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# Nucleon formfactors: a review


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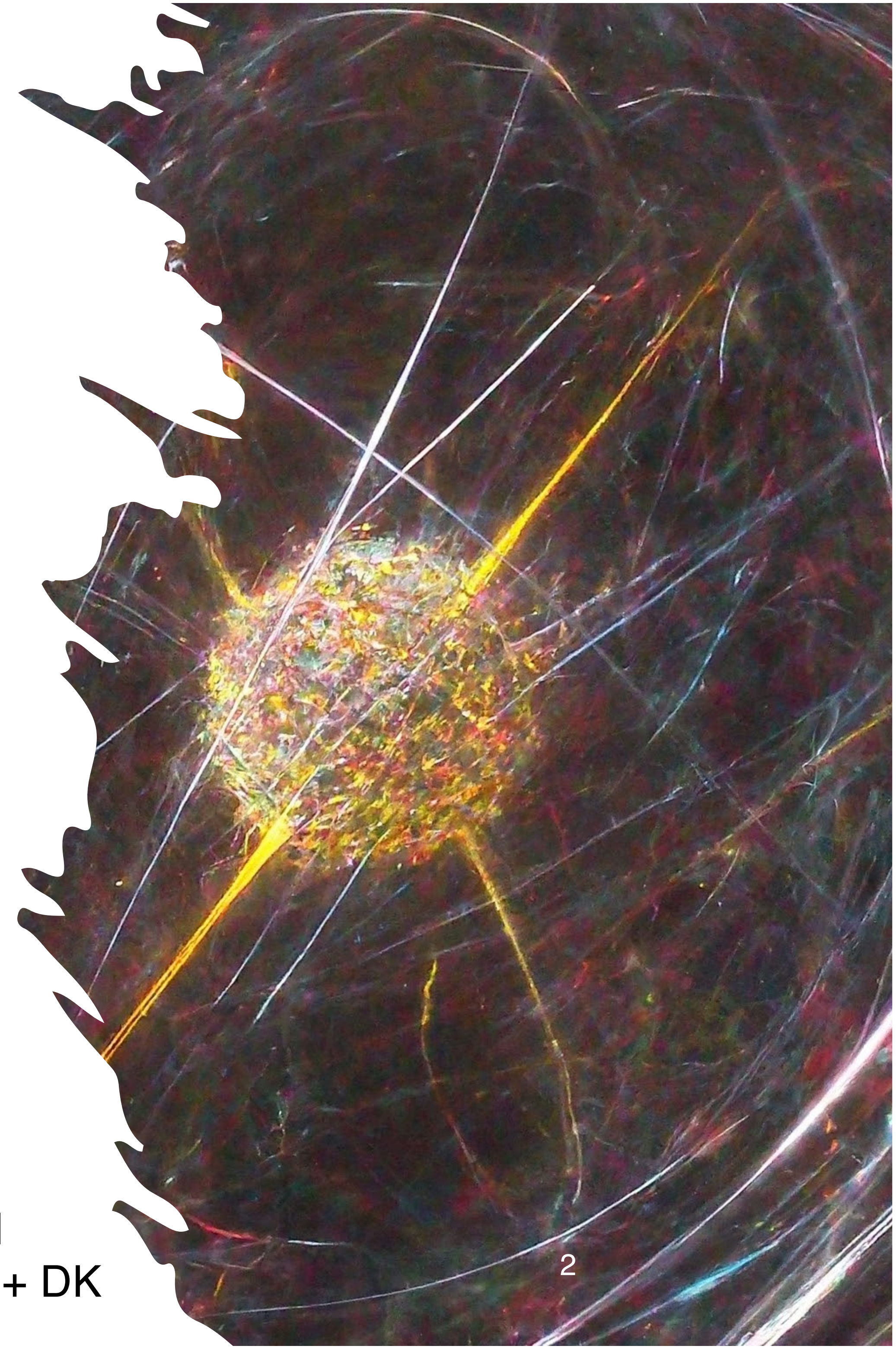
# Outline

- Why formfactors?
- Electromagnetic formfactors
- Gravitational formfactors
- Outlook



Google AI
ImageFX + DK

# Why formfactors?

Fundamental property of a physical state

Classically motivated picture:

formfactors provide information about the spatial distributions of charge, magnetic moment, chirality, mass, …

$$\sigma(\theta_e) = \sigma_{\text{Mott}} \left| \int_{\text{volume}} \rho(\vec{r}) e^{i\vec{q}.\vec{r}} d^3\vec{r} \right|^2 = \sigma_{\text{Mott}} |F(q)|^2$$

Quantum picture:

formfactors provide information about the survival of a pure quantum state after a measurement with resolution 1/q
(“Loschmidt echo” in QIS terminology)

# Formfactors: the beginning

REVIEWS OF MODERN PHYSICS VOLUME 28, NUMBER 3 JULY, 1956

## Electron Scattering and Nuclear Structure*

ROBERT HOFSTADTER

*Department of Physics, Stanford University, Stanford, California*

Robert Hofstadter

as the appropriate model of the nucleus. The results can be summarized in a well-known formula for the radius of a uniform sphere

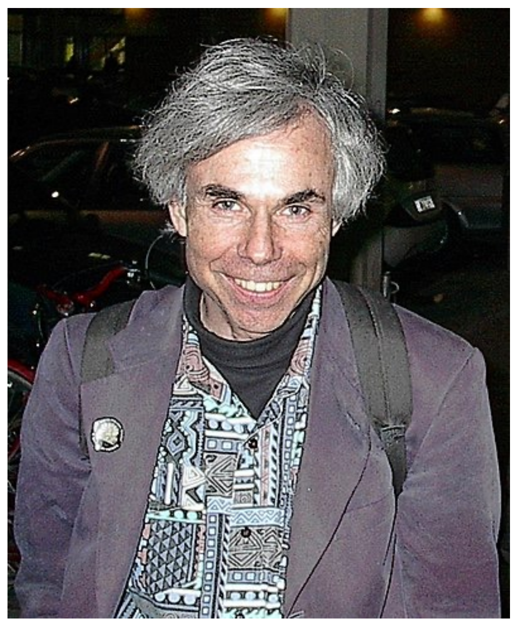
$$R = r_0 A^{\frac{1}{3}} \times 10^{-13} \text{ cm.} \qquad (1)$$

Henceforth, we shall measure all distances in terms of $10^{-13}$ cm as a unit and shall call this unit the fermi. For

Nobel prize
1961

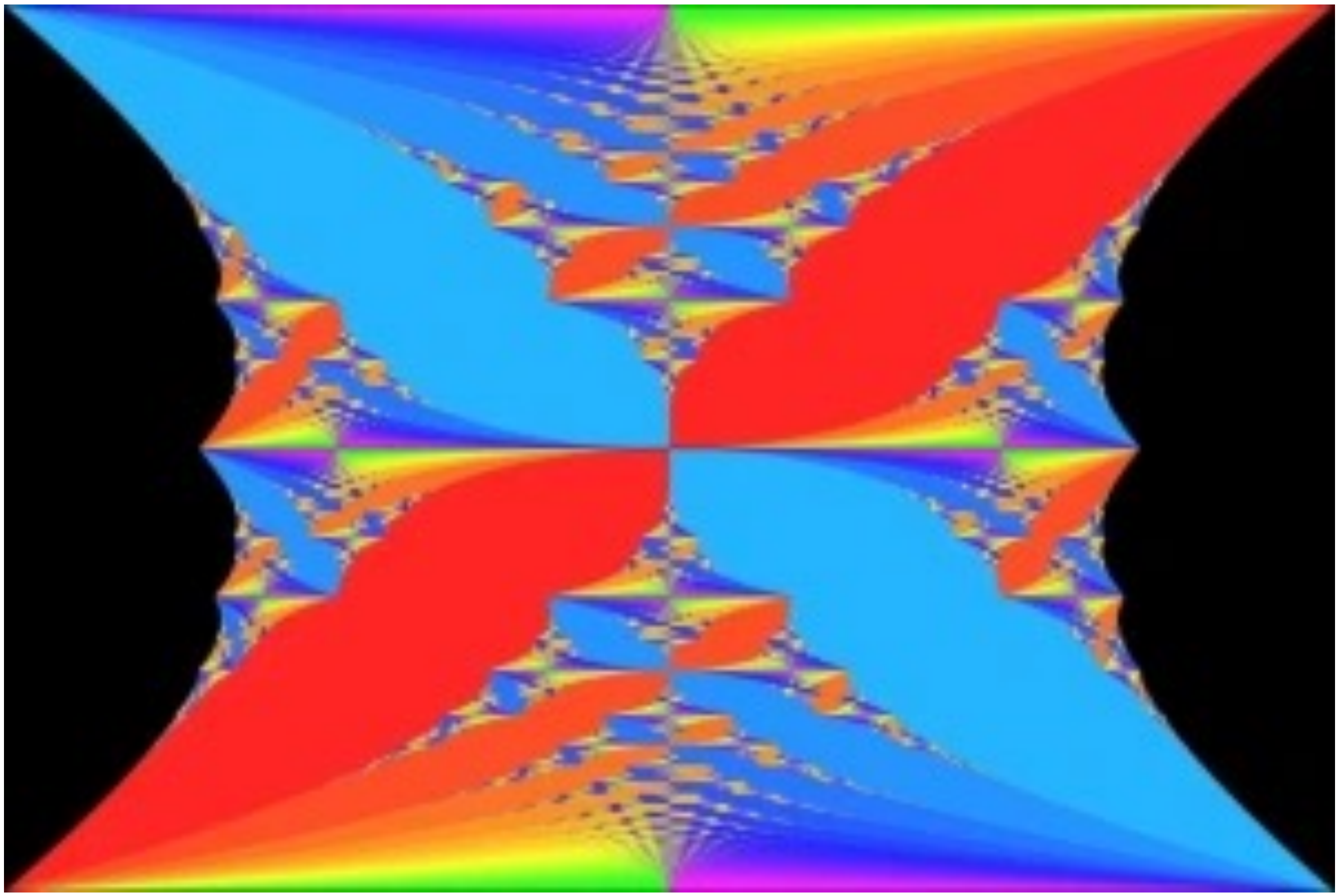
# Hofstadter's law:

*"It always takes longer than you expect,*
*even when you take into account Hofstadter's law"*



Douglas Hofstadter

# Hofstadter’s butterfly



# The proton formfactor

REVIEWS OF MODERN PHYSICS VOLUME 28, NUMBER 3 JULY, 1956

## Electron Scattering and Nuclear Structure*

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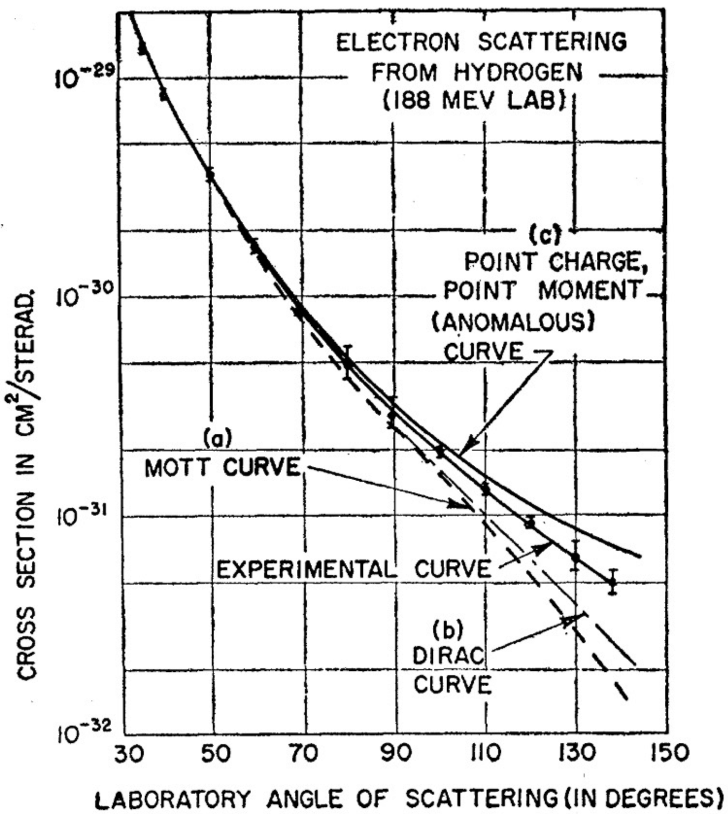
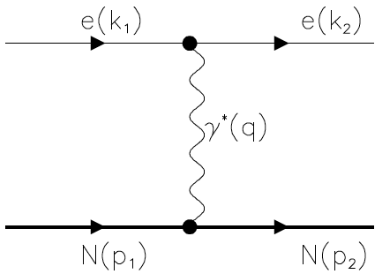



FIG. 24. Electron scattering from the proton at an incident energy of 188 Mev. The experimental points lie below the point-charge point-moment curve of Rosenbluth, indicating finite size effects.

