PRD 2019) but very challenging
- Consider unphysical quark masses where states are stable
- States outside quark model: e.g. glueballs in Yang Mills



Credit: University of Glasgow

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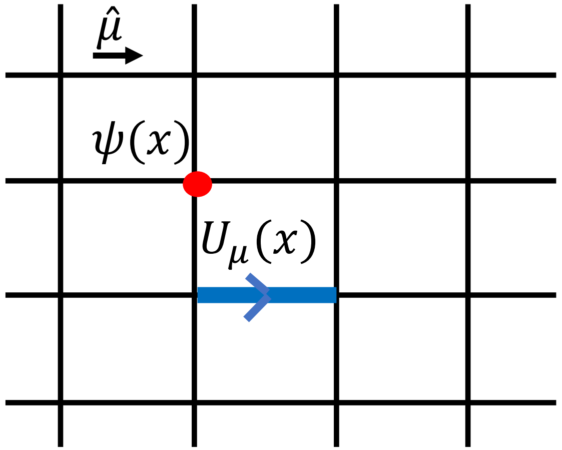
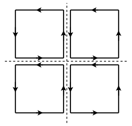
GFFs from lattice QCD: EMT



$$T^{\mu\nu} = -F_a^{\mu\alpha} F_{a,\alpha}^{\nu} + \frac{1}{4} g^{\mu\nu} F_a^{\alpha\beta} F_{a,\alpha\beta} + \sum_f i\bar{\psi}_f \gamma^{\{\mu} D^{\nu\}} \psi_f$$

$$= \sum_{i\,\in\{q,g\}} T_i^{\mu\nu}$$

- $T_i^{\mu\nu}$: write in terms of Euclidean lattice fields

$F_{\mu\nu} \sim$ 

$$(\vec{D}_\mu \psi)(x) = \frac{1}{2}\Big(U_\mu(x)\psi(x+a\hat{\mu}) - U_\mu^\dagger(x-a\hat{\mu})\psi(x-a\hat{\mu})\Big),$$
$$(\bar{\psi}\,\overleftarrow{D}_\mu)(x) = \frac{1}{2}\Big(\bar{\psi}(x+a\hat{\mu})U_\mu^\dagger(x) - \psi(x-a\hat{\mu})U_\mu(x-a\hat{\mu})\Big)$$

- $T_i^{\mu\nu}$: isotropic hypercubic lattice: Lorentz group → $H(4)$

  symmetric traceless components transform under $\tau_1^{(3)}$(diagonal), $\tau_3^{(6)}$(off-diagonal)

  Gockeler et al PRD 1996

- $T_i^{\mu\nu}$: flavor singlet $q = u + d + s + \cdots$ mixes with g

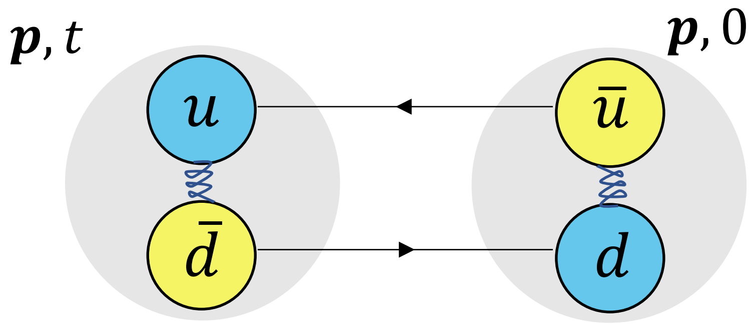
  non-singlet $u - d$, $u + d - 2s$ renormalize multiplicatively

Lattice simulation

	m_π (MeV)	a (fm)	$L^3 \times T$	N_f
Ens. A	450	0.12	$32^3 \times 96$	$2+1$
Ens. B	170	0.09	$48^3 \times 96$	$2+1$

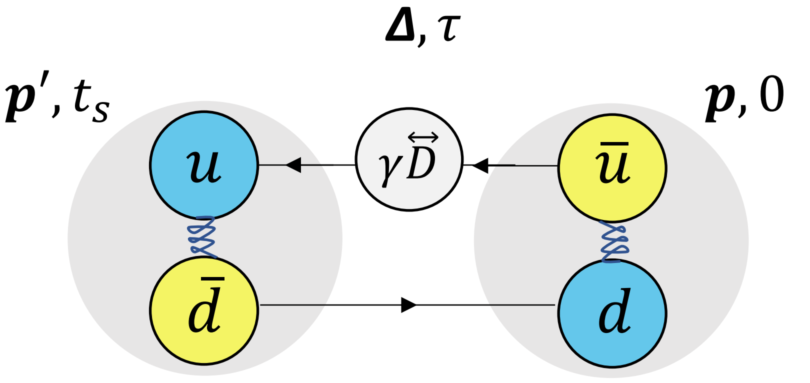
Clover-improved Wilson quarks, Lüscher-Weisz gauge action generated by JLab/LANL/MIT/WM groups

2-point functions ~ $e^{-E_{\boldsymbol{p}} t}, E_{\boldsymbol{p}} = \sqrt{m^2 + |\boldsymbol{p}|^2}$

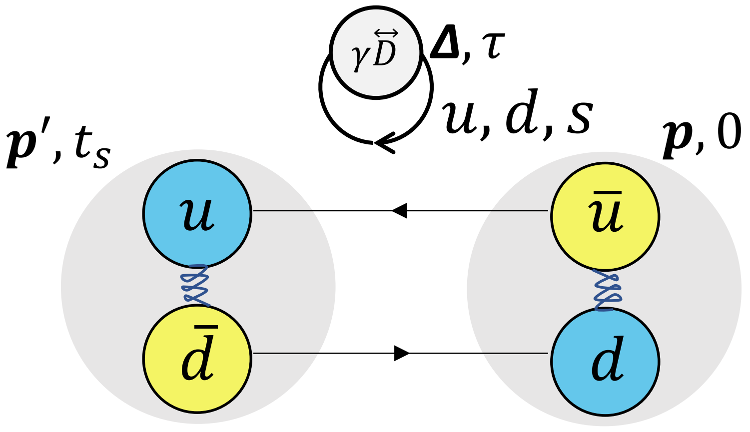


3-point functions ~ Matrix elements $\langle h(p', s') | T^{\mu\nu}_{q,g} | h(p, s) \rangle$

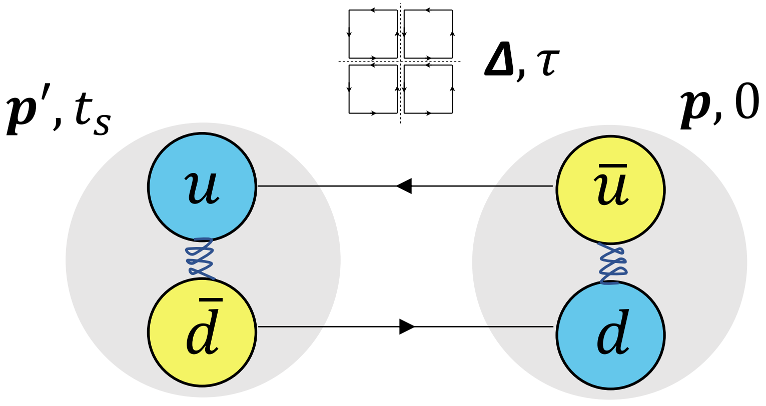
Connected contribution



Disconnected contribution



Gluon contribution

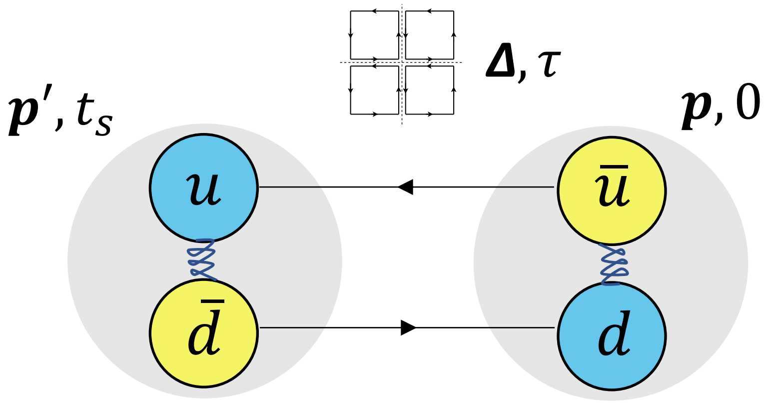


Gluon GFFs of the pion, rho meson, proton, and delta baryon

DAP, Hackett, Shanahan PRD (2022)

	m_π (MeV)	a (fm)	$L^3 \times T$	N_f
Ens. A	450	0.12	$32^3 \times 96$	$2+1$

Clover-improved Wilson quarks, Lüscher-Weisz gauge action generated by JLab/LANL/MIT/WM groups



→ 2820 configurations

→ $\frac{t_{\text{flow}}}{a^2} = 1$

→ 235 sources

→ $|\boldsymbol{\Delta}|^2 \leq 18(\frac{2\pi}{L})^2$

→ $|\boldsymbol{p}'|^2 \leq 10(\frac{2\pi}{L})^2$

Quark and gluon GFFs

	m_π (MeV)	a (fm)	$L^3 \times T$	N_f
Ens. B	170	0.09	$48^3 \times 96$	$2+1$

Pion: Hackett, Oare, **DAP**, Shanahan PRD (2023)
Proton: Hacket, **DAP**, Shanahan 2310.08484

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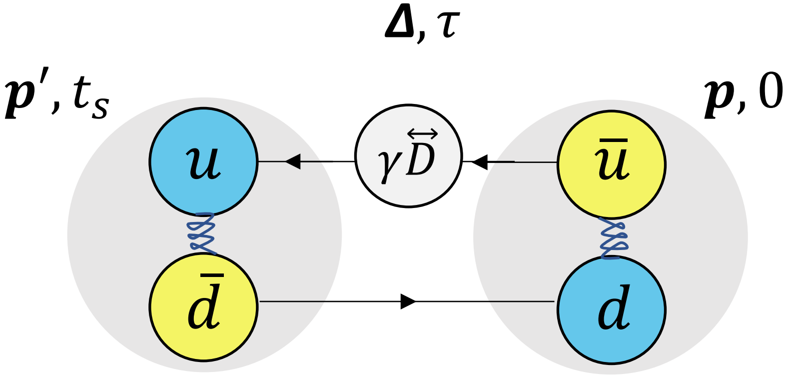
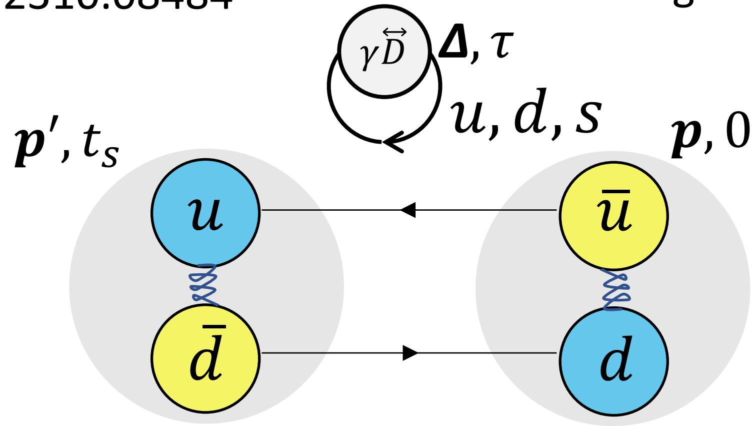
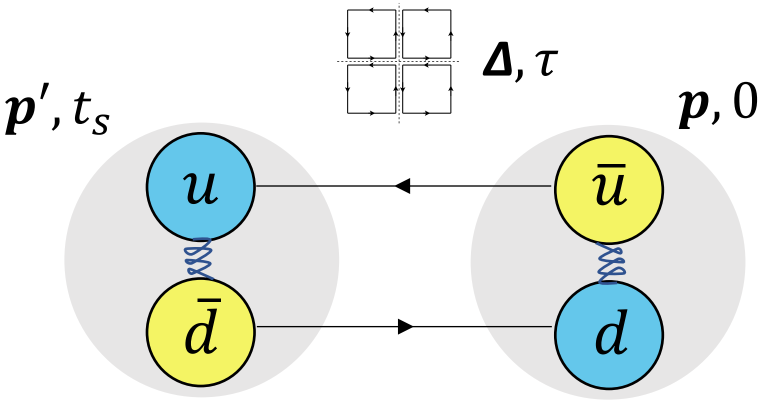


Connected contribution
- → 1381 configurations
- → sequential sources
- → $t_s \in \{6-18\}$
- → $|\mathbf{\Delta}|^2 \le 25(\frac{2\pi}{L})^2$
- → $\boldsymbol{p}' \in \{(1,-1,0),(-2,-1,0),(-1,-1,-1)\}2\pi/L$



Disconnected contribution
- → 1381 configurations
- → Z_4 noise, hierarchical probing, 512 Hadamard vectors
- → 1024 sources
- → $|\mathbf{\Delta}|^2 \le 25(\frac{2\pi}{L})^2$
- → $|\boldsymbol{p}'|^2 \le 10(\frac{2\pi}{L})^2$



Gluon contribution
- → 2511 configurations
- → $\frac{t_{\text{flow}}}{a^2} = 2$
- → 1024 sources
- → $|\mathbf{\Delta}|^2 \le 25(\frac{2\pi}{L})^2$
- → $|\boldsymbol{p}'|^2 \le 10(\frac{2\pi}{L})^2$

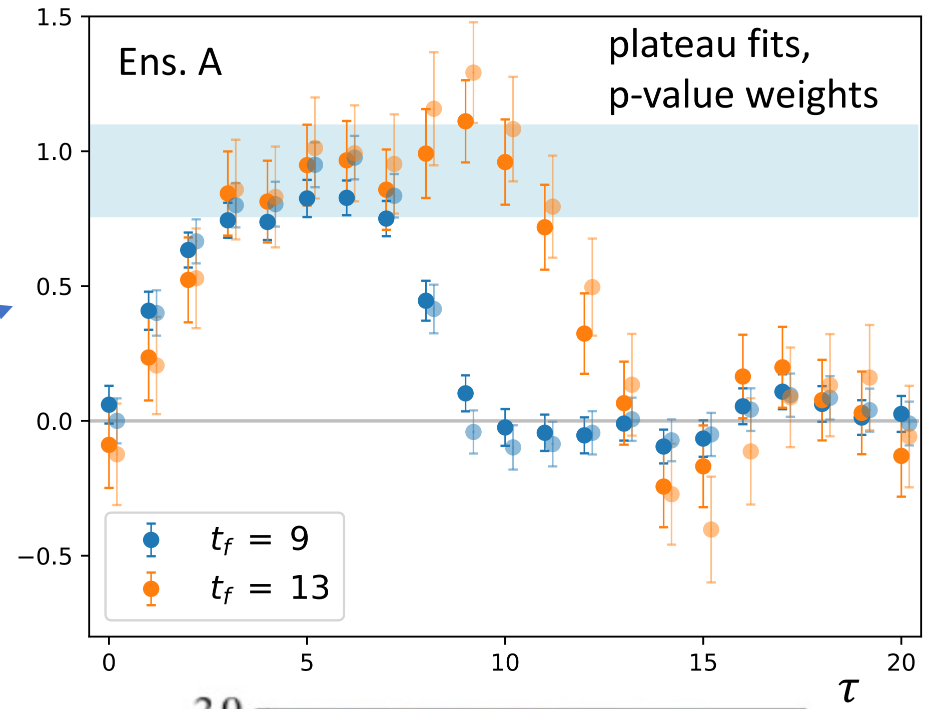
Matrix elements → bare GFFs

- From 2- and 3-point functions, extract $\langle h(\boldsymbol{p},s)|T_i^{\mu\nu}|h(\boldsymbol{p}',s')\rangle$ for several kinematic combinations $\underline{\boldsymbol{p}',\boldsymbol{\Delta},s,s',\mu,\nu}$

$$R_{\mu\nu}(\boldsymbol{p}',t_s,\boldsymbol{\Delta},\tau)=\frac{C_{\mu\nu}^{3\text{pt}}(\boldsymbol{p}',t_s,\boldsymbol{\Delta},\tau)}{C^{2\text{pt}}(\boldsymbol{p}',t_s)}\sqrt{\frac{C^{2\text{pt}}(\boldsymbol{p},t_s-\tau)C^{2\text{pt}}(\boldsymbol{p}',t_s)C^{2\text{pt}}(\boldsymbol{p}',\tau)}{C^{2\text{pt}}(\boldsymbol{p}',t_s-\tau)C^{2\text{pt}}(\boldsymbol{p},t_s)C^{2\text{pt}}(\boldsymbol{p},\tau)}}$$

