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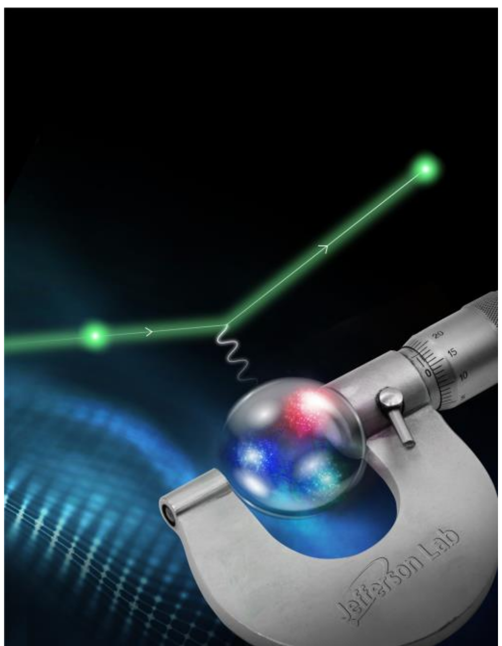
Charge Radius Extractions
of the PRad/PRad-II Experiments and Proposed DRad Experiment at Jefferson Lab



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For the PRad/PRadII-DRad Collaboration

NREC & PREN & µASTI workshop 2024

Acknowledgment: This work is supported in part by the U.S. Department of Energy under Contract No. DE-FG02-03ER41231.

Outline

- Proton charge radius and PRad/PRad-II experiments
- Deuteron charge radius and DRad proposal
- Proton Charge Radius Extraction for PRad/PRad-II
- Deuteron Charge Radius Extraction for DRad

Proton root-mean-square charge radius

- The proton is the primary, stable building block of nearly all visible matter in the Universe.

Proton rms charge radius $\boldsymbol{r_p}$ *— an important quantity of the proton:*

- Understand how QCD works in the non-perturbative region
- Important input to the bound-state QED calculations, the proton finite size contributes to the muonic H Lamb shift ($2S_{1/2} - 2\mathrm{P}_{1/2}$) by as much as 2%
- Impacts the determination of the Rydberg constant R_∞

$$r_p^2 \equiv -6\left.\frac{dG_E^p(Q^2)}{dQ^2}\right|_{Q^2=0}$$

measurements
- lepton-proton elastic scattering (nuclear physics)
  - e-p elastic scattering
  - μ-p elastic scattering
- spectroscopy (atomic physics)
  - ordinary Hydrogen Lamb shift
  - muonic Hydrogen Lamb shift

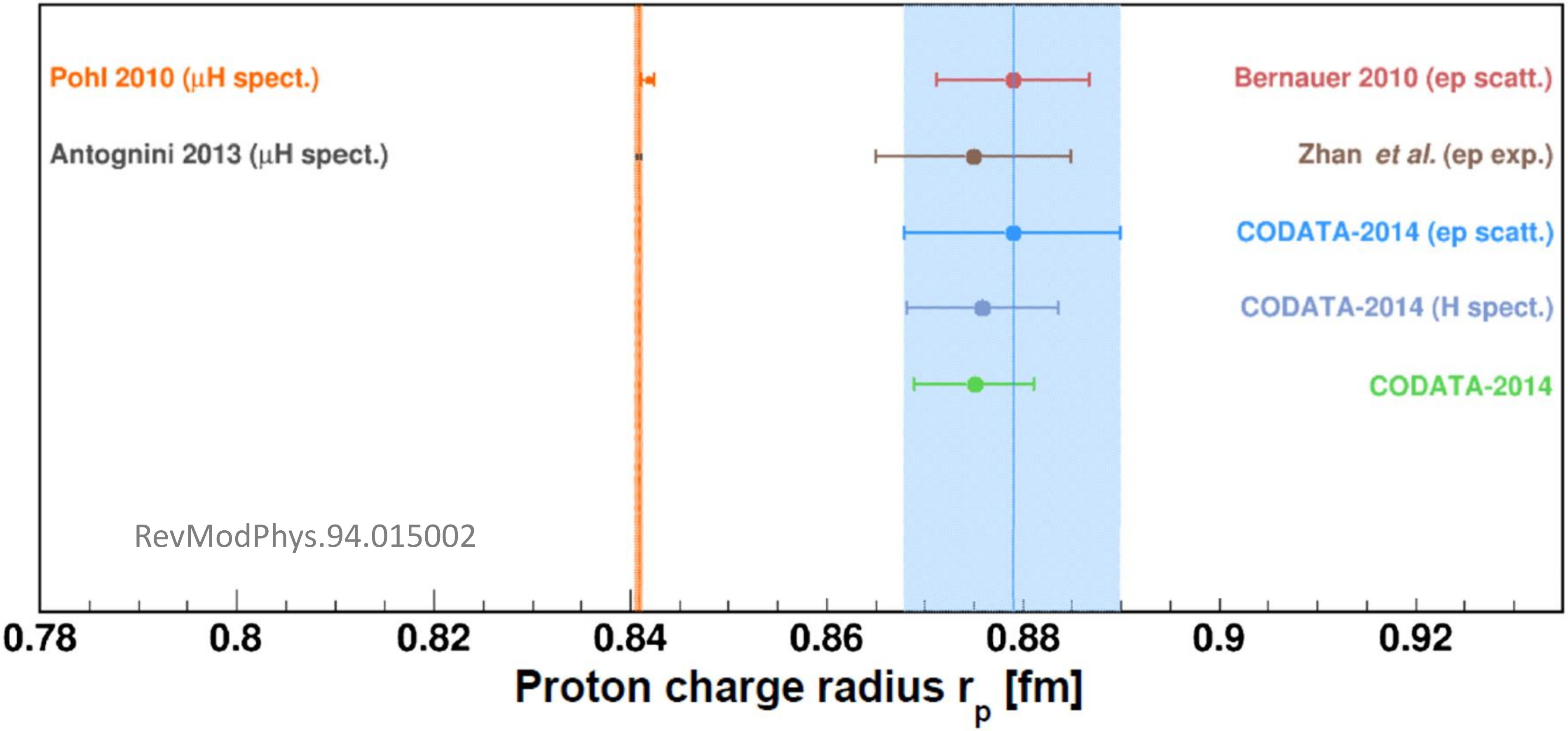
Q^2 : Four momentum transfer

$\mathrm{G_E^p}$: Proton Electric form factor

G. A. Miller, Phys. Rev. C 99, no. 3, 035202 (2019)

Proton charge radius puzzle

~ **8σ** discrepancy between muon and electron based measurements



Proton rms charge radius measured using

electrons: 0.8770 ± 0.0045 (CODATA2010 + Zhan et al.)

