# Form Factors from Lattice QCD

Phiala Shanahan, MIT

Image Credit: 2018 EIC User's Group Meeting



Massachusetts Institute of Technology

#### Nucleon and Nuclear Form Factors from Lattice QCD

- Electromagnetic form factors
- Axial and pseudo-scalar form factors
- Generalised form factors incl. gravitational
- Nuclear form factors

Numerical first-principles approach to non-perturbative QCD

• Discretise QCD onto 4D space-time lattice

- Approximate QCD path integral using Monte-Carlo methods and importance sampling
- Run on supercomputers and dedicated clusters
- Take limit of vanishing discretisation, infinite volume, physical quark masses



#### Numerical first-principles approach to non-perturbative QCD

- Euclidean space-time
  - Non-zero lattice spacing
  - Finite volume
- Some calculations use largerthan-physical quark masses (cheaper)



Approximate the QCD path integral by Monte Carlo

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A\mathcal{D}\overline{\psi}\mathcal{D}\psi\mathcal{O}[A,\overline{\psi}\psi] e^{-S[A,\overline{\psi}\psi]} \longrightarrow \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\text{conf}}} \sum_{i}^{N_{\text{conf}}} \mathcal{O}([U^{i}])$$

with field configurations  $U^i$  distributed according to  $e^{-S[U]}$ 

Numerical first-principles approach to non-perturbative QCD

#### Calculations use world's largest computers

- Many millions of CPU/ GPU/KNL hours
- Specifically designed processors for QCD
   (QCDOC precursor of BlueGene computers)





#### Numerical first-principles approach to non-perturbative QCD

#### INPUT

Lattice QCD action has same free parameters as QCD: quark masses,  $\alpha_S$ 

- Fix quark masses by matching to measured hadron masses, e.g.,  $\pi, K, D_s, B_s$  for u, d, s, c, b
- One experimental input to fix lattice spacing in GeV (and also  $\alpha_S$ ), e.g., 2S-1S splitting in  $\Upsilon$ , or  $f_{\pi}$  or  $\Omega$  mass

#### OUTPUT

Calculations of all other quantities are QCD predictions



### Uncertainties in lattice QCD

#### Differences between calculations

• Uncertainties on a single "ensemble"

- Data volume (statistical sampling)
- Fitting methodology (e.g., treatment of "excited-state contamination")
- Renormalisation procedure (i.e., matching from lattice quantities to MS)
- ullet Uncertainties of result "extrapolated to physical p $\pm$ 
  - Continuum extrapolation: range of lattice spa $^{-}$
  - ullet Infinite-volume extrapolation: range of volumes  $^a$
  - Tuning of the bare quark masses: values of pion, kaon masses, extrapolation/interpolation thereof, isospin-breaking, ...



 $\rightarrow$  ()

 $m_{\pi} 
ightarrow 140 {
m MeV}$   $m_{\pi} 
ightarrow 140 {
m MeV}$   $m_{\pi} 
ightarrow 140 {
m MeV}$ Phiala Shanahan, MIT

# **FLAG**ន៍ Flavour Lattice Averaging Group Group

Many lattice calculations of what are now "simple" flavour physics and hadron structure quantities

FLAG: Flavour Lattice Averaging Group Similar effort to the PDG, for Lattice QCD

- Members from most major lattice QCD collaborations
- ullet Evaluates and grades different aspects of each calculation  $\prec$  O
- Provides averages as the "Lattice QCD community consensus" value for a given quantity
- Includes lattice dictionary and summaries for non-experts
- Summary report every ~2 years: Feb 2024 update at http://flag.unibe.ch
- New version planned for October 2024: coverage expanded in 2019 version to include nucleon charges, continues to expand over time (FFs likely soon)

## Form factors from Lattice QCD

#### **Precision Era**

Fully-controlled w/ few-percent errors now or within ~5y

- Forward limits of FF incl. charges, moments of GPDs
- *t*-dependence of EM, axial, pseudo scalar FFs

