

New Constraints on Proton Size from Spin Structure Data

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Based on current work with D. Ruth, K. Slifer, J.-P. Chen, F. Hagelstein,
V. Pascalutsa, A. Deur, S. Kuhn, M. Ripani, X. Zheng, R. Zielinski, & C. Gu.
Also ancient papers with Nazaryan & Griffioen, PRL 2006, CJP 2007, LNP 2008,
PRA 2008, 2011

Motivation

- General: find the proton structure effects on the hyperfine splitting (HFS) in hydrogen, both μH and $e\text{H}$.
- More specific:
 - Take advantage of excellent new input data
 - Get more accurate prediction of splitting to help in setting up new experiments
 - Get a new measure of the average separation between magnetic and electric material in the proton, the Zemach radius. (Titular motivation)
- Talk will be mostly about how the calculation is done. Will show new results, both for HFS and Zemach radius.

New input data

- For our calculation, input data includes the spin-dependent proton structure functions g_1 and g_2 , measured in polarized inelastic ep scattering
- Functions of W (total CM ep energy) and Q^2 (photon off-shell mass).
- Previously, no g_2 data at all. Now g2p JLab experiment 84 data points, at 4 different Q^2 (Ruth et al., 2022)
- And wonderfully extended set of g_1 data from JLab EG4. 1085 data points, at 25 values of Q^2 , range $\approx [0.01, 1.0] \text{ GeV}^2$ (Zheng et al., 2021)

For information, the old data

- No old data at all for g_2 .
Wilczek-Wandzura relation could give part of g_2
and there were data fits (!)
- JLab EG1b g_1 data, available in 2005
1124 data points at 27 values of Q^2
range $\approx [0.05, 5.0]$ GeV²
(publication Fersch et al., 2017)
- SLAC E155 g_1 data, 24 data points, $Q^2 > 1.2$ GeV²
(Anthony et al., 2000)
- Actual data for g_2 and good lower Q^2 g_1 data creates
opportunity for much improved calculational result

New planned experiments

- CREMA, FAMU, & JPARC propose measurement of HFS in ground state μH
- $1\text{S } \mu\text{H}$ splitting is about 182.636 meV
or wavelength $\approx 6.8 \mu\text{m}$ (infrared)
or frequency $\approx 44.2 \text{ THz}$
- Worry about time to run experiment:
Have laser, frequency width $\approx 100 \text{ MHz}$
- Say spread of prediction is about 0.16 meV (can do better!)
 - spread of frequency prediction is $\approx 40 \text{ GHz}$
 - need ≈ 400 frequency settings of laser to scan HFS region.

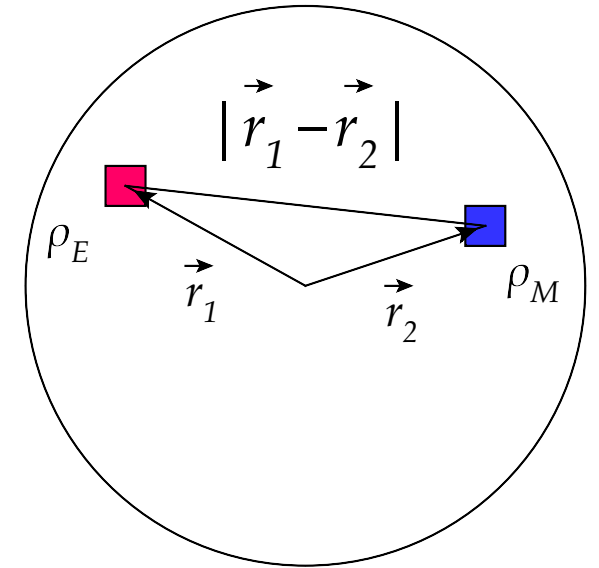
Planned experiments run time

- From talks: need 1.4 hour to get 4σ signal above background, and 1 hour to change laser frequency.
- $2.4 \text{ hours} \times 400 = 960 \text{ hours} \approx 8 \text{ weeks}$ (@ 5 days/week)
Ugh: other groups want the PSI (CREMA's location) also
- \therefore want good theoretical help to reduce the laser scan width
- Anticipate fractional experimental uncertainty upon completion better than $100 \text{ MHz}/44.2 \text{ THz} \approx 2 \text{ ppm}$
- Current best μH HFS splitting measurement is from CREMA (Science, 2013) and is $22.8089 (51) \text{ meV}$ for the 2S state, or $\approx 220 \text{ ppm}$.
- For comparison,
$$E_{1S,\text{HFS}}(eH) = h \times 1420.405\,751\,768 (2) \text{ MHz} \text{ or } 1.4 \text{ ppt}$$

Coming soon: Zemach radius

- Is average separation of magnetic and electric material in proton

$$r_Z = \int d^3r_1 d^3r_2 \rho_E(\vec{r}_1) |\vec{r}_1 - \vec{r}_2| \rho_M(\vec{r}_2)$$



- Nonrelativistically same as

$$r_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[\frac{G_E(Q^2)G_M(Q^2)}{1 + \kappa_p} - 1 \right]$$

- Can obtain from form factors, or by “reverse engineering” from coming work on HFS

Side note

- NR, form factors are Fourier transforms of charge or magnetic densities.

- Relativistically, form factors defined from matrix element of electromagnetic current,

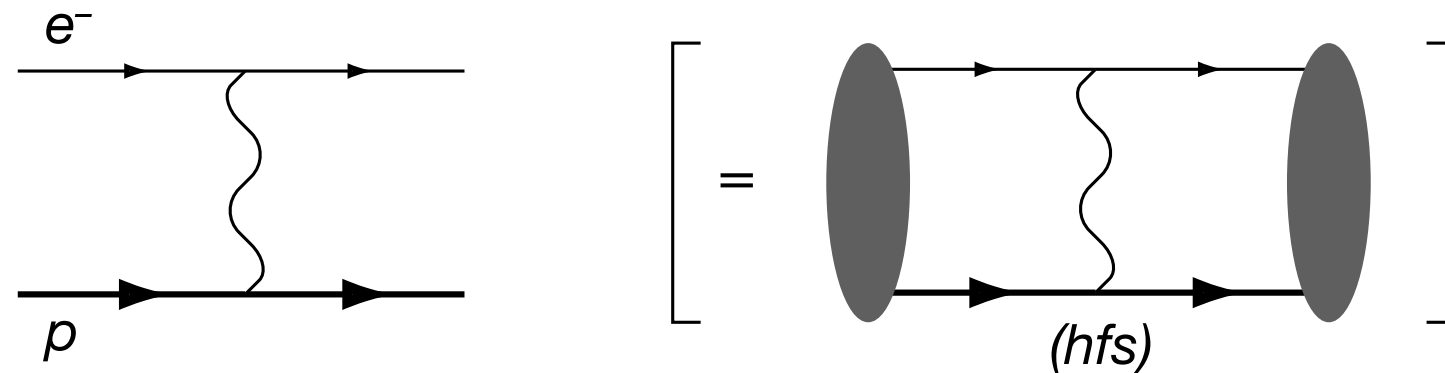
$$\langle p', s' | J_\mu | p, s \rangle = \bar{u}(p', s') \left[\gamma_\mu F_1 + i \sigma_{\mu\nu} q^\nu \frac{F_2}{2m_p} \right] u(p, s)$$

$$(\text{with } G_M = F_1 + F_2 ; \quad G_E = F_1 - (Q^2/4m_p^2)F_2)$$

- Provably same in NR limit, but not in general.
Coming results define r_Z from form factor expression.

The calculation: lowest order

- H-atom, S-state, spin-dependent splitting
UG textbook calculation!



