Threshold resummation for the virtual Compton process

Jakob Schönleber

RIKEN BNL

Based on [J.S., 2022] and [J.S., in preparation]

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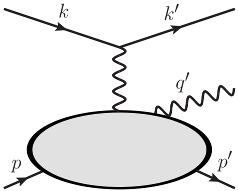
Virtual Compton scattering

$$\gamma^*(q)\ N(p) \longrightarrow \gamma^{(*)}(q')\ N(p')$$



- Kinematical parameters

$$P^\mu = \frac{p^\mu + p'^\mu}{2}, \qquad Q^2 = -\frac{1}{4}(q+q')^2, \qquad t = (p-p')^2, \qquad s = (p+q)^2,$$

$$x_B = -\frac{(q+q')^2}{2(p+p')\cdot(q+q')}, \qquad \xi = \frac{n\cdot(p-p')}{n\cdot(p+p')}, \qquad m^2 = p^2 = p'^2$$

- n is an auxiliary light-cone vector that projects p and p' onto their large light-cone components $p^+ \equiv n\cdot p$ and $p'^+ \equiv n\cdot p'$.
- We consider $Q^2 \to \infty$ limit at fixed x_B and t.
- **DVCS:** $q'^2 = 0$ or equivalently $x_B = \xi$.

Virtual Compton scattering

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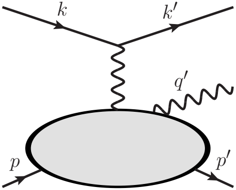


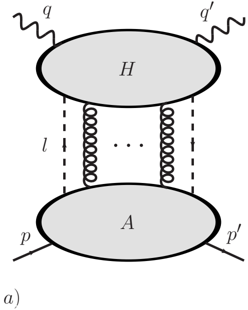
Leading power kinematics in terms of two light-cone vectors $n, \bar{n}$

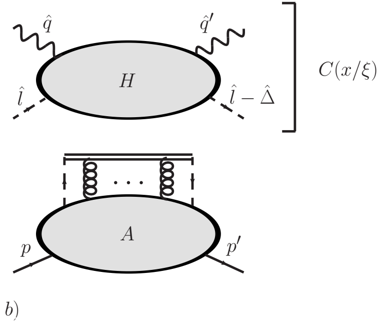
$$p^\mu = (1+\xi)P^+\bar{n}^\mu,$$

$$p'^\mu = (1-\xi)P^+\bar{n}^\mu,$$

$$q^\mu = -(x_B+\xi)P^+\bar{n}^\mu + \frac{Q^2}{2x_B P^+}n^\mu,$$

$$q'^\mu = -(x_B-\xi)P^+\bar{n}^\mu + \frac{Q^2}{2x_B P^+}n^\mu.$$





- a) Leading region of DVCS amplitude, b) Factorized form after performing expansion.
  Dashed lines can be quarks or transversely polarized gluons.
- Collinear approximation: In H set $l = (l^+, l^-, l_\perp) \sim \hat{l} = (l^+, 0, 0_\perp)$ and $M^2 = t = 0$

$$T^{\mu\nu} \sim \int d^4 l \, H(q, q', l) A(p, p', l) \sim \int dl^+ \underbrace{H(\hat{q}, \hat{q}', \hat{l})}_{\text{coefficient function } C} \underbrace{\int dl^- d^2 l_\perp A(p, p', l)}_{\text{GPD}}$$

- C has double logs

$$C = \frac{1}{x - x_B}\left\{1 + \frac{\alpha_s C_F}{2\pi}\left[\log^2\left(\frac{x - x_B}{2x_B}\right) + ...\right]\right.$$
$$\left. + \left(\frac{\alpha_s C_F}{2\pi}\right)^2\left[\frac{1}{2}\log^4\left(\frac{x - x_B}{2x_B}\right) + ...\right] + ...\right\} + O((x - x_B)^0)$$

- Do these logs exponentiate to all orders? Can resum? $\rightarrow$ Yes.
- Are these logs "large"? $\rightarrow$ Probably not.