muons: 0.8409 ± 0.0004

The PRad experiment overview

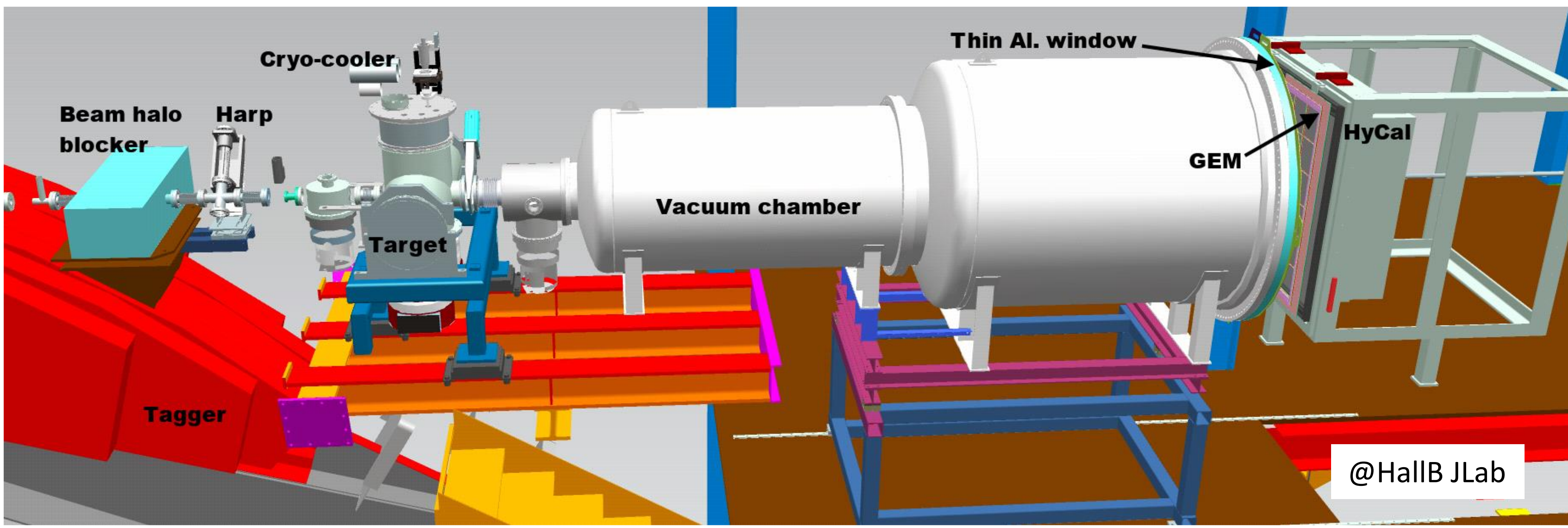
The high-precision proton charge radius measurement at Jefferson Lab

May 7, 2024, 4:00 PM

Speaker

Dipangkar Dutta

- Magnetic-spectrometer-free calorimetric method
- E_{beam} =1.1, 2.2 GeV, $\theta' = 0.7^o \sim 7.0^o$
- Covers two orders of magnitude in low Q^2 range in one fixed setting: $[2 \times 10^{-4} \sim 6 \times 10^{-2}]\,(GeV/c)^2$
- Simultaneous detection of $ee \rightarrow ee$ Møller scattering process for normalization
- Extract the radius with precision from sub-percent cross section measurement



Xiong, W., et al., 2019, “A small proton charge radius from an electron-proton scattering experiment,” Nature (London) 575, 147–150

The PRad-II experiment

PRad result: $r_p = 0.831 \pm 0.007(\text{stat.}) \pm 0.012(\text{sysm.})\text{fm}$ supports a smaller r_p

→ PRad has not reached its ultimate precision for this experimental technique

→ Possible difference between proton radius from electronic vs. muonic system

→ Need higher precision to investigate the discrepancy between PRad and MAMI form factor

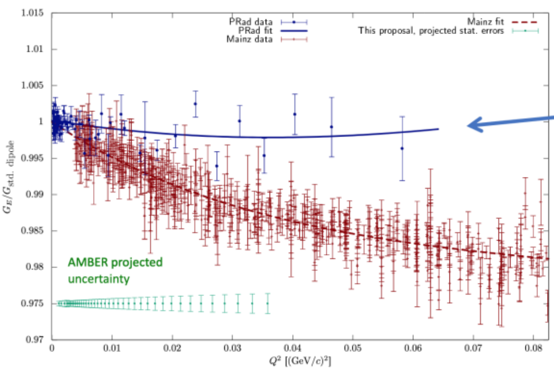
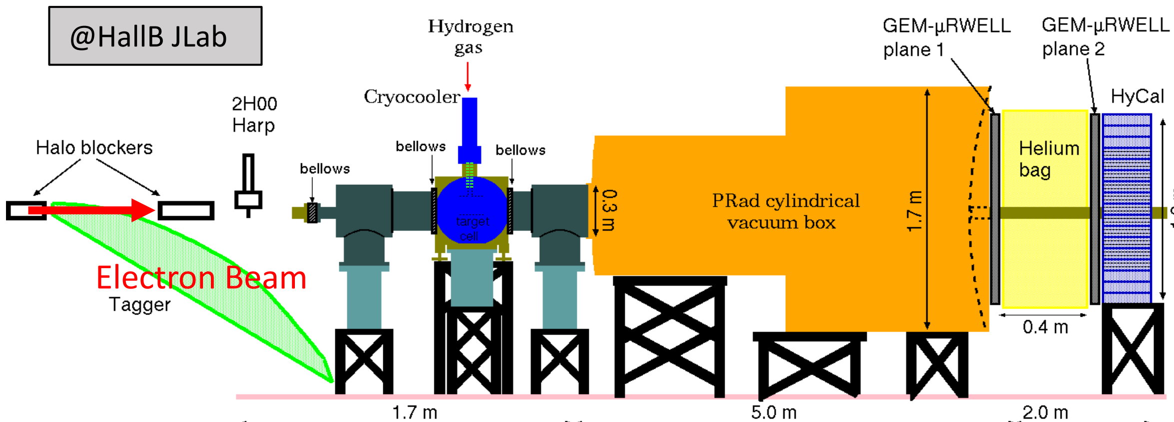


figure: J. Bernauer



- Based on the PRad experimental technique
- Three beam energies, E = 0.7, 2.1 and 3.5 GeV to increase Q^2 range
- Even lower $Q^2 \sim 10^{-5}(\text{GeV/c})^2$
- Upgrades to the original detectors, new detectors, new calculations…
- Overall uncertainty in r_p reduced by **3.5** times compared to PRad

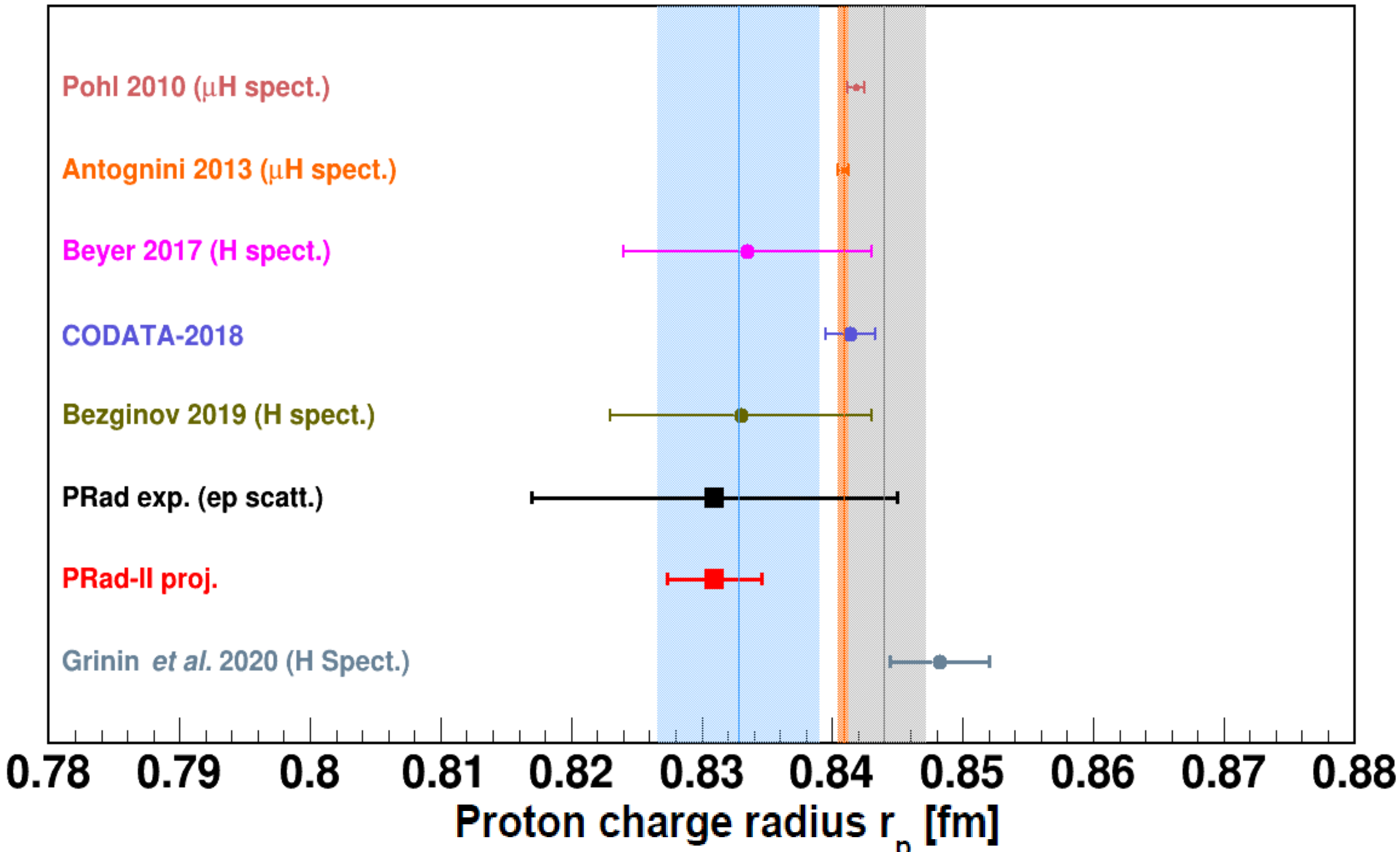
For latest running condition see presentation by Dr. Dipangkar Dutta:

