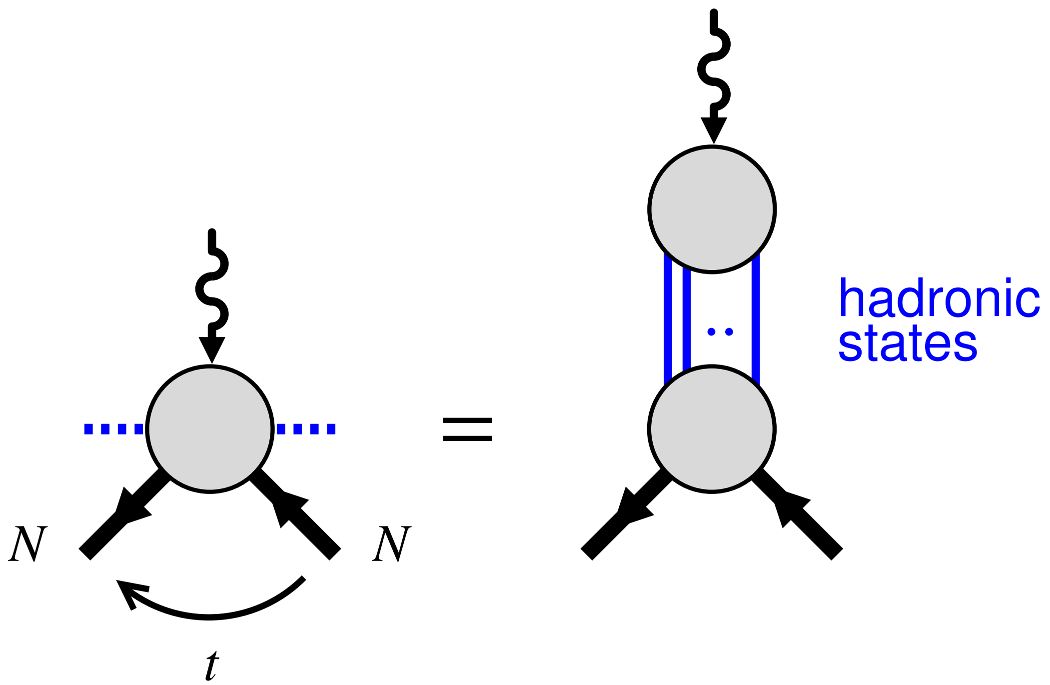
Combining dispersion theory and chiral EFT in low-Q2 form factor analysis

C. Weiss (JLab), NREC 2024 Workshop, Stony Brook U., 08 May 2024

Jefferson Lab



Analytic structure

Correlations $Q^2 = 0 \longleftrightarrow$ finite

DIChEFT: Dispersion theory × chiral EFT

Spectral functions $\pi\pi$

Information flow

Radius extraction

ep Mainz electric/magnetic

μp MUSE

Applications

Transverse densities

Other form factors: EM Tensor, GPDs

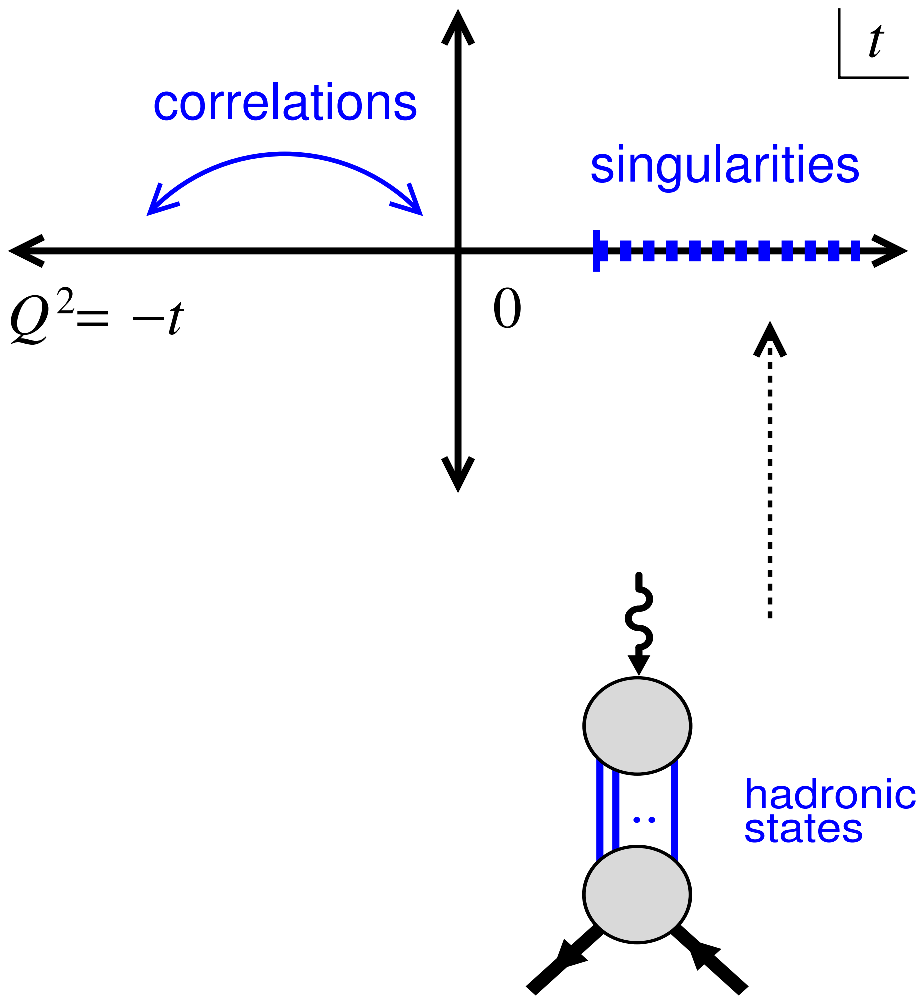
Based on

F. Gil-Dominguez, J.M. Alarcon, C. Weiss PRD 108, 074026 (2023) [INSPIRE]

J.M. Alarcon, D. Higinbotham, C. Weiss, PRC 102, 035203 (2020) [INSPIRE]

J.M. Alarcon, C. Weiss, PRC 96, 055206 (2017) [INSPIRE], PRC97, 055203 (2018) [INSPIRE], PLB784, 373 (2018) [INSPIRE]

Analyticity: Motivation



FFs analytic functions of $t = -Q^2$

Singularities: Branch cuts at $t > 0$
from hadronic exchanges

Position of singularities: Hadron masses
Strength of singularities: Amplitudes → Theory

Implications for radius extraction

Correlates FF at $Q^2 > 0$ with derivatives at $Q^2 = 0$

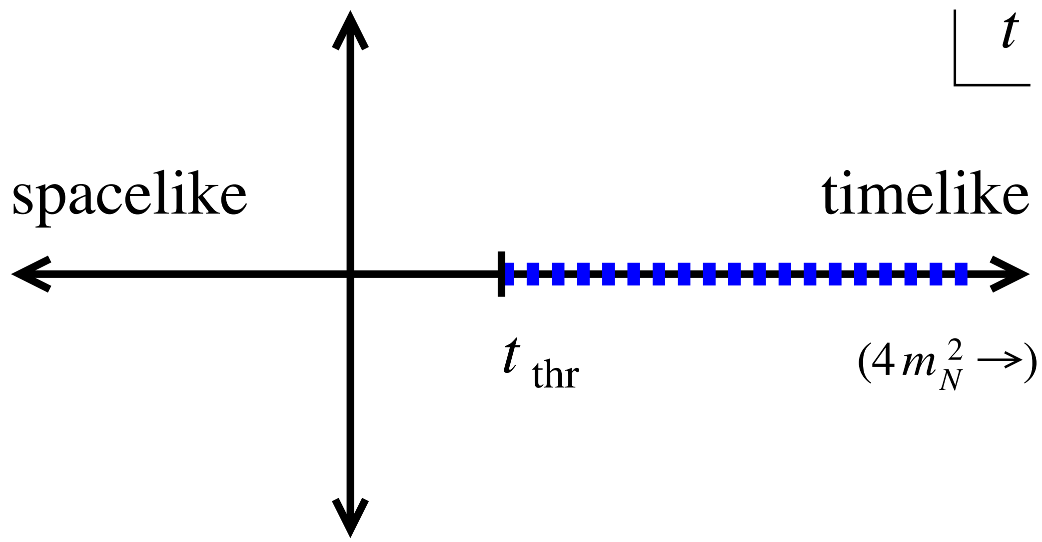
Allows to use data at finite Q^2 for radius extraction,
avoids "extrapolation to zero"

Necessary for magnetic radius extraction

Predicts size and pattern of higher derivatives
from singularities

Should be implemented
and used in radius extraction!

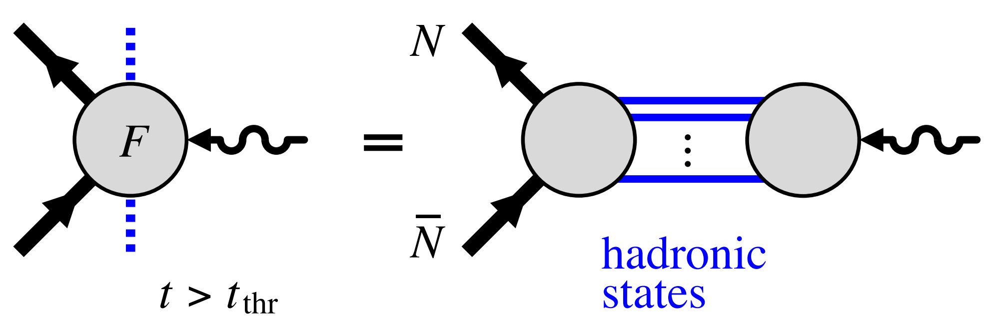
Analyticity: Dispersion theory



Dispersive representation

$$F_i(t) = \int_{t_{\text{thr}}}^{\infty} \frac{dt'}{\pi} \frac{\text{Im}\, F_i(t')}{t' - t - i0}$$