#### **Early Era**

Fully-controlled w/ ~15-percent errors now or within ~5y

- *t*-dependence of GFFs
- Forward limits of nuclear FFs

 $k \tau$ 

#### **Exploratory Era**

First calculations, timeline for controlled calculations unclear

- Many transition and resonance FFs
- t-dependence of nuclear FFs



#### Nucleon and Nuclear Form Factors from Lattice QCD

- Electromagnetic form factors
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**Electromagnetic form factors** encode coupling of the nucleon to an electromagnetic current  $\langle N(p',s')|\bar{q}\gamma^{\mu}q|N(p,s)\rangle = \bar{u}_N(p',s') \begin{bmatrix} \text{Dirac FF} & \text{Pauli FF} \\ F_1(Q^2)\gamma^{\mu} + F_2(Q^2) \frac{i\sigma^{\mu\nu}q_{\nu}}{2M_N} \end{bmatrix} u_N(p,s) \qquad q = p' - p$   $q^2 = -Q^2$ 

• Linear combinations define the **Sachs electric and magnetic** form factors

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_N^2} F_2(Q^2) \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

- LQCD results for EMFFs and associated radii are now fully-controlled for  $0 \le Q^2 \le 1 \text{GeV}^2$ ; physical quark masses, several lattice volumes, spacings
- Few calculations of strange contributions from LQCD, but reasonably precise
- Efforts driving to large  $Q^2 \lesssim 10 \text{GeV}^2$ , but still work-in-progress
- Work ongoing to study more complicated FFs, e.g., transition, resonance FFs

New work in 2023 that separates proton and neutron FFs with complete uncertainty quantification [Djukanovic et al., arXiv:2309.06590 (2023)]



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New work in 2023 that separates proton and neutron FFs with complete uncertainty quantification [Djukanovic et al., arXiv:2309.06590 (2023)]

- Electric (charge) radius of the proton closer to muonic hydrogen spectroscopy & recent *ep*-scattering experiments than to the A1 *ep*-scattering result
- Magnetic radius compatible with the analyses of A1 data, in tension with the other collected world data
- Interesting tension with magnetic neutron radius





New work in 2023 with large lattice volumes (up to  $160^4$ ) and yields many points in the low- $Q^2$  region [Tsuji et al. [PACS Collaboration], arXiv:2311.10345 (2023)]

- Improves control over finite-volume and discretisation uncertainties; consistent with recent *ep*-scattering data
- Isovector combinations only so far (with systematic control)





### High-momentum form factors

#### New efforts pushing towards high- $Q^2$ form factors

[Syritsyn et al. Few-Body Syst 64:72 (2023), see also QCDSF-UKQCD-CSSM], 2202.01366 (2022)]

- Not completely controlled (discretisation, neglected disconnected contributions, ...)
- Significant discrepancies with phenomenology [Alberico et al, PRC79:065204 (2008)]



[Syritsyn et al. Few-Body Syst 64:72 (2023)]

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- Not completely controlled (discretisation, neglected disconnected contributions, ...)
- Ratios qualitatively consistent with phenomenological fits of expt. data [Alberico] & quark+diquark Faddeev equation calculations [Cui]



[Syritsyn et al. Few-Body Syst 64:72 (2023)]

# Odd-parity EN

New work to study helicity amplitudes  $\sqrt[5]{4}^{-0.2}$ the ground state nucleon to the first tw  $_{-0.4}$ [Stokes et al., 2404.07625 (2024)]

- First attempt to establish formalism, r
- Qualitatively consistent with relativised constituent quark model





#### Nucleon and Nuclear Form Factors from Lattice QCD

• Electromagnetic form factors

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#### Axial charges and form factors