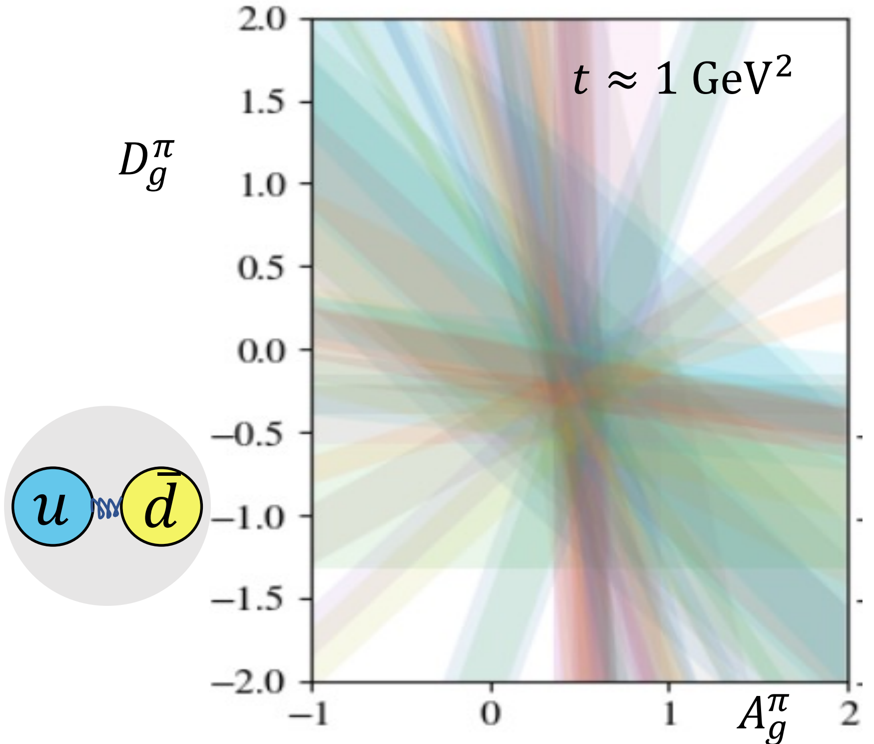
Model average over Euclidean time ranges

Jay Neil PRD 2021
Rinaldi et al PRL 2019
NPLQCD PRL 2015



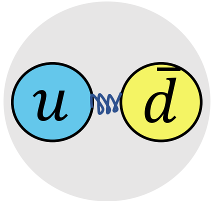
- $\langle h(\boldsymbol{p},s)|T_i^{\mu\nu}|h(\boldsymbol{p}',s')\rangle \sim$ Coefficients × GFFs ($t=\Delta^2$)
  Partition into momentum bins with equal or similar values of t, solve over-constrained linear systems
  → bare GFFs at discrete values of t

  Connected contribution: sequential-source through the sink → limited $\boldsymbol{p}'$
  choose such that GFFs can be resolved

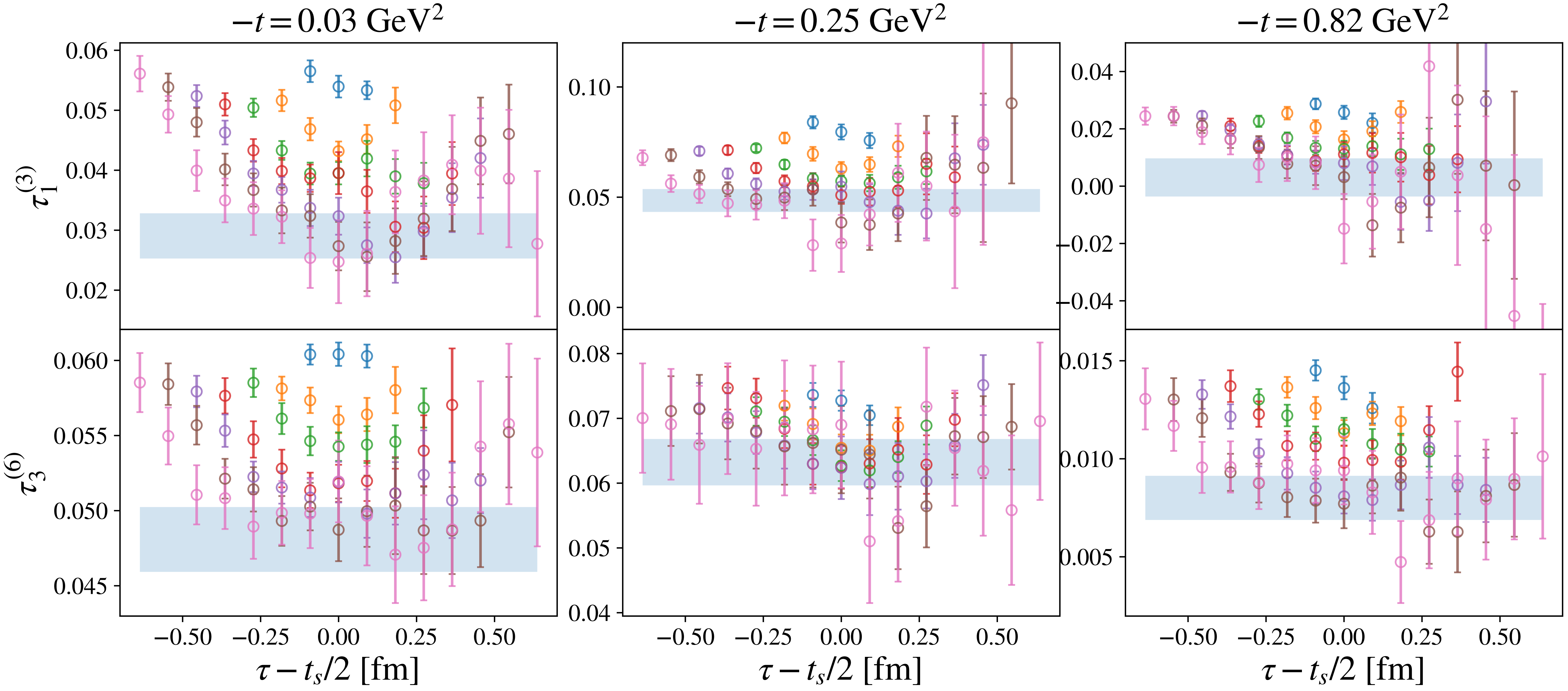


$\tau_1^{(3)}$: diagonal elements irrep
$\tau_3^{(6)}$: off-diagonal elements irrep