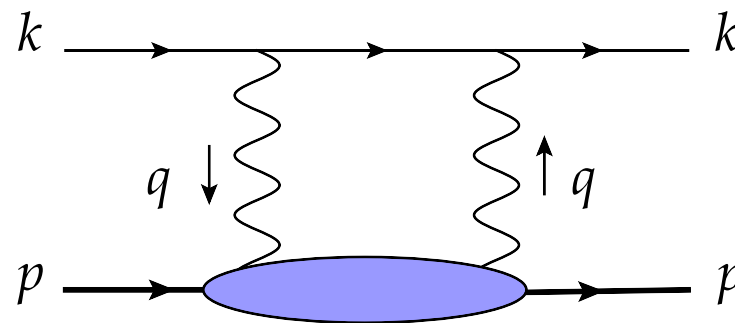
- Get
$$E_F^p = \frac{8\pi}{3} \frac{\mu_B \mu_p}{a_B^3} = \frac{8\pi}{3} (m_r \alpha)^3 \mu_B \mu_p$$
- $\mu_B = e/(2m_\ell)$ Bohr magneton
 $\mu_p = (1 + \kappa_p) e/(2m_p)$ exact magnetic moment for proton
- “Fermi energy” ; Can evaluate to about 10-figure accuracy
- Alternate writings,
$$E_F^p = \frac{8\alpha^4}{3} \frac{m_\ell^2 (1 + \kappa_p)}{m_p (1 + m_\ell/m_p)^3} = \frac{16\alpha^2}{3} \frac{\mu_p}{\mu_B} \frac{R_\infty}{(1 + m_\ell/m_p)^3}$$

Next need corrections

- Write as

$$E_{HFS}^p = E_F^p \left(1 + \Delta_{QED} + \Delta_S + \text{some smaller corrections} \right)$$

- Δ_{QED} well calculated
- “some smaller corrections” won’t be discussed here
- Δ_S = structure dependent corrections,
here meaning corrections from 2- γ exchange,



- Conventionally separate as

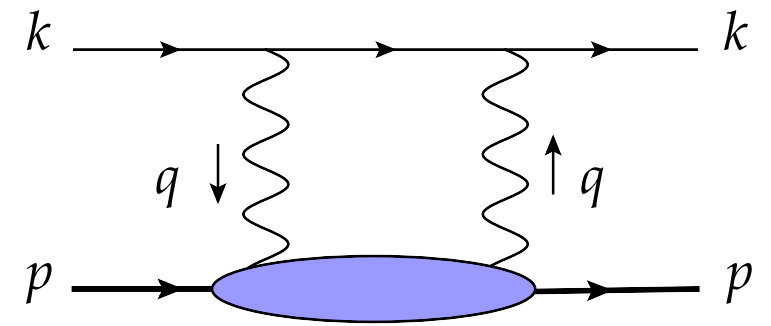
$$\Delta_S = \Delta_Z + \Delta_R + \Delta_{pol}$$

NR elastic Rel. elastic Polarizability
“Zemach” Corrections corrections

To be discussed

- How do we get the 2γ corrections from ep scattering data? (General answer: dispersion relations)
- Believe unsubtracted dispersion relation o.k. Can be discussed if needed.
- Effect of new data.

2 γ corrections



- Not calculable *ab initio*.
But lower part is forward Compton scattering of off-shell photons, algebraically gotten from

$$T_{\mu\nu}(q, p, S) = \frac{i}{2\pi m_p} \int d^4\xi \, e^{iq\cdot\xi} \langle pS | T j_\mu(\xi) j_\nu(0) | pS \rangle$$

- Spin dependence is in the antisymmetric part

$$T_{\mu\nu}^A = \frac{i}{m_p} \epsilon_{\mu\nu\alpha\beta} q^\alpha \left[H_1(\nu, Q^2) S^\beta + H_2(\nu, Q^2) \frac{p \cdot q S^\beta - S \cdot q p^\beta}{p \cdot q} \right]$$

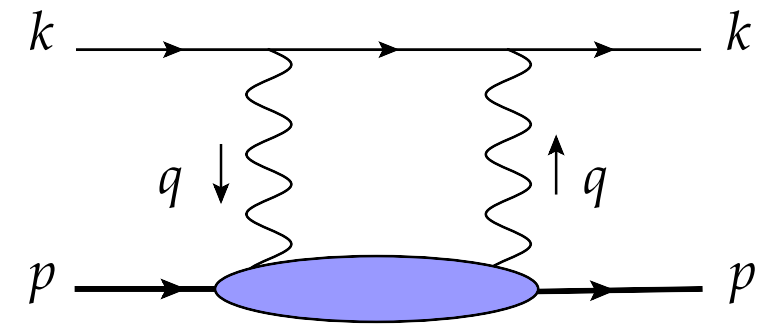
Some use
 $S_{1,2} = 4\pi^2\alpha H_{1,2}$

- Imaginary part of above is related to polarized inelastic *ep* scattering, with

$$\text{Im } H_1(\nu, Q^2) = \frac{1}{\nu} g_1(\nu, Q^2) \quad \text{and} \quad \text{Im } H_2(\nu, Q^2) = \frac{m_p}{\nu^2} g_2(\nu, Q^2)$$

- Emphasize: g_1 and g_2 are measured at SLAC, HERMES, JLab, ...

2 γ corrections



- Combine electron part of diagram with Compton bottom, and energy from 2 γ exchange

$$\Delta_{\text{pol}} = \frac{E_{2\gamma}}{E_F} \Big|_{\text{inel}} = \frac{2\alpha m_e}{(1 + \kappa_p)\pi^3 m_p} \times \int \frac{d^4 Q}{(Q^4 + 4m_e^2 Q_0^2)Q^2} \{ (2Q^2 + Q_0^2)H_1^{\text{inel}}(iQ_0, Q^2) - 3Q^2 Q_0^2 H_2^{\text{inel}}(iQ_0, Q^2) \}$$

- (Wick rotated). Great, but don't know $H_{1,2}$ from data.
- But do know Im parts, and if no subtraction, simple Cauchy (dispersion relation) gives

$$H_1^{\text{inel}}(\nu, Q^2) = \frac{1}{\pi} \int_{\nu_{th}^2}^{\infty} d\nu'^2 \frac{\text{Im } H_1(\nu', Q^2)}{\nu'^2 - \nu^2}$$

and similarly for H_2 .

Do some integrals analytically, getting

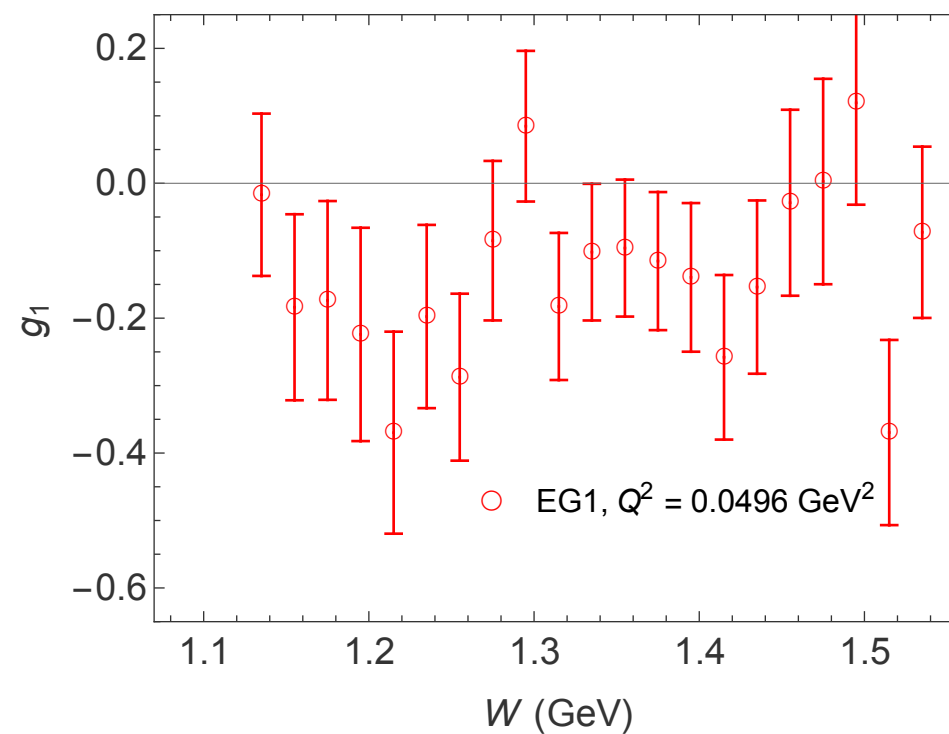
- $\Delta_{\text{pol}} = \frac{\alpha m_\ell}{2(1 + \kappa_p)\pi m_p}(\Delta_1 + \Delta_2)$
- $\Delta_1 = \int_0^\infty \frac{dQ^2}{Q^2} \left\{ \beta_1\left(\frac{Q^2}{4m_\ell^2}\right) F_2^2(Q^2) + 4m_p \int_{\nu_{th}}^\infty \frac{d\nu}{\nu^2} \tilde{\beta}_1(Q^2, \nu, m_\ell) g_1(\nu, Q^2) \right\}$
- $\Delta_2 = -12m_p \int_0^\infty \frac{dQ^2}{Q^2} \int_{\nu_{th}}^\infty \frac{d\nu}{\nu^2} \tilde{\beta}_2(Q^2, \nu, m_\ell) g_2(\nu, Q^2)$
- $\beta_1(\tau) = -3\tau + 2\tau^2 + 2(2 - \tau)\sqrt{\tau(\tau + 1)}$
- $\tilde{\beta}_1$ and $\tilde{\beta}_2$ are known kinematic weighting functions.