Axial and induced pseudoscalar form factors encode coupling of the proton to an axial current  $A^q_\mu = \overline{q}\gamma_\mu\gamma_5 q$ 

$$\langle N(p',s')|A_{\mu}|N(p,s)\rangle = \bar{u}_{N}(p',s') \left[ \gamma_{\mu} G_{A}(Q^{2}) - \frac{Q_{\mu}}{2m_{N}} G_{P}(Q^{2}) \right] \gamma_{5} u_{N}(p,s) \qquad \begin{array}{l} q = p' - p \\ q^{2} = -Q^{2} \end{array}$$

- W-boson interaction incl.  $A_{\mu}^{(u-d)}$ : relevant in quasi-elastic  $\nu$  scattering,  $\mu$  capture
- Z-boson interaction incl.  $A_{\mu}^{(u-d-s)}$ : relevant in elastic  $\nu p$ , parity-violating ep scattering
- Axial charges defined in forward limits  $g_A^q \equiv G_A^q(0)$
- Combination of axial, induced pseudoscalar, pseudoscalar FFs constrained by Axial Ward-Takahashi / Partially Conserved Axial Current identity (PCAC)

$$G_A(Q^2) - \frac{Q^2}{4m_N^2}G_P(Q^2) = \frac{m_q}{m_N}G_5(Q^2) \qquad \langle N(p',s')|P|N(p,s)\rangle \\ = G_5(Q^2)\bar{u}_N(p',s')\gamma_5u_N(p,s)$$
Pseudoscalar FF

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### Nucleon isovector axial charge

FLAG includes summaries and averaged values of axial charges from lattice QCD

• Community consensus for isovector charge, with few-percent uncertainties



[FLAG, arXiv:2111.09849]

## Nucleon flavour-diagonal axial charges

FLAG includes summaries and averaged values of axial charges from lattice QCD  $$_{\rm FLG2021}$$ 

 Flavour-diagonal charges have been less well-studied, but community consensus is developing



[FLAG, arXiv:2111.09849]



#### Isovector axial & pseudoscalar FFs

Isovector axial & pseudoscalar FFs studied by many LQCD collabs

- Systematic control: physical pion mass, control of discretisation systematics etc
- PCAC relation satisfied within uncertainties

Recent example from one collaboration in 2023 [ETMC 2309.05774 (2023)]:



#### Isovector axial & pseudoscalar FFs

Isovector axial & pseudoscalar FFs studied by many LQCD collabs

Consensus between different LQCD calculations/collaborations



- Combined~10% uncertainties for  $0 \le Q^2 \le 1 \text{GeV}^2$
- Consistent with experimental results from MINERvA
- Tension with older *v*-deuterium bubble chamber scattering data
- Dipole fit ansatz insufficient

[Compilation from Gupta 2401.16614] Lattice QCD: PNDME 2305.11330 (2023), RQCD 1911.13150 (2019), ETMC 2309.05774 (2023), NME 2103.05599 (2021), Mainz 2207.03440 (2022) vD fit: Taken from 1603.03048 (2016)

# Lattice QCD compared with MINERIA

Isovector axial & pseudoscalar FFs'studied by many LCD collabs 2.0

- Consensus between different LOGP calculations/collaborations 1.5
- Consistent with experimental results from MINERvA  $Q^2 \,[{
  m GeV}^2]$ antineutrino-hydrogen data

C 0.8 H

elastic differential cross-section  $\theta_{\mu} \leq 20^{\circ}$  1.5 GeV  $\leq p_{\mu} \leq 20$  GeV **Axial FF** 3. deuterium fit  $\overline{\nu}_{\mu}p \rightarrow \mu^{+} n \theta_{\mu} \le 20^{0} \quad 1.5 \text{ GeV} \le p_{\mu} \le 20 \text{ GeV}$ hydrogen fit OME(|OCD)deuterium fit for  $F_{A^{\text{s}}}$ deuterium fit  $cm^2$ 1.0 hydrogen fit  $\overline{\nu}_{\mu}p \rightarrow \mu^{+} \ n$ PNDME  $F_{\Delta}$ PNDME  $GeV^{21}$ deuterium fit for EA  $dQ^2$ dσ  $G_A (Q^2)$  $PNDME \; F_A \; (\texttt{LQCD})$  $\frac{10^{38}}{d\sigma}[$ hydrogen data  $G_{A}\left(Q^{2}\right)$ 0.5  $10^{38}$  $0.5 \cdot$ GeV<sup>2</sup> 0.  $10^{38}$ 0.1 0.5 1.0 Ó  $Q^2$  [GeV<sup>2</sup>] Q<sup>2</sup>'|Ge" 0.1 0.5 1.5 1.0 2.0  $Q^2 [GeV^2]$  $Q^2 [GeV^2]$ [Tomalak, Gupta, Bhattacharya, 2307.14920 (2023)]

