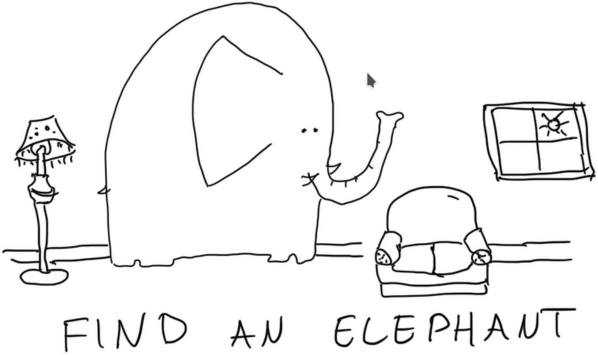
Nuclear radii from the spectra of heavy muonic atoms

Natalia S. Oreshkina
Max Planck Institute for Nuclear Physics, Heidelberg
Workshop on NREC, Stony Brook, May 8 2024

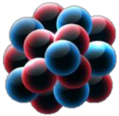
MAX-PLANCK-GESELLSCHAFT

Outline



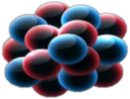
Nuclear corrections' overview

Nuclear size



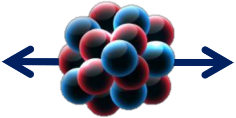
Depends on R_N

Nuclear shape



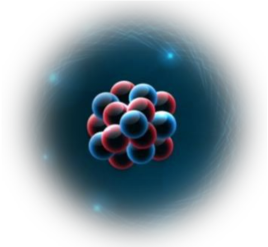
Depends on R_N, nuclear density

Relativistic nuclear recoil



Depends on the mass

Nuclear polarization

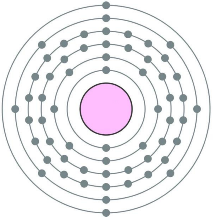


Depends on complete nuclear spectra

Figures courtesy of I. Valuev

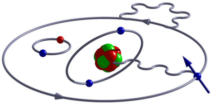
Atomic systems

Atoms



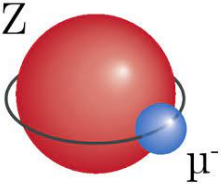
- Coulomb Z
- Coulomb e^-

Highly charged ions



- Coulomb Z
- QED $\sim \alpha$
- nuclear effects

Muonic atoms



- Coulomb Z
- nuclear effects
- QED $\sim \alpha$

How to measure rms radii?

Heavier than hydrogen

$\Downarrow$

Binding and transition energies: $\propto Z^2$

$\Downarrow$

Transition rates: even faster

$\Downarrow$

no precision spectroscopy

$\Downarrow$

Muonic atoms

"Live fast, die young!"

Fig: https://www.particlezoo.net

Muonic spectra importance

Atomic Data and Nuclear Data Tables 99 (2013) 69–95

ELSEVIER

Contents lists available at SciVerse ScienceDirect

Atomic Data and Nuclear Data Tables

journal homepage: www.elsevier.com/locate/adt

Table of experimental nuclear ground state charge radii: An update

I. Angeli [a], K.P. Marinova [b,*]

[a] *Institute of Experimental Physics, University of Debrecen, H-4010 Debrecen Pf. 105, Hungary*

[b] *Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russia*

ARTICLE INFO

Article history:
Received 9 August 2011
Received in revised form
10 November 2011
Accepted 2 December 2011
Available online 12 December 2012

Keywords:
Nuclear charge radii
Radii changes
Optical isotope shifts
K_α X-ray isotope shifts
Electron scattering
Muonic atom spectra

ABSTRACT

The present table contains experimental root-mean-square (*rms*) nuclear charge radii R obtained by combined analysis of two types of experimental data: (i) radii changes determined from optical and, to a lesser extent, K_α X-ray isotope shifts and (ii) absolute radii measured by muonic spectra and electronic scattering experiments. The table combines the results of two working groups, using respectively two different methods of evaluation, published in ADNDT earlier. It presents an updated set of *rms* charge radii for 909 isotopes of 92 elements from $_1$H to $_{96}$Cm together, when available, with the radii changes from optical isotope shifts. Compared with the last published tables of R-values from 2004 (799 ground states), many new data are added due to progress recently achieved by laser spectroscopy up to early 2011. The radii changes in isotopic chains for He, Li, Be, Ne, Sc, Mn, Y, Nb, Bi have been first obtained in the last years and several isotopic sequences have been recently extended to regions far off stability, (e.g., Ar, Mo, Sn, Te, Pb, Po).