Expresses analytic structure of $F_i(t)$ on physical sheet



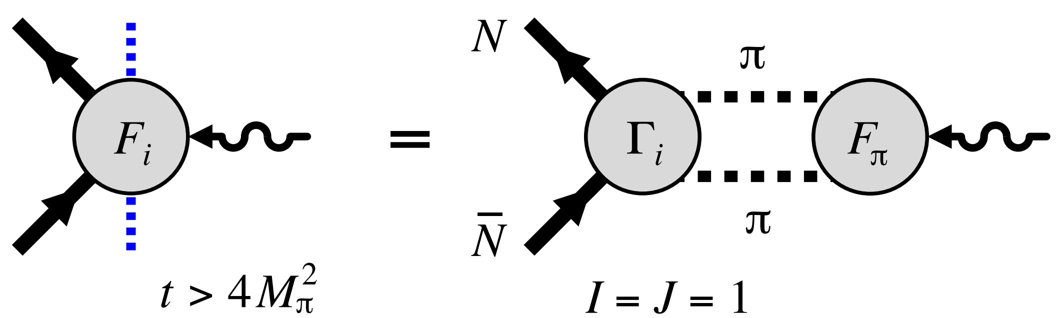
Spectral functions Im $F_i(t)$

Transition amplitude current → hadronic states → $N\bar{N}$

Processes in unphysical region $t < 4m_N^2$ below $N\bar{N}$ threshold

Isovector: $\pi\pi$ (incl. ρ), 4π, $K\bar{K}$, …
Isoscalar: 3π (incl. ω), $K\bar{K}$ (incl. ϕ), …

Needs to be calculated theoretically
Frazer, Fulco 1960; Höhler et al 1975+



Two-pion cut

Appears in isovector vector form factors

Lowest-mass state, dominates low-Q^2 spacelike form factors, peripheral densities

$\pi\pi$ system strongly interacting, ρ resonance

Spectral functions on two-pion cut

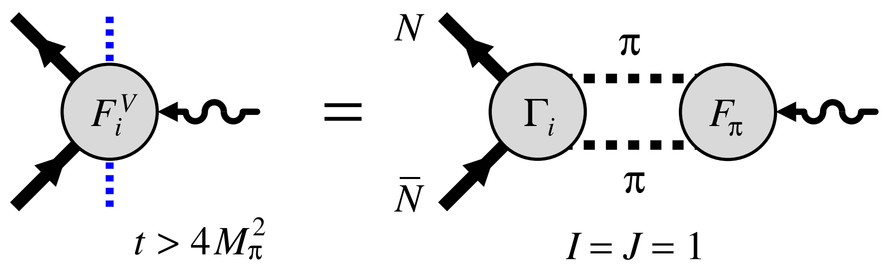
Analytic continuation of πN scattering data
Frazer, Fulco 1960; Höhler et al 1975+

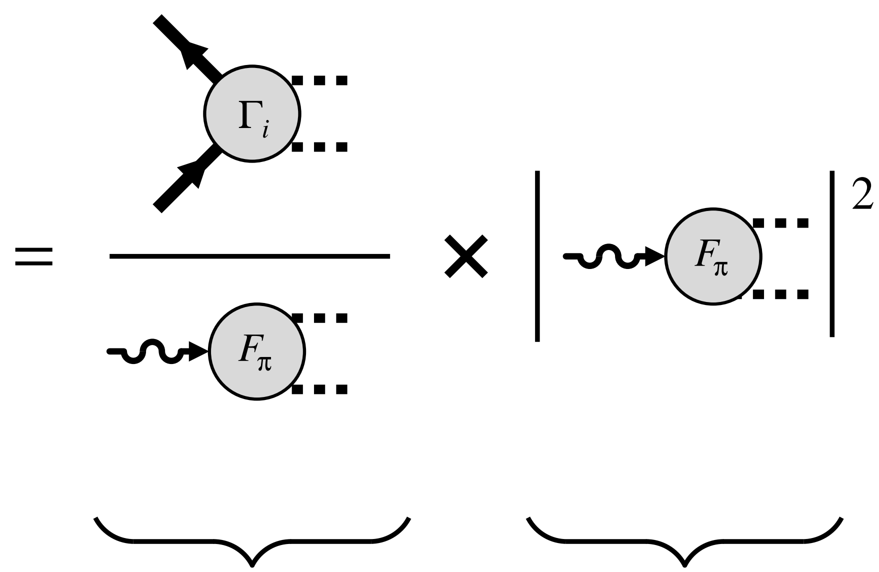
Roy-Steiner equations for πN scattering
Hoferichter, Kubis, Ruiz de Elvira, Hammer, Meissner 2016

Chiral EFT? Direct calculations poorly convergent because of strong $\pi\pi$ interactions
Gasser, Sainio, Svarc 1988; Becher, Leutwyler 1999; Kubis, Meissner 2001; Kaiser 2003; ...

Need different approach!

DIChEFT: Elastic unitarity and $\pi\pi$ interactions





calculated in chiral EFT

measured in $e^+e^- \to \pi^+\pi^-$

$$\text{Im}\, F_i(t) = \frac{k_{\text{cm}}^3}{\sqrt{t}} \Gamma_i(t)\, F_\pi^*(t)$$

$$= \frac{k_{\text{cm}}^3}{\sqrt{t}} \frac{\Gamma_i(t)}{F_\pi(t)} \,|\, F_\pi(t)\,|^2$$

Elastic unitarity relation

$F_\pi(t)$ current $\to$ $\pi\pi$ amplitude = pion timelike FF
$\Gamma_i(t)$ $\pi\pi \to N\bar{N}$ partial-wave amplitude

Amplitudes have same phase from $\pi\pi$ interactions: Watson theorem

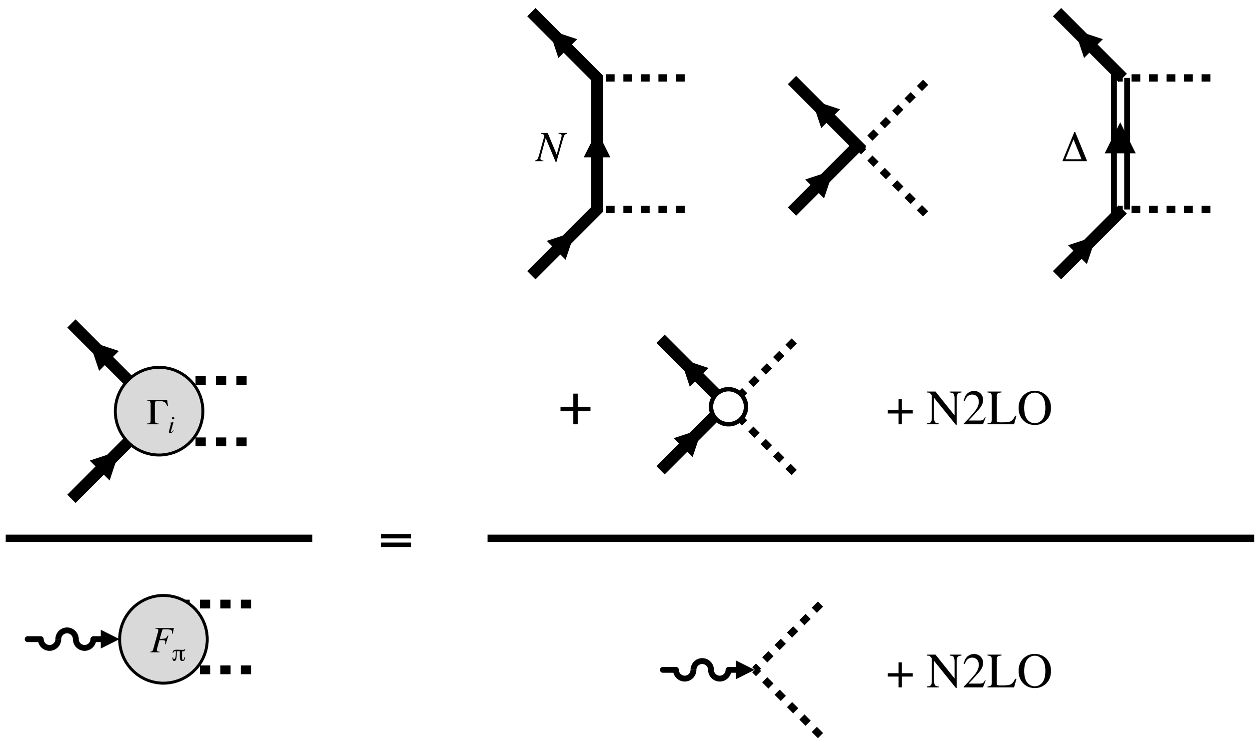
Factorize $\pi\pi$ interactions (N/D representation)

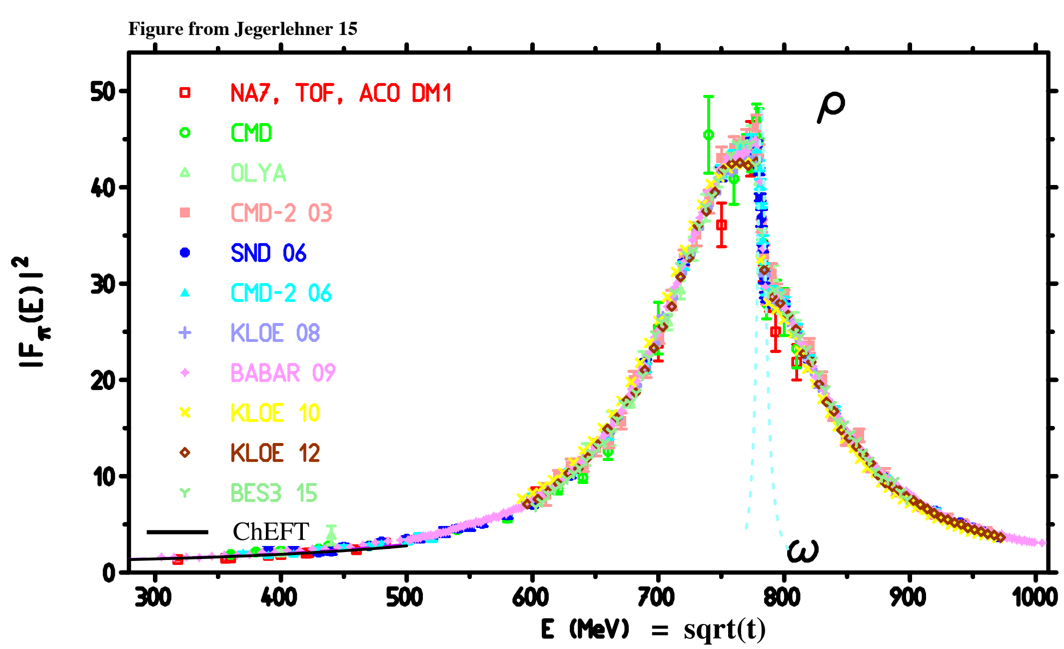
Γ_i / F_π free of $\pi\pi$ interactions
→ calculated in ChEFT with good convergence