# EM formfactor of the nucleon

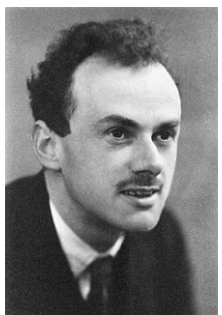


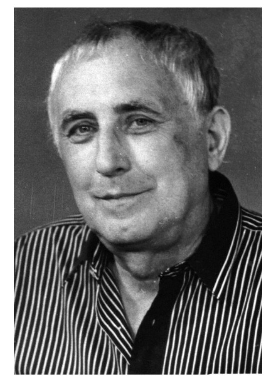

$$\mathcal{M} = \frac{e^2}{q^2}\ell_\mu \mathcal{J}_\mu = \frac{e^2}{q^2}\ell \cdot \mathcal{J},$$

**Polarization effects in elastic electron-proton scattering**
**(Lecture notes)**

Michail P. Rekalo † and Egle Tomasi-Gustafsson

nucl-th/0202025





M.P.Rekalo
1938-2004

$$\mathcal{J}_\mu = \overline{u}(p_2)\left[F_1(q^2)\gamma_\mu - \frac{\sigma_{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p_1),$$

Dirac FF
(el. charge)

Pauli FF
(anomalous magnetic moment)

$$\sigma_{\mu\nu} = \frac{\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu}{2}.$$

Current conservation:

$$\mathcal{J} \cdot q = 0$$

# EM formfactor of the nucleon

$$\mathcal{J}_\mu = \overline{u}(p_2)\left[F_1(q^2)\gamma_\mu - \frac{\sigma_{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p_1),$$

<u>Free</u> Dirac equation for the nucleon

G. Breit
1899-1981

$$\mathcal{J}_\mu = \overline{u}(p_2)\left[(F_1+F_2)\,\gamma_\mu - \frac{(p_1+p_2)_\mu}{2m}F_2\right]u(p_1)$$

Breit reference frame vs Lab and CMS:

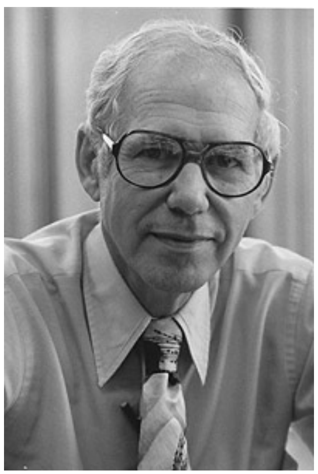
| | **Lab** | **CMS** | **Breit** |
|---|---|---|---|
| $q$ | $(\omega, \mathbf{q})$ | $(\widetilde{\omega}, \widetilde{\mathbf{q}})$ | $(\omega_B = 0, \mathbf{q_B})$ |
| $k_1$ | $(\epsilon_1, \mathbf{k_1})$ | $(\widetilde{\epsilon_1}, \widetilde{\mathbf{k_1}})$ | $(\epsilon_{1B}, \mathbf{k_{1B}})$ |
| $p_1$ | $(m, 0)$ | $(\widetilde{E_1}, -\widetilde{\mathbf{k_1}})$ | $(E_{1B}, \mathbf{p_{1B}})$ |
| $k_2$ | $(\epsilon_2, \mathbf{k_2})$ | $(\widetilde{\epsilon_2}, \widetilde{\mathbf{k_2}})$ | $(\epsilon_{2B}, \mathbf{k_{2B}})$ |
| $p_2$ | $(E_2, \mathbf{p_2})$ | $(\widetilde{E_2}, -\widetilde{\mathbf{k_2}})$ | $(E_{2B}, -\mathbf{p_{1B}})$ |

# EM formfactor of the nucleon

$$\mathcal{J}_\mu = \overline{u}(p_2)\left[(F_1+F_2)\,\gamma_\mu - \frac{(p_1+p_2)_\mu}{2m}F_2\right]u(p_1)$$

In Breit frame:

$$\boxed{\begin{aligned}\mathcal{J}_0 &= 2m\chi_2^\dagger\chi_1\,(F_1-\tau F_2)\\ \vec{\mathcal{J}} &= i\chi_2^\dagger\vec{\sigma}\times\mathbf{q_B}\chi_1\,(F_1+F_2)\end{aligned}}$$



Robert Sachs
1916-1999

Sachs electric and magnetic FFs:

$$\tau = -\frac{q^2}{4m^2} \geq 0$$

$$G_{EN} = F_1 - \tau F_2 \quad G_{MN} = F_1 + F_2.$$

# Rosenbluth formula

$$\left(\frac{d\sigma}{d\Omega}\right)_{CMS} = \frac{\overline{|\mathcal{M}|}^2}{64\pi^2 s}\frac{q_i}{q_f} \qquad \overline{|\mathcal{M}|^2} = \left(\frac{e^2}{q^2}\right)^2 \overline{|\ell \cdot \mathcal{J}|^2} = \left(\frac{e^2}{q^2}\right)^2 L_{\mu\nu} W_{\mu\nu}$$

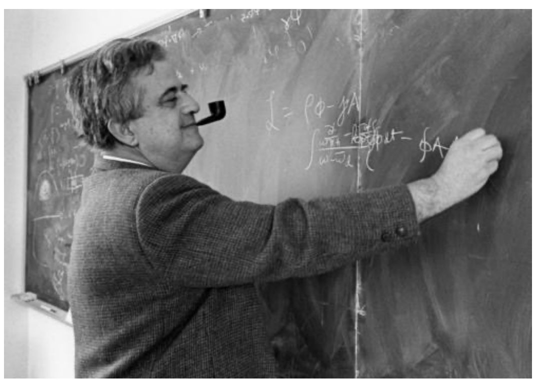
Hadronic tensor:

$$W_{\mu\nu} = \overline{\mathcal{J}_\mu \mathcal{J}_\nu^*}$$

$$W_{00} = 4m^2 G_{EN}^2,$$

$$W_{xx} = -q^2 G_{MN}^2,$$

$$W_{yy} = -q^2 G_{MN}^2.$$

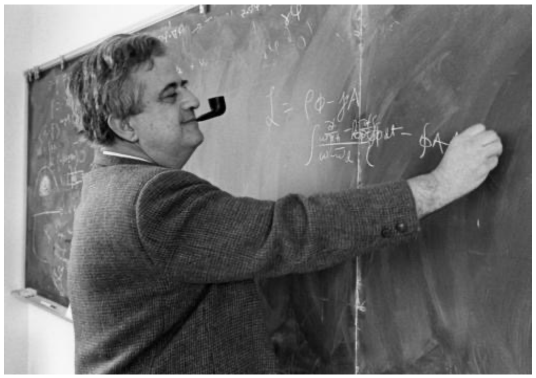


Marshall Rosenbluth
1927-2003

In the lab frame:

$$\frac{d\sigma}{d\Omega_e} = \sigma_M \left[2\tau G_{MN}^2 \tan^2\frac{\theta_e}{2} + \frac{G_{EN}^2 + \tau G_{MN}^2}{1+\tau}\right]$$

# Rosenbluth formula



Marshall Rosenbluth
1927-2003

In the lab frame:

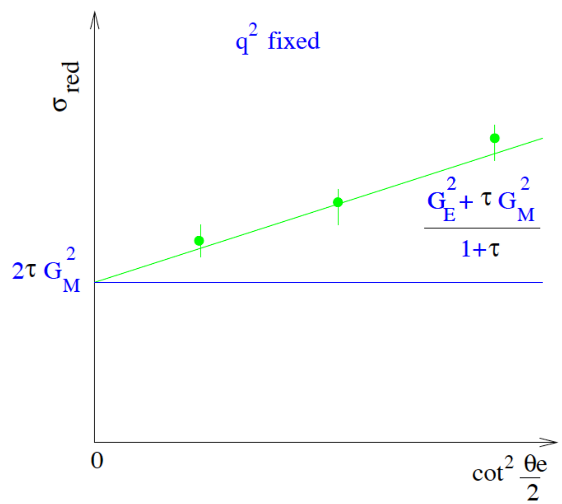
$$\frac{d\sigma}{d\Omega_e} = \sigma_M \left[ 2\tau G_{MN}^2 \tan^2 \frac{\theta_e}{2} + \frac{G_{EN}^2 + \tau G_{MN}^2}{1+\tau} \right]$$

Mott cross section (one photon exchange):

$$\sigma_M = \frac{\alpha^2}{-q^2} \left( \frac{\epsilon_2}{\epsilon_1} \right)^2 \frac{\cos^2 \frac{\theta_e}{2}}{\sin^2 \frac{\theta_e}{2}}$$

"Reduced cross section":

$$\sigma_{red} = \frac{\frac{d\sigma}{d\Omega_e}}{\frac{\alpha^2}{-q^2} \left( \frac{\epsilon_2}{\epsilon_1} \right)^2}$$




Large $q^2$
(large $\tau$):
mostly magnetic

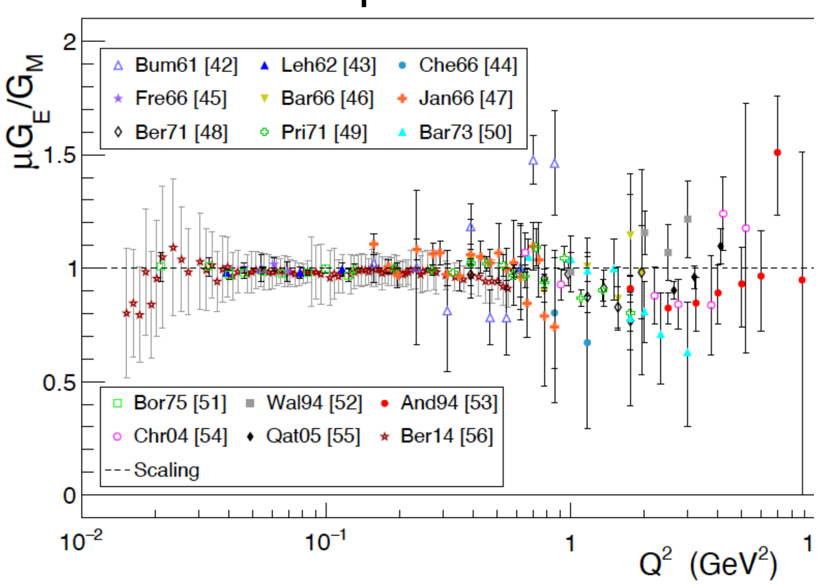
# Rosenbluth formula

Polarized electrons can induce the proton polarization:

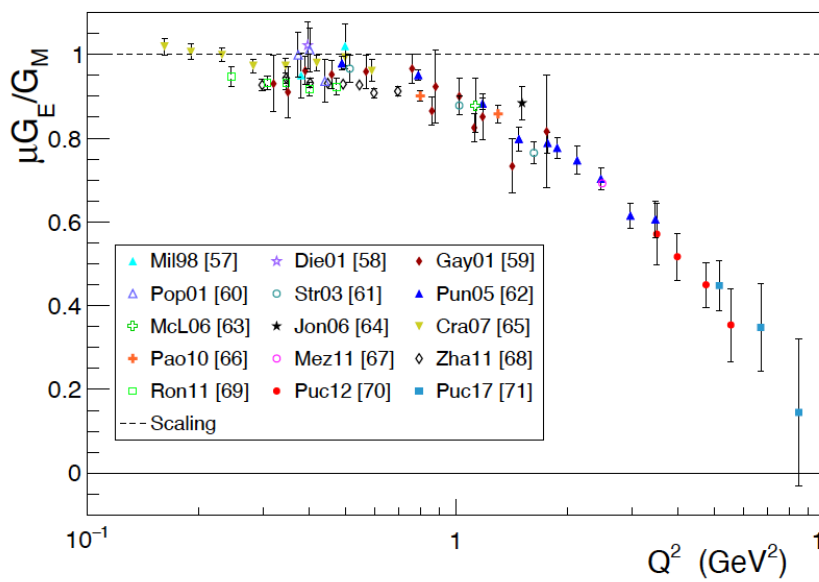
$$\frac{P_x}{P_z} = \frac{P_t}{P_\ell} = -2\cot\frac{\theta_e}{2}\frac{m}{\epsilon_1+\epsilon_2}\frac{G_{EN}(q^2)}{G_{MN}(q^2)}$$

The ratio of transverse and longitudinal polarization of the recoil proton yields a measurement of the ratio of electric and magnetic FFs