- Let l be the parton momentum and $k = l + q$. Then the partonic center-of-mass energy is

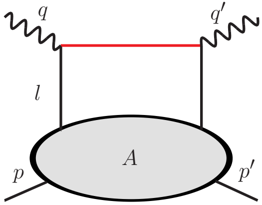
$$k^2 \approx \frac{x - x_B}{2x_B} Q^2,$$

where $x = \frac{l^+}{P^+} - \xi$.

- I define the "threshold region" by $|k^2| \ll Q^2$ or equivalently $|x - x_B| \ll 1$.

- Hard propagator becomes collinear the n-direction

$$k = l + q \approx \left((x - x_B) P^+, \frac{Q^2}{2x_B P^+}, 0_\perp \right)$$



- Consider two parameters

$$\lambda \sim \frac{\sqrt{|t|}}{Q} \sim \frac{m}{Q} \sim \frac{\Lambda_{\mathrm{QCD}}}{Q},$$
$$\eta \sim \frac{\sqrt{|k^2|}}{Q} = \sqrt{\frac{|x_B - x|}{x_B}}.$$

- Main factorization formula is

$$T = \int dl^+ \, C(k, q, q') F(l^+, p, p') + O(\lambda).$$

- Idea: Derive secondary factorization of the coefficient function

$$C(k, q, q', \mu) = H(-q^2, \mu) H(-q'^2, \mu) \mathcal{J}(-k^2, \mu) + \hat{O}(\eta).$$

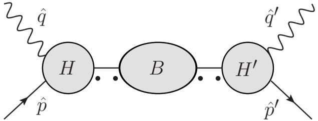
($\hat{O}$ means O up to logarithms)

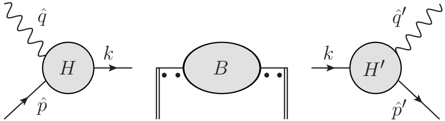
- Solve evolution equations for H and $\mathcal{J}$ to resum power of $\log(x - x_B)$

- Simplification: Consider the coefficient function C as on-shell and massless amplitude.
- This is not quite true, because we also have subtractions. There are arguments why the subtractions also factorize.
- This simplification kills the target-collinear and soft regions (they always lead to scaleless integrals), so we only have two regions: hard and n-collinear.
- Contribution from a given region R that has collinear subgraph B and a hard subgraph H is proportional to $\eta^{p(R)}$, where

$$\begin{aligned} p(R) = -4 + \#(\text{external lines}) + \#(\text{lines } B \text{ to } H) \\ - \#(\text{scalar pol. gluon lines } B \text{ to } H). \end{aligned}$$

- Factorization:





$$C(k,q,q',\mu) = H(-q^2,\mu)H(-q'^2,\mu)\mathcal{J}(-k^2,\mu) + \hat{O}(\eta^0),$$

where $\mathcal{J} \sim \eta^{-1}$ and $H, H' \sim \eta^0$

- H is hard matching coefficient of Sudakov form factor. $\mathcal{J}$ is uncut jet function, i.e. quark propagator in axial gauge.

- Evolution equations

$$\mu\frac{d}{d\mu}H(Q^2,\mu^2) = \left[\Gamma_{\rm cusp}(\alpha_s(\mu))\log\frac{Q^2}{\mu^2} + \gamma_H(\alpha_s(\mu))\right]H(Q^2,\mu^2)$$
$$\mu\frac{d}{d\mu}\mathcal{J}(-k^2,\mu) = \left[-2\Gamma_{\rm cusp}(\alpha_s(\mu))\log\frac{-\hat{s}}{\mu^2} - 2\gamma_{\mathcal{J}}(\alpha_s(\mu))\right]\mathcal{J}(-k^2,\mu)$$