Comments

- Early history: begun by Iddings (1965),
finalized by Drell and Sullivan (1967),
put in present notation by de Rafael (1971).
No spin-dependent data existed,
no nonzero evaluation for > 30 years,
until Faustov and Martynenko (2002),
then modern era starts
- Someone added something: the F_2^2 term. Not inelastic.
(Put in here, taken out somewhere else.)
Thought convenient in 1967, still here in 2024..
- Δ_1 term as written finite in $m_e \rightarrow 0$ limit, because of known sum
rule, $4m_p \int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu^2} g_1(\nu, 0) = -\kappa_p^2$ (DHGHY)

Effect of new data, g_1

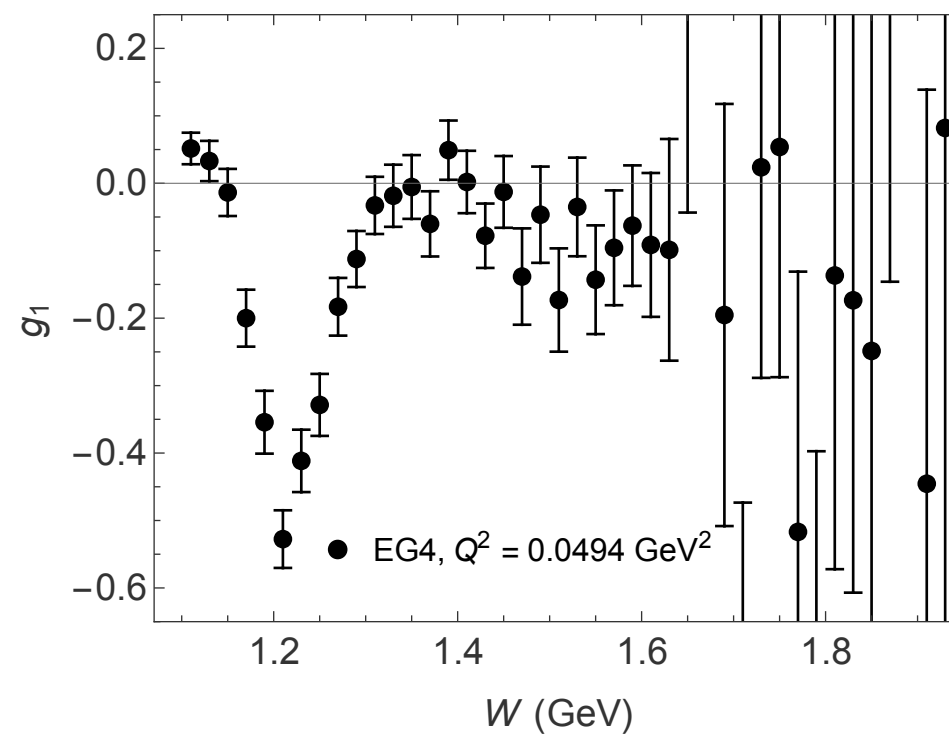
- One set of plots



- 2005 data

Effect of new data, g_1

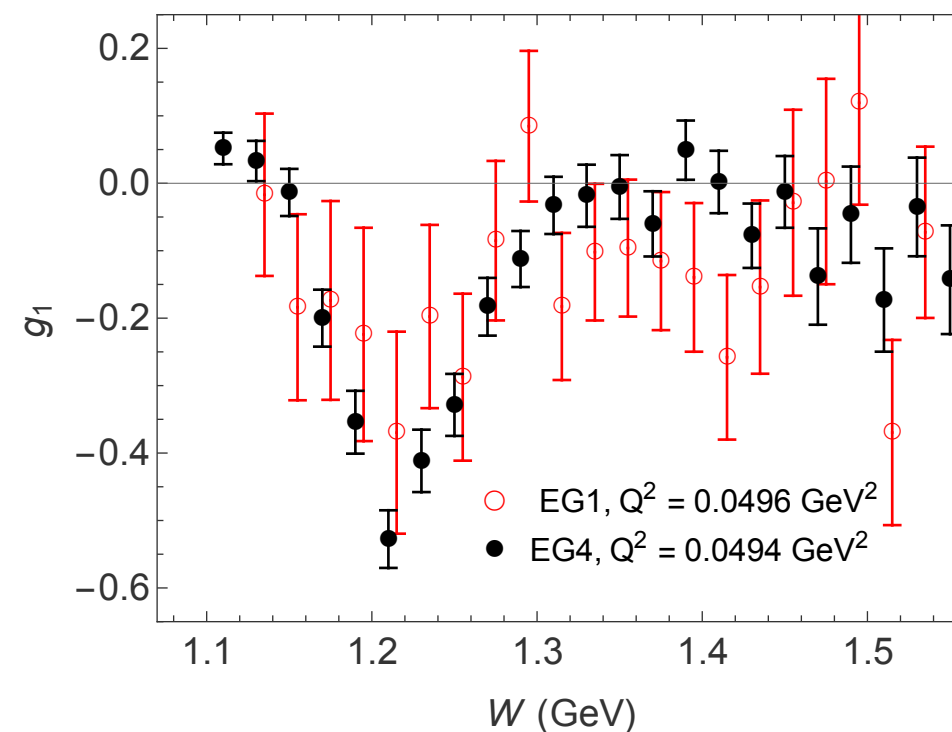
- One set of plots



- 2022 data

Effect of new data, g_1

- One set of plots

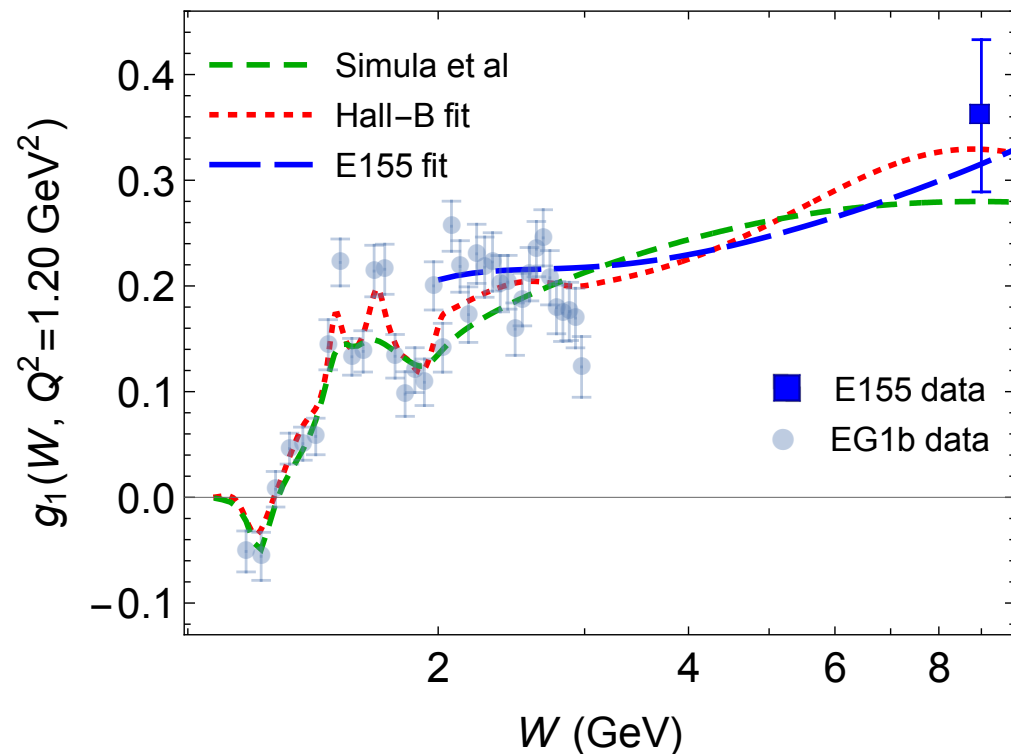
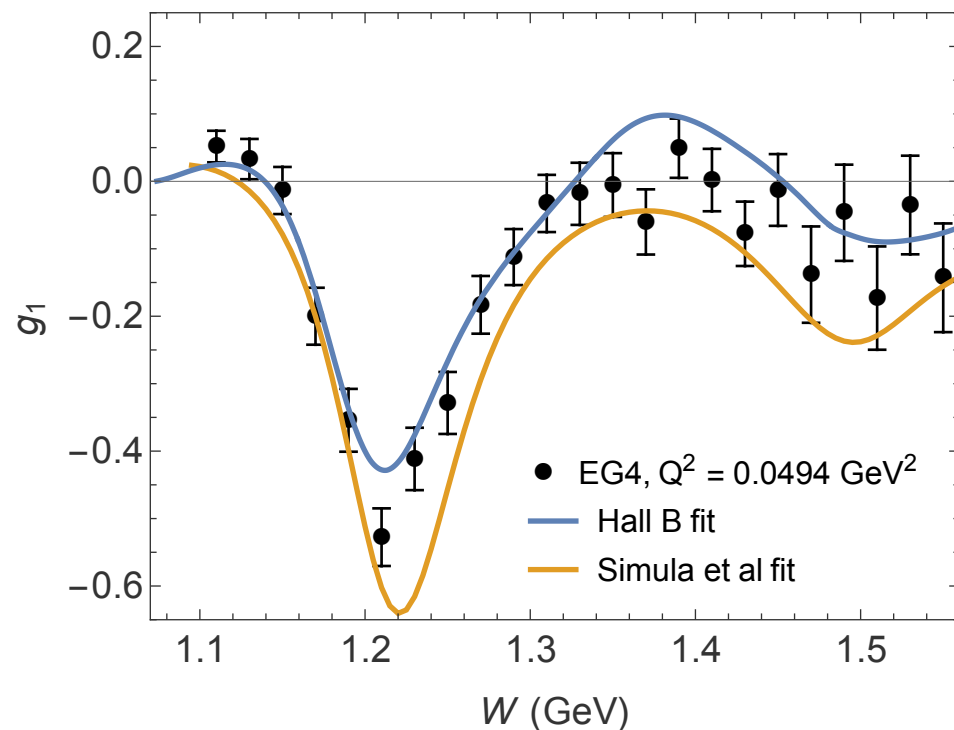