Very basic theory

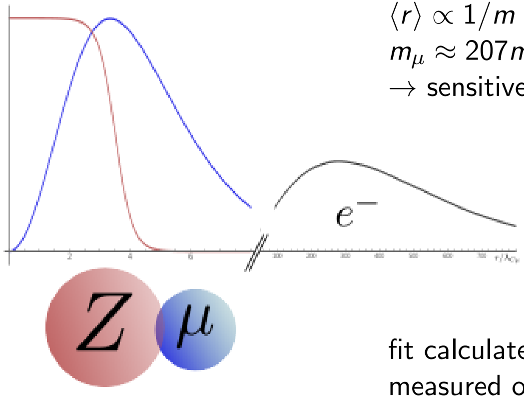


Fig: Niklas Michel

$\langle r \rangle \propto 1/m$

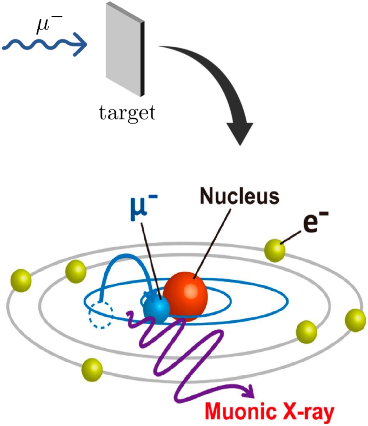
$m_\mu \approx 207 m_e$

$\rightarrow$ sensitive to nuclear structure

fit calculated spectra to measured ones

$\rightarrow$ determine nuclear parameters

Access to muonic atoms

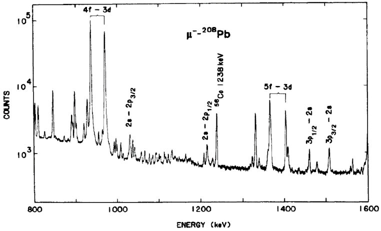


http://www.mdpi.com/2412-382-X/1/1/11/htm

- capture and cascade: $10^{-12} - 10^{-9}$ s
- lifetime: $0.1 - 2.2\ \mu$s
- always H-like
- decay channels

$$\mu^- \to e^- + \overline{\nu}_e + \nu_\mu$$

$$\mu^- + p \to n + \nu_\mu$$

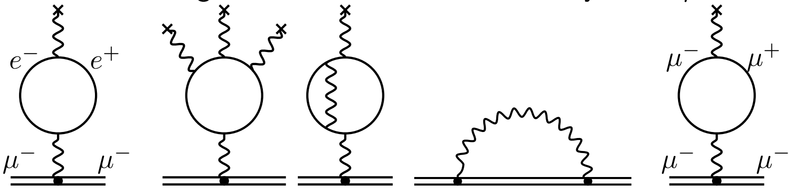


More theory (still basic)

- Muons are close to the nucleus, relativistic → Dirac equation
- Extended nucleus: sphere, Fermi, deformed Fermi distribution

$$\rho_{a,c,\beta}(r_\mu, \vartheta_\mu) = \frac{N}{1 + e^{[r - c(1+\beta Y_{20}(\vartheta_\mu))]/a}}$$