The high-precision proton charge radius measurement at Jefferson Lab

May 7, 2024, 4:00 PM

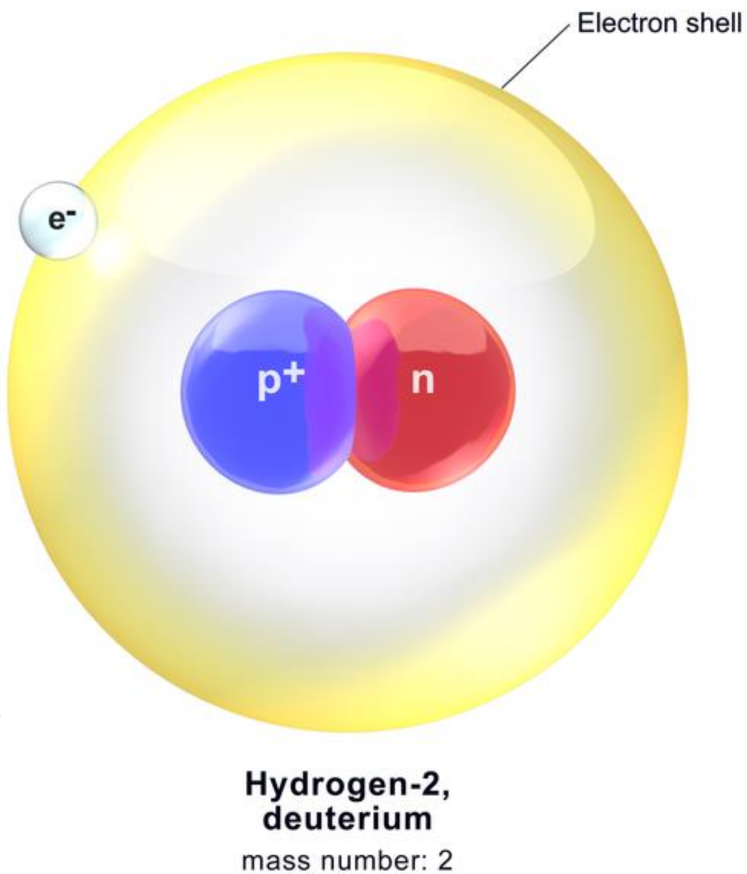
PRad-II Projection

- The mentioned upgrades in hardware combine with the planned NNLO radiative correction calculations reduces the overall uncertainty by a factor of **3.5** compared to PRad
- Form factor measurements reach even lower $Q^2 \sim 10^{-5}(\mathrm{GeV/c})^2$



Deuteron

- Excellent laboratory to study QCD in nuclei
- The simplest and lightest nucleus in nature
- The only bound two-nucleon system
- Effective neutron target
- Various theoretical calculations

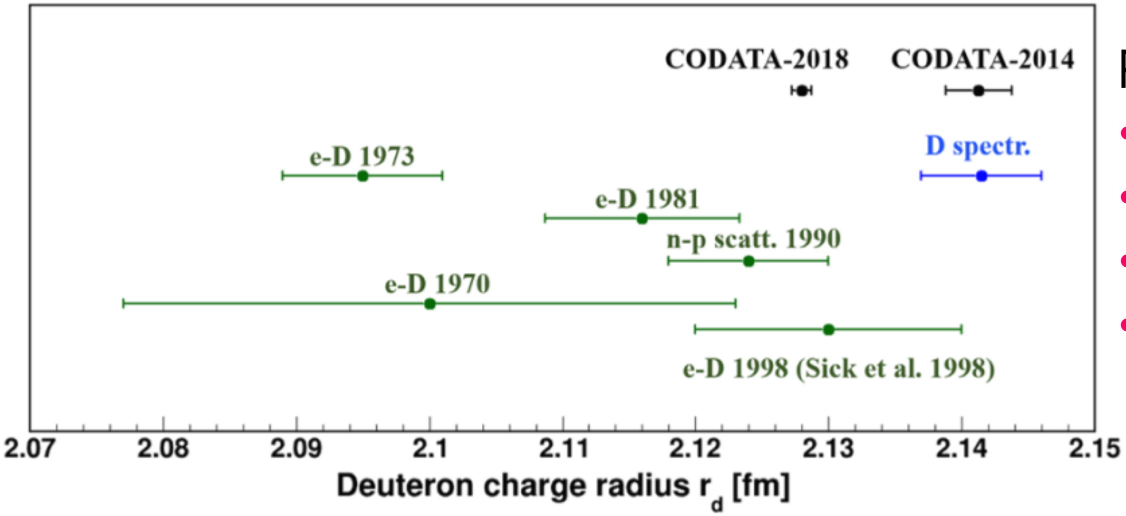


- Deuteron rms charge radius: an ideal observable to compare experiments with theories

$$r_d^2 \equiv -6 \left. \frac{dG_C^d(Q^2)}{dQ^2} \right|_{Q^2=0}$$

Q^2 : Four momentum transfer
G_C^d : Deuteron charge form factor

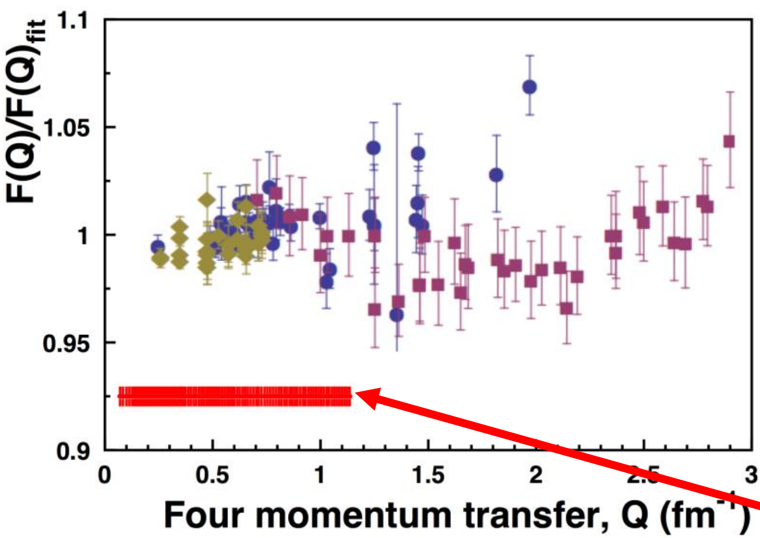
The deuteron charge radius from e-d scattering



