NREC & PREN & μASTI: 6th–10th May 2024

QED corrections to low-energy scattering processes with McMule

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8th May 2024

QED and the mule

McMule

Monte Carlo for MUons and other LEptons

https://mule-tools.gitlab.io

P. Banerjee, A. Coutinho, T. Engel, A. Gurgone, F. Hagelstein, S. Kollatzsch, D. Moreno, L. Naterop, D. Radic, M. Rocco, N. Schalch, V. Sharkovska, A. Signer, Y. Ulrich

$\Rightarrow$ a framework for fully-differential higher-order QED calculations of scattering processes

- fixed-order NNLO QED corrections available/planned for

$\ell \to \ell' \nu\bar{\nu}$	$\ell \to \ell' \nu\bar{\nu}\gamma$	$\ell \to \ell' \nu\bar{\nu}(e^+e^-)$
$e^\pm\mu \to e^\pm\mu$	$e^\pm e^\pm \to e^\pm e^\pm$	$\ell\, p \to \ell\, p$
$e^+e^- \to \gamma^*$	$e^+e^- \to \gamma\,\gamma$	$e^+e^- \to \mu^+\mu^-\gamma$

- full NNLO!!, toying with N^{3}LO, but no dirty protons and no parton shower/YFS (yet)
- fully differential Monte Carlo integrator $\Rightarrow$ generator to follow [Y.Ulrich]

Talk based on Impact of NNLO QED corrections on lepton-proton scattering at MUSE

[2307.16831: T.Engel, F.Hagelstein, M.Rocco, V.Sharkovska, AS, Y.Ulrich]

Three (not necessarily new) messages

- # 1 QED/QCD has seen huge progress, do not use techniques of the 70ties
- # 2 NNLO QED corrections can be larger than TPE
- # 3 For TPE, always do virtual + real

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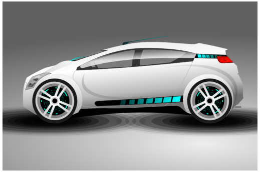
www.flickr.com/photos/jacksnell707/3410447948

1970 Chevrolet Monte Carlo, looks great, but ...



this one is just better

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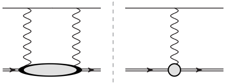
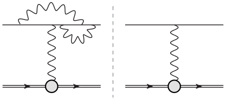
never do  without 

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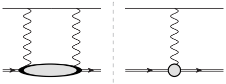

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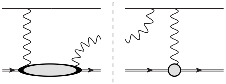
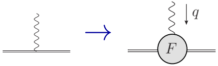


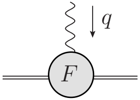
set-up and LO

start with Moller scattering at NNLO [2107.12311: P.Banerjee, T.Engel, N.Schalch, AS, Y.Ulrich]

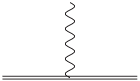
and Muon-electron scattering at NNLO
[2212.06481: A.Broggio, T.Engel, A.Ferroglia, M.Mandal, P.Mastrolia, M.Rocco, J.Ronca, AS, W.Torres Bobadilla, Y.Ulrich, M.Zoller]

From $e^\pm\mu \to e^\pm\mu$ to $\ell\, p \to \ell\, p$: as a simplistic model

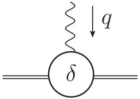




$$= -\, e q_p\, \gamma^\mu + \delta^\mu =$$

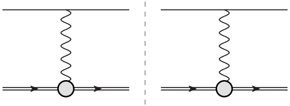


$+$



$$F_{1/2} \sim \left(\frac{\Lambda^2}{\Lambda^2 + Q^2}\right)^2$$ with $0.60\,\mathrm{GeV}^2 \le \Lambda^2 \le 0.86\,\mathrm{GeV}^2$

$$\mathcal{M}_n^{(0)}(q_\ell^2, F^2) = \left|\mathcal{A}_n^{(0)}(q_\ell, F)\right|^2 =$$



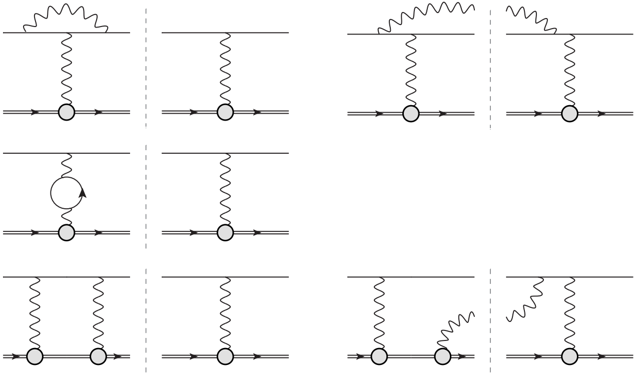
only representative diagrams are shown, but obviously all have been computed

virtual real

electron $q_\ell^4\, F^2$

fermionic $q_\ell^2\, \Pi^{(1)}\, F^2$

TPE $\supset$ mixed $q_\ell^3\, F^3$



! IR singularities !