- together
- Effect on error limits:
new: $\Delta_1(eH, \text{data only}) = 4.72 \pm 1.02$
old: $\Delta_1(eH, \text{data only}) = x.xx \pm 2.69$

Completion of Δ_1 calculation

- More comments on Δ_1 before going to g_2
- \exists noticeable contributions from outside the data region. Need model or fit to extrapolate. Have fit of Simula et al (PRD, 2002) and fit of Hall B collaboration (unpub., ca. 2016) and fit of E155 (PLB, 2000, high Q^2 , high W only).
- Hall B fits best where we have comparison data

Some fit comparisons



- Generally good agreement among the three fits in scaling region (high Q^2 , high W).
- Hall B closer in data region. (They did have EG1b data.)
- We use the Hall B fit for the fill-in contributions (higher W for Q^2 in data region, and Q^2 above and below data region).

Δ_1 results today

- $\Delta_1(eH) = 4.71 \pm 1.02$ from data
+ $1.60 \pm \dots$ high W fill-in, data region
+ $0.12 \pm \dots$ low Q^2
+ $0.34 \pm \dots$ high Q^2

$$= 6.78 \pm 1.02_{\text{data}} \pm 0.23_{\text{fill-in}}$$

- Old $\Delta_1(eH) = 8.85 \pm 2.78$
- About -1 unit from newer data and about -1 from updated fill-in choice.

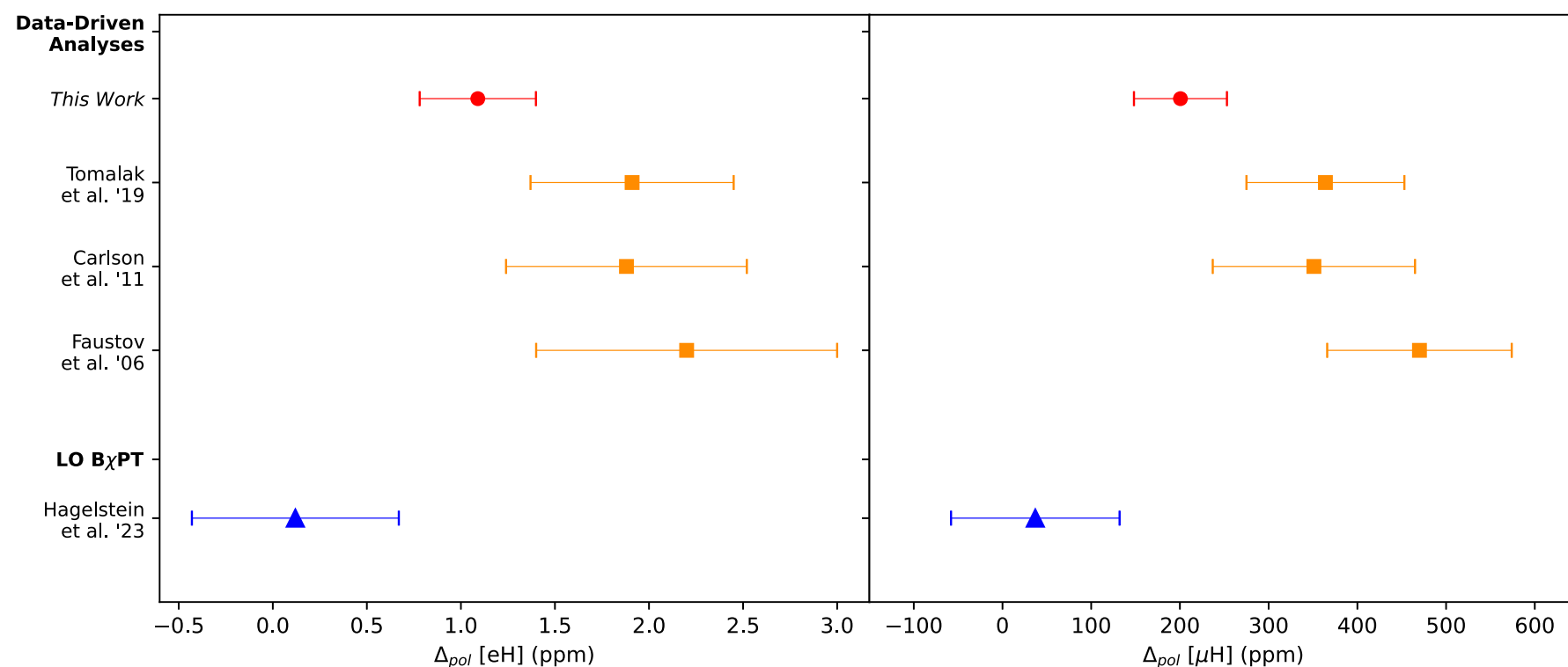
Modern Δ_2 , short version

- Thanks to g2p JLab experiment, have data where there was none before
- $\Delta_2(eH) = -1.20_{\text{data}} \pm 0.16_{\text{data}} + \text{fill-in}$
 $= -1.98 \pm 0.16_{\text{data}} \pm 0.38_{\text{fill-in}}$
- Old $\Delta_2(eH) = -0.57 \pm 0.57$
- Big difference from having data.
- Wilczek-Wandzura close to old value, not to data.

