

Mapping Pion and Kaon Parton Distributions with Lattice QCD

QUANTUM 3

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U.S. DEPARTMENT OF ENERGY

RESEARCH CORPORATION for SCIENCE ADVANCEMENT

@LinQCD

MICHIGAN STATE UNIVERSITY

Motivation

§ Meson structure is crucial to understand the mechanism of emergent hadron mass (EHM)

- ❖ Help decode QCD origin of mass

§ Experimentally, meson structure is harder to study

- ❖ LQCD can provide predictions and better precision inputs
- ❖ Quark and gluon parton distribution functions(PDFs), for example

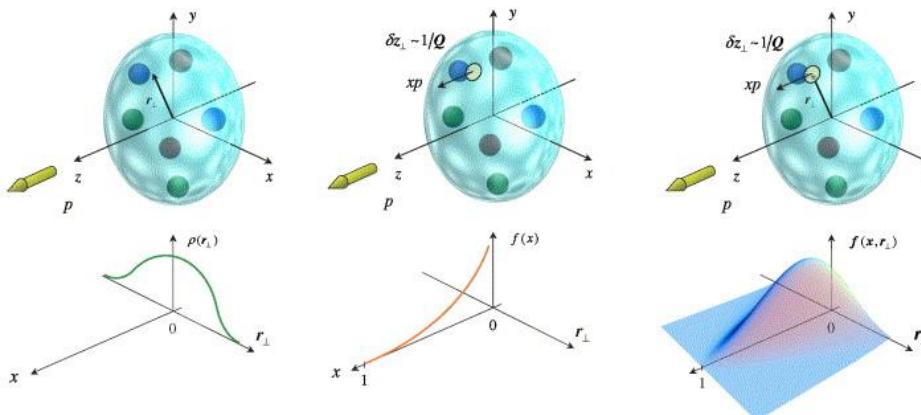
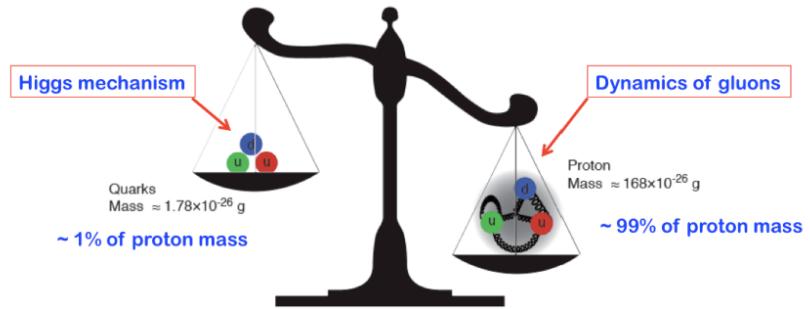


Image from A. Belitskya and A Radyushkin,
Physics Report, 416 (2015)



§ Generalized parton distributions (GPDs) encode information about the spatial structure & the partonic distribution of spin and orbital angular momenta

- ❖ US EIC, EIcC, LHeC, ...

Outline

§ Lattice QCD in a Nutshell

§ x -dependent Meson Structure

- ❖ Valence quark distributions of pion and kaon
- ❖ Pion and Kaon gluon distribution
- ❖ Pion valence-quark GPDs

Biased selected results toward MSULat students and postdocs



What is Lattice QCD?

- § Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories
- § Physical observables are calculated from the path integral

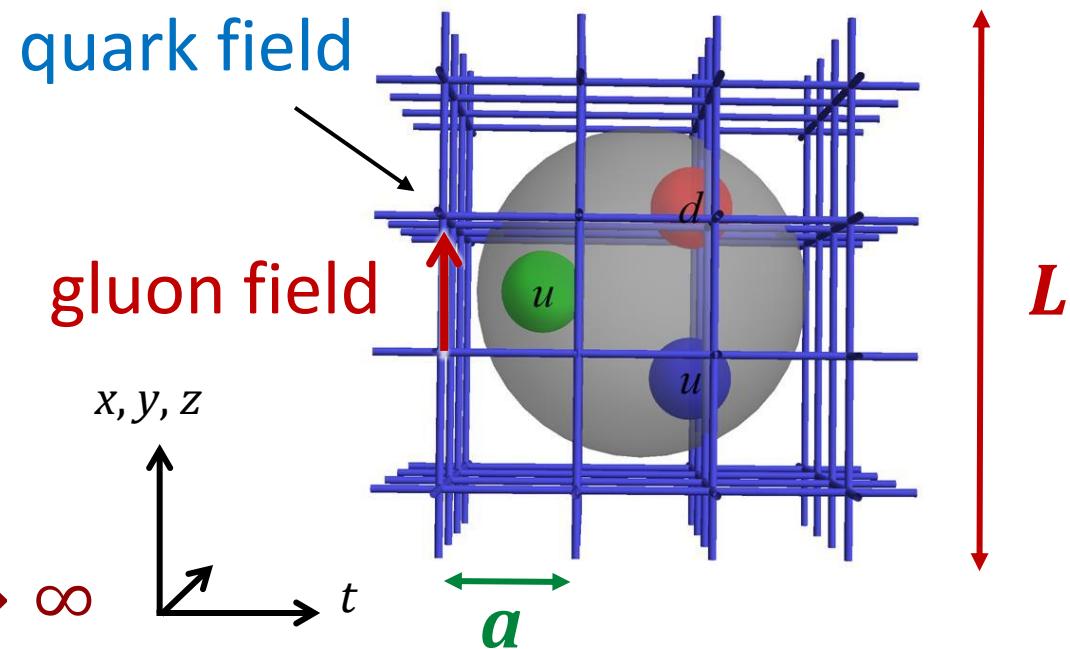
$$\langle 0 | O(\bar{\psi}, \psi, A) | 0 \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS(\bar{\psi}, \psi, A)} O(\bar{\psi}, \psi, A)$$

in **Euclidean** space

- ❖ Quark mass parameter (described by m_π)
- ❖ Impose a UV cutoff
discretize spacetime
- ❖ Impose an infrared cutoff
finite volume

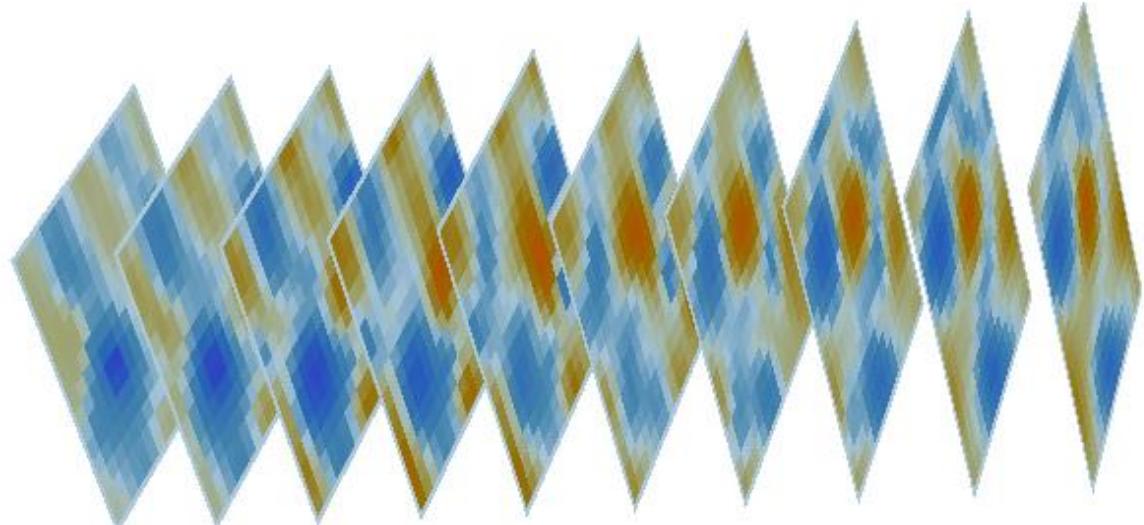
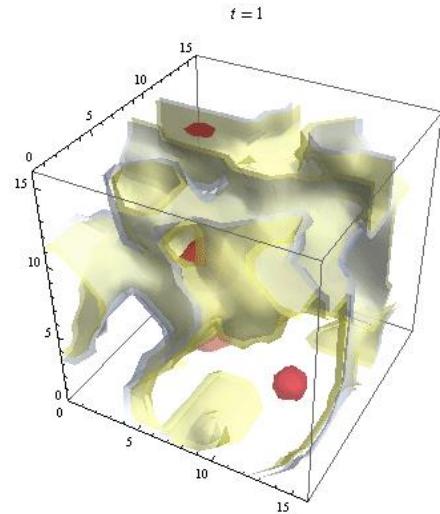
§ Recover physical limit

$$m_\pi \rightarrow m_\pi^{\text{phys}}, a \rightarrow 0, L \rightarrow \infty$$



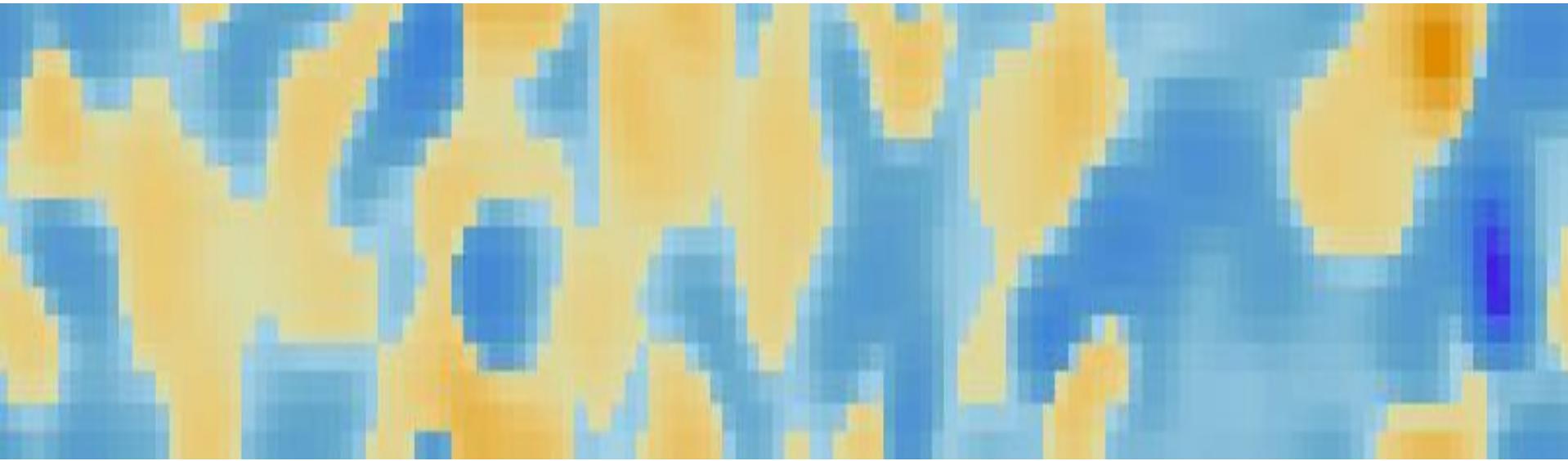
Anatomy of a Lattice Calculation

1. Start with QCD Vacuum (gauge configurations)



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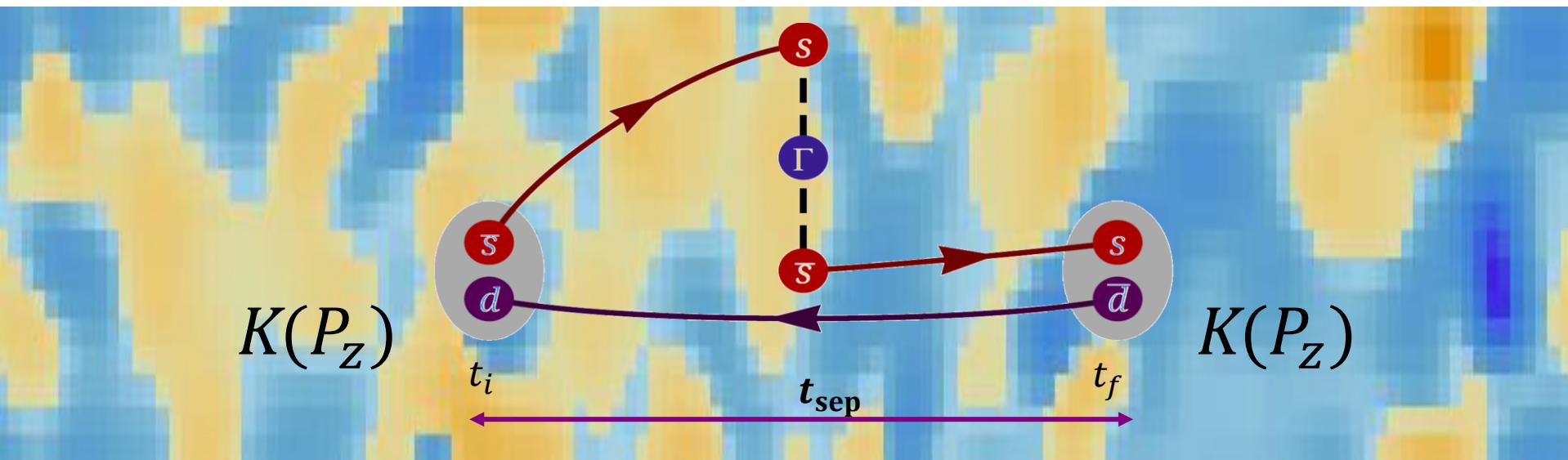


Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

Anatomy of a Lattice Calculation

2. Correlators (hadronic observables)

- ❖ Invert Dirac operator matrix (rank 10^{12})
- ❖ Combine using color, spin and momentum into hadrons



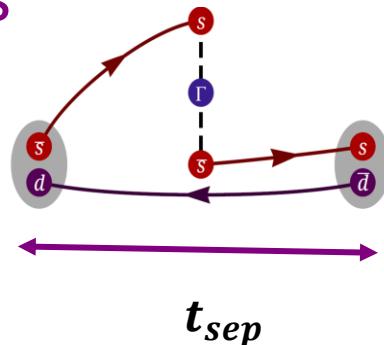
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Anatomy of a Lattice Calculation

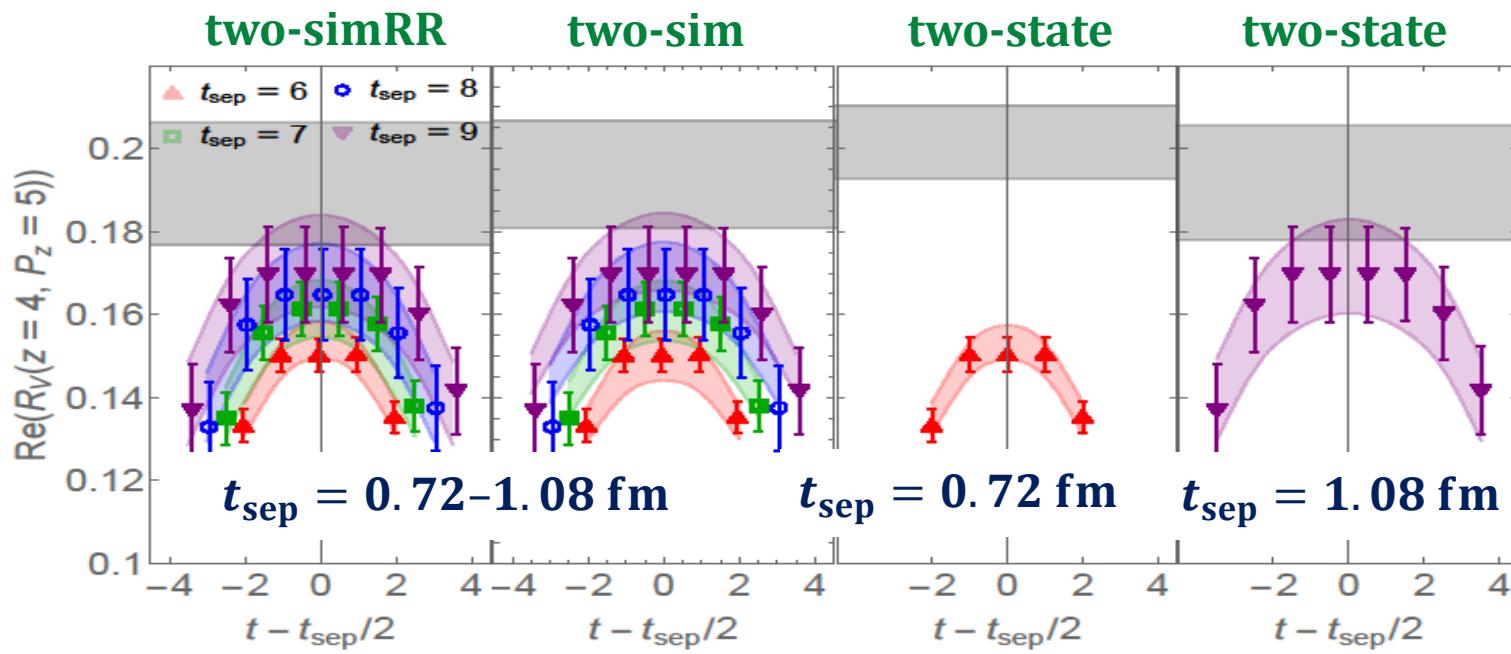
3. Extract reliable ground-state matrix elements

❖ Excited-state removal

❖ For example, kaon matrix element
at $M_\pi \approx 220$ MeV, $a \approx 0.12$ fm



H. Lin et al. (MSULat), 2003.14128

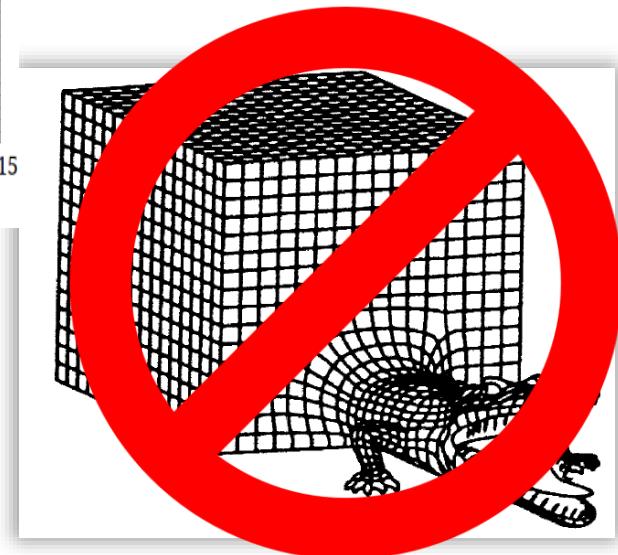
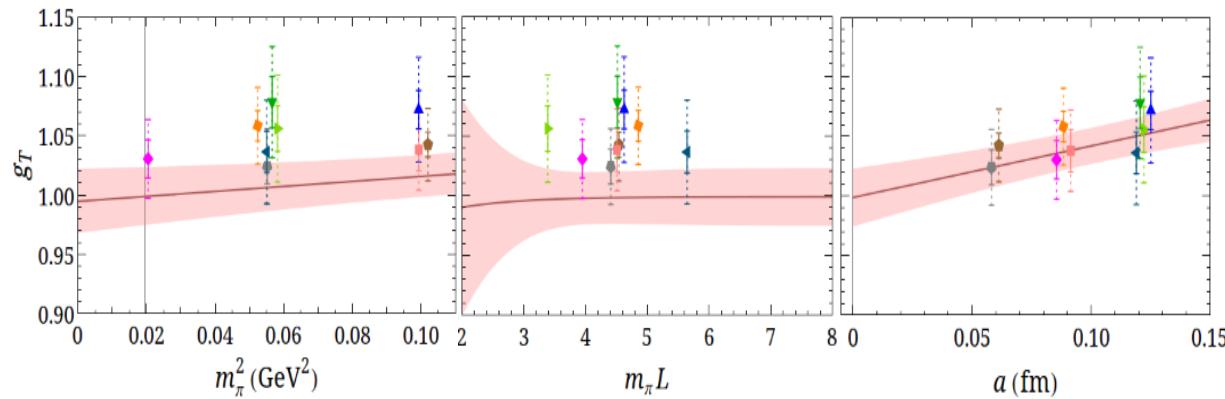


stability in extracting matrix elements

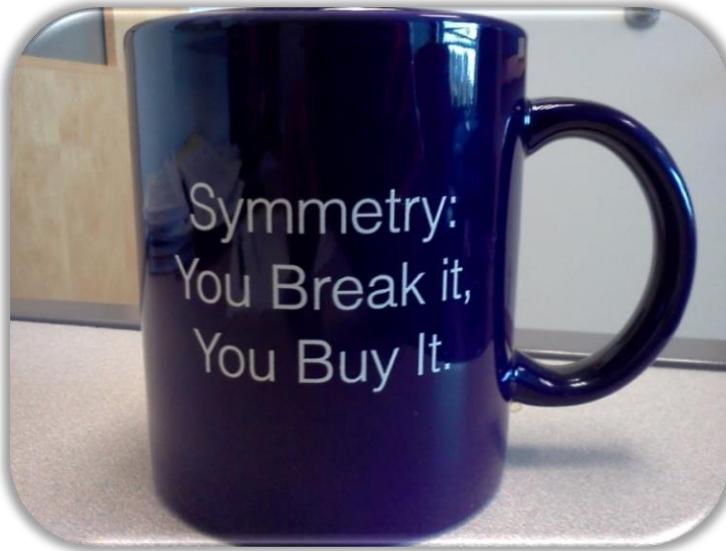
Anatomy of a Lattice Calculation

4. Systematic uncertainty (nonzero a , finite L , etc.)

- ❖ Nonperturbative renormalization, etc
- ❖ Extrapolation to the continuum limit
 $(m_\pi \rightarrow m_\pi^{\text{phys}}, L \rightarrow \infty, a \rightarrow 0)$



Lattice Structure Limitation



§ Lattice calculations rely on operator product expansion, only provide moments

$$\langle x^{n-1} \rangle_q = \int_{-1}^1 dx \ x^{n-1} q(x)$$

§ Longstanding obstacle!

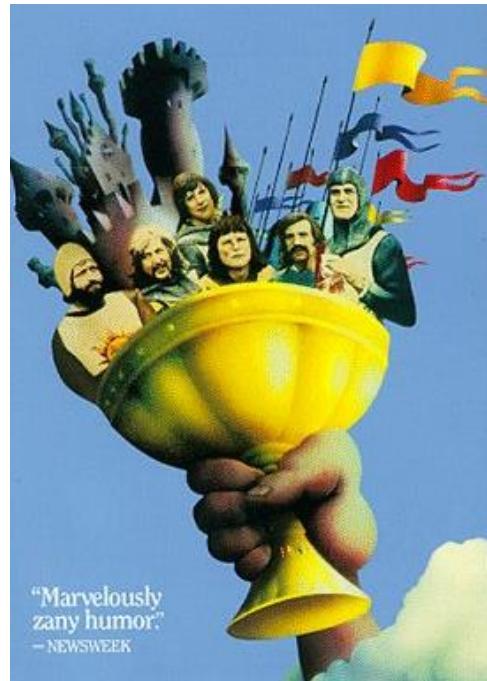
❖ Holy grail of structure calculations

§ Applies to many structure quantities:

❖ Parton Distribution Functions (PDFs)

❖ Generalized parton distributions (GPD)

❖ Transverse-momentum distributions (TMD)



A NEW HOPE

It is a period of war and economic uncertainty.

Tummoil has engulfed the galactic republics.

*Basic truths at foundation of the human civilization
are disputed by the dark forces of the evil empire.*

*A small group of QCD Knights from United Federation
of Physicists has gathered in a remote location on the
third planet of a star called Sol on the inner edge of
the Orion-Cygnus arm of the galaxy.*

*The QCD Knights are the only ones who can tame the
power of the Strong Force, responsible for holding
atomic nuclei together, for giving mass and shape to
matter in the Universe.*

They carry secret plans to build the most powerful

Direct x -Dependent Structure

§ Longstanding obstacle to lattice calculations!



- ❖ **Quasi-PDF**/large-momentum effective theory (LaMET)
(X. Ji, 2013; See 2004.03543 for review)
- ❖ **Pseudo-PDF** method: differs in FT (A. Radyushkin, 2017)
- ❖ Lattice cross-section method (**LCS**) (Y Ma and J. Qiu, 2014, 2017)
- ❖ Hadronic tensor currents (Liu et al., hep-ph/9806491, ... 1603.07352)
- ❖ Euclidean correlation functions (**RQCD**, 1709.04325)
- ❖ ...

Direct x -Dependent Structure

§ Longstanding obstacle to lattice calculations!