Antineutrino-nucleon charged current

2.0

### Strange and charm form factors

Flavour-decomposition of FFs also calculated, but less well-studied



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#### Flavour octet and singlet FFs

Flavour-decomposition of FFs also calculated, but less well-studied



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[Alexandrou et al., 2106.13468 (2021)]

### Axial charges and form factors

- LQCD results for isovector charges & FFs fully-controlled
  - Axial, induced pseudoscalar, pseudoscalar satisfy PCAC relation
  - Results from different collaborations consistent within ~10% uncertainties for  $0 \le Q^2 \le 1 {\rm GeV}^2$
- Consistent with experimental results from MINERvA, tension with older v-deuterium bubble chamber scattering data
- Dipole fit ansatz insufficient
- Complete flavour decomposition (including *s*, *c* quark contributions) also computed, but less well-studied, less precise, c.f. isovector

Community-consensus isovector **axial** and **pseudoscalar form factors** from LQCD likely to reach ~1% precision for  $Q^2 \lesssim 2 \text{GeV}^2$  by ~2030

#### Nucleon and Nuclear Form Factors from Lattice QCD

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#### **Generalised Form Factors**

**Generalised form factors** encode moments of Generalised Parton Distribution functions (GPDs)

e.g., Gravitational form factors

- Encode "graviton scattering" from the nucleon
- Related to leading-twist chiral-even GPDs

$$\begin{split} \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p', s'| G_a^{\{\mu\alpha}(-\frac{\lambda}{2}n) \left[ \mathcal{U}_{\left[-\frac{\lambda}{2}n,\frac{\lambda}{2}n\right]}^{(A)} \right]_{ab}} G_{b\alpha}^{\{\nu\}}(\frac{\lambda}{2}n) |p,s\rangle & \text{ Strength tensor } \\ &= \frac{1}{2} \left( H_g(x,\xi,t) \bar{U}(p',s') P^{\{\mu} \gamma^{\nu\}} U(p,s) + E_g(x,\xi,t) \bar{U}(p',s') \frac{P^{\{\mu}i\sigma^{\nu\}\alpha}\Delta_{\alpha}}{2M} U(p,s) \right) + \dots, \\ & \text{ GPDs(Bjorken x, skewness, mom transfer)} \\ & \int_{0}^{1} dx \ H_g(x,\xi,t) = A_g(t) + \xi^2 D_g(t), \qquad \int_{0}^{1} dx \ E_g(x,\xi,t) = B_g(t) - \xi^2 D_g(t) \\ & \quad t = \Delta^2 \\ n^2 = 0 \end{split}$$

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## Gravitational FFs encode EMT

 Gravitational form factors describe matrix elements of Energy-Momentum Tensor
 e.g., traceless gluon EMT for nucleon:



Sum rules of gluon and quark GFFs in forward limit

• Momentum fraction  $A_a(0) = \langle x \rangle_a$   $\longrightarrow$   $\sum A_a(0) = 1$ 

• Spin  $J_a(t) = \frac{1}{2}(A_a(t) + B_a(t))$ 

$$\sum_{a=q,g} A_a(0) = 1$$
$$\sum_{a=q,g} J_a(0) = \frac{1}{2}$$

• D-terms  $D_a(0)$  less known but equally fundamental!