Δ_{pol} results

- Reminders:
$$\Delta_{\text{pol}} = \frac{\alpha m_\ell}{2(1 + \kappa_p)\pi m_p}(\Delta_1 + \Delta_2)$$

$$E_{HFS}^p = E_F^p \left(1 + \Delta_{QED} + \Delta_Z + \Delta_R + \Delta_{\text{pol}} + \text{some smaller corrections} \right)$$
- New results: $\Delta_{\text{pol}}(eH) = 1.09 \pm 0.31 \text{ ppm}$
 $\Delta_{\text{pol}}(\mu H) = 200.6 \pm 52.4 \text{ ppm}$



More results

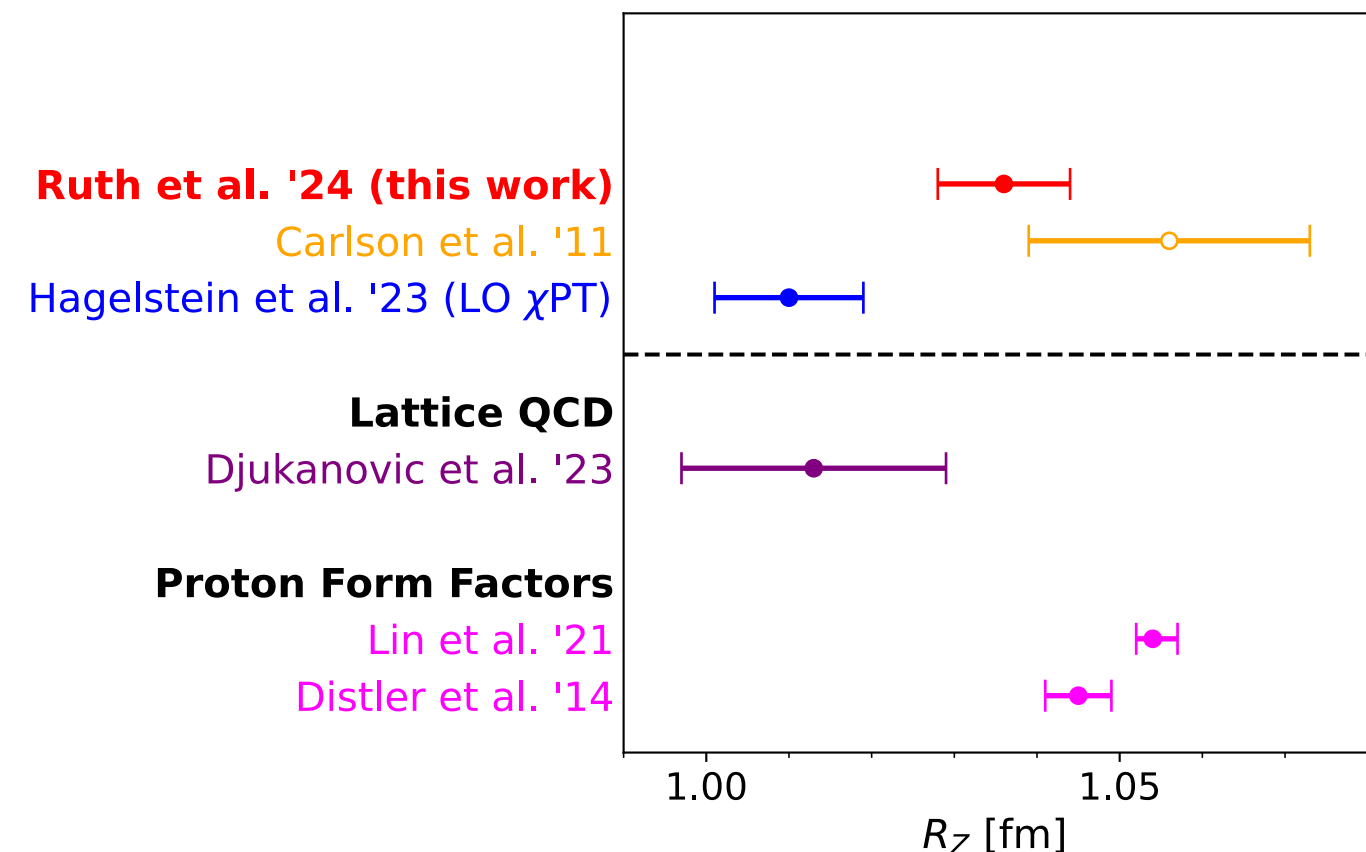
- Want also Δ_Z and Δ_R
- Δ_Z is big (magnitude about 7000 ppm for μ case), and varies with choice of form factors.
- If focusing on muon case, one way to do better is to use electron experimental data and calculation of Δ_R (a smaller term, about 900 ppm) to obtain the Zemach radius.
- Then use Zemach radius so determined to find HFS for μH

Zemach radius

- Reminder:
$$r_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[\frac{G_E(Q^2)G_M(Q^2)}{1 + \kappa_p} - 1 \right]$$

$$\Delta_Z = -2\alpha m_{r\ell} r_Z$$

- “Reverse engineering”: $r_Z = 1.036(8) \text{ fm}$



- More comparisons,
 $r_Z = \{1.080, 1.091, 1.069, 1.049, 1.025\}$
for AMT, AS, Kelly, FW, dipole

Muon results

- Use this r_Z and work out other corrections to find overall HFS for μH .
- Specifically need recoil term, which is a relativistic correction, dependent on G_E , G_M , & F_2 . Surprisingly steady at about 931 ppm.
- $E_{1S\text{-HFS}}^{\text{th.}}(\mu H) = 182.636(16) \text{ meV}$

Note

- Further improveable method of Tomalak and of Peset and Pineda (2018). They realized that the experimental $E_{HFS}^p(eH)$ is known to 12 figures and the bulk of the μH calculation just scales with the $m_{r\mu}/m_{re}$ mass ratio, known to 10 figures. Just need to calculate the smaller pieces that don't scale this way, leading to a final result with smaller overall uncertainty. Will see again ready soon. Relatively modern number obtained this way already available in Antognini et al (2022). Error bar (8).

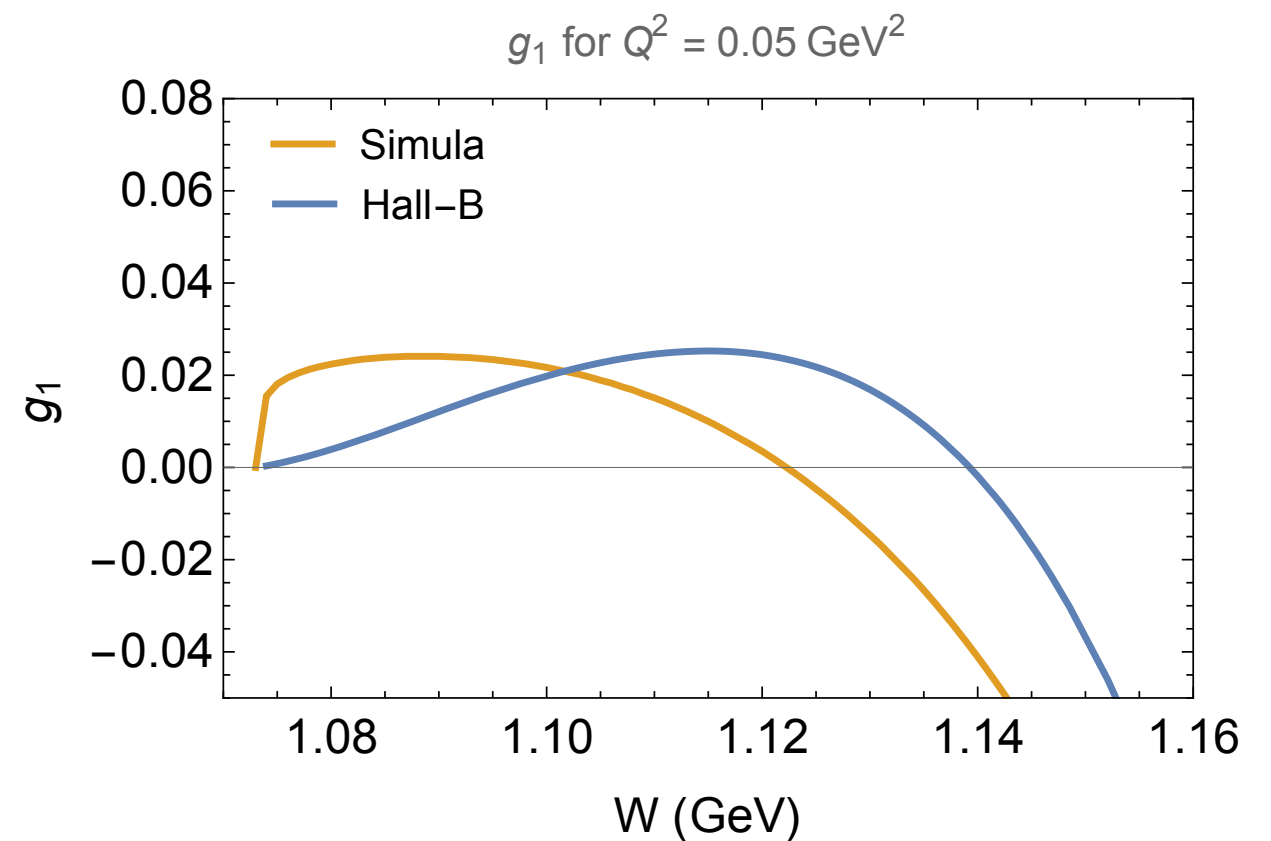
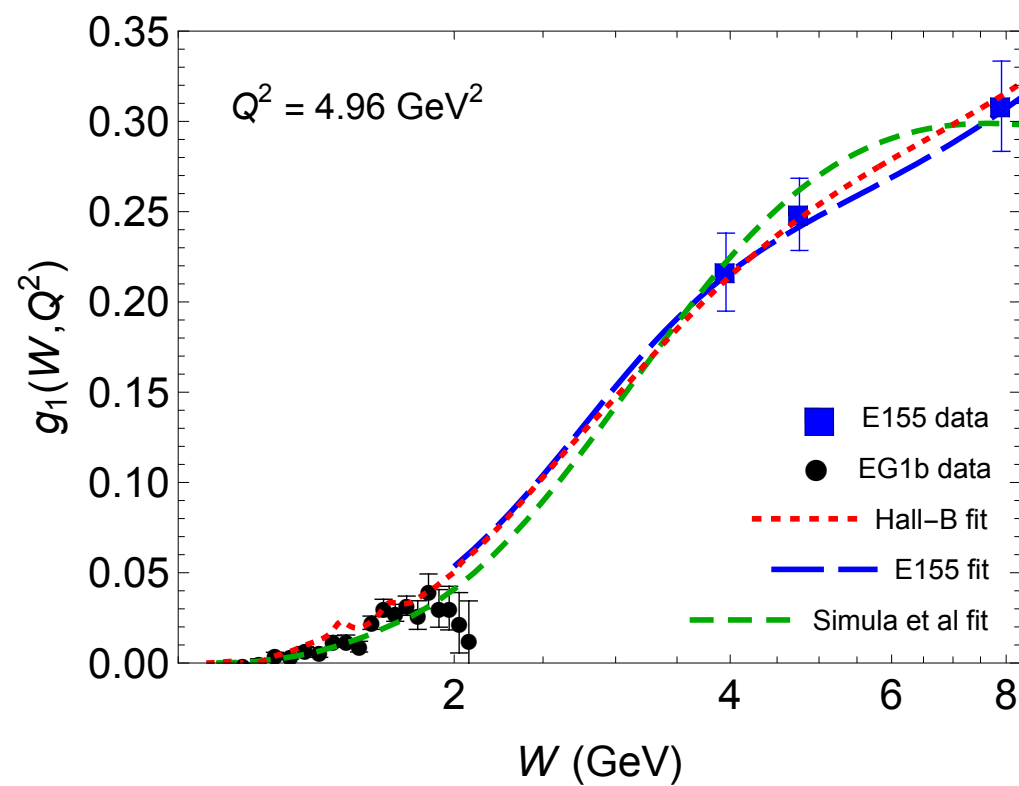
Summary