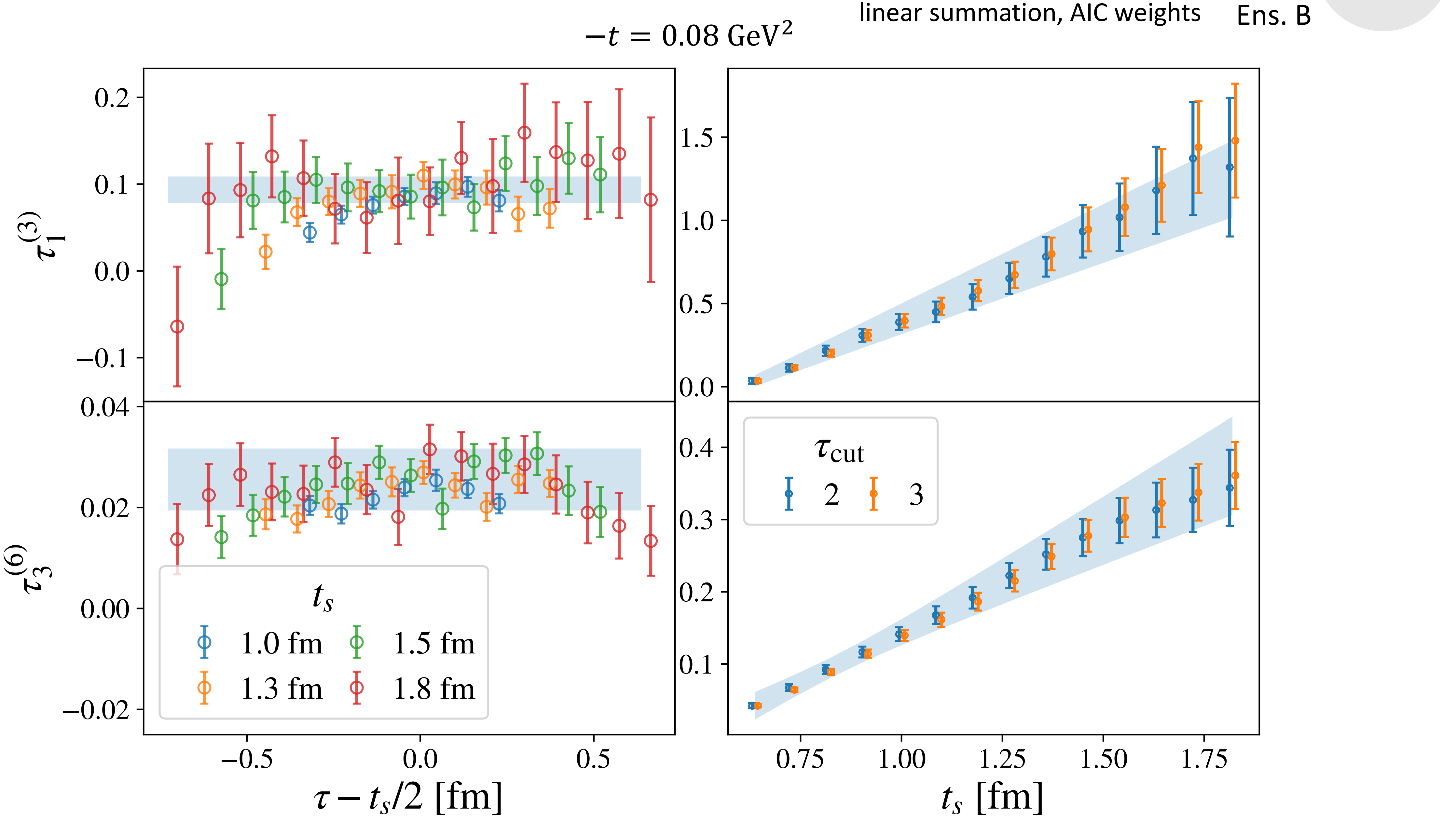
Pion connected quark contribution



linear summation, summation + exponential, AIC weights   Ens. B

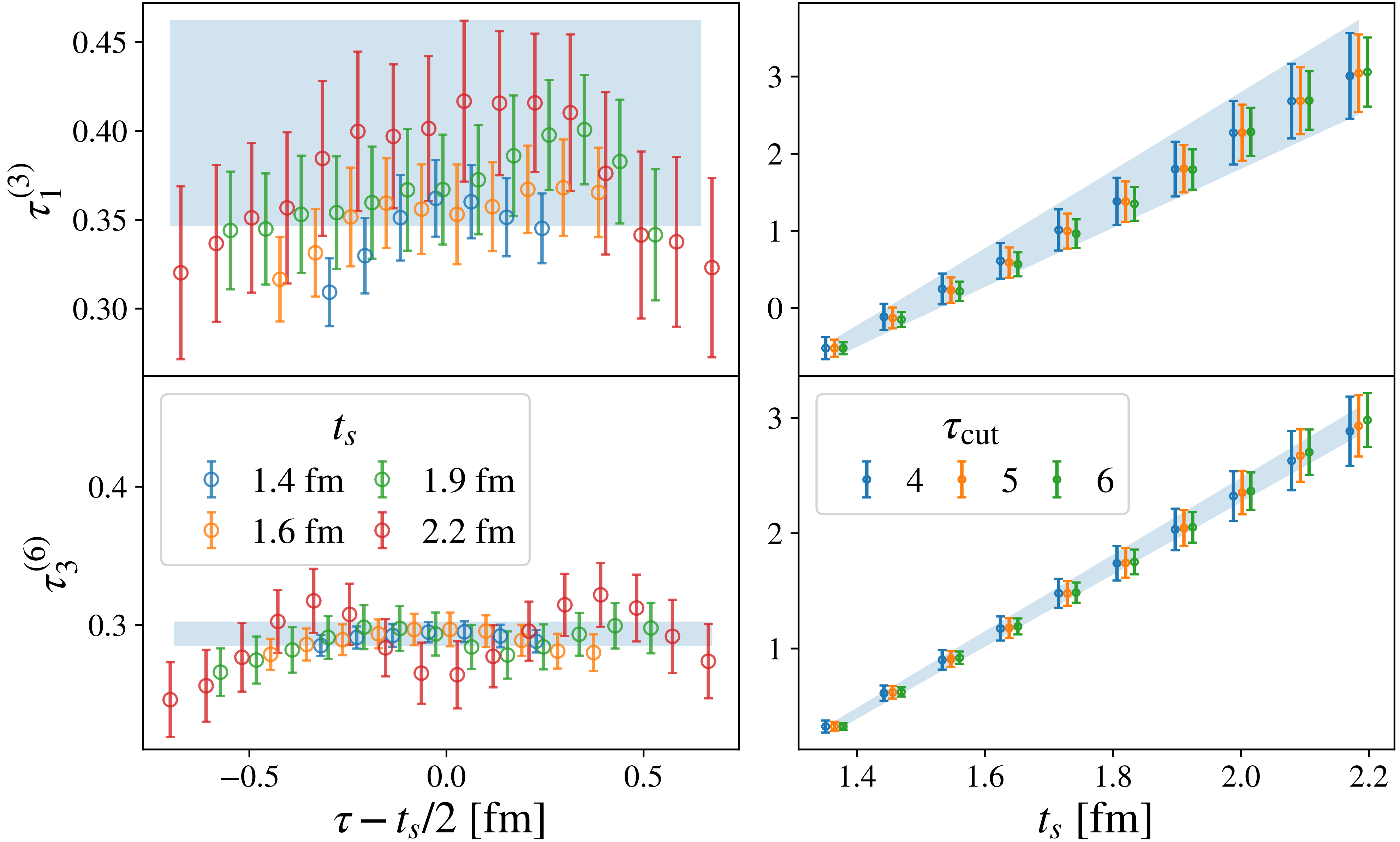


Pion disconnected quark contribution

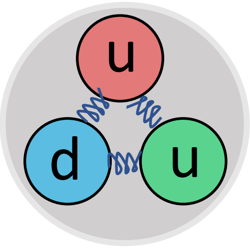


Pion gluon contribution

$-t = 0.13\ \mathrm{GeV}^2$ linear summation, AIC weights Ens. B

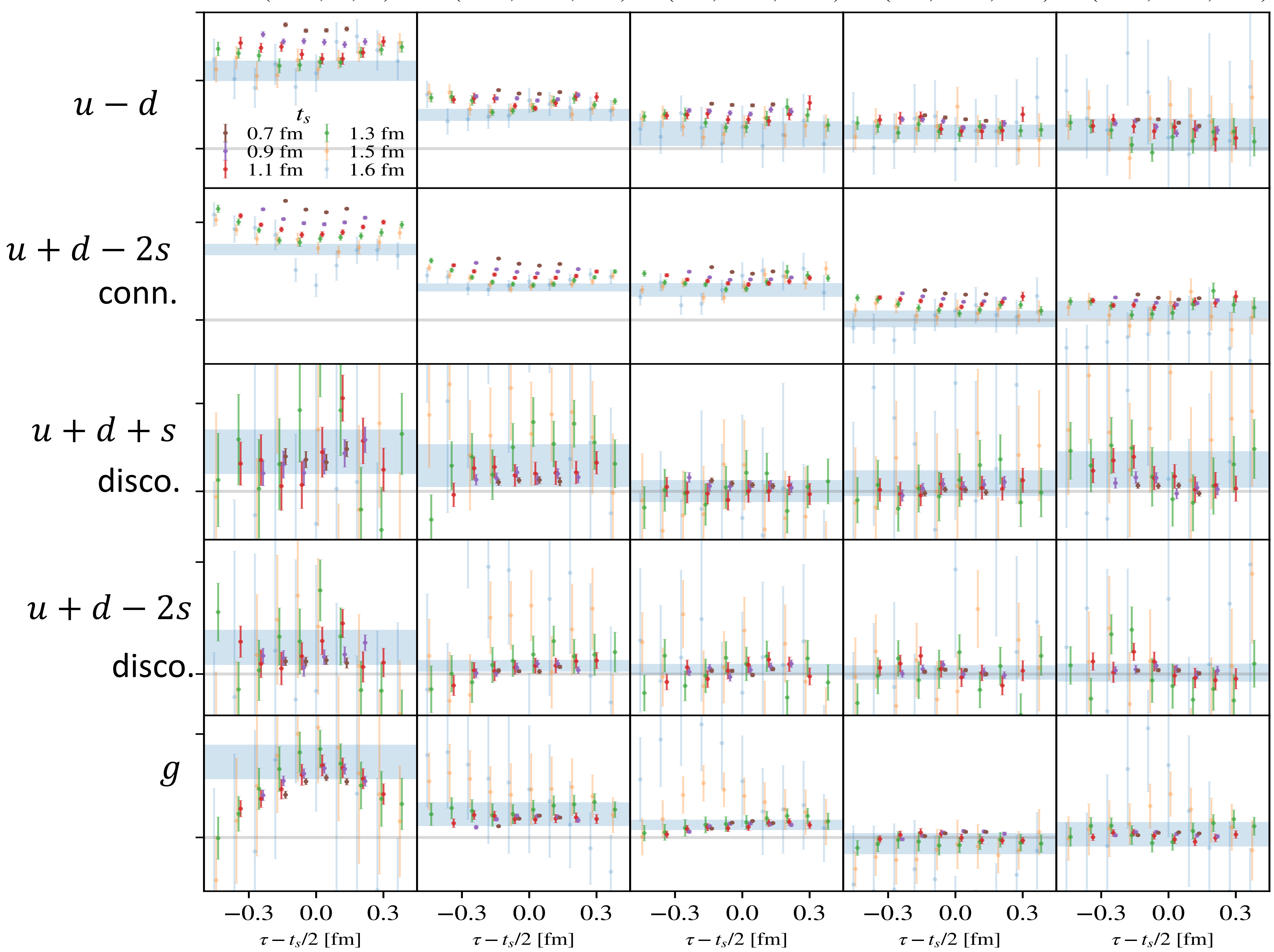


Proton



Ens. B $\tau_1^{(3)}$

linear summation,
AIC weights



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Renormalization

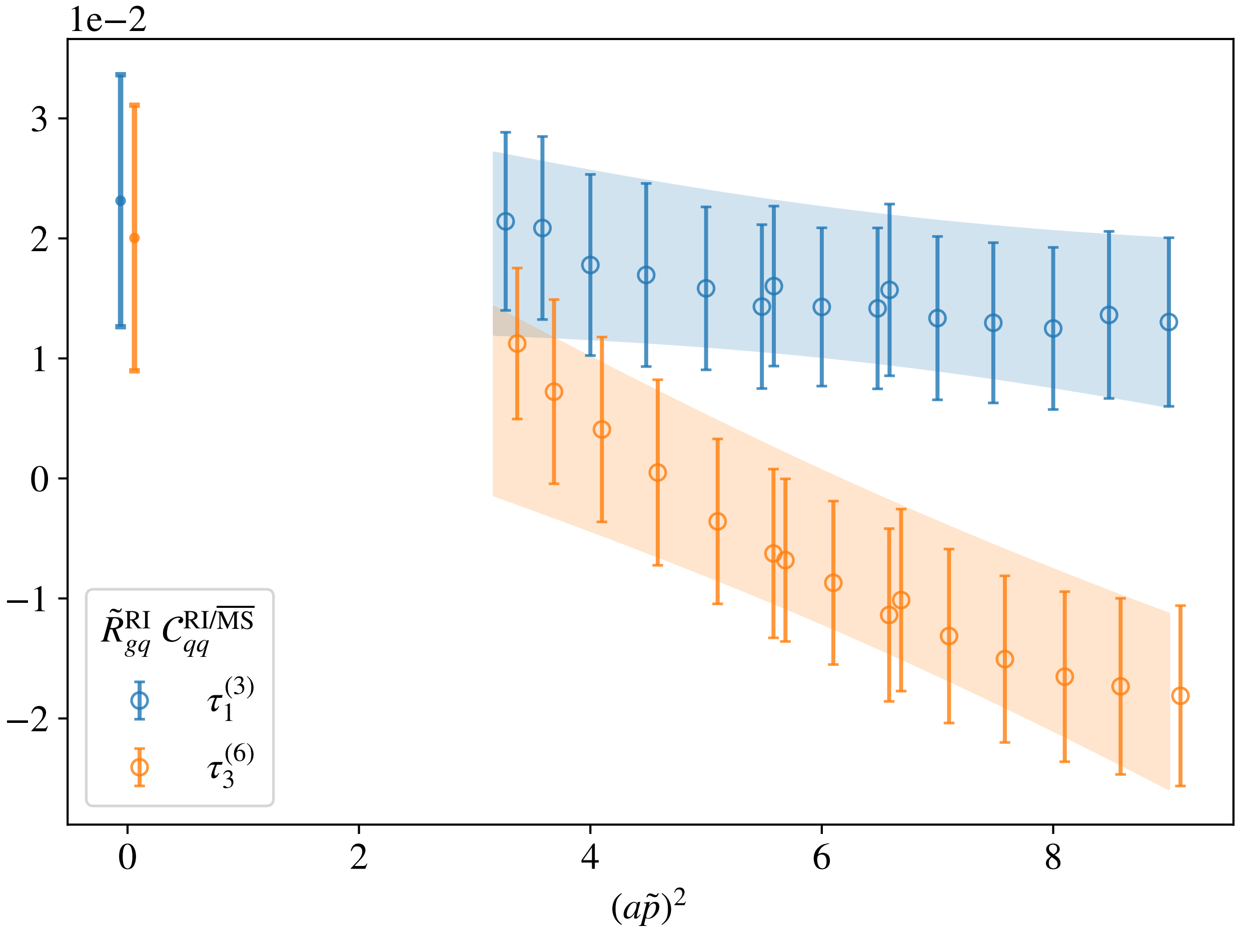
$\mathcal{R} \in \{\tau_1^{(3)}, \tau_3^{(6)}\}$

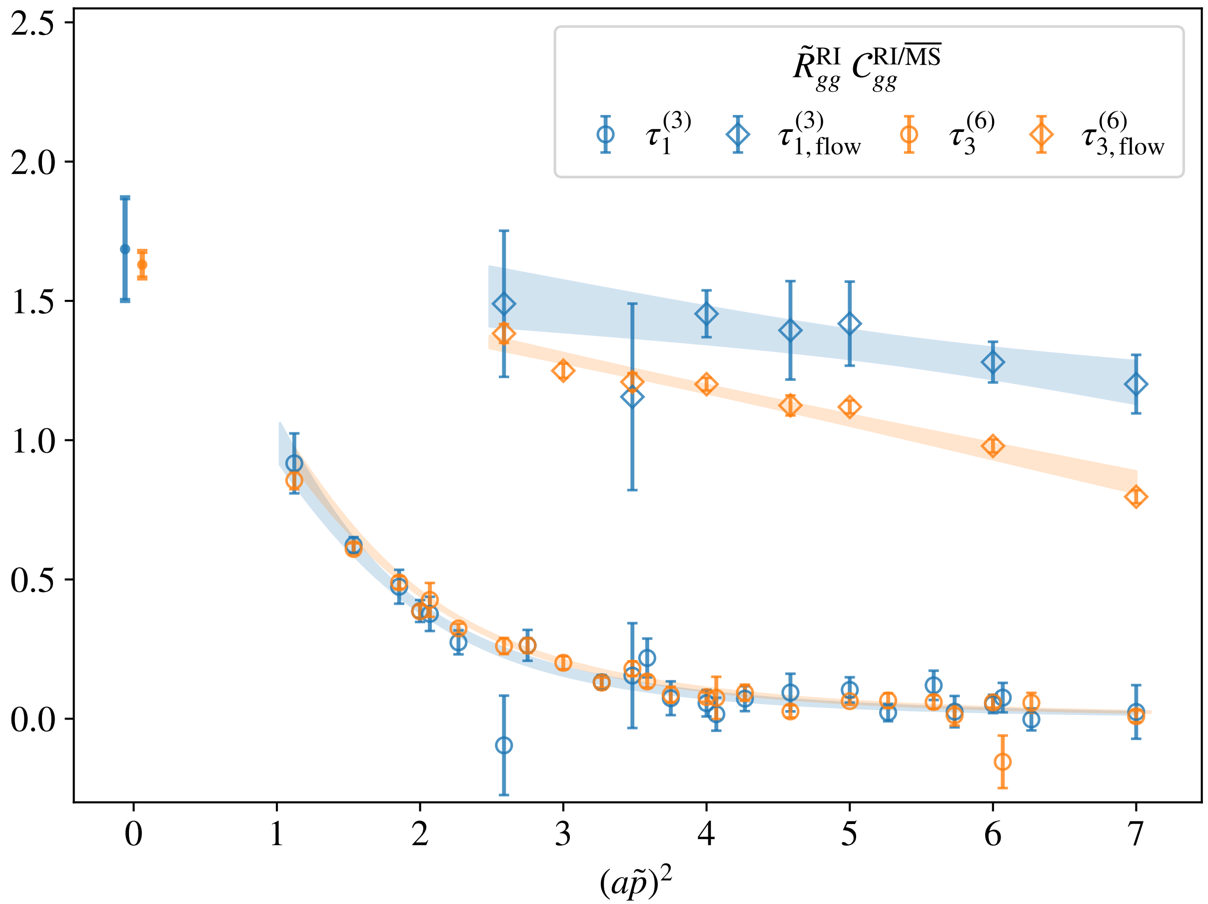
m_π (MeV)	a (fm)	$L^3 \times T$	N_f
450	0.12	$12^3 \times 24$	$2+1$

- $\begin{pmatrix} T_q^{\overline{\mathrm{MS}}} \\ T_g^{\overline{\mathrm{MS}}} \end{pmatrix} = \begin{pmatrix} Z_{qq\mathcal{R}}^{\overline{\mathrm{MS}}} & Z_{qg\mathcal{R}}^{\overline{\mathrm{MS}}} \\ Z_{gq\mathcal{R}}^{\overline{\mathrm{MS}}} & Z_{gg\mathcal{R}}^{\overline{\mathrm{MS}}} \end{pmatrix} \begin{pmatrix} T_{q\mathcal{R}}^{\mathrm{bare}} \\ T_{g\mathcal{R}}^{\mathrm{bare}} \end{pmatrix}$: quark isosinglet and gluon mix under renormalization
- $T_v^{\overline{\mathrm{MS}}} = Z_{v\mathcal{R}}^{\overline{\mathrm{MS}}} T_{v\mathcal{R}}^{\mathrm{bare}}$, $T_v = T_u + T_d - 2T_s$: non-singlet does not mix in the chiral limit
- Compute non-perturbatively via the RI-MOM scheme, convert to $\overline{\mathrm{MS}}$ scheme at $\mu = 2$ GeV using two-loop matching coefficients (Panagopoulos et al PRD 2021)
- For regular volume ensembles, gluon and disconnected have intractable noise
  → Use smaller volume ensemble to get renormalization factors (different spacing)

$$\begin{pmatrix} Z_{qq\mathcal{R}}^{\overline{\mathrm{MS}}} & Z_{qg\mathcal{R}}^{\overline{\mathrm{MS}}} \\ Z_{gq\mathcal{R}}^{\overline{\mathrm{MS}}} & Z_{gg\mathcal{R}}^{\overline{\mathrm{MS}}} \end{pmatrix}^{-1} (\mu^2) = \begin{pmatrix} R_{qq\mathcal{R}}^{\mathrm{RI}} & R_{qg\mathcal{R}}^{\mathrm{RI}} \\ R_{gq\mathcal{R}}^{\mathrm{RI}} & R_{gg\mathcal{R}}^{\mathrm{RI}} \end{pmatrix} (\mu_R^2) \times \begin{pmatrix} \mathcal{C}_{qq}^{\mathrm{RI}/\overline{\mathrm{MS}}} & \mathcal{C}_{qg}^{\mathrm{RI}/\overline{\mathrm{MS}}} \\ \mathcal{C}_{gq}^{\mathrm{RI}/\overline{\mathrm{MS}}} & \mathcal{C}_{gg}^{\mathrm{RI}/\overline{\mathrm{MS}}} \end{pmatrix} (\mu^2, \mu_R^2)$$