• D<sub>a</sub>(t) GFFs encodes pressure and shear distributions

## D-term from JLab DVCS

Experimental determination of DVCS D-term and extraction of proton pressure distribution [Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)]

$$s(r) = -\frac{r}{2}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r), \quad p(r) = \frac{1}{3}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}(r)$$

- Peak pressure near centre ~10<sup>35</sup> Pascal, greater than pressure estimated for neutron stars
- Key assumptions: gluon D-term same as quark term, tripole form factor model,  $D_u(t,\mu) = D_d(t,\mu)$

#### EXP + LQCD complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]



#### Radial pressure distribution



#### Nucleon D-term GFFs from LQCD



### Proton GFFs from lattice QCD

- First complete decomposition of proton gravitational form factors into u, d, s, g contributions from lattice QCD in 2023
- Physical pion mass
- Non-pert. renormalisation incl. mixing
- [Still a single ensemble, no control of discretisation effects]

Lattice QCD: Pefkou, Hackett, Shanahan, PRD 105, 054509 (2022), PRD 108, 114504 (2023), 2310.08484 (2023)



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#### Gravitational FFs c.f. experiment

Compare quark *D* GFF with 2018 results from DVCS [Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)]

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• Consistency and complementarity: lattice result more precise at large t, experimental constraints are at small  $t^{\circ} \phi^{\circ} = \frac{1}{2} \phi^{\circ} \phi^{\circ} = \frac{1}{2} \phi^{\circ} \phi^{\circ} \phi^{\circ}$ 



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PRD 108, 114504 (2023), 2310.08484 (2023)

## Gravitational FFs c.f. experiment

Synergy between lattice QCD and experiment continues!

- First experimental constraint on gluon "generalised form factors" in 2023 from J/ψ photoproduction [Duran et al., Nature 615, 813-816 (2023)]
- Lattice calculation important in distinguishing between models based on
  - Holographic QCD (method 1)
  - Generalised Parton Distributions (method 2)

Lattice QCD: Pefkou, Hackett, Shanahan, PRD 105, 054509 (2022), PRD 108, 114504 (2023), 2310.08484 (2023) Experiment: Duran et al., Nature 615, 813-816 (2023) Guo et al., 2308.13006 (2023); BEG 2310.11568 (2023)



## Proton quark and gluon radii

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Define quark and gluon radii from energy and longitudinal force densities

$$\left\langle r_i^2 \right\rangle^{\rm mass} = \frac{\int d^3 \mathbf{r} \, r^2 \varepsilon_i(r)}{\int d^3 \mathbf{r} \, \varepsilon_i(r)} \,, \quad \left\langle r_i^2 \right\rangle^{\rm mech} = \frac{\int d^3 \mathbf{r} \, r^2 F_i^{||}(r)}{\int d^3 \mathbf{r} \, F_i^{||}(r)}.$$

- Mass and mechanical radii of proton comparable to charge radius
- Gluons act to extend radius defined
   by quark contributions

$$r^2 > mech$$
  
 $r total$   
 $r q$ 

1.5

2.0

$$\varepsilon_i(r) = m \left[ A_i(t) - \frac{t(D_i(t) + A_i(t) - 2J_i(t))}{4m^2} \right]_{\rm FT} \;, \label{eq:expansion}$$

$$F_i^{||}(r) = p_i(r) + 2s_i(r)/3$$

Lattice QCD: Pefkou, Hackett, Shanahan, PRD 105, 054509 (2022), PRD 108, 114504 (2023), 2310.08484 (2023) Experiment: Duran et al., Nature 615, 813-816 (2023) otal Guo et al., 2308.13006 (2023); BEG 2310.11568 (2023)



### Glue trace anomaly form factor

 Matrix elements of the trace of the energy-momentum tensor are related to hadron mass

$$m_{\rm H} = \langle \frac{\beta}{2g} F^2 + \sum_{f} \gamma_m m_f \overline{\psi}_f \psi_f \rangle_{\rm H} + \sum_{f} m_f \langle \overline{\psi}_f \psi_f \rangle_{\rm H}$$
  