$|\, F_\pi\,|^2$ contains $\pi\pi$ interactions
→ measured in e^+e^- annihilation

Alarcon, Hiller Blin, Vicente Vacas, Weiss, NPA 96, 18 (2017)
Alarcon, Weiss, PLB 784 (2018) 373; PRC 97 (2018) 055203
Alt. formulation: Granados, Leupold, Perotti 2017

DIChEFT: Calculation





Relativistic ChEFT

Expansion in $\{M_\pi, k_\pi\}/\Lambda_{\text{chiral}}$

Include Δ isobar

ChEFT calculation of Γ_i/F_π

LO: Born terms + Weinberg-Tomozawa

NLO: Contact term in Γ_i $(i = 2)$

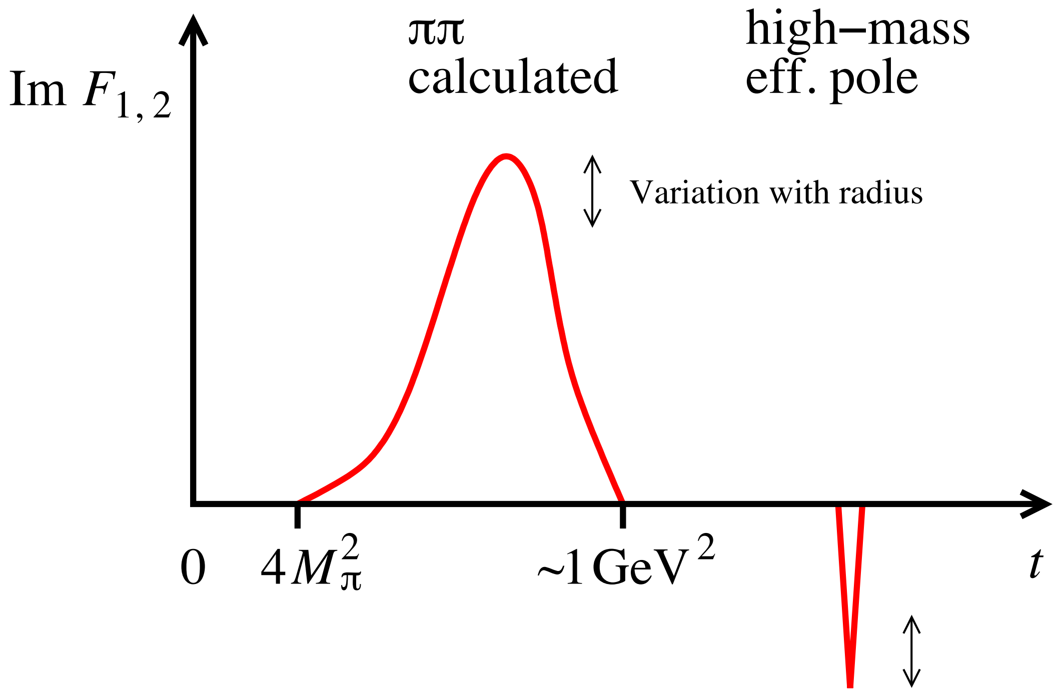
N2LO: Contact term and pion loops
Presently use partial result
Contains LEC, to be determined

Good convergence

Pion timelike form factor $|F_\pi|^2$

Measured accurately in $e^+e^- \to \pi^+\pi^-$

DIChEFT: Sum rules and parameters



Spectral functions

$\pi\pi$ region calculated from unitarity + ChEFT

High-mass region parametrized by effective poles
Pole positions → theoretical uncertainty

Sufficient for low-Q^2 form factors

Sum rules and parameters

$$\frac{1}{\pi}\int_{t_{\rm thr}}^{\infty} dt\, \frac{\text{Im}\, F_1(t)}{t} = Q$$

$$\frac{1}{\pi}\int_{t_{\rm thr}}^{\infty} dt\, \frac{\text{Im}\, F_1(t)}{t^2} = \frac{1}{6}\langle r^2\rangle_1$$

$$\frac{1}{\pi}\int_{t_{\rm thr}}^{\infty} dt\, \frac{\text{Im}\, F_2(t)}{t} = \kappa$$

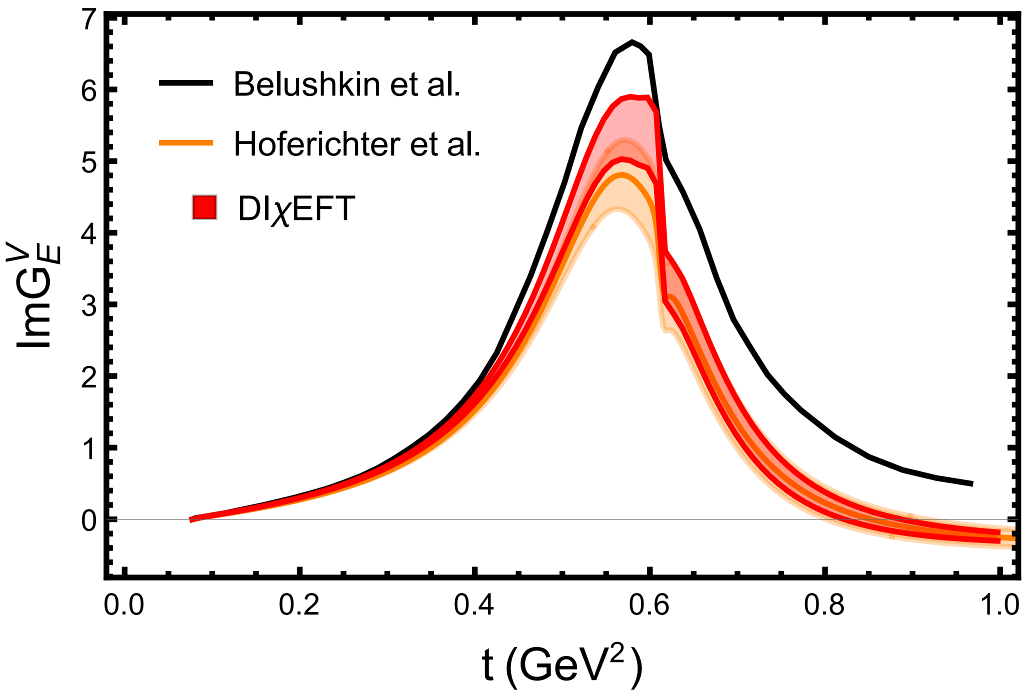
$$\frac{1}{\pi}\int_{t_{\rm thr}}^{\infty} dt\, \frac{\text{Im}\, F_2(t)}{t^2} = \frac{1}{6}\kappa\langle r^2\rangle_2$$

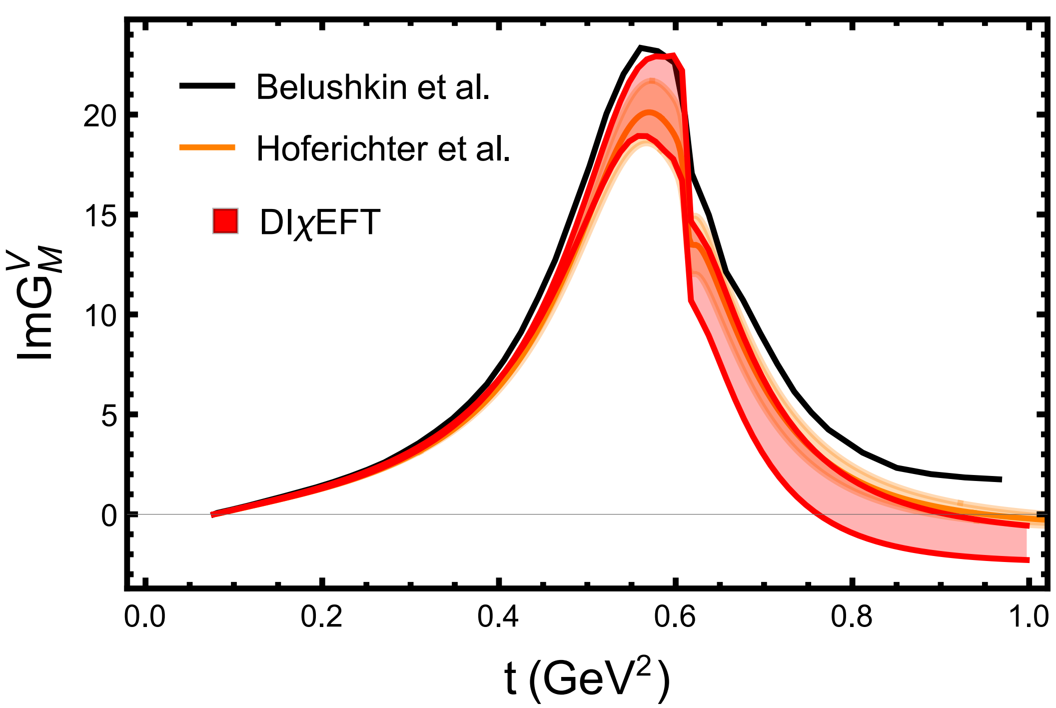
[+ asymptotic conditions]

Spectral functions constrained by sum rules
for $F(0), F'(0)$ = charges, radii

Sum rules connect ChEFT LECs ↔ nucleon radii

Nucleon radii appear directly as parameters,
control finite-Q^2 behavior of form factors





Alarcon, Weiss, PLB784, 373 (2018) [INSPIRE]

