Fantômas: Analysis of parton distributions in a pion with Bézier parametrizations

From PDFs to the underlying QCD

Fred Olness
SMU

*Thanks for substantial input
from my friends & colleagues*

Aurore Courtoy, Tim Hobbs, Lucas Kotz,
Pavel Nadolsky, Maximiliano Ponce-Chavez

FANTÔMAS

CFNS
Stony Brook U
25 June 2024


STONY BROOK, L. I. DEPOT
STONY BROOK

The Goal

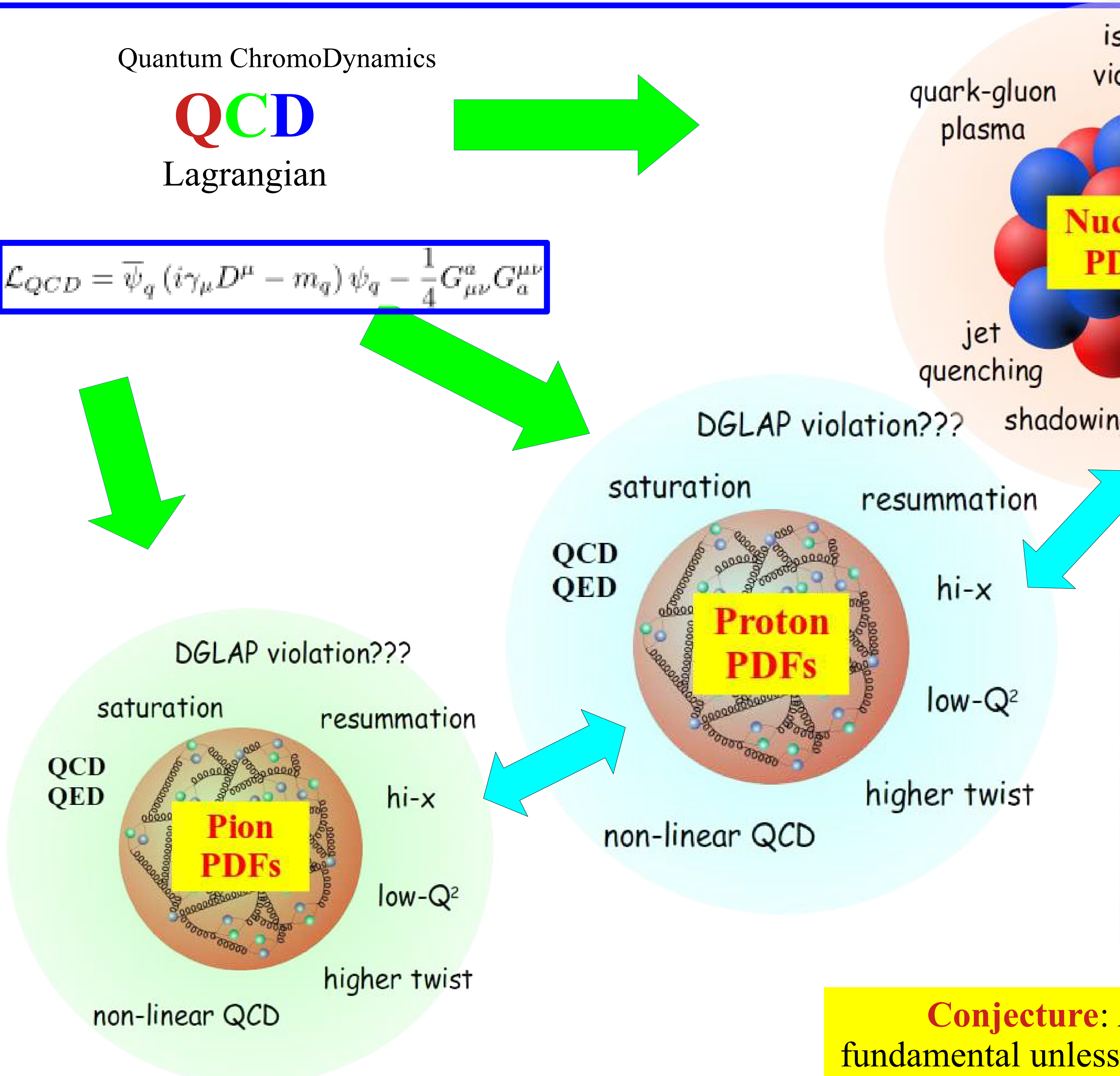
QCD: From Parameterization to a Deeper Understanding

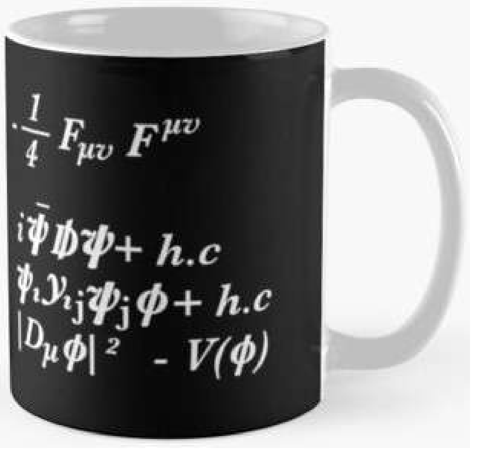
Quantum ChromoDynamics

QCD

Lagrangian

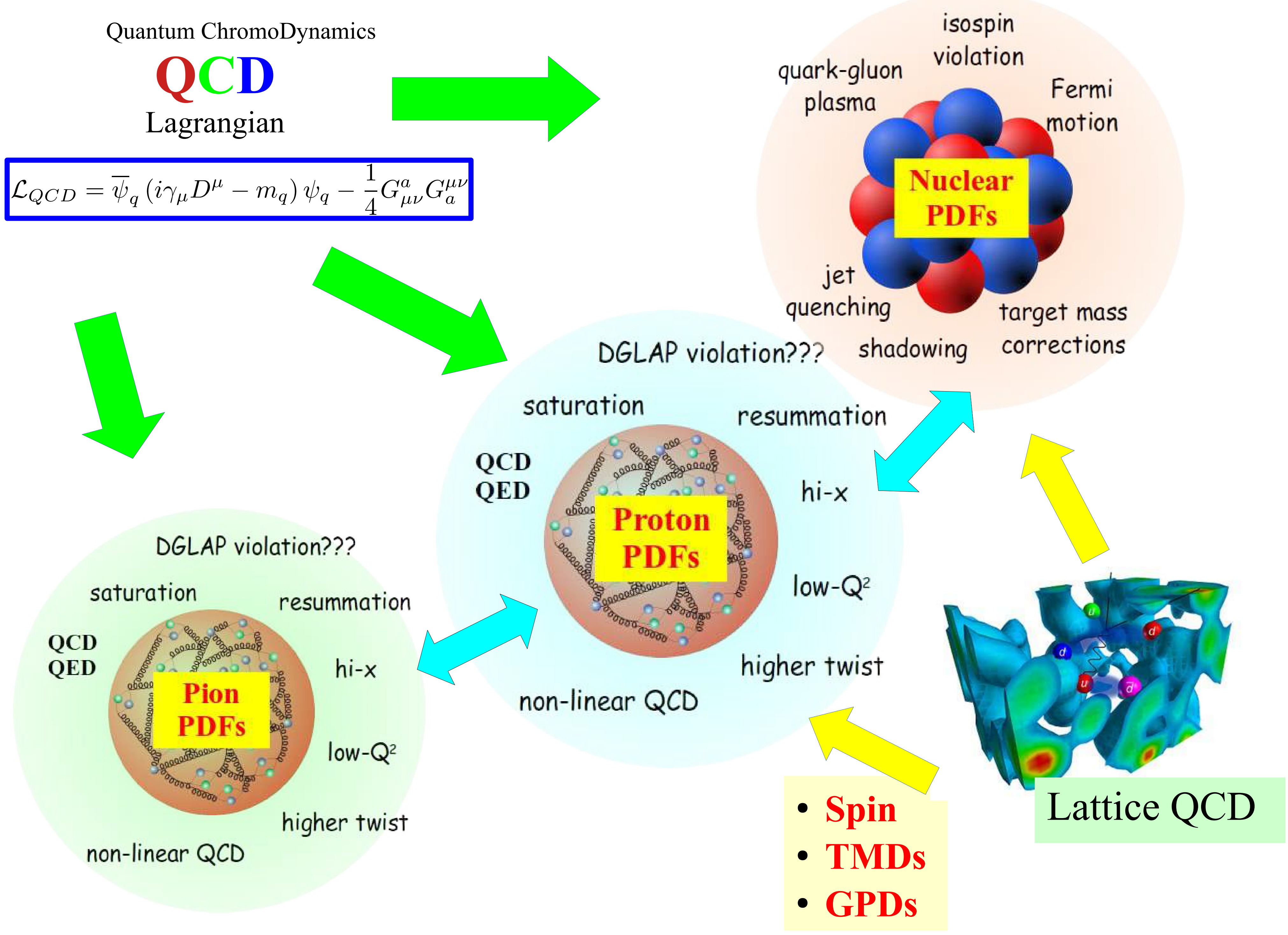
$$\mathcal{L}_{QCD} = \overline{\psi}_q \left(i\gamma_\mu D^\mu - m_q \right) \psi_q - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$



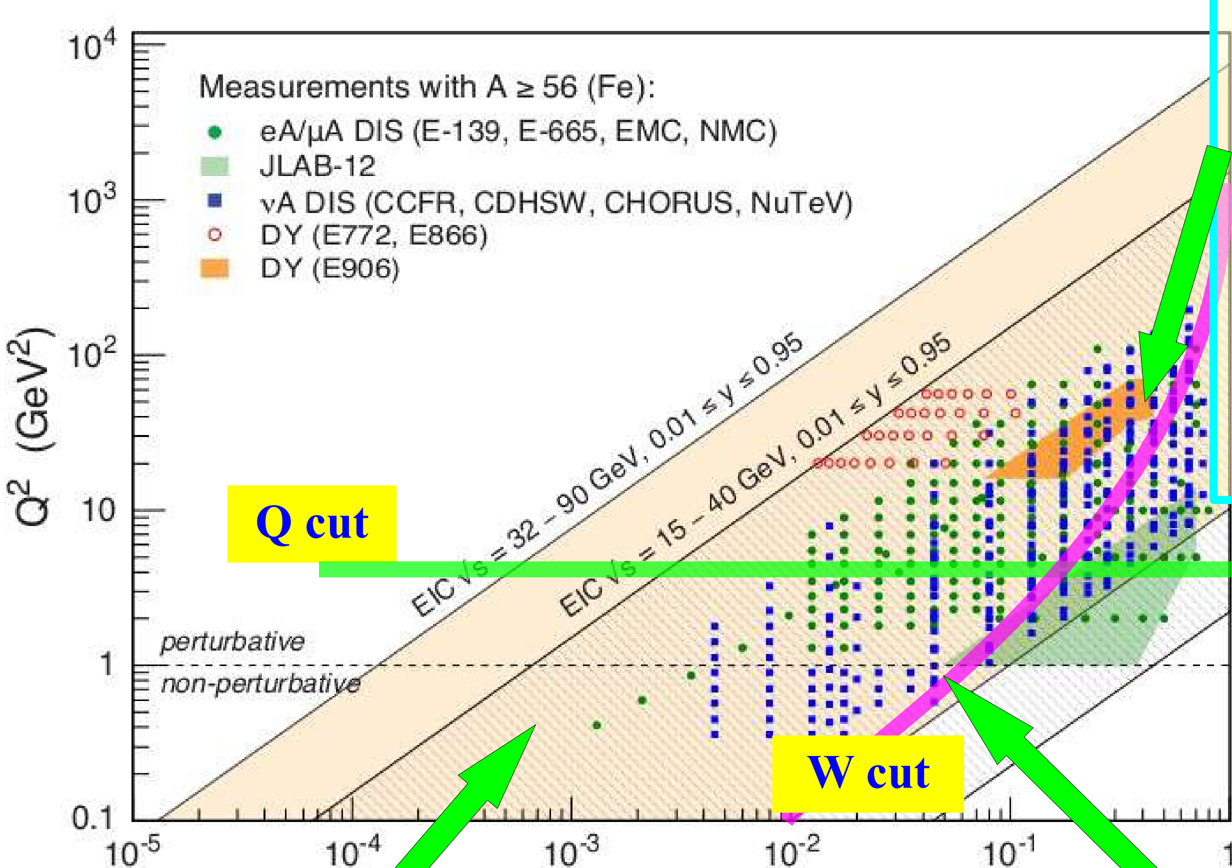


Conjecture: A theory can't be fundamental unless it fits on a coffee mug.

QCD: From Parameterization to a Deeper Understanding



To boldly go where no analysis has gone before ...



High-x:

Nuclear PDFs: x>1 allowed;
impacts F_2^{Nuc}/F_2^{Iso} in Fermi region
Target Mass Corrections
pick up M^2/Q^2 higher twist
Deuteron Corrections
impacts $F_2^{Nuc}/F_2^{Deuteron}$ ratio

Low-x:

Shadowing
Recombination
Resummation
BFKL
Saturation

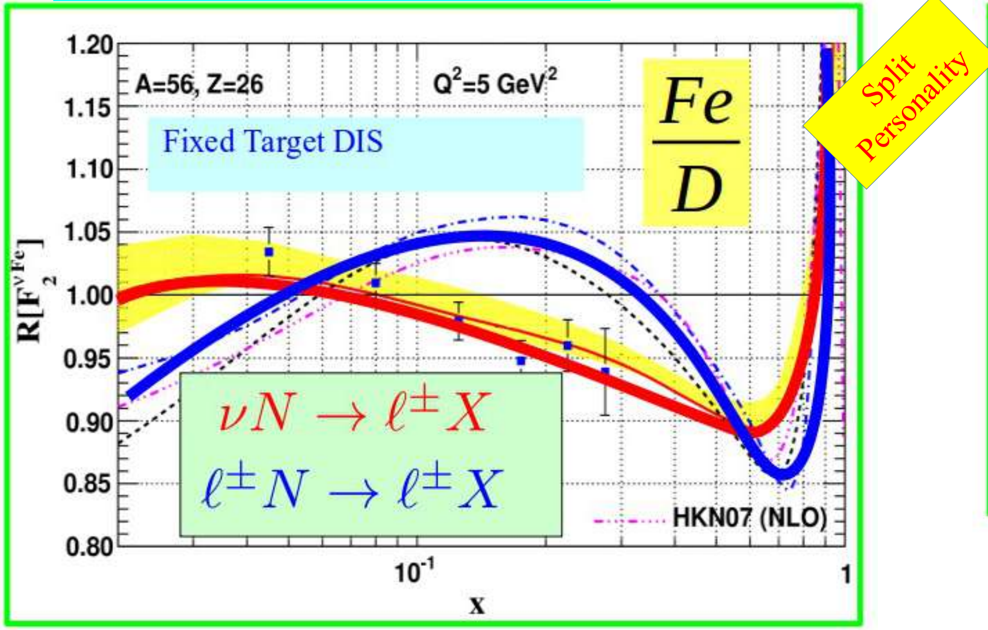
Low-Q^2:

Non-Perturbative inteface
collective effects
Target Mass Corrections
pick up M^2/Q^2 higher twist
F_L at low Q^2 access to g(x)

Are we just looking under the lamppost

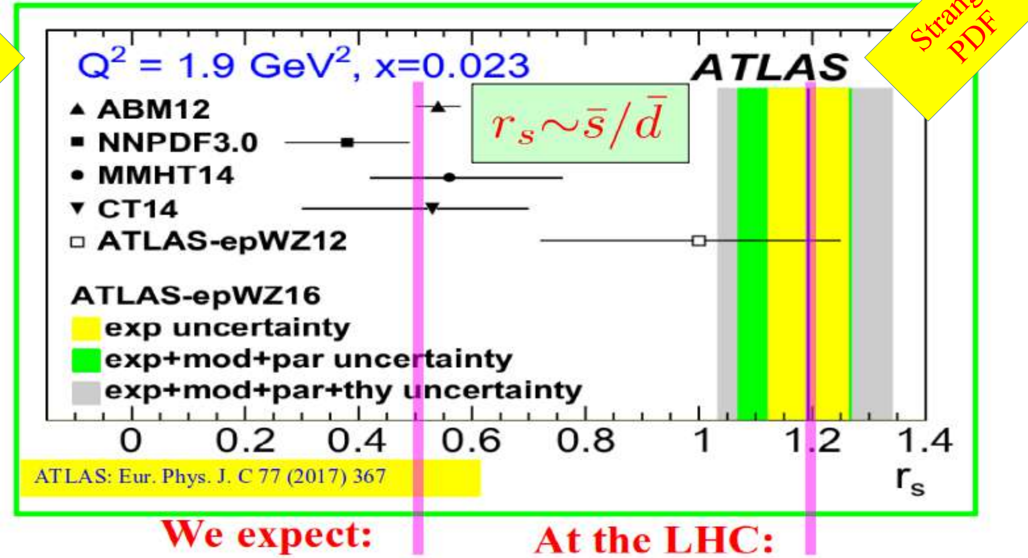
Need theoretical guidance in these regions

nCTEQ15ν



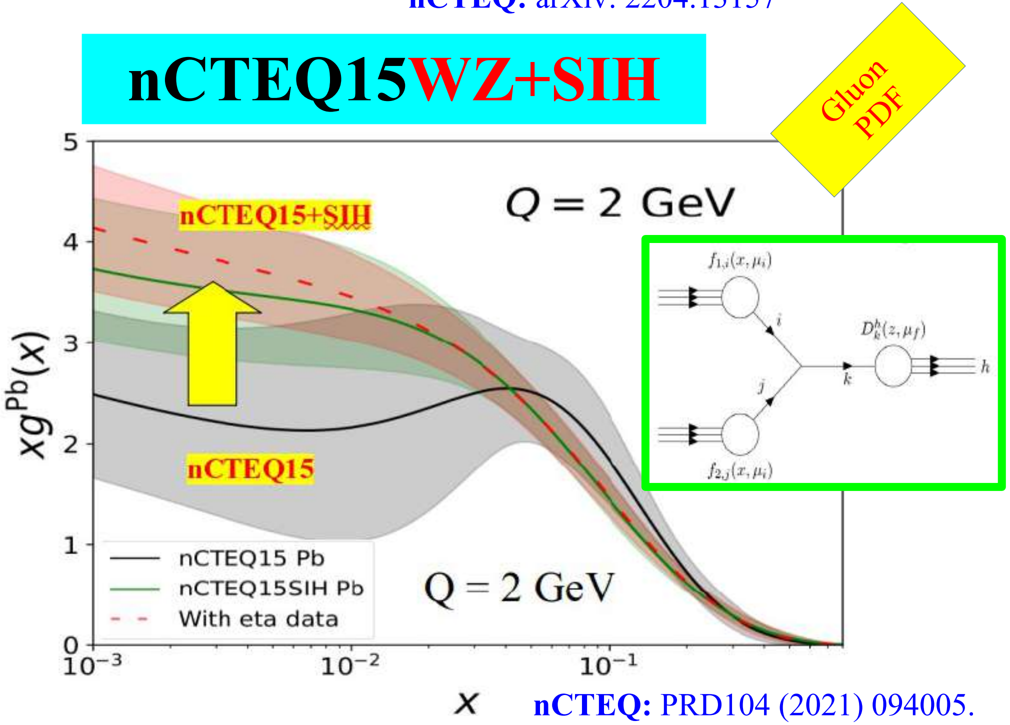
nCTEQ: arXiv: 2204.13157

nCTEQ15WZ



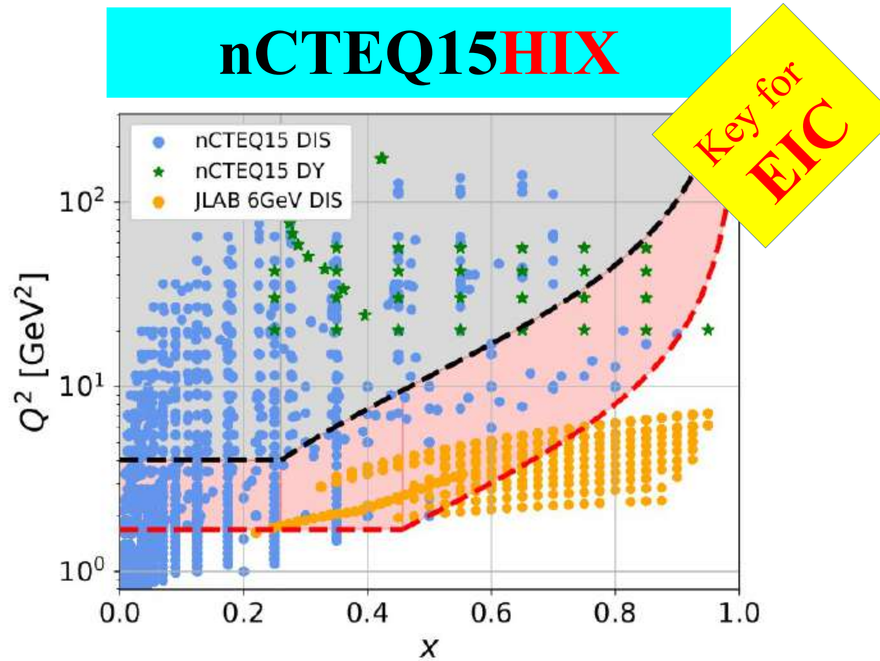
nCTEQ: Phys.Rev.D 104 (2021) 094005

nCTEQ15WZ+SIH



nCTEQ: PRD104 (2021) 094005.