NLO

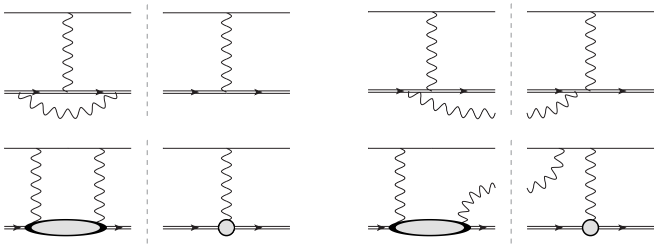
fully included   approximately included   not included

i.e. only elastic TPE is implemented in McMule so far

virtual   real

proton $q_\ell^2\, q_p^4$

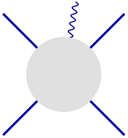
mixed $\to 0$



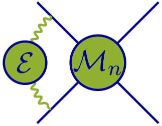
proton-line corrections (IR divergences) not treated as part of form factor

even if mixed is IR finite, still have virtual + real

$m \neq 0$ $\quad\Rightarrow$ only soft singularities use dim reg for IR singularities



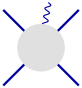
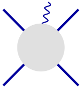
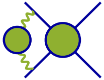
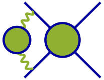
$\xrightarrow{E_\gamma \to 0}$



$$\mathcal{M}^{(\ell)}_{n+1} = \mathcal{E}\, \mathcal{M}^{(\ell)}_{n} \; + \; \mathcal{O}(E_\gamma^{-1})$$

eikonal $\mathcal{E} = \sum_{i,j} \frac{p_i \cdot p_j}{(p_\gamma \cdot p_i)\; (p_\gamma \cdot p_j)} \sim \mathcal{O}(E_\gamma^{-2})$

$\Rightarrow$ subtraction scheme (FKS$^\ell$) at any order in QED [1909.10244; T.Engel, AS, Y.Ulrich]

$\int d\Phi_\gamma$  $= \int d\Phi_\gamma \Big($  $-$  $\Big) + \int d\Phi_\gamma$ 

divergent and complicated — complicated but finite — divergent but easy

subtraction scheme

we do not write $\sigma_n^{(1)} = \sigma_n^{(v)}(\lambda) + \sigma_n^{(s)}(\lambda,\omega) + \sigma_{n+1}^{(h)}(\omega)$ photon mass λ, resolution ω

we do write $\sigma_n^{(1)} = \sigma_n^{(1)}(\xi_c) + \sigma_{n+1}^{(1)}(\xi_c)$ at NLO

no approximation at all

$\sigma_n^{(2)} = \sigma_n^{(2)}(\xi_c) + \sigma_{n+1}^{(2)}(\xi_c) + \sigma_{n+2}^{(2)}(\xi_c)$ at NNLO

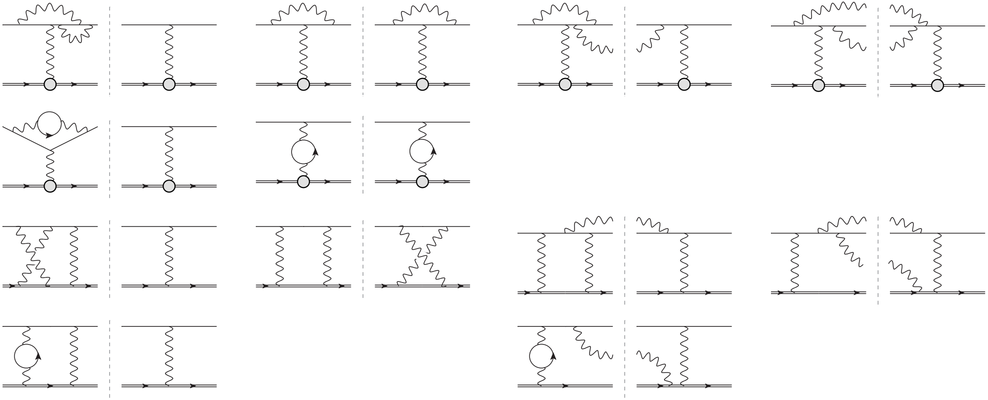
$$\sigma_n^{(1)}(\xi_c) = \int d\Phi_n^{d=4} \Big(\underbrace{\mathcal{M}_n^{(1)}}_{1/\epsilon} + \underbrace{\hat{\mathcal{E}}(\xi_c)}_{1/\epsilon} \mathcal{M}_n^{(0)} \Big) = \int d\Phi_n^{d=4} \underbrace{\mathcal{M}_n^{(1)f}(\xi_c)}_{\text{finite}}$$