Extraction of renormalization coefficients

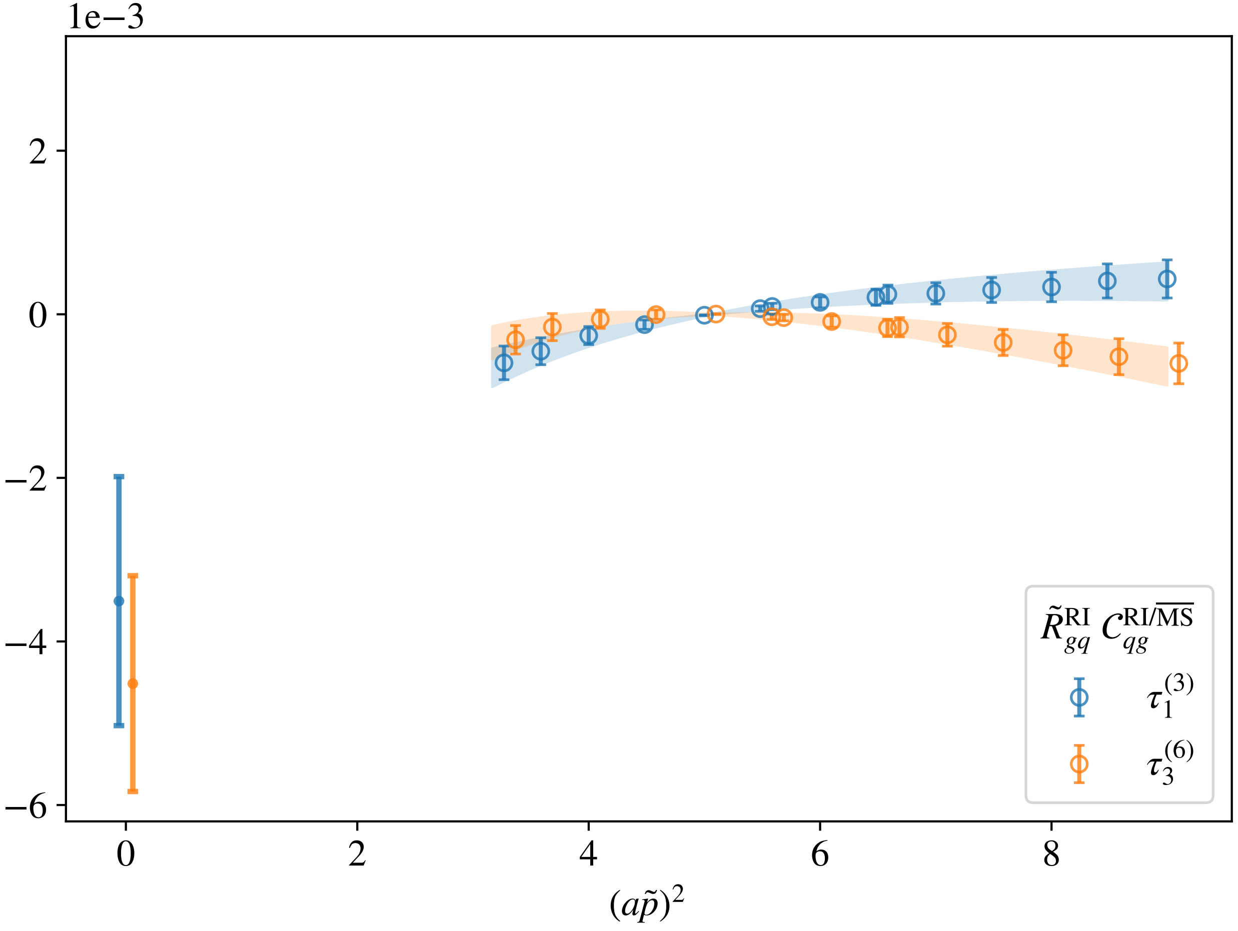


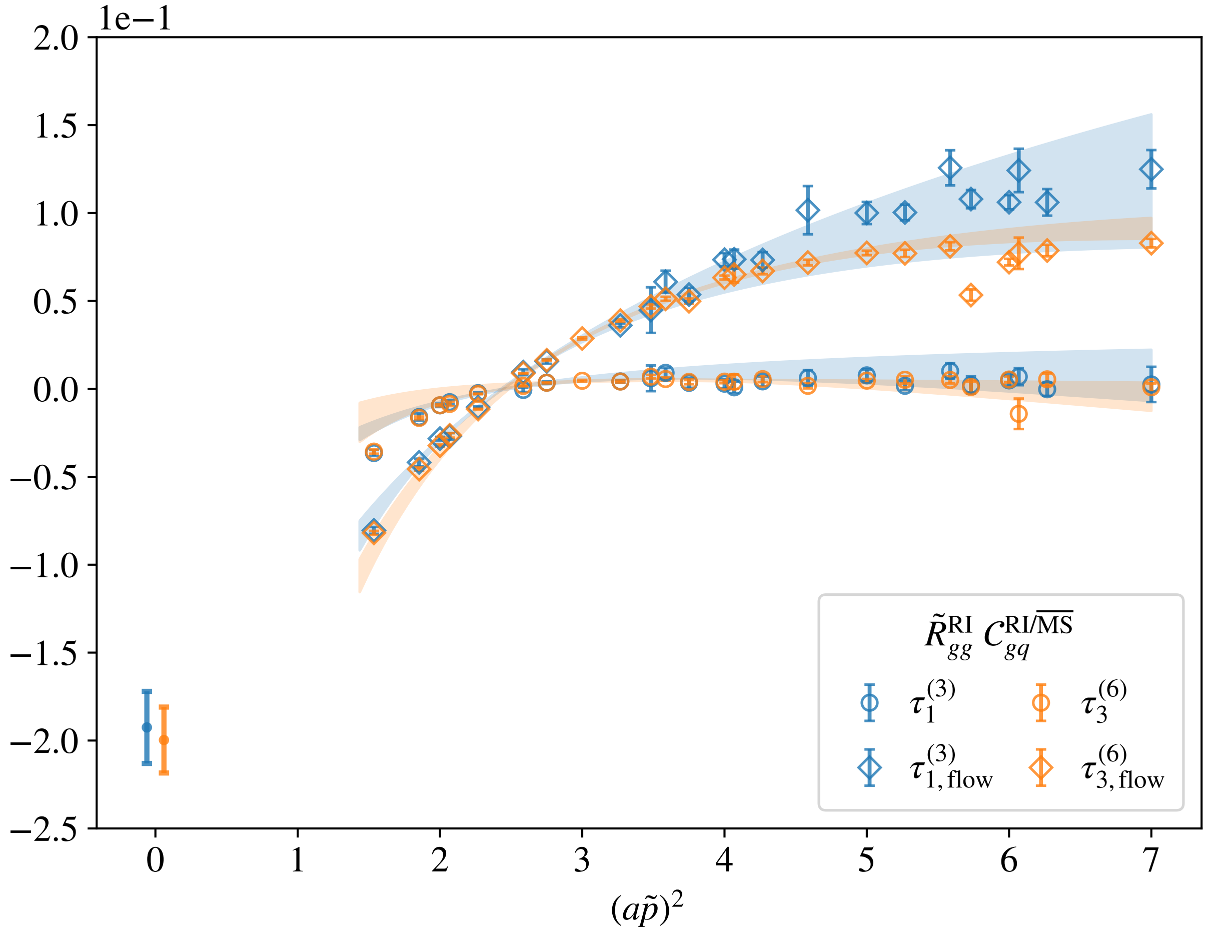


Fit $(a\tilde{p})$ dependence due to discretization artifacts, non-perturbative effects, etc.

(inverse) polynomial

Extraction of renormalization coefficients





Fit $(a\tilde{p})$ dependence due to discretization artifacts, non-perturbative effects, etc.

logarithmic

Finally: obtain renormalized GFFs

We have: 1) bare matrix elements $\langle h | T_i^{\mu\nu} | h \rangle, i \epsilon \{g, q, v\}$ grouped in t-bins for each irrep $\mathcal{R}$

2) mixing matrix renormalization $\begin{pmatrix} Z_{qq\mathcal{R}}^{\overline{\text{MS}}} & Z_{qg\mathcal{R}}^{\overline{\text{MS}}} \\ Z_{gq\mathcal{R}}^{\overline{\text{MS}}} & Z_{gg\mathcal{R}}^{\overline{\text{MS}}} \end{pmatrix}^{-1}$, non-singlet $Z_{v\mathcal{R}}^{\overline{\text{MS}}^{-1}}$ for each $\mathcal{R}$

→ recast into a simultaneous combined-irrep system of equations, solve by linear regression

Beware of d'Agostini bias!

D'Agostini Phys.Res.Sect.A 1994

Fit with 1) multipole : $F_n = \frac{\alpha}{(1+\frac{t}{\Lambda^2})^n}$,

2) z-expansion : $F = \sum_k \alpha_k [z(t)]^k$ (less restrictive)

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  [**DAP** Hackett Shanahan PRD (2022) arXiv:2107.10368]
- Bonus: GFFs of scalar glueball at pure Yang-Mills

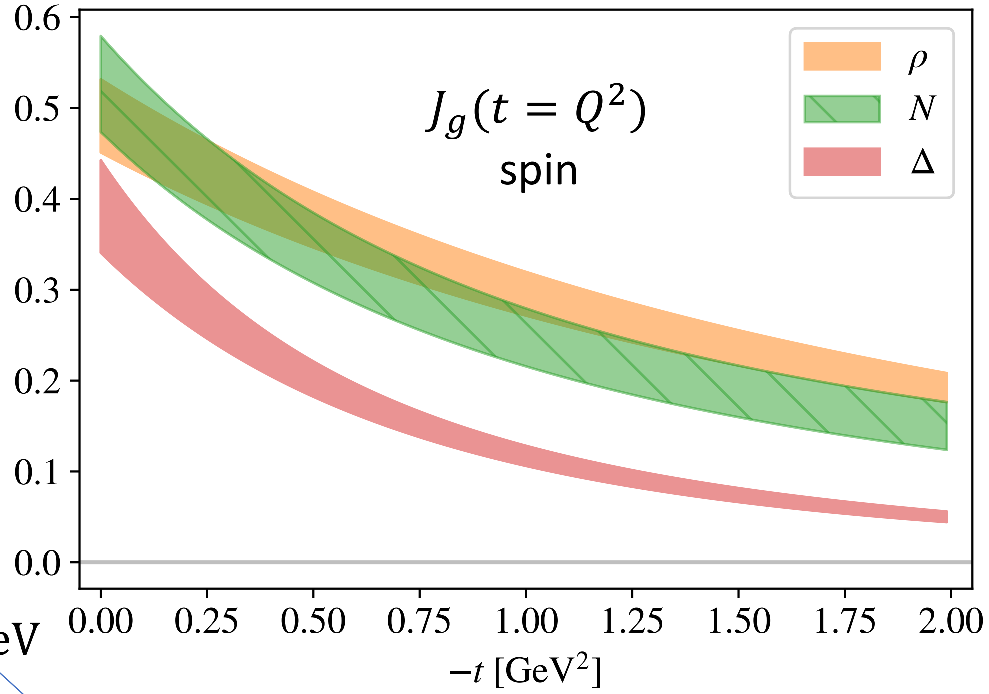
  [Abbott Hackett DAP Romero-Lopez Shanahan Urban, in preparation]

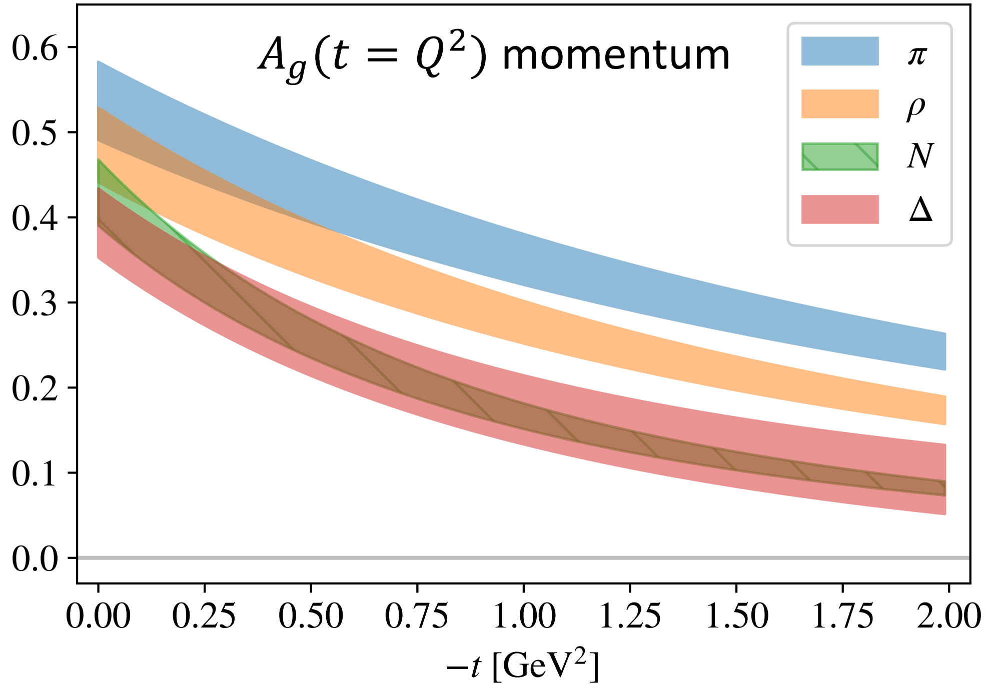
Gluon gravitational structure hadrons of different spin
($m_\pi \approx 450$ MeV, mixing neglected)

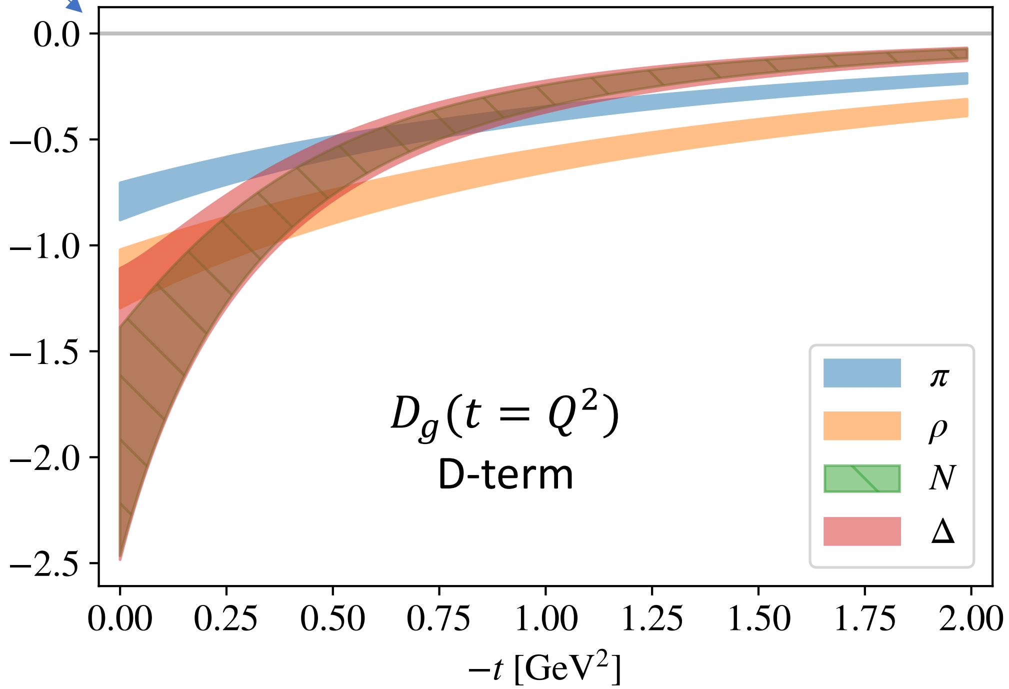
DAP, Hackett, Shanahan PRD (2022)

Hadron	π	ρ	N	Δ
Spin	0	1	1/2	3/2
GFF #	2	7	3	8

$\overline{\text{MS}}, \mu = 2$ GeV

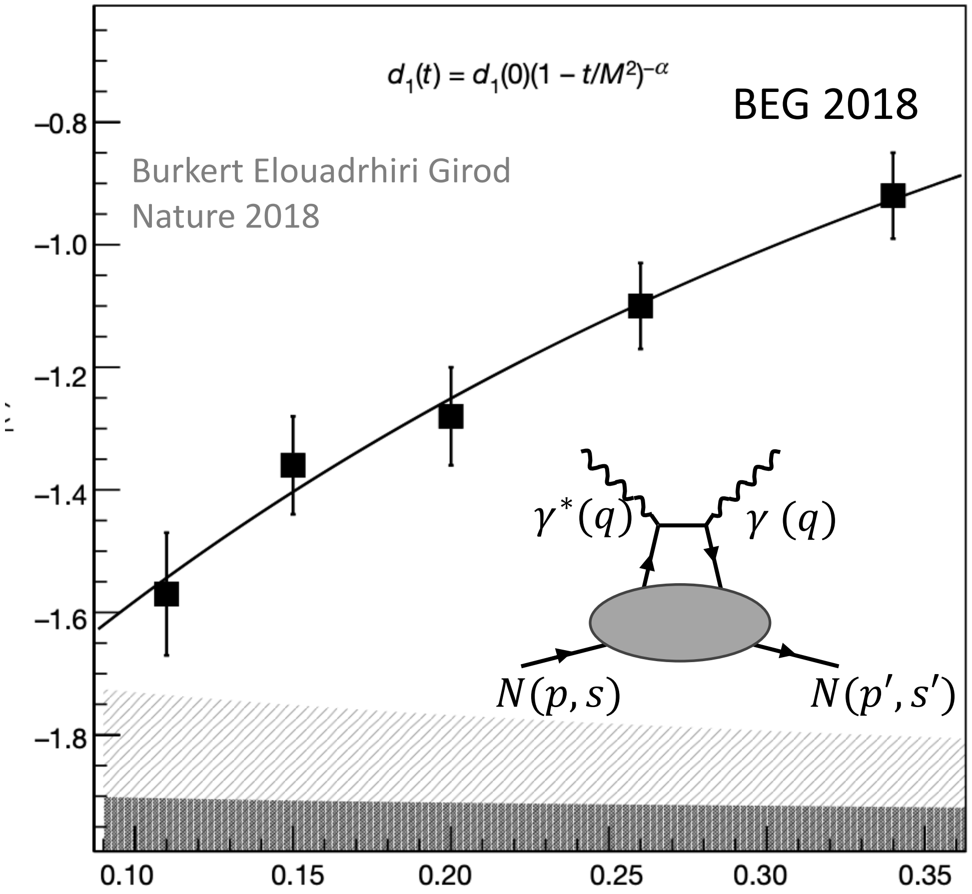




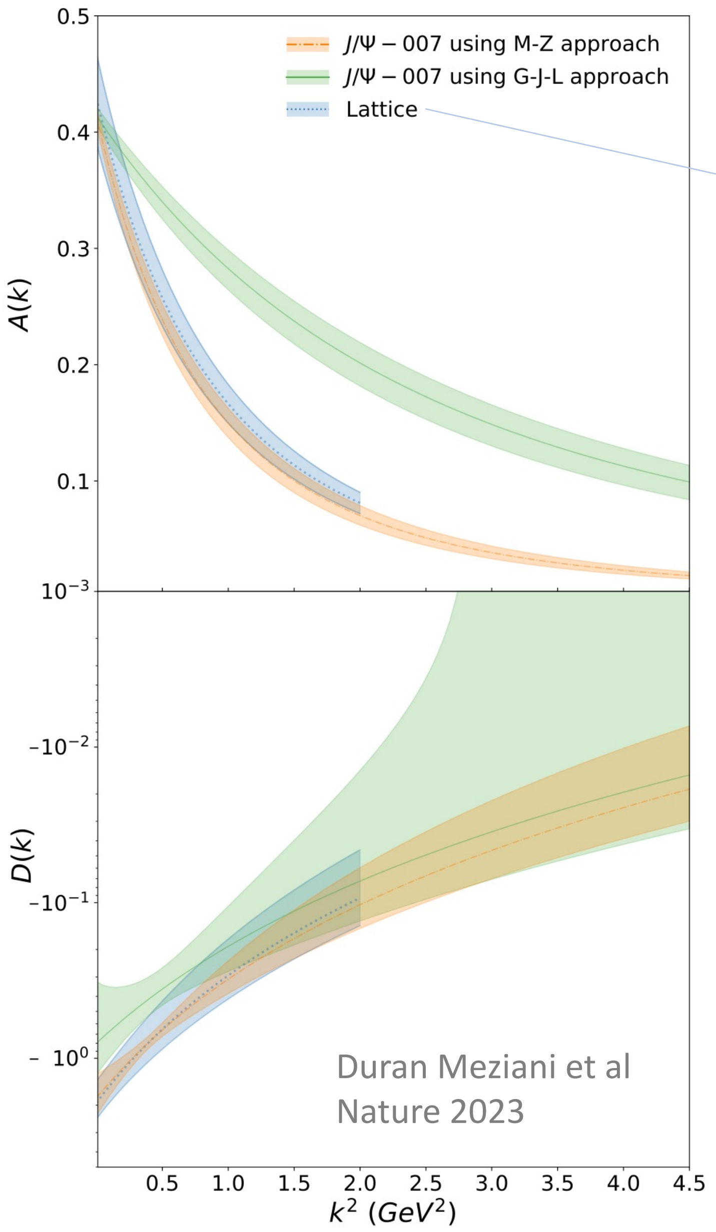


Proton: first experimental results

Quark D^N_{u+d} from DVCS

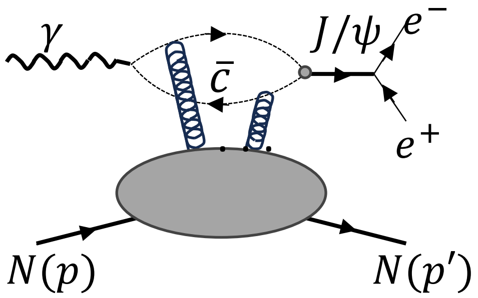


Gluon A^N_g and D^N_g from J/ψ photoproduction



See Shivangi Prasad's talk on Wednesday

Lattice: **DAP** Hackett Shanahan *PRD* (2022)
heavier pion mass + neglecting mixing with quark