- Dispersive calculation, assuming no subtractions are needed, is complete, well defined, and unambiguous.
- Gets value of HFS using spin-dependent ep scattering data as input.
- New data reduces uncertainty limits in calculated HFS by more than factor 2.
- Result already useful for pinning down starting point of laser settings in new μH HFS measurements.
- Also have an improved (Zemach) radius determination.
- Can still do somewhat better.
- EFT calculations not much mentioned in this talk, but there is a “tension” that requires resolution.

Beyond the end

More fit comparisons

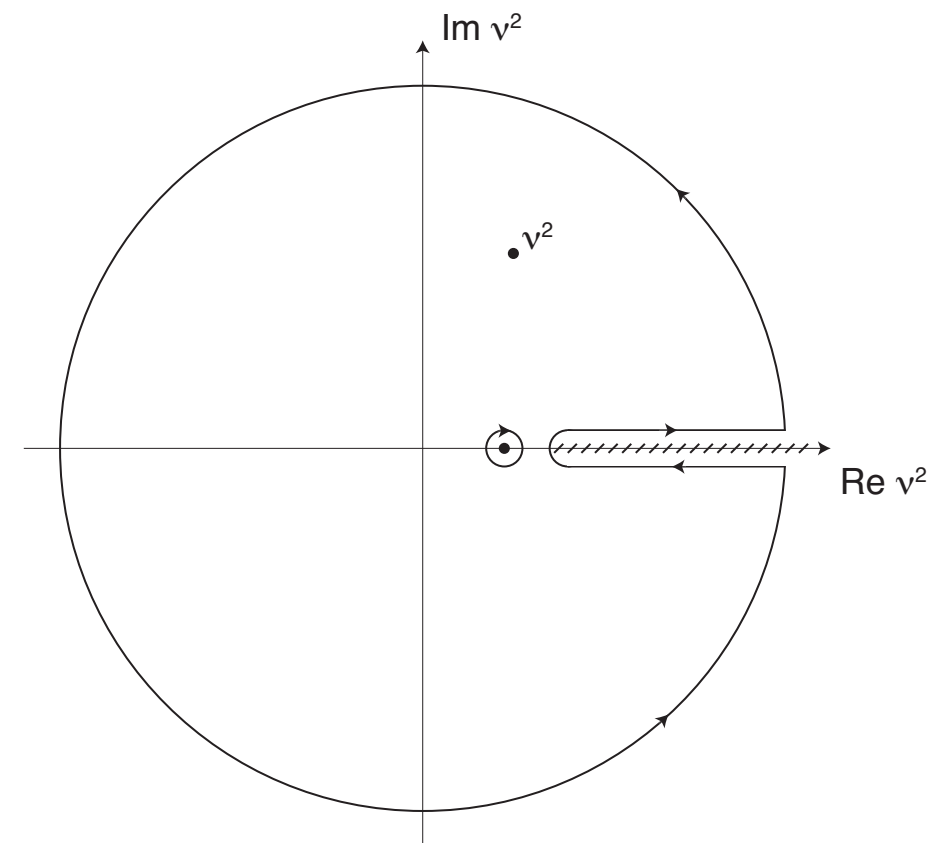
- Scaling region; near threshold W .



Unsubtracted dispersion relation (DR)?

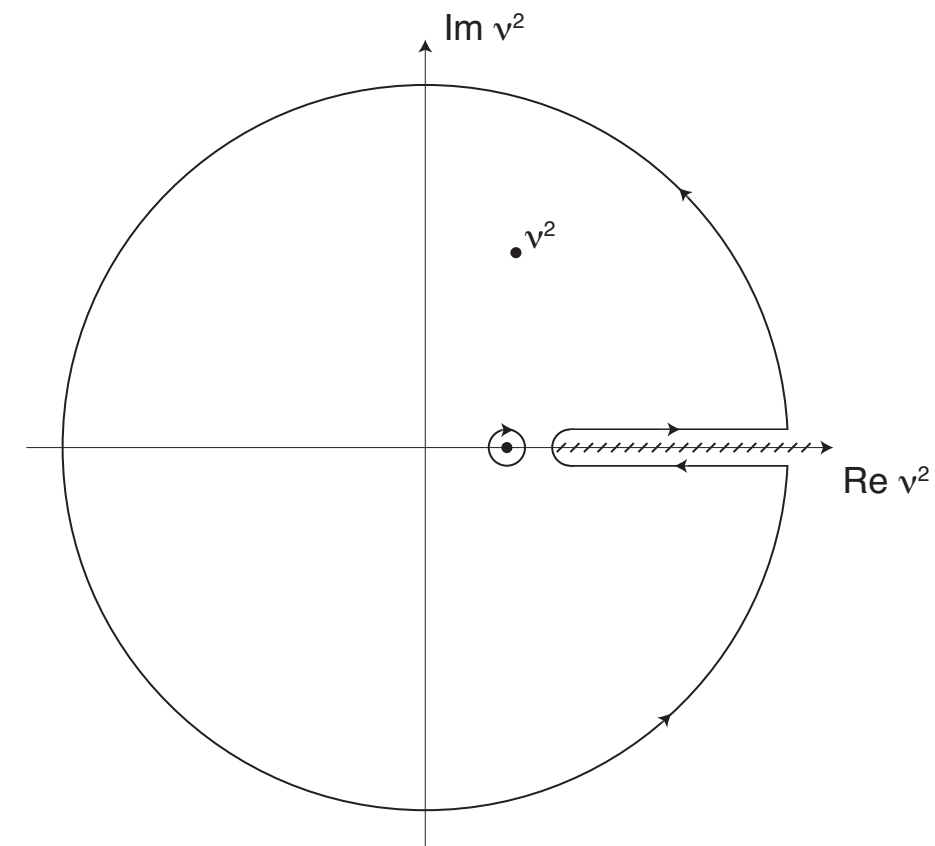
- Was once openly discussed (< 2006, say), now seems generally thought o.k.
- DR comes from Cauchy integral formula applied with some contour (closed integration path)

$$H_1(\nu, Q^2) = \frac{1}{2\pi i} \oint \frac{H_1(\nu', Q^2)}{\nu'^2 - \nu^2} d\nu'^2$$