nCTEQ15HIX



nCTEQ: Phys.Rev.D 103 (2021) 11, 114015

precision $f_A(x,Q)$ can serve as Boundary Condition for $f_A(x,Q,k_T,b_T,\sigma)$

xFitter

The xFitter project is an open source
QCD fit framework ready to extract PDFs
and assess the impact of new data

...

including mesons

xFitter

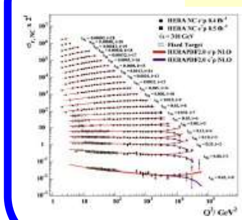
PROTON NUCLEON MESON

www.xFitter.org

Sample data files:

- **LHC:** ATLAS, CMS, LHCb
- **Tevatron:** CDF, D0
- **HERA:** H1, ZEUS, Combined
- **Fixed Target:** …
- **User Supplied:** ...

Experimental Data



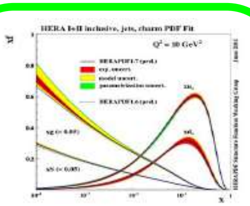
Data: HERA, Tevatron, LHC, fixed target experiments

Processes: Inclusive DIS, Jets, Drell-Yan, Diffraction, Top production W and Z production

Theory Calculations

HQ Schemes:	MSTW, NNPDF, ABM, ACOT
Jets, W, Z:	FastNLO, ApplGrid
Top:	Hathor
Evolution:	QCDNUM, APFEL, k_T
Other:	NNPDF reweighting TMDs, Dipole Model, ...

xFitter



Parton Distribution Functions: PDF, Updf, TMD

$\alpha_s(M_Z)$, m_c,m_b,m_t ...

Theoretical Cross Sections

Comparisons to other PDFs (LHAPDF)

extensions include nuclear PDFs

Features & Recent Updates:

- **NNLO DGLAP**
- Photon PDF & **QED**
- Pole & MS-bar masses
- Profiling and Re-Weighting
- **BFKL interface**
- Heavy Quark Variable Treshold
- Improvements in χ^2 and correlations
- **TMD** PDFs (uPDFs)
- *... and many other*

xFitter 2.2.0 Future Freeze

xFitter: Tools for nucleon and meson PDF fits

... xFitter with nucleons & pions ...

https://www.xfitter.org/

xFitter Pion Fit

xFitter Meson PDFs

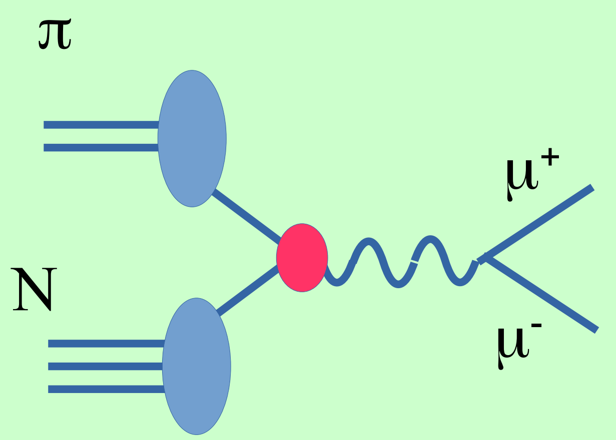
Special thanks to: Ivan Novikov, Alexander Glazov, Oleksandr Zenaiev

xFitter: open-source framework for global fits to meson PDFs

https://www.xfitter.org/

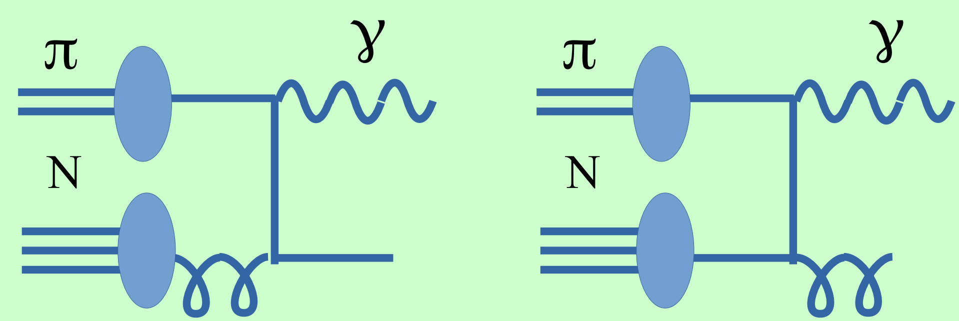
xFitter

Experiment	χ^2/N_{points}
E615	206/140
NA10 (194 GeV)	107/67
NA10 (286 GeV)	95/73
WA70	64/99



E615 & NA10

Pions on Tungsten



WA70

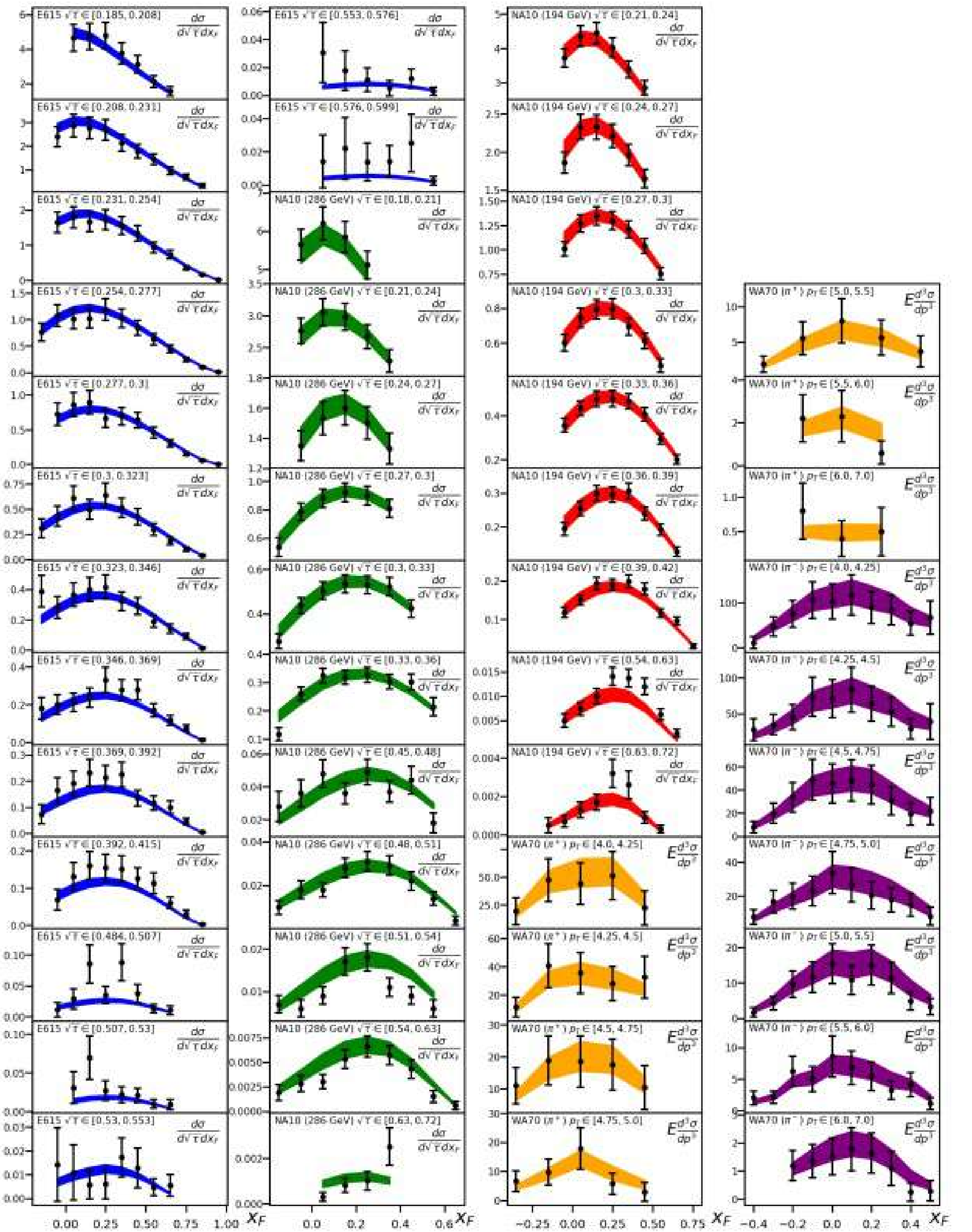
Pions on Proton

Parton Distribution Functions of the Charged Pion Within The xFitter Framework

xFitter Developers' team: Ivan Novikov,[1,2,*] Hamed Abdolmaleki,[3] Daniel Britzger,[4] Amanda Cooper-Sarkar,[5] Francesco Giuli,[6] Alexander Glazov,[2,†] Aleksander Kusina,[7] Agnieszka Luszczak,[8] Fred Olness,[9] Pavel Starovoitov,[10] Mark Sutton,[11] and Oleksandr Zenaiev[12]

Phys.Rev.D 102 (2020) 1, 014040

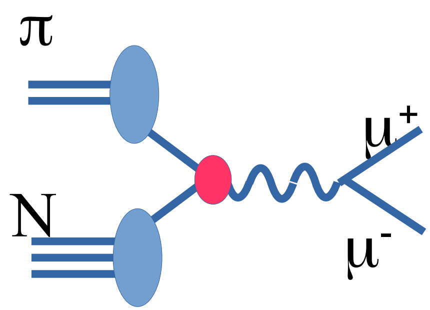
Pion Data:



Pions (π^{-}) on Tungsten

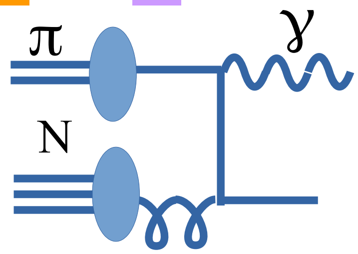
E615 E_π = **252 GeV**

NA10 E_π = **194 GeV** & **286 GeV**



Pions ($\pi^{\pm}$) on Proton

WA70 π^{+} π^{-}



NLO computation with MCFM / APPLGRID

- theory errors from α_s, and nPDF uncert
- uncertainties include scale variations.
  - for factorization scale variation modify APPLGRID for two PDFs

xFitter Pion PDFs

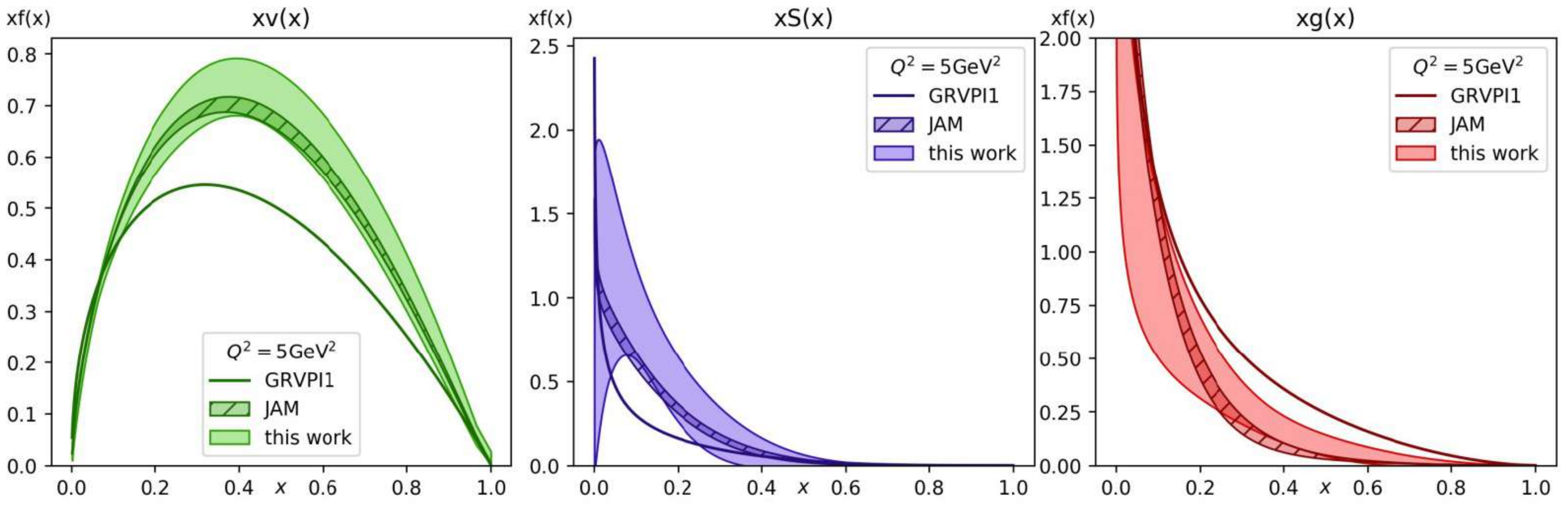
Experiment	Normalization uncertainty	χ^2/N_{points}
E615	15 %	206/140
NA10 (194 GeV)	6.4%	107/67
NA10 (286 GeV)	6.4%	95/73
WA70	32%	64/99

$$xv(x) = A_v x^{B_v}(1-x)^{C_v}(1+D_v x^{\alpha}),$$

$$xS(x) = A_S x^{B_S}(1-x)^{C_S}/\mathcal{B}(B_S+1, C_S+1),$$

$$xg(x) = A_g(C_g+1)(1-x)^{C_g},$$

	$\langle xv \rangle$	$\langle xS \rangle$	$\langle xg \rangle$	Q^2 (GeV^2)
JAM [26]	0.54 ± 0.01	0.16 ± 0.02	0.30 ± 0.02	1.69
JAM (DY)	0.60 ± 0.01	0.30 ± 0.05	0.10 ± 0.05	1.69
this work	0.55 ± 0.06	0.26 ± 0.15	0.19 ± 0.16	1.69
Lattice-3 [16]	0.428 ± 0.030			4
SMRS [20]	0.40 ± 0.02			4
Han et al. [42]	0.428 ± 0.03			4
DSE [7]	0.52			4
this work	0.50 ± 0.05	0.25 ± 0.13	0.25 ± 0.13	4
JAM	0.48 ± 0.01	0.17 ± 0.01	0.35 ± 0.02	5
this work	0.49 ± 0.05	0.25 ± 0.12	0.26 ± 0.13	5
Lattice-1 [14]	0.558 ± 0.166			5.76
Lattice-2 [15]	0.48 ± 0.04			5.76
this work	0.48 ± 0.05	0.25 ± 0.12	0.27 ± 0.13	5.76
WRH [21]	0.434 ± 0.022			27
ChQM-1 [11]	0.428			27
ChQM-2 [13]	0.46			27
this work	0.42 ± 0.04	0.25 ± 0.10	0.32 ± 0.10	27
SMRS [20]	0.49 ± 0.02			49
this work	0.41 ± 0.04	0.25 ± 0.09	0.34 ± 0.09	49