Trace anomaly term Sigma term

 First calculation of glue part of trace anomaly FF in 2024
 [Wang et al., 2401.05496 (2024)]

$$G_{\rm H} \equiv \frac{\beta(g)}{2g} \langle F^2 \rangle_{\rm H} / m_{\rm H}$$

- One lattice ensemble (fixed sea quark masses  $m_{\pi} \sim 340 {\rm MeV}$ )
- Complications with renormalisation and mixing to be further explored



[Wang et al., 2401.05496 (2024)]

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### Other generalised form factors

#### Other GFFs can also be computed from LQCD

- New calculation in 2022 of moments of isovector transverse quark spin densities
   [Alexandrou et al., arXiv:2202.09871 (2022)]
- Physical pion mass, 3 lattice spacings
- Multipole fits in t

$$F(t) = \frac{F(0)}{(1 - t/m_p^2)^p}$$

#### Isovector GFFs from LQCD



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# Nuclear physics from lattice QCD

# Nuclei on the lattice are HARD

• Noise:

Statistical uncertainty grows exponentially with number of nucleons

• Complexity: Number of contractions grows factorially





#### Calculations possible for A<5

#### Larger nuclei

• What about larger (phenomenologically-relevant) nuclei?



#### Nuclear matrix elements

"Nuclear physics from LQCD Collaboration" NPLQCD

- Nuclei with A<5 unphysical quark masses
- Physical-mass calculations
   begun 2021



Scalar, axial, tensor matrix elements [Phys.Rev.Lett. 120 (2018), Phys.Rept. 900 (2021), Phys.Rev.D 103, 074511(2021)]

nsorDouble β-decay[Phys.Rev.Lett. 119 (2017),SPhys.Rev.D 96 (2017),8),Phys.Rev.D 107 (2023),2402.09362 (2024)]



Nuclear parton distribution functions [Phys.Rev.D 96 (2017), Phys.Rev.Lett. 126 (2021)]



Proton-proton fusion and tritium β-decay [Phys.Rev.Lett. 119, 062002 (2017)]

#### Baryon-baryon interactions, incl. QED

[Phys.Rev.D 96 (2017), Phys.Rev.D 103 (2021), Phys.Rev.D 103 (2021), Phys.Rev.D 107 (2023), 2403.00672 (2024)]

## Momentum fraction of nuclei

Matrix elements of the Energy-Momentum Tensor in light nuclei first QCD determination of momentum fraction of nuclei

 Bounds on EMC effect in moments at ~few percent level, consistent with phenomenology

Ratio of quark momentum fraction in nucleus to nucleon



## Momentum fraction of nuclei

Matrix elements of the Energy-Momentum Tensor in light nuclei first QCD determination of momentum fraction of nuclei

Match isovector moments to low-energy constants of EFT, extrapolate to physical quark masses





# Axial charge of the triton

- No axial form factors of nuclei from lattice QCD yet
- Axial charge of He: first extrapolation to the physical quark masses in 2021



[NPLQCD 2102.03805 (2021)]

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# Nuclear physics from lattice QCD

#### Nuclei on the lattice are HARD

Constraints on nuclear matrix elements are possible:

- Pipeline well-defined and tested
- Still quite far from controlled calculations

Controlled calculations achievable for nuclei with A<5 with ~10-20% uncertainty in 10-year timeframe:

- Axial MEs, including form factors
- Scalar MEs relevant for e.g., dark matter direct detection
- Double beta-decay matrix elements
- Constraints on PDFs, GPDs of nuclei via moments



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#### Early Era

Fully-controlled w/ ~15-percent errors now or within ~5y

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- Forward limits of nuclear FFs

 $k \tau$ 

#### **Exploratory Era**

First calculations, timeline for controlled calculations unclear

- Many transition and resonance FFs
- t-dependence of nuclear FFs