- Solve evolution equations for H and $\mathcal{J}$ to get

$$C(q^2,q'^2,k^2,\mu) = H(-q^2,\mu_h)H(-q^2,\mu_h')\mathcal{J}(-k^2,\mu_i) \times U_H(q^2,\mu_h)U_H(q'^2,\mu_h') \underbrace{U_{\mathcal{J}}(k^2,\mu_i)}_{\text{threshold logs resummed here}}$$

- $\mu_h, \mu_h' \sim Q$ but $\mu_i \sim\, ?$

- μ_i should be choosen such that logs are minimized, so an obvious choice is $\mu_i \sim \eta Q$. Problem: η depends on the loop momentum variable x
- The resummed result depends on $\alpha_s(\mu_i)$. Need to provide analytic continuation of α_s into the complex plane.
- Integrating above or below the Landau pole in the complex x-plane gives different results. This ambiguity is power-suppressed $\sim (\Lambda_{\mathrm{QCD}}/Q)^p$.
- The Landau pole can be avoided by choosing a fixed μ_i (i.e. independent of x). But what value?

- Take imaginary part of the amplitude

$$J(k^2,\mu) = \frac{1}{\pi}\mathrm{Im}\,\mathcal{J}(-k^2 - i0,\mu).$$

  Supported for $k^2 > 0$ or equivalently $x > x_B$.

- Get factorization formula for DIS as $x_B \to 1$

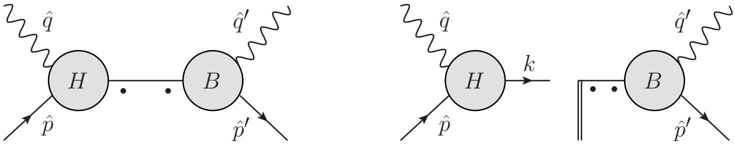
$$\frac{1}{\pi}\mathrm{Im}\,T^{\mathrm{forward}} = (H(Q^2,\mu))^2 Q^2 \int_{x_B}^{1} \frac{dx}{x_B}\, J(k^2,\mu) q(x,\mu) + \hat{O}((1-x_B)q(x_B,\mu))$$

- Well-known, usually used in moment space: [Sterman, 1987], [Catani, Trentadue, 1989], [Korchemsky, Marchesini, 1993]. Momentum space using SCET: [Becher, Neubert, Pecjak, 2007].

- Integration is limited to $[x_B, 1] \Rightarrow$ the expansion in η corresponds to an expansion in $1 - x_B \Rightarrow$ Threshold resummation is relevant for the endpoint region $x_B \to 1$ in DIS.

- The fixed scale choice $\mu_i = \sqrt{1-x_B}\,Q$ makes sense for the imaginary part of the amplitude. What about the real part?

- Consider now DVCS, where $q'^2 = 0$ or equivalently $x_B = \xi$. The threshold region $x \sim x_B$ then coincides with the soft parton region $x \sim \xi$.
- Factorization of the coefficient function:



$$C(k, q, \mu) = H(-q^2, \mu)\mathcal{I}(-k^2, \mu) + \hat{O}(\eta)$$

- This factorization has been confirmed at two-loop accuracy.
- $\mathcal{I} = \langle \hat{p}' | T(\bar{\psi} W_n) j^{\mu}_{\text{em}} | 0 \rangle$ is a new function. It has the standard double log evolution

$$\mu \frac{d}{d\mu} \mathcal{I}(-k^2, \mu) = \left[-\Gamma_{\text{cusp}}(\alpha_s(\mu)) \log \frac{-k^2}{\mu^2} - \gamma_{\mathcal{I}}(\alpha_s(\mu)) \right] \mathcal{I}(-k^2, \mu)$$

- A subtelty: CF has pole $\frac{1}{x-\xi}$ and GPD has discontinuous derivative at $x=\xi$. Fortunately, it is continuous.