- QED: e-Uehling, Wichmann-Kroll, Källén-Sabry, SE, μ-Ue

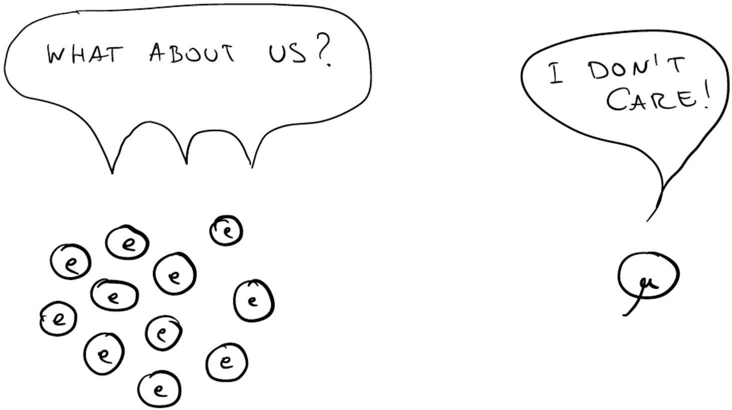


- HFS: electric quadrupole (dominant) and magnetic dipole

A. S. M. Patoary and NSO, EPJD **72**, 54 (2018)
N. Michel, NSO, and C. H. Keitel, PRA **96**, 032510 (2017)
N. Michel and NSO, PRA **99**, 042501 (2019)

A frequently asked question:

What's about electrons correlation and electron-muon interaction?



Two leptons $\Rightarrow$ $1s$ for muons and $1s$ for electrons

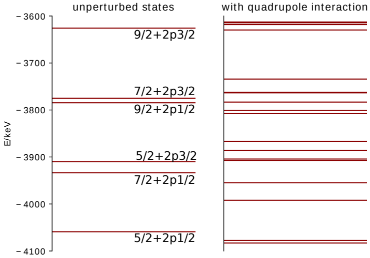
- Electrons are far
- The electron screening effect has been calculated and negligible

Dynamical splitting

- similar energy scale

$$|\mu\rangle \otimes |N\rangle \to |\mu N\rangle$$

- E2- and M1-HFS mixing

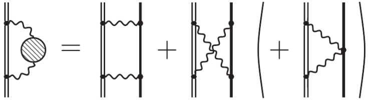


Naively:

$$V_{\mathrm{NP}}(r) = -\alpha \sum_{Z} \frac{1}{|\mathbf{r} - \mathbf{r}_{\mathbf{N_i}}|}$$

$$\Delta E_{\mathrm{NP}} = \sum_{nN} \frac{|\langle aA|\delta V|nN\rangle|^2}{E_{aA} - E_{nN}}$$

- QFT for muon-nucleus interaction



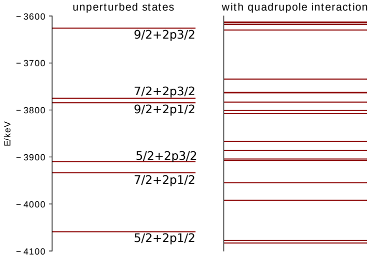
- precise muonic description
- state-of-art nuclear input

Dynamical splitting and Nuclear polarization

- similar energy scale

$$|\mu\rangle \otimes |N\rangle \to |\mu N\rangle$$

- E2- and M1-HFS mixing

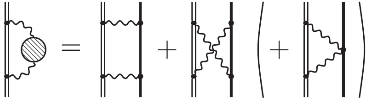


Naively:

$$V_{\mathrm{NP}}(r) = -\alpha \sum_{Z} \frac{1}{|\mathbf{r} - \mathbf{r_{N_i}}|}$$

$$\Delta E_{\mathrm{NP}} = \sum_{nN} \frac{|\langle aA|\delta V|nN\rangle|^2}{E_{aA} - E_{nN}}$$

- QFT for muon-nucleus interaction



- precise muonic description
- state-of-art nuclear input

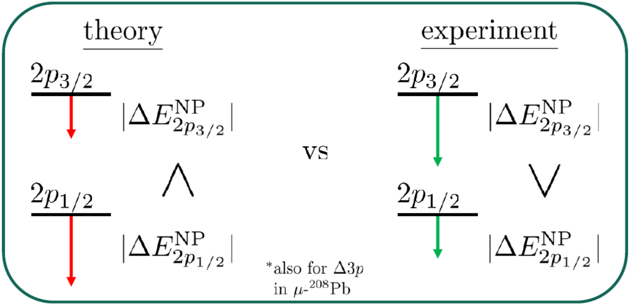
A fine-structure anomaly

A simultaneous fit of $2p_{3/2} - 1s_{1/2}$ and $2p_{1/2} - 1s_{1/2}$

muonic ^{90}Zr, $^{112-124}$Sn, ^{208}Pb: very poor fit, $\chi^2/\mathrm{DF} = 187$