Previous e-d scattering experiments:

- magnetic spectrometer method
- different types of targets
- normalized to *e-p* cross sections
- the most recent result in 1998 is a reanalysis of old data

I. Sick and D. Trautmann, NPA 637, 559 (1998)



◆ R.W. Berard et al. Phys. Rev. Lett. B47,355 (1973):

cooled H2 and D2 gas, measured ratio of *ed/ep* cross sections
$Q^2 = [4 \times 10^{-2} - 5 \times 10^{-2}]\mathrm{fm}^{-2}$

● G.G. Simon et al. Nucl. Phys. A364, 285 (1981):

different gas and liquid targets:
$Q^2 = [4 \times 10^{-2} - 4]\mathrm{fm}^{-2}$

■ S. Platchkov, et al. Nucl. Phys. A510, 740, (1990):

different LH2 and LD2 targets
$Q^2 = [5 \times 10^{-2} - 20]\mathrm{fm}^{-2}$

We propose a new independent method to measure *e-d* elastic cross sections with high precision

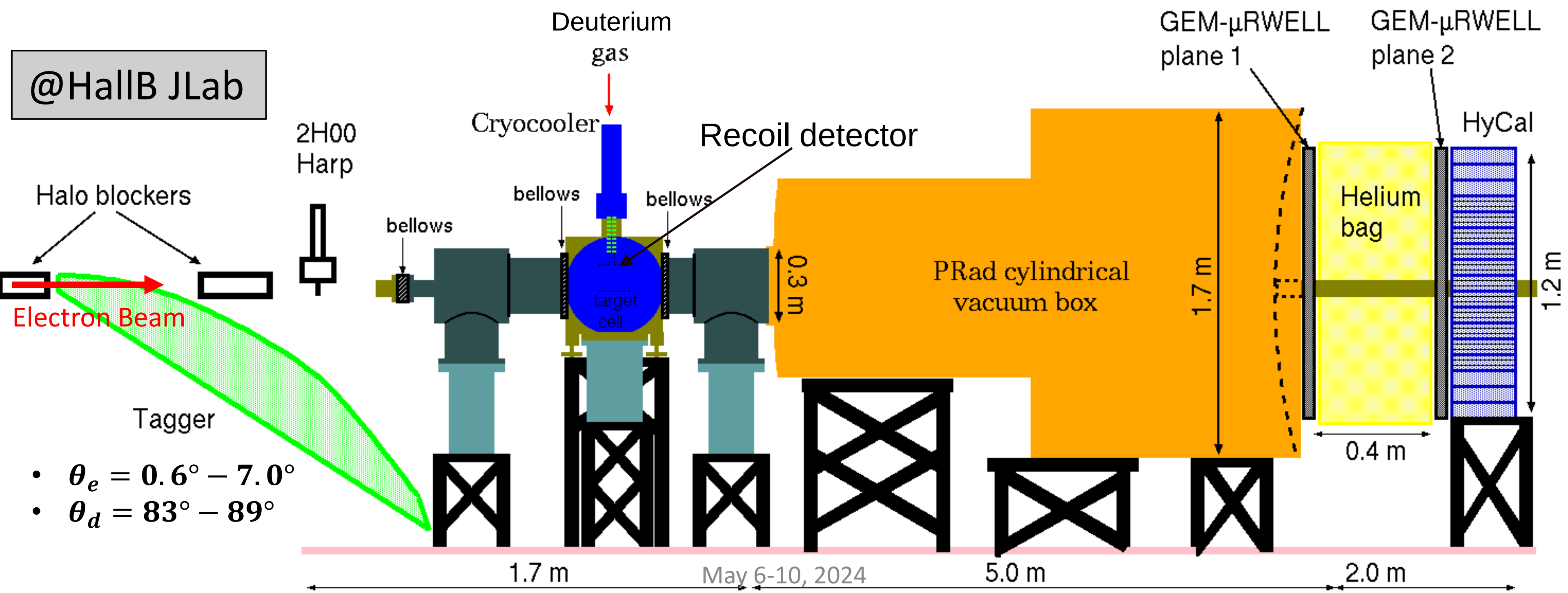
The highlight of DRad proposal

- DRad proposal(PR12-23-011): calorimetric method with windowless gas flow target based on PRad-II experiment(E12-20-004)
- Measure *e-d* elastic cross sections at very low Q^2 range:

$$[5 \times 10^{-3} - 1.3]\mathrm{fm}^{-2} \,/\, [2 \times 10^{-4} - 5 \times 10^{-2}]\,\mathrm{GeV}^2$$

- Two beam energies, E = 1.1 and 2.2 GeV to increase Q^2 range
- A new two-layer cylindrical recoil detector for reaction elasticity
- Simultaneous detection of *ee* → *ee* Møller scattering process to control systematics

@HallB JLab



- $\theta_e = 0.6° - 7.0°$
- $\theta_d = 83° - 89°$

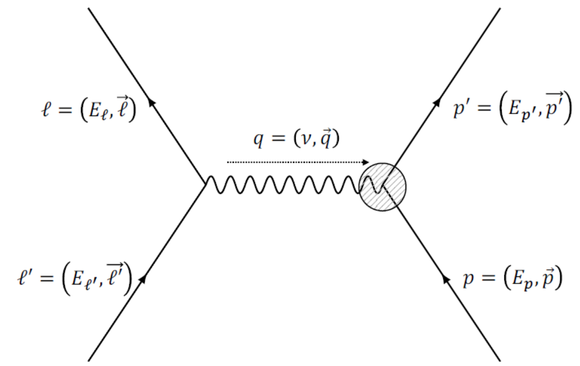
Unpolarized e-p elastic scattering

- In the Born approximation (one photon exchange):

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left(\frac{E'}{E}\right) \frac{1}{1+\tau} \left({G_E^p}^2(Q^2) + \frac{\tau}{\epsilon} {G_M^p}^2(Q^2) \right)$$

$$Q^2 = 4EE' \sin^2(\theta/2)$$

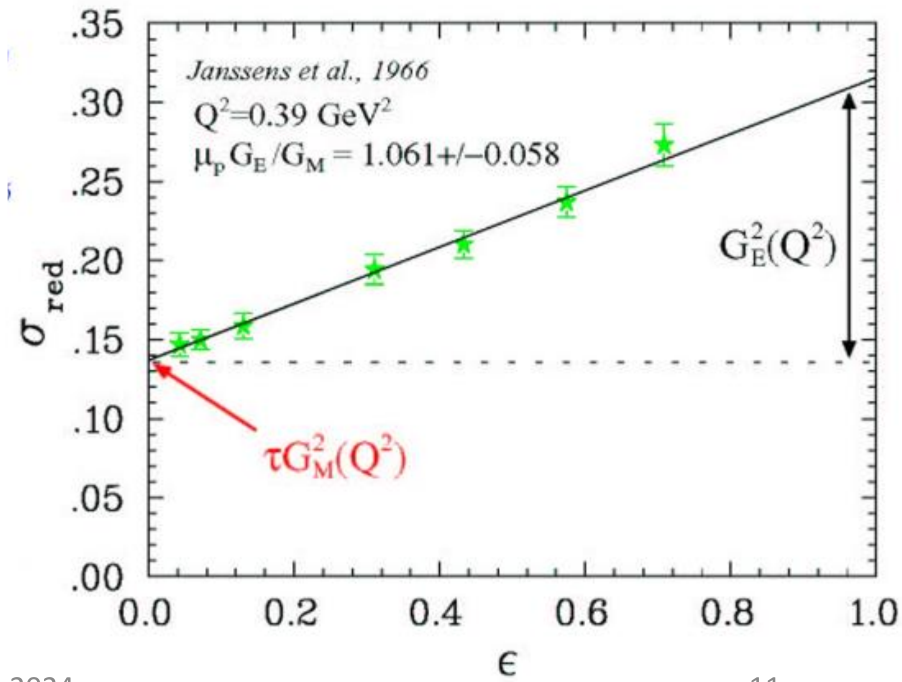
$$\tau = Q^2/(4M_p^2) \quad \epsilon = [1 + 2(1+\tau)\tan^2(\theta/2)]^{-1}$$



- G_E^p and G_M^p can be extracted using Rosenbluth separation

$$\left(\frac{d\sigma}{d\Omega}\right)_{reduced} = \tau {G_M^p}^2(Q^2) + \epsilon {G_E^p}^2(Q^2)$$

- At very low Q^2 region, cross section dominated by G_E^p, one may also extract G_E^p assuming G_M^p in certain form.