Unpolarized data



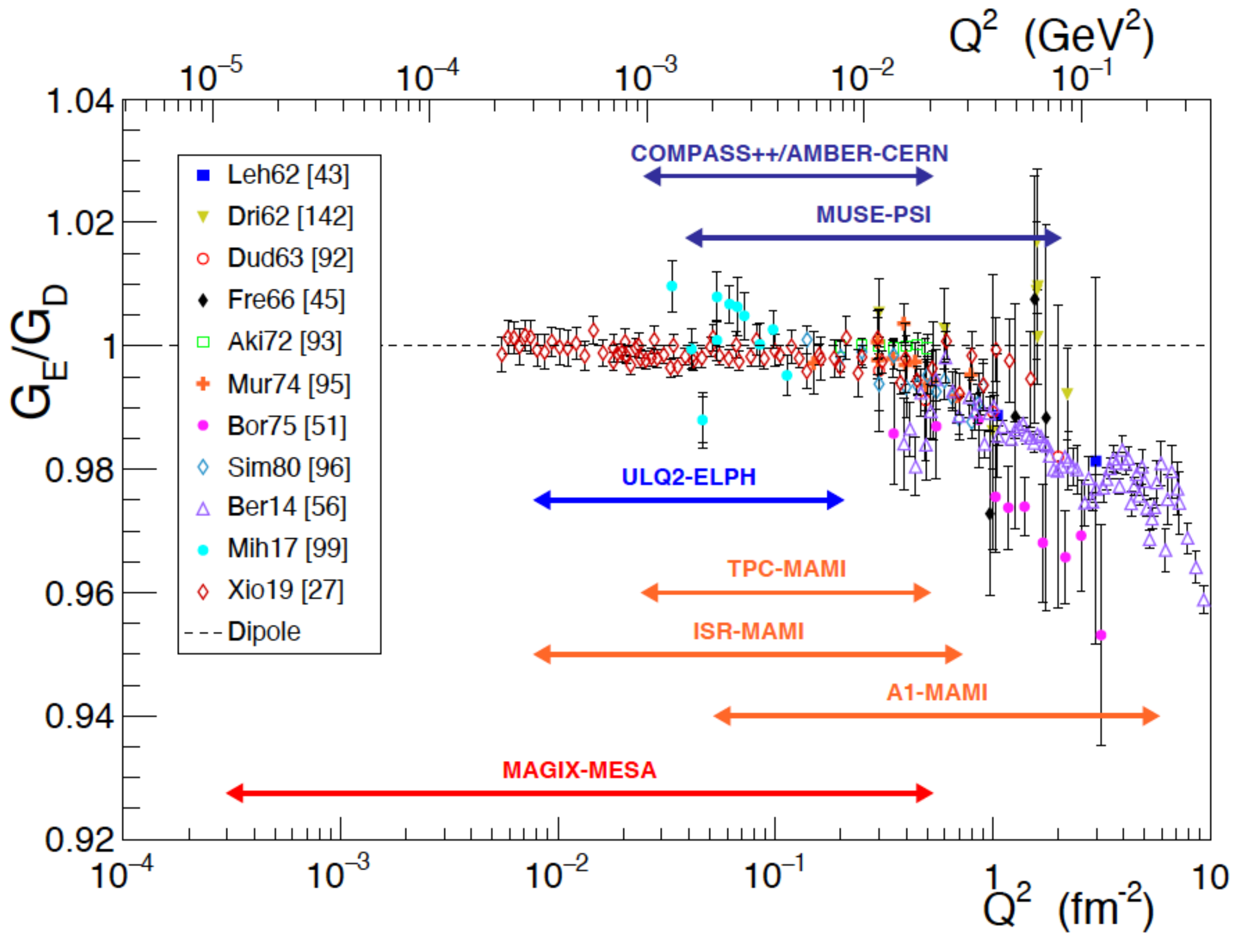

Polarized data




J.-P.Karr, D.Marchand, E.Voutier, Nature Rev.Phys. 2020,2,601

What is the origin of the discrepancy?

Two-photon exchange?...




J.-P.Karr, D.Marchand, E.Voutier, Nature Rev.Phys. 2020,2,601

**Figure 4. Measurements of the electric form factor.** Past and planned measurements of the proton electric form factor ($G_E$) measurements at low squared four-momentum transfer ($Q^2$) compared with the historical dipole parameterization $G_D = (1 + (Q^2/0.71))^{-2}$, with $Q^2$ in GeV$^2$ units. The $Q^2$ coverage of each experiment is indicated by a line labelled with the experiment name and location. The dipole parameterization is also shown for comparison. The error bars represent one standard deviation. Data from refs[27,43,45,51,56,92,93,95,96,99,142]. The data from ref.[56] are restricted to the Rosenbluth subset.

# The proton mass puzzle

Proton radius from atomic spectroscopy

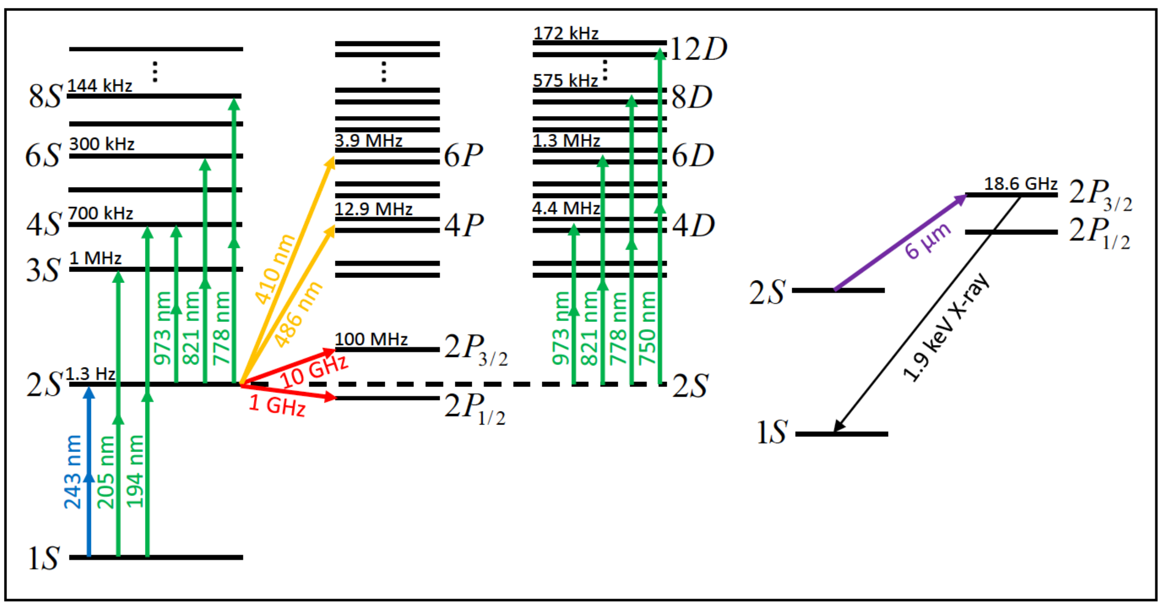
energy levels of s-states in the hydrogen atom depend on the proton radius:

$$E_{n,l} \simeq -\frac{R_\infty}{n^2} + \delta_{l,0} \frac{L_{1S} + a r_p^2}{n^3}$$

J.Bernauer
EPJ 234(2020)

$R_\infty \simeq 3.29 \times 10^6$ MHz is the Rydberg constant, $L_{1S} \simeq 8.172$ MHz is the Lamb shift

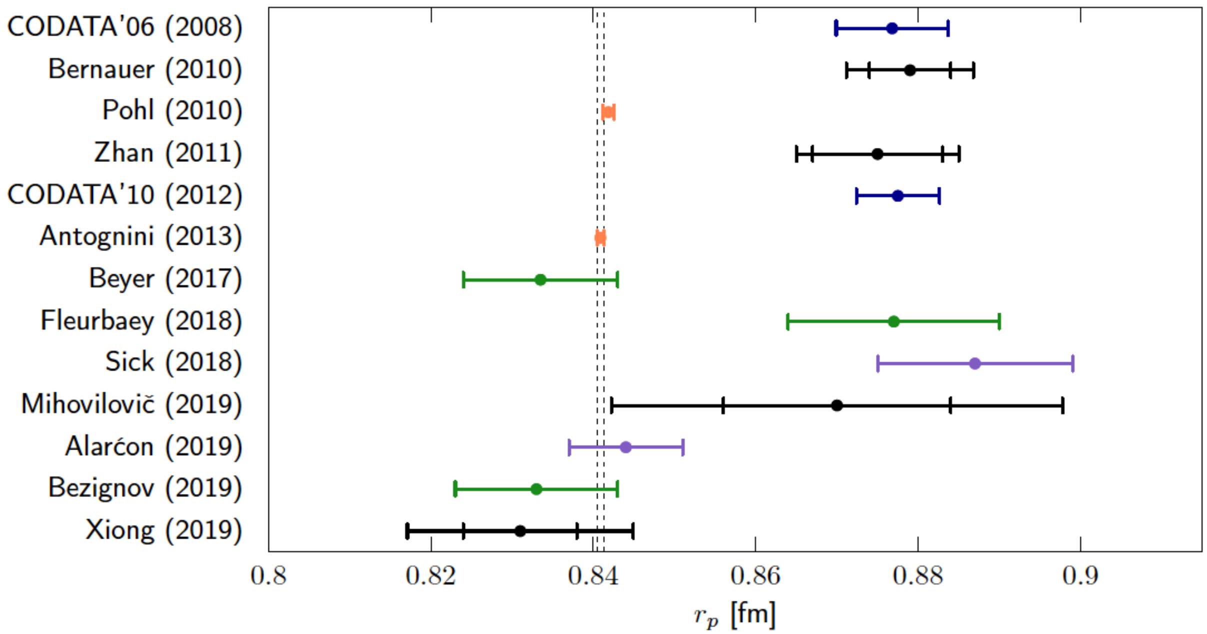
Muonic hydrogen: $\Delta E_{2S-2P} = 206.0808(61)\ \text{meV} - 5.2272\, r_p^2\ \text{meV fm}^{-2}$.




J.-P.Karr,
D.Marchand,
E.Voutier,
Nature
Rev.Phys.
2020,2,601

**Transitions in electronic and muonic hydrogen.** Spectra showing the transitions in electronic (left) and muonic

# The proton mass puzzle

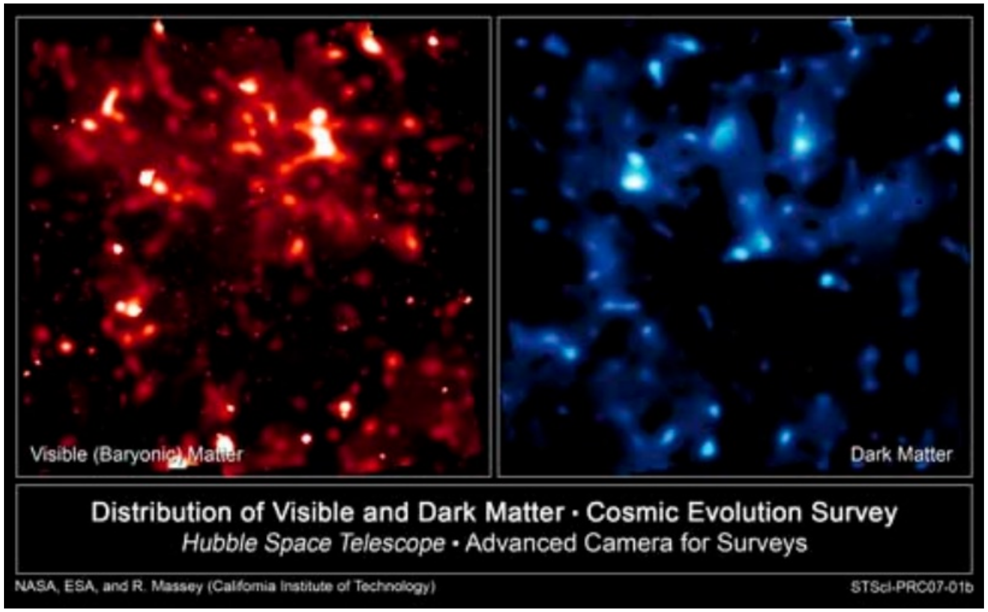



J.Bernauer
EPJ 234(2020)

**Figure 1.** Collection of experimental results on the proton charge radius, and a (small) selection of fits by other authors. CODATA values (dark blue) [3, 46] are global fits, using electron spectroscopy and scattering data as input. Bernauer [2], Zhan [47], Mihovilovič [44] and Xiong [45] (black) are results from scattering experiments, sometimes including the world data set. Sick [21] and Alarćon [17] (purple) are refits of existing data, in the latter case based on dispersion relations. Beyer, Fleurbaey and Bezignov [40–42] (green) are electron spectroscopy results, Pohl and Antognini [1, 5] (orange) are the results from the muon spectroscopy experiment.

Two-photon? Better description of atomic shift? Beyond standard model?

# Gravitational formfactors and the mass distribution



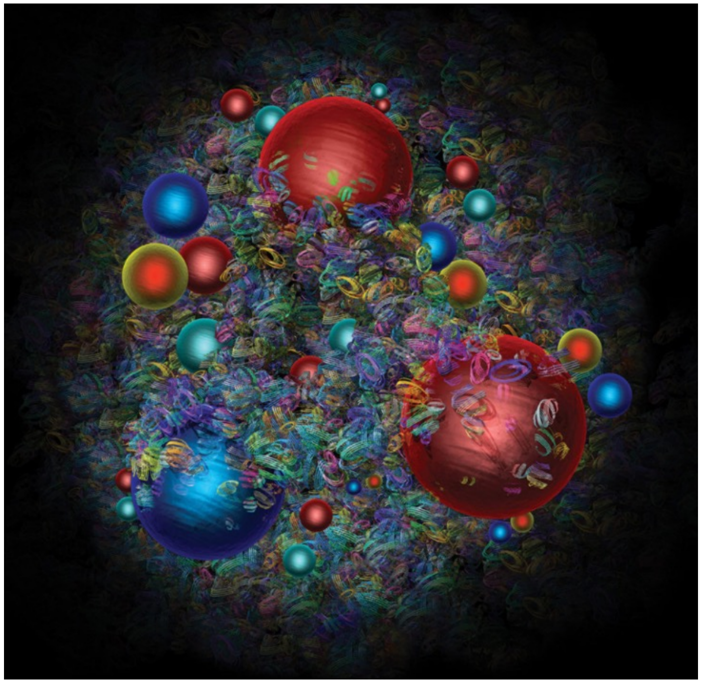



Image: CERN

What is the origin of the proton mass?

How is the mass distributed inside the proton?

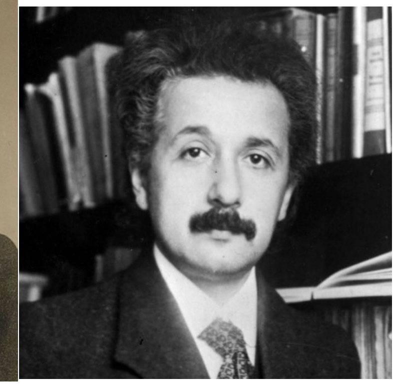
Is it associated with quarks (“visible matter”)
or with gluons (“dark matter”)?