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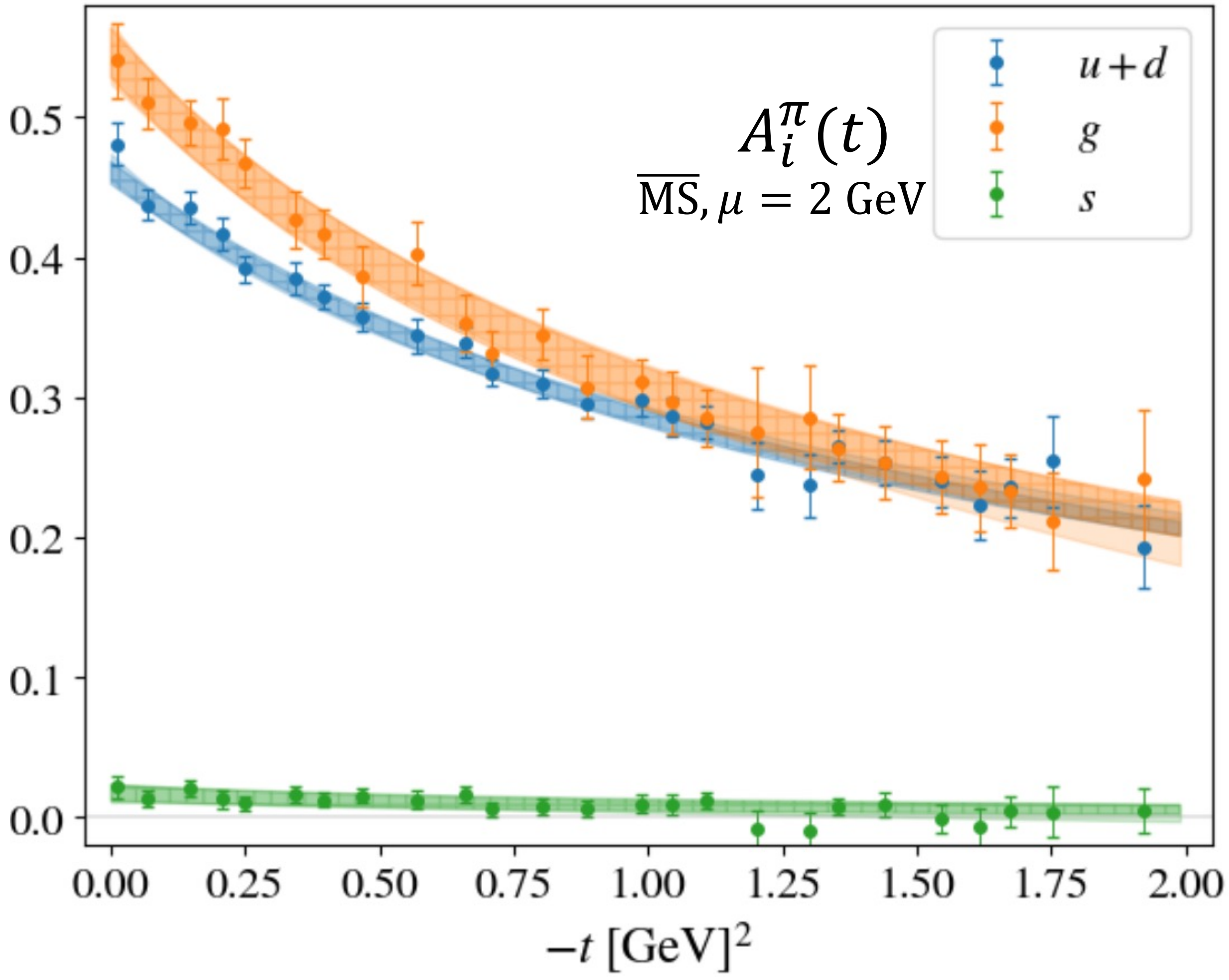
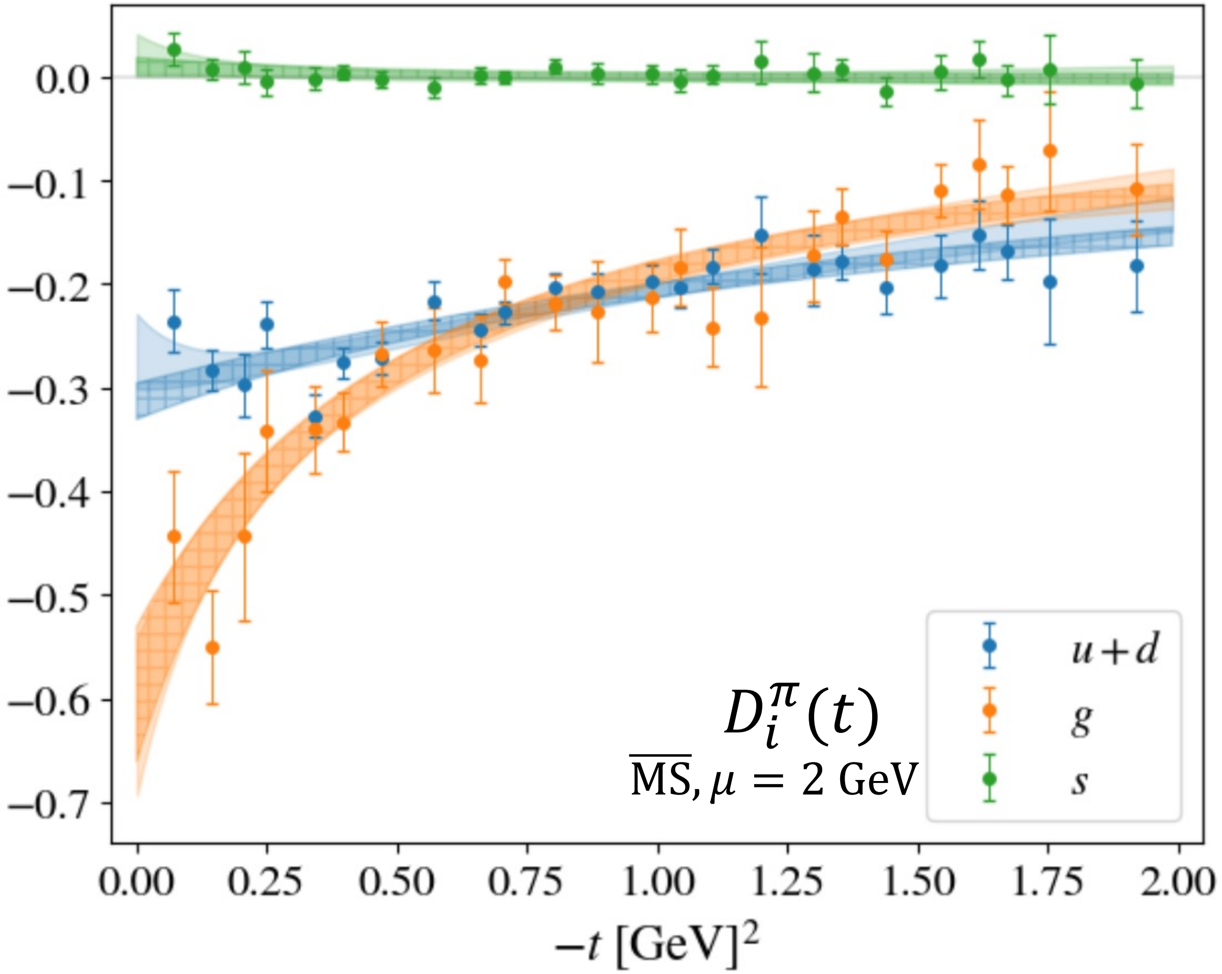
Quark and gluon GFFs of the pion

($m_\pi \approx 170$ MeV, including mixing)

Hackett Oare **DAP** Shanahan PRD 2023

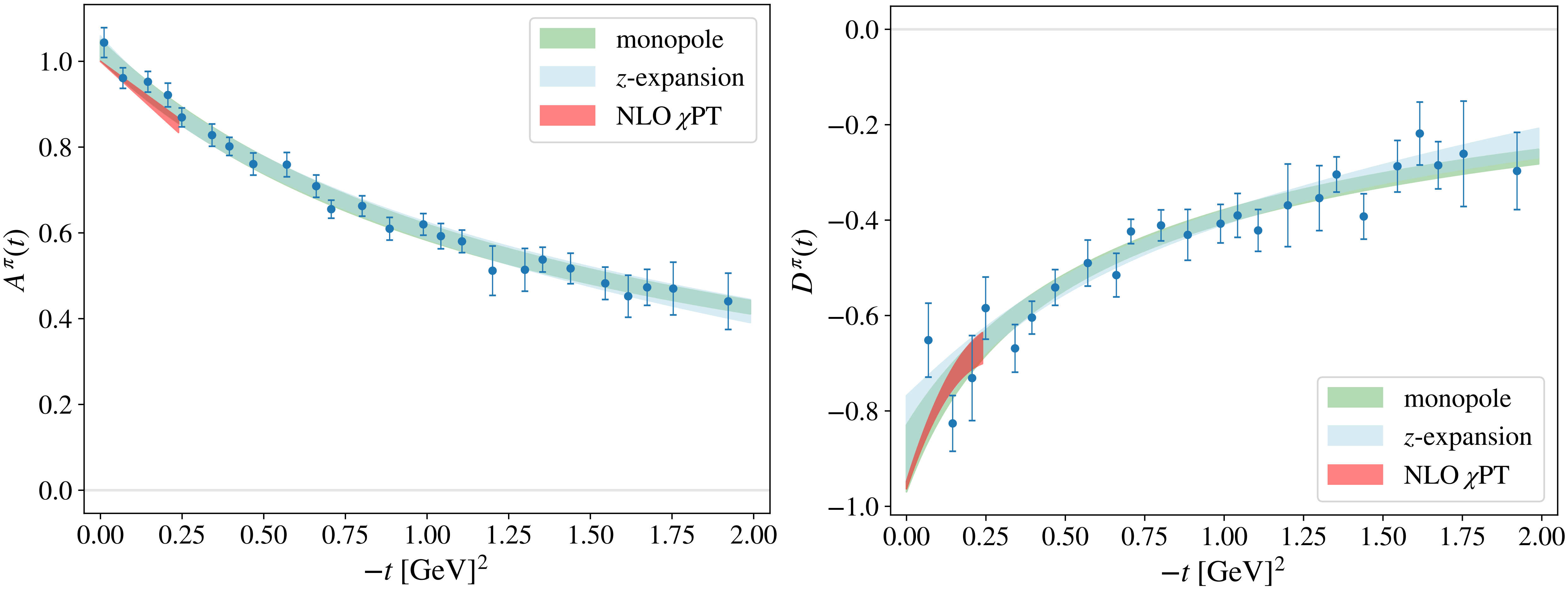




hatched bands : monopole, opaque bands : z-expansion with $k_{\text{max}} = 2$

Pion : total GFFs

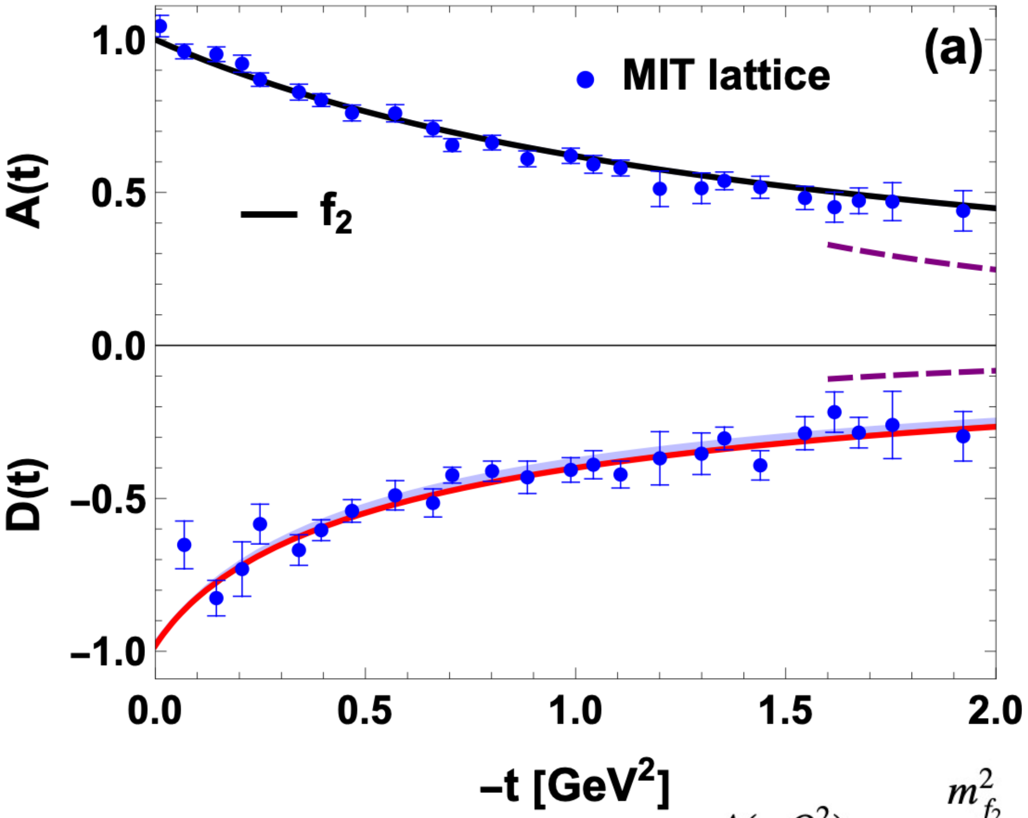
($m_\pi \approx 170$ MeV, including mixing)
Hackett Oare **DAP** Shanahan PRD 2023

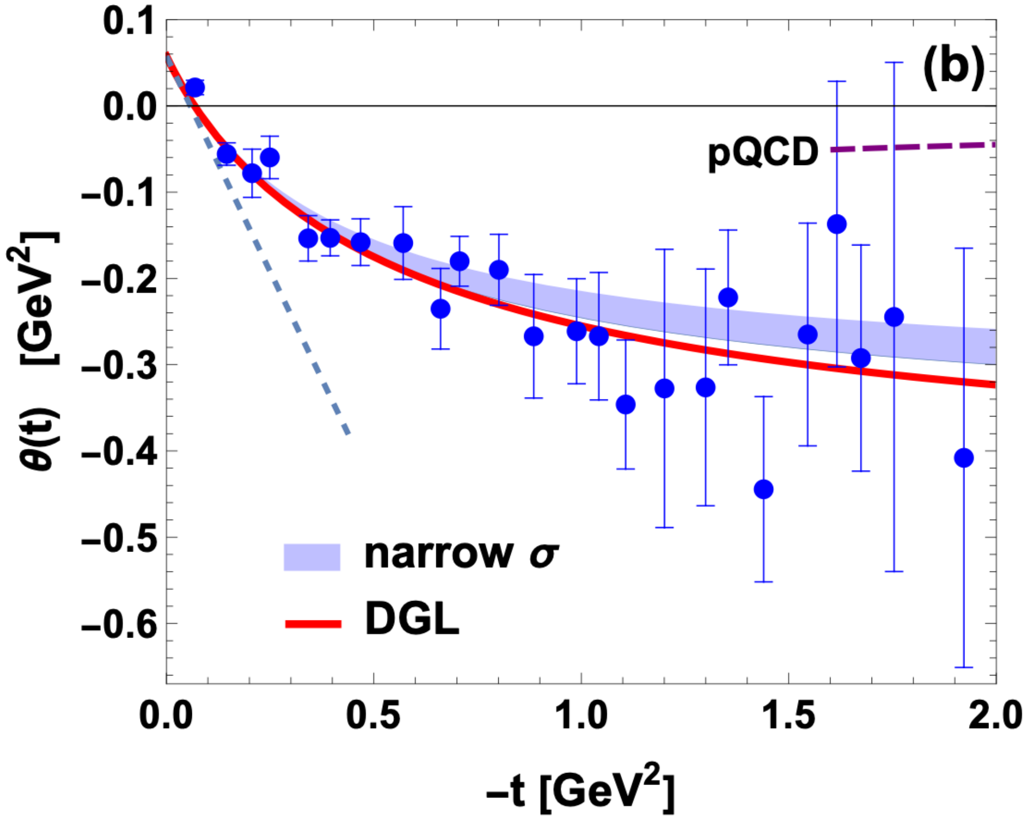


Red band spread due to different estimates for low energy constants [Donoghue Leutwyler Z.Phys.C 1991]