Quantities
that can be
calculated
on the lattice
today

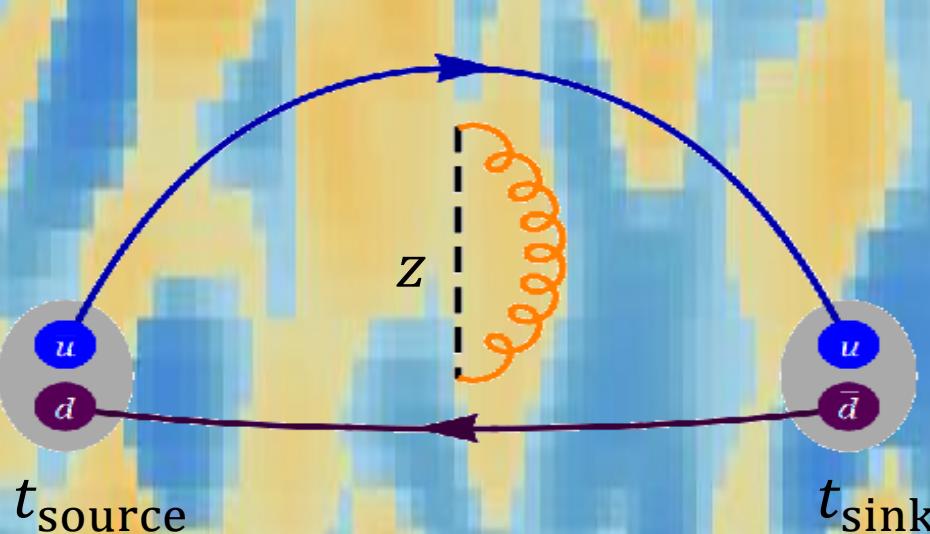
$$= \sum$$

Wanted
PDFs,
GPDs,
etc.

pQCD-
calculate
d kernel

Quasi-PDF & Pseudo-PDF method

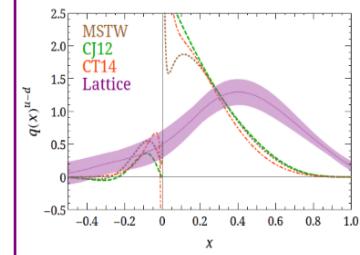
Quantities
that can be
calculated on
the lattice



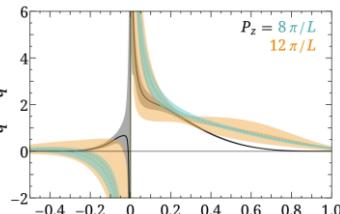
Lattice Parton Calculations

§ Physics quantity milestones

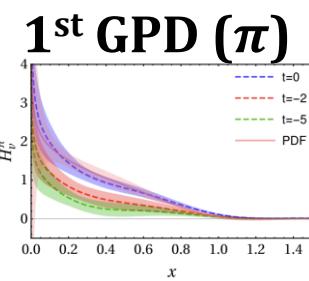
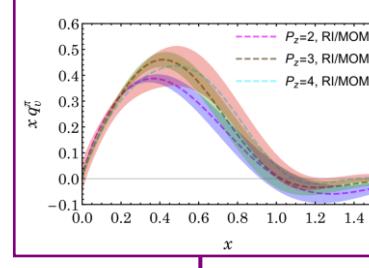
First unpol. lattice PDF



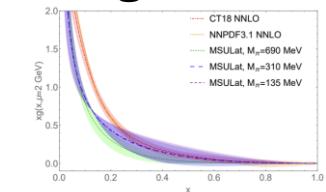
First PDFs at M_π^{phys}



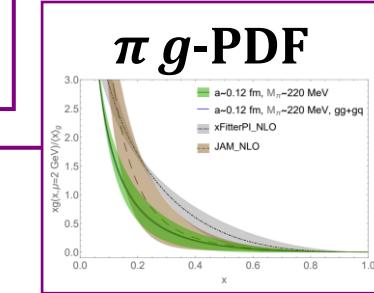
Pion v-PDF



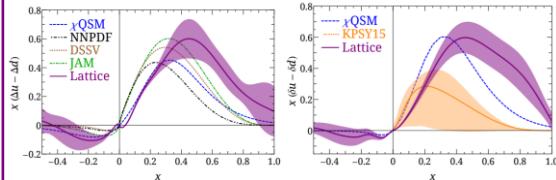
$N g$ -PDF



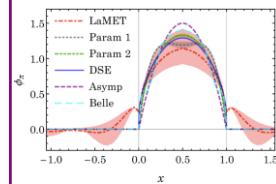
πg -PDF



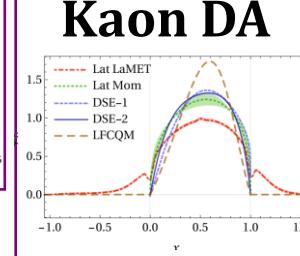
Pol. PDFs and mass corrections



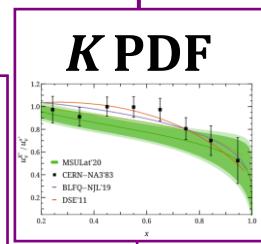
Pion DA



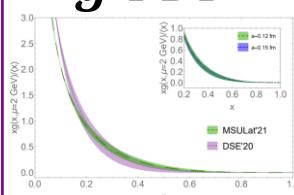
Kaon DA



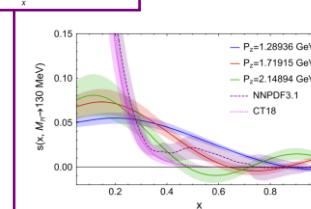
K PDF



Kaon g -PDF



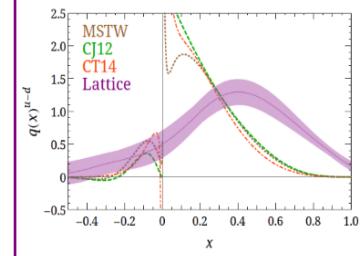
s,c PDF



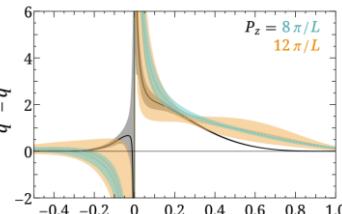
Lattice Parton Calculations

§ Physics quantity milestones

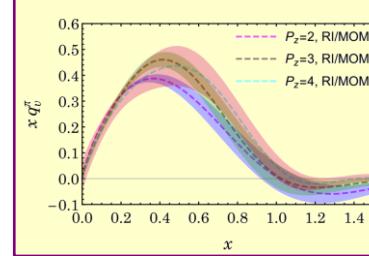
First unpol. lattice PDF



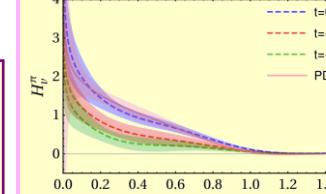
First PDFs at M_π^{phys}



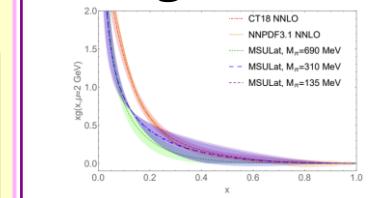
Pion v-PDF



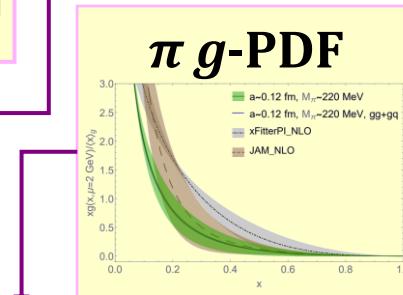
1st GPD (π)



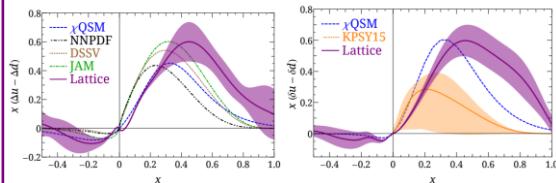
$N g$ -PDF



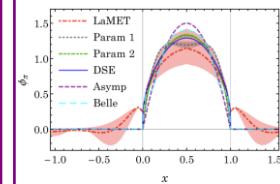
πg -PDF



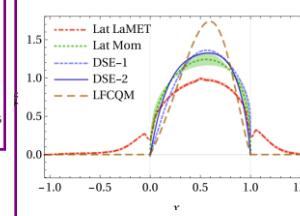
Pol. PDFs and mass corrections



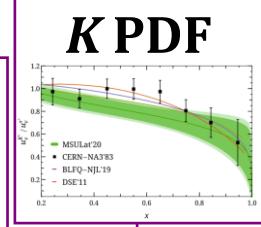
Pion DA



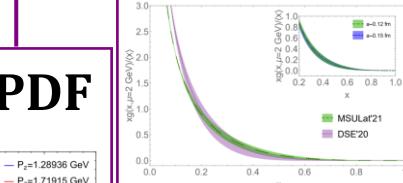
Kaon DA



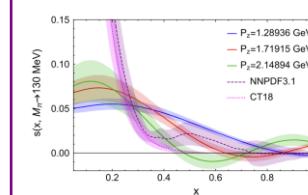
K PDF



Kaon g -PDF



s, c PDF



\mathcal{MSU} lat Pion/Kaon Structure

§ Meson distribution amplitude

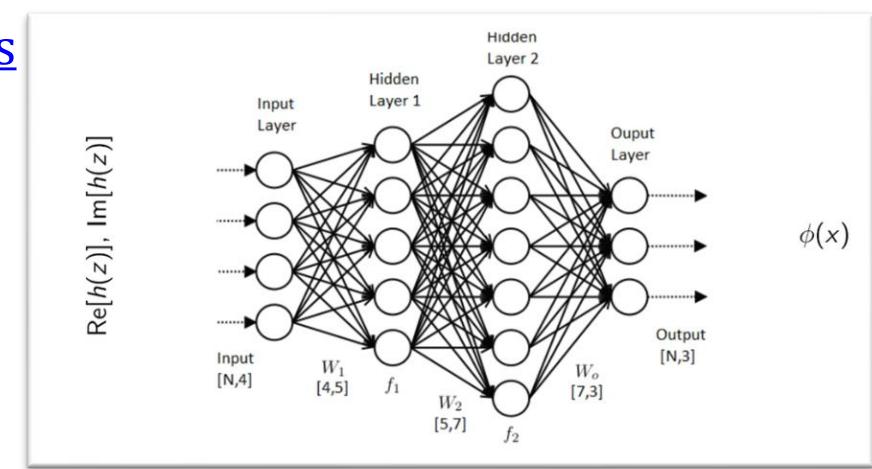
- ❖ [Pion Distribution Amplitude from Lattice](#),

Phys. Rev. D 95 (2017) 9, 094514

- ❖ [Kaon Distribution Amplitude from Lattice QCD and the Flavor SU\(3\) Symmetry](#), Nucl. Phys. B 939 (2019) 429-446

- ❖ [Pion and kaon distribution amplitudes in the continuum limit](#),

Phys. Rev. D 102 (2020) 9, 094519



- ❖ [Precision control in lattice calculation of x-dependent pion distribution amplitude](#), Nucl. Phys. B 993 (2023) 116282

§ Miscellaneous

- ❖ [Machine-learning prediction for quasiparton distribution function matrix elements](#), Phys. Rev. D 101 (2020) 3, 034516

MSULat Pion/Kaon Structure

§ Pion/kaon PDFs

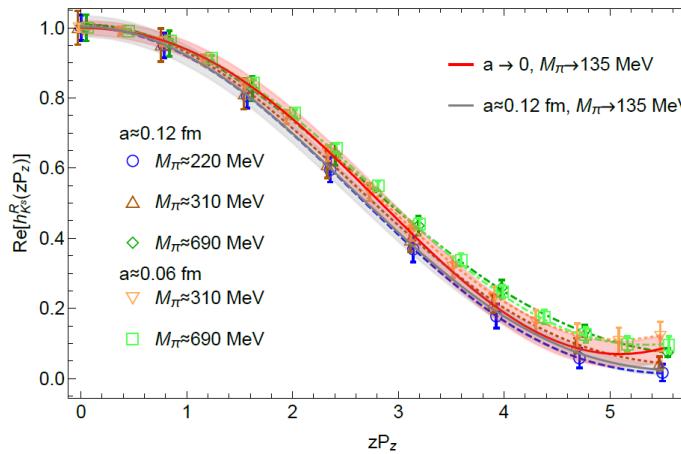
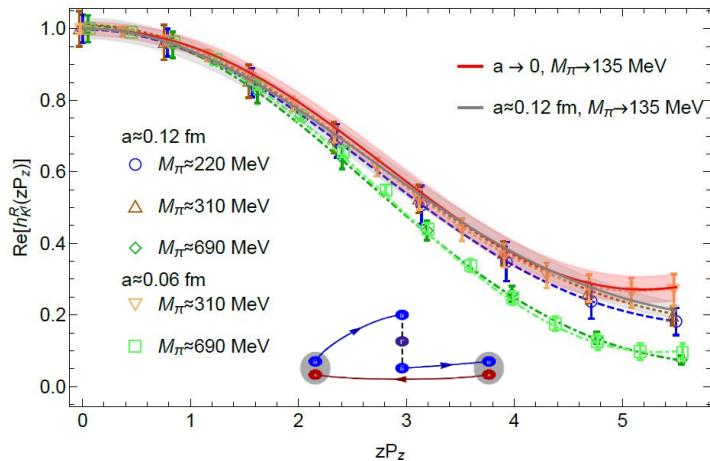
- ☞ [First direct lattice-QCD calculation of the \$x\$ -dependence of the pion parton distribution function](#), Phys. Rev. D 100 (2019) 3, 034505
- ☞ [Valence-Quark Distribution of the Kaon and Pion from Lattice QCD](#), Phys. Rev. D 103 (2021) 1, 014516
- ☞ [Gluon parton distribution of the pion from lattice QCD](#), Phys. Lett. B 823 (2021) 136778
- ☞ [First glimpse into the kaon gluon parton distribution using lattice QCD](#), Phys. Rev. D 106 (2022) 9, 094510
- ☞ [The Gluon Moment and Parton Distribution Function of the Pion from \$N_f=2+1+1\$ Lattice QCD](#), 2310.12034 [hep-lat]
- ☞ [Pion valence quark distribution at physical pion mass of \$N_f=2+1+1\$ LQCD](#)

§ Pion GPD

- ☞ [Pion generalized parton distribution from lattice QCD](#), Nucl. Phys. B 952 (2020) 114940
- ☞ [Pion valence-quark generalized parton distribution at physical pion mass](#), Phys. Lett. B 846 (2023) 138181

Meson Valence-quark PDFs

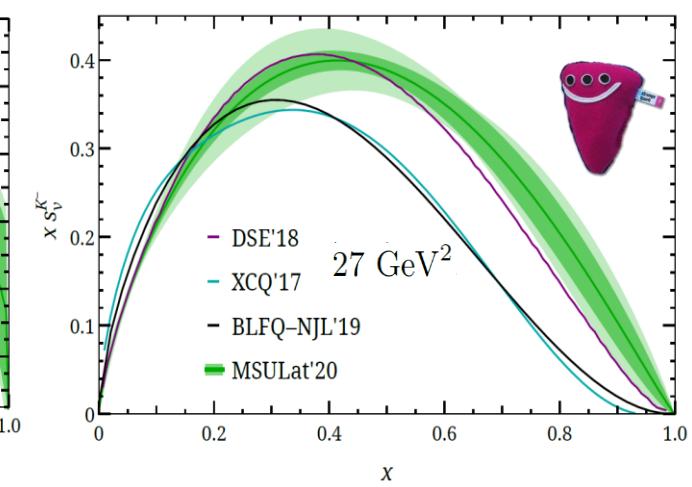
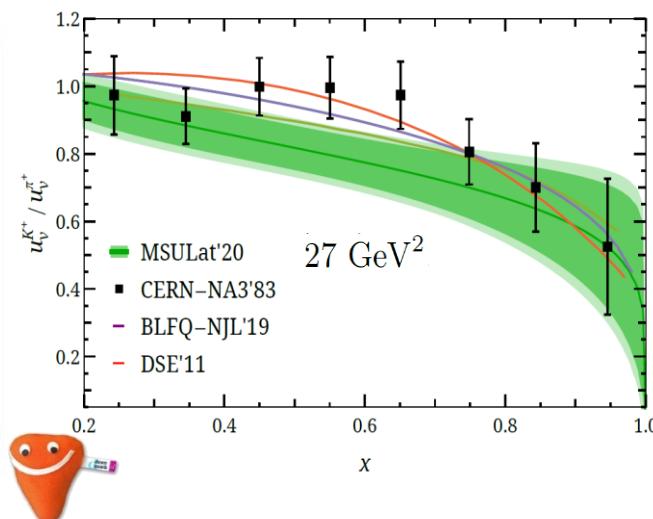
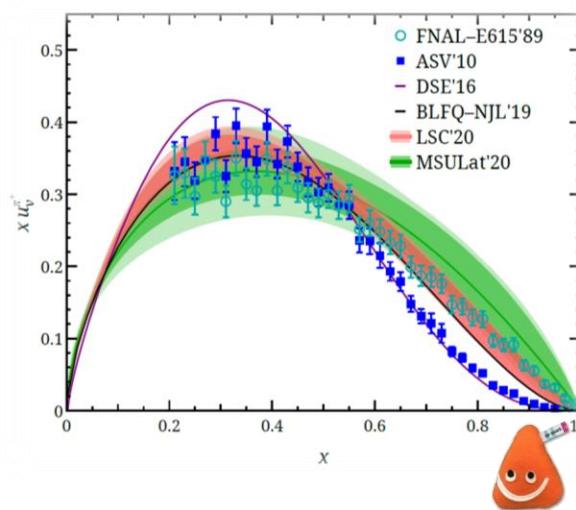
§ Pion/kaon PDFs using quasi-PDF in the continuum limit



Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc...

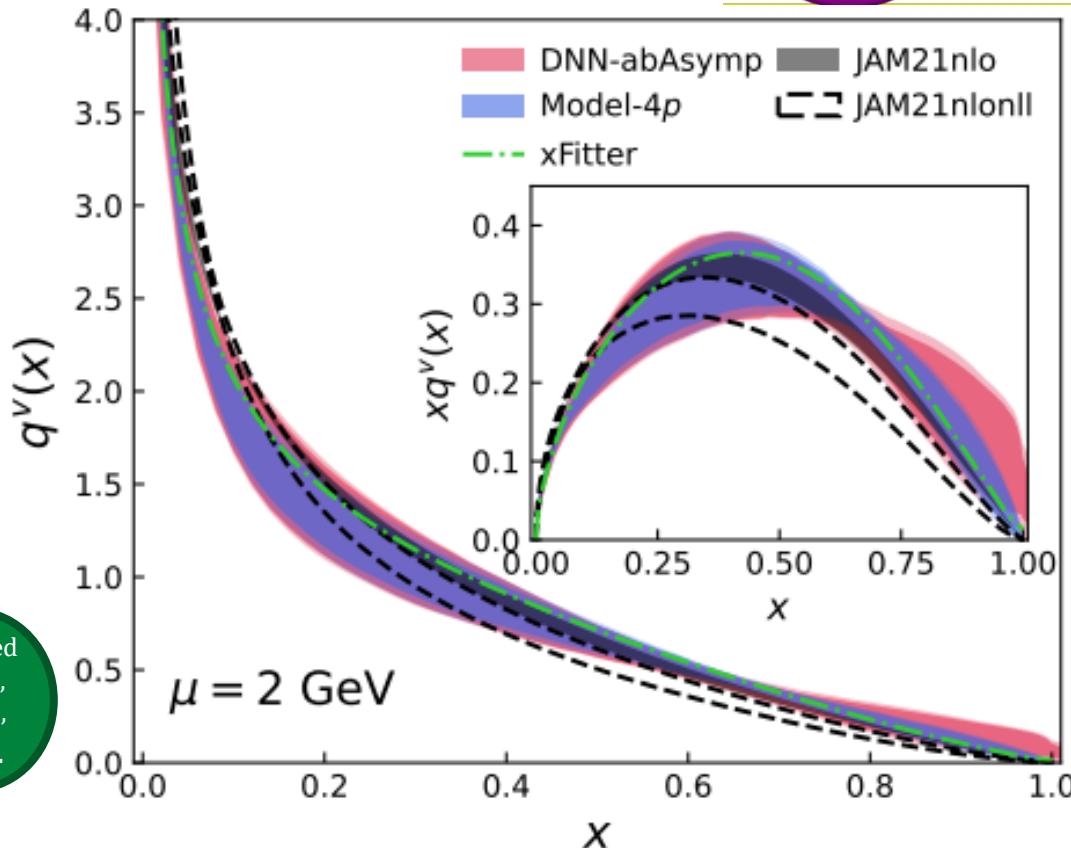
MSULat, 2003.14128



Valence-quark PDFs Update

§ Pion PDFs calculated directly at physical pion mass

❖ with NNLO matching



❖ $N_f=2+1$ clover/HISQ
 $a \sim 0.076 \text{ fm}$

ANL/BNL, Phys. Rev.
D 106, 114510 (2022)

Wanted
PDFs,
GPDs,
etc...

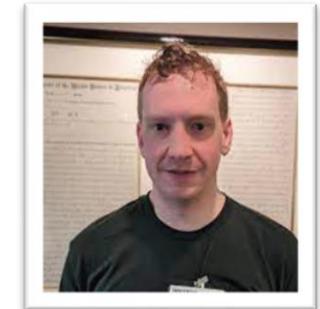
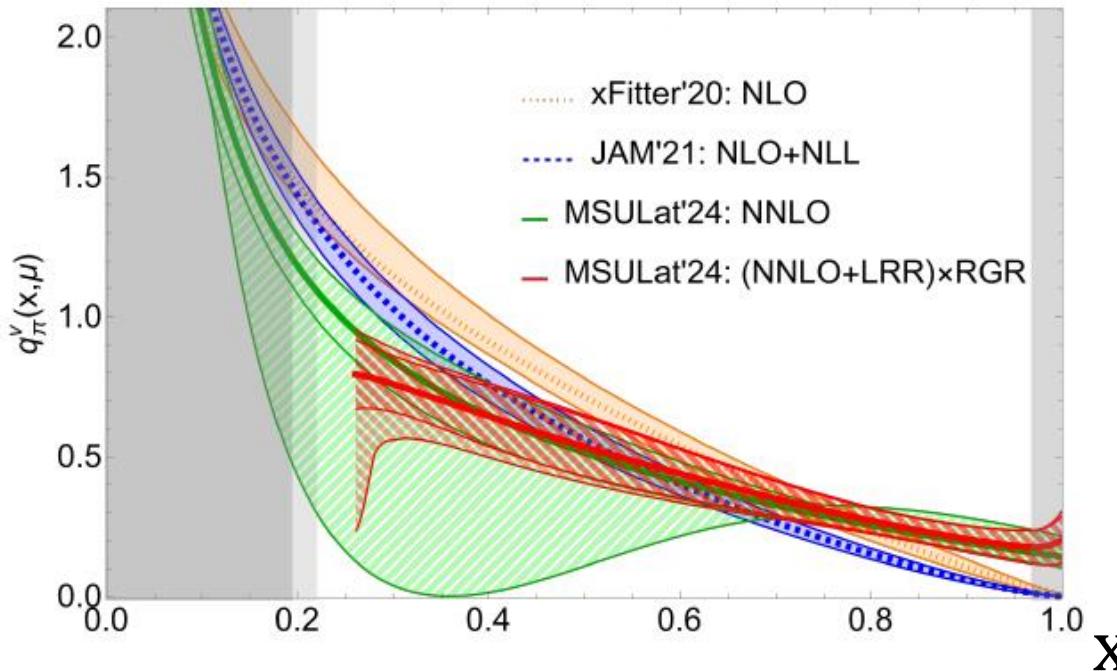
Valence-quark PDFs Update

§ Pion PDFs calculated directly at physical pion mass

- ❖ NNLO matching & treat leading-renormalon effects
 - ❖ Leading-renormalon resummation (LRR) R. Zhang, et. al.
 - ❖ Renormalization-group resummation (RGR) PLB 844, 138081 (2023)
 - ❖ $N_f=2+1+1$ clover/HISQ, $a \sim 0.09$ fm
- J. Holligan, HL (MSULat), [10.1088/1361-6471/ad3162](https://doi.org/10.1088/1361-6471/ad3162)



Wanted
PDFs,
GPDs,
etc...