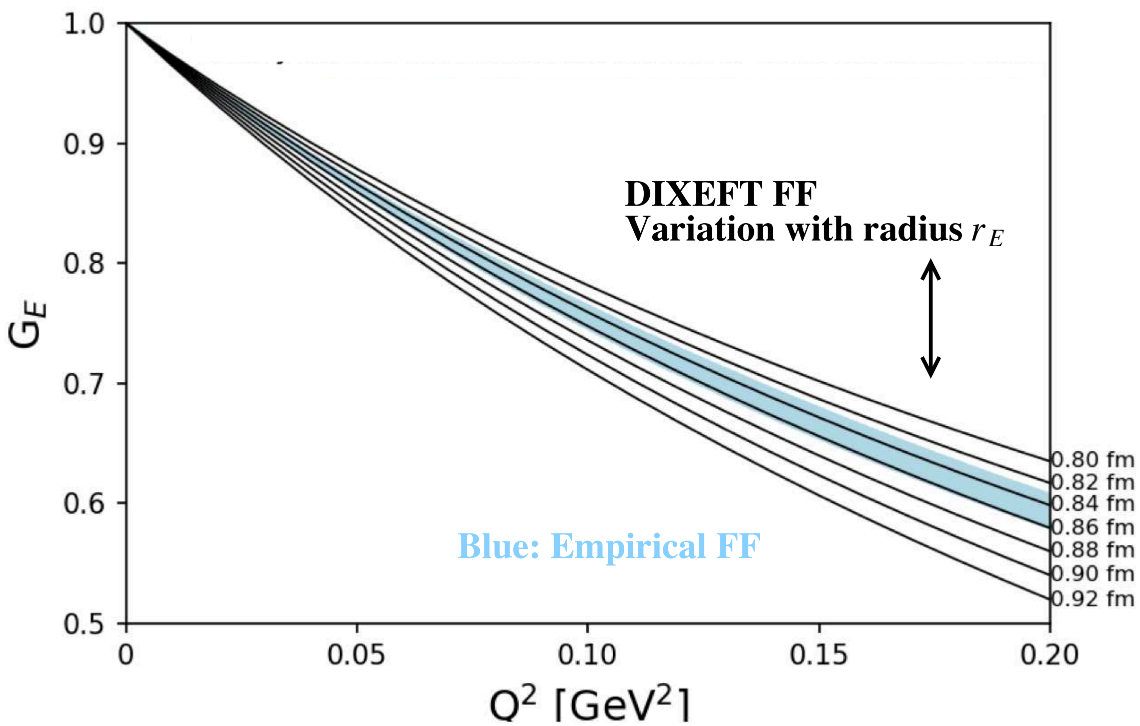
Depend on radii as parameters

Bands show uncertainty from radii (PDG range)
Uncertainty from high-mass pole position → later

Good agreement with Roy-Steiner results
Hoferichter et al. 2017

Here: $G_E, G_M \leftrightarrow F_1, F_2$

DIChEFT: Form factors



$$G_i(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{\pi} \frac{\text{Im}\, G_i(t')}{t' - t - i0}$$

Family of FF predictions depending on radii as parameters

Each member respects analyticity, sum rules

Each member has intrinsic theoretical uncertainty from high-mass states

Radius correlated with finite-Q^2 behavior!

Radius extraction using DIChEFT

Compare DIChEFT FF predictions with data, for various values of radius parameter

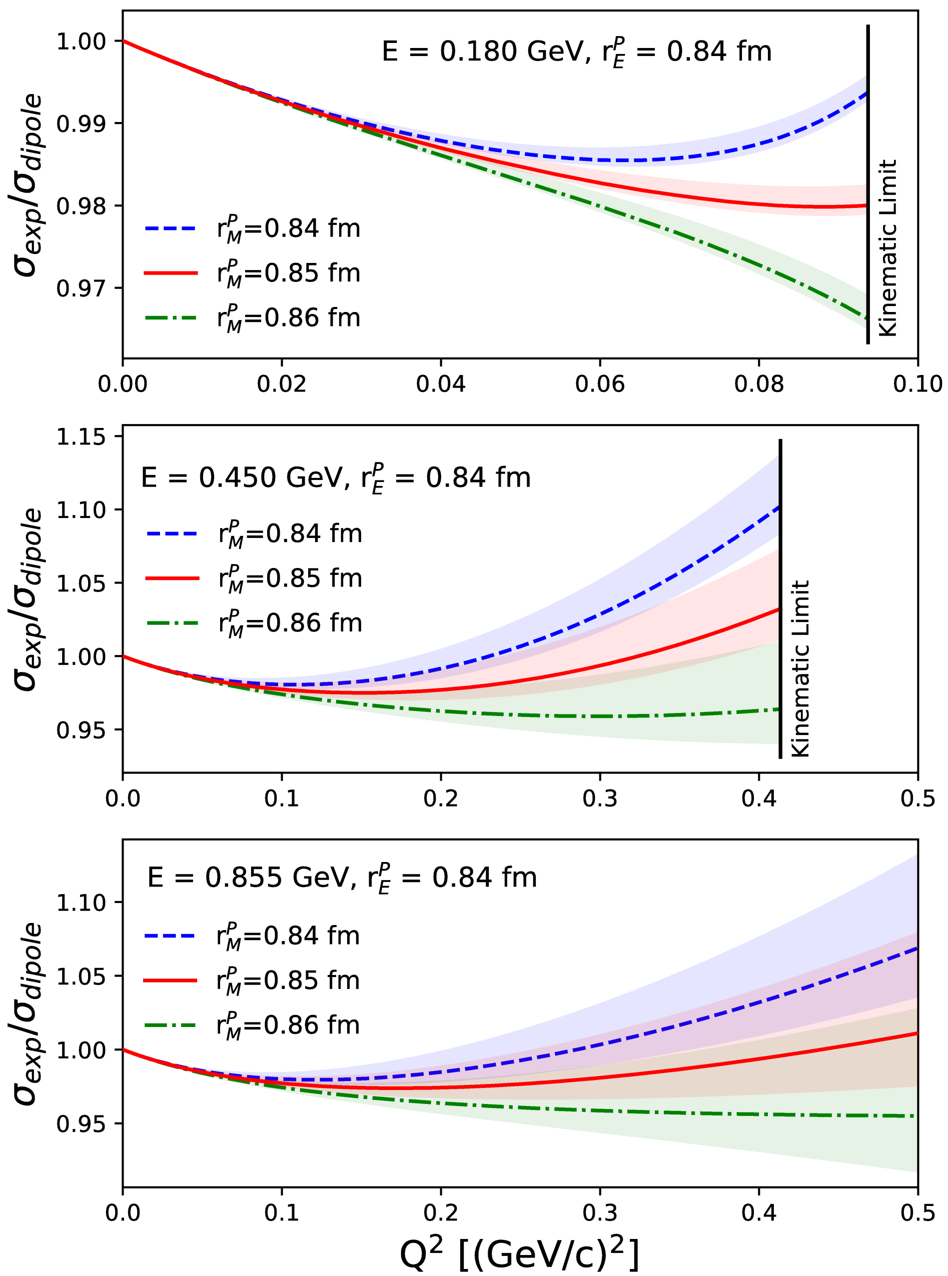
Radius constrained by finite-Q2 data

Optimal Q2 range determined by interplay of radius sensitivity and exp+thy uncertainties

Example: Charge radius $r_E^p = 0.844(7)$ fm extracted from fit to FF data, Q2 range up to ~0.5 GeV2, uncertainties estimated

Alarcon, Higinbotham, Weiss, Ye PRC 99, 044303 (2019) [INSPIRE]

Radius extraction: ep Mainz



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{\epsilon\,[G_E^p]^2 + \tau[G_M^p]^2}{\epsilon(1+\tau)}$$

Extracted electric + magnetic radii from fit to cross section data

Used DIChEFT $G_{E,M}$ depending on r_E^p, r_M^p

Fitted cross sections with floating normalizations

Quantified fit and theoretical uncertainties

$r_E^p = 0.842 \pm 0.002$ (fit 1σ) ${}^{+0.005}_{-0.002}$ (theory full-range) fm

$r_M^p = 0.850 \pm 0.001$ (fit 1σ) ${}^{+0.009}_{-0.004}$ (theory full-range) fm

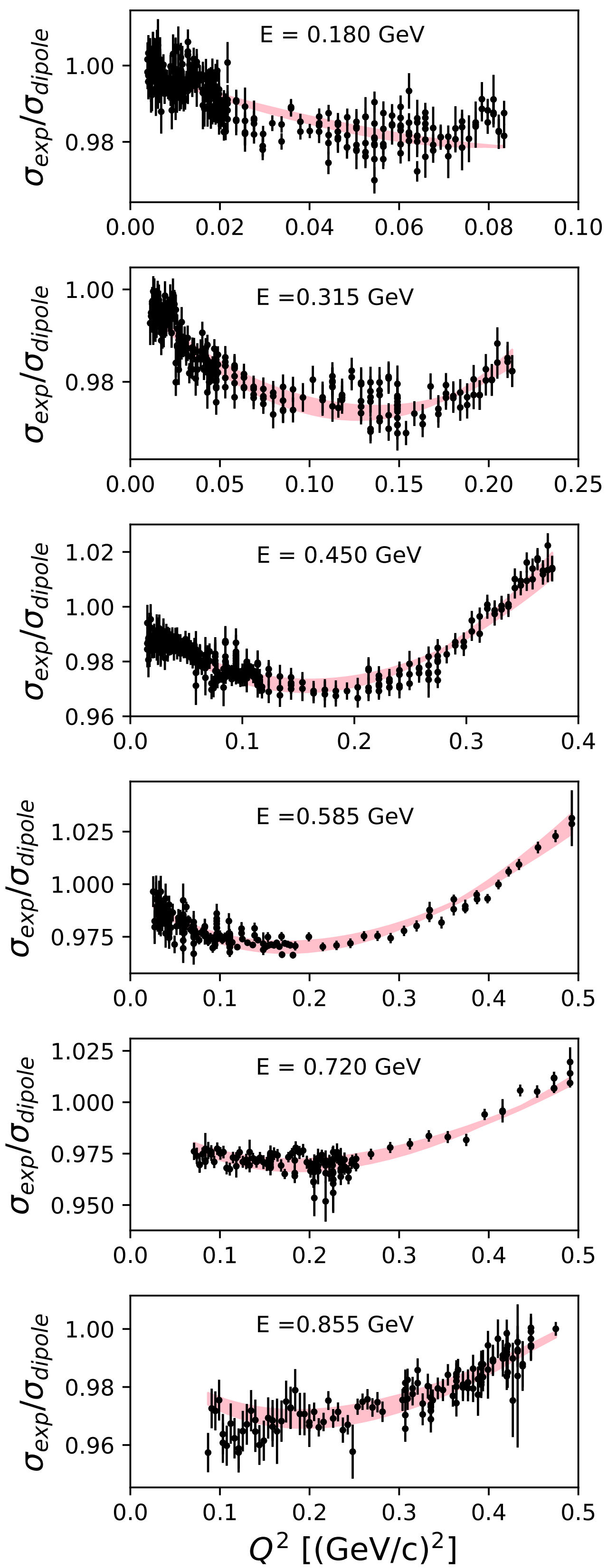
Sensitivity to G_M only at $Q^2 > 0$, needs theory