$\rightarrow$ nuclear polarization correction as variable parameters

$\rightarrow$ the root of the problem



$2p_{1/2}$ is closer to a nucleus and should be affected more strongly

P. Bergem *et al.*, Phys. Rev. C **37** 2821 (1988)

Nuclear polarization effect

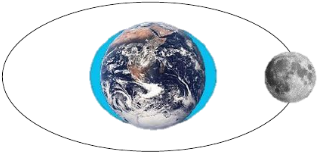


Image source: www.universetoday.com

$$V_{\mathrm{Coul}}(r) = -\frac{\alpha Z}{r}$$

$$V_{\mathrm{ext}}(r) = -\alpha \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r}'$$

$$V_{\mathrm{NP}}(r) = -\alpha \sum_{Z} \frac{1}{|\mathbf{r} - \mathbf{r}_{\mathbf{N_i}}|}$$

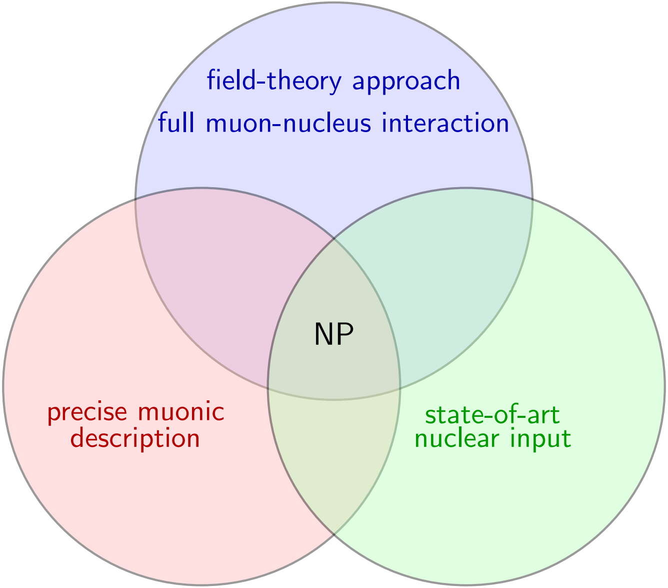
Longitudinal (Coulomb) part

$$H = H_N + \boldsymbol{\alpha}\mathbf{p} + \beta m_\mu + V(\mathbf{r}, \mathbf{r}_{N_i})$$

$$\Delta E_I = {\sum_N}' \frac{\langle I|\Delta V|N\rangle\langle N|\Delta V|I\rangle}{E_I - E_N}$$

Transverse part: only via field-theory approach

Our goal


field-theory approach
full muon-nucleus interaction
NP
precise muonic
description
state-of-art
nuclear input

Transverse part of muon-nucleus interaction

$$H = H_N + \boldsymbol{\alpha}\mathbf{p} + \beta m_\mu + V(\mathbf{r}, \mathbf{r}_{N_i})$$

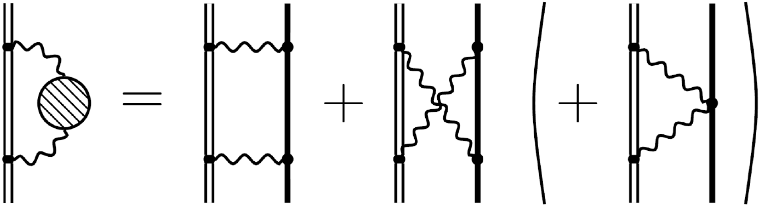
$$\Downarrow$$

$$H = H_N + \boldsymbol{\alpha}(\mathbf{p} - e\mathbf{A}(\mathbf{r}, \mathbf{r}_{N_i})) + \beta m_\mu + V(\mathbf{r}, \mathbf{r}_{N_i})$$