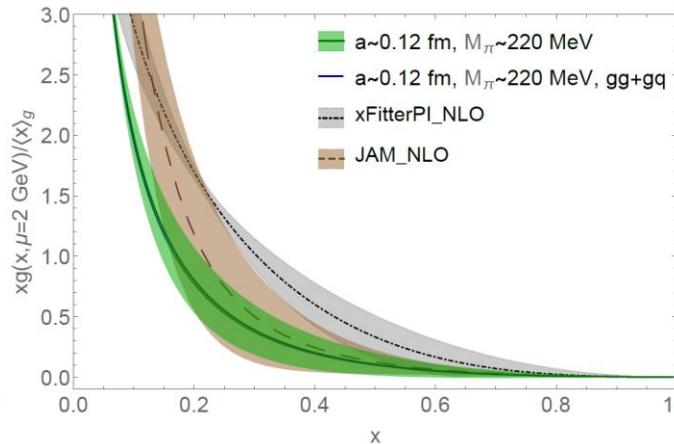
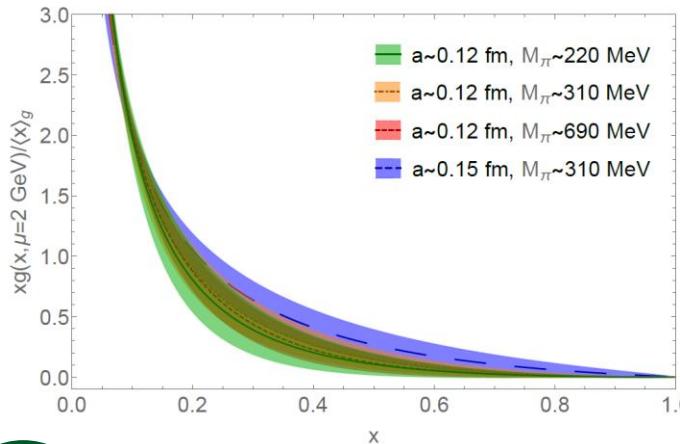


P: Jack Holligan

Meson Gluon PDFs

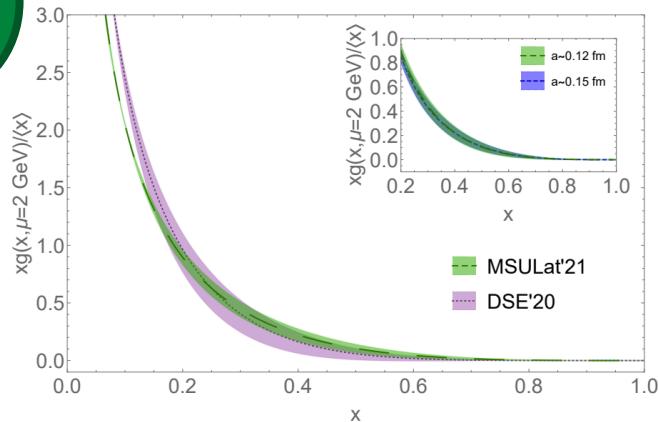


§ First pion and kaon gluon PDFs $g(x)/\langle x \rangle$ using pseudo-PDF

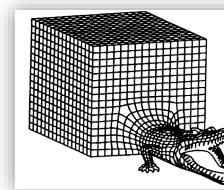
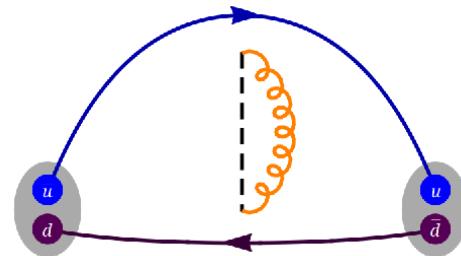


G: Zhouyou Fan

2104.06372, Fan et al. (MSULat); 2112.03124, Salas-Chavira et al. (MSULat)



G: Alejandro
Salas-Chavira



finite-volume,
discretization,
heavy quark
mass, ...

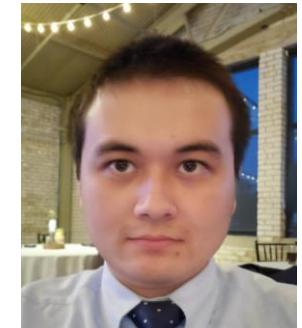
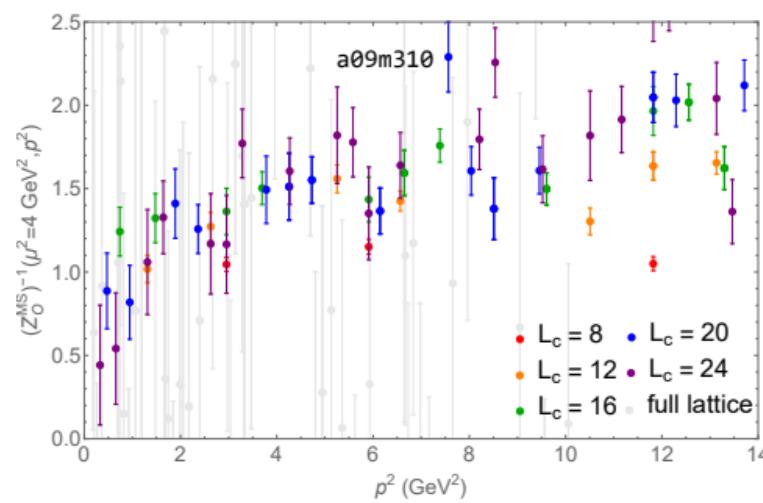
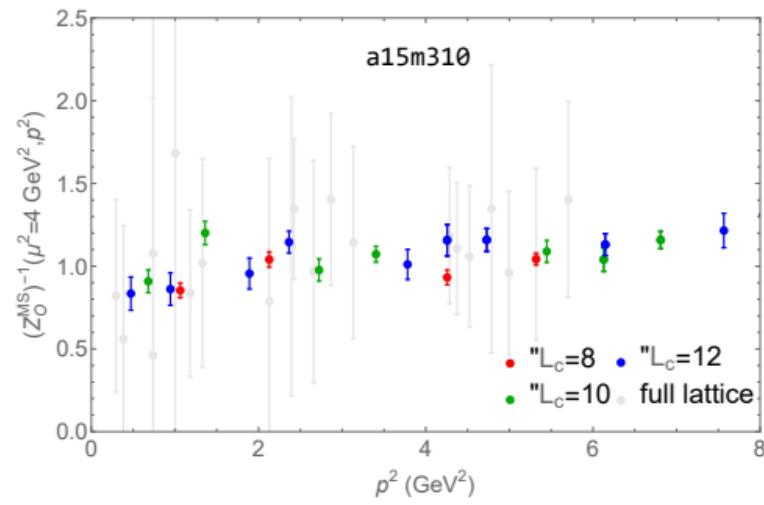
§ What does lattice QCD say about $g(x)$?



Pion Gluon PDF Update

§ Nonperturbatively renormalized $\langle x \rangle_{\{\pi,g\}}$ at the finer lattice spacing at lighter pion mass is nontrivial

- ❖ Using cluster-decomposition error reduction (CDER) to enhance the signal-to-noise ratio 1805.00531, Y. Yang et al. (χ QCD)
- ❖ Lattice details: clover/HISQ, $a \sim \{0.15, 0.12, 0.09\}$ fm 2208.00980, Fan et al. (MSULat)

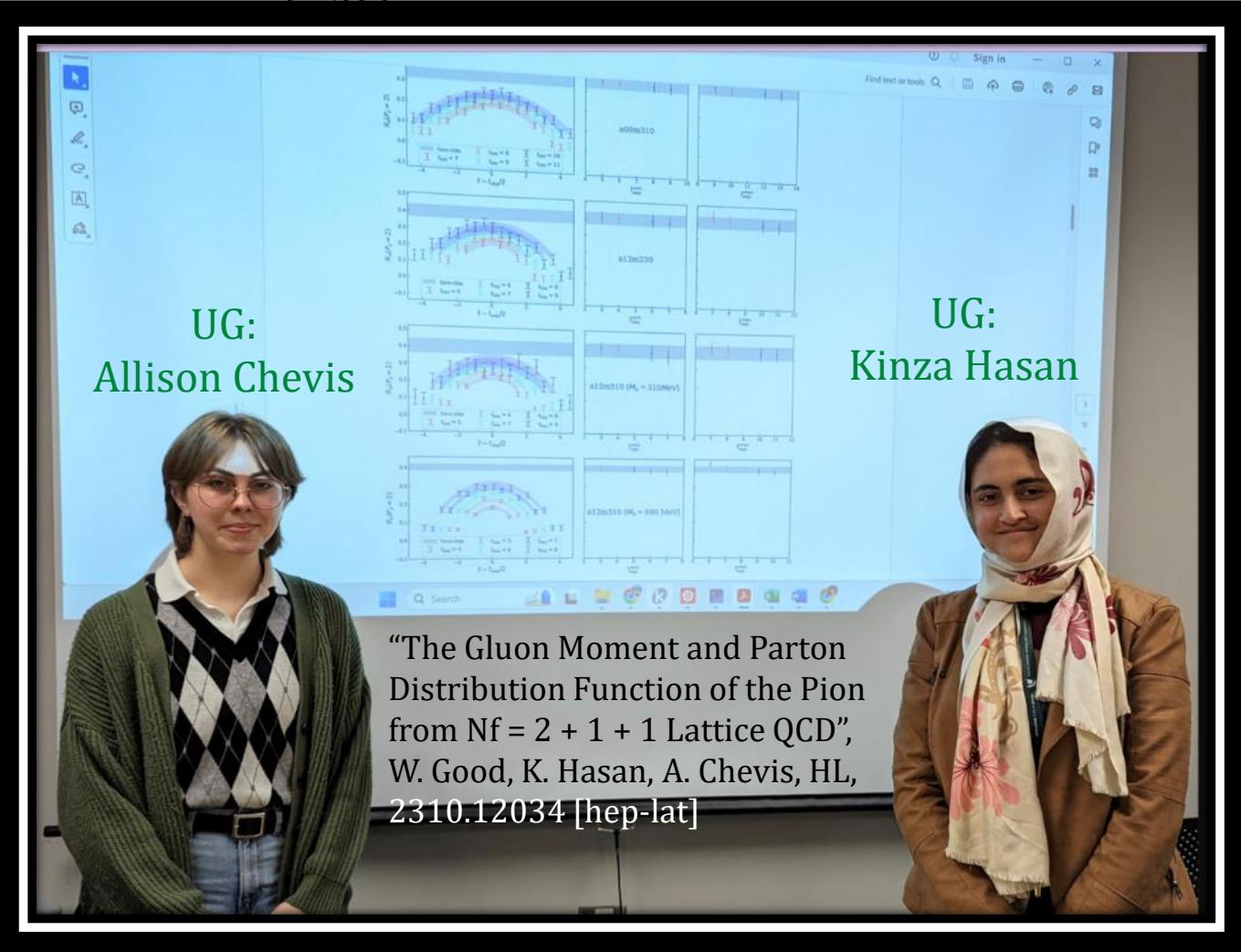
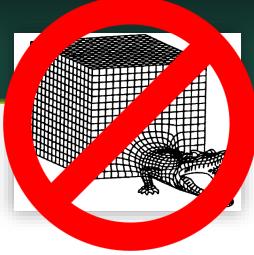


M: Matthew Zeilbeck



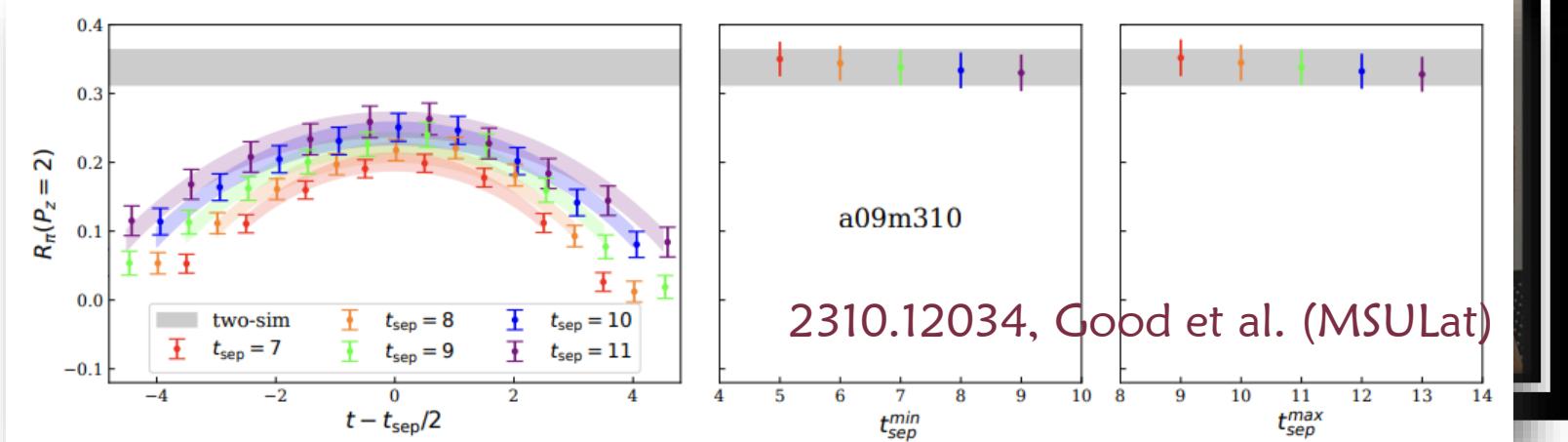
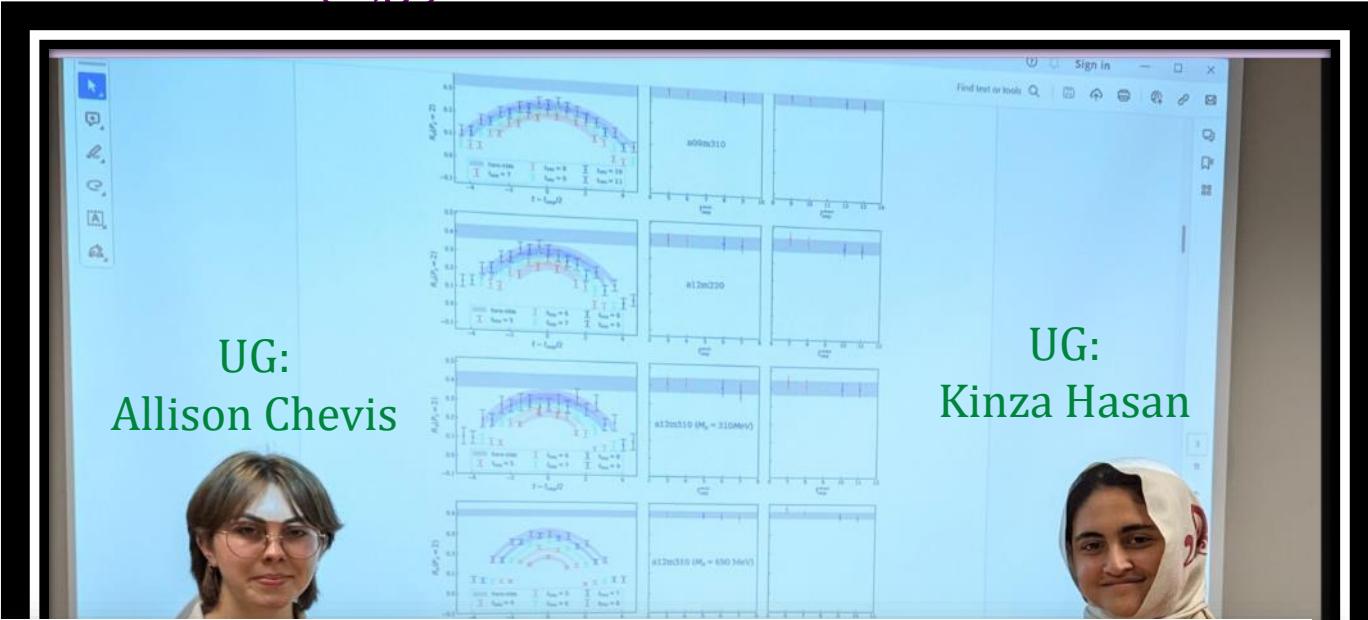
Pion Gluon PDF Update

§ Study Bare $\langle x \rangle_{\{\pi,g\}}$



Pion Gluon PDF Update

§ Study Bare $\langle x \rangle_{\{\pi,g\}}$



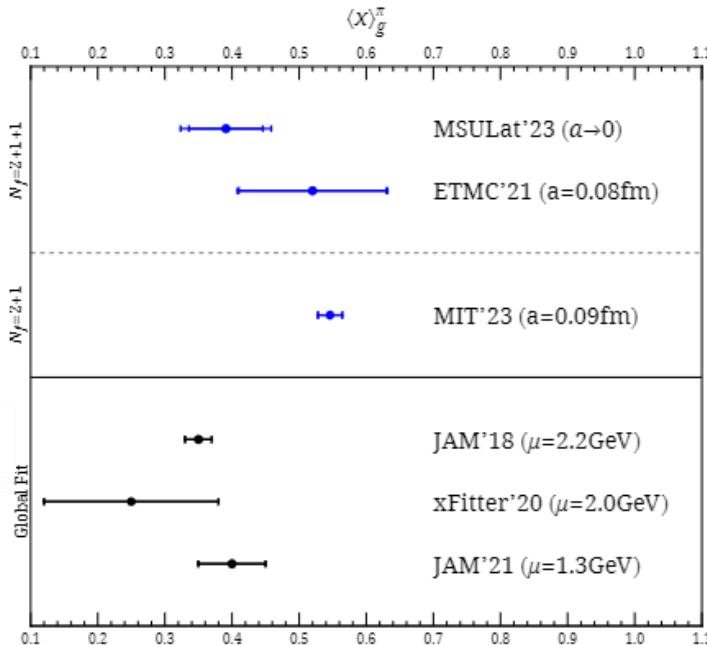
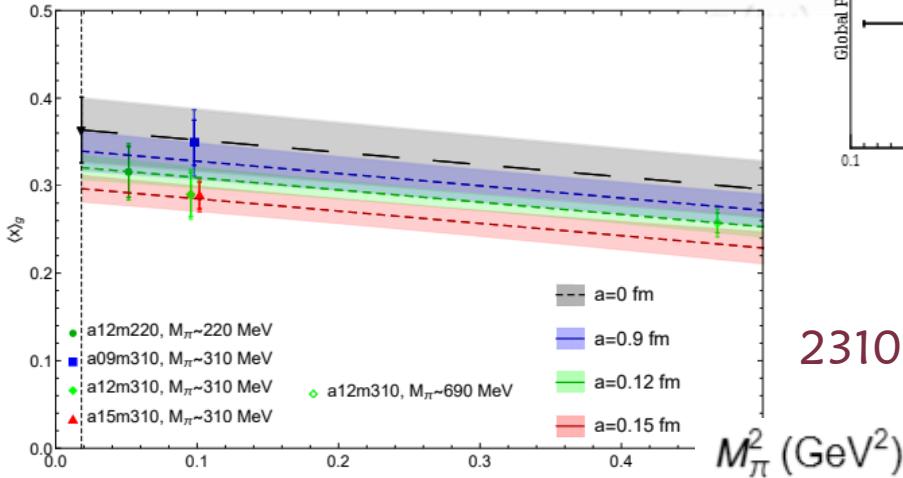
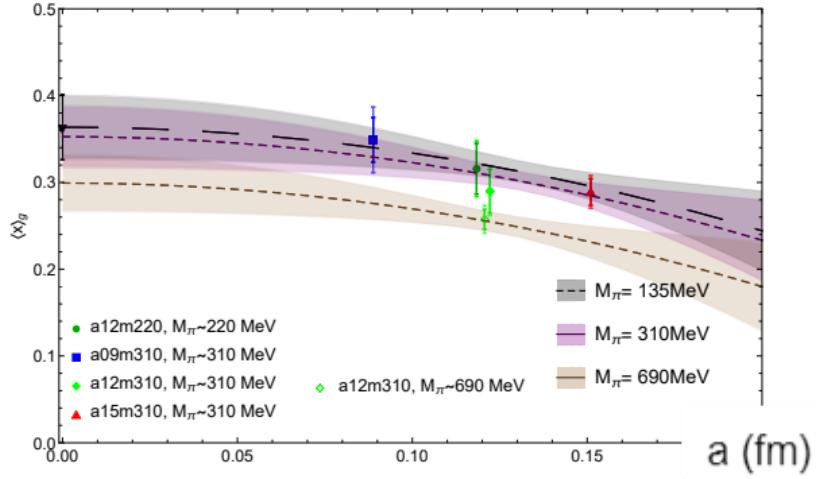


Pion Gluon PDF Update



§ Study discretization systematic in $\langle x \rangle_{\{\pi,g\}}$

❖ Lattice details: clover/HISO. HISO. $a \sim \{0.15, 0.12, 0.09\} \text{ fm}$



2310.12034, Good et al. (MSULat)



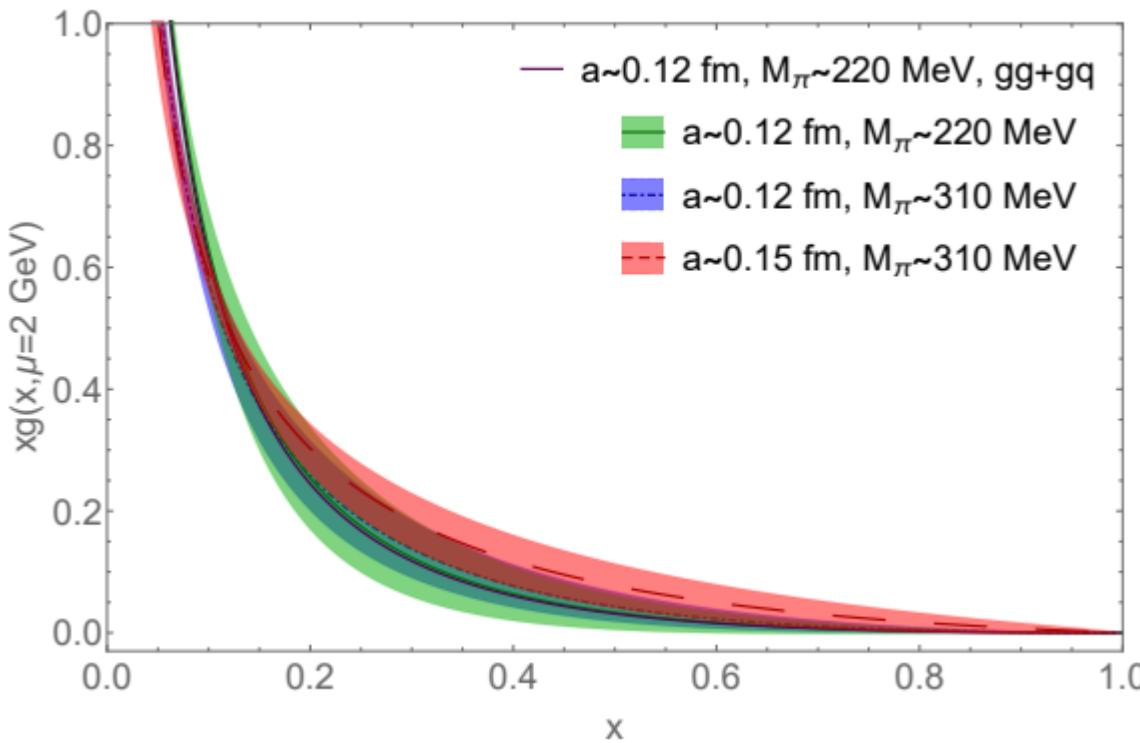
G: Bill Good



Pion Gluon PDF Update

§ Back to Pion gluon PDF $g(x)$

❖ Update previous calculated $g(x)/\langle x \rangle$ in 2021



2310.12034, Good et al. (MSULat)

G: Bill Good

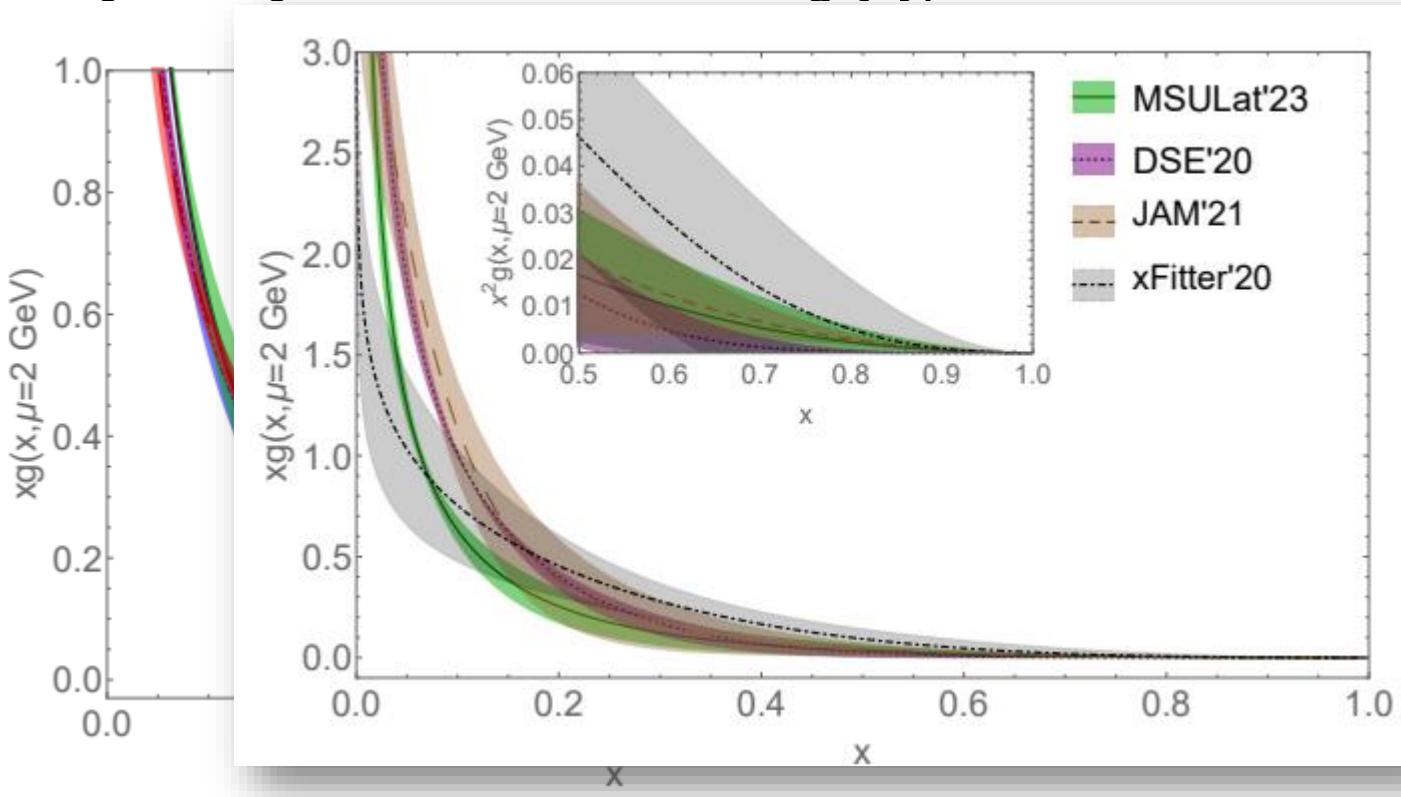




Pion Gluon PDF Update

§ Back to Pion gluon PDF $g(x)$

❖ Update previous calculated $g(x)/\langle x \rangle$ in 2021



2310.12034, Good et al. (MSULat)