How can we measure the mass distribution?

# Gravitational formfactors and the mass distribution:

## scalar (Nordstrom) vs tensor (Einstein) gravity

Gunnar Nordstrom
1881-1923



Albert Einstein
1879-1955

Consider Einstein gravity:

Ricci curvature tensor → $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$$

Take the trace with metric tensor:

$$-R = 8\pi G\, T \qquad T \equiv T^{\mu}_{\mu}$$

Cf Nordstrom 1912;
Einstein 1913; Einstein-Fokker 1914

Non-relativistic, weak gravitational field limit:

$$g_{00} = 1 + 2\varphi, \qquad T^{\nu}_{\mu} = \mu\, u_\mu u^\nu, \qquad u_0 = u^0 = 1, \quad u_i = 0.$$

Therefore, in this limit, the distributions of mass and of T coincide:

$$T^0_0 = \mu; \qquad T \equiv T^{\mu}_{\mu} = T^0_0 = \mu$$



[45] Einstein, Albert and Fokker, Adriann, D., "Die Nordströmsche Gravitationstheorie vom Standpunkt des absoluten Differentialkalküls", *Annelen der Physik* 44, 1914, pp. 321-328; p. 321.

# Gravitational formfactors and the mass distribution

Newtonian limit:

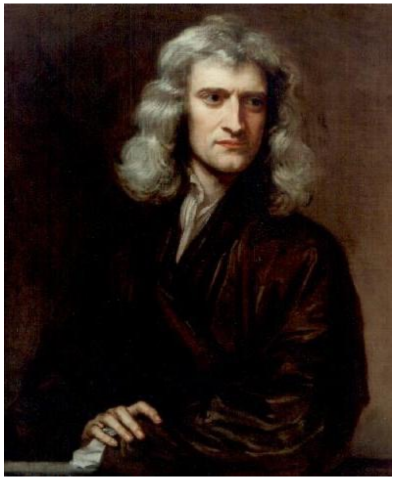
$$R_0^0 = \frac{\partial^2 \varphi}{\partial x^{\mu 2}} \equiv \Delta\varphi,$$

Einstein equation:

$$R_{\mu\nu} = 8\pi G(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T);$$

The only non-vanishing component:

$$R_0^0 = 4\pi G\mu,$$



Isaac Newton
1643-1727

Therefore, the distribution of mass determines the gravitational potential:

$$\Delta\varphi = 4\pi G\mu. \quad \Rightarrow \quad \varphi = -G \int \frac{\mu\, dV}{R} \quad \Rightarrow \quad F_g = -m\, \partial\varphi/\partial R$$

$$M = \int \mu dV$$

$$\boxed{F_g = -G\, \frac{mM}{R^2}.}$$

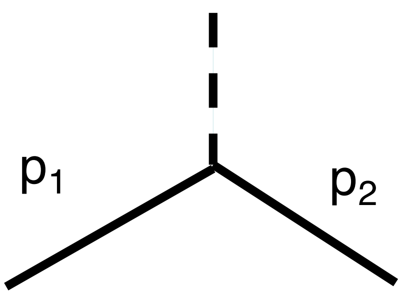
# Gravitational formfactors and the mass distribution

The mass distribution is encoded in the gravitational formfactors.

For the spin ½ nucleon, 3 formfactors appear:

H. Pagels ‘66,
A. Pais, S. Epstein ‘49

$$\langle \mathbf{p}_1 | T_{\mu\nu} | \mathbf{p}_2 \rangle = \left( \frac{M^2}{p_{01}\ p_{02}} \right)^{1/2} \frac{1}{4M}\ \bar{u}(p_1, s_1) \Big[ G_1(q^2)(p_\mu \gamma_\nu + p_\nu \gamma_\mu) + G_2(q^2) \frac{p_\mu p_\nu}{M} + \\ + G_3(q^2) \frac{(q^2 g_{\mu\nu} - q_\mu q_\nu)}{M} \Big] u(p_2, s_2),$$




Energy-momentum conservation:

$$\partial^\mu T_{\mu\nu} = 0$$

$$\Rightarrow \quad q^\mu \langle \mathbf{p}_1 | T_{\mu\nu} | \mathbf{p}_2 \rangle = 0;$$

Satisfied for on-shell nucleons (use Dirac equation)

$$\textstyle\sum_s \bar{u}(p, s) u(p, s) = (\hat{p} + M)/2M$$

$$p_1^2 = p_2^2 = M^2$$

# Gravitational formfactors and the mass distribution

For the spin ½ nucleon, 3 formfactors appear: (no $G_1$ for spin 0)

$$\langle \mathbf{p}_1 | T_{\mu\nu} | \mathbf{p}_2 \rangle = \left( \frac{M^2}{p_{01}\ p_{02}} \right)^{1/2} \frac{1}{4M}\ \bar{u}(p_1, s_1) \Big[ G_1(q^2)(p_\mu \gamma_\nu + p_\nu \gamma_\mu) + G_2(q^2) \frac{p_\mu p_\nu}{M} + $$
$$+ G_3(q^2) \frac{(q^2 g_{\mu\nu} - q_\mu q_\nu)}{M} \Big] u(p_2, s_2),$$

Compare to
the macroscopic energy-momentum
tensor in relativistic hydrodynamics:

C. Eckart, 1940

**The Thermodynamics of Irreversible Processes**

**III. Relativistic Theory of the Simple Fluid**

CARL ECKART
*Ryerson Physical Laboratory, University of Chicago, Chicago, Illinois*
(Received September 26, 1940)

$u_\nu$ - matter velocity

$$\theta_{\mu\nu} = \underbrace{(w_\mu u_\nu + w_\nu u_\mu)}_{\text{Heat flow}} + \underbrace{w\ u_\mu u_\nu}_{\text{Energy density}} + \underbrace{w_{\mu\nu}}_{\text{Stress tensor}}$$

Heat flow: $u_\mu\ w^\mu = 0$

Energy density: $w \sim \epsilon$

Stress tensor

# Gravitational formfactors and the mass distribution

For the spin ½ nucleon, 3 formfactors appear: (no $G_1$ for spin 0)

$$\langle \mathbf{p}_1 | T_{\mu\nu} | \mathbf{p}_2 \rangle = \left( \frac{M^2}{p_{01}\; p_{02}} \right)^{1/2} \frac{1}{4M} \, \bar{u}(p_1, s_1) \Big[ G_1(q^2)(p_\mu \gamma_\nu + p_\nu \gamma_\mu) + G_2(q^2) \frac{p_\mu p_\nu}{M} + \\ + G_3(q^2) \frac{(q^2 g_{\mu\nu} - q_\mu q_\nu)}{M} \Big] u(p_2, s_2),$$

Zero momentum transfer $q \to 0$ :

$$\langle \mathbf{p} | T_{\mu\nu} | \mathbf{p} \rangle = \left( \frac{M^2}{p_0^2} \right)^{1/2} \bar{u}(p, s) u(p, s) \, \frac{p_\mu p_\nu}{M^2} \left[ G_1(0) + G_2(0) \right]$$

(no "stress" $G_3$)

In the rest frame of the nucleon:

the Hamiltonian

$$\langle \mathbf{p} = 0 | T_{00} | \mathbf{p} = 0 \rangle = M$$

$$\Downarrow$$

$$H = \int d^3x \; T_{00}(x)$$

$$G_1(0) + G_2(0) = M.$$

# Formfactor of the trace of the energy-momentum tensor

Let us call it "scalar gravitational formfactor", as it would be a gravitational formfactor in a scalar model of gravity:

Nordstrom 1912
Einstein 1913

$$T \equiv T^\mu_\mu$$

$$\langle \mathbf{p}_1 | T | \mathbf{p}_2 \rangle = \left( \frac{M^2}{p_{01} \; p_{02}} \right)^{1/2} \bar{u}(p_1, s_1) u(p_2, s_2) \; G(q^2),$$

Scalar gravitational formfactor:

$$G(q^2) = G_1(q^2) + G_2(q^2) \left( 1 - \frac{q^2}{4M^2} \right) + G_3(q^2) \frac{3q^2}{4M^2}$$

In the rest frame of the nucleon:

$$\langle \mathbf{p} = 0 | T | \mathbf{p} = 0 \rangle = \langle \mathbf{p} = 0 | T_{00} | \mathbf{p} = 0 \rangle = M,$$

$$\Downarrow$$

$$G(0) = M$$

# How to define the mass distribution in the nucleon?

At small momentum transfer $|q^2| \ll M^2$ ,

the formfactor of $\theta_{00}$ and the scalar gravitational formfactor coincide if

$$\frac{G_i(0)}{4M} \ll \frac{dG_i}{dt}\Big|_{t=0} \equiv G_i(0)/m_i^2$$

The origin of the difference is frame dependence of $\theta_{00}$ :

In Breit frame, $\mathbf{p}_2 = \frac{1}{2}\mathbf{q},\ \mathbf{p}_1 = -\frac{1}{2}\mathbf{q}$ the proton is moving with

$$\gamma = E/M = \sqrt{M^2 + (q^2/4)}/M = \sqrt{1 + q^2/(4M^2)},$$

so for $q \equiv |\mathbf{q}| \simeq m_i$ it is Lorentz-contracted with

$$1/\gamma \simeq (1 + m_i^2/(4M^2))^{-1/2}$$

For massive bodies, $m_i << 2M$ – size much larger than the Compton wavelength! In this limit, the formfactors of $T_{00}$ and T coincide.



[the proton: $8M^2_p >> M^2_s$ ] See A. Freese, G.A. Miller (2022) + R.L. Jaffe (2021)

# How to define the mass distribution in the nucleon?

At small momentum transfer $|q^2| \ll M^2$ , $\quad \frac{G_i(0)}{4M} \ll \frac{dG_i}{dt}\bigg|_{t=0}$

the formfactor of $\theta_{00}$ and the scalar gravitational formfactor are close,

thus the scalar gravitational formfactor can be used to define

the **mass radius of the proton**:

$$\langle R_C^2 \rangle = 6 \frac{dG_{EM}}{dt}\bigg|_{t=0} . \Rightarrow \boxed{\langle R_M^2 \rangle = \frac{6}{M} \frac{dG}{dt}\bigg|_{t=0},}$$

In the relativistic region (mass -> energy), it is natural to consider the scalar gravitational formfactor, as T is the Lorentz scalar

# How close is the scalar radius to the mass radius?

As argued above, the difference between the scalar radius $<R_S^2>$ and a ``true" mass radius $<R_M^2>$ should be suppressed by $1/M^2$ (M is the nucleon mass).

But how small is really this difference?

$G_3(0)$

$$\langle R_S^2 \rangle - \langle R_M^2 \rangle = -12 \frac{C(0)}{M^2}$$

$$\langle P' | T^{\mu\nu} | P \rangle = \bar{u}(P'S') \Big[ A(q^2) \gamma^{(\mu} \bar{P}^{\nu)} + B(q^2) \frac{\bar{P}^{(\mu} i \sigma^{\nu)\alpha} q_\alpha}{2M} + C(q^2) \frac{q^\mu q^\nu - g^{\mu\nu} q^2}{M} \Big] u(PS)$$

Y.Guo, X.Ji, Y.Liu, arXiv:2103.11506 [PRD]

$C(0) = -0.84 \pm 0.82$

Extracted value is consistent with zero;

But if C(0)=-1, the difference is big:

$$\langle R_S^2 \rangle - \langle R_M^2 \rangle \simeq 0.47 \text{ fm}^2$$

**Need a reliable value of C(0) from lattice!**

# Trace of $T^{\mu\nu}$ plays a fundamental role: link to scale invariance

Scale transformations (dilatations) are defined by

$$x \to e^{\lambda}\ x$$

the corresponding dilatational current is

$$s^{\mu} = x_{\nu} T^{\mu\nu}$$

Hermann Weyl (1885-1955)

It is conserved (a theory is scale-invariant) if the energy-momentum is traceless:

$$\partial_{\mu} s^{\mu} = T^{\mu}_{\mu} \equiv T$$

# Scale invariance

A scale-invariant theory cannot contain massive particles, all particles must be massless

For example, in Maxwell electrodynamics with action

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

the energy-momentum is traceless: (massless photons)

$$T^{\mu}_{\mu} = 0$$

Note: because of this, in scalar gravity (Nordstrom, 1912; Einstein, 1913) there would be no light bending by massive bodies!