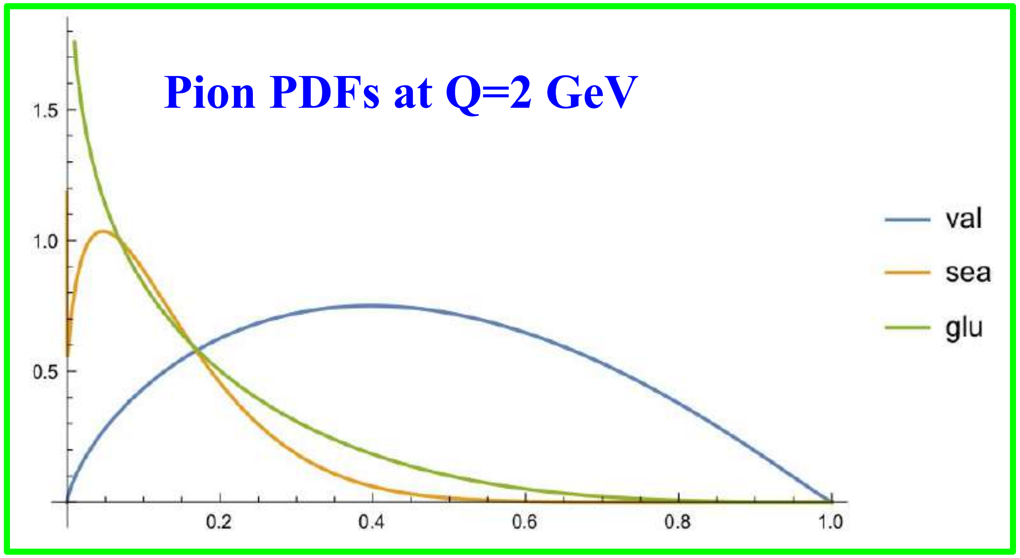
The Tools

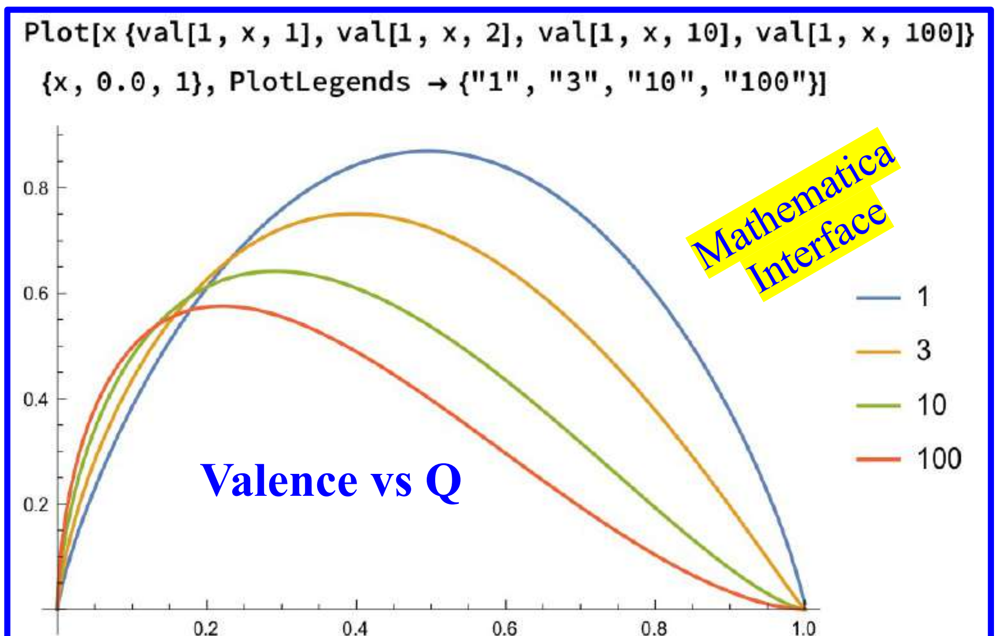
xFitter-draw

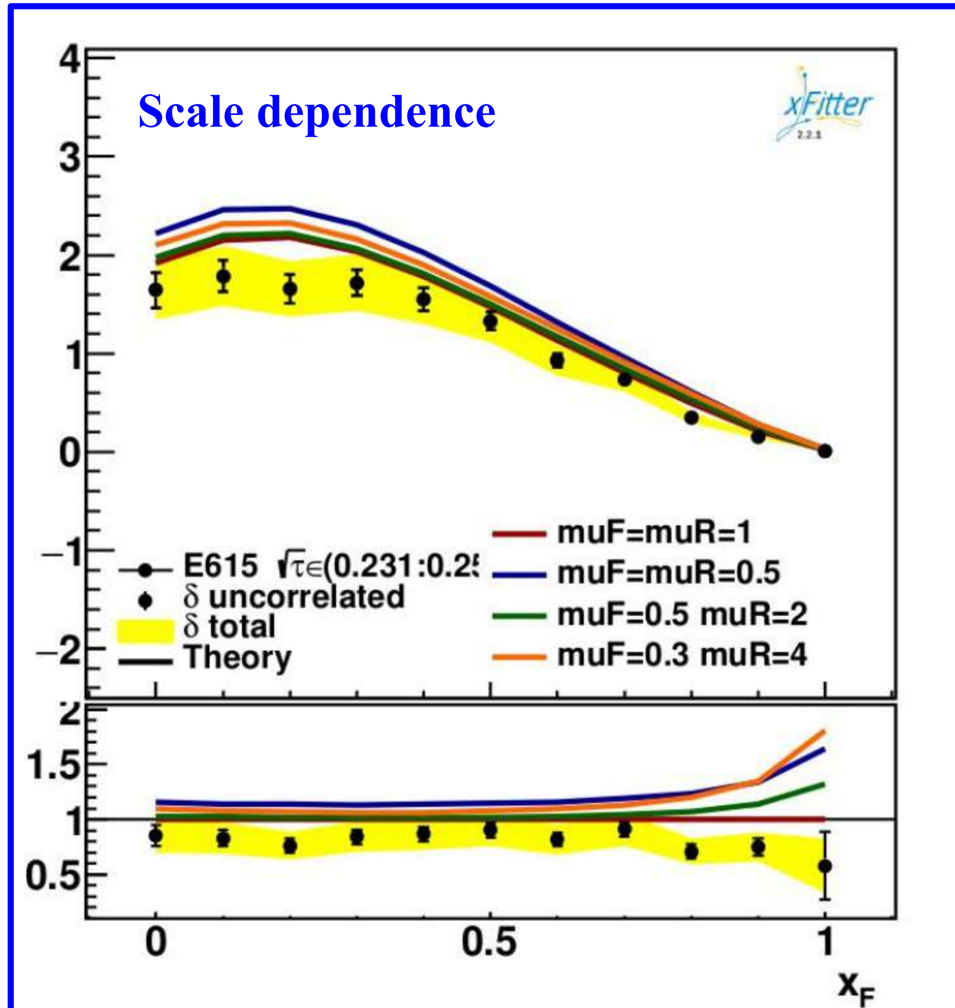
Python Jupyter

Mathematica: ManeParse

xFitter Pion PDFs

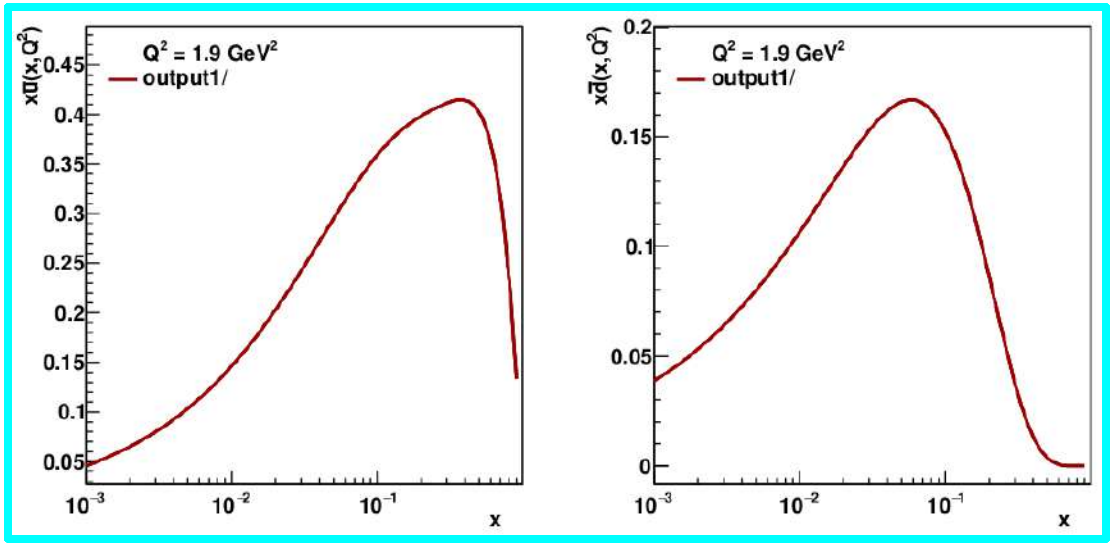






xFitter Tools:

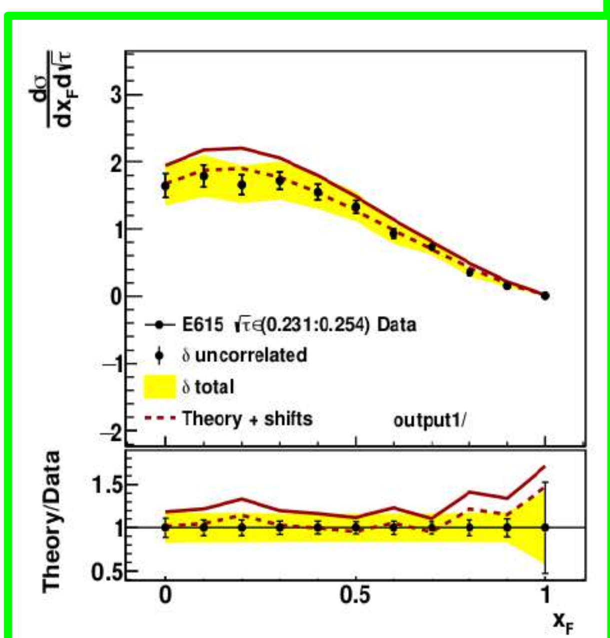
xfitter-draw
xfitter-process

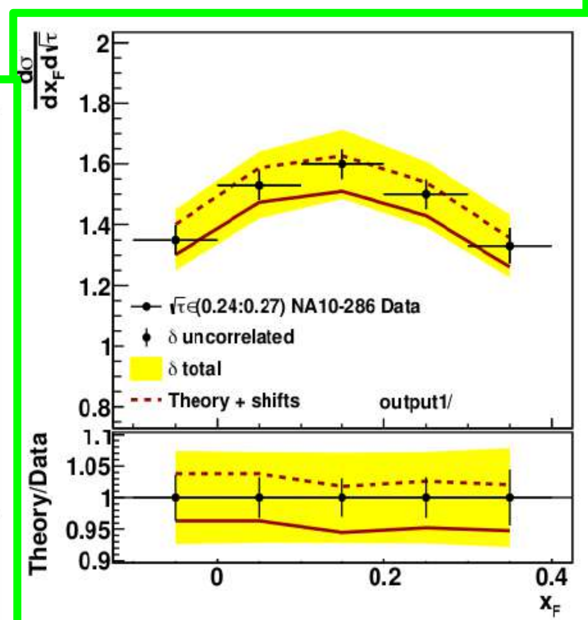


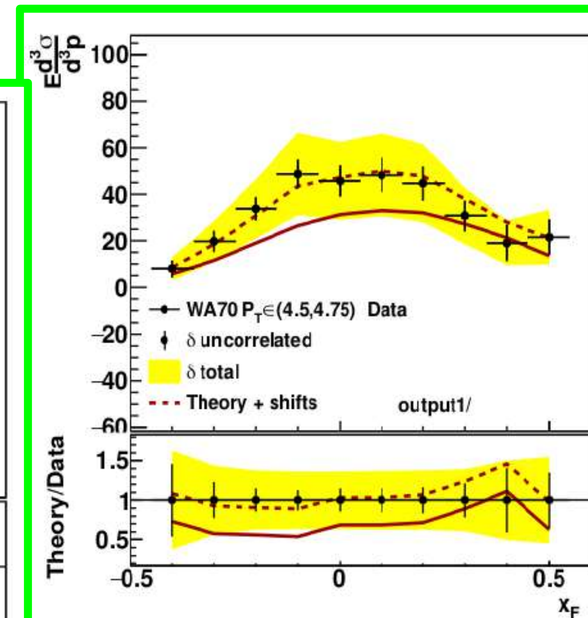
PDF Plots

Chi2 Tables

Dataset	output1
E615-0	13 / 7
E615-1	9.5 / 10
E615-2	13 / 11
E615-3	15 / 12
E615-4	8.9 / 11
E615-5	9.7 / 11
E615-6	12 / 11
E615-7	16 / 11
E615-8	16 / 11
E615-9	18 / 10
E615-13	32 / 8
E615-14	18 / 7
E615-15	3.5 / 8
E615-16	8.0 / 6
E615-17	16 / 6
NA10-194-0	8.8 / 6
NA10-194-1	6.5 / 6
NA10-194-2	8.3 / 7
NA10-194-3	5.1 / 7
NA10-194-4	2.7 / 8
NA10-194-5	6.0 / 8
NA10-194-6	21 / 9
NA10-194-7	40 / 8
NA10-194-8	9.8 / 8
NA10-286-0	10 / 4
NA10-286-1	1.7 / 5
NA10-286-2	3.6 / 5
NA10-286-3	1.5 / 6
NA10-286-4	4.7 / 7
NA10-286-5	15 / 8
NA10-286-6	10 / 8
NA10-286-7	5.9 / 9
NA10-286-8	20 / 8
NA10-286-9	13 / 9
NA10-286-10	13 / 4
WA70plus-0	1.1 / 5
WA70plus-1	4.9 / 5
WA70plus-2	2.3 / 5
WA70plus-3	4.5 / 5
WA70plus-4	1.7 / 5
WA70plus-5	2.2 / 3
WA70plus-6	3.5 / 3
WA70-0	3.8 / 10
WA70-1	5.4 / 10
WA70-2	4.0 / 10
WA70-3	8.8 / 10
WA70-4	8.9 / 10
WA70-5	9.5 / 10
WA70-6	3.0 / 8
Correlated χ^2	3.5
Log penalty χ^2	-36.91
Total χ^2 / dof	445 / 373
χ^2 p-value	0.01







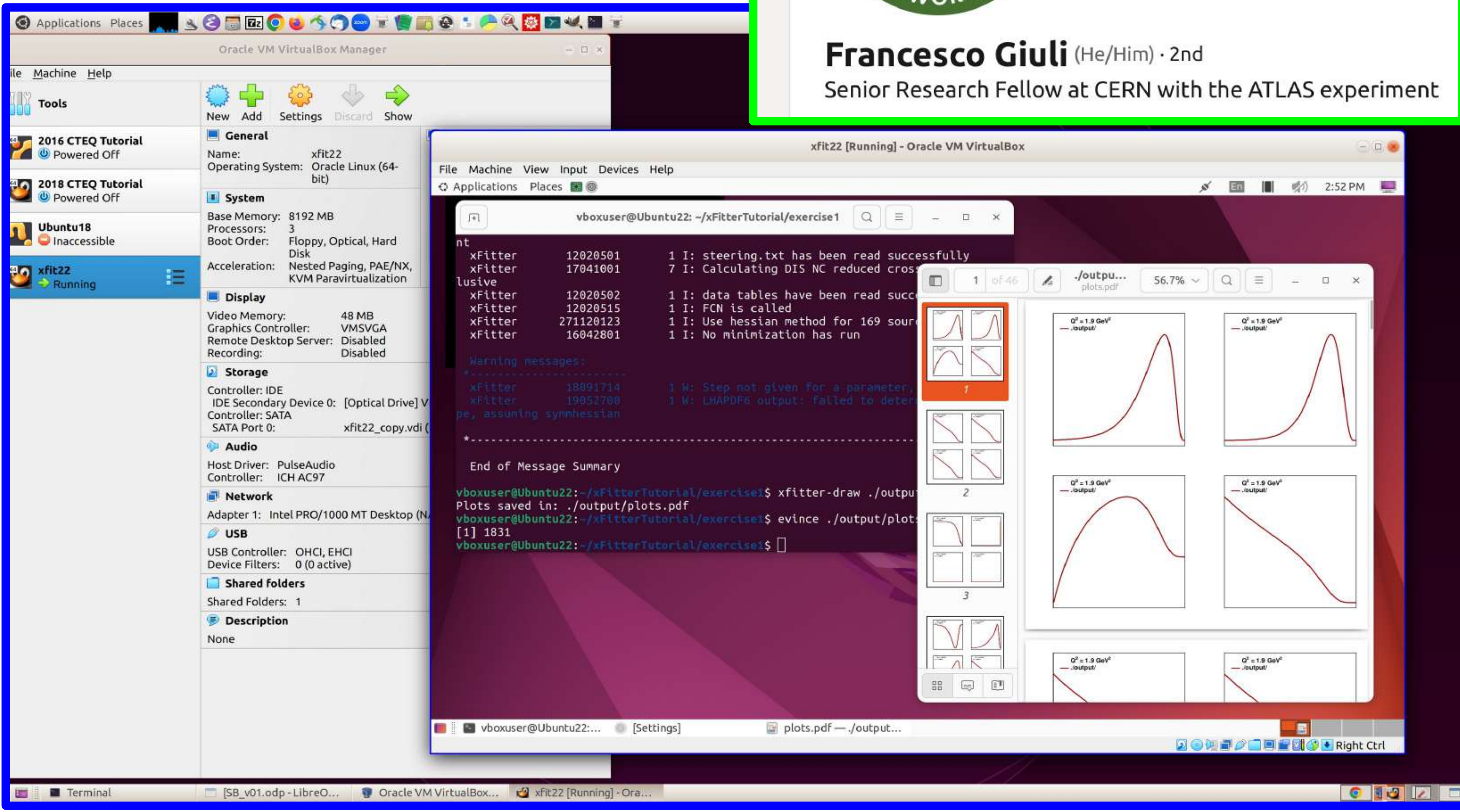
Data vs. Theory

The Goal

Demonstrate xFitter for Meson PDFs
so that others can use this code

xFitter

VirtualBox



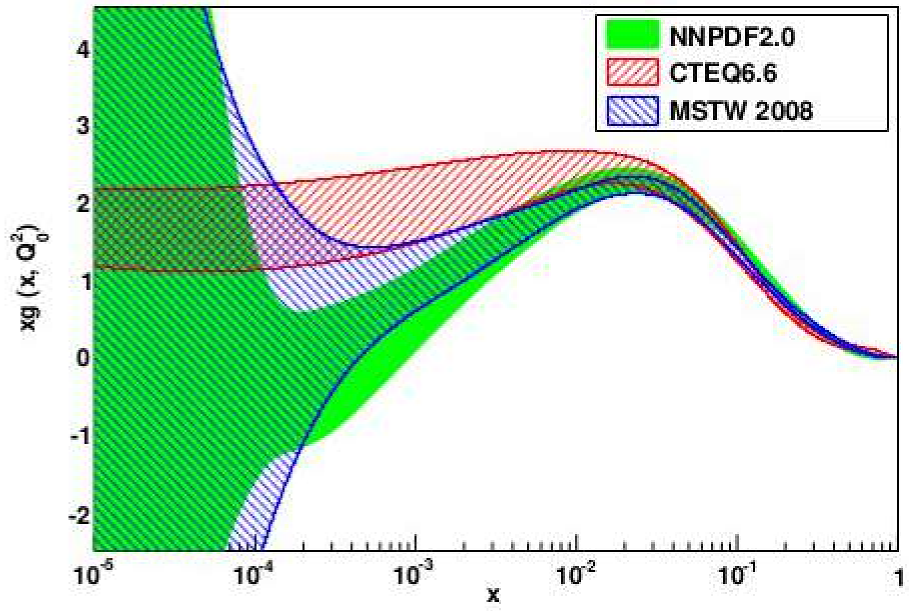
https://smu.box.com/s/78k1jr8l1ahrtd1h1t3khz05l1amh2g7

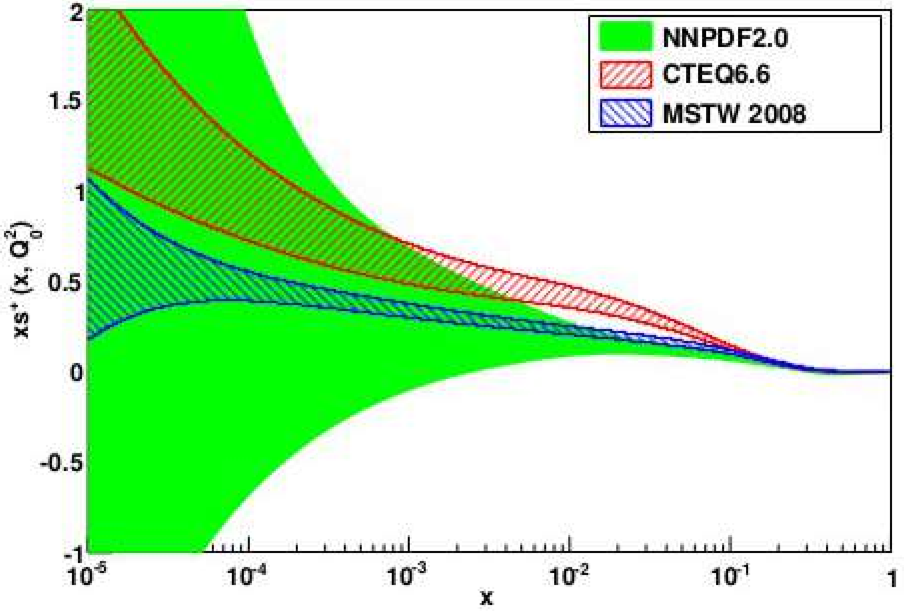
Others did use this code: … Fantômas

Motivation: Standard polynomial forms can be restrictive. Desire more flexibility

$$x f_i(x, Q_0^2) = A_i x^{B_i} (1-x)^{C_i} \times Poly(x)$$

Example: Strange PDF at small x





NNPDF: Nucl.Phys.B838:136-206,2010

Aurore Courtoy

Fantômas4QCD

Main idea: to quantify the rôle of parametrization form in global analyses.