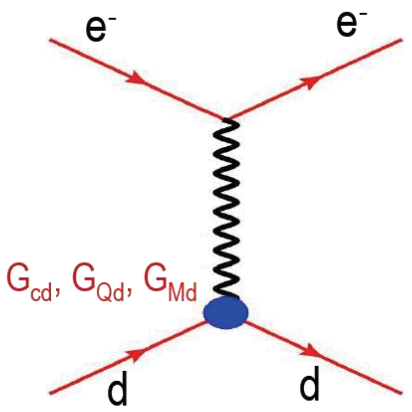


Unpolarized e-d elastic scattering

- In the Born approximation (one photon exchange):

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left[A(Q^2) + B(Q^2)\tan^2\frac{\theta}{2}\right] \quad Q^2 = 4EE'\sin^2(\theta/2)$$

A and B are structure functions related to the deuteron charge ($\mathbf{G_C^d}$), magnetic ($\mathbf{G_M^d}$) and quadrupole ($\mathbf{G_Q^d}$) form factors:



$$A(Q^2) = \mathrm{G_C^d}^2(Q^2) + \frac{2}{3}\tau\mathrm{G_M^d}^2(Q^2) + \frac{8}{9}\tau^2\mathrm{G_Q^d}^2(Q^2)$$

$$B(Q^2) = \frac{4}{3}\tau(1+\tau)\mathrm{G_M^d}^2(Q^2) \qquad \tau = Q^2/(4M_d^2)$$

- At very low Q^2(DRad), cross section dominated by $\mathrm{G_C^d}$, one may extract $\mathrm{G_C^d}$ by assuming $\mathrm{G_M^d}$ and $\mathrm{G_Q^d}$ in certain forms from parametrizations based on the data.

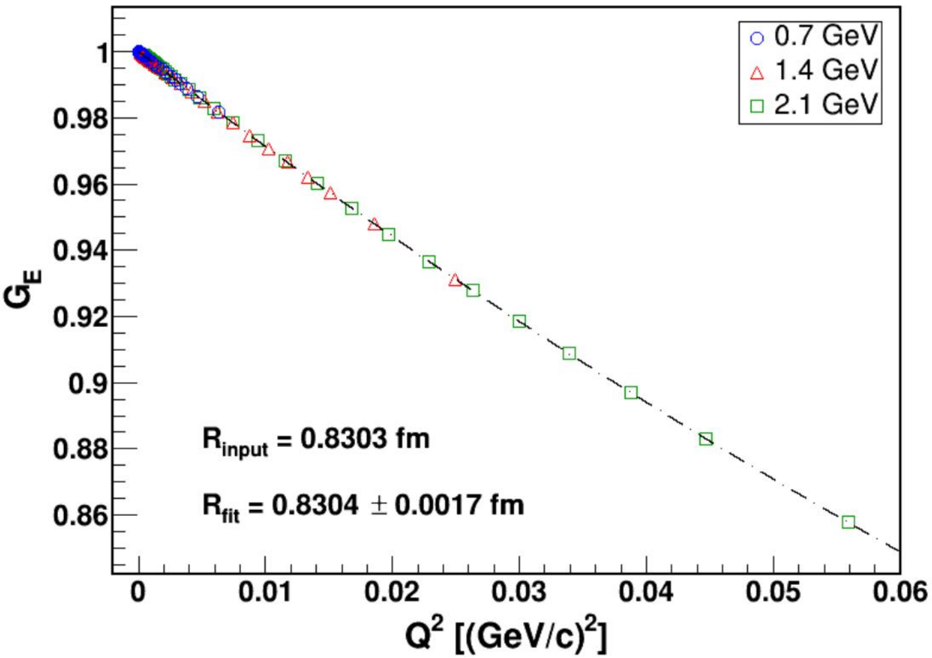
- The rms charge radius can be obtained from the slope of the electric/charge form factor $G_E^p\ /\mathrm{G_C^d}$ at $\mathrm{Q}^2 = 0$:

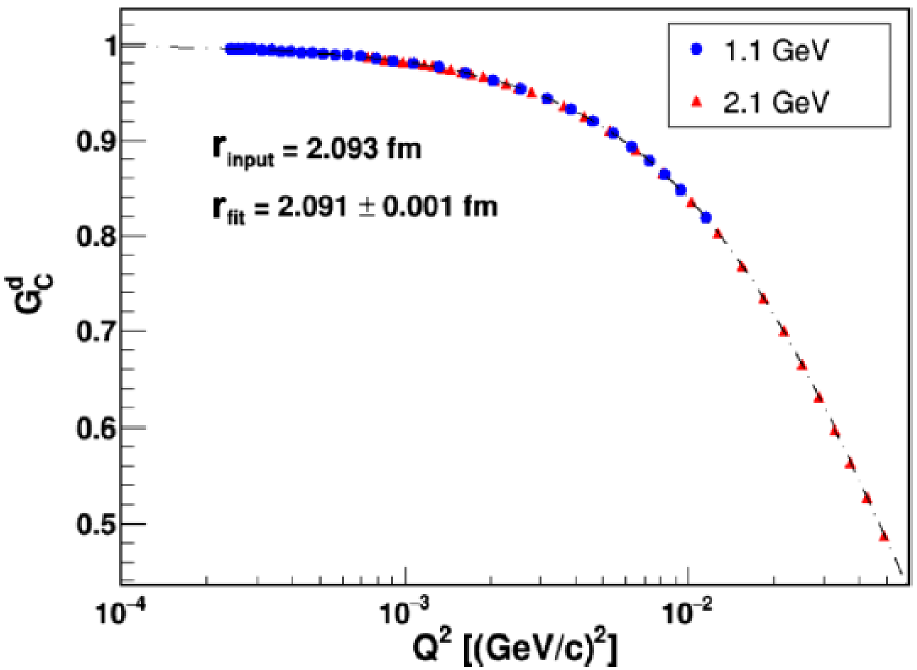
$$r_{p/d}^2 \equiv -6\left.\frac{dG_{E/C}^{p/d}(Q^2)}{dQ^2}\right|_{Q^2=0}$$

A robust fitter for the extraction of the radius

- We do not have measured data all the way to $Q^2 = 0$
- We never know the true function of $G_E^p(Q^2)/G_C^d(Q^2)$
- Fit a selected function to the G_E^p/G_C^d data, and extrapolate the function to $Q^2 = 0$ to obtain r_p/r_d

$$r_\text{p} \equiv \sqrt{\langle r_p^2 \rangle} \equiv \sqrt{-6\frac{dG_E^p(Q^2)}{dQ^2}\bigg|_{Q^2=0}} \qquad r_d \equiv \sqrt{\langle r_d^2 \rangle} \equiv \sqrt{-6\frac{dG_C^d(Q^2)}{dQ^2}\bigg|_{Q^2=0}}$$