Pion data in support of meson dominance principle

Broniowski Arriola arXiv:2405.07815





$$A(-Q^2) = \frac{m_{f_2}^2}{m_{f_2}^2 + Q^2},$$

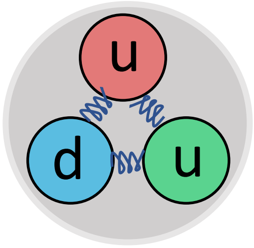
$$\Theta(-Q^2) = 2m_\pi^2 - \frac{m_\sigma^2 Q^2}{m_\sigma^2 + Q^2},$$

$$D = -\frac{2}{3t}\left[\Theta - \left(2m_\pi^2 - \tfrac{1}{2}\,t\right)A\right]$$

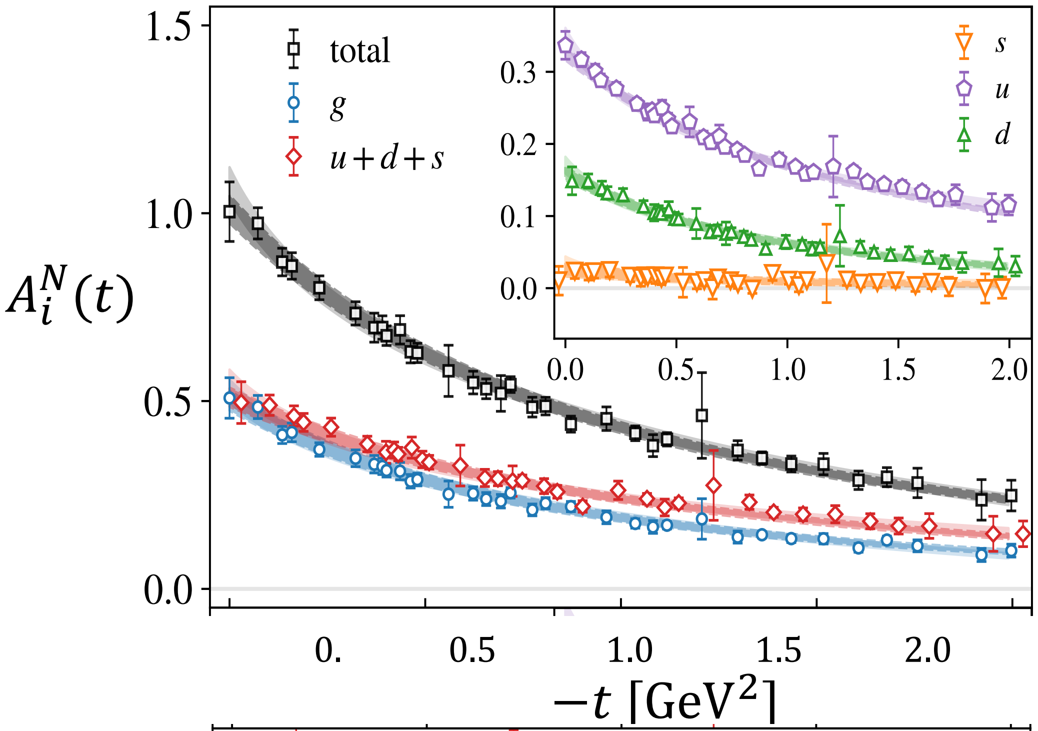
Quark and gluon GFFs of the proton

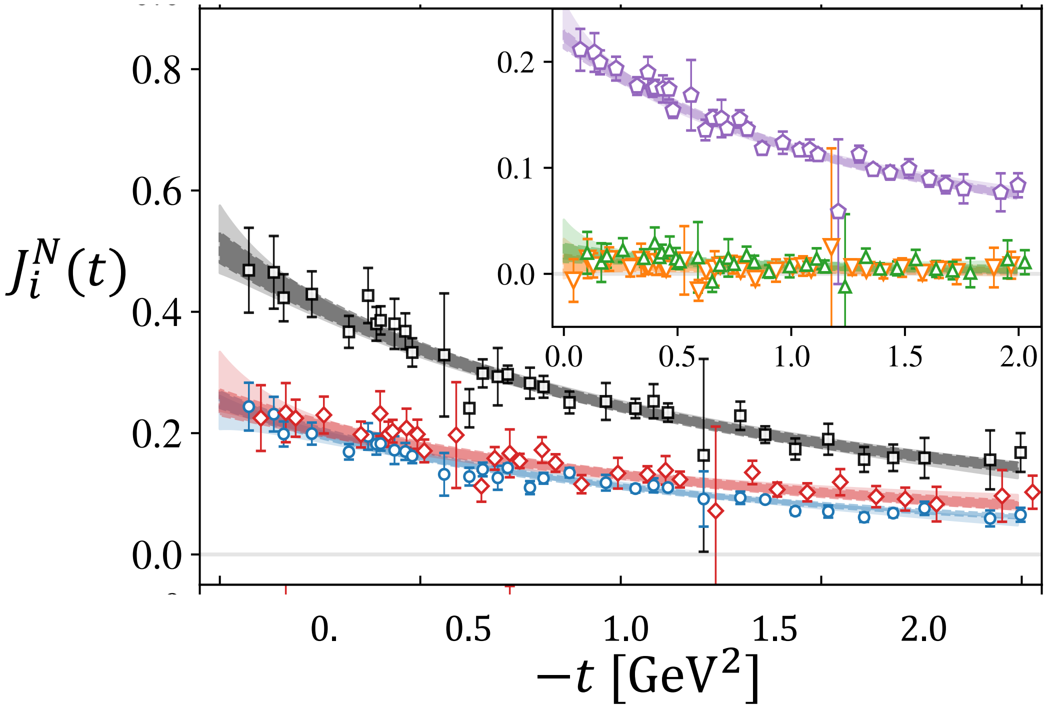
($m_\pi \approx 170$ MeV, including mixing)

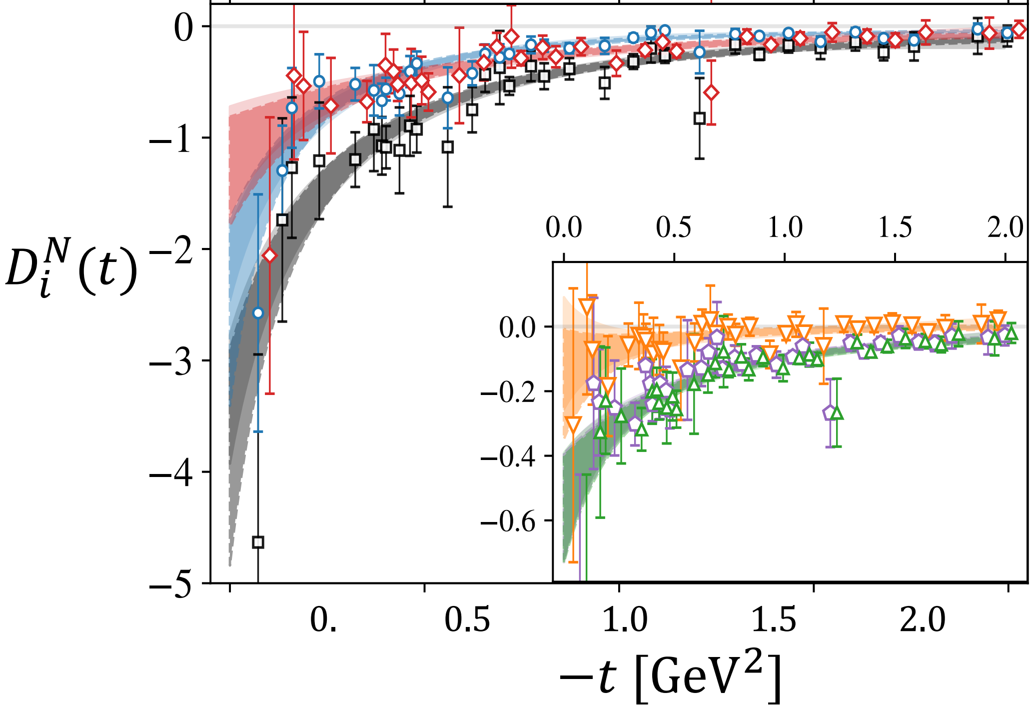
Hackett **DAP** Shanahan arXiv:2310.08484



$\overline{\text{MS}}, \mu = 2$ GeV



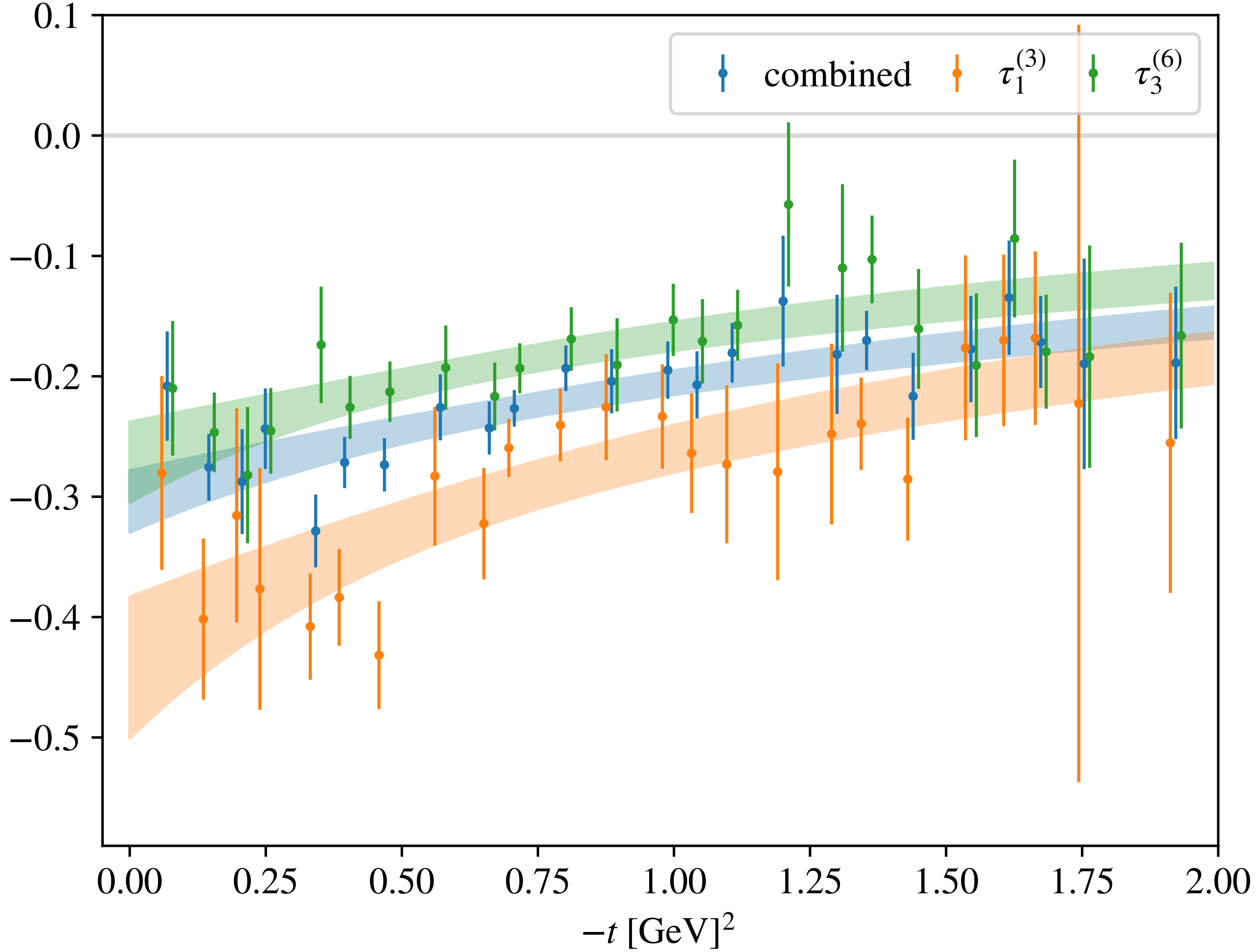




Renormalized pion GFFs

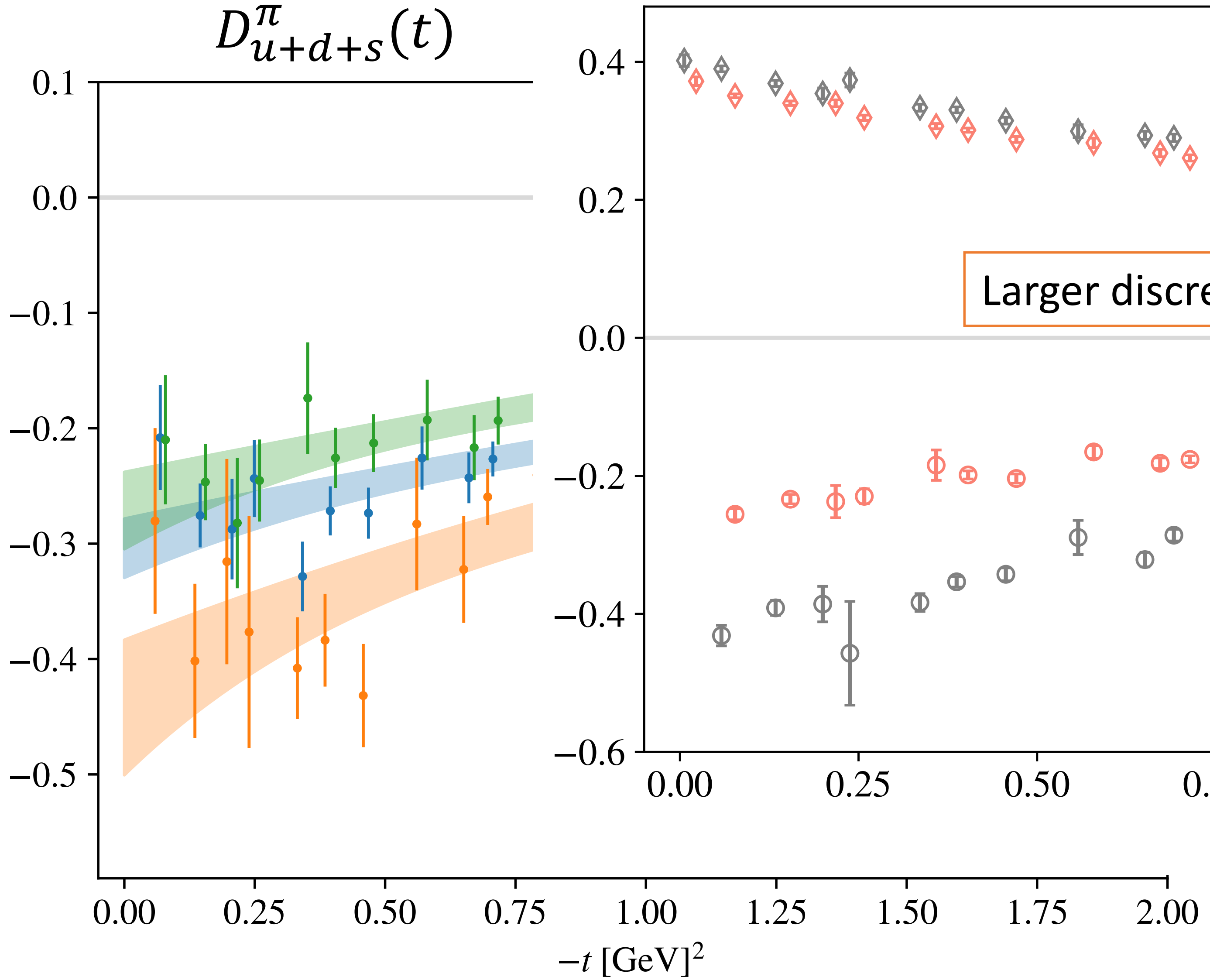
Comparison between final results (combined fits to both irreducible representations) and individual irrep fit results