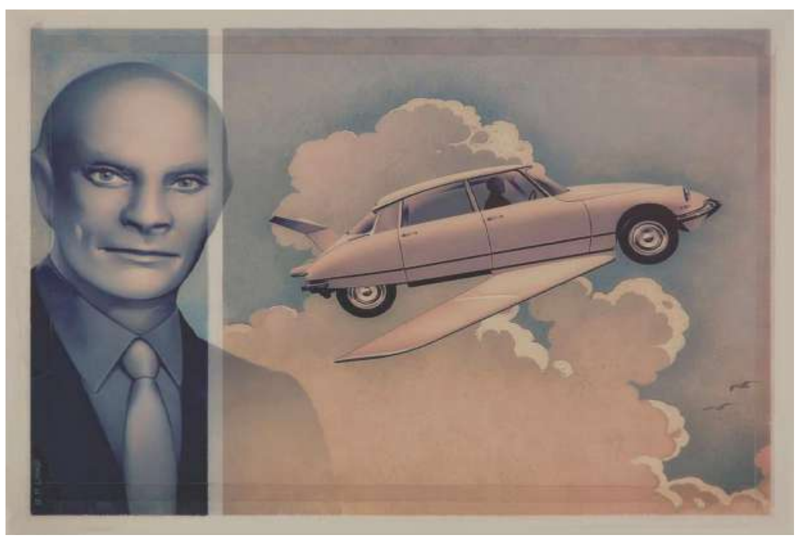
Fantômas4QCD: Our new c++ code, Fantômas, automates series of fits using multiple functional forms.

Just like neural networks, these polynomial functional forms can approximate any arbitrary PDF shape.

This code facilitates unbiased estimates of parametrization dependence.

CONACYT
Consejo Nacional de Ciencia y Tecnología

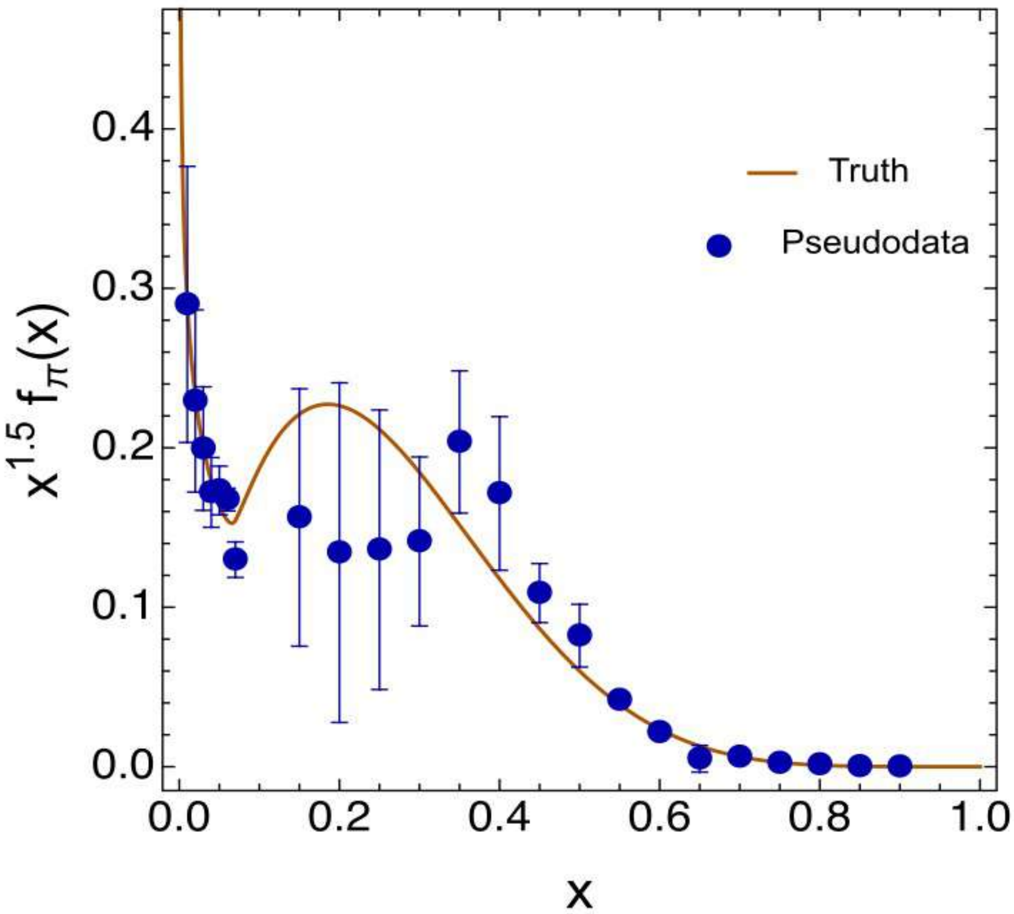
dgapa
Dirección General de Asuntos del Personal Académico

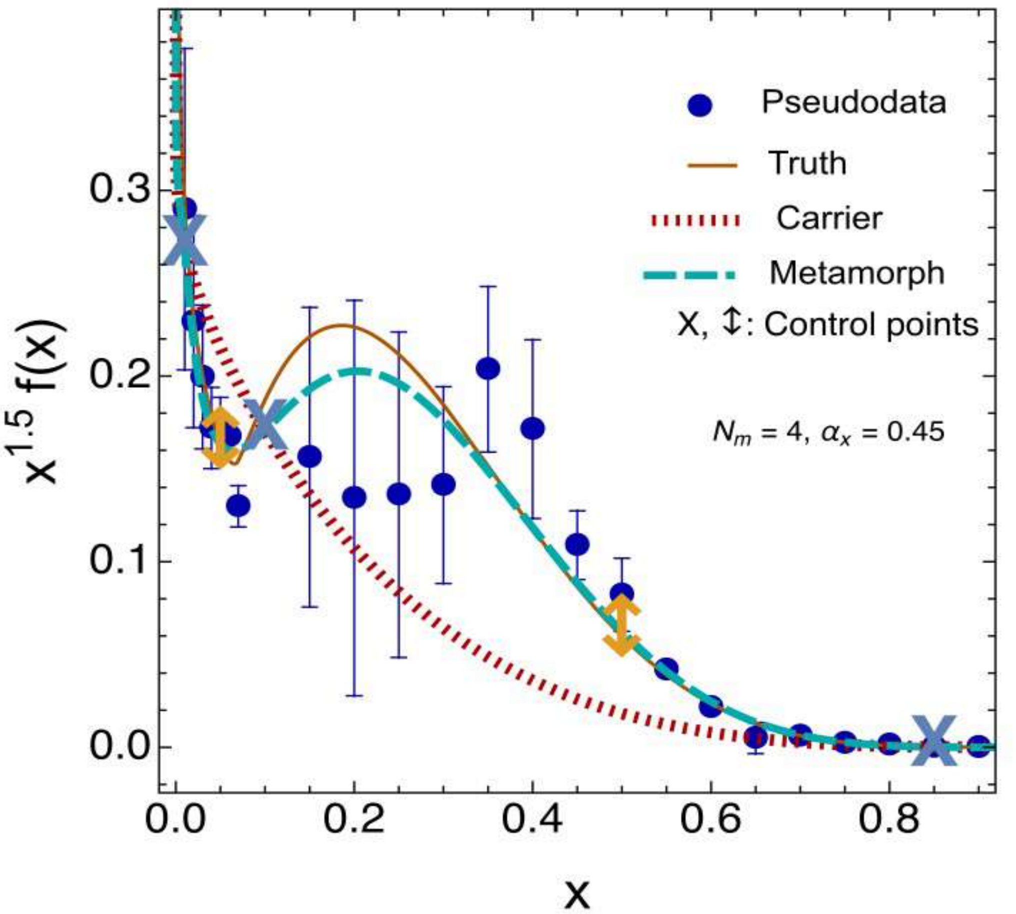


PHYSICAL REVIEW D **109,** 074027 (2024)

Analysis of parton distributions in a pion with Bézier parametrizations

Fantômas: A more flexible parameterization





$$xf_i(x, Q_0^2) = A_i x^{B_i}(1-x)^{C_i}[1 + \mathcal{B}^{(N_m)}(y(x))]$$

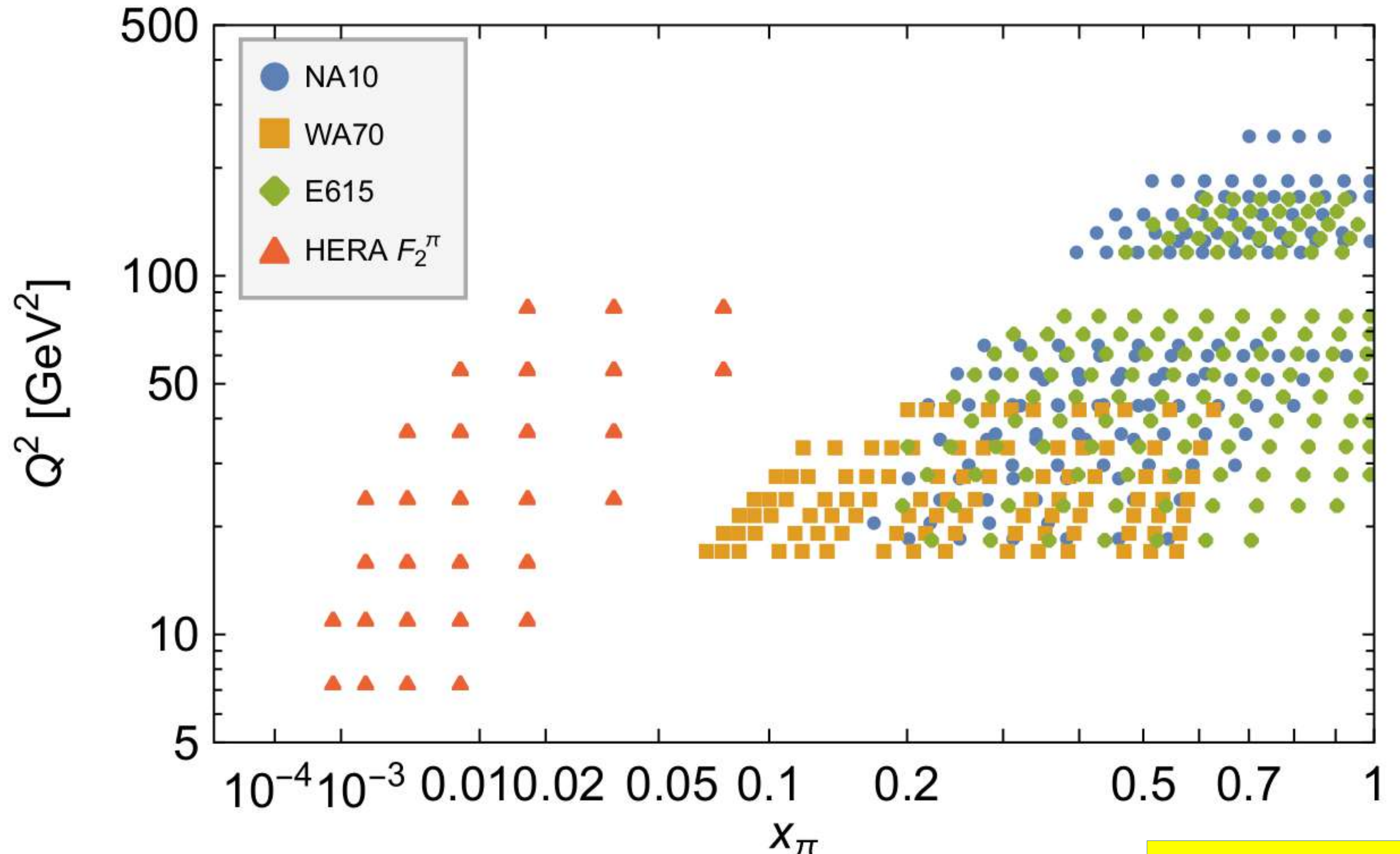
Parametric Form

Carrier: $x^{B_i}(1-x)^{C_i}$

Bézier Curve

$$B_{N_m,l}(y) \equiv \binom{N_m}{l} y^l (1-y)^{N_m - l}$$

Data Sets: Start from xFitter Analysis and add H1 LN Set



H1 Leading Neutron = (pion flux factor) × (pion DIS SF)

$$F_2^{LN(3)}(Q^2, x, x_L = 0.73) = f_{\pi N}(x_L = 0.73) F_2^\pi(x_\pi, Q^2)$$

E615 & NA10

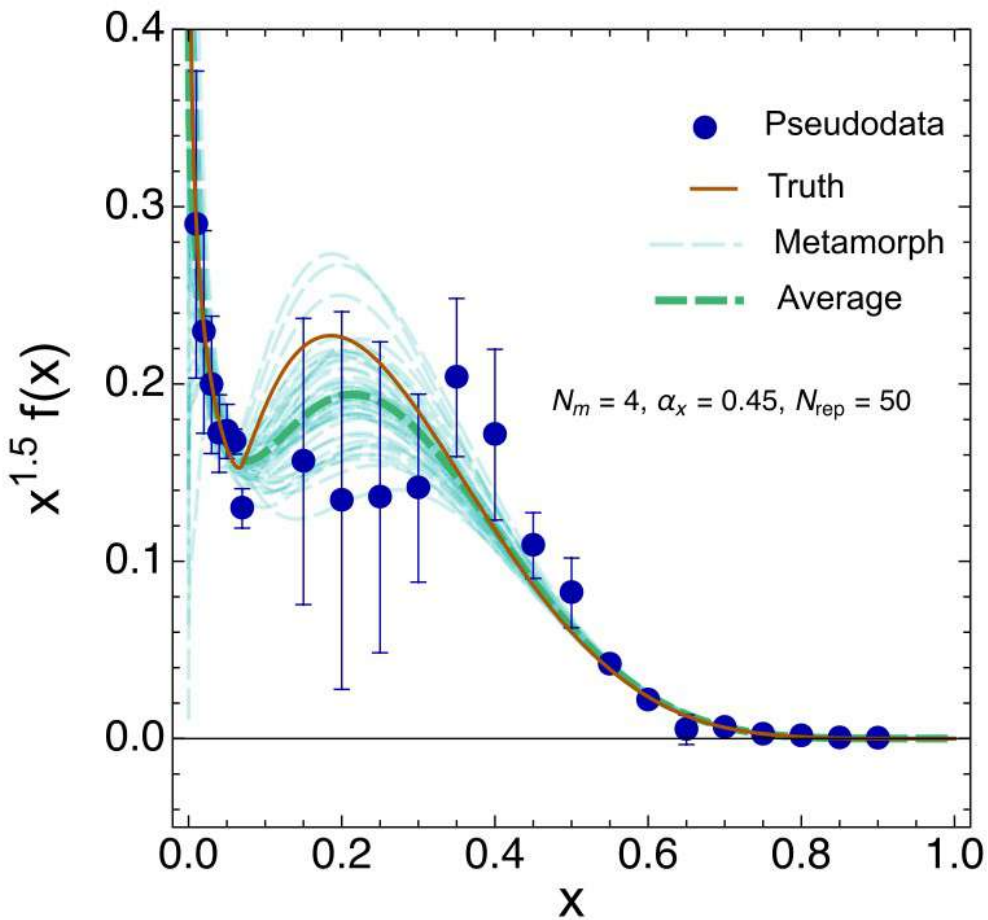
Pions on Tungsten

WA70

Pions on Proton

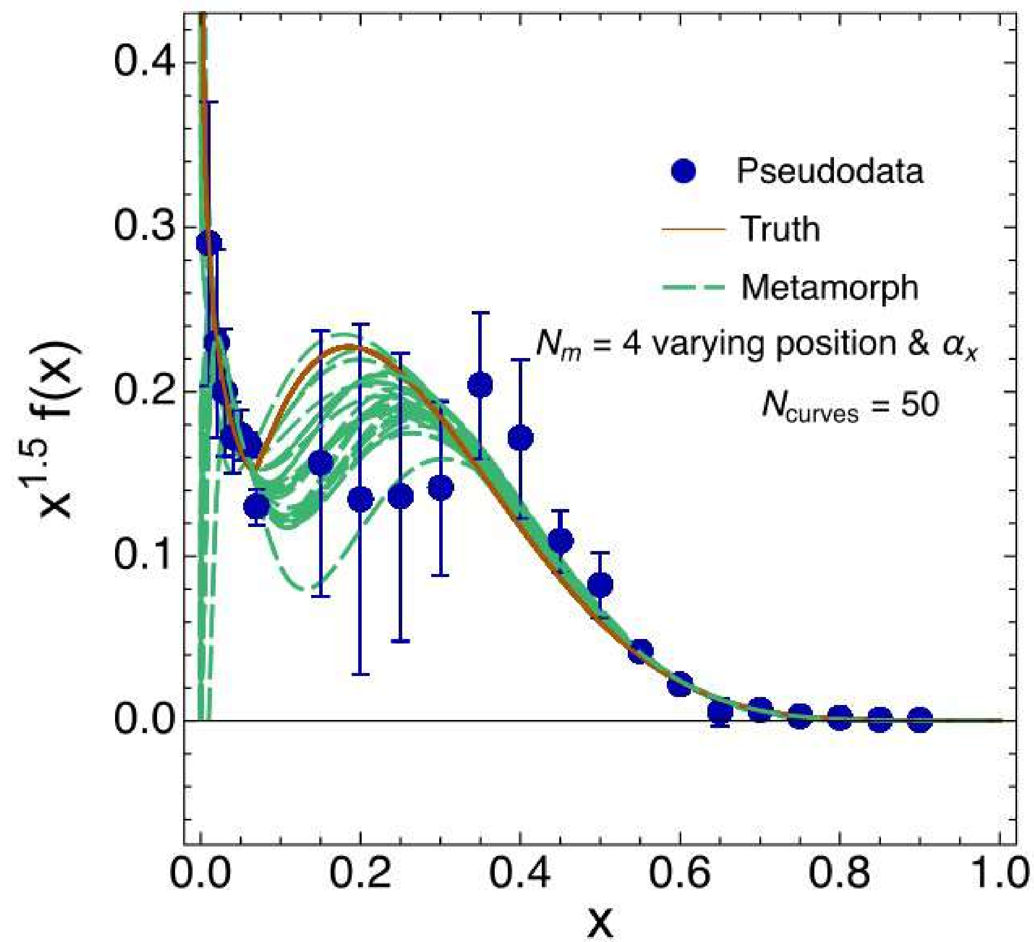
PDF Uncertainties: Variation of: 1) Data Values & 2) Theory Values

Experimental Uncertainty



Bootstrap; resampling or importance sampling, it involves generating N rep = 50 replicas fluctuating the **central data** values according to their respective standard deviations.