X.Yan *et al.* PRC98,025204 (2018)
J.Zhou *et al.* PRC**103**, 024002 (2021)

A robust fitter for the extraction of the radius

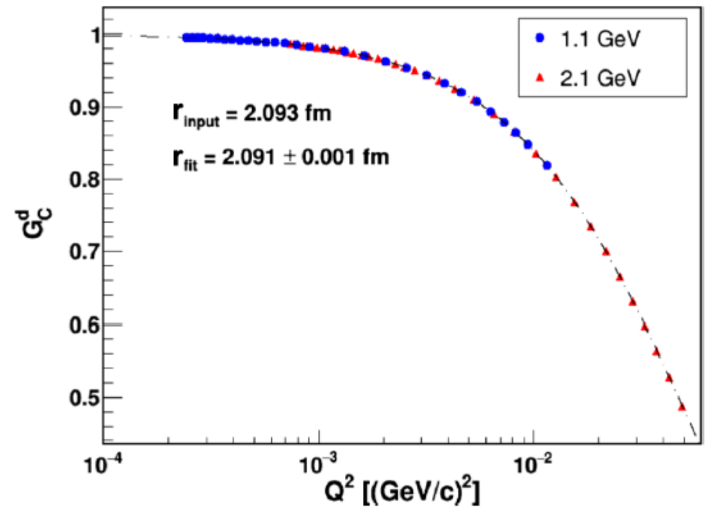
How to know the selected function (fitter) is good?
Describe the data, predict behavior at $Q^2 = 0$

Robust extraction of the proton charge radius from electron-proton scattering data

Xuefei Yan, Douglas W. Higinbotham, Dipangkar Dutta, Haiyan Gao, Ashot Gasparian, Mahbub A. Khandaker, Nilanga Liyanage, Eugene Pasyuk, Chao Peng, and Weizhi Xiong
Phys. Rev. C **98**, 025204 – Published 21 August 2018

Advanced extraction of the deuteron charge radius from electron-deuteron scattering data

Jingyi Zhou, Vladimir Khachatryan, Haiyan Gao, Douglas W. Higinbotham, Asia Parker, Xinzhan Bai, Dipangkar Dutta, Ashot Gasparian, Kondo Gnanvo, Mahbub Khandaker, Nilanga Liyanage, Eugene Pasyuk, Chao Peng, and Weizhi Xiong
Phys. Rev. C **103**, 024002 – Published 3 February 2021



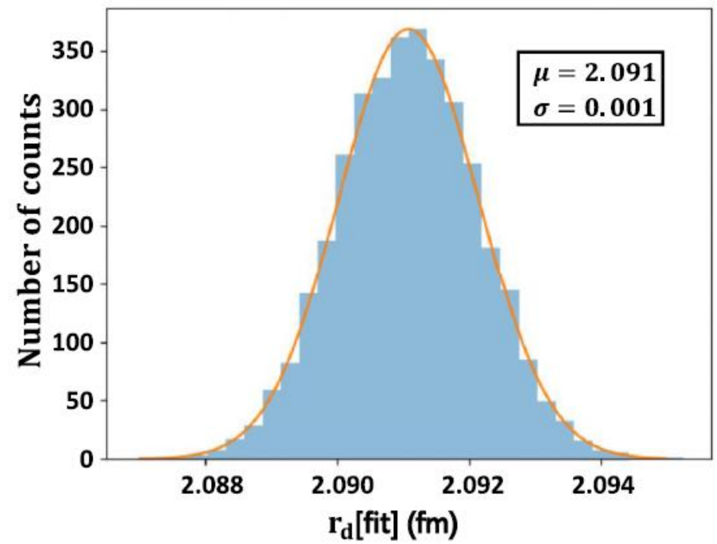
- **Robustness**: the fitter can extract $r_{p/d}$ precisely from a variety of pseudo-data generated from plausible form-factor parametrizations (with $r_{p/d}$ as the input)

→Qualitative Standard of **Robustness**: $\boldsymbol{\delta r < \sigma}$

- **Bias**: $\delta\mathrm{r} = \mathrm{r_{fit}} - \mathrm{r_{input}} = \mathrm{r_{fit}[mean]} - \mathrm{r_{input}}$
- $\boldsymbol{\sigma}$: uncertainty from one curve fitting ≅ root-mean-square width (reflect point-to-point δG_C^d)

→Standard of **Goodness**:

$$\mathrm{RMSE} = \sqrt{\delta r^2 + \sigma^2}$$



X.Yan et al. PRC98,025204 (2018)
*J.Zhou et al. PRC**103**, 024002 (2021)*

The robust fitter study for PRad and PRad-II

- 9 models to reflect various reasonable approximations to the unknown true function of G_E^p
- Fitters: dipole, monopole, gaussian, rational, polynomial, poly-z, and continued fraction…

$$f_{monopole}(Q^2) = \frac{p_0}{1 + Q^2R^2/6}$$

$$f_{dipole}(Q^2) = \frac{p_0}{\left(1 + \frac{Q^2R^2}{12}\right)^2}$$

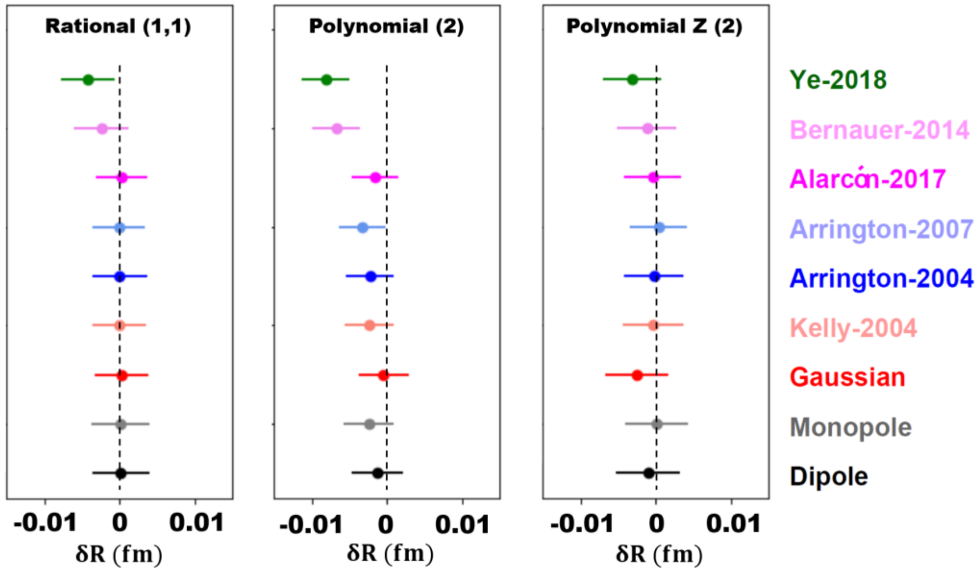
$$f_{gaussian}(Q^2) = p_0 Exp(-Q^2R^2/6)$$

$$f_{polyQ}(Q^2) = p_0(1 + \sum_{i=1}^{N} p_i Q^{2i})$$

$$f_{CF}(Q^2) = p_0 \frac{1}{1 + \frac{p_1 Q^2}{1 + \frac{p_2 Q^2}{1 + \cdots}}}$$

$$f_{rational\_a\_b}(Q^2) = p_0 \frac{1 + \sum_{i=1}^{N} p_i^a Q^{2i}}{1 + \sum_{j=1}^{M} p_j^b Q^{2j}}$$

$$f_{polyz}(Q^2) = p_0(1 + \sum_{i=1}^{N} p_i z^i) \qquad z = \frac{\sqrt{4m_\Pi^2 + Q^2} - \sqrt{4m_\Pi^2}}{\sqrt{4m_\Pi^2 + Q^2} + \sqrt{4m_\Pi^2}}$$