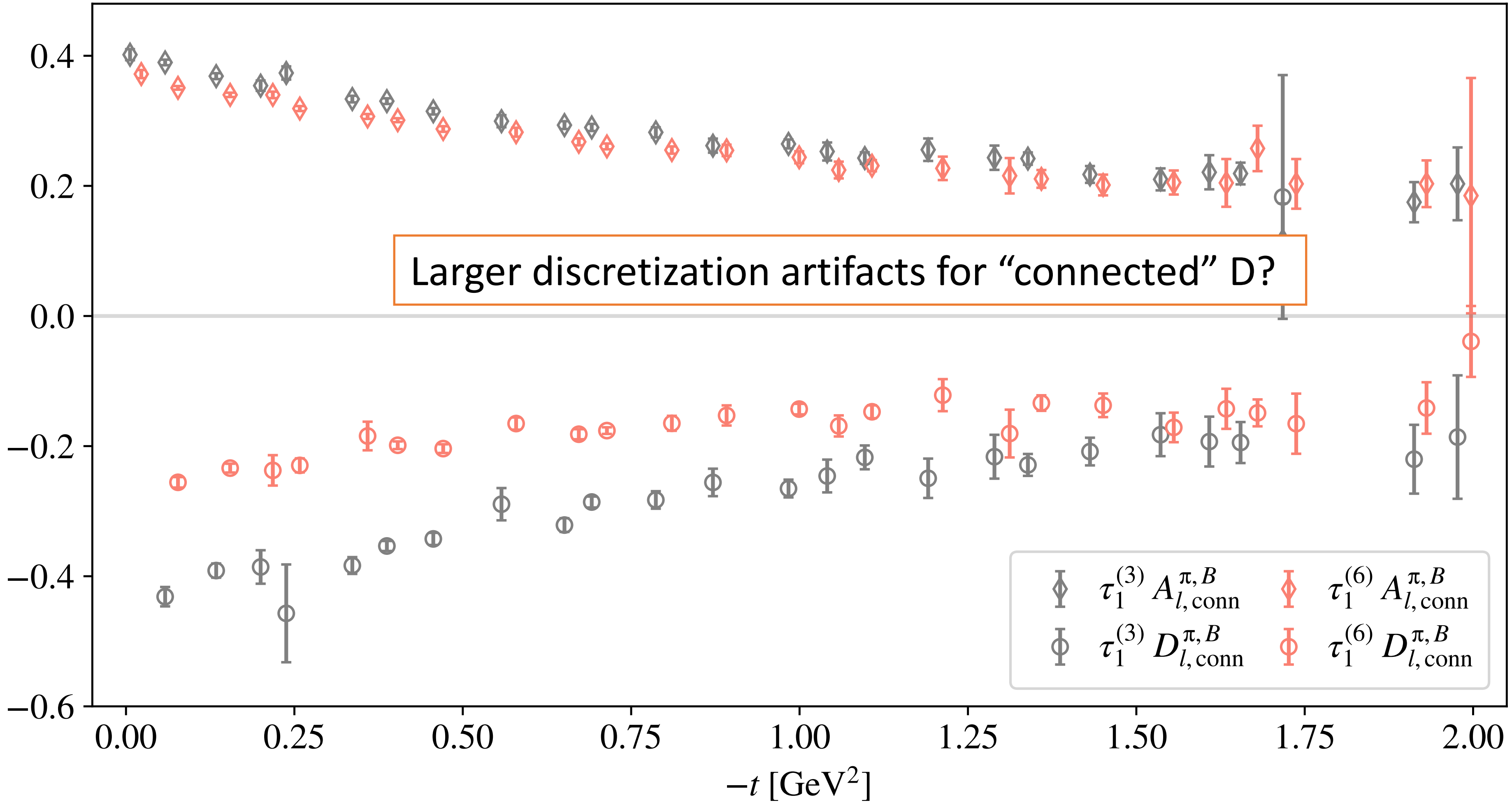
$D^{\pi}_{u+d+s}(t)$ $\overline{\text{MS}}, \mu = 2$ GeV



Renormalized pion GFFs

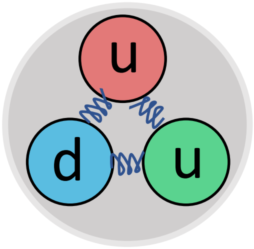
Comparison between final results (combined fits to both irreducible representations) and individual irrep fit results

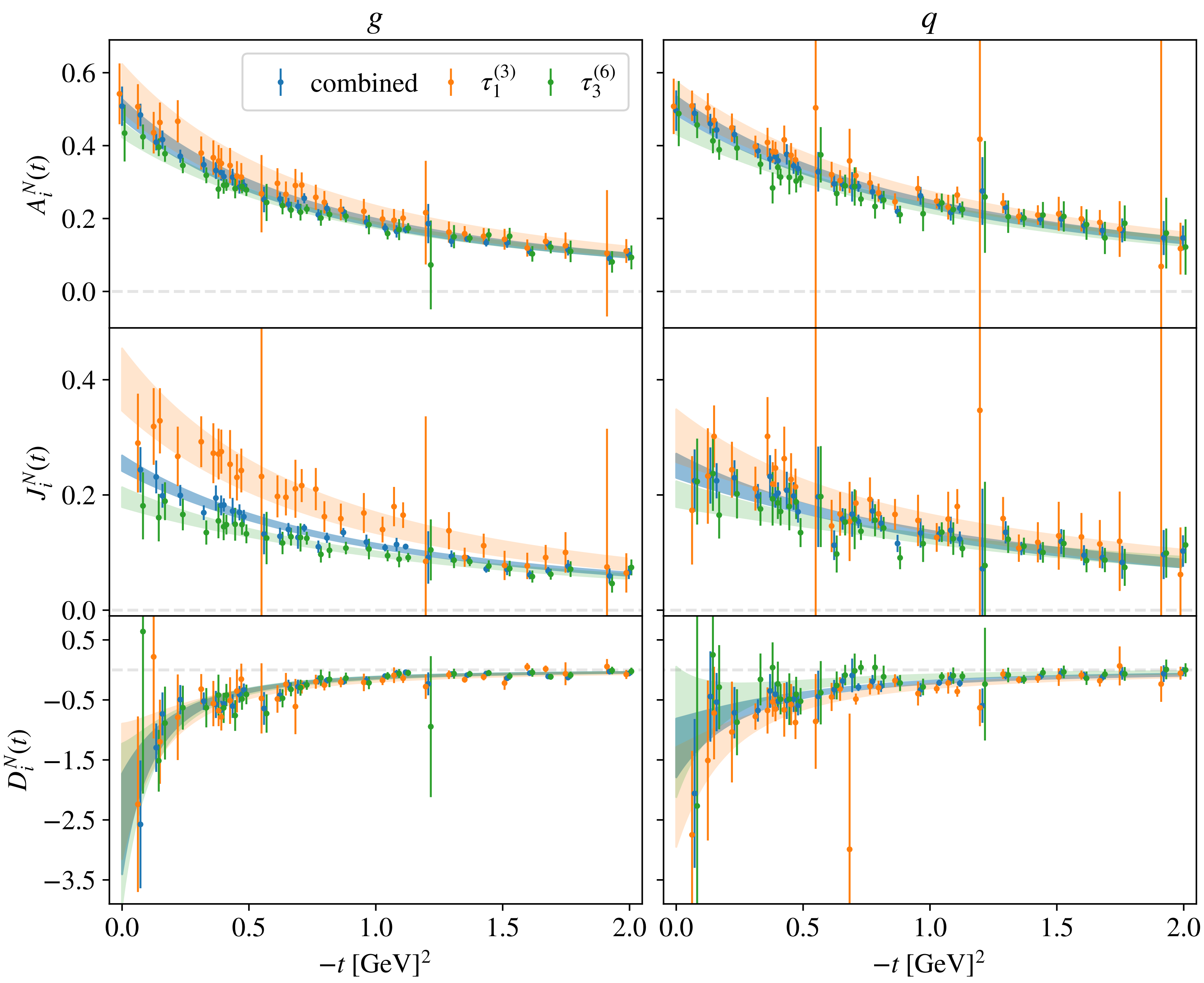




Renormalized proton GFFs – comparison with single-irrep fits

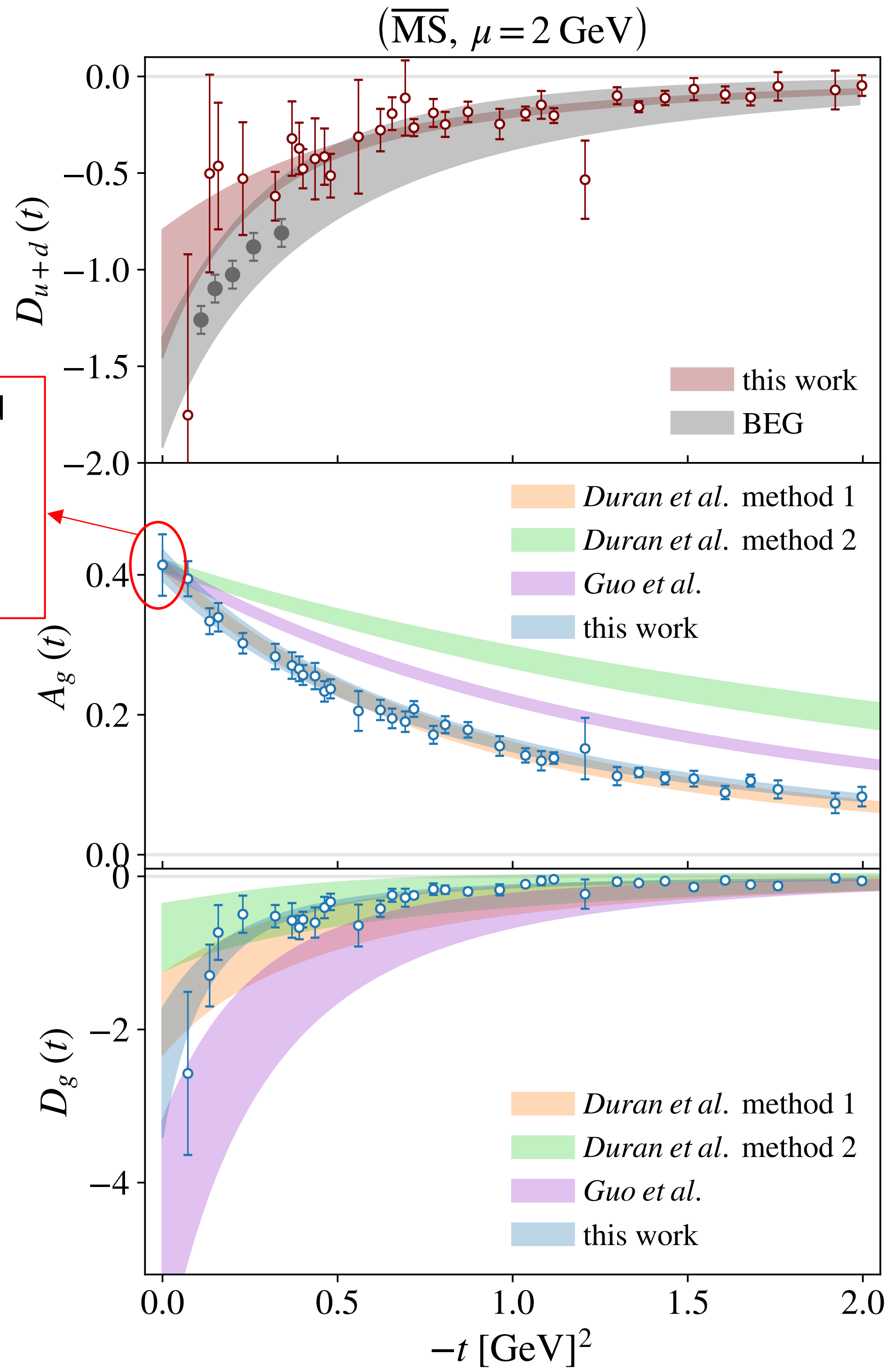
$\overline{\text{MS}}, \mu = 2$ GeV

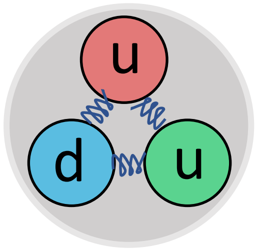




Renormalized nucleon GFFs – comparison to experiments

all normalized to $\langle x \rangle_g$ from global fit
Hou et al PRD 2021

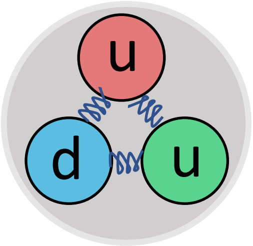




Burkert Elouardhiri Girod Nature 2018 (DVCS)

Duran et al Nature 2023 (J/ψ)
method 1: holographic QCD (Mamo Zahed PRD 2021+2022)
method 2: GPDs (Guo Ji Liu PRD 2021)

Guo et al PRD 2023 (+ GlueX data)
method 2 updated formula

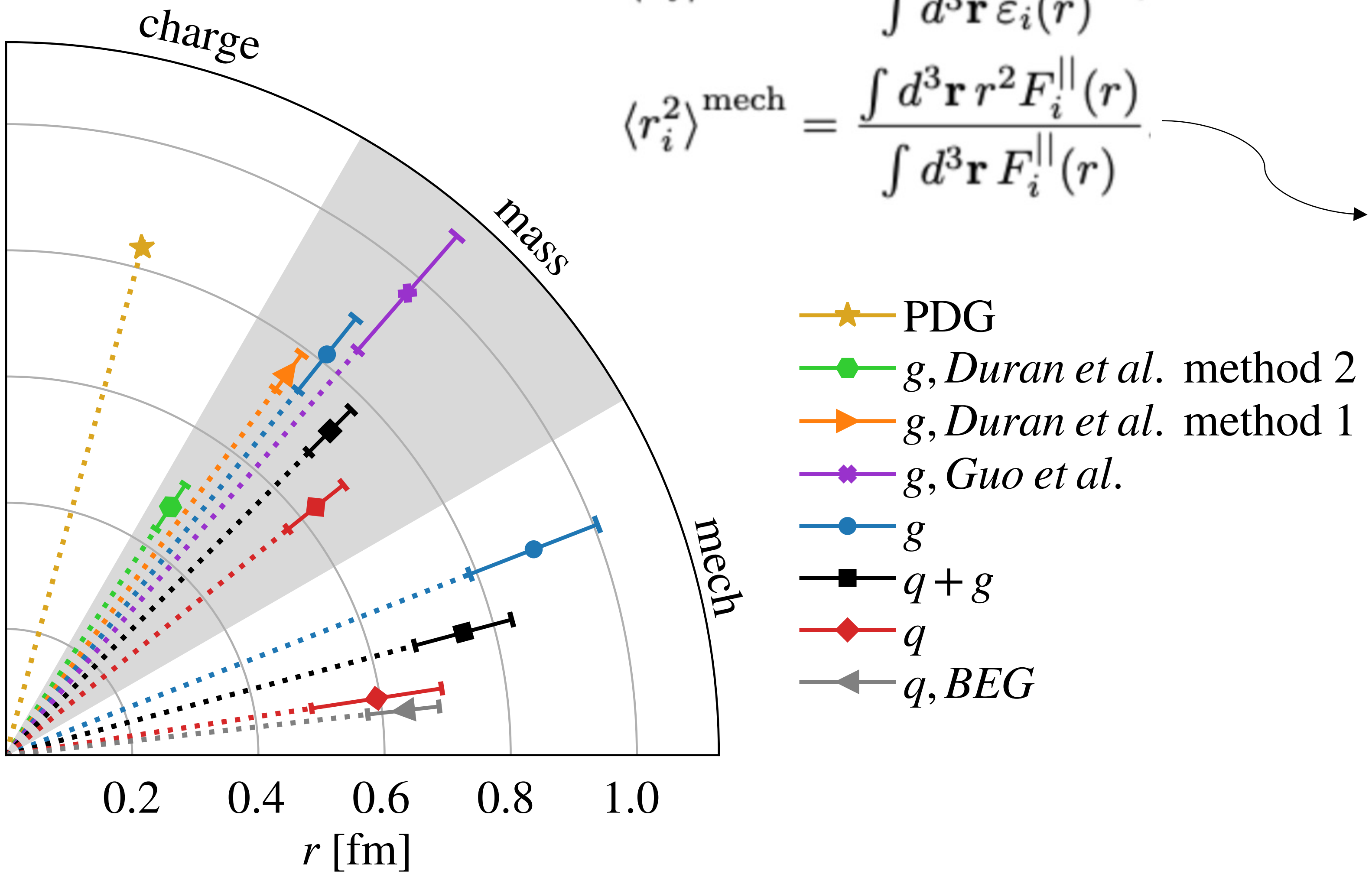


Nucleon size

$$\langle r_i^2\rangle^{\text{mass}} = \frac{\int d^3\mathbf{r}\, r^2 \varepsilon_i(r)}{\int d^3\mathbf{r}\, \varepsilon_i(r)}, \quad \longrightarrow \quad \varepsilon_i(r) = m\left[A_i(t) - \frac{t(D_i(t)+A_i(t)-2J_i(t))}{4m^2}\right]_{\text{FT}}$$

$$\langle r_i^2\rangle^{\text{mech}} = \frac{\int d^3\mathbf{r}\, r^2 F_i^{||}(r)}{\int d^3\mathbf{r}\, F_i^{||}(r)}$$

$$\begin{cases} p_i(r) = \dfrac{1}{6m}\dfrac{1}{r^2}\dfrac{d}{dr}r^2\dfrac{d}{dr}[D_i(t)]_{\text{FT}} \\ s_i(r) = -\dfrac{1}{4m}r\dfrac{d}{dr}\dfrac{1}{r}\dfrac{d}{dr}[D_i(t)]_{\text{FT}} \\ F_i^{||}(r) = p_i(r) + 2s_i(r)/3 \end{cases}$$