# Scale invariance in QCD

The trace of the energy-momentum tensor in QCD (computed in classical field theory) is

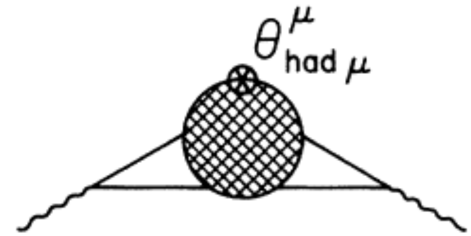
$$T^{\mu}_{\mu} = \sum_{l=u,d,s} m_l \; \bar{q}_l q_l + \sum_{h=c,b,t} m_h \; \bar{Q}_h Q_h$$

Two problems:

1. Potentially large contribution from heavy quarks to the masses of light hadrons

2. If we forget about heavy quarks, all hadron masses must be equal to zero in the chiral limit

# Scale anomaly in QCD




The quantum effects (loop diagrams) modify the expression for the trace of the energy-momentum tensor:

$$T^{\mu}_{\mu} = \frac{\beta(g)}{2g} G^{\mu\nu a} G^{a}_{\mu\nu} + \sum_{l=u,d,s} m_l (1+\gamma_{m_l}) \bar{q}_l q_l + \sum_{h=c,b,t} m_h (1+\gamma_{m_h}) \bar{Q}_h Q_h$$

Running coupling -> dimensional transmutation -> mass scale

Gross, Wilczek; Politzer

$$\beta(g) = -b \frac{g^3}{16\pi^2} + \ldots, \quad b = 9 - \frac{2}{3} n_h,$$

Ellis, Chanowitz; Crewther; Collins, Duncan, Joglecar; …

At small momentum transfer, heavy quarks decouple:

$$\sum_h m_h \bar{Q}_h Q_h \to -\frac{2}{3} n_h \frac{g^2}{32\pi^2} G^{\alpha\beta a} G^{a}_{\alpha\beta} + \ldots$$

so only light quarks enter the final expression

Shifman, Vainshtein Zakharov '78

$$T^{\mu}_{\mu} = \frac{\tilde{\beta}(g)}{2g} G^{\mu\nu a} G^{a}_{\mu\nu} + \sum_{l=u,d,s} m_l (1+\gamma_{m_l}) \bar{q}_l q_l,$$

0

# The proton mass

At zero momentum transfer, the matrix element of the trace of the energy-momentum tensor defines the mass of the proton:

$$\langle \mathbf{p} = 0 | T | \mathbf{p} = 0 \rangle = \langle \mathbf{p} = 0 | T_{00} | \mathbf{p} = 0 \rangle = M,$$

$$T^{\mu}_{\mu} = \frac{\tilde{\beta}(g)}{2g} G^{\mu\nu a} G^{a}_{\mu\nu} + \sum_{l=u,d,s} m_l (1 + \gamma_{m_l}) \bar{q}_l q_l,$$

In the chiral limit, the only contribution is from gluons!

PHYSICAL REVIEW D **104,** 074507 (2021)

# Demonstration of the hadron mass origin from the QCD trace anomaly

Fangcheng He,[1,*] Peng Sun,[2,†] and Yi-Bo Yang[1,3,4,5,‡]

($\chi$QCD Collaboration)

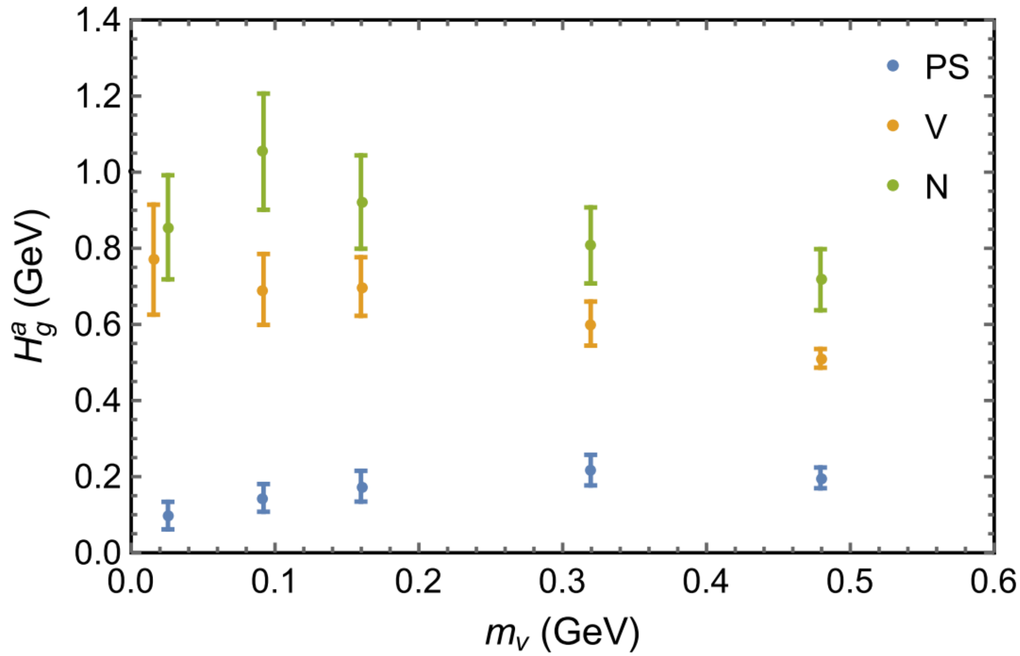



FIG. 3. The gluon trace anomaly contribution to the hadron mass. For five different quark masses, the corresponding pion masses are 0.340, 0.647, 0.864, 1.277, and 1.640 GeV. We can see that it is always small for the PS meson, while it approaches ~800 MeV for the nucleon and vector mesons in the chiral limit $m_v \to 0$.

# Confinement due to scale anomaly?

$$T^\mu_\mu = \frac{\tilde{\beta}(g)}{2g}\, G^{\mu\nu a} G^a_{\mu\nu} + \sum_{l=u,d,s} m_l (1+\gamma_{m_l}) \bar{q}_l q_l,$$

In quantum theory, gluons gravitate; scale anomaly induces conformally flat deformation of space-time. Can this be used to describe confinement?

QCD in curved space-time: A conformal bag model   Also: JHEP06(2009)055

Dmitri Kharzeev, Eugene Levin, and Kirill Tuchin
Phys. Rev. D **70**, 054005 – Published 3 September 2004

$$g_{\mu\nu}(x) \;=\; e^{h(x)}\, \delta_{\mu\nu}$$

$$S = \int d^4x \left( \frac{4\,|\epsilon_v|}{m^2}\, e^h\, (\partial_\mu h)^2 - \frac{1}{4}(F^a_{\mu\nu})^2 + |\epsilon_v|\, e^{2h} - \frac{1}{4}\, e^{2h} \left[ -\frac{b\, g^2}{32\,\pi^2} (F^a_{\mu\nu})^2 \right] \right)$$

This model belongs to the class of confining models proposed in
'''t Hooft hep-th/0207179:   It describes gluons in the dilaton background:

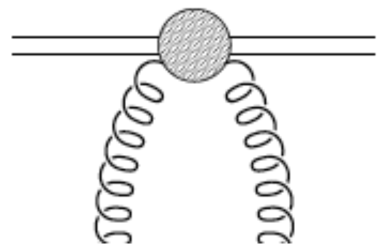
$$\mathcal{L} = -V(\chi) - Z(\chi)\frac{1}{4}(F^a_{\mu\nu})^2 \qquad Z(\chi) = -e^\chi (1-\chi)\, c + 1\,, \quad V(\chi) = -|\epsilon_v|\, e^\chi (1-\chi)$$

# How to measure the mass distribution inside the proton?

No dilatons available…
next best thing: a heavy quarkonium

M.B. Voloshin
1953-2020

QCD multipole expansion:



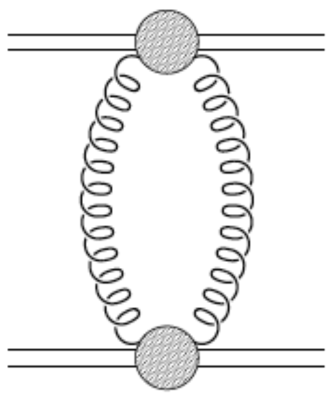
Voloshin '78; Appelquist, Fischler '78; Gottfried '78; Peskin '79; Novikov, Shifman '81; Leutwyler '81, Luke, Manohar, Savage '92, …

$$g^2\mathbf{E}^{a2} = \frac{g^2}{2}(\mathbf{E}^{a2} - \mathbf{B}^{a2}) + \frac{g^2}{2}(\mathbf{E}^{a2} + \mathbf{B}^{a2})$$

$$= -\frac{1}{4}g^2 G^a_{\alpha\beta}G^{a\alpha\beta} + g^2(-G^a_{0\alpha}G^{a\alpha}_0 + \frac{1}{4}g_{00}G^a_{\alpha\beta}G^{a\alpha\beta}) = \frac{8\pi^2}{b}\theta^\mu_\mu + g^2\theta^{(G)}_{00}$$

$$\theta^\mu_\mu \equiv \frac{\beta(g)}{2g}G^{a\alpha\beta}G^a_{\alpha\beta} = -\frac{bg^2}{32\pi^2}G^{a\alpha\beta}G^a_{\alpha\beta}\,, \quad \theta^{(G)}_{\mu\nu} \equiv -G^a_{\mu\alpha}G^{a\alpha}_\nu + \frac{1}{4}g_{\mu\nu}G^a_{\alpha\beta}G^{a\alpha\beta}$$

# Quarkonium interactions at low energy



Perturbation theory:

at large distances,
the Casimir-Polder
interaction (retardation)

Bhanot, Peskin '78

$$V^{\mathrm{pt}}(R) = -g^4 \left( \bar{d}_2 \frac{a_0^2}{\epsilon_0} \right)^2 \frac{23}{8\pi^3} \frac{1}{R^7}$$

Fujii, DK '99

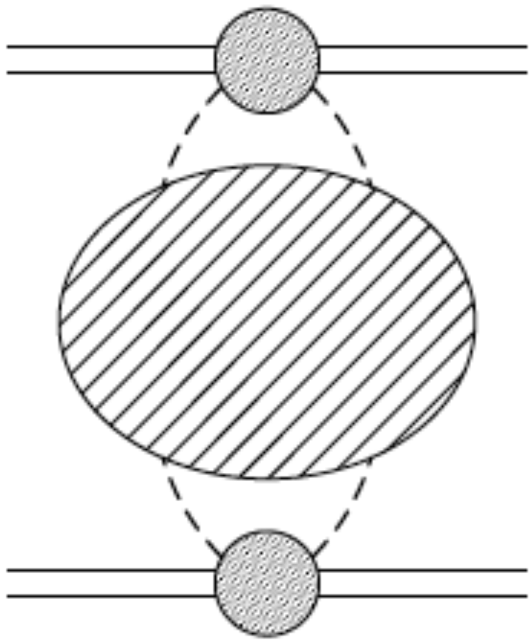
$23 = 15 + 8$

scalar $0^{++}$ tensor $2^{++}$

Beyond perturbation theory, scalar is strongly enhanced due to scale anomaly

# Quarkonium interactions at low energy and the scale anomaly

But, at very large distances, the interaction must be dominated by the lightest physical states - pions



conversion of gluons to pions is a (hopeless?) non-perturbative problem

…but, can use scale anomaly matching!

Voloshin, Zakharov ’80
Novikov, Shifman ‘82

# Quarkonium interactions at low energy and the scale anomaly

Use RG invariance to match the EMT computed in QCD and in the chiral theory:

$$\theta^\mu_\mu = -2\,\frac{f_\pi^2}{4}\,\mathrm{tr}\,\partial_\mu U \partial^\mu U^\dagger \;-\; m_\pi^2 f_\pi^2\,\mathrm{tr}\left(U + U^\dagger\right)$$

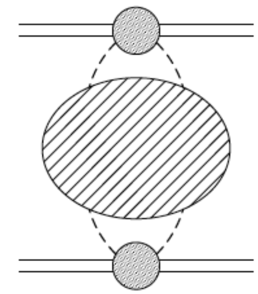
to lowest order in the pion field

$$\theta^\mu_\mu = -\partial_\mu \pi^a \partial^\mu \pi^a + 2m_\pi^2 \pi^a \pi^a + \cdots$$

In the chiral limit scale anomaly yields:

$$\langle \pi^+ \pi^- | \theta^\mu_\mu | 0 \rangle = q^2$$

# Quarkonium interactions at low energy and the scale anomaly

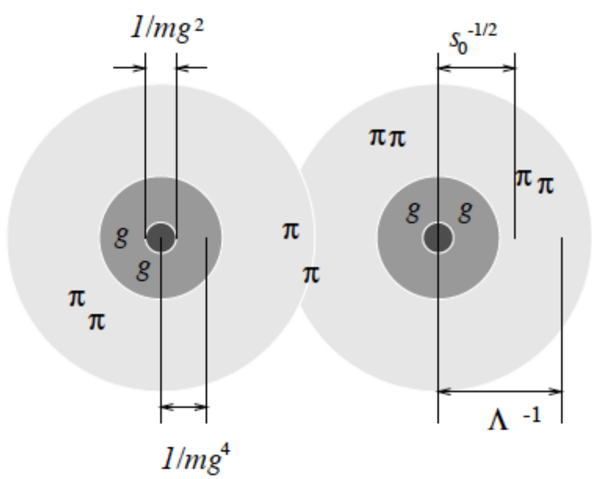


The result (long distances):

$$V^{\pi\pi}(R) \to -\left(\bar{d}_2 \frac{a_0^2}{\epsilon_0}\right)^2 \left(\frac{4\pi^2}{b}\right)^2 \frac{3}{2}(2m_\pi)^4 \; \frac{m_\pi^{1/2}}{(4\pi R)^{5/2}} \; e^{-2m_\pi R}.$$

Fujii, DK, PRD (1999)

See also A.Belitsky and X.Ji, PLB (2002)




1. Not a Yukawa potential (retardation)
2. The QCD coupling has disappeared at large distance (but not b from the beta-function)
3. Entirely due to scalar $0^{++}$ exchange

# This two-pion tail in quarkonium interactions has just been clearly observed on the lattice:

arXiv:2205.10544

**Attractive $N$-$\phi$ Interaction and Two-Pion Tail from Lattice QCD near Physical Point**

Yan Lyu,[1,2,*] Takumi Doi,[2,†] Tetsuo Hatsuda,[2,‡] Yoichi Ikeda,[3,§]
Jie Meng,[1,4,¶] Kenji Sasaki,[3,**] and Takuya Sugiura[2,††]

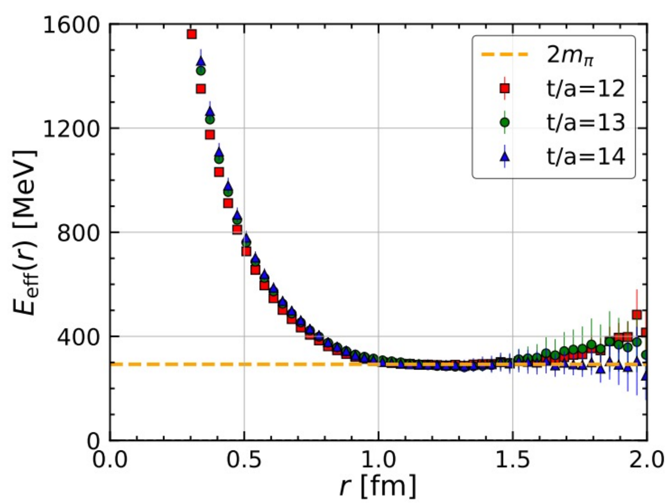



FIG. 2. (Color online). The spatial effective energy $E_{\mathrm{eff}}(r)$ as a function of separation $r$ at Euclidean time $t/a = 12$ (red squares), 13 (green circles) and 14 (blue triangles). The orange dashed line corresponds to $2m_\pi$ with lattice pion mass $m_\pi = 146.4$ MeV.