- Robust fitters: Rational(1,1), 2nd order polynomial, 2nd order polynomial-Z function
- Best fitter for PRad and PRad-II:

$$\boxed{f_{Rational(1,1)}(Q^2) = p_0 \frac{1 + p_1^a Q^2}{1 + p_1^b Q^2}}$$

$$r_{fit} = \sqrt{6(p_1^a - p_1^b)}$$

X.Yan et al. PRC98,025204 (2018)

The robust fitter study for DRad

- $Rational(1{,}1)$ does not match G_C^d data at higher Q^2 range (0.1~1.5 GeV^2) → search for possible new fitters
- Limited number of data-driven G_C^d parameterizations, can not reflect different approximations to the unknown true function as comprehensively → generalize the robustness test method

Abbott I: $$G_C^d(Q^2) = G_{C,0} \cdot \left[1 - \left(\frac{Q}{Q_C^0}\right)^2\right] \cdot \left[1 + \sum_{i=1}^{5} a_{ci} Q^{2i}\right]^{-1}$$

Abbott II:

$$G_C^d(Q^2) = \frac{G^2(Q^2)}{(2\tau+1)} \cdot \left[\left(1 - \frac{2}{3}\tau\right) g_{00}^+ + \frac{8}{3}\sqrt{2\tau}\, g_{+0}^+ + \frac{2}{3}(2\tau - 1) g_{+-}^+\right]$$

$$g_{00}^+ = \sum_{i=1}^{n} \frac{a_i}{\alpha_i^2 + Q^2} \qquad g_{+0}^+ = Q \sum_{i=1}^{n} \frac{b_i}{\beta_i^2 + Q^2} \qquad g_{+-}^+ = Q^2 \sum_{i=1}^{n} \frac{c_i}{\gamma_i^2 + Q^2}$$

Parker: $$G_C^d(Q^2) = G_{C,0} \cdot \left[1 - \left(\frac{Q}{Q_C^0}\right)^2\right] \cdot \left[\prod_{i=1}^{5} (1 + |a_i| Q^2)\right]^{-1}$$

Sum-of-Gaussian(SOG):

$$G_C^d(Q^2) = G_{C,0} \cdot e^{-\frac{1}{4}Q^2\gamma^2} \cdot \sum_{i=1}^{N} \frac{A_i}{1 + 2R_i^2/\gamma^2} \cdot \left[\cos(QR_i) + \frac{2R_i^2}{\gamma^2} \frac{\sin(QR_i)}{QR_i}\right]$$

A data-driven method to search for new fitters

$$\boxed{f_{\mathrm{Rational}(1,3)}(Q^2) = p_0 \frac{1 + a_1 Q^2}{1 + b_1 Q^2 + b_2 Q^4 + b_3 Q^6}}$$

$$\boldsymbol{r_{\mathrm{fit}}} = \sqrt{\boldsymbol{6(a_1 - b_1)}}$$

- Good asymptotic behaviors: $G_C^d = 1$ at $Q^2 = 0$, $G_C^d \to 0$ at $Q^2 \to \infty$

$$\boxed{f_{\mathrm{fixed\ Rational}(1,3)}(Q^2) = p_0 \frac{1 + a_1 Q^2}{1 + b_1 Q^2 + b_{2,\mathrm{fixed}} Q^4 + b_{3,\mathrm{fixed}} Q^6}}$$

$$\boldsymbol{r_{\mathrm{fit}}} = \sqrt{\boldsymbol{6(a_1 - b_1)}}$$

- Two free parameters in the fit of DRad pseudo-data to control the variance
- $b_{2,\mathrm{fixed}}$ and $b_{3,\mathrm{fixed}}$ are determined from the fit to the existing data at higher Q^2 range

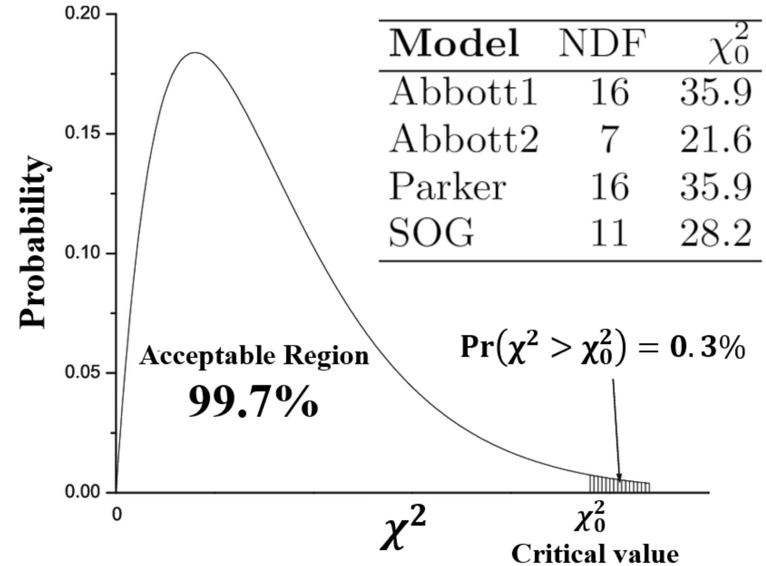
$$b_{2,\mathrm{fixed}} = 0.0416 \pm 0.0152$$
$$b_{3,\mathrm{fixed}} = 0.00474 \pm 0.000892$$
$$(\ \chi^2/\mathrm{NDF} \cong 1.25\ )$$

Q (fm^{-1})	G_C
0.86	.627 ($\pm$.011)
1.15	.474 ($\pm$.008)
1.58	.289 ($\pm$.006)
1.74	.238 ($\pm$.005)
2.026	.160 ($\pm$.005)
2.03	.163 ($\pm$.005)
2.352	.100 ($\pm$.004)
2.49	.087 ($\pm$.004)
2.788	$3.71\ (^{+1.47}_{-0.11})\times10^{-2}$
2.93	$3.45\ (^{+1.22}_{-0.39})\times10^{-2}$
3.566	$1.53\ (^{+0.06}_{-1.38})\times10^{-2}$
3.78	$1.25\ (^{+.05}_{-.55})\times10^{-2}$
4.09	$-1.14\ (\pm1.6)\times10^{-3}$
4.22	$1.63\ (^{+1.61}_{-1.44})\times10^{-3}$
4.46	$-2.39\ (\pm.61)\times10^{-3}$
4.62	$-1.63\ (\pm1.14)\times10^{-3}$
5.09	$-3.87\ (\pm0.30)\times10^{-3}$
5.47	$-3.48\ (\pm0.32)\times10^{-3}$
6.15	$-3.19\ (\pm0.55)\times10^{-3}$
	$-4.20\ (^{+.42}_{-.32})\times10^{-3}$
6.64	$-1.89\ (\pm0.38)\times10^{-3}$
	$-3.13\ (^{+.24}_{-.19})\times10^{-3}$

D. Abbott *et al.* (JLab t20 Collaboration), Eur. Phys. J. A **7**, 421 (2000).

A smearing method to estimate the bias

Make sure the smeared model is not too far away from the existing data (Q_0^2, G_{C0}^d) in D. Abbott *et al.* ($0.1\sim1.5\ \mathrm{GeV}^2$)



Model	NDF	χ_0^2
Abbott1	16	35.9
Abbott2	7	21.6
Parker	16	35.9
SOG	11	28.2

Mimic various approximations to the unknown true function

Smearing parameters → $(Q_0^2, G_C^{d'})$

$$\chi^2 = \sum \frac{\left(G_{C0}^d - G_C^{d'}\right)^2}{\left(\delta G_{C0}^d\right)^2}$$