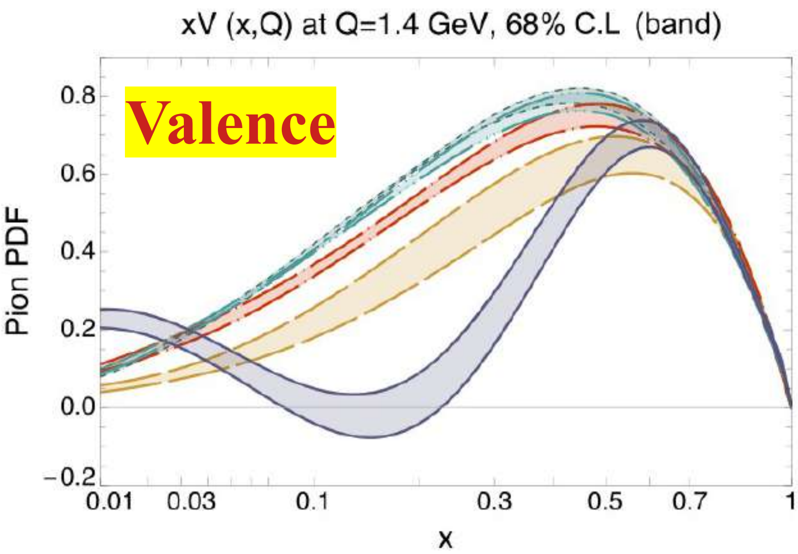
Theoretical Uncertainty

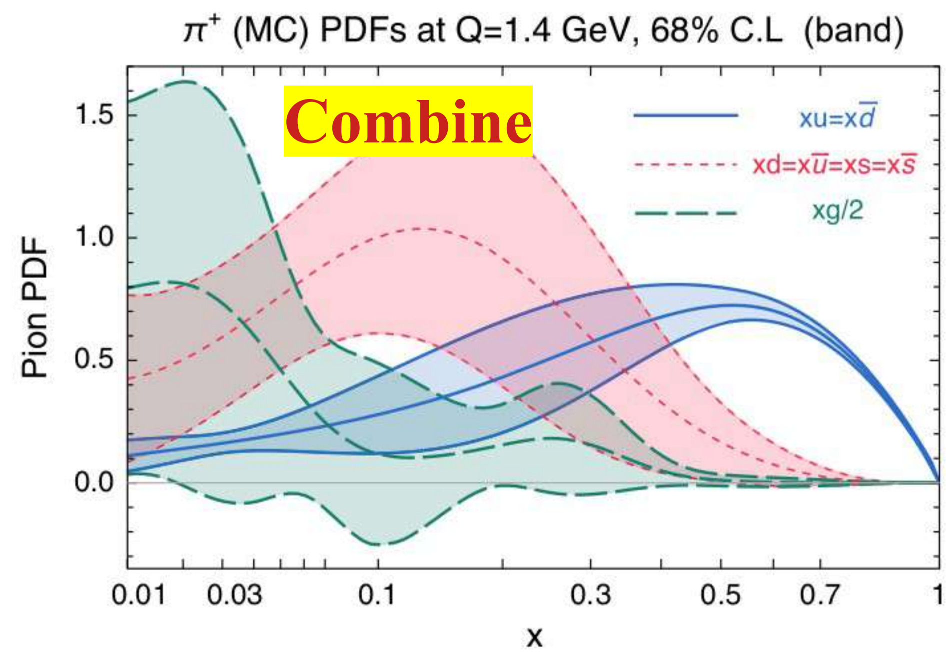


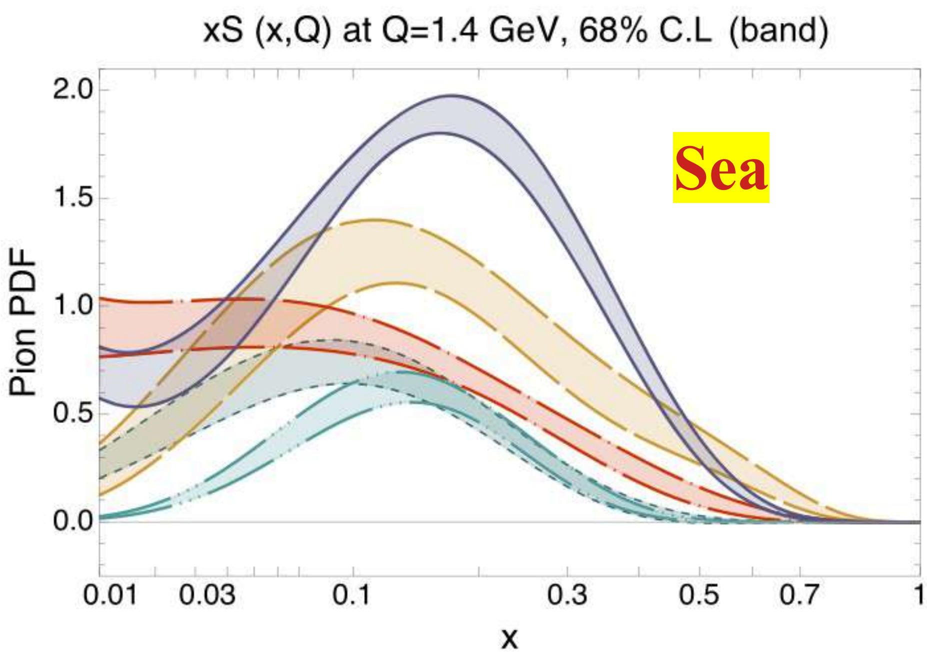
Parameter-space sampling: uncertainties for replicas sampled over the **space of models.**

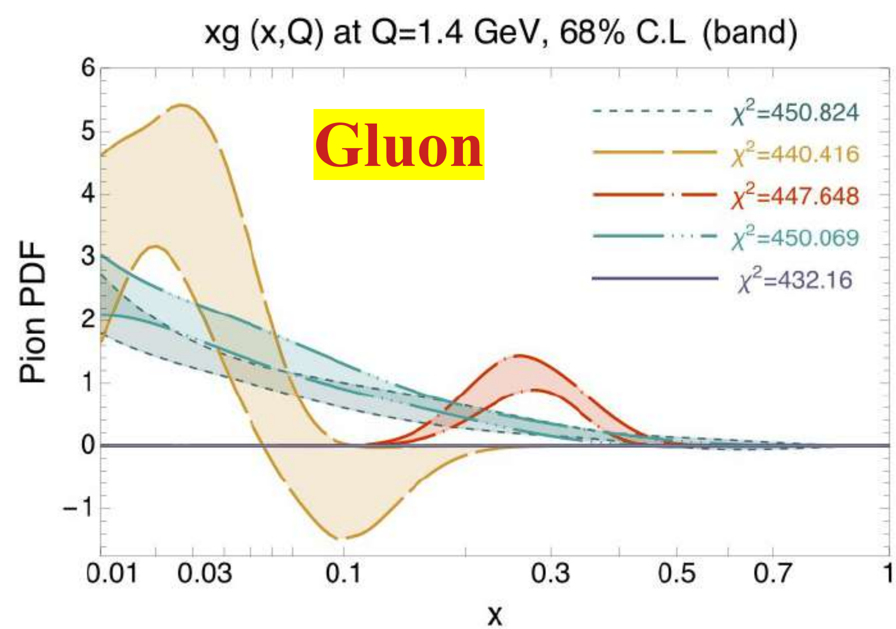
The total uncertainty can be estimated by combining the curves from both sources, which can be done using the METAPDF