This is a consequence of **non-perturbative** mixing between the scalar gluon and quark operators induced by spontaneous breaking of chiral symmetry. It is controlled by scale anomaly:

CERN-TH/99-278
RIKEN-BNL preprint
UT-Komaba preprint

**Scalar Glueball–Quarkonium Mixing and the Structure of the QCD Vacuum**

**John Ellis**[a], **Hirotsugu Fujii**[b] and **Dmitri Kharzeev**[c]

$$\lim_{q\to 0} i\int dx\; e^{iqx}\langle 0|T\left\{\frac{\beta(\alpha_s)}{4\alpha_s}G^2(x),\;\mathcal{O}(0)\right\}|0\rangle = (-d)\langle\mathcal{O}\rangle + O(m_q)$$

General LET:

Novikov, Shifman, Vainshtein, Zakharov '81

For a scalar quark operator and a single resonance:

$$\frac{1}{m_\sigma^2}\langle 0|\frac{\beta(\alpha_s)}{4\alpha_s}G^2|k\rangle\langle k|\sum_i m_i\bar{q}_i q_i|0\rangle = -3\langle\sum_i m_i\bar{q}_i q_i\rangle.$$

# Probing the proton mass

## The quarkonium-proton scattering amplitude

$$F_{\Phi h} = r_0^3 \epsilon_0^2 \sum_{n=2}^{\infty} d_n \langle h | \frac{1}{2} G^a_{0i} (D^0)^{n-2} G^a_{0i} | h \rangle$$

Wilson coefficients

$$d_n^{(1S)} = \left(\frac{32}{N}\right)^2 \sqrt{\pi}\, \frac{\Gamma(n+\frac{5}{2})}{\Gamma(n+5)}$$

M.Peskin '78

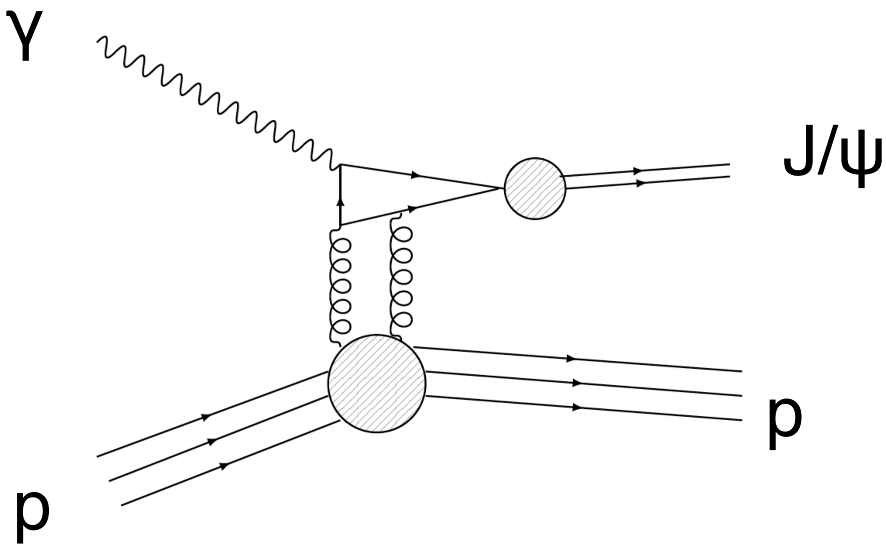
$$d_n^{(2S)} = \left(\frac{32}{N}\right)^2 4^n \sqrt{\pi}\, \frac{\Gamma(n+\frac{5}{2})}{\Gamma(n+7)} (16n^2 + 56n + 75)$$

$$d_n^{(2P)} = \left(\frac{15}{N}\right)^2 4^n 2\sqrt{\pi}\, \frac{\Gamma(n+\frac{7}{2})}{\Gamma(n+6)}$$

DK, '96

nucl-th/9601029

# Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton




Near threshold, dominance of

$$g^2\mathbf{E}^{a2} = \frac{8\pi^2}{b}\theta^\mu_\mu + g^2\theta^{(G)}_{00}$$

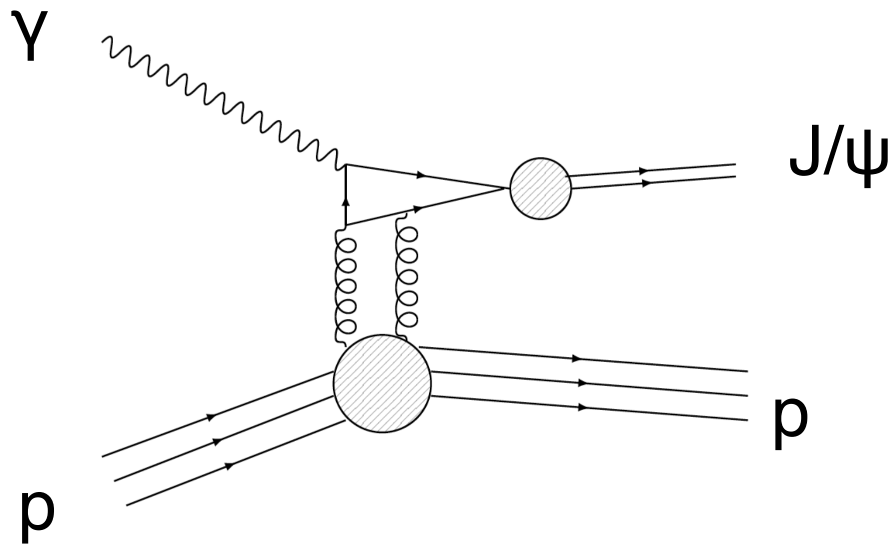
Assuming the validity of vector meson dominance, can relate photoproduction to quarkonium scattering amplitude and probe the mass of the proton

DK '96; DK, Satz, Syamtomov, Zinovjev '99

Other approaches to threshold photoproduction:

Hatta, Yang '18; Hatta, Rajan, Yang '19; Mamo, Zahed '19-'22;
Ji, 2102.07830; Gao, Ji, Liu, 2103.11506; Sun, Tong, Yuan, 2103.12047…

# Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton

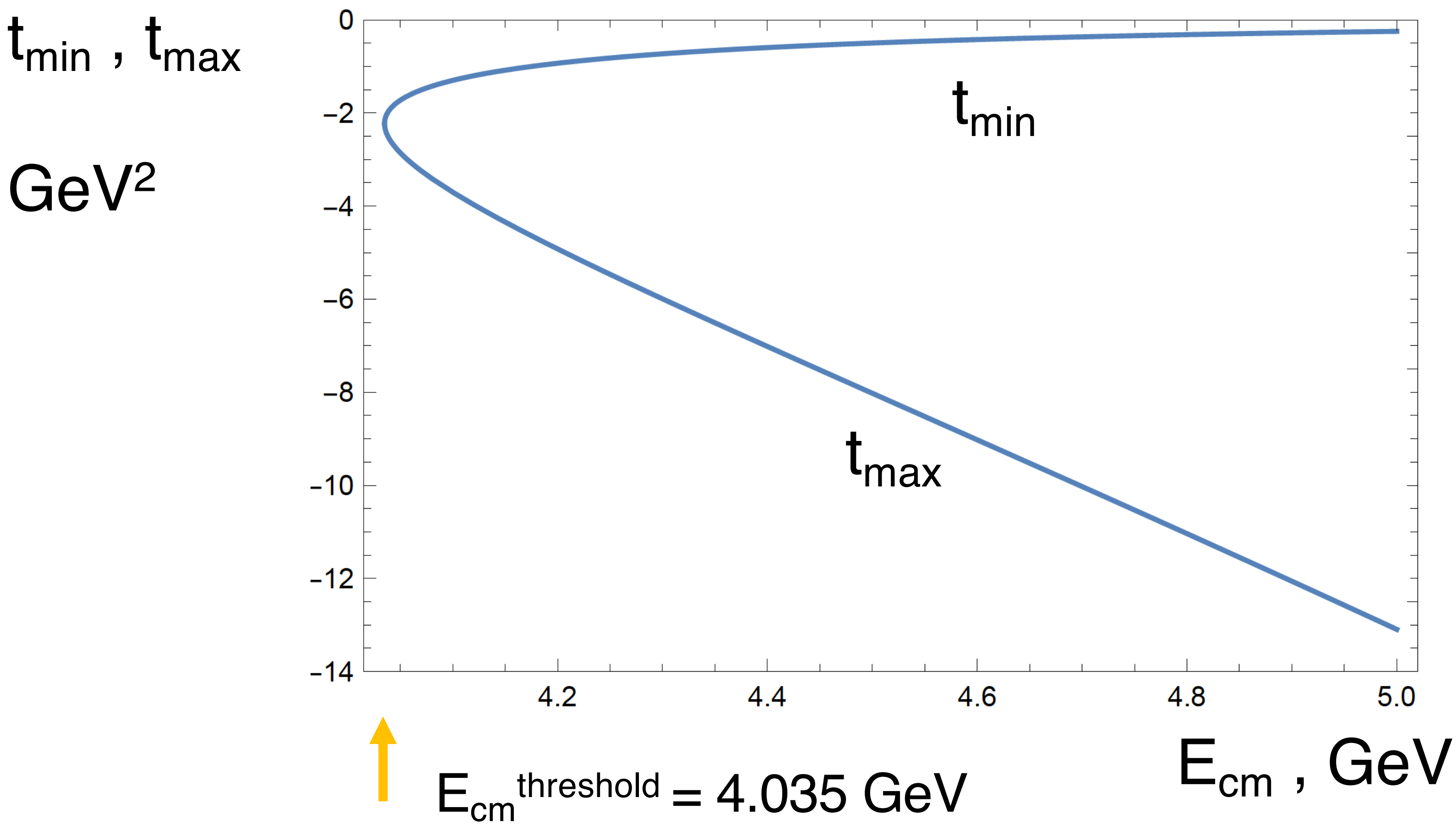



Large minimum momentum transfer at threshold:

$$t_{min} = -\frac{M_\psi^2 M}{M_\psi + M} \simeq -2.23\ \mathrm{GeV}^2 \simeq -(1.5\ \mathrm{GeV})^2$$

–> VDM questionable.
but, scanning the energy range near the threshold,
we measure the scalar gravitational formfactor –
can extract the proton mass distribution!

# Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton

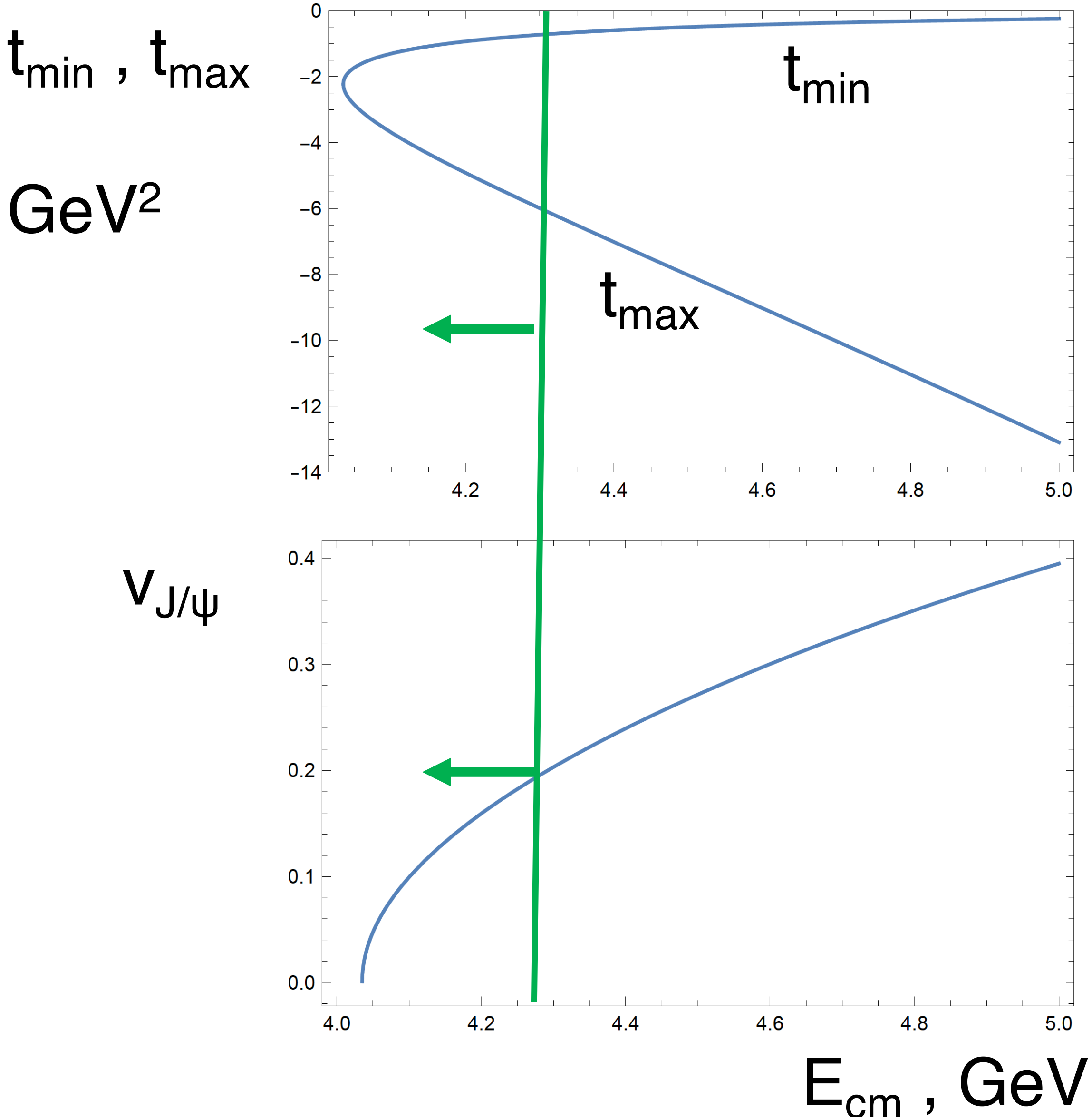



Can measure scalar gravitational formfactor in a broad kinematical domain!

At threshold:

$$t_{min} = -\frac{M_\psi^2 M}{M_\psi + M} \simeq -2.23 \text{ GeV}^2 \simeq -(1.5 \text{ GeV})^2$$

# Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton




The scalar operator dominates for small velocity of heavy quarkonium;

Limiting $v_{J/\psi} < 0.2$, (corrections ~ $v_{J/\psi}^2$ ) the optimal kinematical region is:

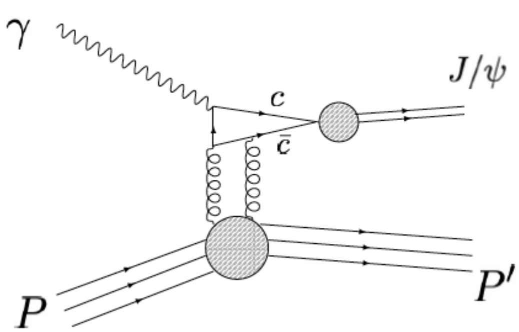
$E_{cm} < 4.25$ GeV
$E_{\gamma} < 9.2$ GeV

$-t < 6$ GeV$^2$

# Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton

The amplitude:

$$\mathcal{M}_{\gamma P\to\psi P}(t) = -Qe\ c_2\ 2M\ \langle P'|g^2\mathbf{E}^{a2}|P\rangle,$$




$$Qe = 2e/3$$

$$\mathcal{M}_{\gamma P\to\psi P}(t) = -Qe\ c_2\ \frac{16\pi^2 M}{b}\ \langle P'|T|P\rangle$$

Differential cross section:

$$\frac{d\sigma_{\gamma P\to\psi P}}{dt} = \frac{1}{64\pi s}\ \frac{1}{|\mathbf{p}_{\gamma cm}|^2}\ |\mathcal{M}_{\gamma P\to\psi P}(t)|^2$$

DK, PRD'21

$$\sigma_{\gamma P\to\psi P}(s) = \int_{t_{min}}^{t_{max}} dt\ \frac{d\sigma_{\gamma P\to\psi P}}{dt},$$

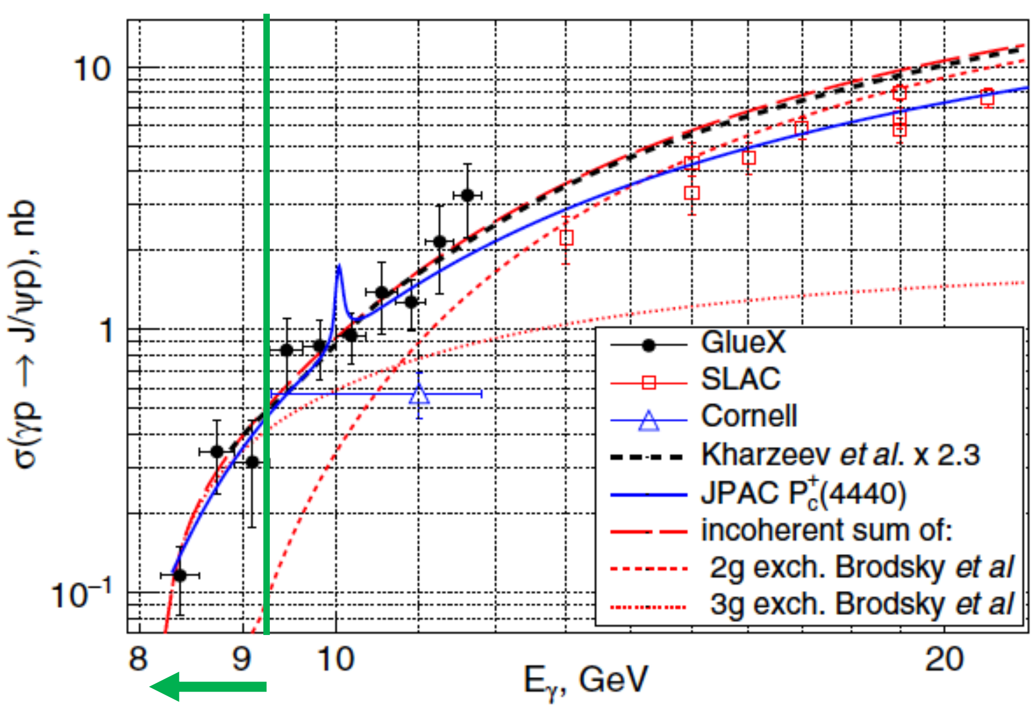
PHYSICAL REVIEW LETTERS 123, 072001 (2019)

Editors' Suggestion

# First Measurement of Near-Threshold $J/\psi$ Exclusive Photoproduction off the Proton

A. Ali,[10] M. Amaryan,[22] E. G. Anassontzis,[2] A. Austregesilo,[3] M. Baalouch,[22] F. Barbosa,[14] J. Barlow,[7] A. Barnes,[3] E. Barriga,[7] T. D. Beattie,[23] V. V. Berdnikov,[17] T. Black,[20] W. Boeglin,[6] M. Boer,[4] W. J. Briscoe,[8] T. Britton,[14] W. K. Brooks,[24] B. E. Cannon,[7] N. Cao,[11] E. Chudakov,[14] S. Cole,[1] O. Cortes,[8] V. Crede,[7] M. M. Dalton,[14] T. Daniels,[20] A. Deur,[14] S. Dobbs,[7] A. Dolgolenko,[13] R. Dotel,[6] M. Dugger,[1] R. Dzhygadlo,[10] H. Egiyan,[14] A. Ernst,[7] P. Eugenio,[7] C. Fanelli,[16] S. Fegan,[8] A. M. Foda,[23] J. Foote,[12] J. Frye,[12] S. Furletov,[14] L. Gan,[20] A. Gasparian,[19] V. Gauzshtein,[25,26] N. Gevorgyan,[27] C. Gleason,[12] K. Goetzen,[10] A. Goncalves,[7] V. S. Goryachev,[13] L. Guo,[6] H. Hakobyan,[24] A. Hamdi,[10] S. Han,[29] J. Hardin,[16] G. M. Huber,[23] A. Hurley,[28] D. G. Ireland,[9] M. M. Ito,[14] N. S. Jarvis,[3] R. T. Jones,[5] V. Kakoyan,[27] G. Kalicy,[4] M. Kamel,[6] C. Kourkoumelis,[2] S. Kuleshov,[24] I. Kuznetsov,[25,26] I. Larin,[15] D. Lawrence,[14] D. I. Lersch,[7] H. Li,[3] W. Li,[28] B. Liu,[11] K. Livingston,[9] G. J. Lolos,[23] V. Lyubovitskij,[25,26] D. Mack,[14] H. Marukyan,[27] V. Matveev,[13] M. McCaughan,[14] M. McCracken,[3] W. McGinley,[3] J. McIntyre,[5] C. A. Meyer,[3] R. Miskimen,[15] R. E. Mitchell,[12] F. Mokaya,[5] F. Nerling,[10] L. Ng,[7] A. I. Ostrovidov,[7] Z. Papandreou,[23] M. Patsyuk,[16] P. Pauli,[9] R. Pedroni,[19] L. Pentchev,[14,*] K. J. Peters,[10] W. Phelps,[8] E. Pooser,[14] N. Qin,[21] J. Reinhold,[6] B. G. Ritchie,[1] L. Robison,[21] D. Romanov,[17] C. Romero,[24] C. Salgado,[18] A. M. Schertz,[28] R. A. Schumacher,[3] J. Schwiening,[10] K. K. Seth,[21] X. Shen,[11] M. R. Shepherd,[12] E. S. Smith,[14] D. I. Sober,[4] A. Somov,[14] S. Somov,[17] O. Soto,[24] J. R. Stevens,[28] I. I. Strakovsky,[8] K. Suresh,[23] V. Tarasov,[13] S. Taylor,[14] A. Teymurazyan,[23] A. Thiel,[9] G. Vasileiadis,[2] D. Werthmüller,[9] T. Whitlatch,[14] N. Wickramaarachchi,[22] M. Williams,[16] T. Xiao,[21] Y. Yang,[16] J. Zarling,[12] Z. Zhang,[29] G. Zhao,[11] Q. Zhou,[11] X. Zhou,[29] and B. Zihlmann[14]

(GlueX Collaboration)




Need to focus on the threshold region!

$E_{cm} < 4.25$ GeV
$E_\gamma < 9.2$ GeV

# Threshold photoproduction of quarkonium: the effect of the scalar gravitational formfactor

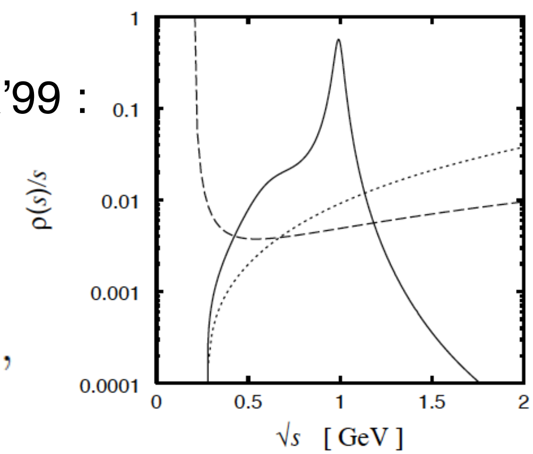
The scalar gravitational formfactor can be constrained theoretically by using:

i) dispersion relations;

ii) low-energy theorems of broken scale invariance;

iii) experimental data on ππ phase shifts and scalar mesons

See e.g. Fujii, DK'99 :