- Can write GPD for $x>0$ as

$$F(x,\xi) = \Theta(x-\xi)\underbrace{F_{x>\xi}(x,\xi)}_{\text{analytic}} + \Theta(\xi-x)\underbrace{F_{x<\xi}(x,\xi)}_{\text{analytic}}$$

and then

$$\int_0^1 dx\; C(x/(\xi-i0))F(x,\xi)$$
$$= \underbrace{\int_0^1 dx\; C(x/(\xi-i0))F_{x<\xi}(x,\xi)}_{\text{deform contour here}} + \int_\xi^1 dx\; C(x/\xi)\underbrace{\Big(F_{x>\xi}(x,\xi) - F_{\xi<x}(x,\xi)\Big)}_{\text{gives additional suppression at } x=\xi}$$

- Get analytic term where contour deformation can be performed and term that is suppressed at least linearly at $x=\xi$, similar to soft-end suppression for distribution amplitudes.

- Pole at $x=\xi$ moves to the endpoint as $\xi\to 1$. Integral still convergent since F vanishes power-like as $x\to 1$.

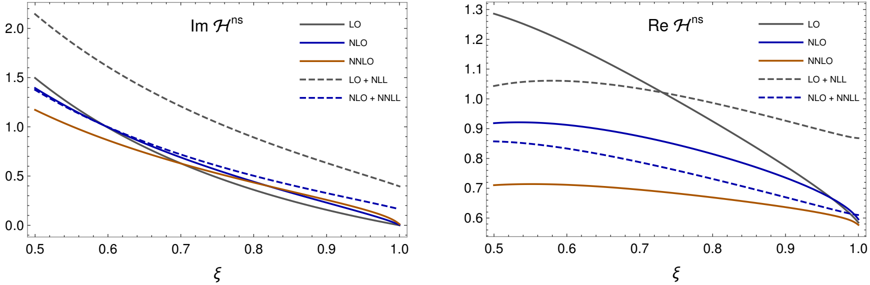
- Similarly, we can avoid the Landau pole by contour deformation.
- For this we have to choose an analytic continuation of the running coupling into the complex plane.

$$\alpha_s(\sqrt{-k^2}) = \frac{\alpha_s(Q)}{r}\left\{1 - \frac{\alpha_s(Q)}{4\pi r}\frac{\beta_1}{\beta_0}\log r + \left(\frac{\alpha_s(Q)}{4\pi r}\right)^2 \times \left[\frac{\beta_1^2}{\beta_0^2}\left(\log^2 r - \log r - 1 + r\right) + \frac{\beta_2}{\beta_0}(1-r)\right]\right\} + O(\alpha_s(Q)^4)$$

where $r = 1 + \frac{\alpha_s(Q)}{4\pi}\beta_0 \log(-k^2/Q^2)$.

- Leading log Landau pole at $x = \xi + \underbrace{e^{-\frac{4\pi}{\alpha_s(Q)\beta_0}}}_{(\frac{\Lambda_{\mathrm{QCD}}}{Q})^2}$ never coincides with $x = \xi$.

- Quark Compton form factor $\mathcal{H}^{\rm ns} = \int_{-1}^{1} \frac{dx}{\xi} C_q(x/\xi) H_q(x,\xi)$ for simple model for the unpolarized GPD H_q



Gray: LO/NLL, Blue: NLO/NNLL, Brown: NNLO
Straight lines: Fixed order, Dashed lines: with resummation
$\mu_i = \eta Q, \mu = \mu_h = Q$.

- Small corrections at small ξ. Substantial corrections at moderately large ξ. Non-zero imaginary part at $\xi \to 1$ due to Landau pole.

- In DVCS the maximum value of ξ is determined by t, by the inequaltiy $\xi \leq \sqrt{\frac{-t}{-t+4m^2}} \Rightarrow$ corrections are only relevant at large t.

Thank you!