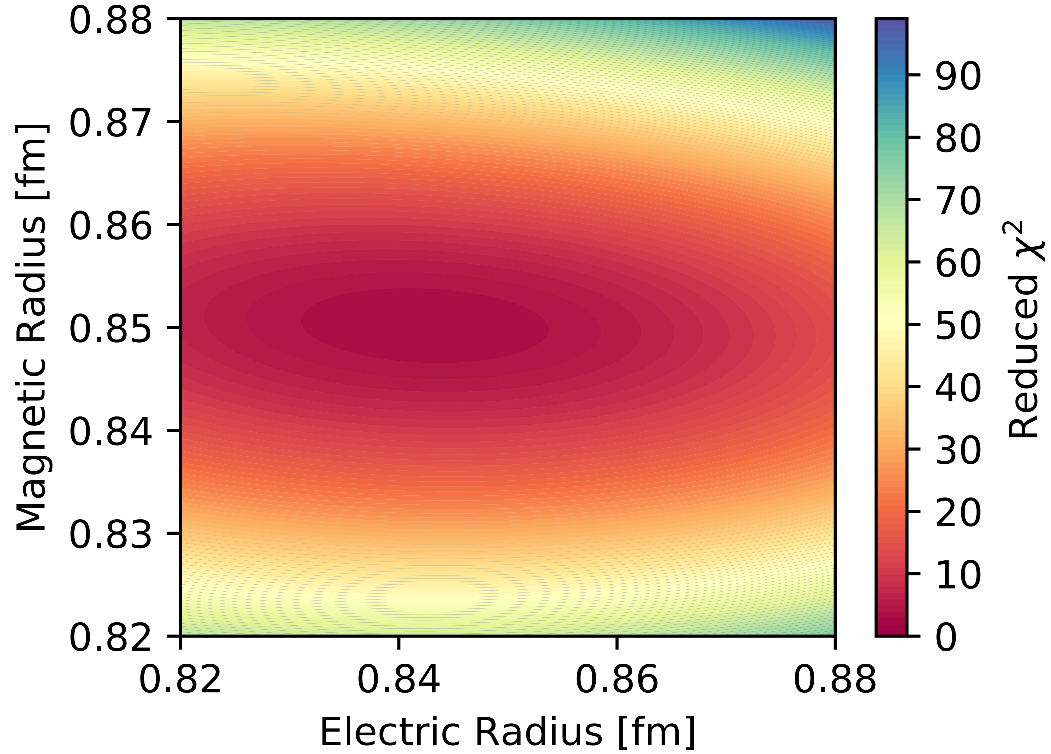
DIChEFT enables accurate magnetic radius extraction

Conventional dispersion analysis: Lorenz, Hammer, Meissner 2012

Alarcon, Higinbotham, Weiss, PRC 102, 035203 (2020) [INSPIRE]

Radius extraction: ep Mainz



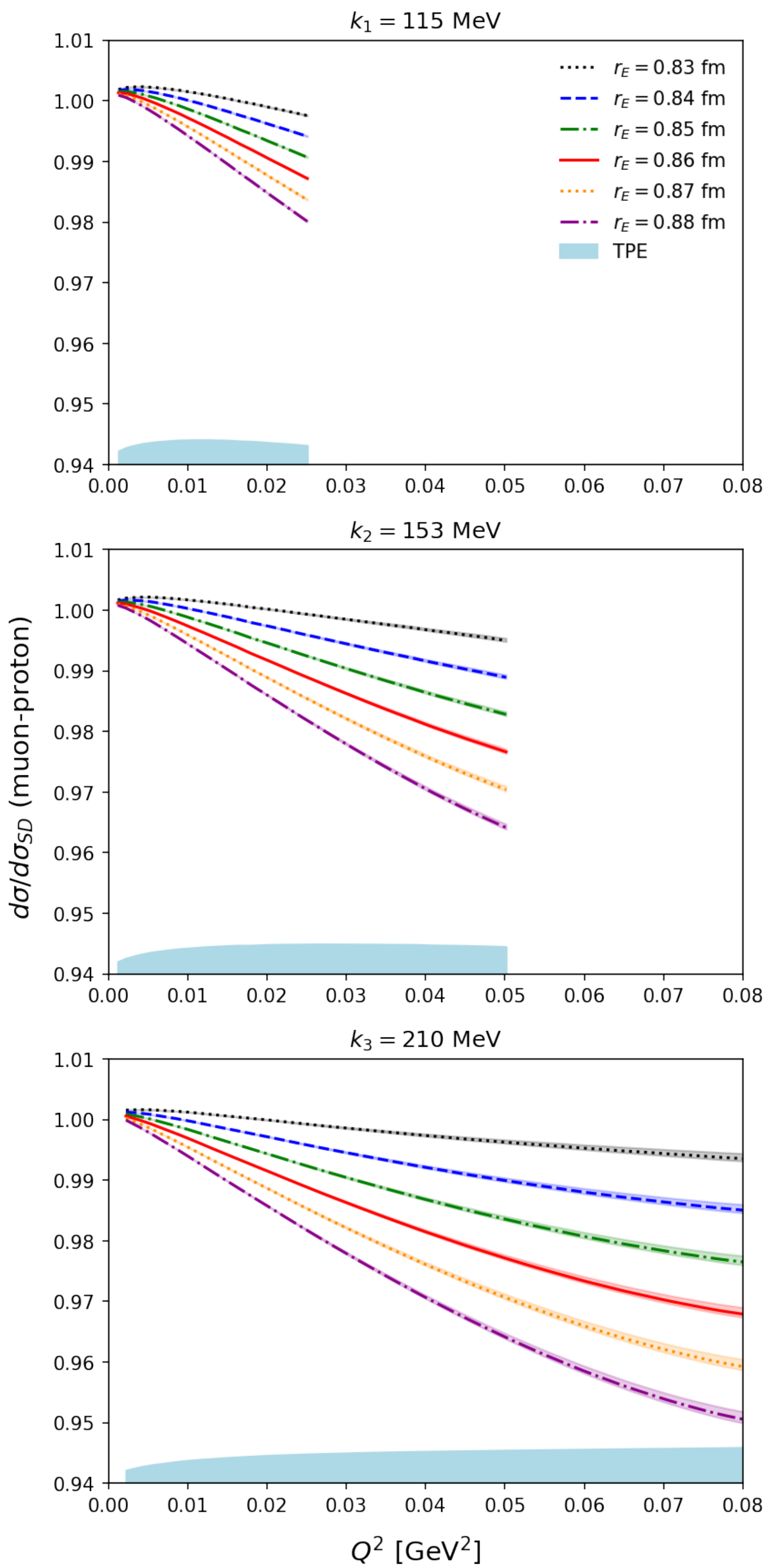


χ^2 profile in electric and magnetic radius

Mainz A1 data and DIχEFT fit
Bands: Fit uncertainty

Alarcon, Higinbotham, Weiss, PRC 102, 035203 (2020) [INSPIRE]

Radius extraction: μp MUSE



First measurement of proton radius in $\mu p + ep$ scattering
k = 115—210 MeV, Q2 = 0.001—0.08 GeV2

Studied radius extraction with DIChEFT

What kinematics has most impact on radius?
What is the overall uncertainty including theory?

Tradeoff between:
Sensitivity of cross section to radius
Theoretical uncertainty
Two-photon exchange corrections

Findings

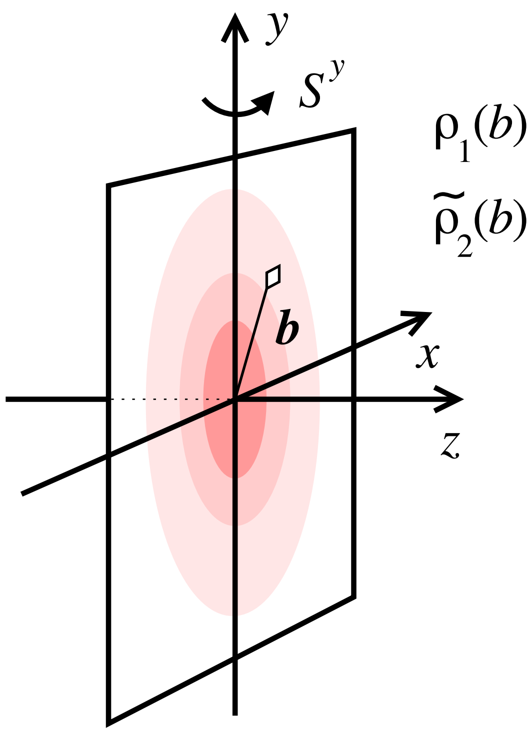
Influence of TPE on radius extraction diminished at higher Q2

Optimal kinematics for radius extraction
k = 210 GeV, Q2 = 0.05–0.08 GeV2

F. Gil-Dominguez, J.M. Alarcon, C. Weiss PRD 108 074026 (2023) [INSPIRE]

Two-photon exchange: Tomalak, Vanderhaeghen 2018

Applications: Transverse densities



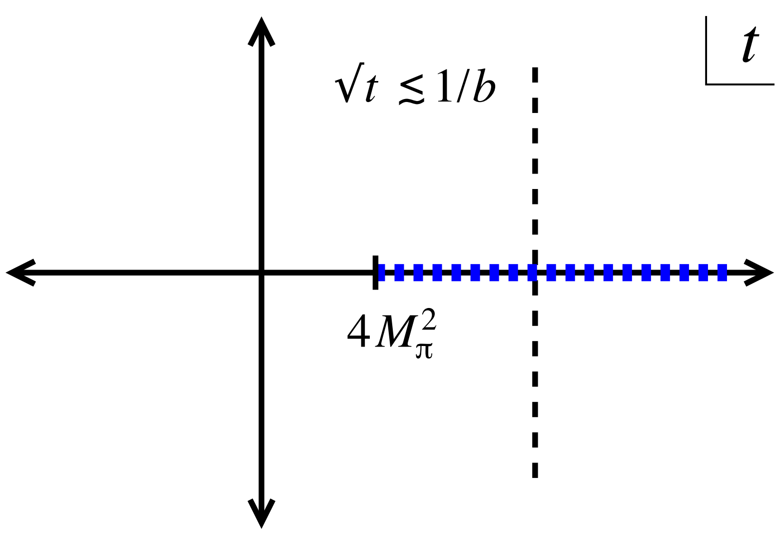
$$F_{1,2}(t = -\boldsymbol{\Delta}_T^2) \;=\; \int d^2b\, e^{i\boldsymbol{\Delta}_T \mathbf{b}}\, \rho_{1,2}(b)$$

Charge/magnetization densities at light-front time x^+
Frame-independent, appropriate for relativistic systems
Soper 1976, Burkardt 2000, Miller 2007