However, as a first step, can try a simple dipole formfactor of the type used for electromagnetic formfactor:

$$G(t) = \frac{M}{(1 - t/M_s^2)^2}$$

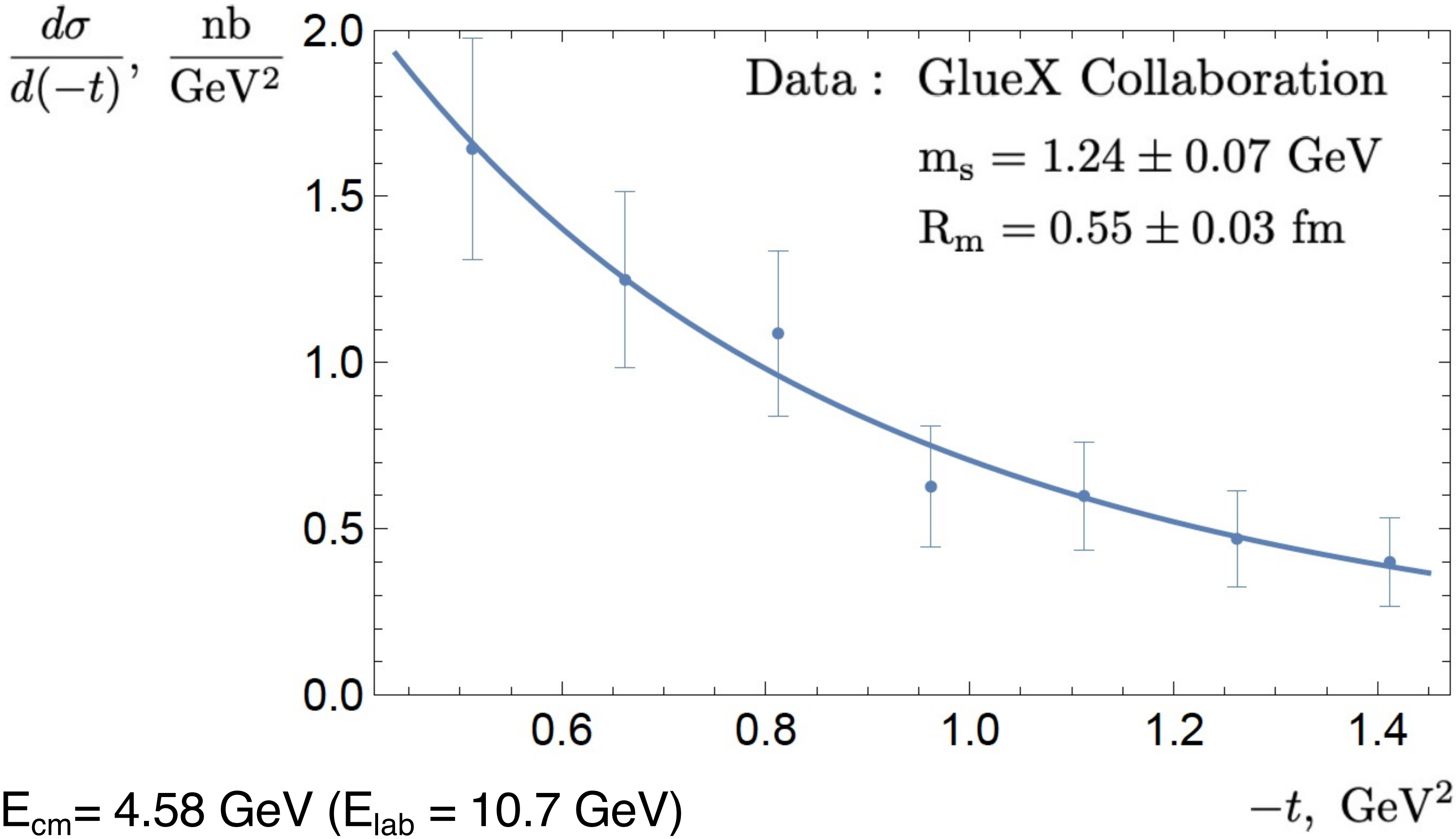
radius $\langle R_{\rm M}^2 \rangle = \frac{6}{M} \left. \frac{dG}{dt} \right|_{t=0},$

Dipole formfactor was also used for 2-gluon coupling in perturbative models

See e.g. Frankfurt, Strikman '02

# Differential cross section

DK, arXiv:2102.00110




$E_{cm}$= 4.58 GeV ($E_{lab}$ = 10.7 GeV)

$$|c_2|^2 = 0.043 \pm 0.006 \text{ fm}^4$$

Lattice QCD, P. Shanahan, W. Detmold PRD'19:

$$m_s = 1.13 \pm 0.06 \text{ GeV}$$ (Traceless gluon operator)

$$c_2 \sim \pi r_{c\bar{c}}^2 \qquad r_{c\bar{c}} \simeq 0.1 \text{ fm}$$

# The proton mass radius

The r.m.s. “proton mass radius” from GlueX data:

DK, arXiv:2102.00110

$$R_{\rm m} \equiv \sqrt{\langle R^2_{\rm m} \rangle} = 0.55 \pm 0.03 \ {\rm fm}$$

Compare to the proton charge radius:

$$\bar{\rm R}_{\rm c} \equiv \sqrt{R^2_c} = 0.8409 \pm 0.0004 \quad {\rm fm}$$

See J.Bernauer, EPJ 234 (2020) for review

A more compact mass distribution? Need more data!

| VALUE (fm) | | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|---|
| **0.8409 ± 0.0004** | OUR AVERAGE | | | | |
| 0.833 ±0.010 | 1 | BEZGINOV | 2019 | LASR | 2S-2P transition in H |
| 0.831 ±0.007 ±0.012 | 2 | XIONG | 2019 | SPEC | $e\,p \rightarrow ep$ form factor |
| 0.84087 ±0.00026 ±0.00029 | | ANTOGNINI | 2013 | LASR | $\mu p$ -atom Lamb shift |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| 0.877 ±0.013 | 3 | FLEURBAEY | 2018 | LASR | 1S-3S transition in H |
| 0.8335 ±0.0095 | 4 | BEYER | 2017 | LASR | 2S-4P transition in H |
| 0.8751 ±0.0061 | | MOHR | 2016 | RVUE | 2014 CODATA value |
| 0.895 ±0.014 ±0.014 | 5 | LEE | 2015 | SPEC | Just 2010 Mainz data |
| 0.916 ±0.024 | | LEE | 2015 | SPEC | World data, no Mainz |
| 0.8775 ±0.0051 | | MOHR | 2012 | RVUE | 2010 CODATA, $ep$ data |
| 0.875 ±0.008 ±0.006 | | ZHAN | 2011 | SPEC | Recoil polarimetry |
| 0.879 ±0.005 ±0.006 | | BERNAUER | 2010 | SPEC | $e\,p \rightarrow ep$ form factor |

**2020 Review of Particle Physics.**
P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

Some day:
p MASS RADIUS in PDG?

# When Color meets Gravity; Near-Threshold Exclusive $J/\psi$ Photoproduction on the Proton

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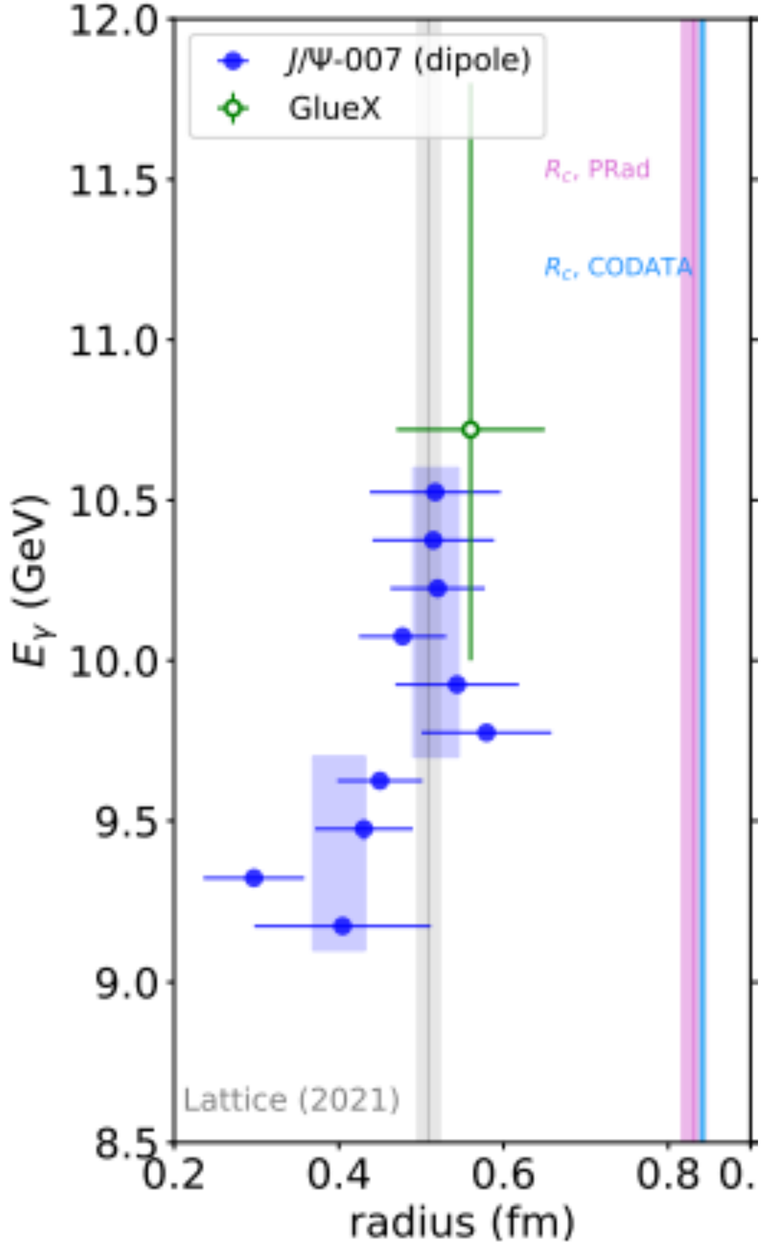
J/ψ 007 Coll.

arXiv:2207.05212

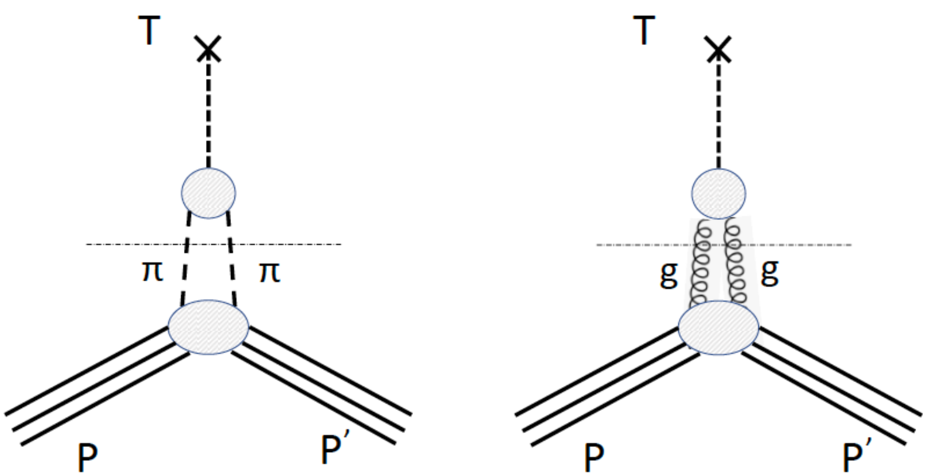



below 9.7 GeV. The radius determined in the energy independent region averages to $\sqrt{\langle r_m^2 \rangle} = 0.52 \pm 0.03$fm. This result

# Theoretical uncertainties

- Higher dimensional operators (suppressed by $1/m_c$)
- Chiral limit (we omitted the scalar quark operator)
- Gluon operators with derivatives (~ 5% close to threshold)
- t-dependence of short-distance coefficient $c_2$ ($\sim t/4m_c^2$)
- Dipole parameterization of formfactor

# Why is proton mass radius smaller than the charge radius?




Spectral representation –

EM formfactor: $M_\rho = 0.77$ GeV

Scalar gravitational formfactor: scalar glueball M = 1.5 GeV

But: scalar gluon current mixes with the scalar quark current – σ(500) is lighter than the ρ!

The real reason (?) – decoupling of Goldstone bosons:

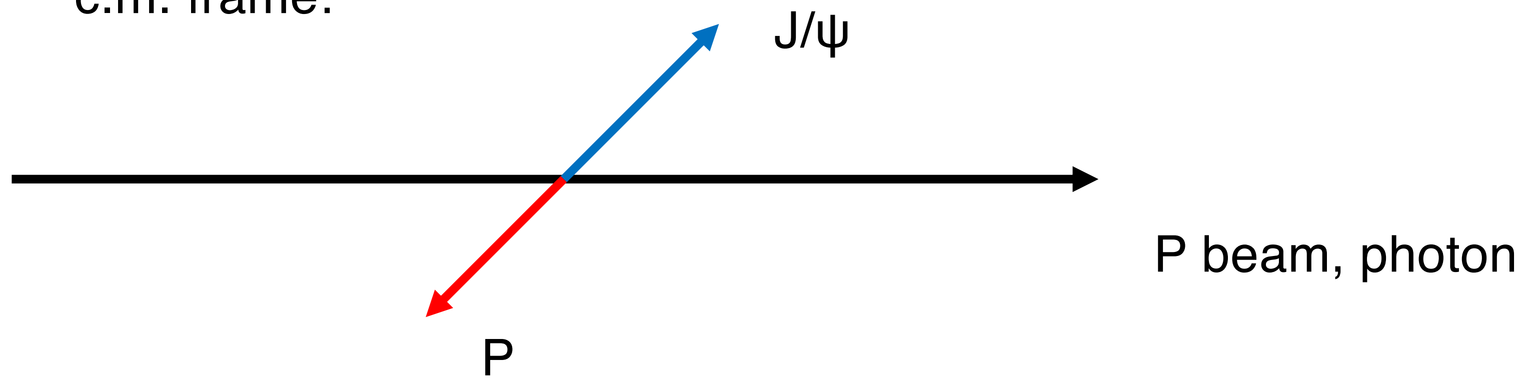
$$\langle 0|T|\pi^+\pi^-\rangle = q^2$$

DK, arXiv:2102.00110

# Future measurements

- GlueX has 10 times more data
- Future: SoLID@Jlab (~ 2028),  EIC (including Υ !)
- Polarization? (scalar vs tensor)
- Also: ultra-peripheral collisions at RHIC?

c.m. frame:




For a fixed invariant mass (cms energy), measure the angular distribution – differential cross section of photoproduction

# Summary

- The formfactors are fundamental properties of nucleons, and provide a unique window into QCD
- After 70 years of experimental and theoretical studies, much progress has been made
- (At least) three puzzles have recently emerged:

  1. $G_{E,}$ $G_M$ : unpolarized vs polarized results

  2. Proton charge radius: scattering vs spectroscopy

  3. Proton mass radius vs proton charge radius

- Vibrant field, with a lot of work that has to be done!