- Longitudinal (or Coulomb) interaction $V(\mathbf{r}, \mathbf{r}_{N_i})$ always $|\Delta E^{\mathrm{NP}}_{2p_1/2}| > |\Delta E^{\mathrm{NP}}_{2p_3/2}|$
- Transverse interaction $\mathbf{A}(\mathbf{r}, \mathbf{r}_{N_i})$ contributes with the opposite muon-spin dependence
- However, the anomalies still persisted (for more than 40 years)

Tanaka and Horikawa, Nucl. Phys. **A580**, 291 (1994)

Total leading-order nuclear polarization



$$\Delta E_{\rm NP}^{\rm L} = -i(4\pi\alpha)^2 \sum_{i'I'} \iint \frac{d\boldsymbol{q}\,d\boldsymbol{q}'}{(2\pi)^6} \int \frac{d\omega}{2\pi} \frac{D_{\mu\xi}(\omega,\boldsymbol{q}) D_{\zeta\nu}(\omega,\boldsymbol{q}') \langle iI|j_m^\mu(-\boldsymbol{q}) J_N^\xi(\boldsymbol{q})|i'I'\rangle \, \langle i'I'|J_N^\zeta(-\boldsymbol{q}') j_m^\nu(\boldsymbol{q}')|iI\rangle}{(\omega+\omega_m - iE_{i'}\epsilon)(\omega-\omega_N+i\epsilon)},$$

$$\Delta E_{\rm NP}^{\rm X} = +i(4\pi\alpha)^2 \sum_{i'I'} \iint \frac{d\boldsymbol{q}\,d\boldsymbol{q}'}{(2\pi)^6} \int \frac{d\omega}{2\pi} \frac{D_{\mu\xi}(\omega,\boldsymbol{q}) D_{\zeta\nu}(\omega,\boldsymbol{q}') \langle iI'|j_m^\mu(-\boldsymbol{q}) J_N^\xi(\boldsymbol{q})|i'I\rangle \, \langle i'I|J_N^\zeta(-\boldsymbol{q}') j_m^\nu(\boldsymbol{q}')|iI'\rangle}{(\omega+\omega_m - iE_{i'}\epsilon)(\omega+\omega_N-i\epsilon)};$$

$$\Delta E_{\rm NP}^{\rm SG} = -i(4\pi\alpha)^2 \sum_{i'} \iint \frac{d\boldsymbol{q}\,d\boldsymbol{q}'}{(2\pi)^6} \int \frac{d\omega}{2\pi} \frac{D_{\mu\xi}(\omega,\boldsymbol{q}) \delta^{\xi\zeta} D_{\zeta\nu}(\omega,\boldsymbol{q}') \langle i|j_m^\mu(-\boldsymbol{q})|i'\rangle \, \langle i'|j_m^\nu(\boldsymbol{q}')|i\rangle}{(\omega+\omega_m - iE_{i'}\epsilon)} \frac{\langle I|\rho_N(\boldsymbol{q}-\boldsymbol{q}')|I\rangle}{m_p}.$$

summations over entite muonic (i') and nuclear (I') spectra

Muonic spectrum

Dirac equation:

$$[\boldsymbol{\alpha}\mathbf{p} + \beta m_\mu + V_0(\mathbf{r})]\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

V_0 from Fermi nuclear charge distribution

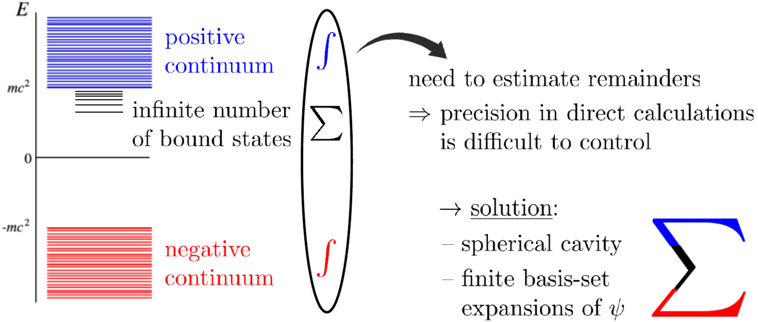


Fig: Igor Valuev

Nuclear spectrum

effective nuclear force
→ Skyrme interaction

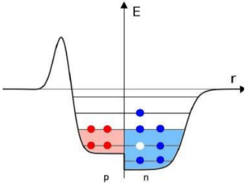
↓

Hartree-Fock
calculations

↓

single-nucleon states
(occupied and unoccupied)