$\chi^2 > \chi_0^2$

$\chi^2 < \chi_0^2$

Generating pseudo-data in DRad kinematics $[2 \times 10^{-4} - 5 \times 10^{-2}]\ \mathrm{GeV}^2$

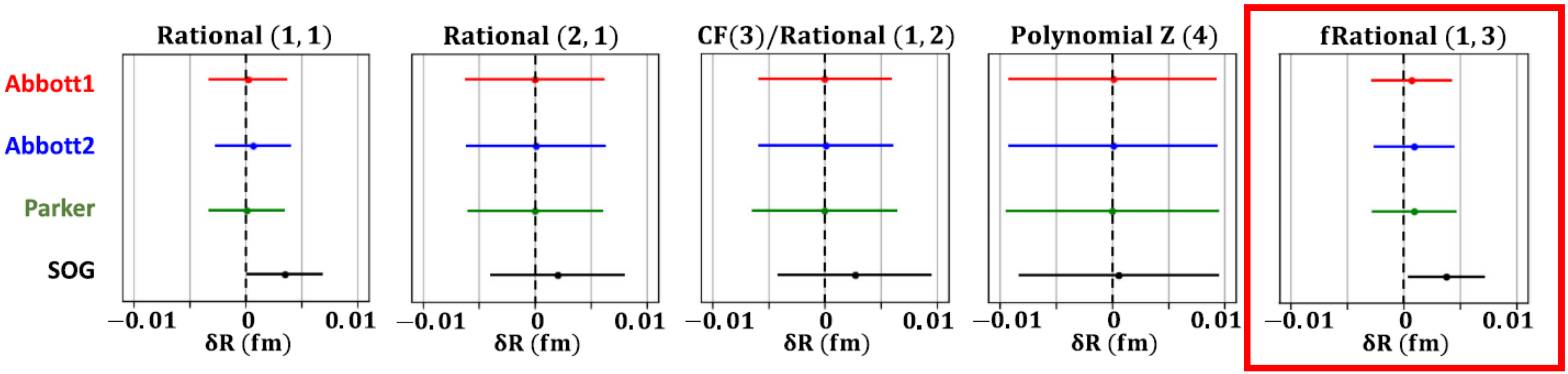
Fitting and obtain $\delta r_d / r_d$

repeat for 10,000 times

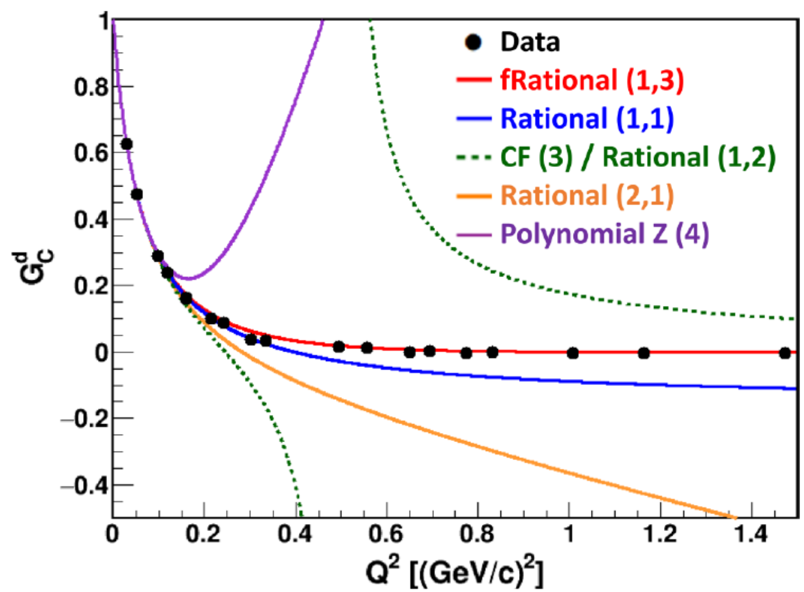
The rms value of the $\boldsymbol{\delta r_d / r_d}$ distribution → $\boldsymbol{\delta r_d}$

- The smearing method used with limited models cannot precisely reflect the behavior of other models, it can exhibit more comprehensively how a fitter controls the bias.

DRad Fitter Results



J.Zhou *et al*. PRC**103**, 024002 (2021)



- ❖ DRad experiment: RMSE(overall uncertainty) dominated by the point-to-point uncertainties
- ❖ Proposed fitter: fixed Rational(1,3)
  - Good ability to control the variance
  - Acceptable bias
  - Describe the G_C^d data at high Q^2 much better than the other fitters

Outlook:

- Use the data from the ongoing Mainz experiment to better constrain the fixed parameters
- New methods…

- Curves: Functional fits to the pseudo-data generated by the Abbott1 model in the DRad Q^2 range
- Data from D. Abbott *et al.* (JLab t20 Collaboration), Eur. Phys. J. A **7**, 421 (2000).

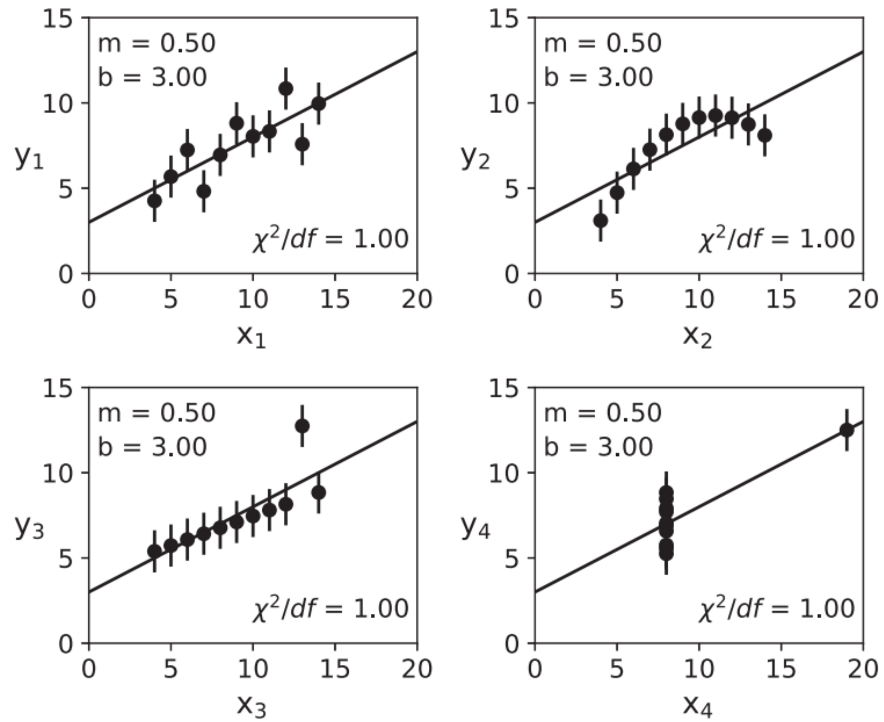
Zhu-Fang Cui et al 2022 Chinese Phys. C 46 122001

Summary and Outlook

- Analytic choices can affect the extraction of radius and electromagnetic form factors from elastic electron scattering cross section data
- Uncertainty in the radius extraction has become a non-negligible factor in a sub-percent level measurement
- A robust method was developed for PRad/Prad-II experiments and generalized for DRad proposal

X.Yan et al. PRC98,025204 (2018)

J.Zhou *et al.* PRC**103**, 024002 (2021)



S. K. Barcus, D. W. Higinbotham, and R. E. McClellan, Phys. Rev. C 102, 015205 (2020).

Outlook:

- More robust methods in extraction
- More comprehensive testing methods
- Reduce human bias in data analysis

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