Explore wide variety of solutions …

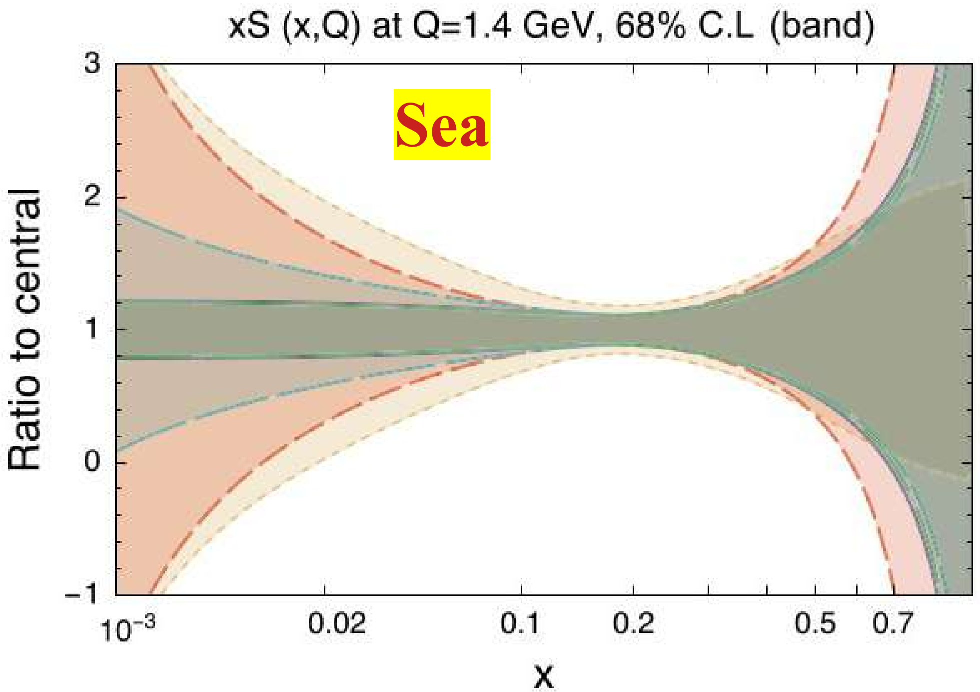


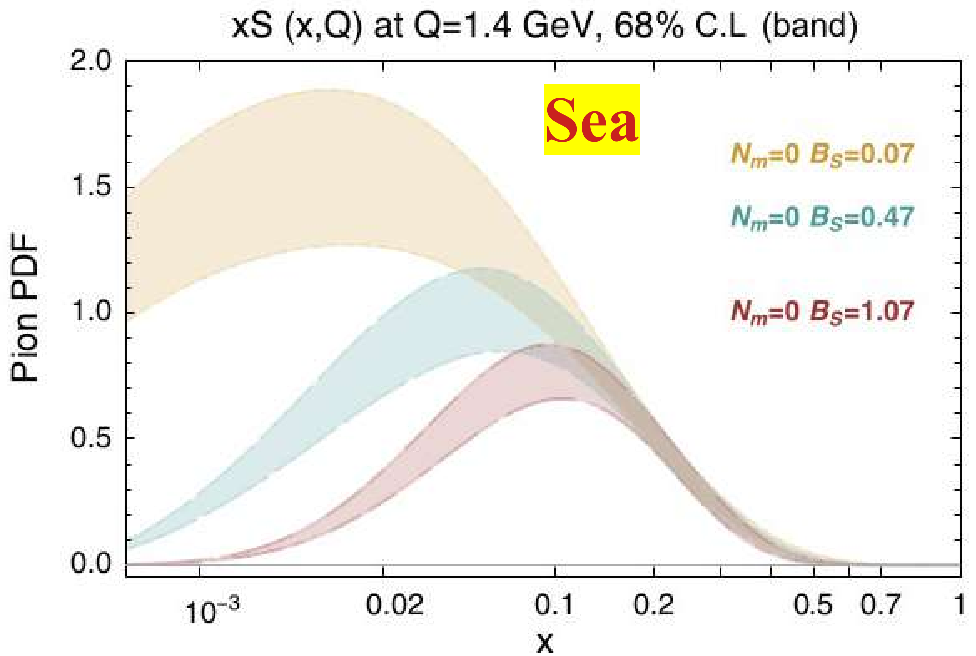


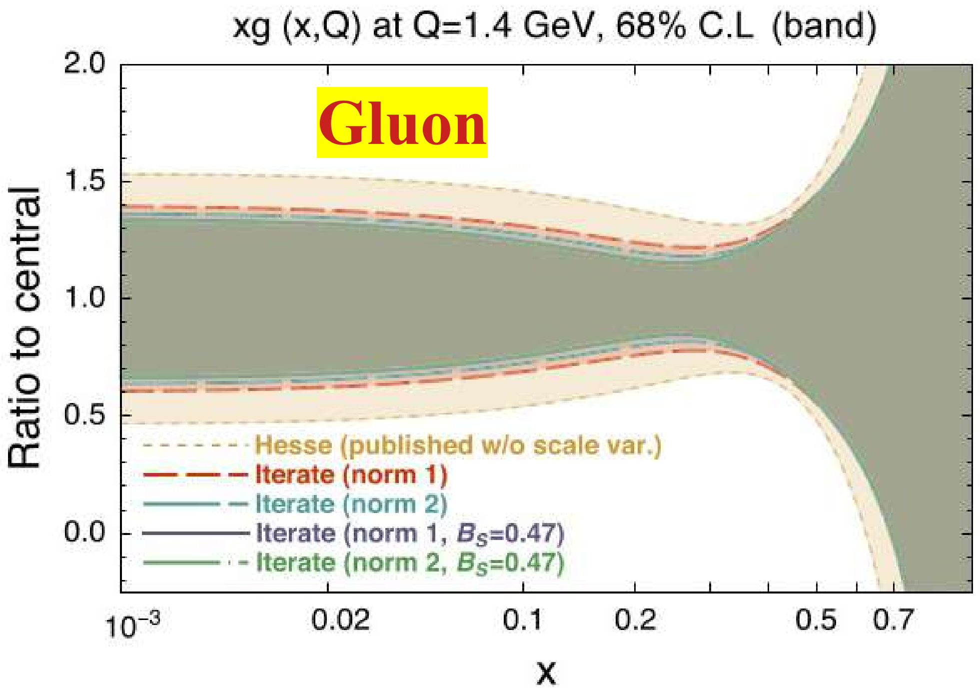


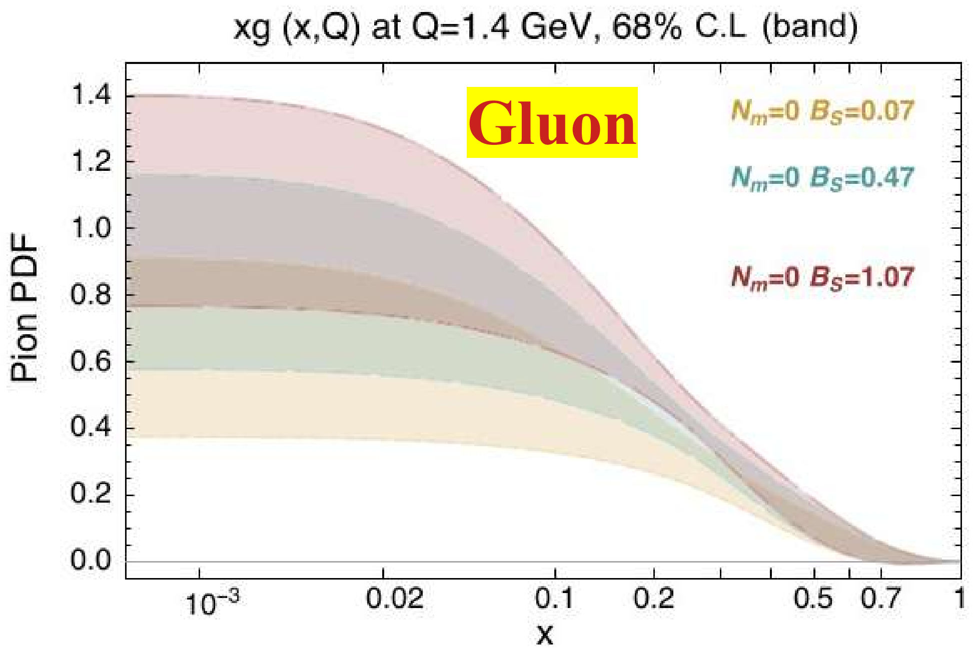


Interplay between Gluon and Sea

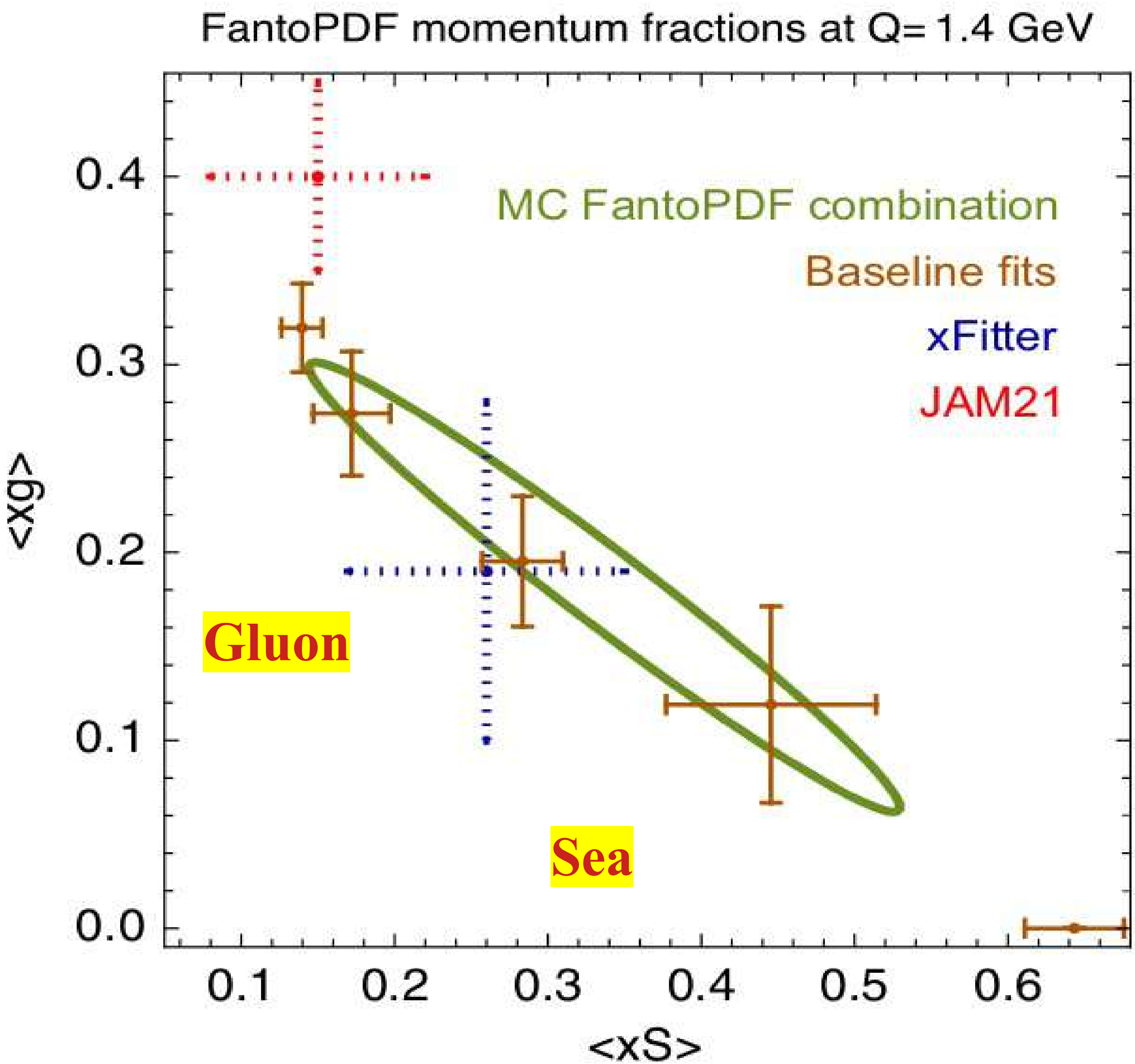






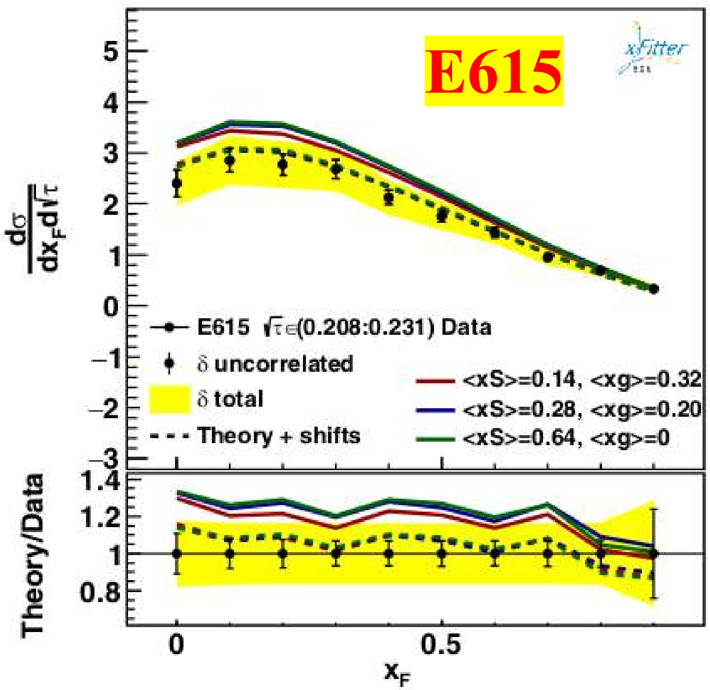


Interplay between Gluon and Sea

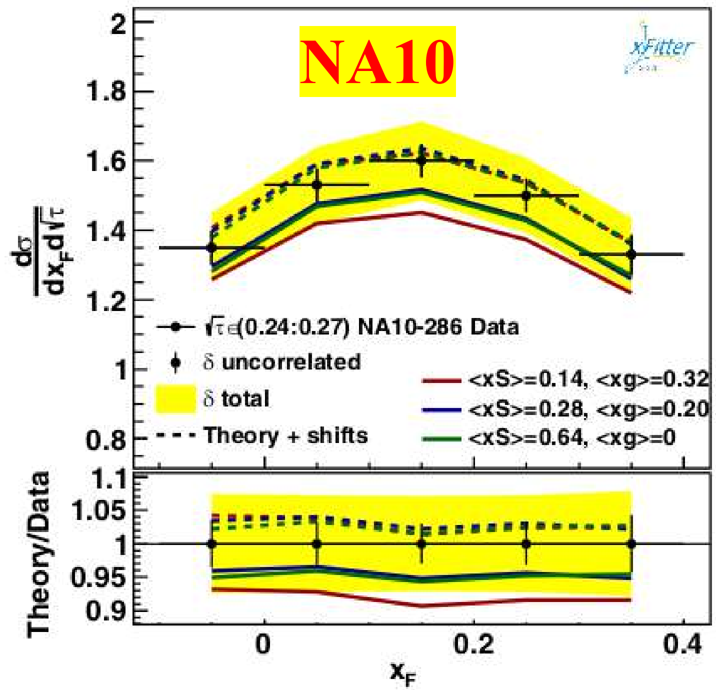


χ^2(d.o.f. = 379 − 5)	$\langle xV \rangle$	$\langle xS \rangle$	$\langle xg \rangle$
445.70	0.556	0.268	0.177
445.38	0.557	0.239	0.204
445.29	0.558	0.217	0.225
445.36	0.559	0.199	0.243
445.52	0.559	0.184	0.257
445.76	0.559	0.172	0.269

Interplay between Gluon and Sea

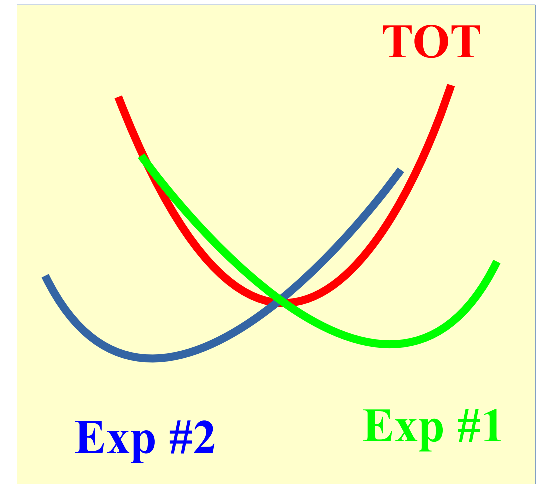


(a)

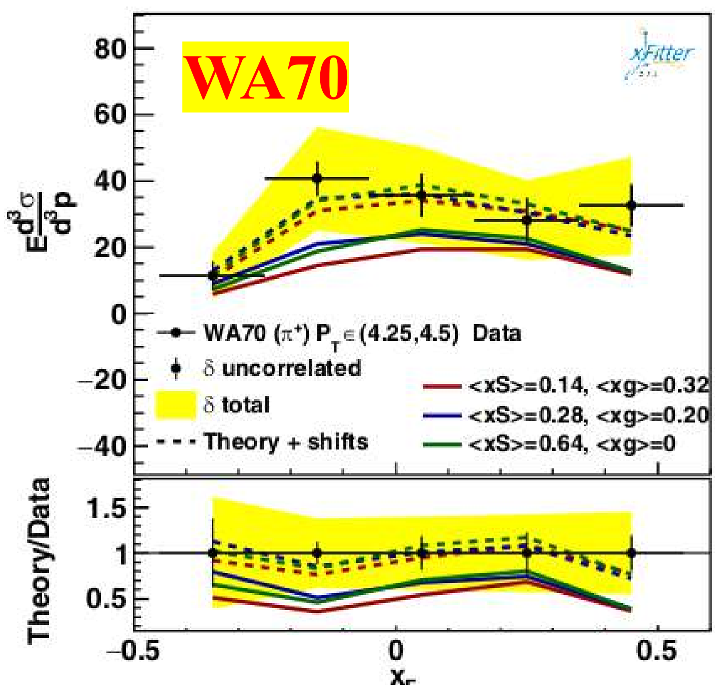


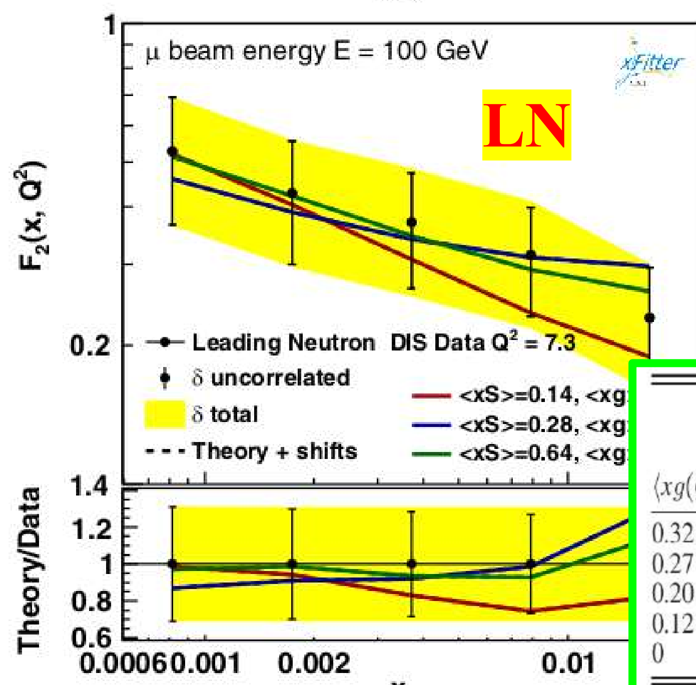
(b)

Balancing separate data sets



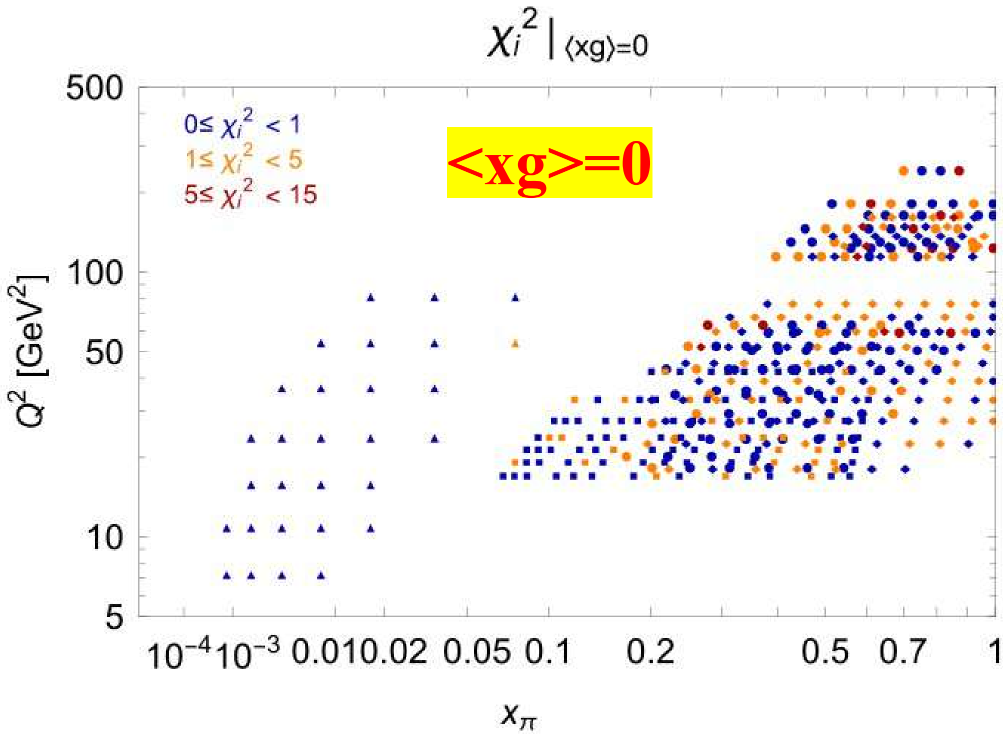
Strong constraints from the pion-induced Drell-Yan pair production on a tungsten target.

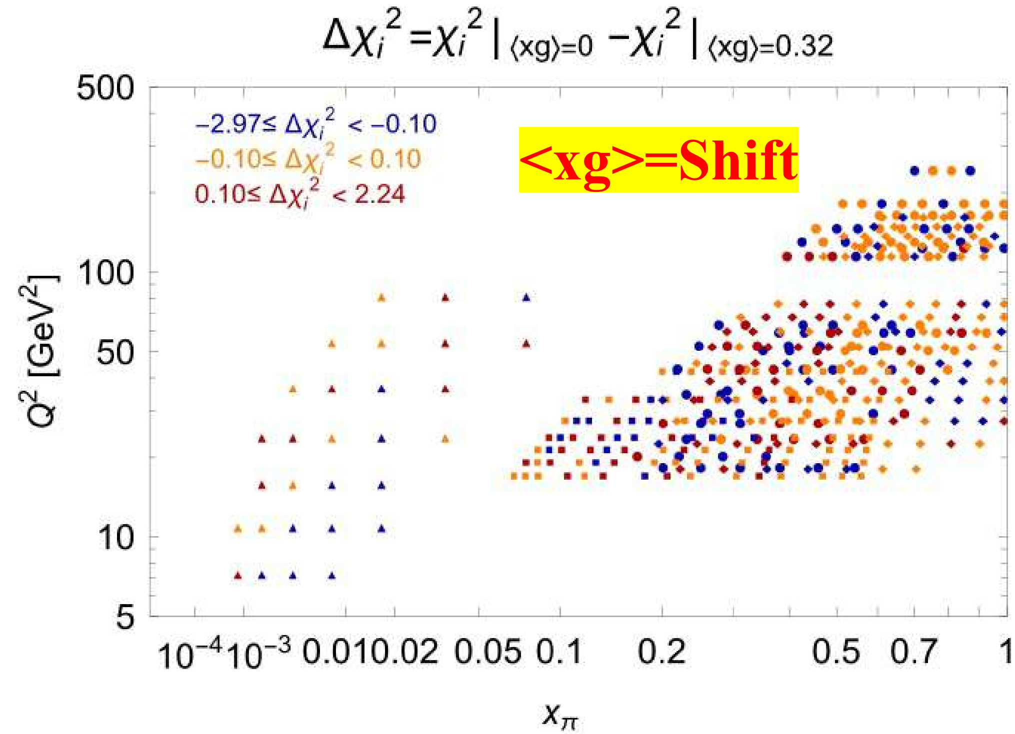


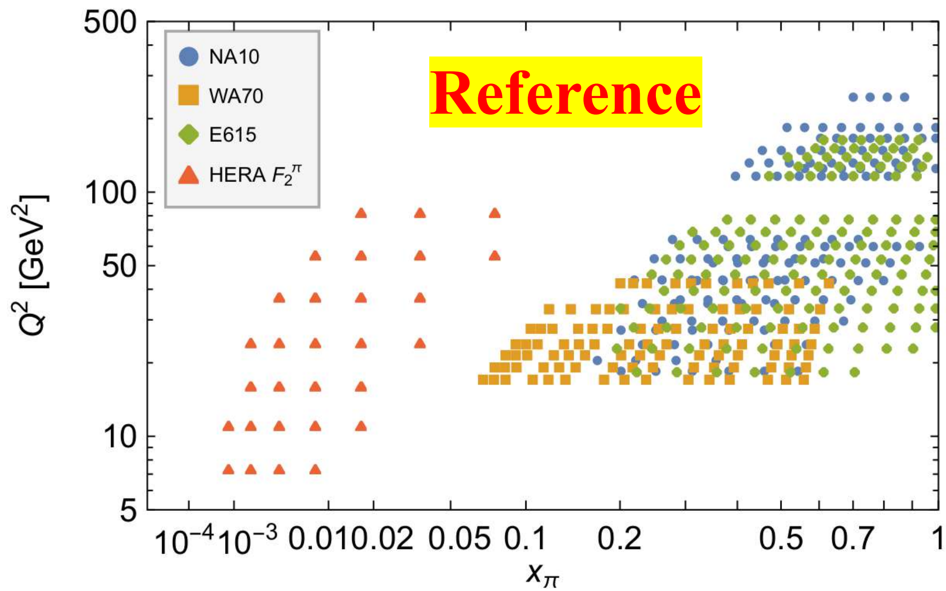


		E615	NA10 -194	NA10 -286	WA70	HERA F_2^π	$\chi^2_{\rm tot}$
$\langle xg(Q_0)\rangle$	$\langle xS(Q_0)\rangle$	(140)	(67)	(73)	(99)	(29)	(408)
0.32	0.14	204.7	107.7	101.5	25.6	4.7	450.1
0.27	0.17	205.5	106.4	101.5	28.8	4.3	450.8
0.20	0.28	207.5	100.7	102.5	27.0	7.1	447.6
0.12	0.45	208.4	99.6	94.9	31.6	2.1	440.4
0	0.64	204.6	95.8	93.6	29.6	4.94	432.2