Fourier transform of form factor data
Miller 2007; Carlson Vanderhaeghen 2008; Venkat, Arrington, Miller, Zhan 2010

Dispersive representation

$$\rho(b) \;=\; \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi^2}\, K_0(\sqrt{t}b)\, \mathrm{Im}\, F(t) \qquad K_0 \sim e^{-b\sqrt{t}}$$



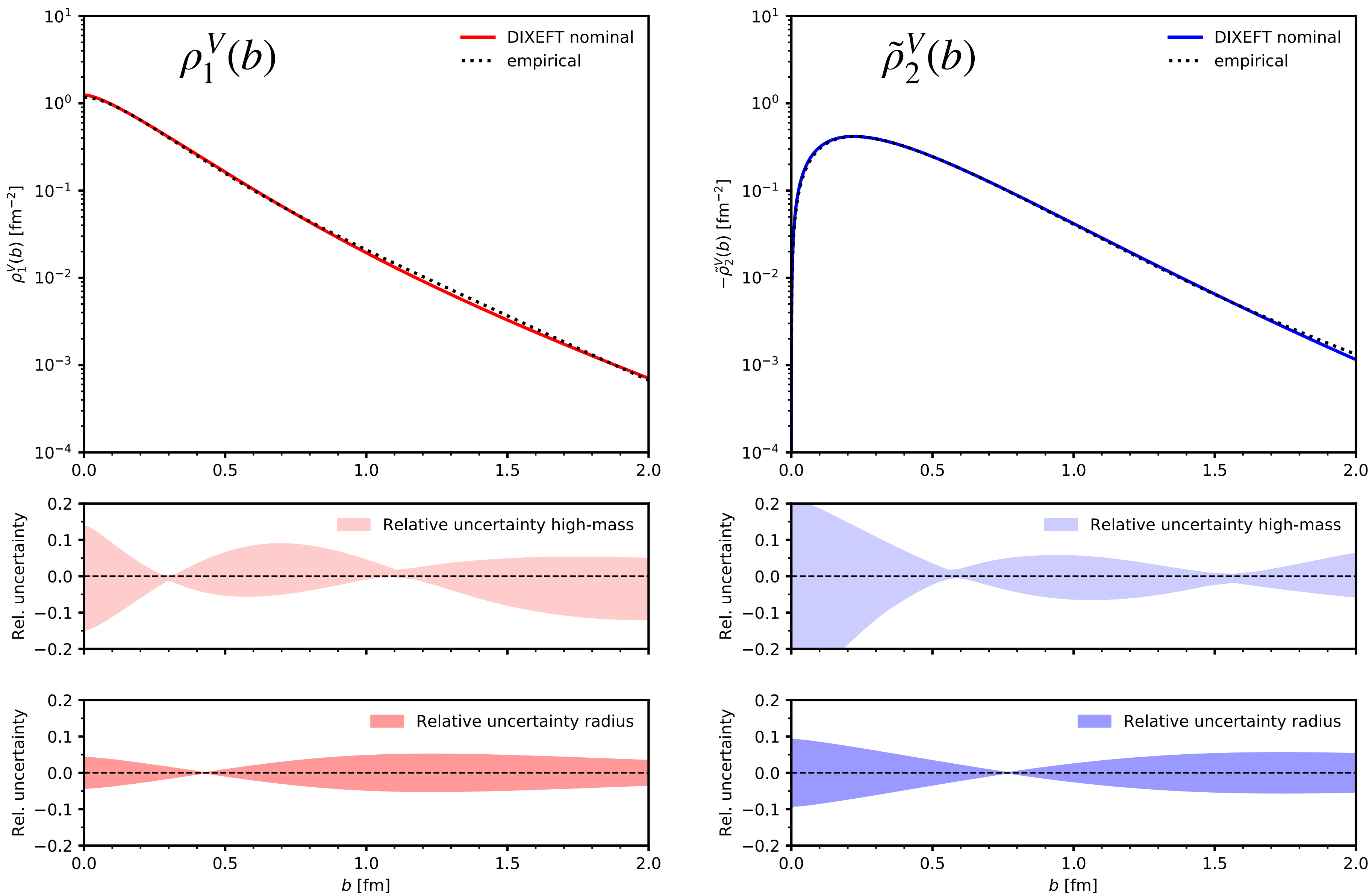
Exponentially convergent, acts as filter $\sqrt{t} \lesssim 1/b$
Large distances $b \leftrightarrow$ low masses $\sqrt{t}$

→ Peripheral densities
→ Uncertainty quantification

} with analyticity

Strikman, Weiss PRC 82, 042201 (2010) [INSPIRE]; Miller, Strikman, Weiss PRC 84 , 045205 (2011) [INSPIRE]; Granados, Weiss JHEP 01, 092 (2014) [INSPIRE]

Transverse densities: DIChEFT results



Alarcon, Weiss PRD 106, 054005 (2022) [INSPIRE]

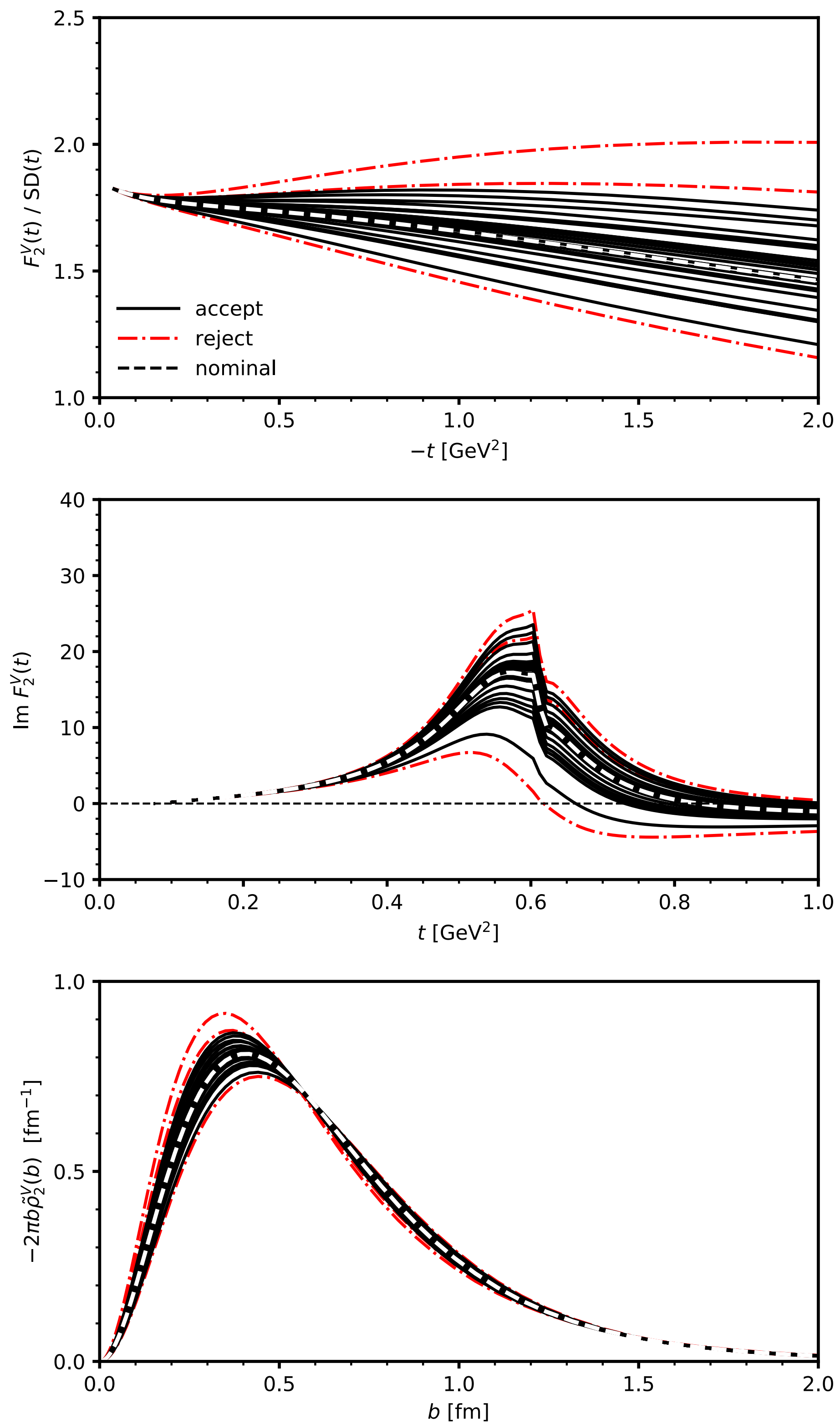
Large-b asymptotics governed by spectral properties

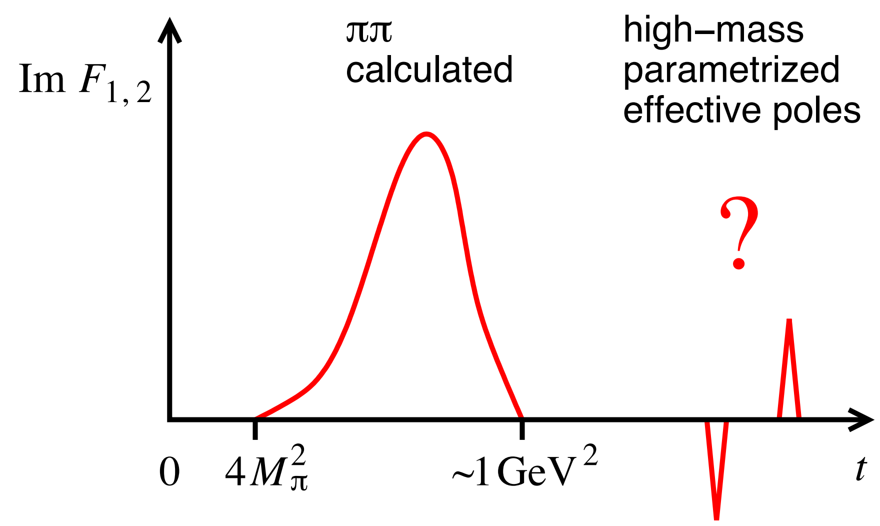
Difficult to obtain from Fourier transform. Requires proper analyticity of form factor!

Densities predicted with relative uncertainties $\lesssim 10\,\%$ at $b > 0.3$ fm

Excellent agreement with empirical densities

Transverse densities: Uncertainty quantification





Shape of high-mass spectral function unknown
→ treat as theoretical uncertainty

Parametrize through effective poles
$\text{Im}\, F_1[\text{high-mass}] = a_0 \delta(t - t_0) + a_1 \delta'(t - t_1)$

Pole positions considered unknown (in reasonable range)
Pole coefficients fixed by sum rules

Generate MC ensemble of spectral functions
Propagate variation into form factors, densities, etc.