$$\sigma_{n+1}^{(1)}(\xi_c) = \int d\Phi_{n+1}^{d=4} \left(\frac{1}{\xi_1}\right)_c \left(\xi_1 \mathcal{M}_{n+1}^{(0)f}\right)$$

ξ_c dependence cancels between the two/three terms (implementation/stability check)

NNLO

two-loop virtual | one-loop squared | real virtual | double real



e

VP

mix

VP

also pointlike proton NNLO corrections, but they are tiny

challenges @ NNLO

Many people are involved, directly McMule AND indirectly beyond McMule!

- many diagrams $\to$ automate generation of diagrams, algebra, reduction to master integrals $\Rightarrow$ amplitude $\sim$ 60 Mb

  [Bonciani, Broggio, Di Vita, Ferroglia, Mandal, Mastrolia, Mattiazzi, Primo, Ronca, Schubert, Torres Bobadilla, Tramontano]
- MUonE two-loop integrals with $m_e = 0$ expressed in terms of generalised polylogs $\to$ develop Fortran tool for fast numerical evaluation [handyG]
- include effects of $m_e \neq 0$ in mixed NNLO approximately (massification)

  [Penin; Becher, Melnikov; Engel, Gnendiger, AS, Ulrich]
- delicate numerics for one-loop (up to pentagon) diagrams $\to$ use [OpenLoops]
- delicate numerics in phase space integration $\to$ use next-to-soft approach, extension of LBK theorem beyond NLO [Engel, AS, Ulrich]
- . . . no 1970 Chevrolet Monte Carlo has been used . . . recall message # 1

toy observables

- how important are the various contributions?
- can we just neglect NNLO ?
- do we really want to make approximations at NLO ?
- how relevant are specifics of TPE vs NNLO ?

- answers depend on precise definition of observables
- let's look at some toy observables for MAGIX, MUSE, AMBER, PRad ...
- if your favourite observable is not here, ask the mule

$\Rightarrow$ McMule is keen to exchange benchmarks with experimental collaborations

MAGIX, $E = 105$ MeV

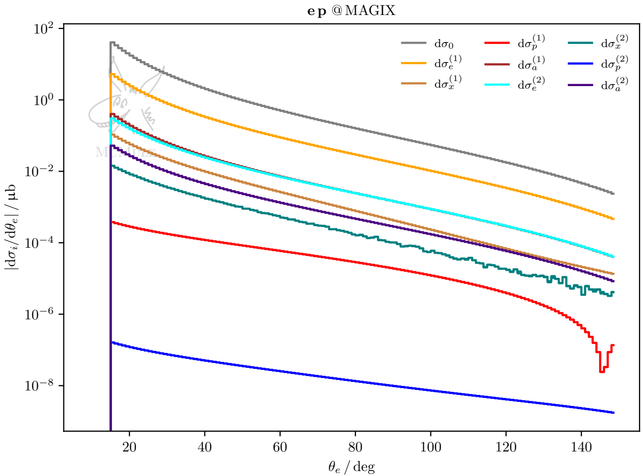
$e^- \, p \to e^- \, p$
with $E_{\text{in}}(e^-) = 105$ MeV

$15^\circ \le \theta_e \le 150^\circ$

73.5 MeV $\le p_e \le 105$ MeV

$E_{\gamma\text{tot}} < 1$ MeV (!)