FT = Fourier transform

3D Breit frame

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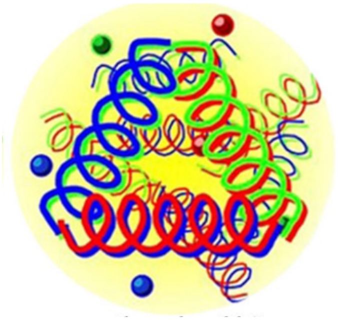
  [Abbott Hackett DAP Romero-Lopez Shanahan Urban, in preparation]

GFFs of Scalar glueball $G\ [0^+]$ in Yang-Mills

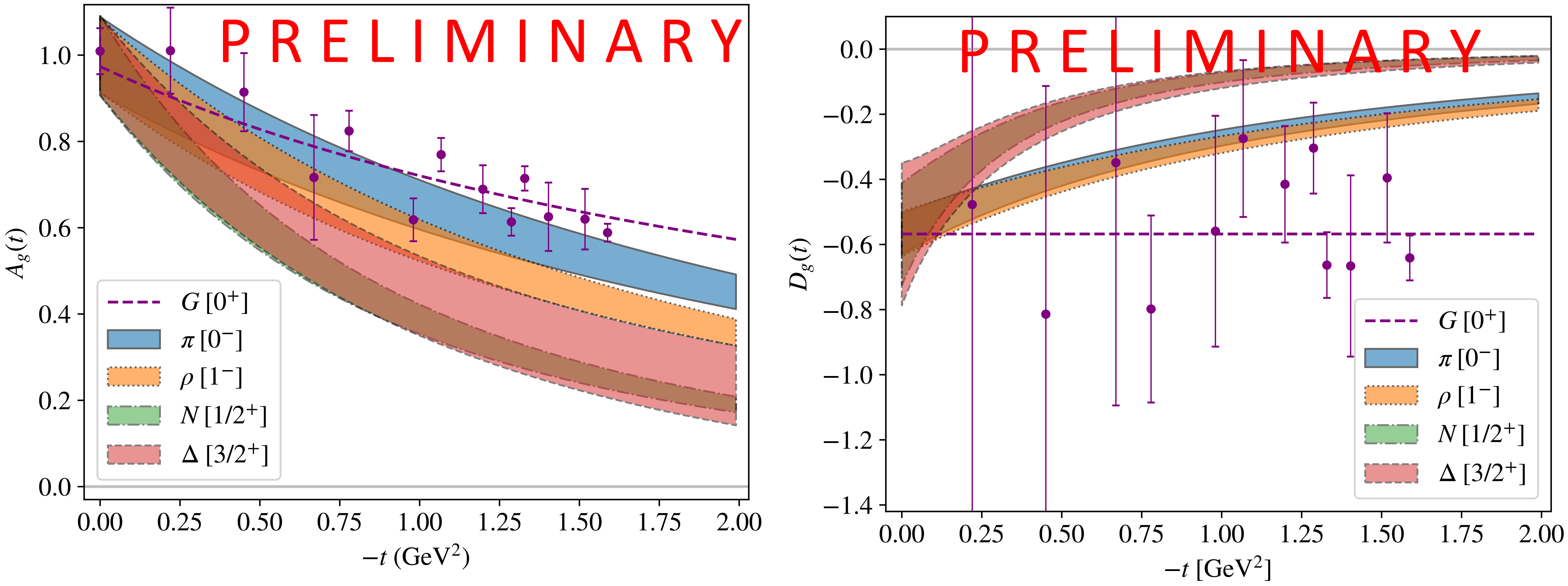
Abbott, Hackett, **DAP**, Romero-Lopez, Shanahan, Urban
in preparation

- SU(3) (no dynamical quarks) with $\beta = 5.95$
- $L^3 \times T = 24^3 \times 48, a \approx 0.1 \text{ fm}$
- Lattice ensemble generation algorithm: heatbath + overrelaxation
- $|\boldsymbol{\Delta}|^2 \leq 8\left(\frac{2\pi}{L}\right)^2, |\boldsymbol{p}'|^2 \leq 6(\frac{2\pi}{L})^2$
- 2, 3, 5 steps of stout-smearing
- Current preliminary results ~20M measurements

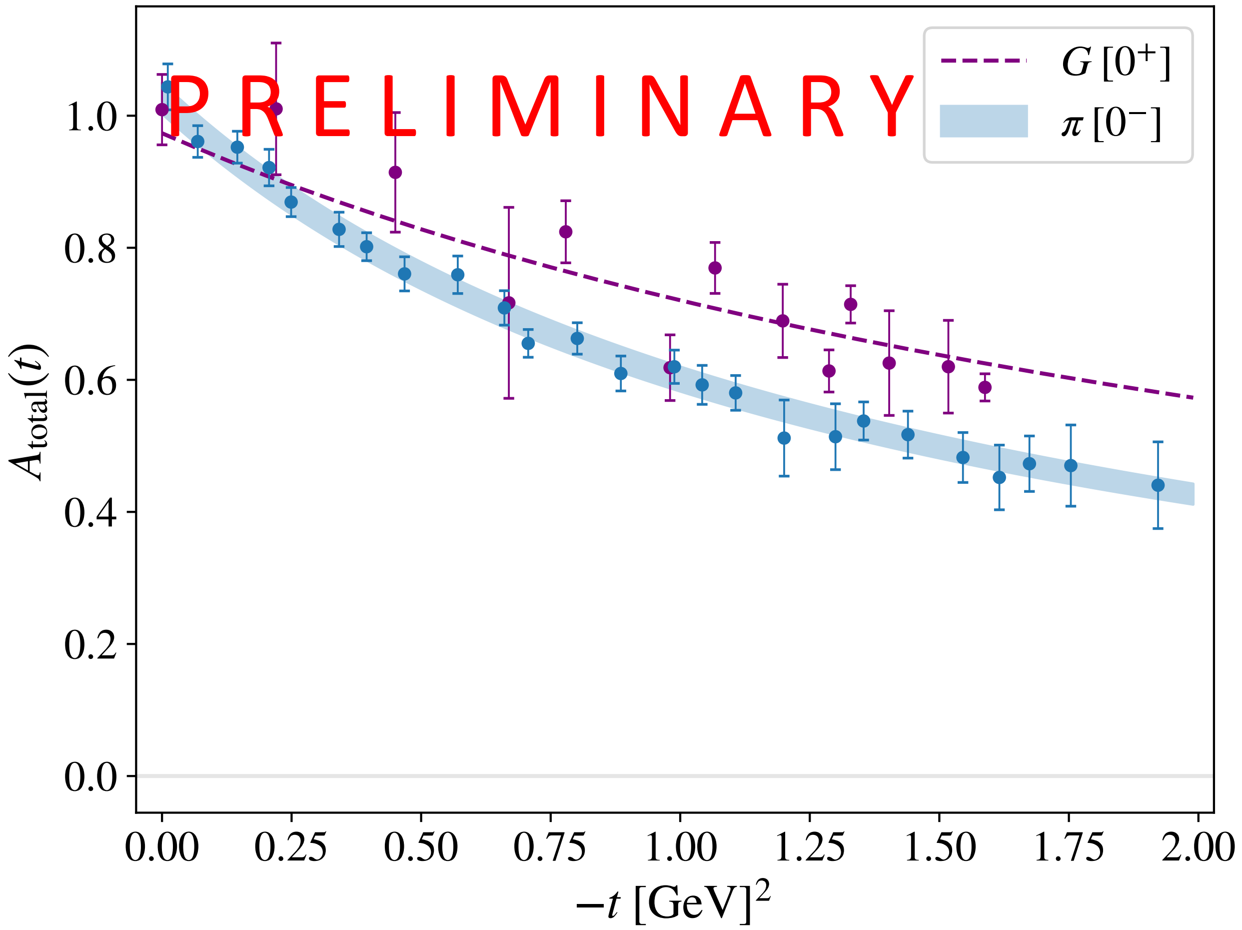


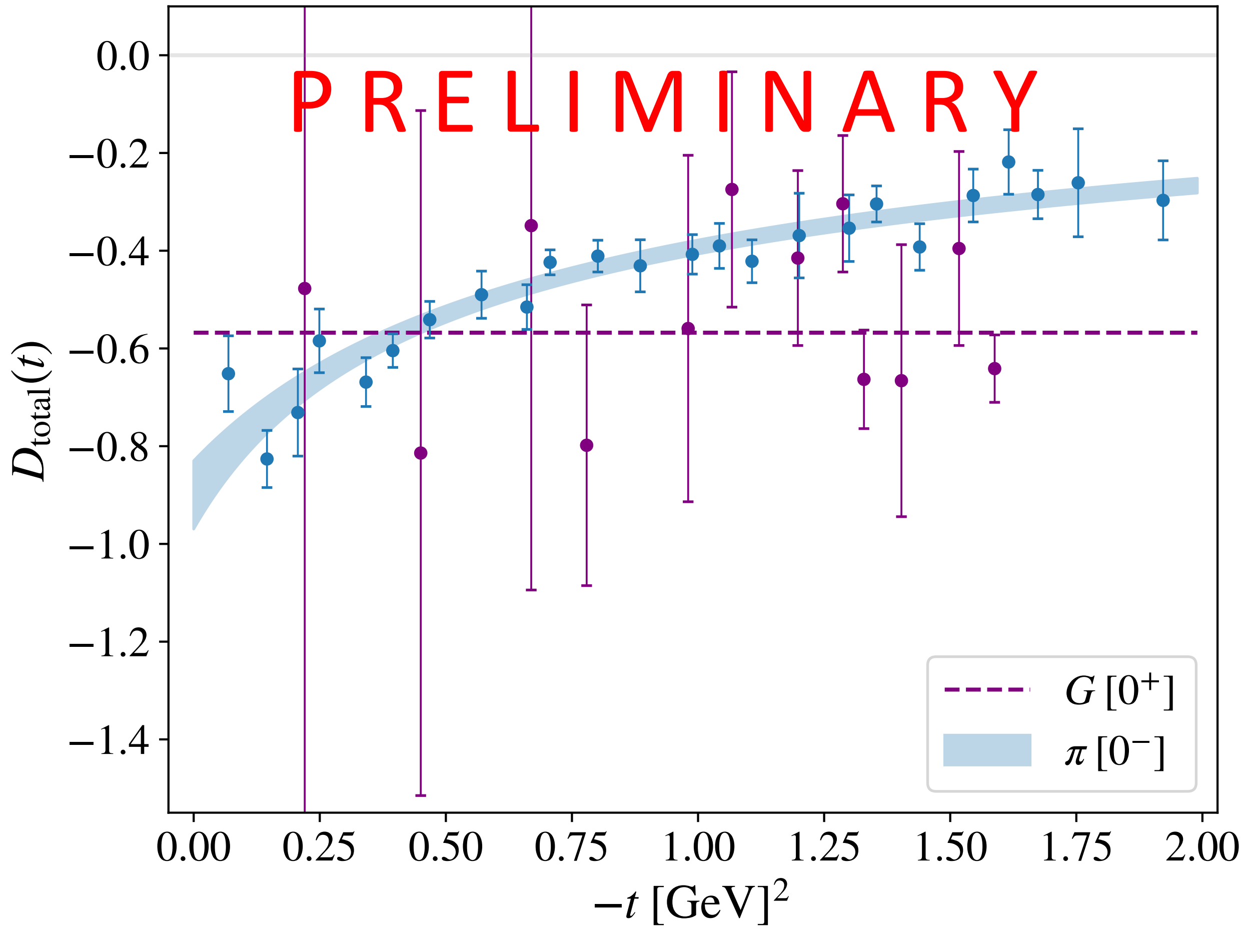
Glueball comparison with hadron gluon GFFs at $m_\pi \approx 450$ MeV

(single-interpolator only, more to be added)



Glueball comparison with pion <u>total</u> GFFs at $m_\pi \approx 170$ MeV

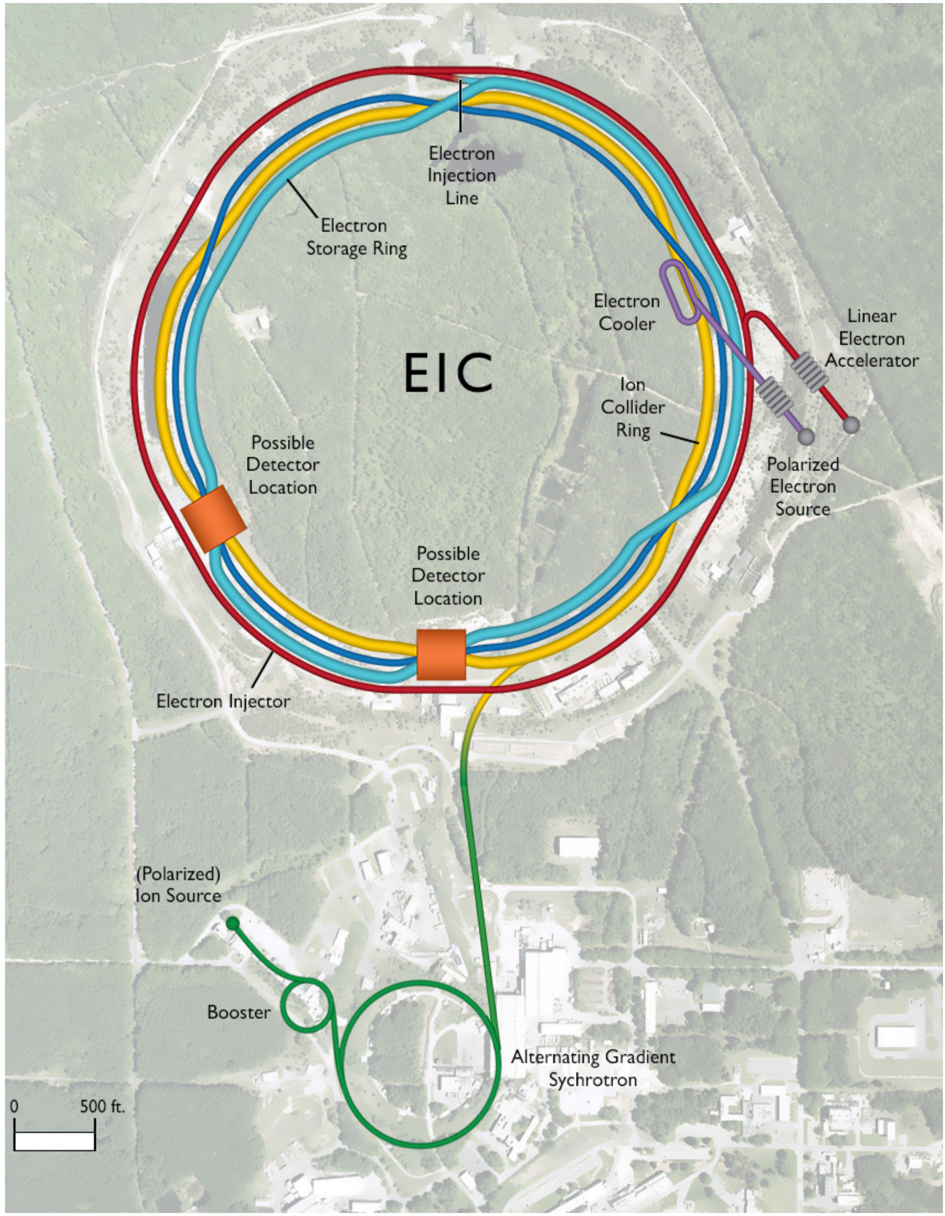




Summary and remarks

- Gravitational form factors: the form factors of the energy-momentum tensor.
- Encode how energy, angular momentum, and mechanical properties are distributed inside hadrons. Moments of GPDs (generalized form factors) and PDFs in the forward limit (e.g momentum fraction).
- Lattice QCD constraints to the GFFs of the pion, proton, … More results are coming from lattice and experiments!
- Beyond measuring: Much more to understand about the QCD EMT and GFFs

See, e.g., Adam Freese's talk on Thursday



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THANK YOU!