Interplay between Gluon and Sea

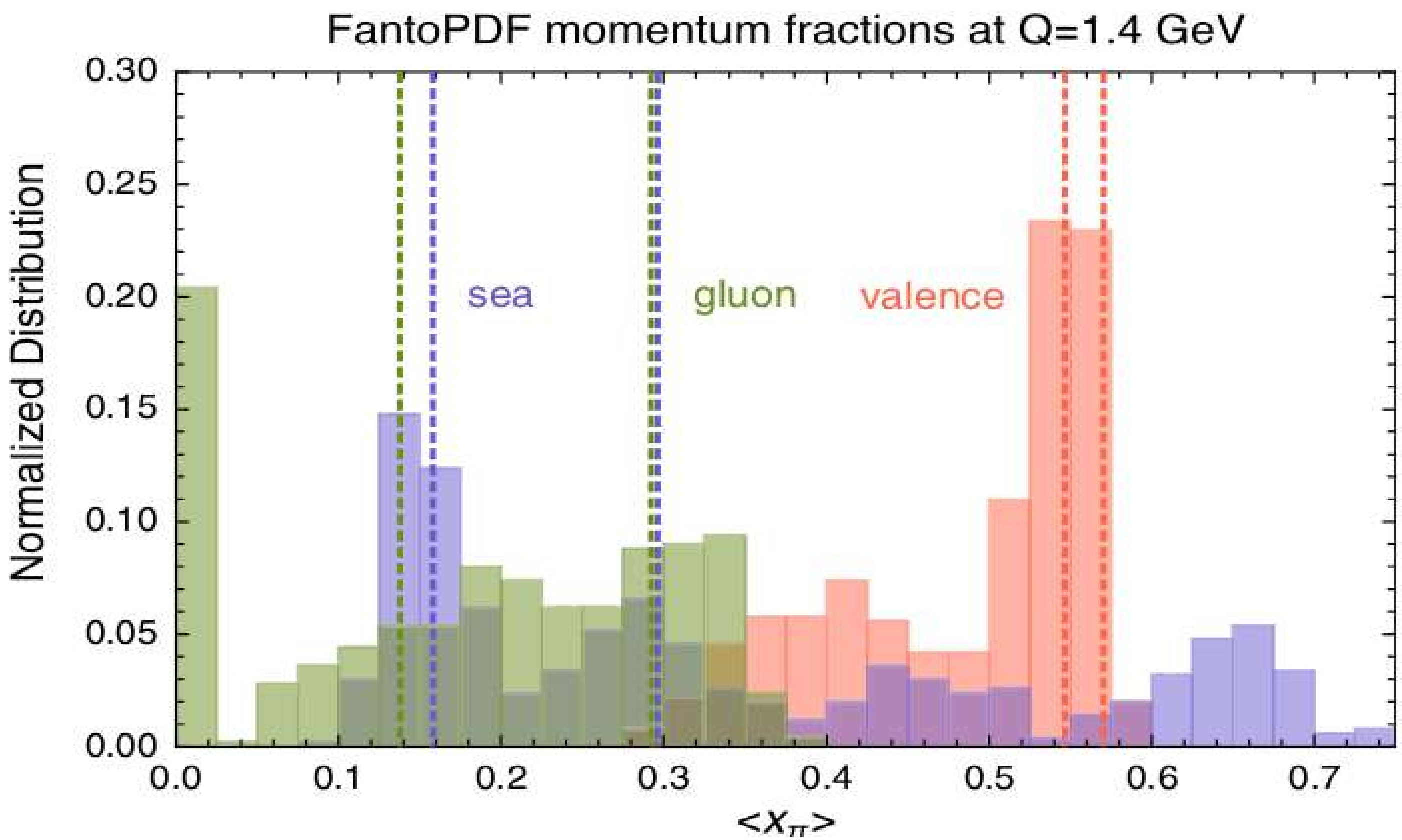






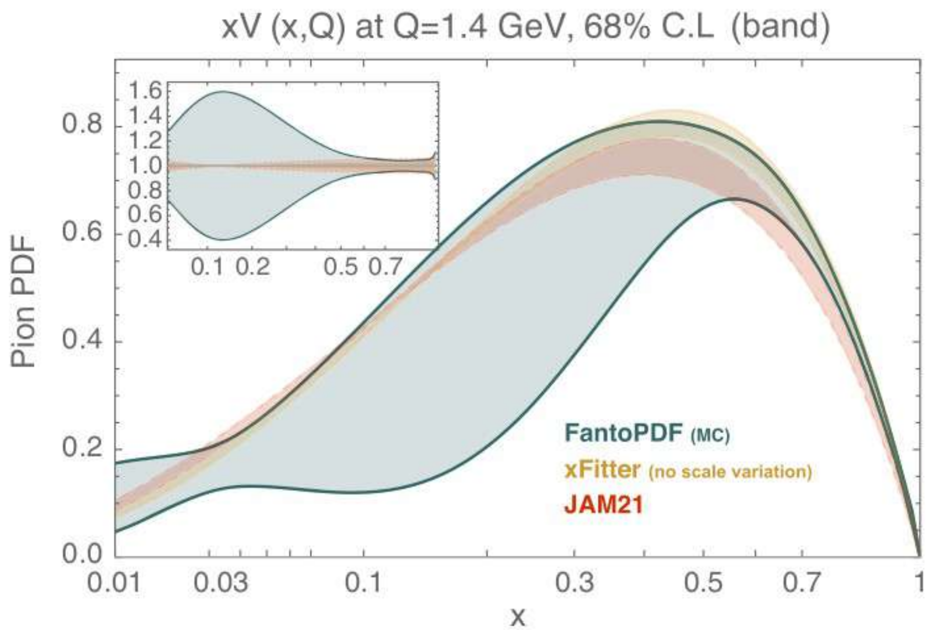
Considering multiple functional forms. The most constraining set of the experimental data, coming from the pion-induced Drell-Yan pair production on a tungsten target, is characterized by large momentum fractions for the pion beam.

Pion Momentum Fractions



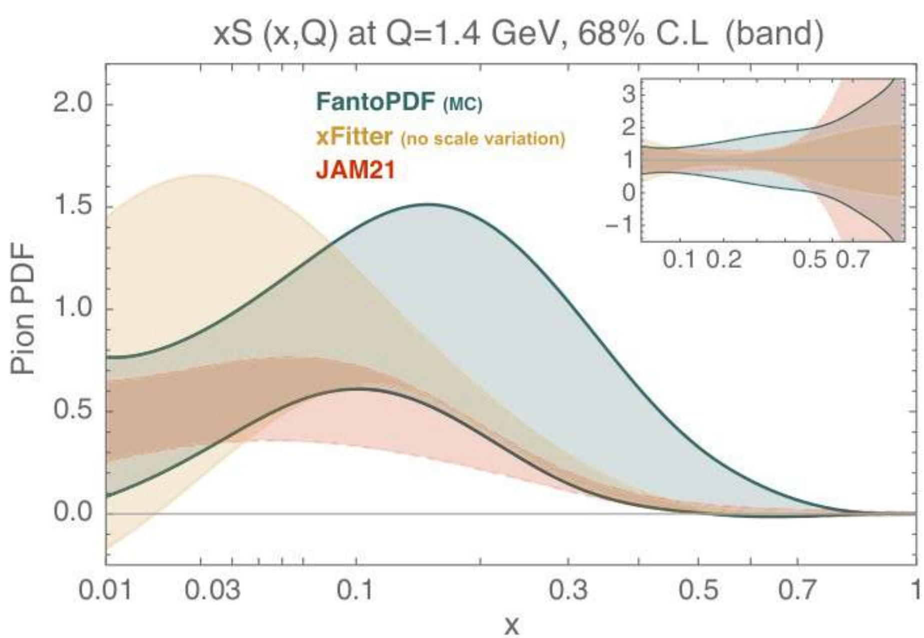
Name	Q (GeV)	$\langle xV \rangle$	$\langle xS \rangle$	$\langle xg \rangle$
FantoPDF (DY + γ + LN)	$\sqrt{1.9}$	0.49(8)	0.34(19)	0.18(12)
xFitter [9] (DY + γ)	$\sqrt{1.9}$	0.55(6)	0.26(15)	0.19(16)
xFitter w/o scale variation	$\sqrt{1.9}$	0.55(2)	0.26(9)	0.19(9)
JAM'18 [8] (DY)	1.27	0.60(1)	0.30(5)	0.10(5)
JAM'18 [8] (DY + LN)	1.27	0.54(1)	0.16(2)	0.30(2)
JAM'21 [11] (DY + LN)	1.27	0.53(2)	0.14(4)	0.34(6)
JAM'21 [11] (DY + LN) +NLL double Mellin	1.27	0.46(3)	0.15(7)	0.40(5)
CT18 NLO (proton)	$\sqrt{1.9}$	0.443(6)	0.160(10)	0.396(10)
CT18 NNLO (proton)	$\sqrt{1.9}$	0.451(5)	0.157(10)	0.390(10)

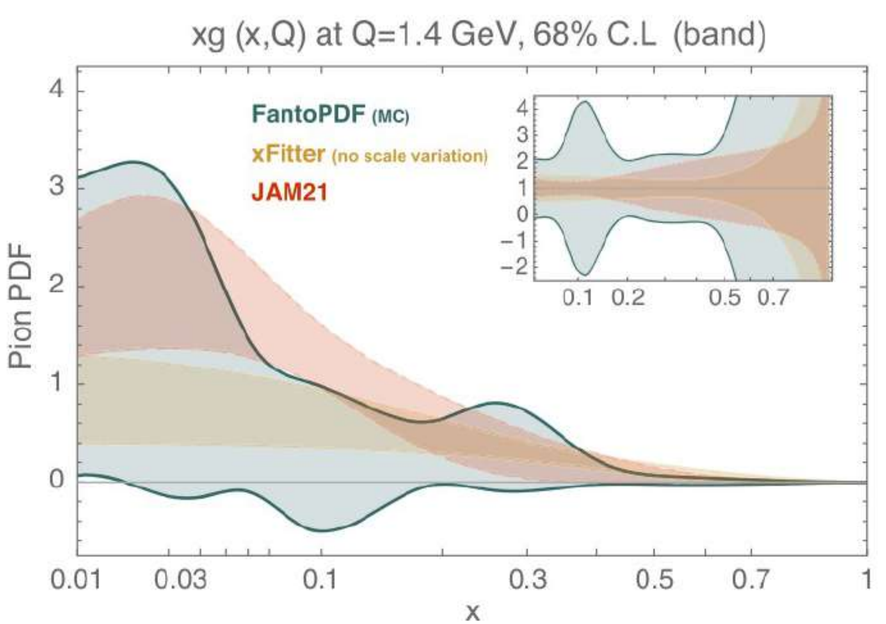
Comparison with other sets



- **General agreement w/ others sets**
- **Increased uncertainties**
- **Parm/theory uncertainties included**

Should we include other constraints?





Comparison to Lattice Results

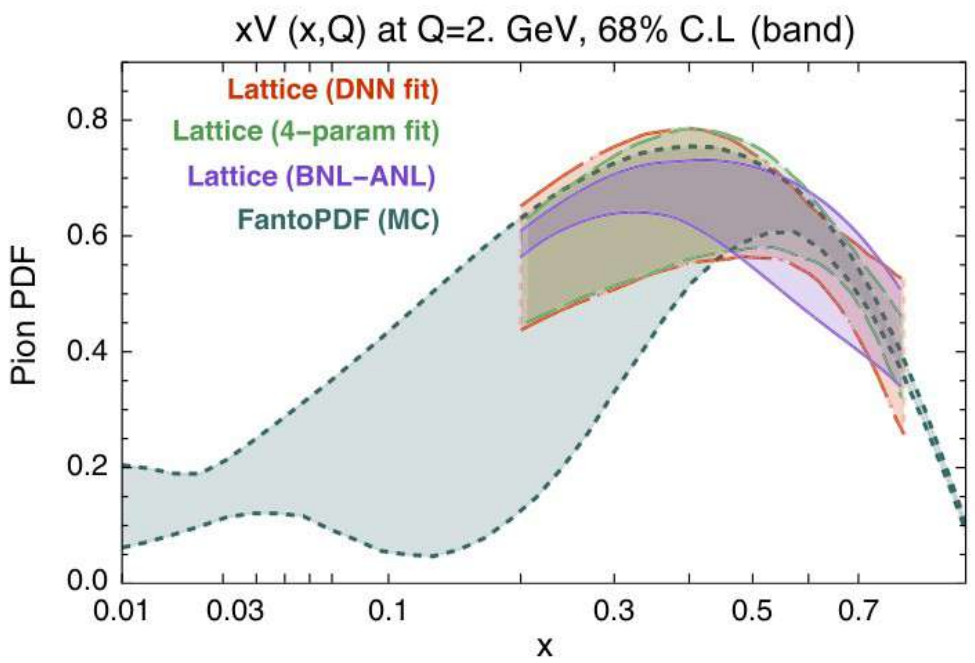


FIG. 15. Valence pion PDF $xV(x, 2\text{ GeV})$ for the FantoPDF in dark cyan, as well as for the lattice result of [25] in red [deep neural network (DNN) version] and green (four-parameter version), and BNL-ANL21 [98] in purple.

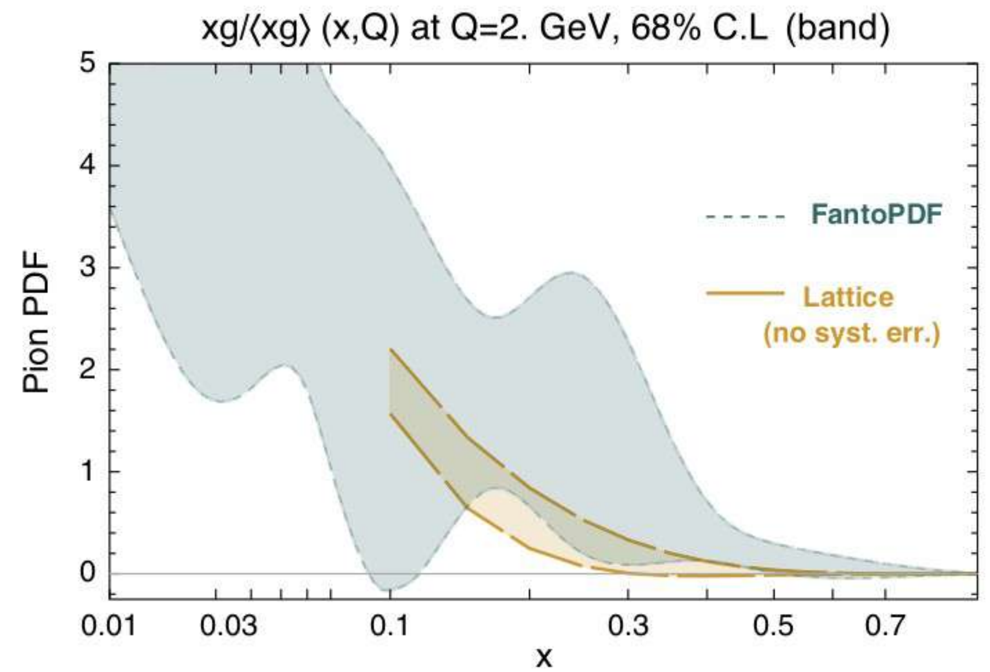


FIG. 16. Gluon pion PDF normalized to its momentum fraction $xg(x, 2\text{ GeV})/\langle xg(x, 2\text{ GeV})\rangle$ for the FantoPDF in dark cyan and for the lattice result of [23] in yellow mustard. Only statistical uncertainties are quoted for the lattice prediction.

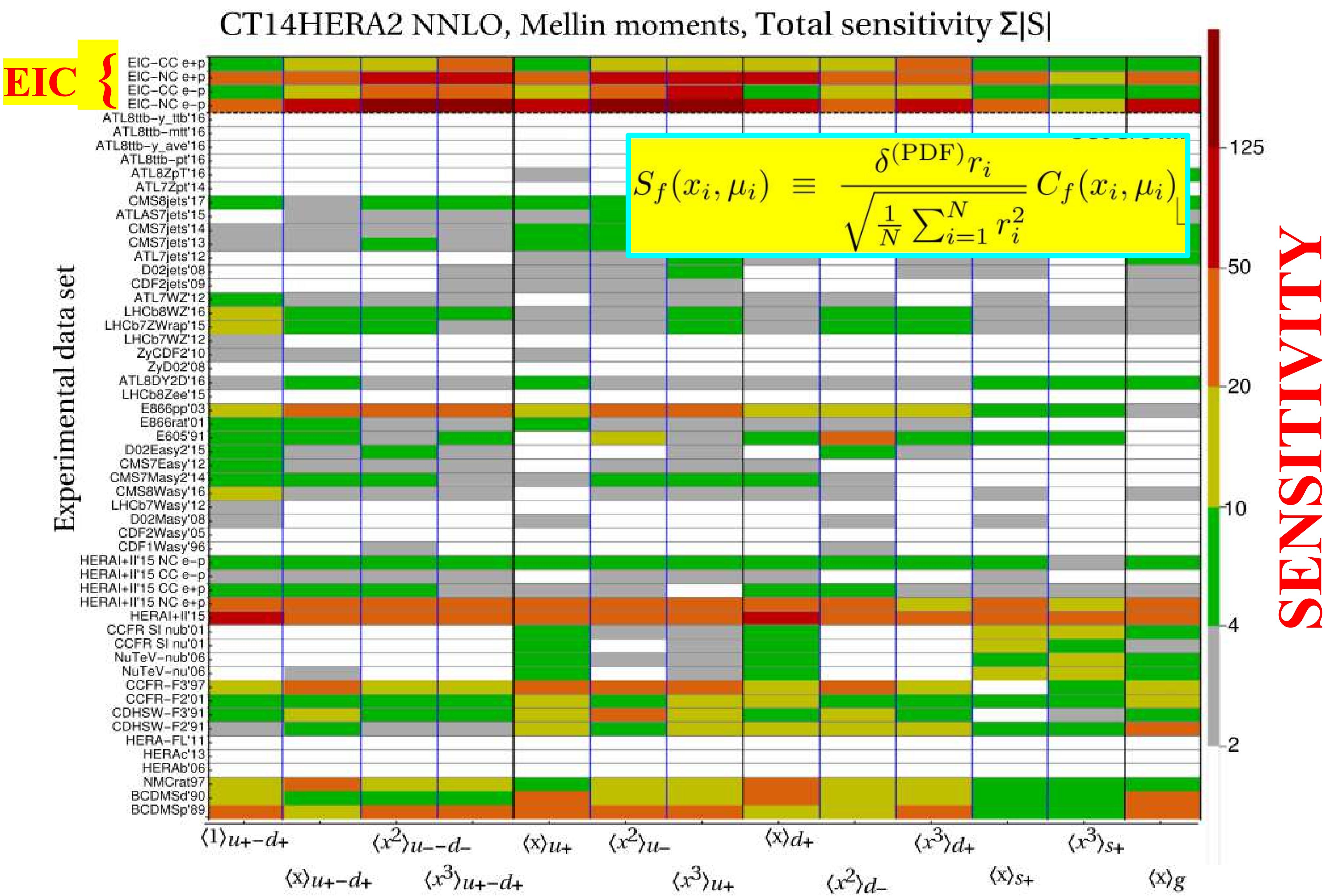
TABLE IV. Odd Mellin moments compared to the lattice evaluation of Ref. [25], at 2 GeV.

Name	$\langle x^2V/2\rangle$	$\langle x^4V/2\rangle$	$\langle x^6V/2\rangle$
FantoPDF	0.110(10)	0.040(2)	0.021(1)
Ref. [25]	0.1104(73)(48)	0.0388(46)(57)	0.0118(48)(48)

Lattice can help here

Future: impose moment constraints

NEW TOOLS: (Proton) Sensitivity to Moments



$$S_f(x_i, \mu_i) \equiv \frac{\delta^{(\mathrm{PDF})} r_i}{\sqrt{\frac{1}{N}\sum_{i=1}^{N} r_i^2}} C_f(x_i, \mu_i)$$

Hobbs, Wang, Nadolsky, Olness PRD 100, 094040 (2019)

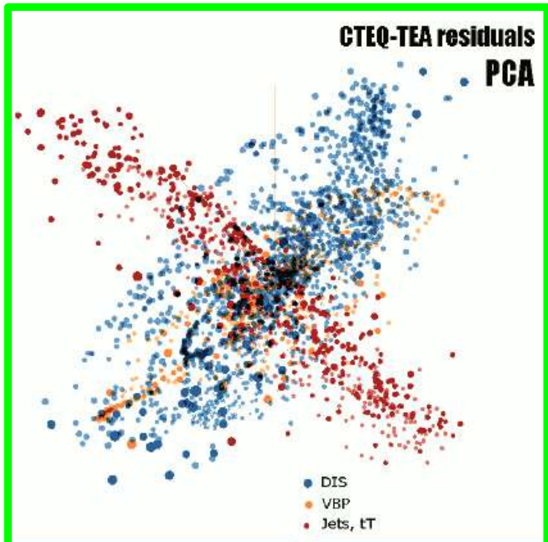
PDFSense:

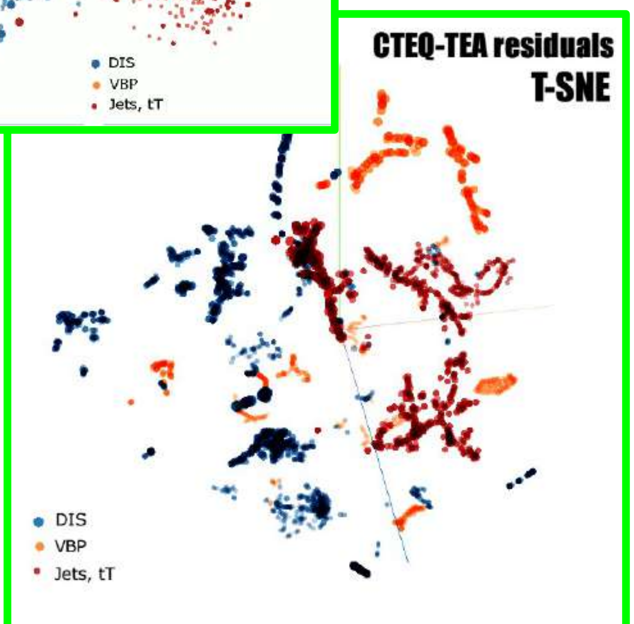
http://metapdf.hepforge.org/PDFSense/

Visualizing sensitivity of hadronic experiments to the nucleon structure

arXiv:1803.02777

Bo-Ting Wang, T.J. Hobbs, Sean Doyle,
Jun Gao, Tie-Jiun Hou,
Pavel Nadolsky, Fred Olness.