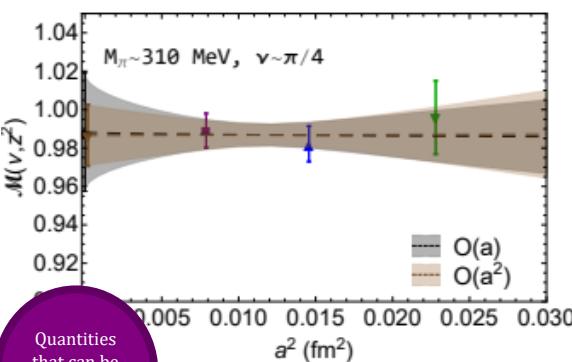
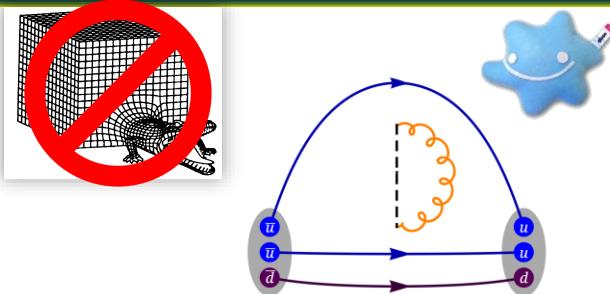
G: Bill Good



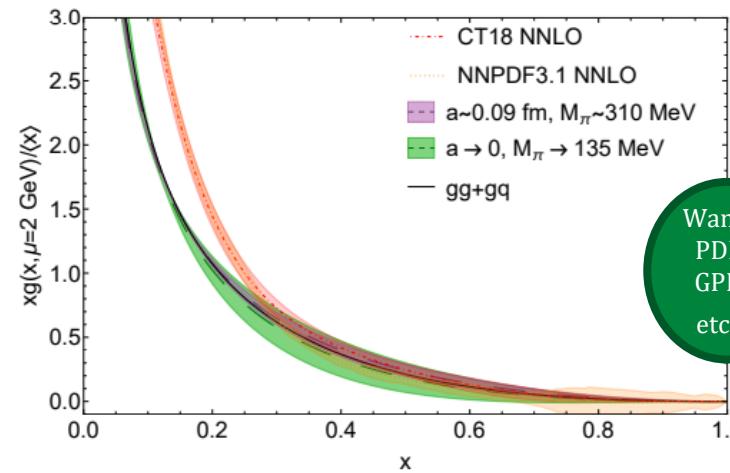
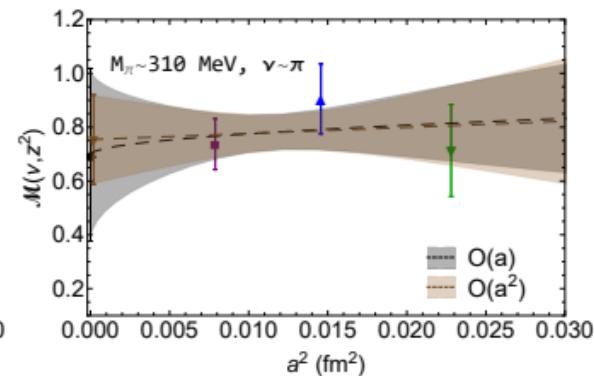
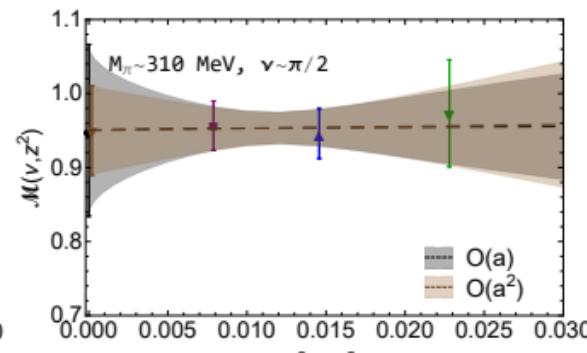
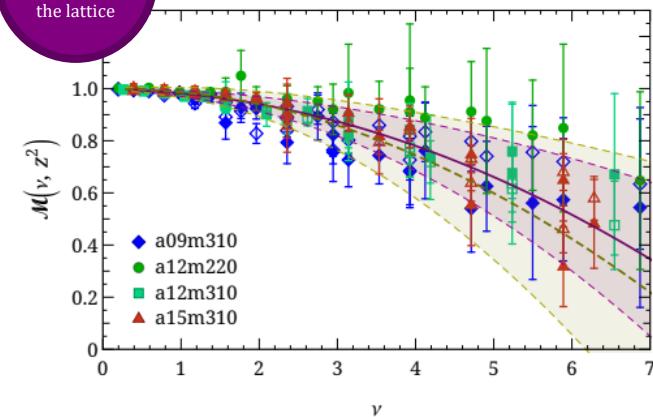
Gluon PDF in Nucleon

§ Continuum Gluon PDF w/ pseudo-PDF

❖ 2+1+1 HISQ {0.09, 0.12, 0.15} fm,
 [220,310,700]-MeV pion, 10^5 - 10^6 statistics
[arXiv:2210.09985](https://arxiv.org/abs/2210.09985)



Quantities that can be calculated on the lattice



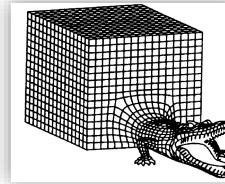
Wanted
PDFs,
GPDs,
etc...



G: Bill Good

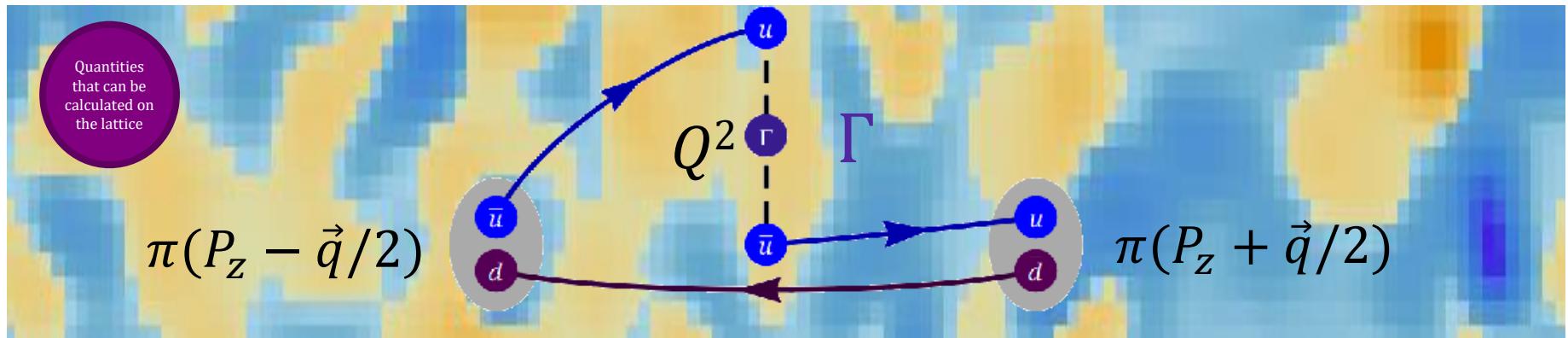
Generalized Parton Distributions

Single-ensemble result



finite-volume,
discretization,
heavy quark mass,

...



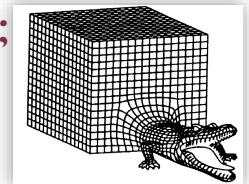
First Lattice GPDs

§ First glimpse into pion GPD using Quasi-PDF/LaMET

❖ Lattice details: clover/HISQ, **0.12fm, 310-MeV** pion mass

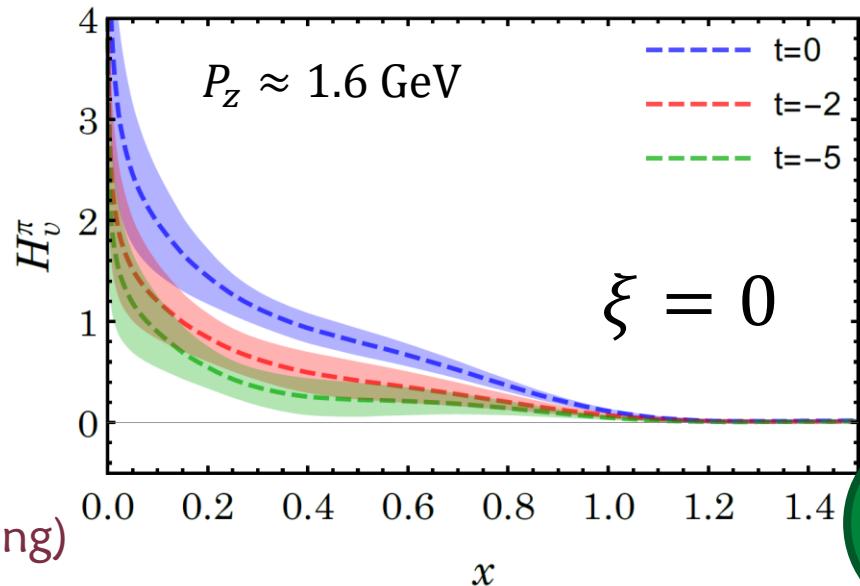
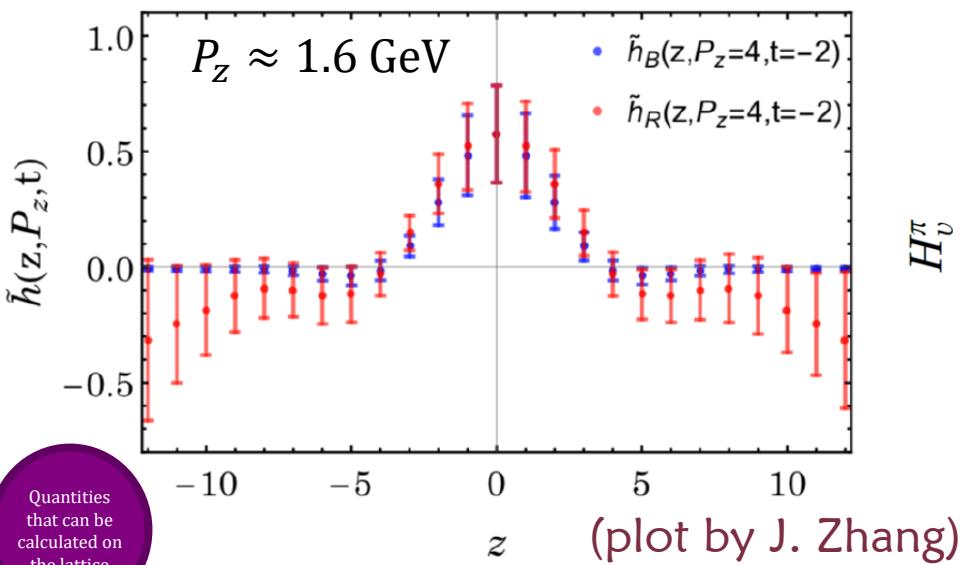
$$P_z \approx 1.3, 1.6 \text{ GeV}$$

MILC, Phys. Rev. D, 82 (2010), 074501;
Phys. Rev. D, 87 (2013), 0545056



J. Chen, HL, J. Zhang, 1904.1237;

$$H_q^\pi(x, \xi, t, \mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P + \Delta/2) \left| \bar{q} \left(\frac{\eta^-}{2} \right) \gamma^+ \Gamma \left(\frac{\eta^-}{2}, -\frac{\eta^-}{2} \right) q \left(-\frac{\eta^-}{2} \right) \right| \pi(P - \Delta/2) \right\rangle$$



Quantities
that can be
calculated on
the lattice

Wanted
PDFs,
GPDs,
etc...

Valence-Quark Pion GPD

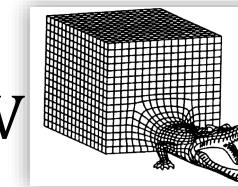
§ Pion GPD (H^π) using quasi-PDFs at physical pion mass

❖ Lattice details: clover/2+1+1 HISQ

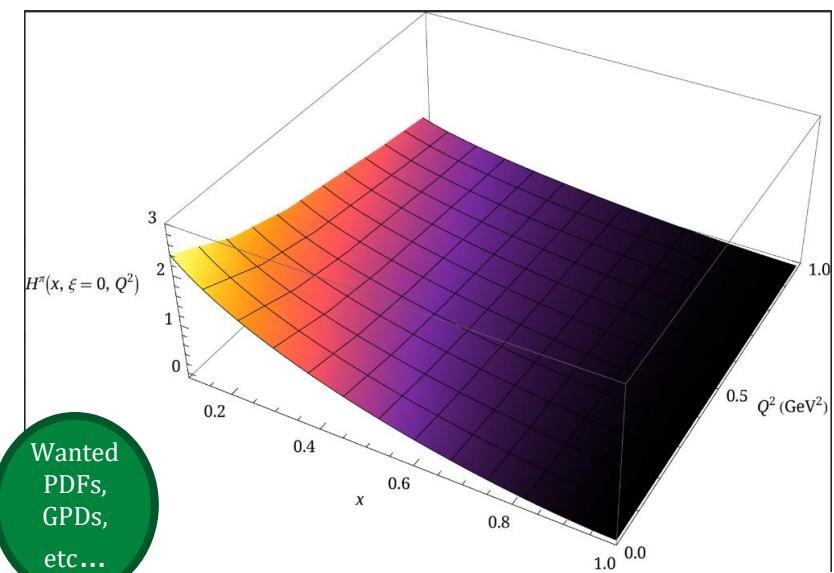
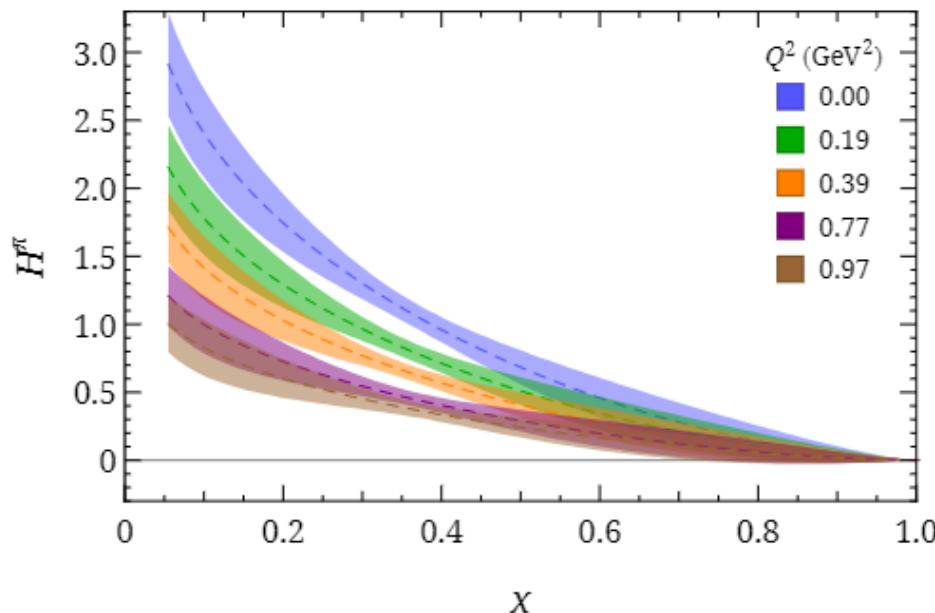
0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV

❖ $\xi = 0$ valence-quark Pion GPD results

HL (MSULat), Phys. Lett. B 846 (2023) 138181



finite-volume,
discretization,



Valence-Quark Pion GPD

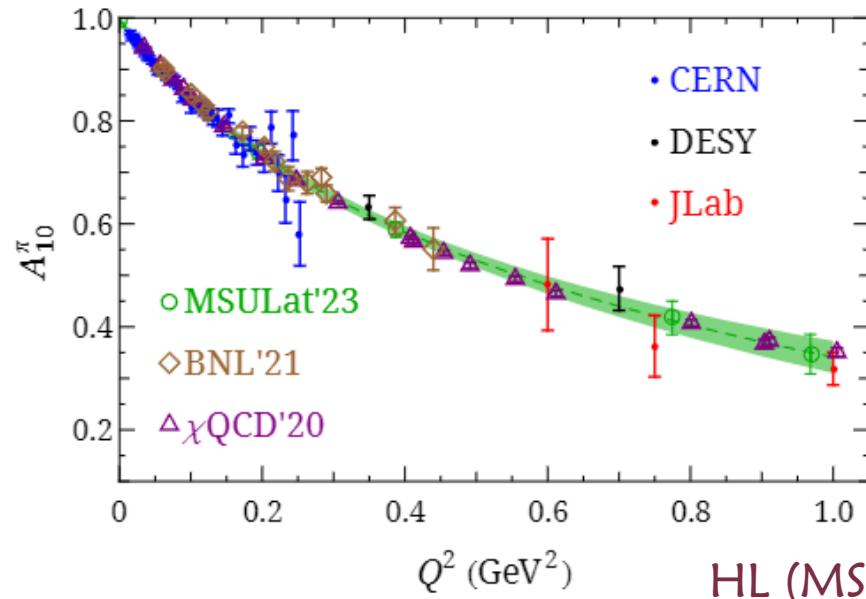
§ Pion GPD (H^π) using quasi-PDFs at physical pion mass

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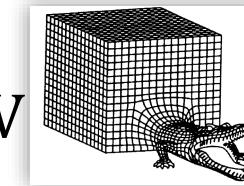
0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV

❖ $\xi = 0$ valence-quark Pion GPD results

$$\int_{-1}^{+1} dx x^{n-1} = A_{ni}^\pi(t)$$



HL (MSULat), Phys. Lett. B 846 (2023) 138181



finite-volume,
discretization,



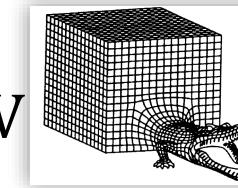
Pion Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

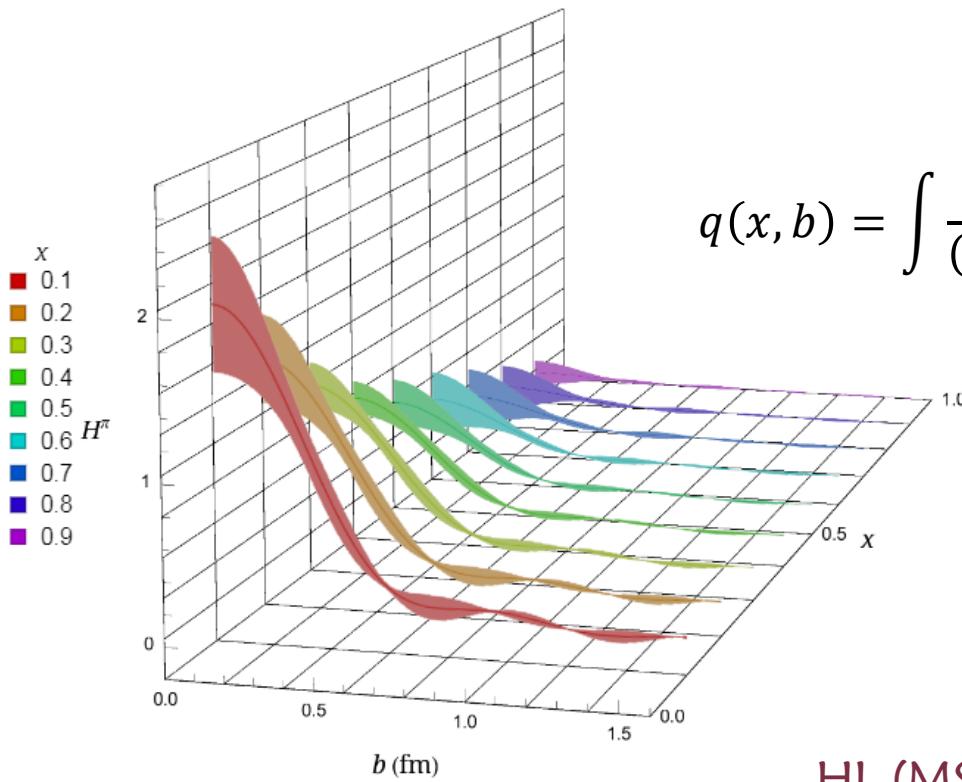
❖ Lattice details: clover/2+1+1 HISQ

0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV

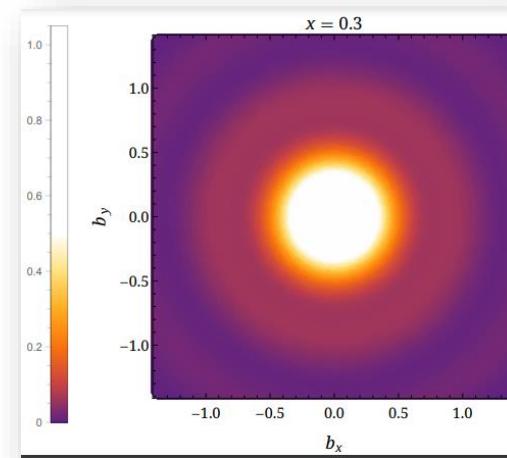
❖ $\xi = 0$ valence-quark Pion GPD results



finite-volume,
discretization,



$$q(x, b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$



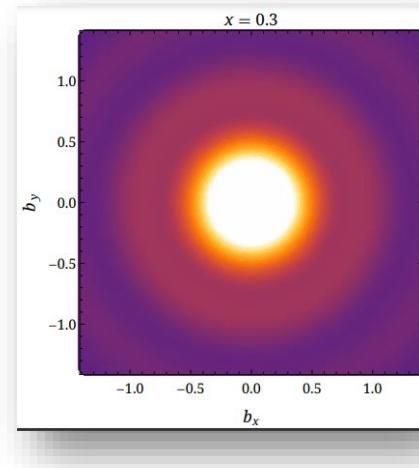
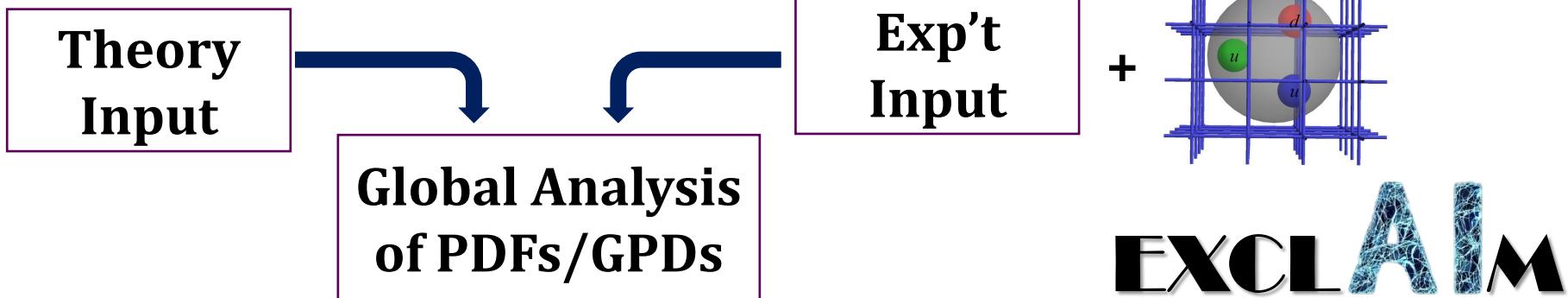
HL (MSULat), Phys. Lett. B 846 (2023) 138181

Summary and Outlook

- § Exciting era using LQCD to study x -dependent pion/kaon structure
- § Overcoming longstanding limitations
 - ❖ Nonperturbative renormalization for gluon operators
(not limited to small volume nor too-heavy pion mass)
 - ❖ Bjorken- x dependence of parton distributions
now widely studied
 - ❖ More study of systematics planned for the near future

§ Precision and progress are limited by resources

§ In the future



Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources
This work is partially sponsored by grants NSF PHY 1653405 & 1653405, DOE DE-SC0024053 & RCSA Cottrell Scholar