Uncertainty quantification consistent with analyticity!

Peripheral densities not sensitive to unknown
high-mass spectral function - robust predictions!

Alarcon, Weiss PRD 106, 054005 (2022) [INSPIRE]

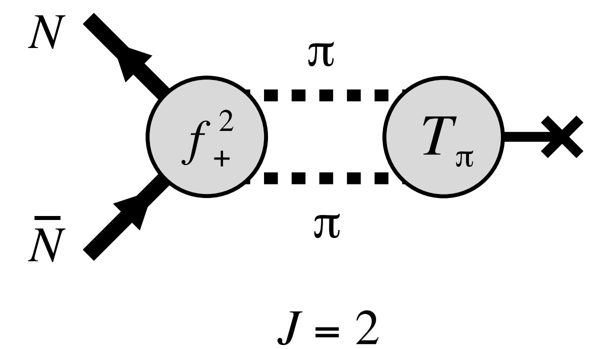
Extensions: Other operators and transitions

Energy-momentum tensor

Nucleon form factors describe distributions of mass, momentum, spin, forces - much interest

Pion matrix elements constrained by chiral symmetry
Voloshin, Dolgov 1982; Polyakov, Weiss 1999

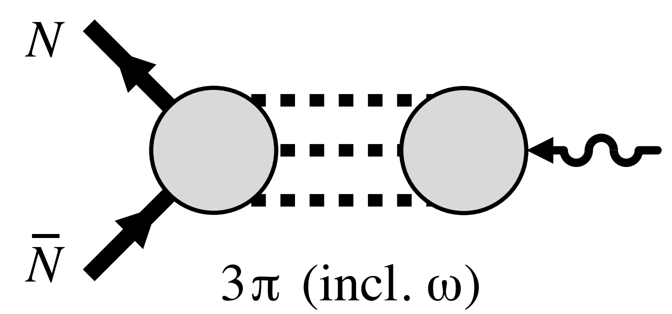
Spin $J > 2$: Generalized FFs = GPD moments



Nucleon FFs with 3π cut

Isoscalar vector current, isovector axial current

Use methods of 3-body unitarity
Szczepaniak, Jackura, Pilloni, Doering et al.



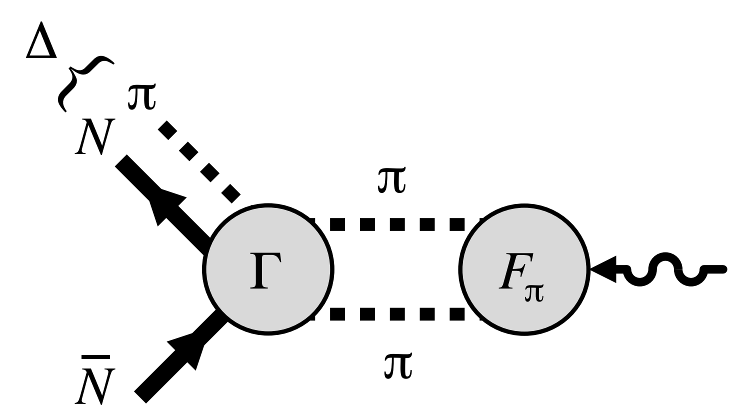
$N \to \Delta$ transition form factors

Compute transition matrix element $\langle N\pi | J^\mu | N \rangle$
Continue to pole in $s_{\pi N} = m_\Delta^2$, extract residue

ChEFT calculations
Ledwig et al. 2010

LQCD results
Alexandrou et al. 08; Aubin, Orginos, Pascalutsa, Vanderhaeghen 08

Large-N_c spin-flavor symmetry connects $N \to N$ and $N \to \Delta$



Summary

- Analyticity correlates FF at Q2 = 0 and finite Q2, plays essential role in radius extraction

- DIChEFT: Combines dispersion theory (analyticity, unitarity) with ChEFT (long-range dynamics), permits first-principles calculations of ππ spectral functions and low-Q2 form factors

- DIChEFT-based radius extraction implements analyticity and information flow

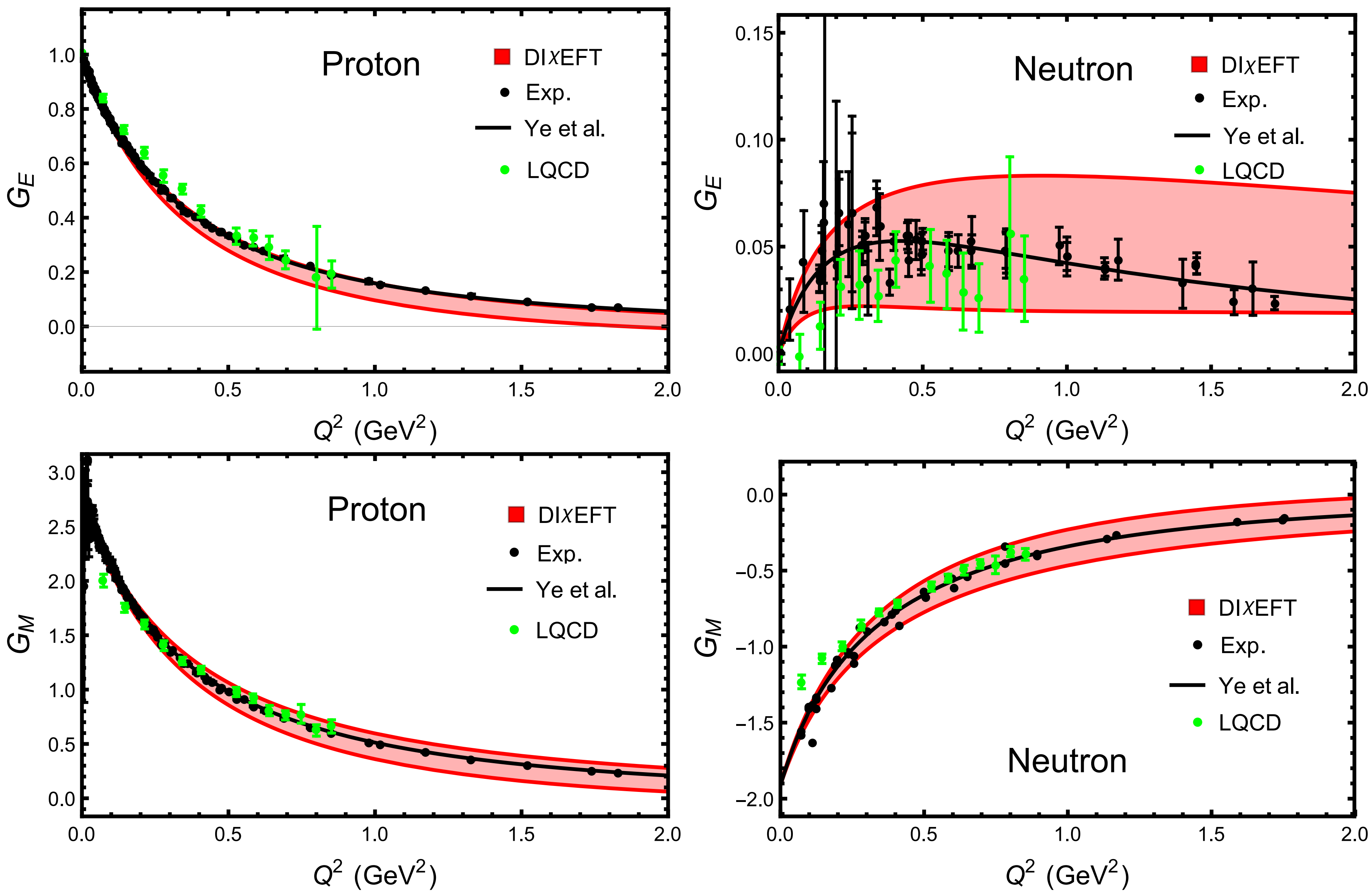
- Highest impact on radius from finite Q2 data, no need for “extrapolation to zero”. Assessments depend on actual exp + thy uncertainties, can be updated

- DIChEFT: Peripheral densities determined with quantified uncertainties, analyticity essential

- Many applications and extensions

Supplemental material

Method: Form factor results



$$G_i(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{\pi} \frac{\mathrm{Im}\, G_i(t')}{t' - t - i0}$$

Isovector: DIChEFT
Isoscalar: ω+high-mass

Alarcon, Weiss,
PLB784, 373 (2018)