PDFSENSE project: sensitivities to PDFs by Expt measurements

On this webpage, we present the sensitivity of hadronic experiments to PDFs using **PDFsense [download here]**. The PDFsense enables users to compute the sensitivity and correlation of experimental data sets and CTEQ PDF sets.

Citation policy:

if you use results from this website, please cite

Visualizing the sensitivity of hadronic experiments to nucleon structure
Bo-Ting Wang, T. J. Hobbs, Sean Doyle, Jun Gao, Tie-Jiun Hou, Pavel Nadolsky, and Fredrick I. Olness
arXiv:1803.02777

Website development: **Bo-Ting Wang, Pavel Nadolsky**

TSV Files

The .tsv file records the residuals of all replicas (normalized by the root-mean-square of th central values of residuals in each experiment as shown in the paper) for each point in each data set. Users can play the .tsv file with Excel or load .tsv file into **Embedding Projector** for PCA and T-SNE analysis. **[download]**

Figures

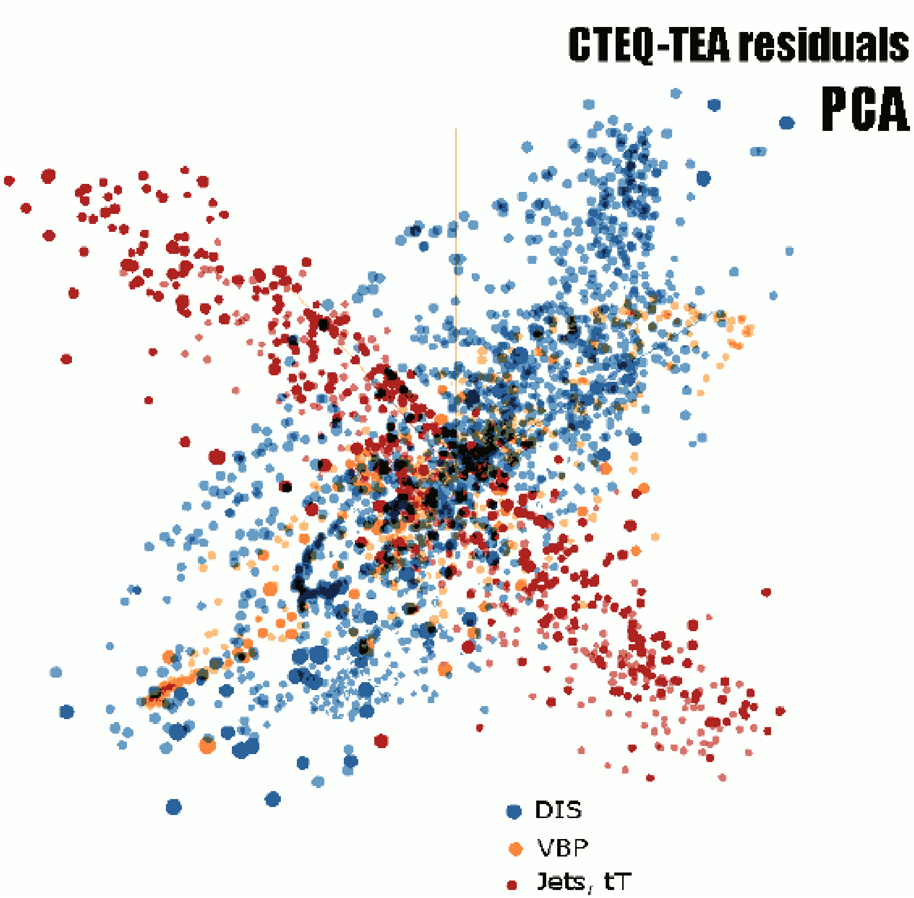
CT14HERA2 NNLO sensitivities

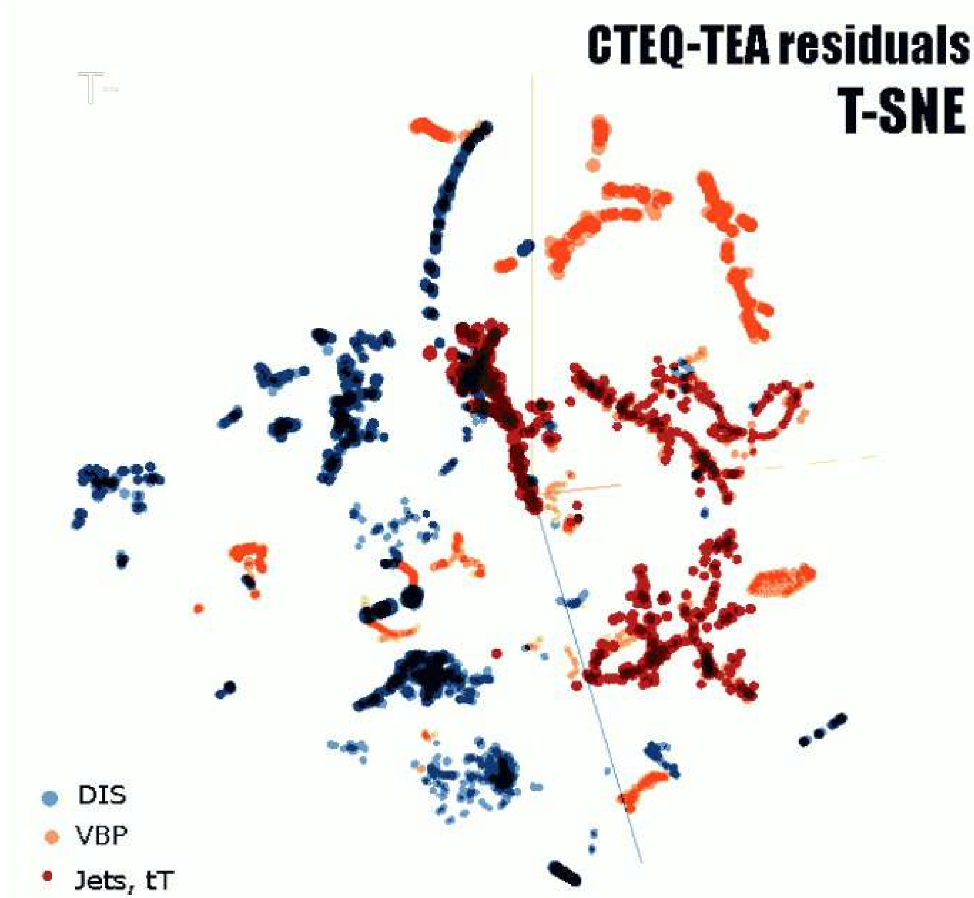
all experiments
Jet measurements at the LHC
pT distribution of Z measurements at the LHC
ttbar measurements at the LHC
lepton asymmetry in W measurements at the LHC

TensorFlow Embedding Projector

http://projector.tensorflow.org

Reads 2 .tsv files with vectors and metadata (descriptions of data points)





Principal Component Analysis (PCA) visualizes the 56-dim. manifold by reducing it to 10 dimensions (à la META PDFs)

t-distributed stochastic neighbor embedding (**t-SNE**) sorts vectors according to their similarity

$$r_i(\vec{a}) = \frac{1}{s_i}\left(T_i(\vec{a}) - D_{i,sh}(\vec{a})\right);$$

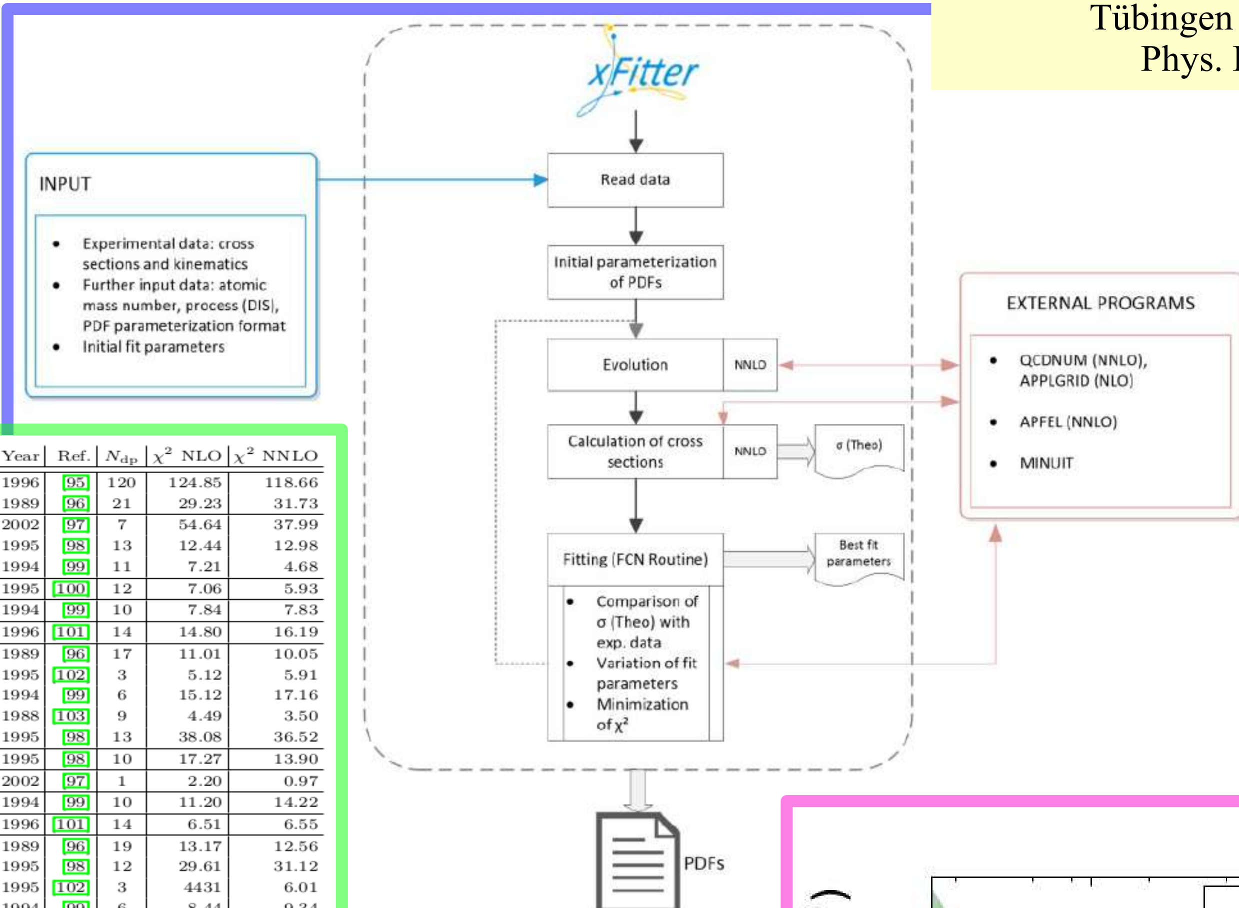
xFitter Nuclear Fit

xFitter Nuclear PDFs

Open-source QCD analysis of nuclear parton distribution functions at NLO and NNLO

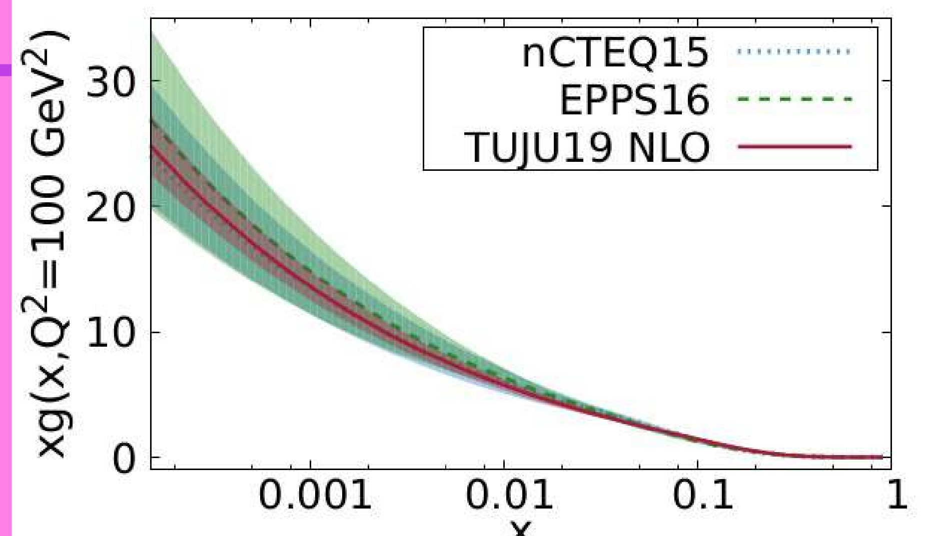
Marina Walt, Ilkka Helenius, Werner Vogelsang
Tübingen U, Jyvaskyla U (TUJU)

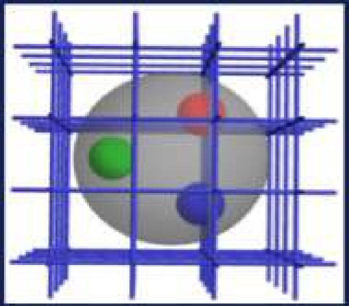
Phys. Rev. D 100, 096015 (2019)

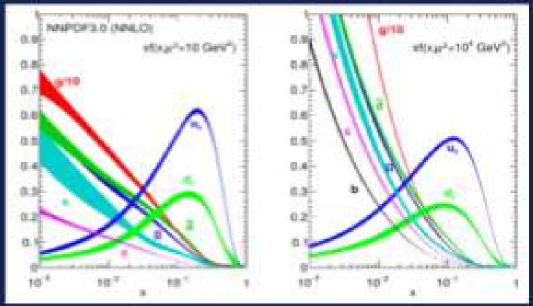


Nucleus	Exp.	Year	Ref.	N_{dp}	χ^2 NLO	χ^2 NNLO
D	NMC 97	1996	[95]	120	124.85	118.66
	EMC 90	1989	[96]	21	29.23	31.73
He/D	HERMES	2002	[97]	7	54.64	37.99
	NMC 95, re.	1995	[98]	13	12.44	12.98
	SLAC E139	1994	[99]	11	7.21	4.68
Li/D	NMC 95	1995	[100]	12	7.06	5.93
Be/D	SLAC E139	1994	[99]	10	7.84	7.83
Be/C	NMC 96	1996	[101]	14	14.80	16.19
C	EMC 90	1989	[96]	17	11.01	10.05
C/D	FNAL E665	1995	[102]	3	5.12	5.91
	SLAC E139	1994	[99]	6	15.12	17.16
	EMC 88	1988	[103]	9	4.49	3.50
	NMC 95, re.	1995	[98]	13	38.08	36.52
C/Li	NMC 95, re.	1995	[98]	10	17.27	13.90
N/D	HERMES	2002	[97]	1	2.20	0.97
Al/D	SLAC E139	1994	[99]	10	11.20	14.22
Al/C	NMC 96	1996	[101]	14	6.51	6.55
Ca	EMC 90	1989	[96]	19	13.17	12.56
Ca/D	NMC 95, re.	1995	[98]	12	29.61	31.12
	FNAL E665	1995	[102]	3	4431	6.01
	SLAC E139	1994	[99]	6	8.44	9.34
Ca/Li	NMC 95, re.	1995	[98]	10	7.36	5.16
Ca/C	NMC 95, re.	1995	[98]	10	6.47	6.70
	NMC 96	1996	[101]	14	7.14	6.99
Fe	SLAC E140	1993	[104]	2	0.05	0.05
Fe/D	SLAC E139	1994	[99]	14	34.08	34.18
Fe/C	NMC 96	1996	[101]	14	9.82	9.96
ν Fe	CDHSW	1991	[105]	464	347.74	365.14
$\bar{\nu}$ Fe	CDHSW	1991	[105]	462	423.06	398.25
Cu/D	EMC 93					
	EMC 88					
Kr/D	HERMES					
Ag/D	SLAC E13					
Sn/D	EMC 88					
Sn/C	NMC 96					
	NMC 96, (					
Xe/D	FNAL E66					
Au/D	SLAC E13					
Pb/D	FNAL E66					
Pb/C	NMC 96					
ν Pb	CHORUS					
$\bar{\nu}$ Pb	CHORUS					
			In total:	2336	2072.29	2014.02

Date	Version	Files	Remarks
 02/2020	2.0.1N Nuclear Daiquiri	xfitter-2.0.1N.tgz	Nuclear xFitter based on OldFashioned 2.0.1

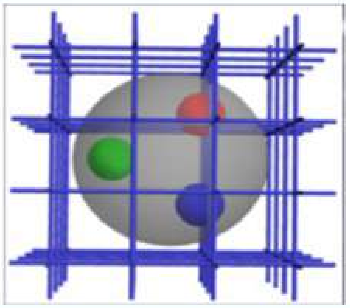






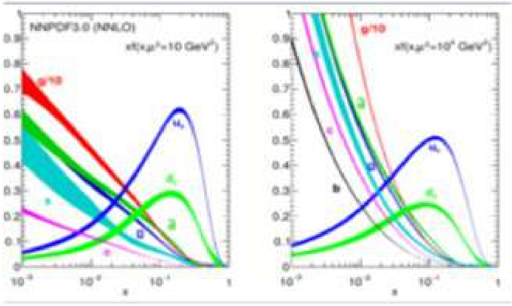
Parton Distributions and Lattice Calculations in the LHC era (PDFLattice 2017)

22-24 March 2017, Oxford, UK

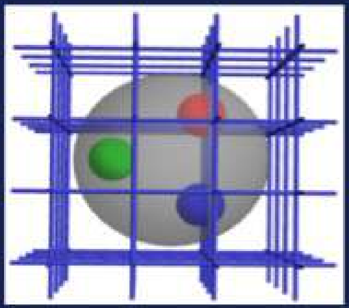


W. K . Kellogg Biological Station

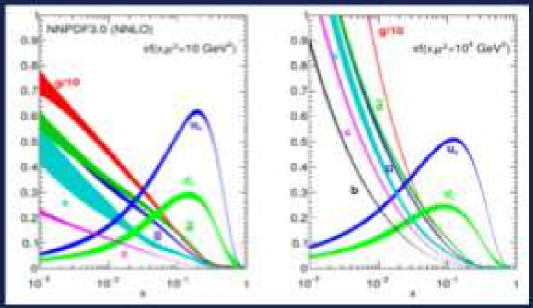
MICHIGAN STATE UNIVERSITY



Parton Distributions and Lattice Calculations (PDFLattice 2019)



Jefferson Lab



Parton Distributions and Lattice Calculations (PDF Lattice 2024)

18-20 November 2024

QCD: From Parameterization to a Deeper Understanding