Students Wanted

LGT4HEP website: <https://lgt4hep.github.io/>



High Energy Physics Computing Traineeship for Lattice Gauge Theory

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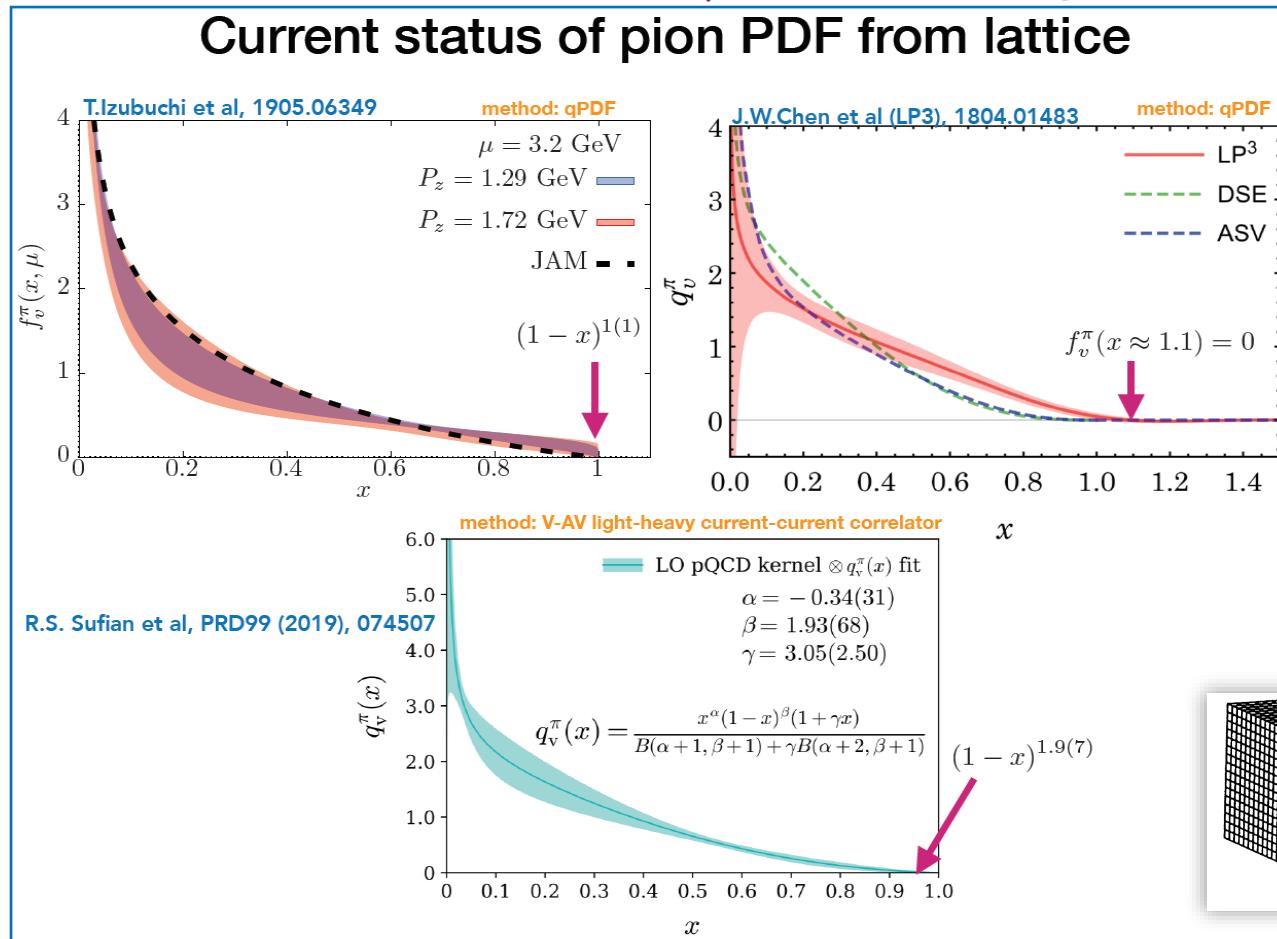
Backup Slides



Pion Valence-Quark PDF

§ Status as of Summer 2019 Slide by Nikhil Karthik @ Lattice 2019

$$M_\pi \approx 310 \text{ MeV}$$



§ Single-ensemble calculation

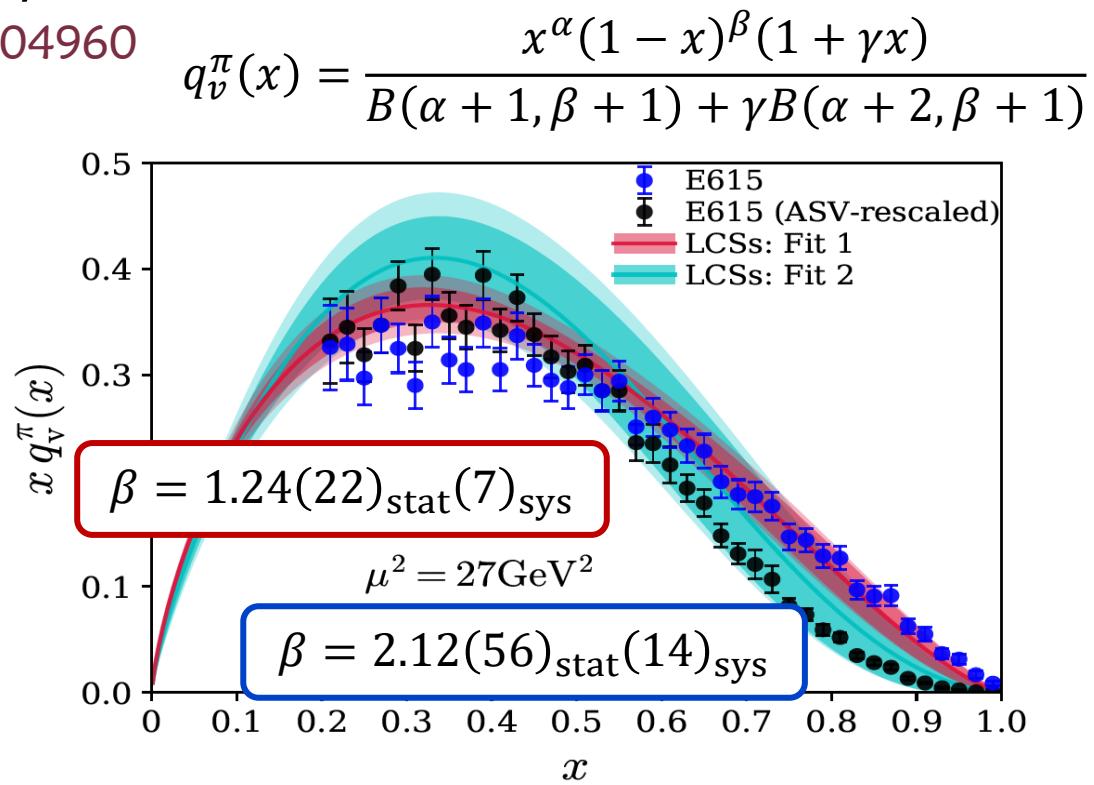
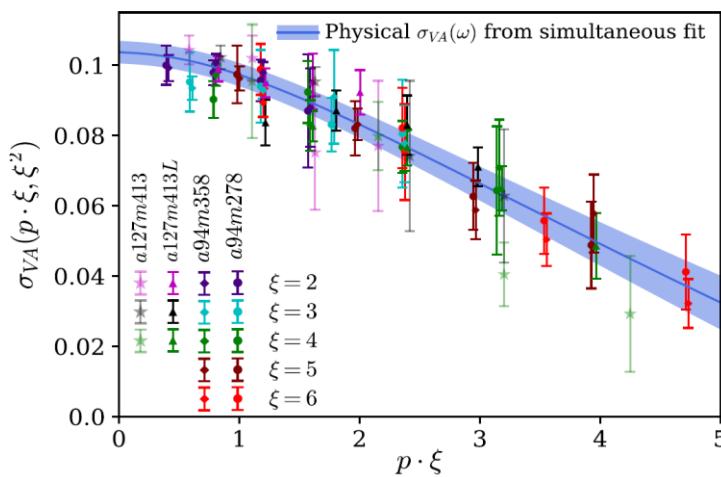
❖ Non-physical pion mass, single lattice spacing, single volume

Pion Valence-Quark PDF

§ Results from JLab-W&M/ LCS method

- ❖ $M_\pi = 278, 358, 413$ MeV with $a = 0.094, 0.127$ fm
- ❖ Extrapolated to physical limit (shown as blue band)
- ❖ Renormalized $Z_{V,A}$ in RI/MOM, matched to $\overline{\text{MS}}$, run to 27 GeV 2

R. S. Sufian, et al, 2001.04960

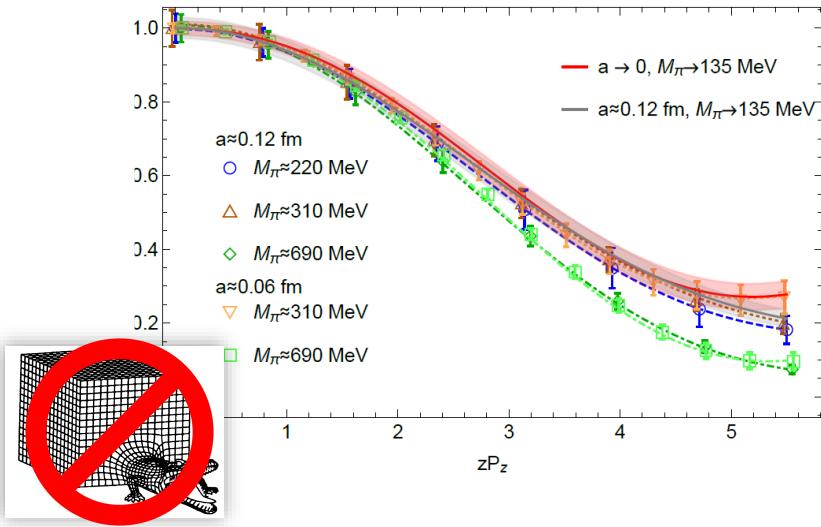


Pion Valence-Quark PDF

§ Results from MSULat/quasi-PDF method

- ~ $M_\pi = 220, 310, 790$ MeV with $a = 0.06, 0.12$ fm
- ~ Extrapolated to physical limit (shown as pink/green band)
- ~ Renormalized in RI/MOM, matched to $\overline{\text{MS}}$, run to 27 GeV 2

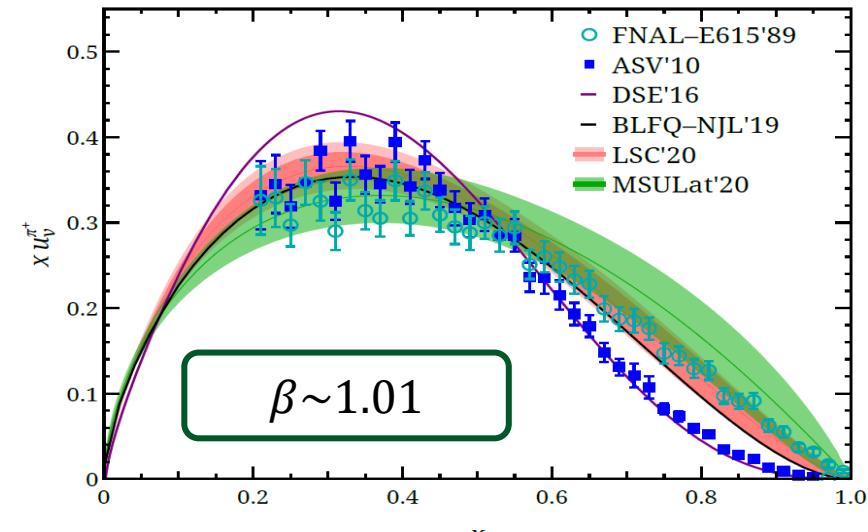
H. Lin et al. (MSULat), 2003.14128



J. S. Conway et al., PRD39, 92 (1989).

M. Aicher et al, PRL105, 252003 (2010), 1009.2481.

C. Chen et al, PRD93, 074021 (2016), 1602.01502.



J. Lan, et al, PRL122, 172001 (2019), 1901.11430;

PRD101, 034024 (2020), 1907.01509.

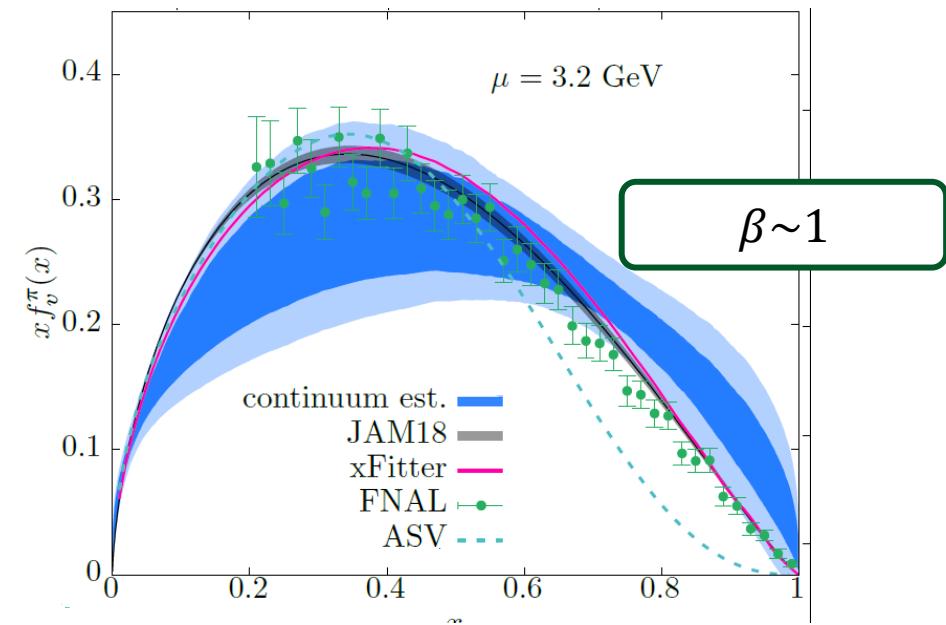
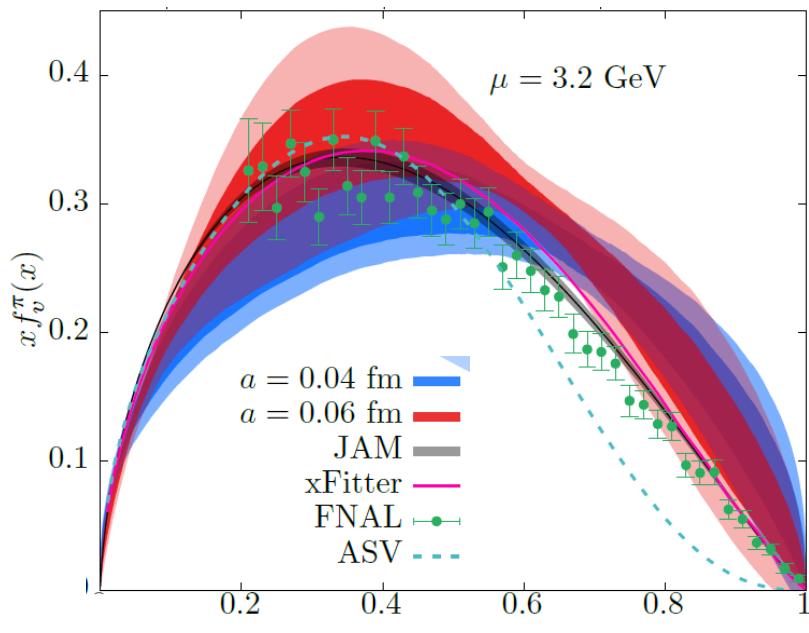
R. S. Sufian, et al, 2001.04960

Pion Valence-Quark PDF

§ Results from BNL/quasi-PDF method

- ❖ $M_\pi = 300$ MeV with $a = 0.04, 0.06$ fm
- ❖ Extrapolated to continuum limit
- ❖ Renormalized in RI/MOM, matched to $\overline{\text{MS}}$ at 10 GeV 2

X. Gao et al. 2007.06590



Kaon Valence-Quark PDFs

§ Pion/kaon PDFs using quasi-PDF in the continuum limit

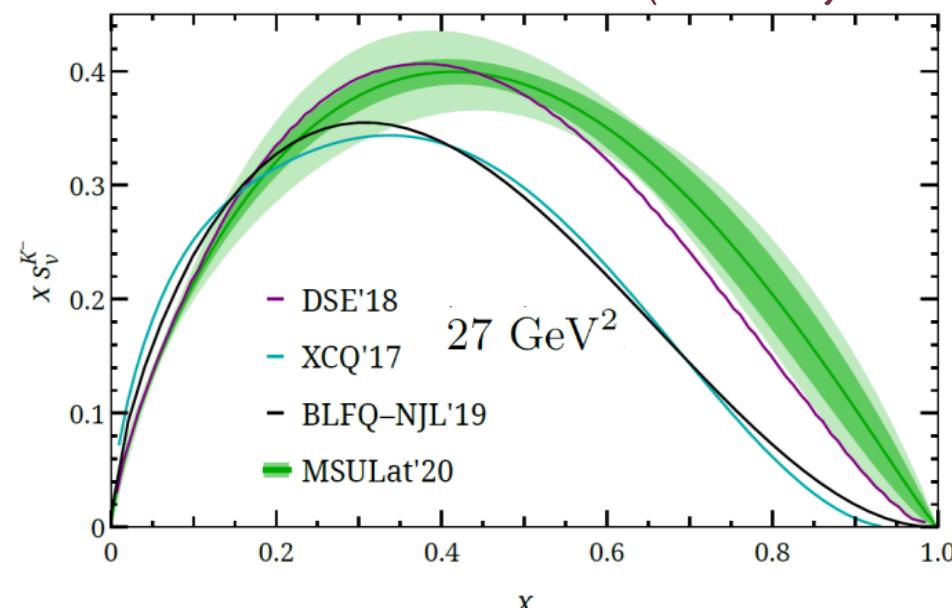
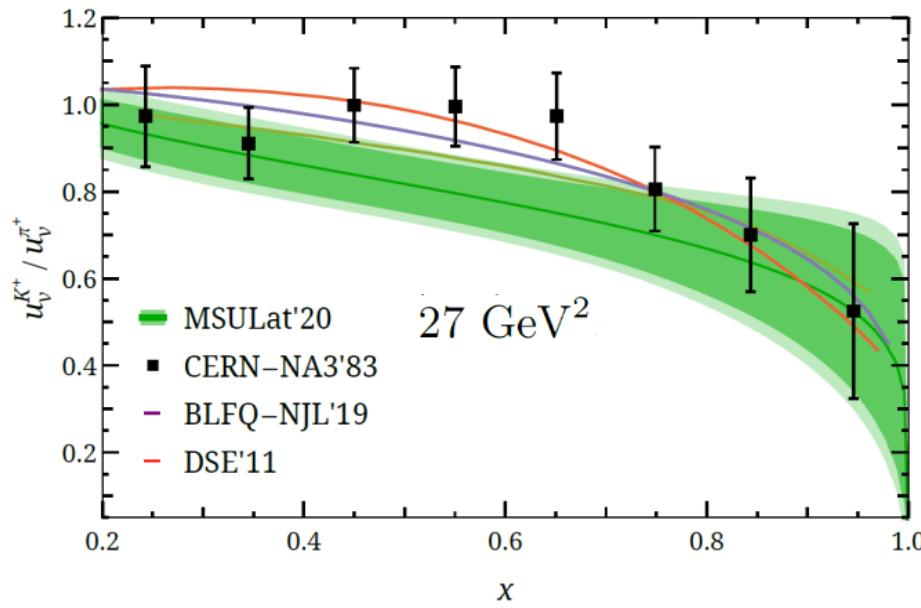
❖ Lattice details: clover/2+1+1 HISQ (MSULat)

$$a \approx \{0.06, 0.12\} \text{ fm},$$

$$M_\pi \in \{220, 310, 690\}-\text{MeV pion}$$

$$P_z \approx \{1.3, 1.7\} \text{ GeV}$$

2003.14128 HL et al (MSULat)



Kaon Valence-Quark PDFs

§ Pion/kaon PDFs using quasi-PDF in the continuum limit

❖ Lattice details: clover/2+1+1 HISQ (MSULat)

$$a \approx \{0.06, 0.12\} \text{ fm},$$

$$M_\pi \in \{220, 310, 690\}\text{-MeV pion}$$

$$P_z \approx \{1.3, 1.7\} \text{ GeV}$$



§ First LQCD calculation $\langle x^n \rangle$ of $u_v^{K^+}$ and $s_v^{K^-}$ 2003.14128
HL et al (MSULat)

n	$\langle x^n \rangle(u_v^{K^+})$	$\langle x^n \rangle(s_v^{K^-})$
1	$0.192(8)_{\text{stat}}(6)_{\text{syst}}$	$0.261(8)_{\text{stat}}(8)_{\text{syst}}$
2	$0.080(7)_{\text{stat}}(6)_{\text{syst}}$	$0.120(7)_{\text{stat}}(9)_{\text{syst}}$
3	$0.041(6)_{\text{stat}}(4)_{\text{syst}}$	$0.069(6)_{\text{stat}}(8)_{\text{syst}}$

Kaon Valence-Quark PDFs

§ Pion/kaon PDFs using quasi-PDF in the continuum limit

❖ Lattice details: clover/2+1+1 HISQ (MSULat)

$$a \approx \{0.06, 0.12\} \text{ fm},$$

$$M_\pi \in \{220, 310, 690\}\text{-MeV pion}$$

$$P_z \approx \{1.3, 1.7\} \text{ GeV}$$



§ First LQCD calculation $\langle x^n \rangle$ of $u_v^{K^+}$ and $s_v^{K^-}$

§ Later ETMC **260**-MeV results on $\langle x^n \rangle$ of $u_v^{K^+}$ and $s_v^{K^-}$

2003.14128 HL et al (MSULat)

2010.0349, 2104.02247

n	$\langle x^n \rangle(u_v^{K^+})$	$\langle x^n \rangle(s_v^{K^-})$
1	$0.192(8)_{\text{stat}}(6)_{\text{syst}}$	$0.261(8)_{\text{stat}}(8)_{\text{syst}}$
2	$0.080(7)_{\text{stat}}(6)_{\text{syst}}$	$0.120(7)_{\text{stat}}(9)_{\text{syst}}$
3	$0.041(6)_{\text{stat}}(4)_{\text{syst}}$	$0.069(6)_{\text{stat}}(8)_{\text{syst}}$

n	$\langle x^n \rangle(u_v^{K^+})$	$\langle x^n \rangle(s_v^{K^-})$
1	$0.246(2)_{\text{stat}}(2)_{\text{syst}}$	$0.317(2)_{\text{stat}}(1)_{\text{syst}}$
2	$0.093(5)_{\text{stat}}(3)_{\text{syst}}$	$0.134(5)_{\text{stat}}(2)_{\text{syst}}$
3	$0.035(6)_{\text{stat}}(3)_{\text{syst}}$	$0.075(5)_{\text{stat}}(1)_{\text{syst}}$

First Pion Gluon PDF

§ Pion GLUON PDFs using pseudo-PDF

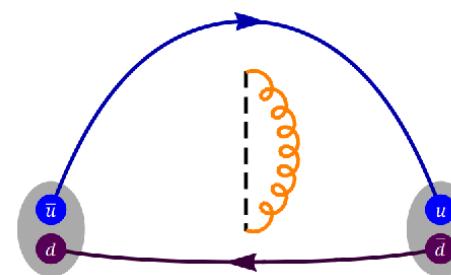
❖ Lattice details: clover/2+1+1 HISQ (MSULat)

$$a \approx \{0.12, 0.15\} \text{ fm},$$

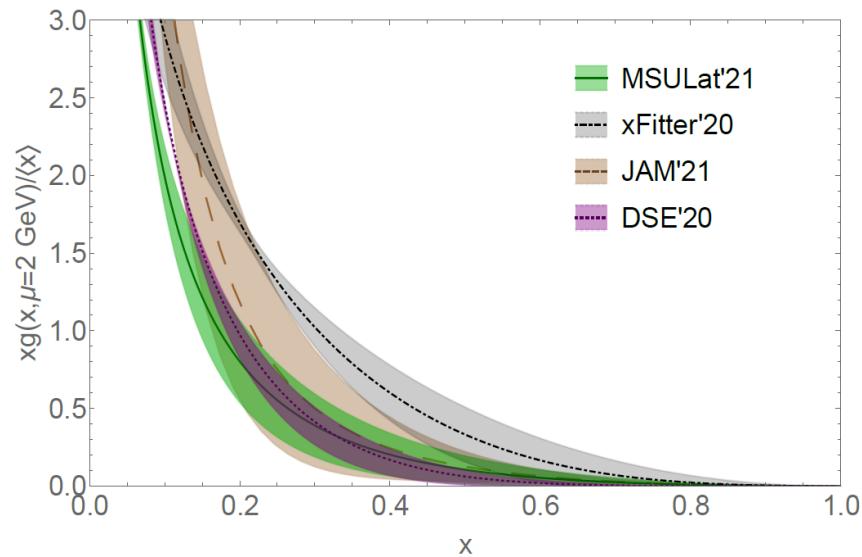
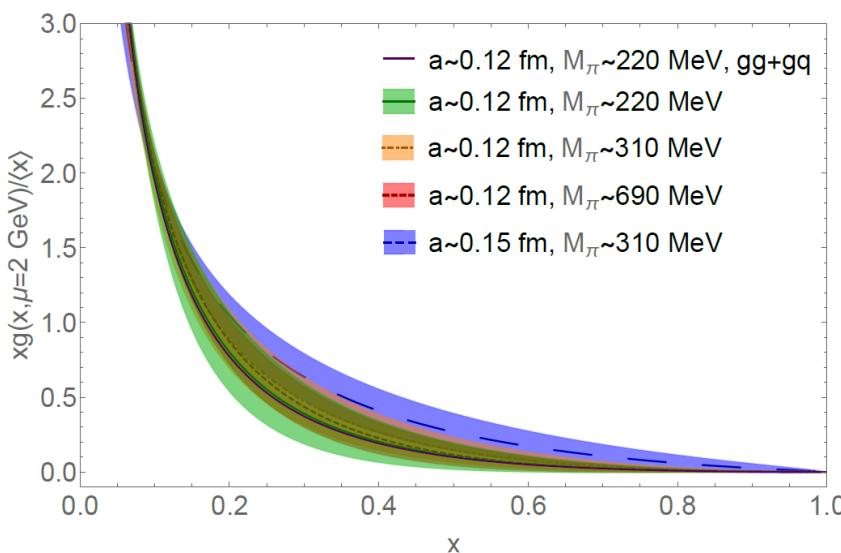
$M_\pi \in \{220, 310, 690\}$ -MeV pion

$$P_{z,\max} \approx 2.3 \text{ GeV}$$

2104.06372, Fan, HL(MSULat)