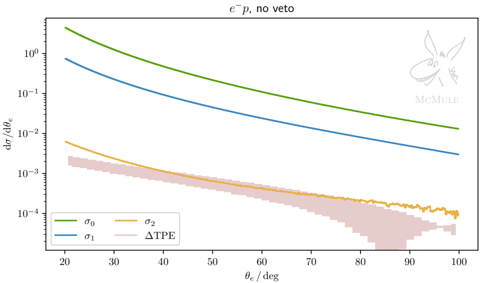
recall message # 2

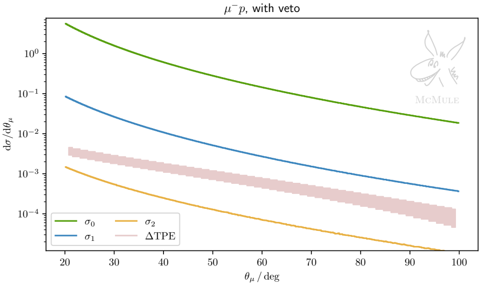


MUSE, $p = 210\,\mathrm{MeV}$

$e\,p$: no cuts on γ

$\mu\,p$: cut on forward γ





QED corrections large

QED corrections small

avoiding initial state collinear emission and $e \to \mu$ helps

$e\,p$ with cut looks similar to $\mu\,p$ (with/out) cut

AMBER, $E = 100$ GeV

$\mu^- \, p \to \mu^- \, p$

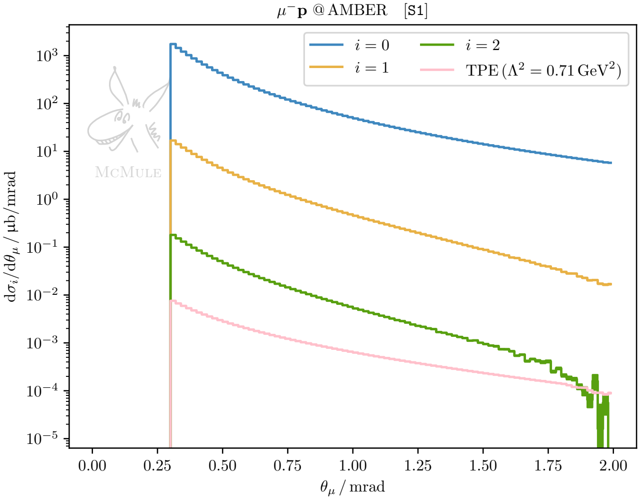
with $E_{\text{in}} = 100$ GeV

0.3 mrad $\leq \theta_\ell \leq 2$ mrad

$E_\mu > 99$ GeV

our TPE is questionable

recall message # 2



PRad, $E = 1.1$ GeV

$e^- p \to e^- p$

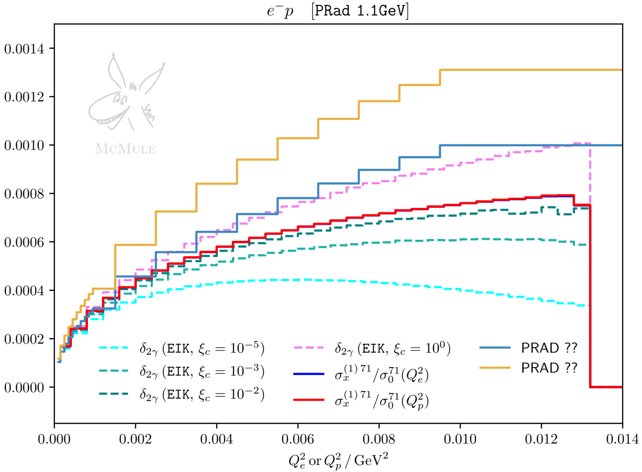
with $E_{\text{in}}(e^-) = 1.1$ GeV

only TPE $=$ mixed NLO

$0.7^\circ \leq \theta_e \leq 6^\circ$

$E_\gamma < 20$ MeV if $\theta_\gamma > 6$ mrad

recall message # 3



outlook / homework

homework for the mule

- implement a decent TPE (beyond elastic TPE implemented so far)
- other targets than proton
- maybe think about polarisation

homework for the rest of us

- recite the three messages every evening before going to sleep
  - # 1 QED/QCD has seen huge progress, do not use techniques of the 70ties
  - # 2 NNLO QED corrections can be larger than TPE
  - # 3 For TPE, always do virtual + real