LQCD: Alexandrou et al. 2017

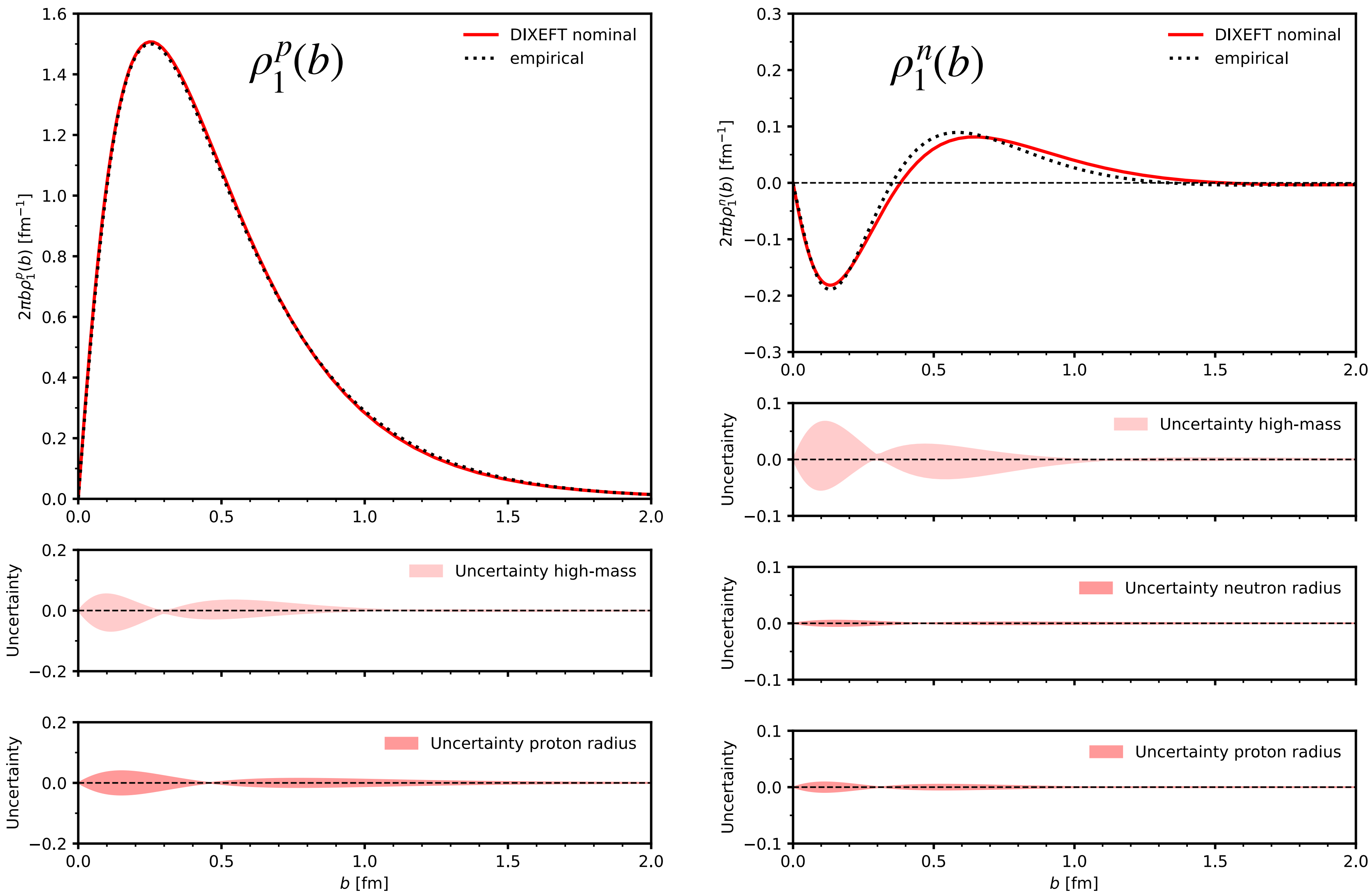
Form factors

Dispersion integral evaluated with spectral functions (including $\pi\pi$ and high-mass part)

Band shows uncertainty from radii (uncertainty from high-mass pole position → later)

Excellent agreement with data. No fit, but prediction based on dynamics

Transverse densities: DIChEFT results



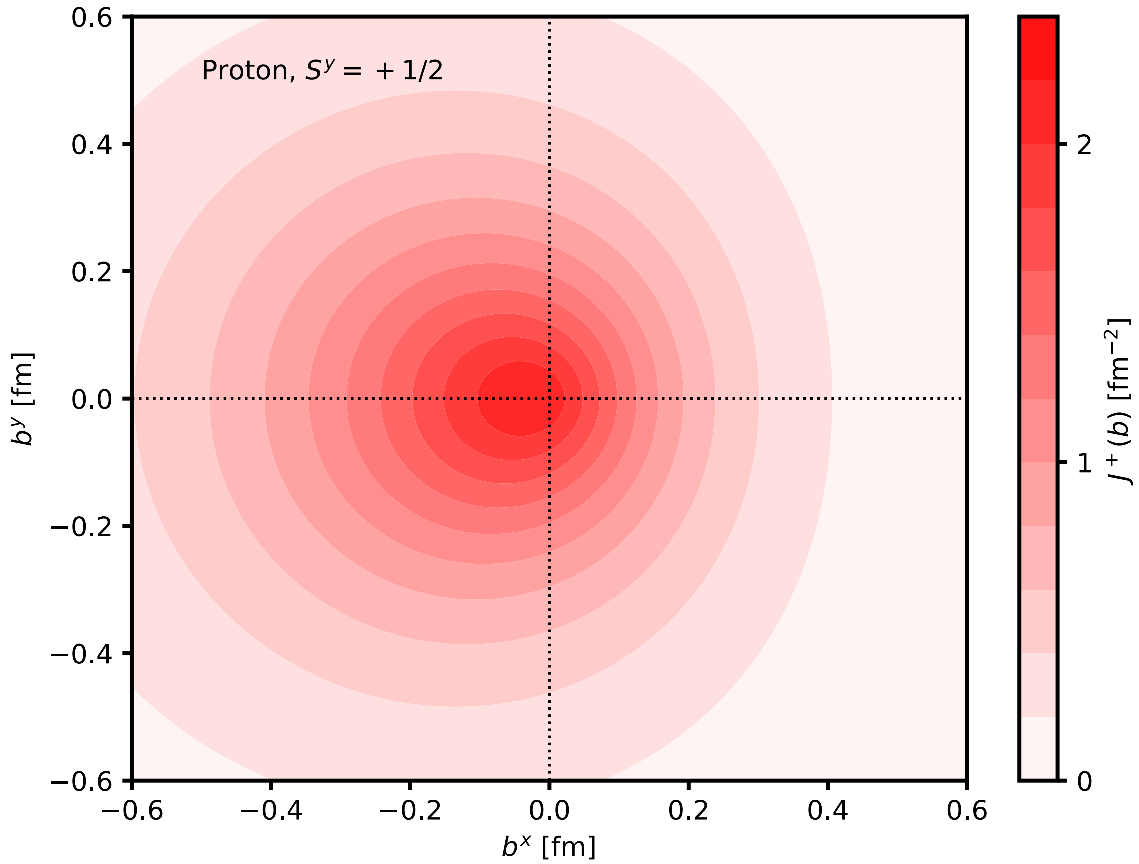
Alarcon, Weiss PRD 106, 054005 (2022)

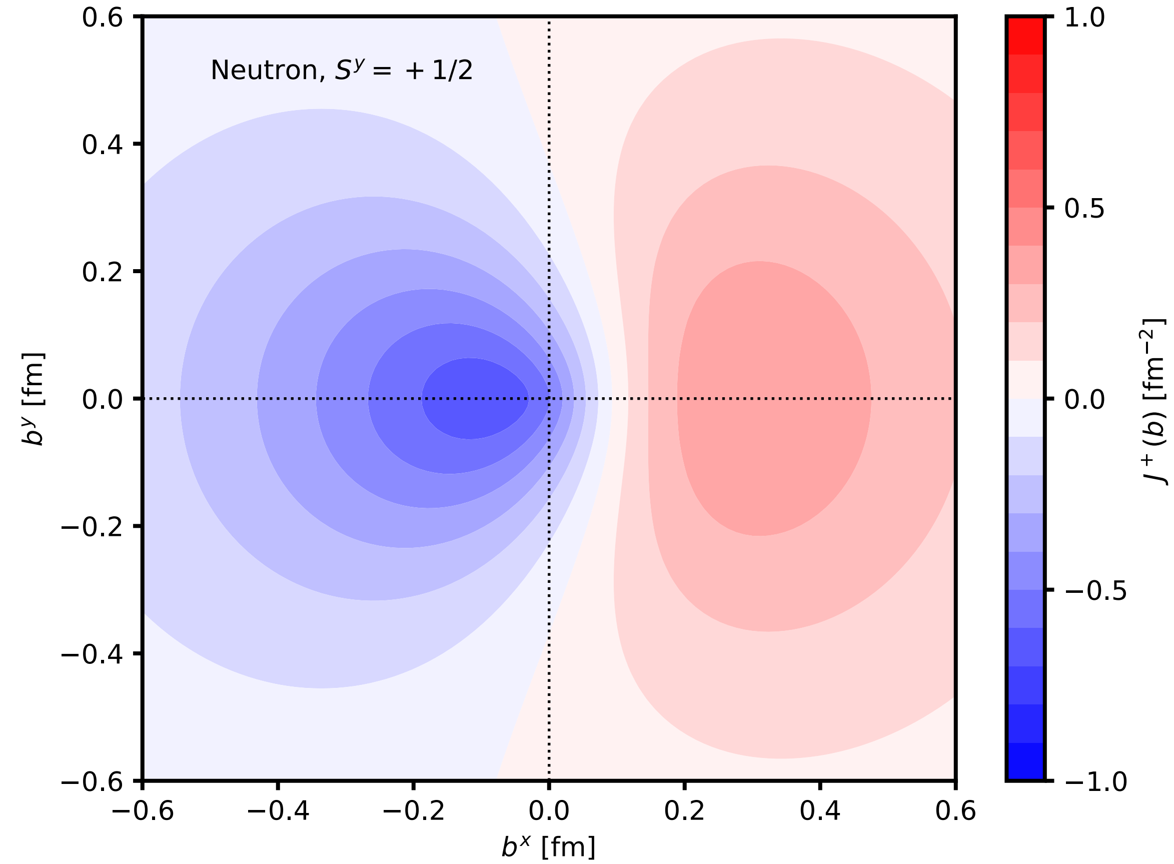
Radial charge densities $2\pi b\, \rho_1^{p,n}$ of proton and neutron

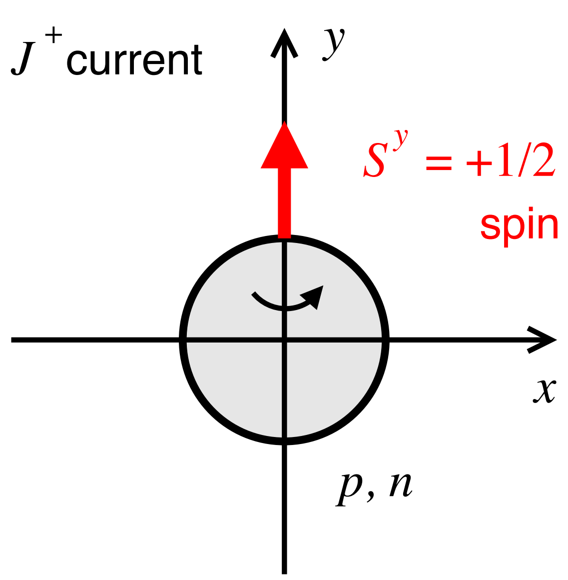
Obtained realistic nucleon densities with controlled uncertainties

Reproduced positive charge density in neutron at intermediate $b \sim 0.5 - 1$ fm
Miller 2007

Transverse densities: J^+ current and spin effects







Plus current density in transversely polarized nucleon localized at $x = y = 0$

$$\langle J^+(\mathbf{b})\rangle = \rho_1(b) + (2S^y)\cos\phi\,\tilde{\rho}_2(b)$$

This is how an electromagnetic probe coupling to J^+ "sees" the nucleon in transverse space

Computed from DIChEFT results

Alarcon, Weiss PRD 106, 054005 (2022)