Zhouyou Fan
(MSU)



Pion and Kaon DA

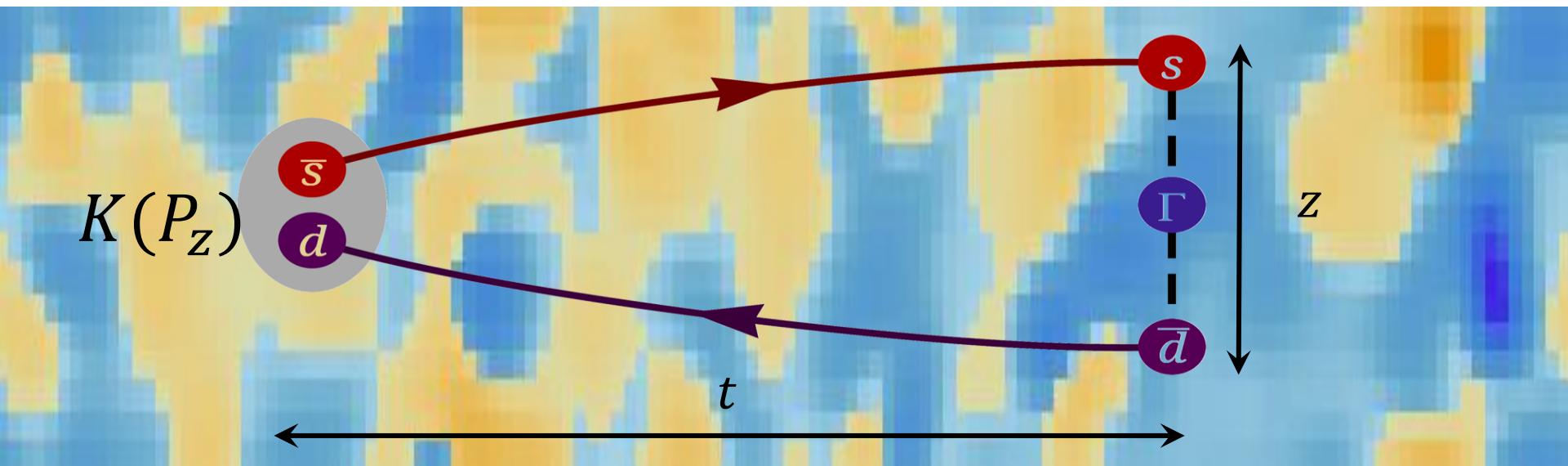
§ The first continuum-limit study of x -dependent meson DA on the lattice

≈ $M_\pi \in \{310, 690 (\eta_s)\}$ MeV

≈ $a \in \{0.06, 0.09, 0.12\}$ fm

≈ $M_\pi^{\min} L = 4.5$

$$C_M^{\text{DA}}(z, P, t) = \left\langle 0 \left| \int d^3y e^{i \vec{P} \cdot \vec{y}} \bar{\psi}_1(\vec{y}, t) \gamma_z \gamma_5 U(\vec{y}, \vec{y} + z \hat{z}) \psi_2(\vec{y} + z \hat{z}, t) \bar{\psi}_2(0,0) \gamma_5 \psi_1(0,0) \right| 0 \right\rangle$$



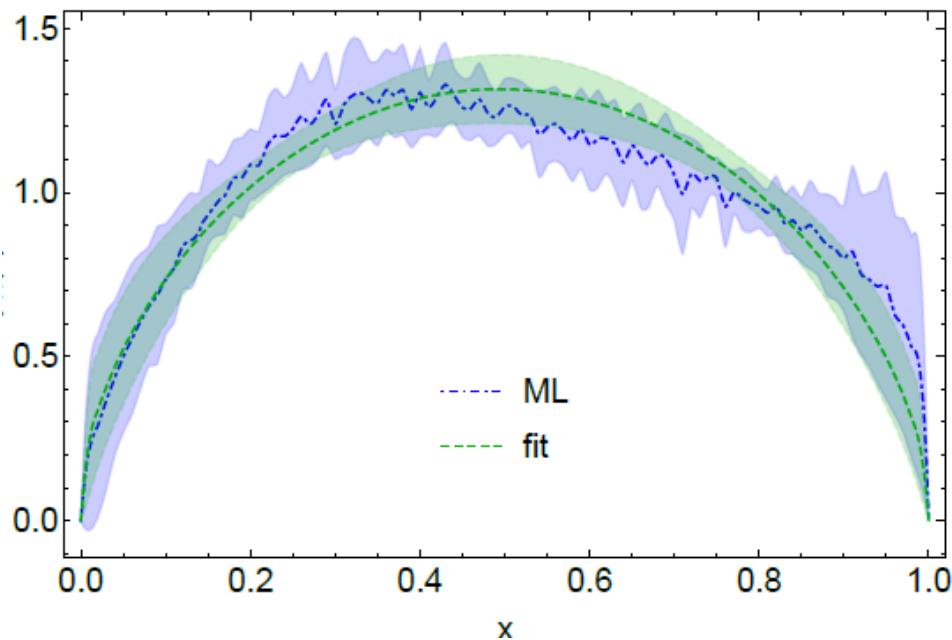
Pion and Kaon DA

§ Extract the DA distribution from the physical-continuum matrix elements

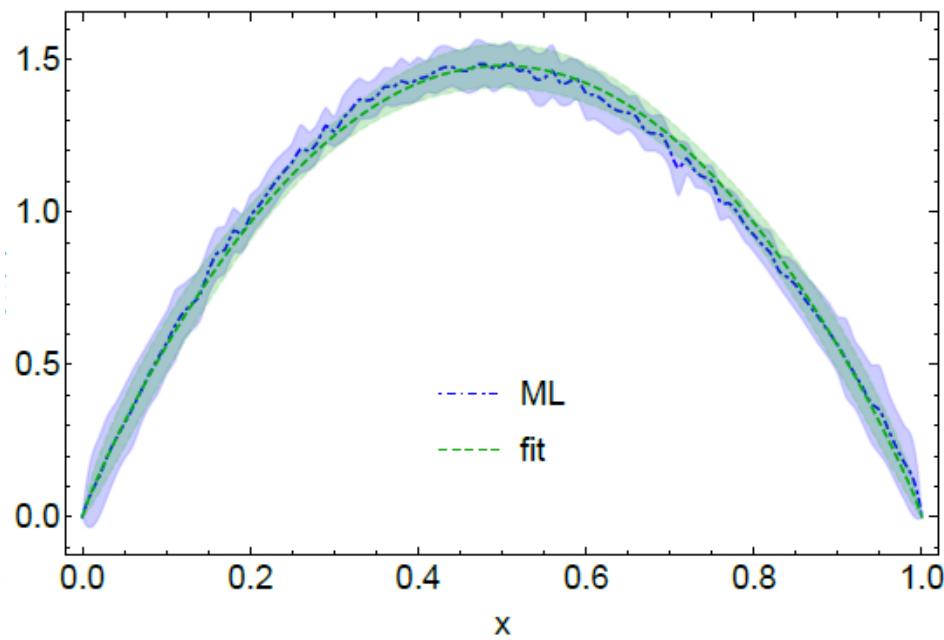
R. Zhang et al. (MSULat), 2005.13955

$$h(z, \mu^R, p_z^R, P_z) = \int_{-\infty}^{\infty} dx \int_0^1 dy C\left(x, y, \left(\frac{\mu^R}{p_z^R}\right)^2, \frac{P_z}{\mu^R}, \frac{P_z}{p_z^R}\right) f_{m,n}(y) e^{i(1-x)zP_z}$$

Pion



Kaon

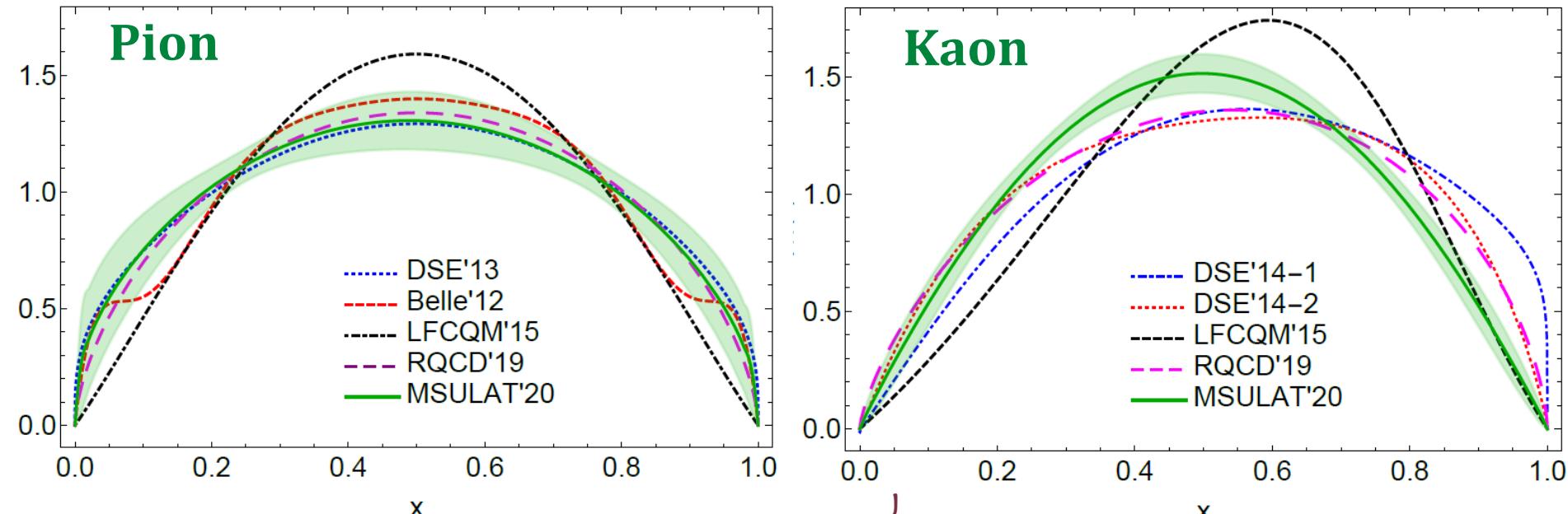


Pion and Kaon DA

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R. Zhang et al. (MSULat), 2005.13955

$$h(z, \mu^R, p_z^R, P_z) = \int_{-\infty}^{\infty} dx \int_0^1 dy C\left(x, y, \left(\frac{\mu^R}{p_z^R}\right)^2, \frac{P_z}{\mu^R}, \frac{P_z}{p_z^R}\right) f_{m,n}(y) e^{i(1-x)zP_z}$$



DES'13: L. Chang et al., Phys. Rev. Lett. 110, 132001 (2013); C. Shi et al., Phys. Lett. B738, 512 (2014)

Belle'12: S. Agaev et al., Phys. Rev. D86, 077504 (2012);

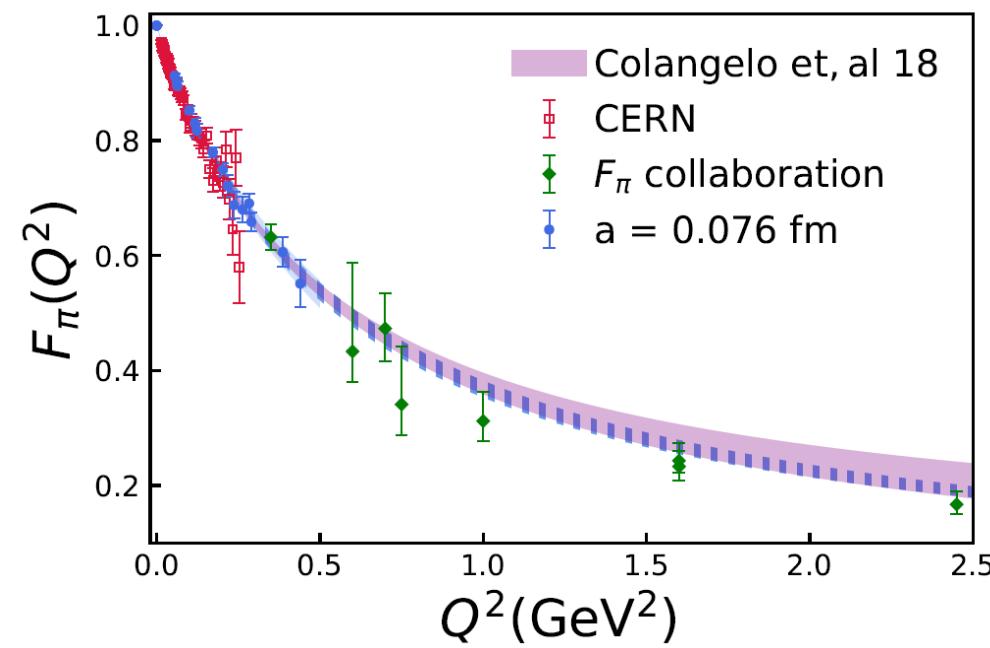
LFCQM'15: J. P. B. C. de Melo et al., AIP Conf. Proc. 1735, 080012 (2016);

RQCD'19: G. S. Bali et al., JHEP 08, 065 (2019); DSE'14:

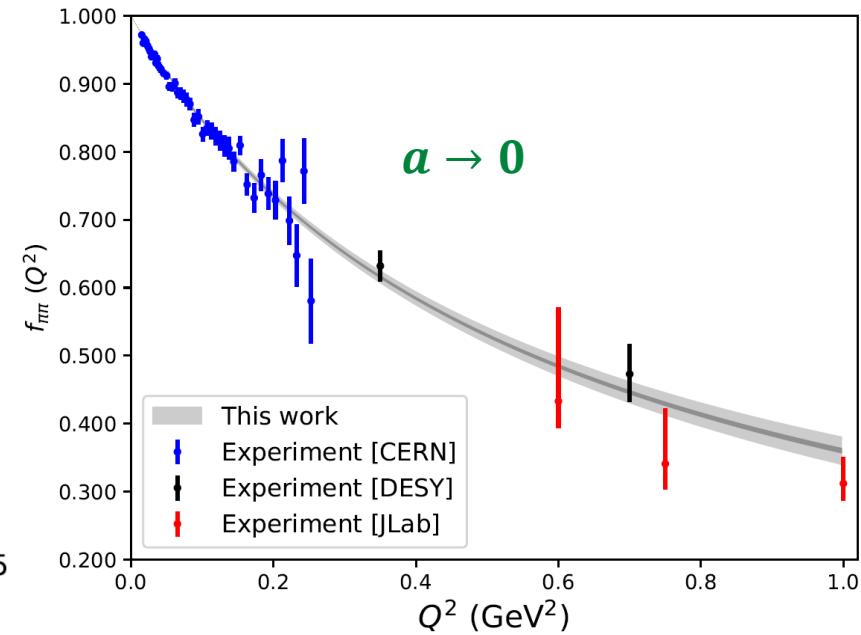
Pion Form Factors

§ Two new lattice pion form factors calcs at physical pion

- ❖ χ QCD: 2+1f, overlap/DWF,
 $a \approx [0.08, 0.2]$ fm, $M_\pi \in [139, 340]$ -MeV
- ❖ BNL: 2+1+1,clover/HISQ,
 $a \approx [0.04, 0.08]$ fm, $M_\pi \in \{135, 300\}$ -MeV



X. Gao et al (BNL), 2102.06047



G. Wang et al (XQCD), 2006.05431

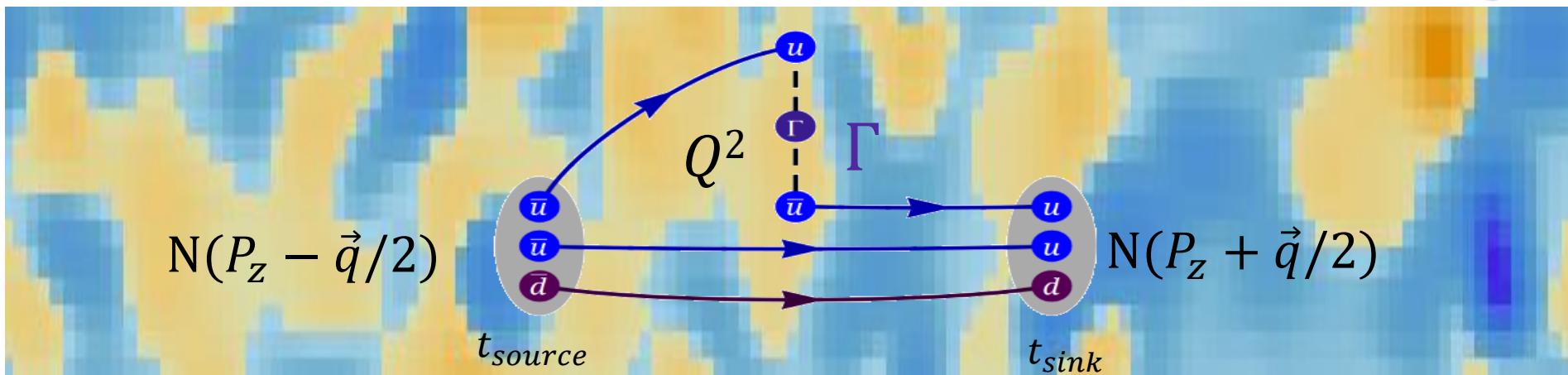
2020: Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at physical pion mass

❖ MSULat: clover/2+1+1 HISQ

0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV

❖ $\xi = 0$ isovector nucleon GPD results



$$\tilde{F}(x, \tilde{\xi}, t, \bar{P}_Z) = \frac{\bar{P}_z}{\bar{P}_0} \int \frac{dz}{4\pi} e^{ixz\bar{P}_Z} \langle P' | \tilde{O}_{\gamma_0}(z) | P \rangle = \frac{\bar{u}(P')}{2\bar{P}^0} \left(H(x, \tilde{\xi}, t, \bar{P}_Z) \gamma^0 + E(x, \tilde{\xi}, t, \bar{P}_Z) \frac{i\sigma^{0\mu}\Delta_\mu}{2M} \right) u(P'')$$

$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \quad \Delta^\mu = p''^\mu - p'^\mu, \quad t = \Delta^2, \quad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

HL, Phys.Rev.Lett. 127 (2021) 18, 182001

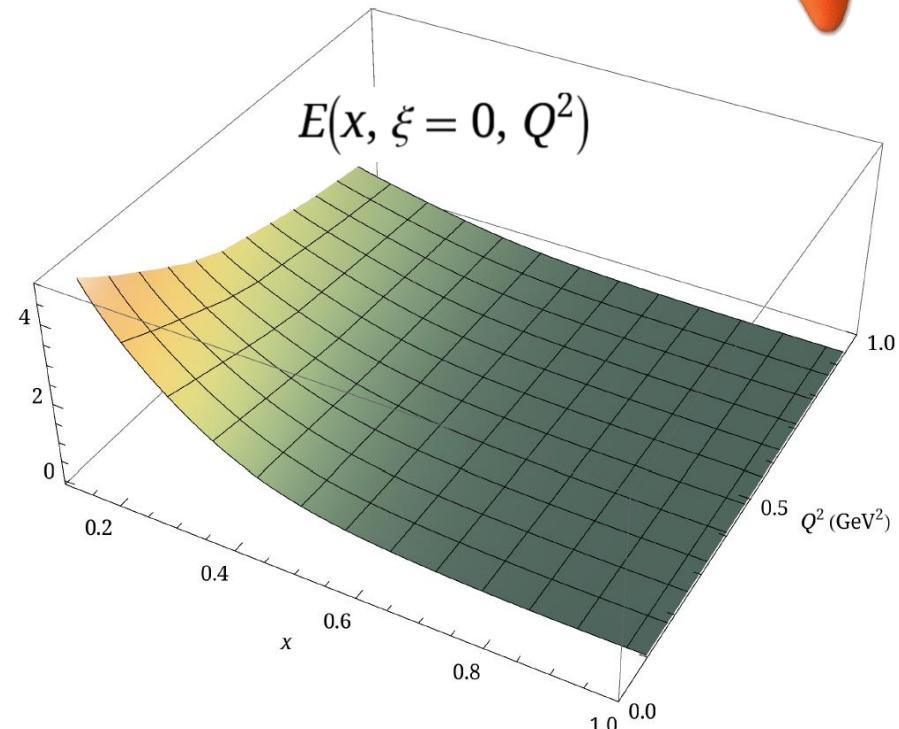
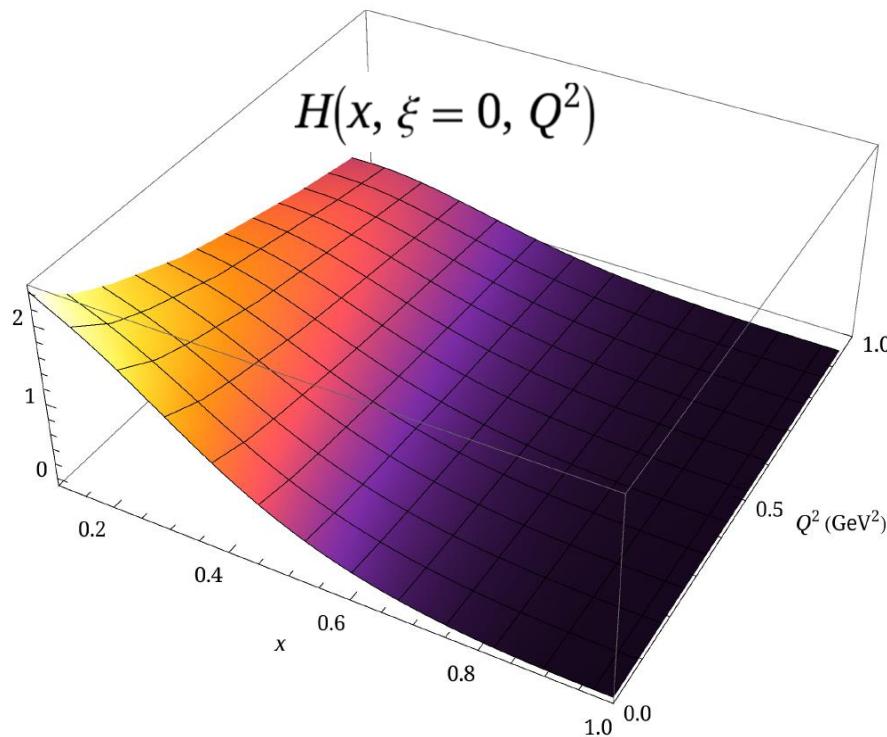
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HL, Phys.Rev.Lett. 127 (2021) 18, 182001



2020: Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at physical pion mass

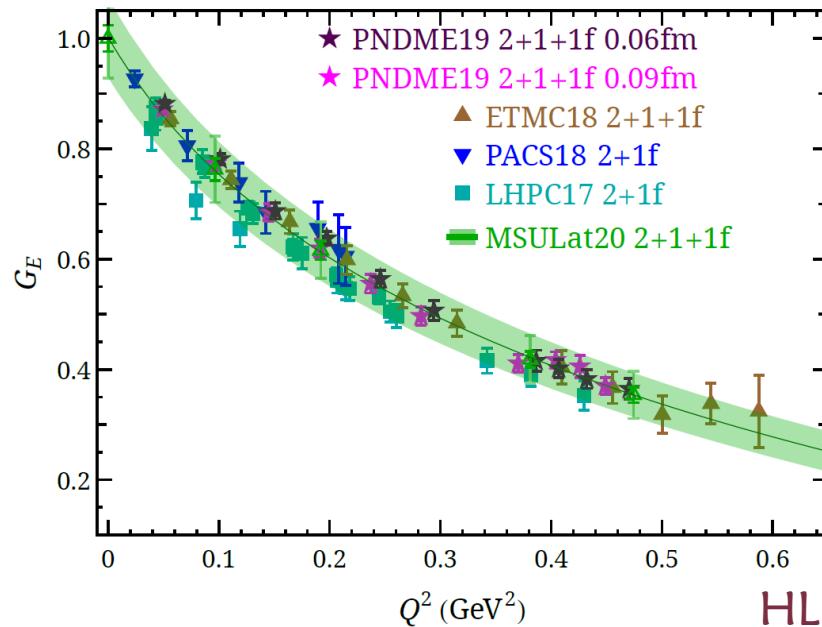
❖ Lattice details: clover/2+1+1 HISQ (MSULat)