- (DR in ν (or ν^2) with Q^2 fixed)

Dispersion relation



- Work into

$$H_1(\nu, Q^2) = \frac{\text{Res } H_1(\nu, Q^2) \big|_{el}}{\nu_{el}^2 - \nu^2} + \frac{1}{\pi} \int_{cut} \frac{\text{Im } H_1(\nu', Q^2)}{\nu'^2 - \nu^2} d\nu'^2 + \frac{1}{2\pi i} \int_{|\nu'|=\infty} \frac{H_1(\nu', Q^2)}{\nu'^2 - \nu^2} d\nu'^2$$

- Drop the $|\nu| = \infty$ term. O.k. if H_1 falls at high ν .
- Can view as standard or as dramatic assumption.

H_1

- The elastic term can be worked out, sticking on-shell form factors at the γp vertices,

$$H_1^{el} = \frac{2m_p}{\pi} \left(\frac{Q^2 F_1(Q^2) G_M(Q^2)}{(Q^2 - i\epsilon)^2 - 4m_p^2 \nu^2} - \frac{F_2^2(Q^2)}{4m_p^2} \right)$$

- The second term does not fall with ν at fixed Q^2 .
- Unsubtracted DR fails for H_1^{el} alone. Overall success requires exact cancelation between elastic and inelastic contributions.

- (In case of interest: $H_2^{el} = -\frac{2m_p}{\pi} \frac{m_p \nu F_2(Q^2) G_M(Q^2)}{(Q^2 - i\epsilon)^2 - 4m_p^2 \nu^2}$.)

But then,

- Free quarks if there is at least one large momentum scale. So at high ν , Compton amplitude for proton should be sum of Compton amplitudes for free quarks, which have zero F_2 .
- Regge theory suggests H_1 must fall with ν . See Abarbanel and Nussinov (1967), who show $H_1 \sim \nu^{\alpha-1}$ with $\alpha < 1$.*
- Very similar DR derivation gives GDH sum rule, which is checked experimentally and works, within current experimental uncertainty.
- GDH sum rule also checked in LO and NLO order perturbation theory in QED. Appears to work.

Resolution?

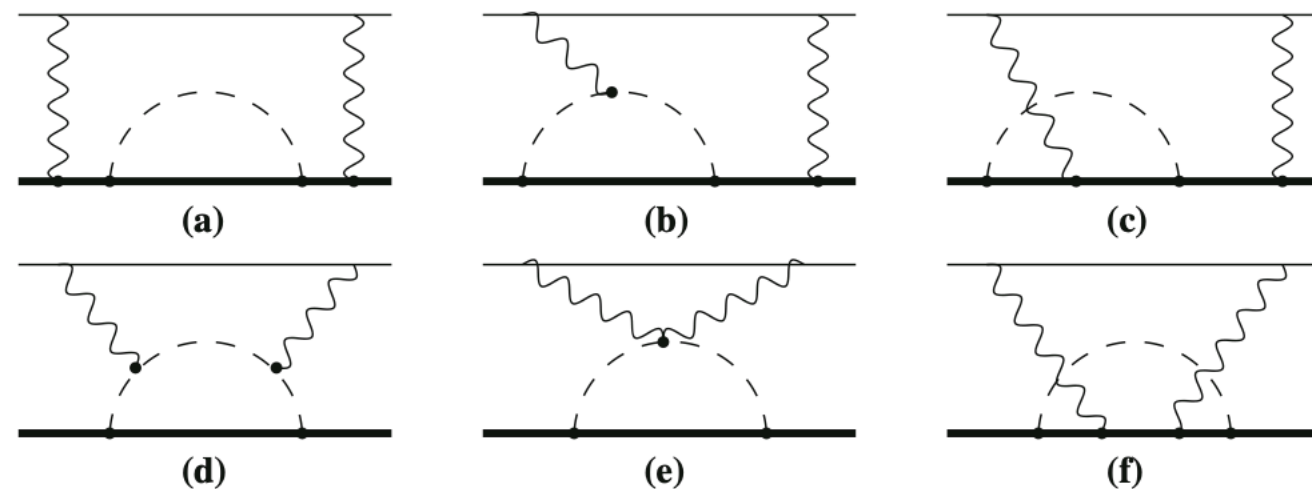
- In modern times, authors who use experimental scattering data and DR to calculate the 2γ corrections assume an unsubtracted DR works for all of H_1 .
- Reevaluation always possible.
- Proceed to next topic, comparison of data driven evaluations of HFS to evaluations using B χ PT to obtain $H_{1,2}$.
- See if subtraction comments come into play.

Side note: how good need we be?

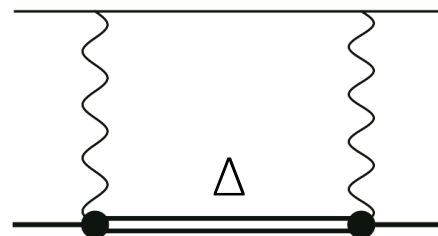
- New measurements of HFS in μH in 1S state are planned.
- May measure to 0.1 ppm (as fraction of Fermi energy). But need theory prediction to help determine starting point of laser frequency scan.
- From 2018 conference at MITP (Mainz), want theory prediction to 25 ppm or better. Better is what we should look for.
- Believe state of art for HFS in 1S μH is from Antognini, Hagelstein, Pascalutsa (2022),
$$E_{\text{HFS}}^{1\text{S}} = 182.634(8) \text{ meV}$$
or 44 ppm.

Application of $B\chi PT$

- Using chiral perturbation theory, one can calculate beyond the elastic case diagrams like