0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV

❖ $\xi = 0$ isovector nucleon GPD results

$$\int_{-1}^{+1} dx x^{n-1} = \sum_{i=0, \text{even}}^{n-1} (-2\xi)^i A_{ni}^q(t) + (-2\xi)^n C_{n0}^q(t) \Big|_{n \text{ even}}$$

$n = 1$



HL, Phys.Rev.Lett. 127 (2021) 18, 182001



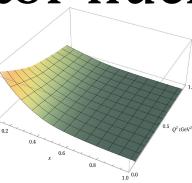
2020: Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at physical pion mass

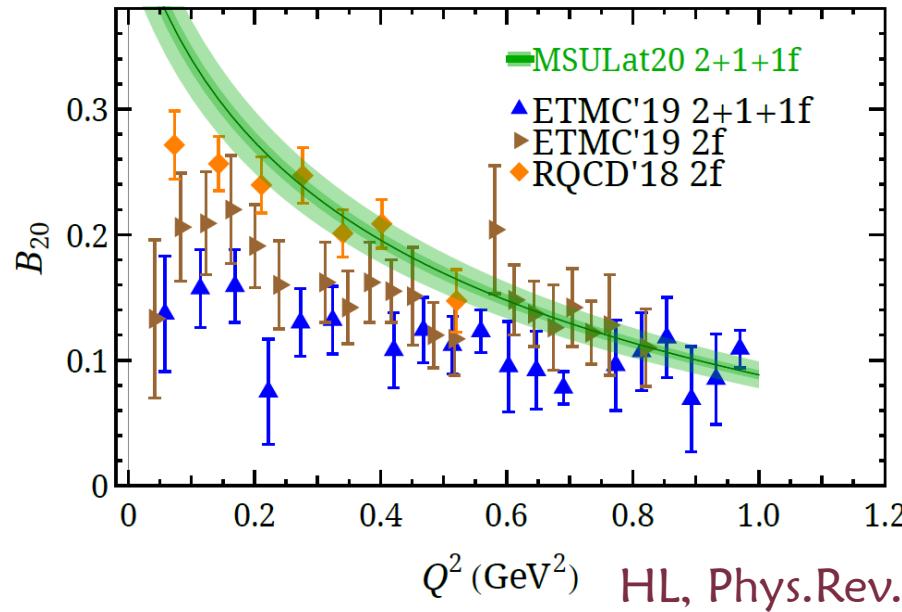
❖ Lattice details: clover/2+1+1 HISQ (MSULat)

0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV

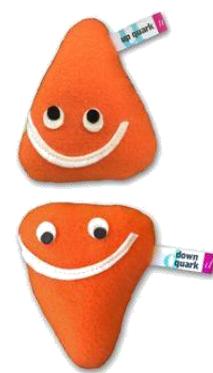
❖ $\xi = 0$ isovector nucleon GPD results

$$\int_{-1}^{+1} dx x^{n-1} = \sum_{i=0, \text{even}}^{n-1} (-2\xi)^i B_{ni}^q(t) - (-2\xi)^n C_{n0}^q(t) \Big|_{n \text{ even}}$$


$n = 2$



HL, Phys.Rev.Lett. 127 (2021) 18, 182001



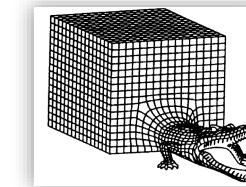
2020: Nucleon Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

❖ Lattice details: clover/2+1+1 HISQ

0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV

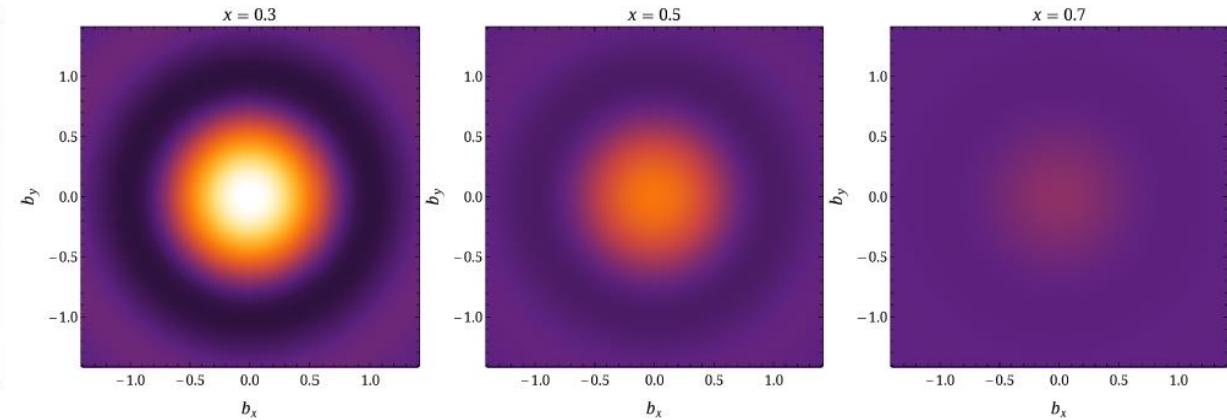
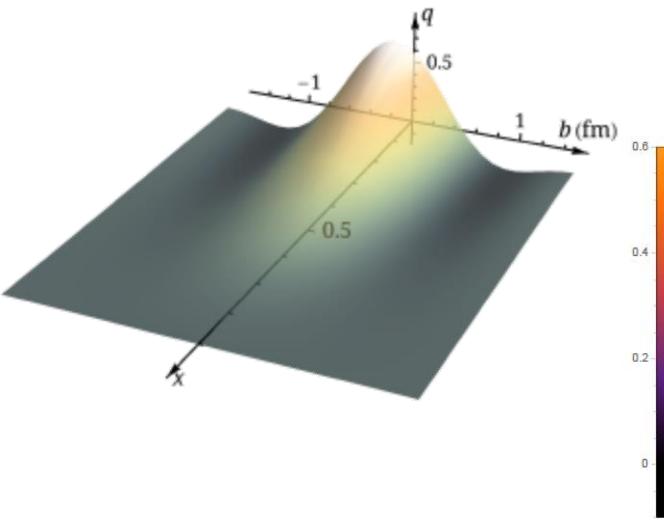
❖ $\xi = 0$ isovector nucleon GPD results



finite-volume,
discretization,



$$q(x, b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q} \cdot \vec{b}}$$



HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Nucleon Polarized GPDs

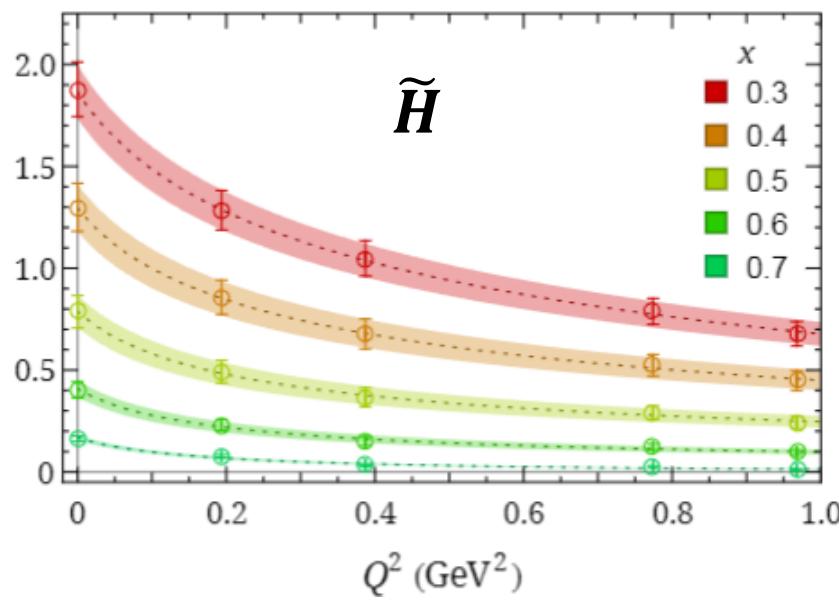
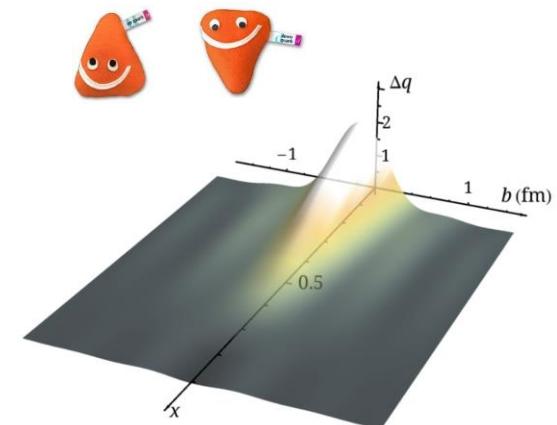
§ Helicity GPD (\tilde{H}) using quasi-PDFs at physical pion mass

❖ MSULat: clover/2+1+1 HISQ

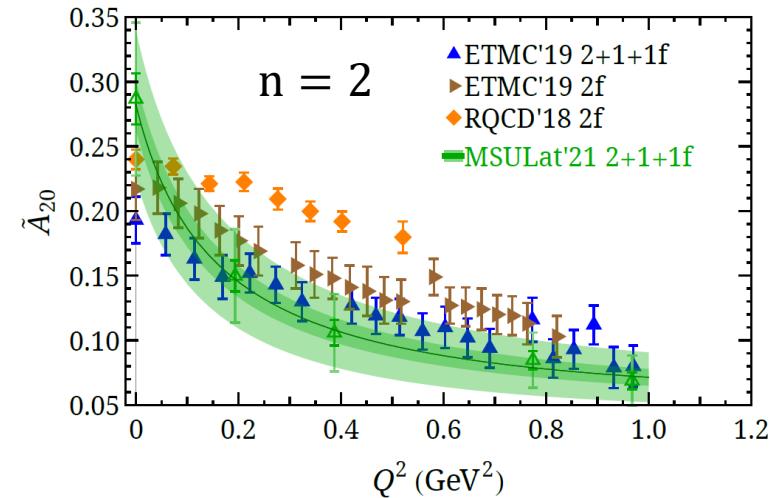
0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV

❖ $\xi = 0$ isovector nucleon (quasi-)GPD results

HL (MSULat), Phys.Lett.B 824 (2022) 136821



❖ Take the integral to form moments



Caveats

§ Systematics in our earlier quasi-PDF calculation

❖ Renormalization: non-perturbative RI/MOM renormalization

❖ State of the art: hybrid-ratio renormalization

X. Ji et. al. NPB 964, 115311 (2021)

❖ Next-leading order (NLO) matching only

❖ State of the art: NNLO matching kernel available

X. Gao, PRL 128, 142003 (2022)

❖ Did not treat leading-renormalon effects

❖ Leading-renormalon resummation (LRR)

❖ Renormalization-group resummation (RGR)

R. Zhang, et. al. PLB 844, 138081 (2023)

❖ For the rest of this presentation, we will focus on the uncertainties from the above (rather than typical lattice-calculation precision or systematics)

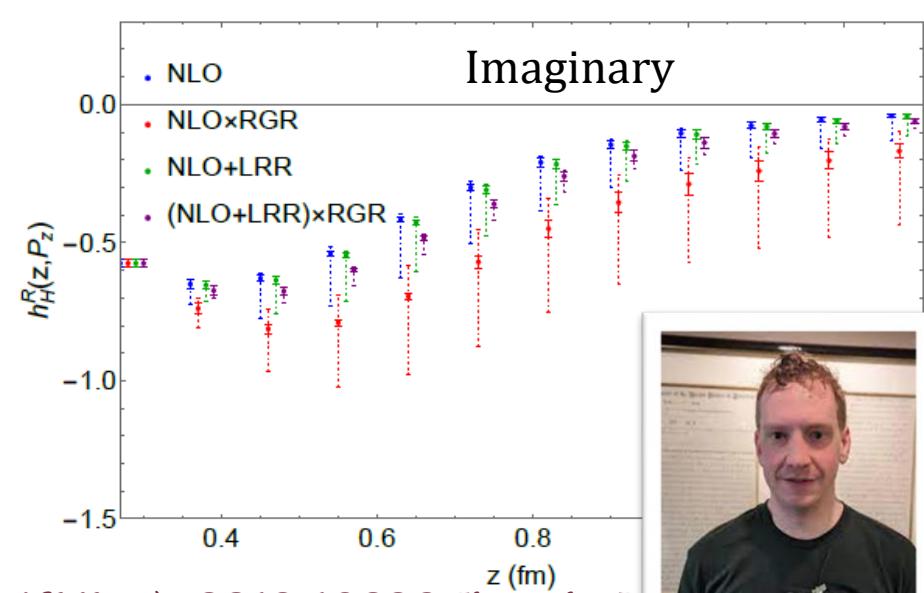
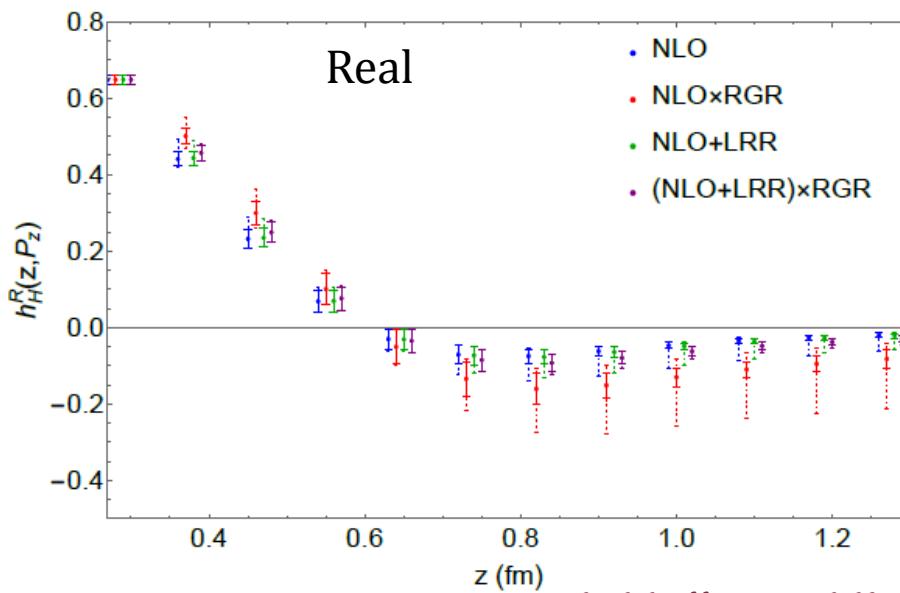
Forward-Limit Case: $\mathcal{P}\mathcal{D}\mathcal{F}$

§ NLO hybrid-ratio renormalized matrix elements

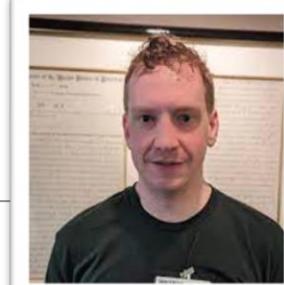
$$h^R(z, P_z) = \begin{cases} N \frac{h^B(z, P_z)}{h^B(z, P_z=0)} & \text{for } z < z_s \\ Ne^{(\delta m + m_0)(z-z_s)} \frac{h^B(z, P_z)}{h^B(z, P_z=0)} & \text{for } z \geq z_s \end{cases}$$

Remove the **linear divergence**
& **renormalon ambiguity**
at large distances

- ❖ Vary the scale within $[0.75, 1.5]$: $\approx 15\%$ variation $\alpha_s(\mu = 2.0 \text{ GeV})$
- ❖ Systematic errors shown below:



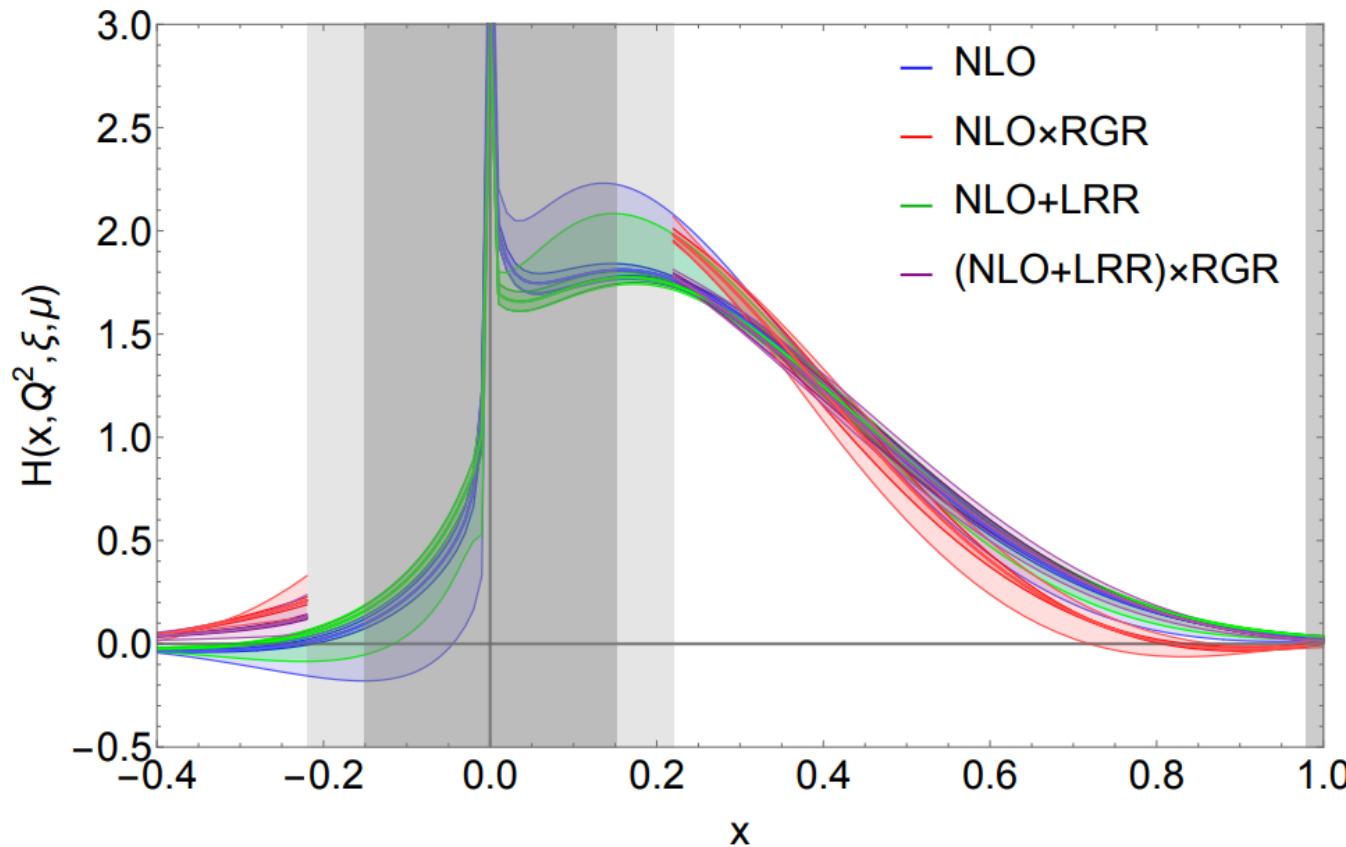
J. Holligan, HL (MSULat), 2312.10829 [hep-lat]



Forward Limit Case: PDF

§ NLO isovector nucleon $H(\xi = 0, Q^2 = 0, x)$

☞ RGR process: DGLAP equation breaks down for $|x| \lesssim 0.2$ with $\mu = c' \times 2xP_z$



J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

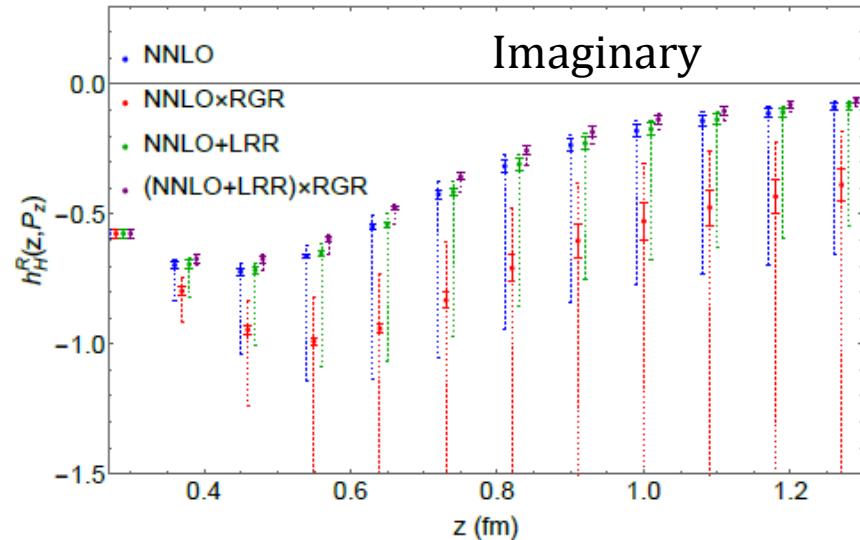
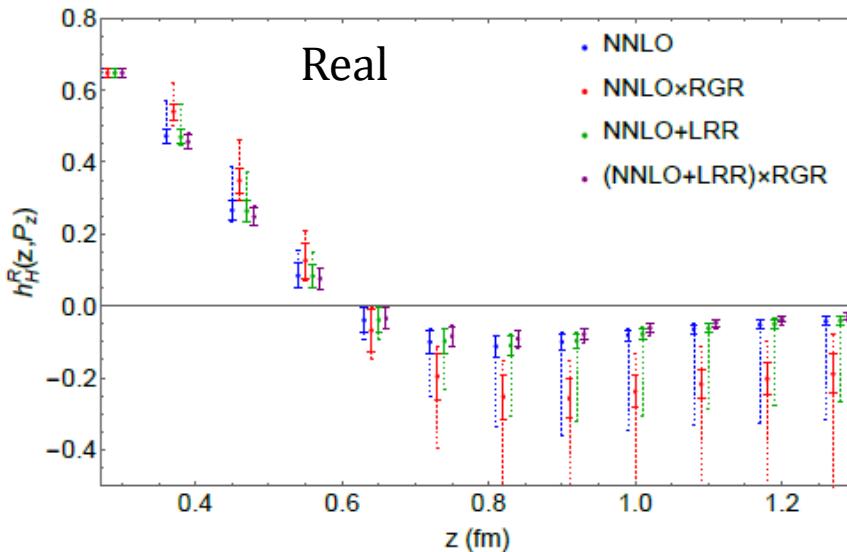
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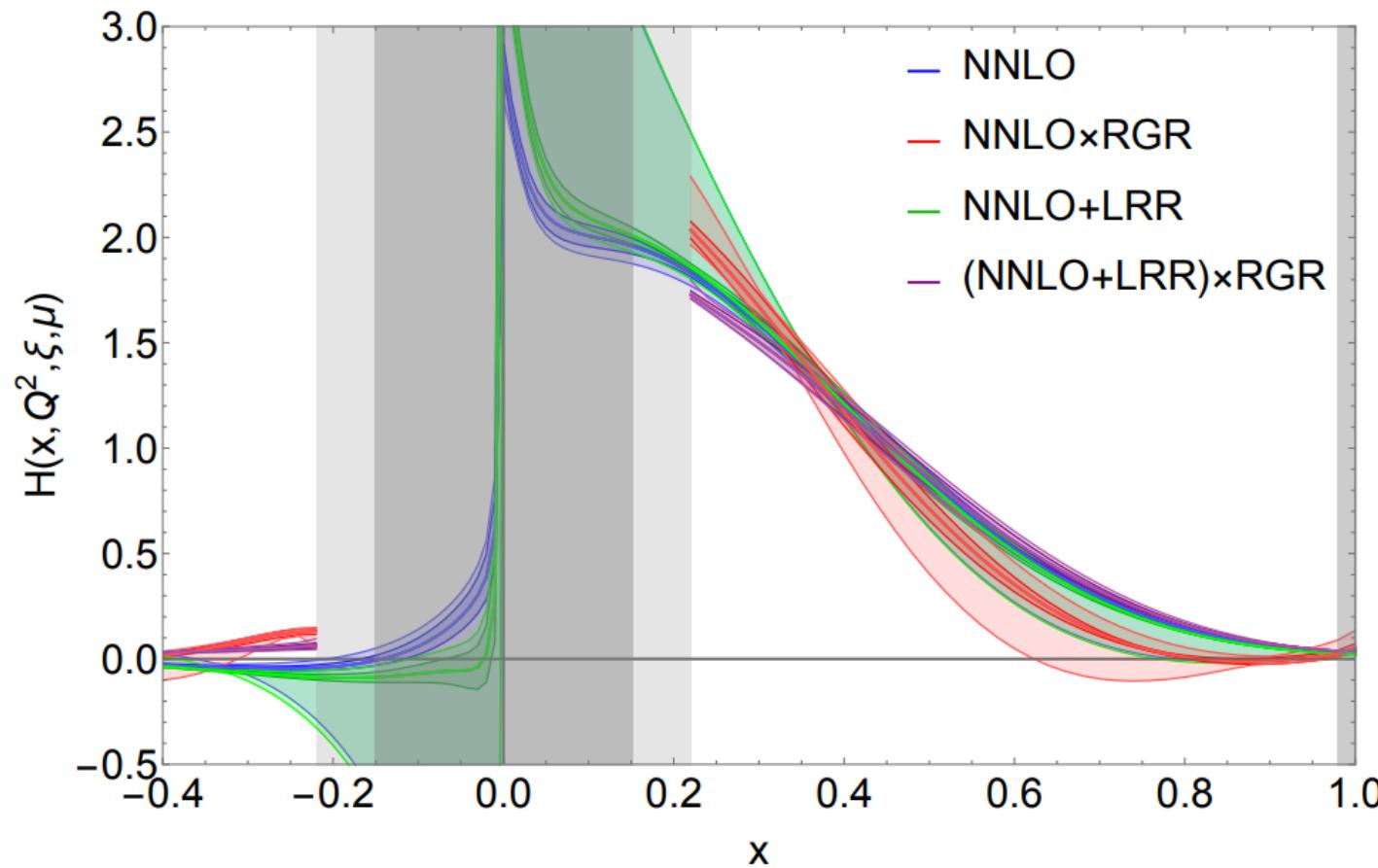