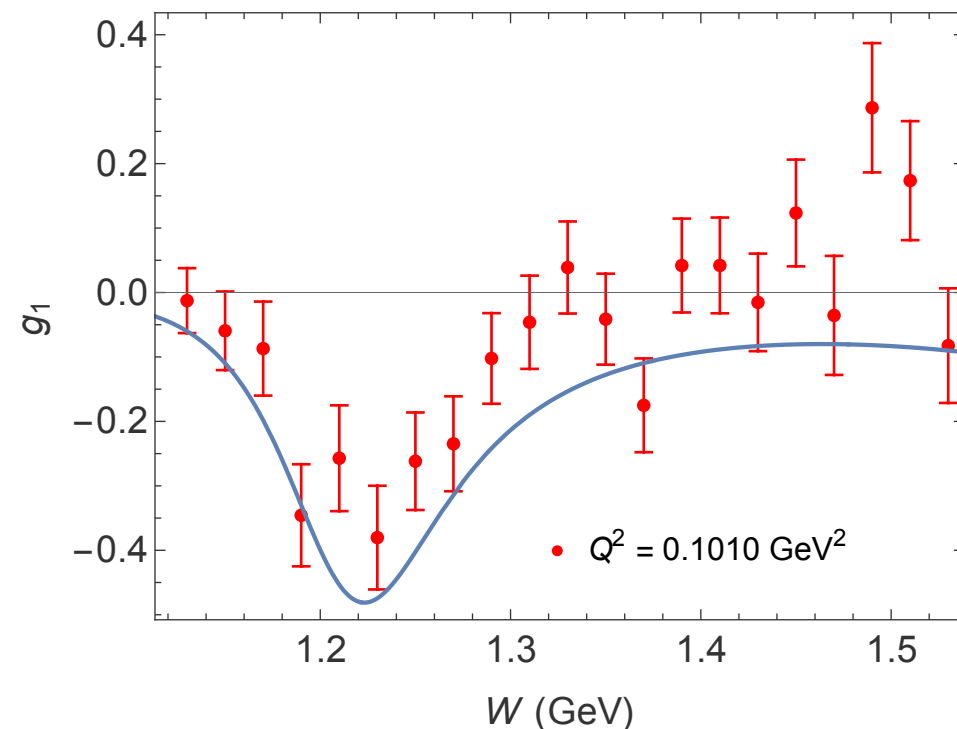
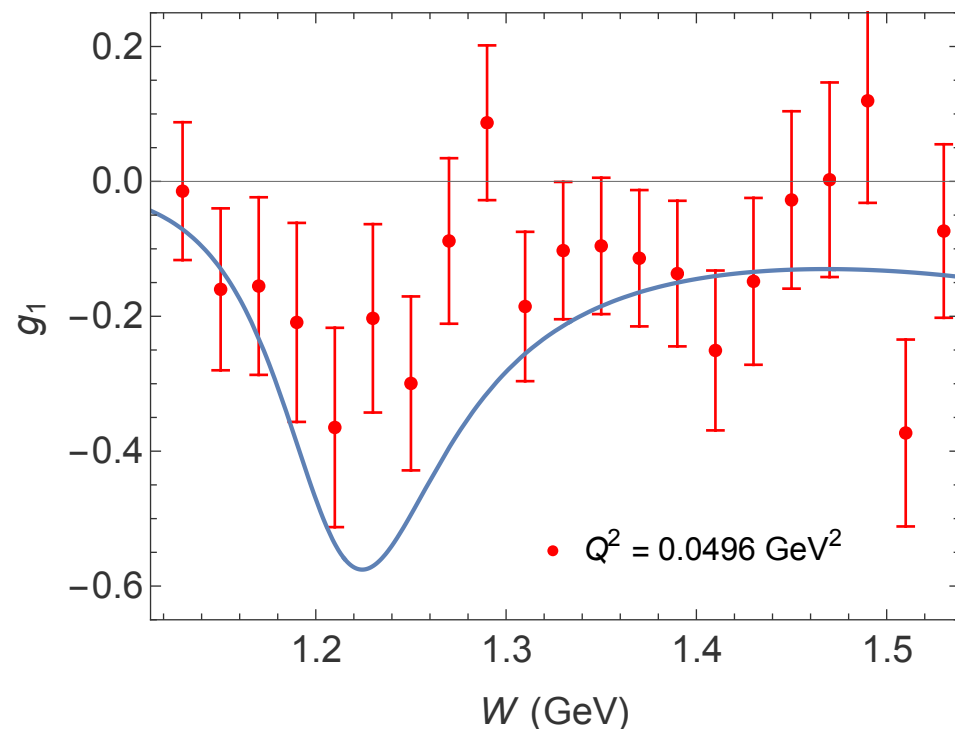
- Or diagrams where there is a Δ -baryon on the hadronic leg,



- These can be used to calculate $H_{1,2}$, at low Q^2 and CM energy W not too far from threshold. Also can get $\gamma^* N \rightarrow \pi N$ or $\gamma^* N \rightarrow \Delta$ and from them obtain $g_{1,2}$ at similarly low kinematics.

g_1 comparison

- Compare g_1 from B χ PT (blue lines) to actual JLab data



- Plots are “unofficial”: Made by me* and involve spreading Δ pole out using Lorentzian of same total area.

*With greatest thanks to Pascalutsa and Hagelstein for providing code for their $\gamma N \rightarrow \pi N$

- O.k. This won't explain difference in Δ_{pol} results.

Non-pole terms

- Non-pole means ν independent terms in $H_{1,2}$.
- Recall elastic $H_1^{el} = \frac{2m_p}{\pi} \left(\frac{Q^2 F_1(Q^2) G_M(Q^2)}{(Q^2 - i\epsilon)^2 - 4m_p^2 \nu^2} - \frac{F_2^2(Q^2)}{4m_p^2} \right)$.
- The B χ PT results for H_1 with π - N and Δ intermediate states also have non-pole terms.
- To calculate energies for the non-pole terms, cannot use the DR (at least not un-subtracted ones), but can use the expressions on slide 7, which were before any Cauchy trickery was used

Pole and non-pole

- One part: The Δ contribution to μH HFS for 2S state*

$$\begin{aligned} E_{pol}^{HFS} &= -40.69 \mu\text{eV} && \text{pole} \\ &= 39.54 \mu\text{eV} && \text{non-pole} \\ &= -1.15 \mu\text{eV} && \text{total} \end{aligned}$$

- Lot of cancellation.
- But from asymptotic freedom, or from Regge analysis, or from success of DHG sum rule, expect zero non-pole term. Totality, from elastic and resonances and inelastic terms, needs to add to zero for the ν independent terms.
- Something to talk about.

One point

- How should one deal with non-zero non-pole terms that result from partial information, when one knows that the non-pole terms are zero when one has complete information?

Δ_{pol} with newest $g_{1,2}$

- Defer to David Ruth (next after next talk).
- Except for comment on handling regions outside the data range.
- Mostly, because of the kinematic factors, the need is for data at low Q^2 and low ν (or W near threshold), and this is where the data is.
- Again, mostly, where there is no data and we use models or interpolations, the contributions to $\Delta_{1,2}$ are not great and the accruing uncertainty is not great.

Δ_{pol} with newest $g_{1,2}$

- An exception may be the very low Q^2 region, where there is no data. For the 2003 data, this was $Q^2 < 0.0452 \text{ GeV}^2$.
- And there may be a problem when comparing to χ^{PT} .

- What we did: reminder

$$\Delta_1 = \frac{9}{4} \int_0^\infty \frac{dQ^2}{Q^2} \left\{ F_2^2(Q^2) + \frac{8m_p^2}{Q^2} B_1(Q^2) \right\}$$

with

$$B_1(Q^2) = \frac{4}{9} \int_0^{x_{\text{th}}} dx \beta_1(\tau) g_1(x, Q^2) \quad .$$

- For very low Q^2 we used

$$B_1(Q^2) = -\frac{\kappa_p^2}{8m_p^2} Q^2 + c_{1B} Q^4 = -\frac{\kappa_p^2}{8m_p^2} Q^2 + 4.94 Q^4 / \text{GeV}^4$$

got by fitting to data $Q^2 < 0.3 \text{ GeV}^2$

Δ_{pol} with newest $g_{1,2}$

- The region $Q^2 < 0.0492 \text{ GeV}^2$ contributed about 15% of Δ_1 and (by our estimate) 30% of the uncertainty.

- Use standard expansion for the form factor,

$$F_2(Q^2) = \kappa_p \left(1 - \frac{1}{6} R_{Pauli}^2 Q^2 + \dots \right)$$

- Get Integrand =

$$\frac{9}{4} \frac{1}{Q^2} \left(F_2^2 + \frac{8m_p^2}{Q^2} B_1 \right) = -\frac{3}{4} \kappa_p^2 R_{Pauli}^2 + 8m_p^2 c_{1B}$$

- And $\Delta_1(0 \rightarrow Q_{low\ data}^2) \approx \text{Integrand} \cdot Q_{low\ data}^2 \approx 1.35$

Δ_{pol} with newest $g_{1,2}$

- χ PT has knowledge of g_1 at low Q^2 , and can do the integrals. Do good approximation by expanding the β_1 function for low Q^2 .

- Work for a while to get Integrand =

$$-\frac{3}{4}\kappa_p^2 R_{Pauli}^2 + 8m_p^2 c_1 - \frac{5m_p^2}{4\alpha}\gamma_0 + \mathcal{O}(Q^2),$$

- Where $\gamma_0 = \frac{2\alpha}{m_p^2} \int \frac{d\nu}{\nu^4} g_1(\nu, 0)$

and c_1 came from

$$I(Q^2) \equiv 4m_p \int \frac{d\nu}{\nu^2} g_1(\nu, Q^2) = -\kappa_p^2 + c_1 Q^2 + \mathcal{O}(Q^4)$$

Δ_{pol} with newest $g_{1,2}$

- Value for known, and doing integrals to get c_1 , find

$$\Delta_1(0 \rightarrow Q_{low\ data}^2) \approx \text{Integrand} \cdot Q_{low\ data}^2 \approx -0.45$$

thanks again to F. Haglestein et al.

- Not even same sign!
- Corresponding numbers for μ are ≈ 0.86 and -0.20
- Remembering $\Delta_{pol} = \frac{\alpha m_\mu}{2(1 + \kappa_p)\pi m_p}(\Delta_1 + \Delta_2)$, difference gives about 50 ppm or about 15% of discrepancy.
- More to talk about!