J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Forward Limit Case: PDF

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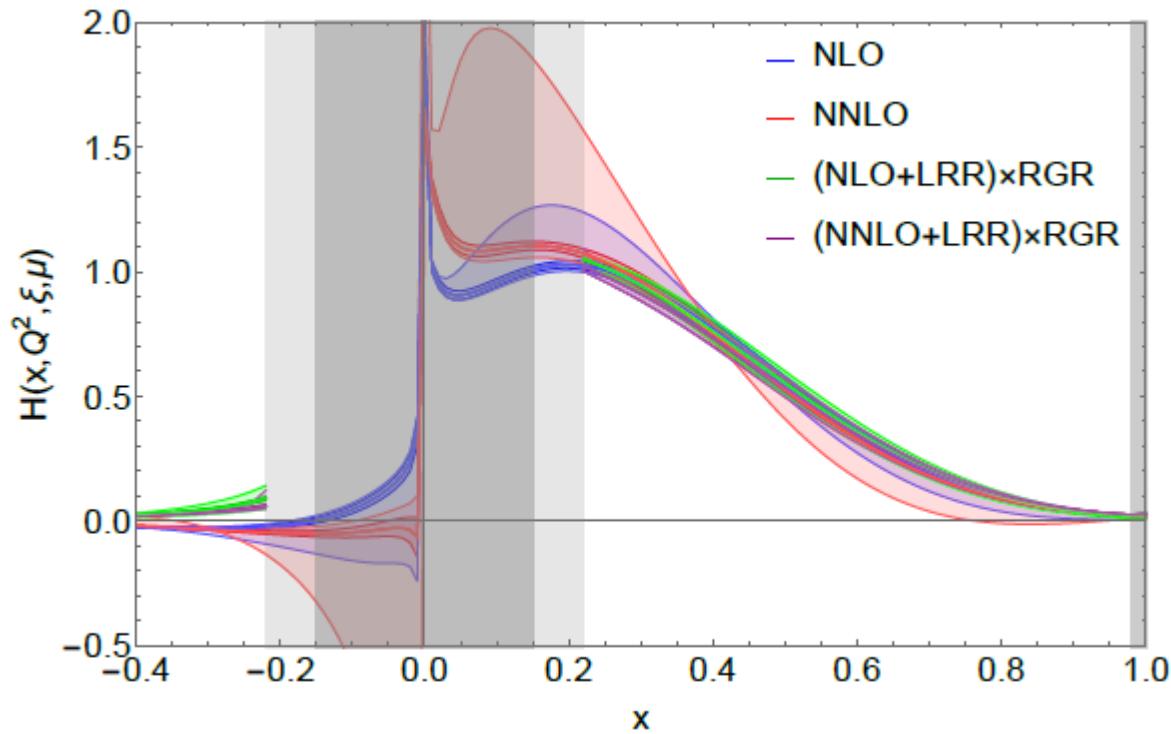
☞ RGR process: DGLAP equation breaks down for $|x| \lesssim 0.2$ with $\mu = c' \times 2xP_z$



J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

$\xi=0, Q^2=0.39 \text{ GeV}^2$ GPDs

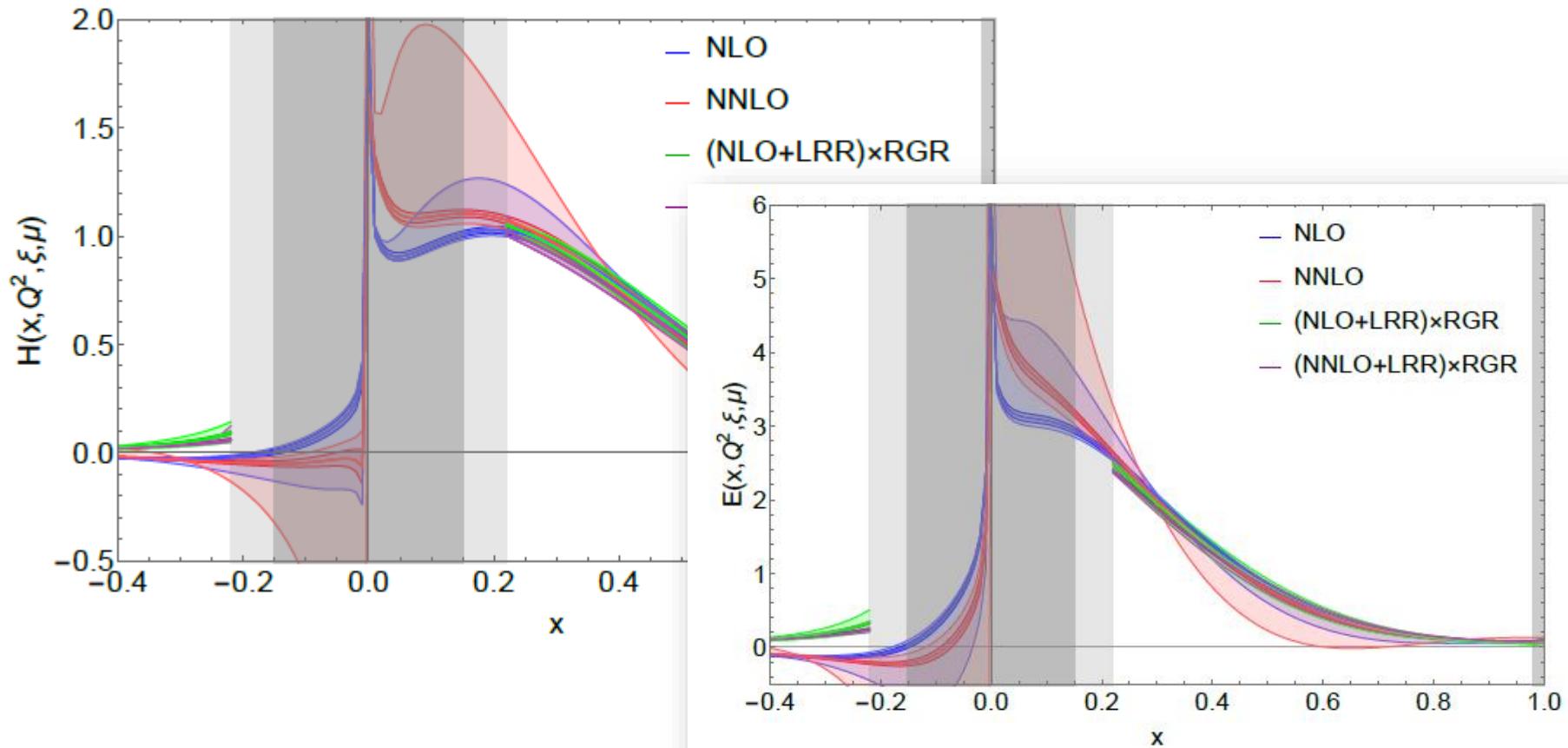
§ Repeat the procedure for nonzero transfer momentum



J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

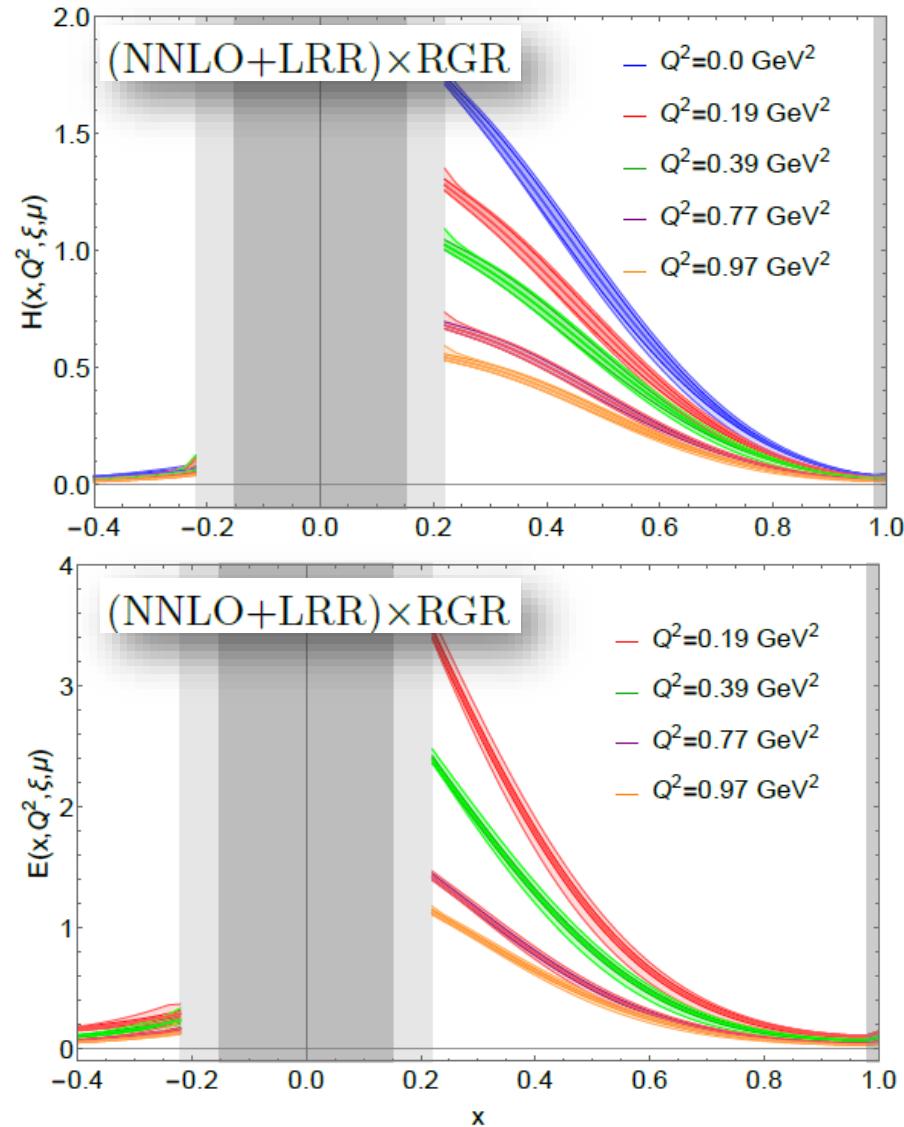
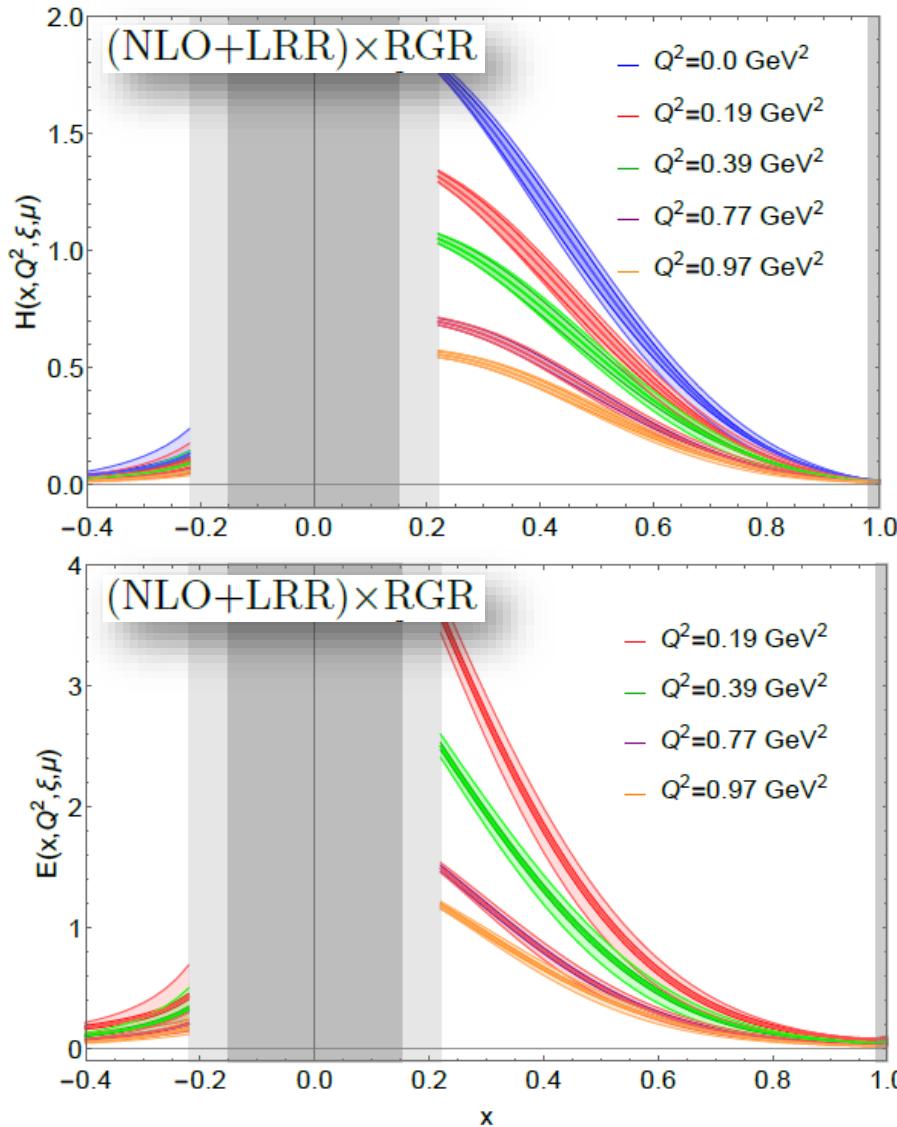
$\xi=0, Q^2=0.39 \text{ GeV}^2$ GPDs

§ Repeat the procedure for nonzero transfer momentum



J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

$\xi=0, Q^2=0.39 \text{ GeV}^2$ GPDs



$\xi \neq 0$ GPDS

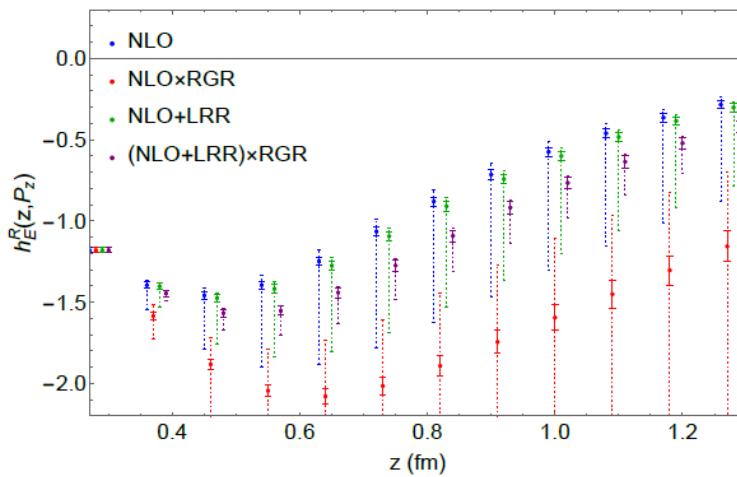
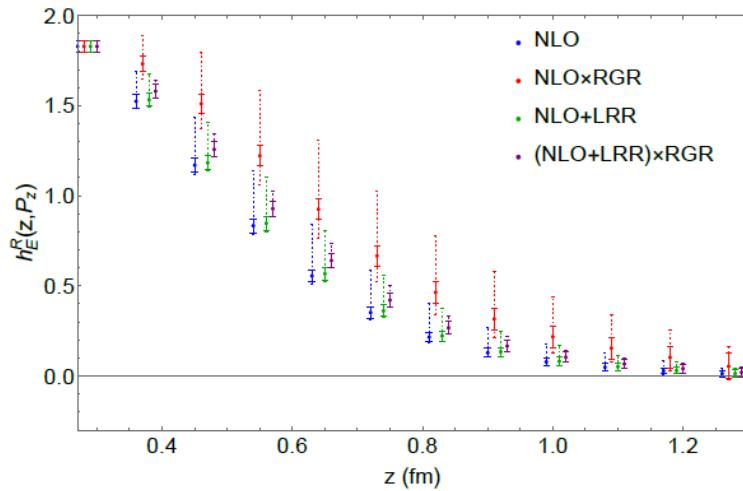
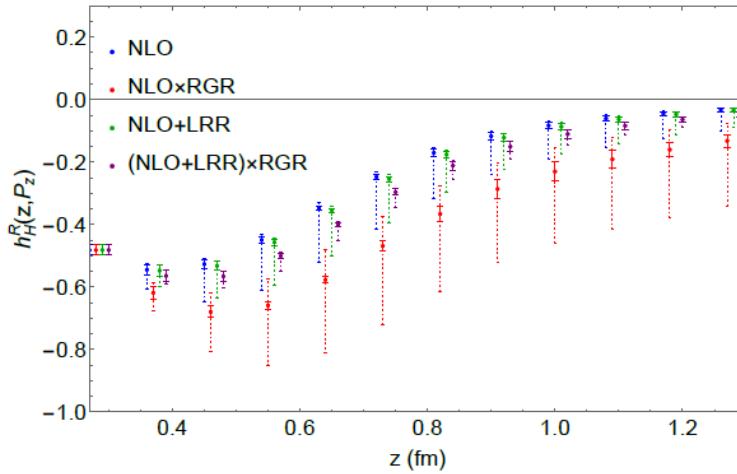
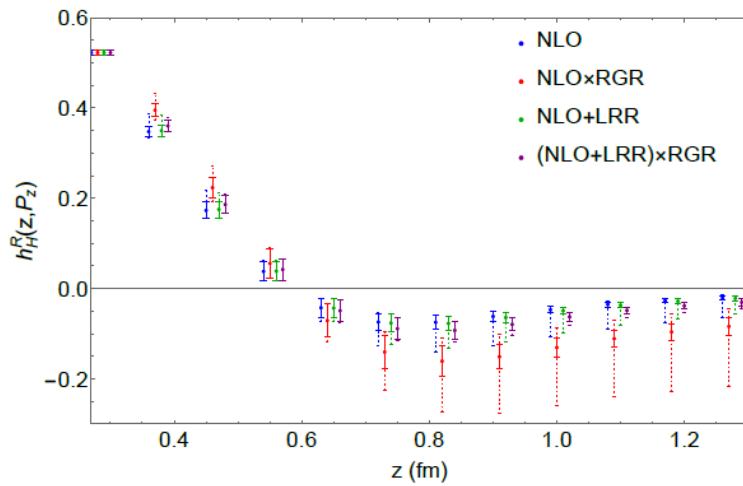
§ Only the NLO matching kernel is available

$$\begin{aligned} & \mathcal{K}(x, y, \mu, \xi, P_z) \\ &= \delta(x - y) \\ &+ \frac{\alpha_s C_F}{4\pi} \left[\left(\frac{|\xi + x|}{2\xi(\xi + y)} + \frac{|\xi + x|}{(\xi + y)(y - x)} \right) \left(\ln \left(\frac{4y^2(\xi + x)^2 P_z^2}{\mu^2} \right) - 1 \right) \right. \\ &+ \left(\frac{|\xi - x|}{2\xi(\xi - y)} + \frac{|\xi - x|}{(\xi - y)(x - y)} \right) \left(\ln \left(\frac{4y^2(\xi - x)^2 P_z^2}{\mu^2} \right) - 1 \right) \\ &+ \left. \left(\left(\frac{\xi + x}{\xi + y} + \frac{\xi - x}{\xi - y} \right) \frac{1}{|x - y|} - \frac{|x - y|}{\xi^2 - y^2} \right) \left(\ln \left(\frac{4y^2(x - y)^2 P_z^2}{\mu^2} \right) - 1 \right) \right] \end{aligned}$$

F. Yoa et al, JHEP 11(2023) 021

$\xi \neq 0$ GPDs

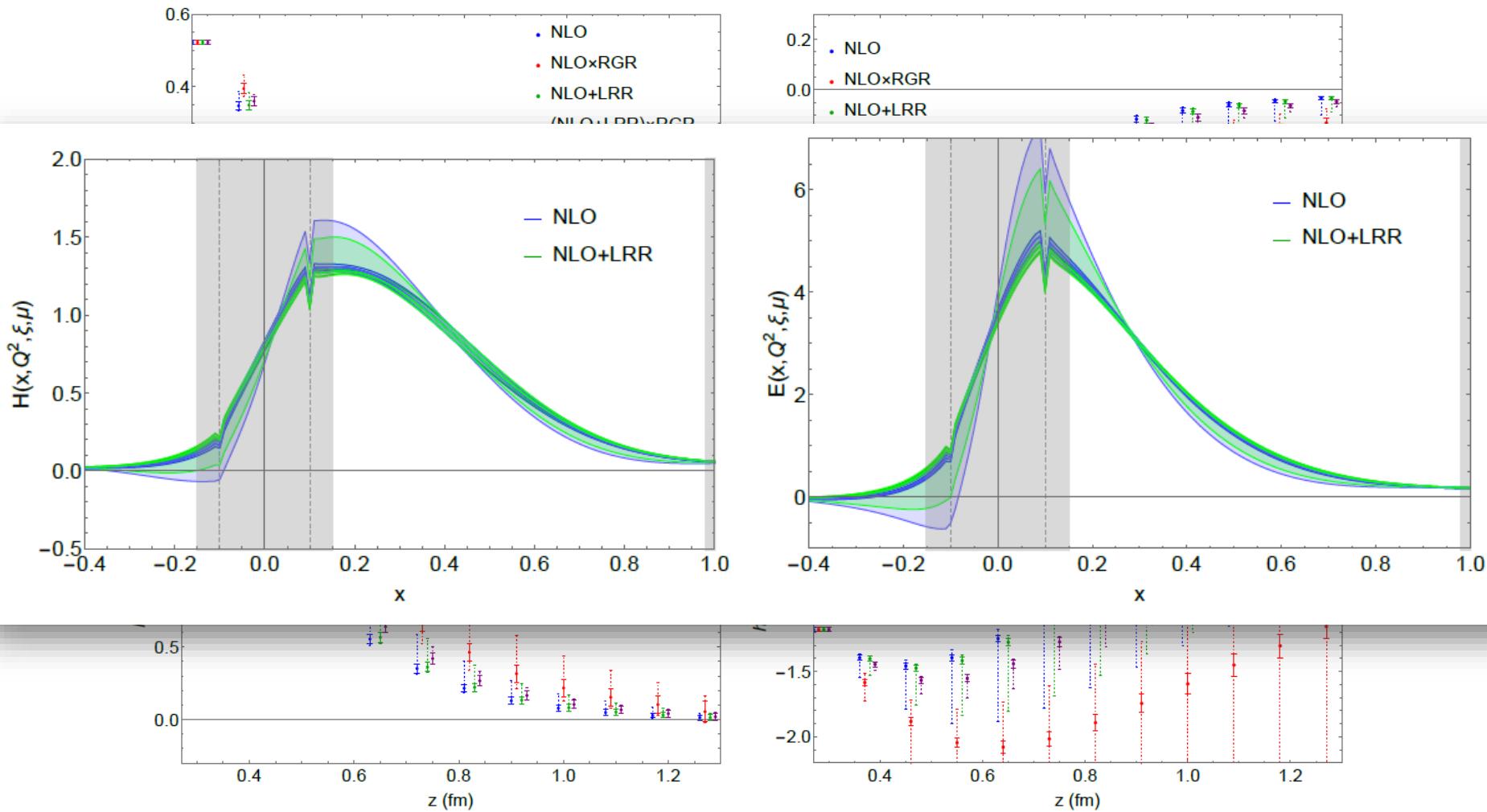
§ NLO $\xi = 0.1$, $Q^2 = 0.23 \text{ GeV}^2$



J. Holligan, HL (MSULat), in preparation

$\xi \neq 0$ GPDs

§ NLO $\xi = 0.1, Q^2 = 0.23 \text{ GeV}^2$



J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Challenges

§ Large momentum is essential

- ❖ With sufficient statistics nucleons may reach 5 GeV

§ Renormalization of linear divergence

- ❖ Wilson-line ops have linear divergences that must be subtracted

§ Methods for signal-to-noise improvement

- ❖ Gluonic observables, new ideas for large momentum

§ Inverse problems PDF extraction in SDF

- ❖ Remove the model/preconditioner-choice dependence

§ Reaching long-range correlations in LaMET

- ❖ For small- x physics, new methods for calculating longer-range correlations must be developed

Whitepaper: Lattice QCD Calculations of Parton Physics, 2202.07193