→

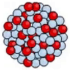


basis of particle-hole
configurations

Image source: arXiv:1410.7167

↓

nuclear excited states $|\nu\rangle$
→ random phase approximation



GIF source:
gfycat.com

↓

matrix elements
$\langle\nu|\hat{J}_N^{\xi}(\mathbf{r})|\mathcal{O}\rangle$

Fig: Igor Valuev

Skyrme-type nuclear interaction

Tony Skyrme in 1946

Fig:

https://en.wikipedia.org/wiki/Tony_Skyrme

$$V(\mathbf{r}_1,\mathbf{r}_2) = t_0(1+\chi_0 P_\sigma)\delta(\mathbf{r}) + \frac{1}{2}t_1(1+\chi_1 P_\sigma)[\mathbf{P}^{\dagger 2}\delta(\mathbf{r}) + \delta(\mathbf{r})\mathbf{P}^2] + t_2(1+\chi_2 P_\sigma)\mathbf{P}^\dagger\cdot\delta(\mathbf{r})\mathbf{P} + \frac{1}{6}t_3(1+\chi_3 P_\sigma)\rho^\lambda(\mathbf{R})\delta(\mathbf{r})$$

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$$

$$\mathbf{P} = \frac{1}{2i}(\nabla_1 - \nabla_2), P_\sigma = \frac{1}{2}(1 + \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$$

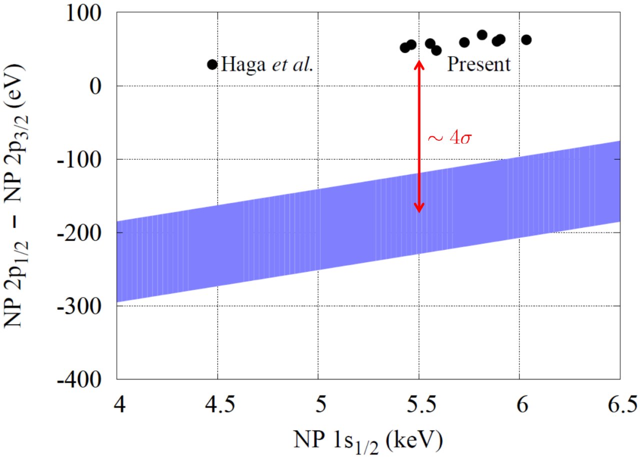
10 parameters

Nuclear wave functions dependence

Calculations details

- complete muonic Dirac spectrum
- 9 different parametrizations of the Skyrme interaction
- Covers all realisitic ranges for nuclear properties
- $0^+, 1^-, 2^+, 3^-, 4^+, 5^-$ and 1^+ excitation modes
- RMS value changes the NP predictions
- Comparison between theory and free-parameter fit of the experimental data

40 years later: Nuclear polarization correction ^{208}Pb



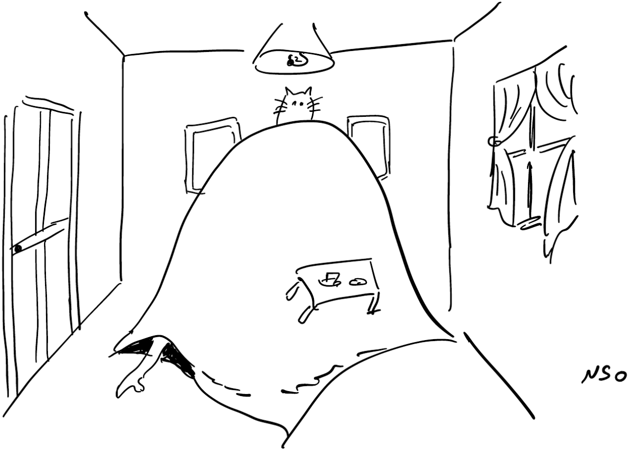
around 150 eV, or 4σ standard deviations gap

$\Rightarrow$ have to go back to $\chi^2/\mathrm{DF} = 187$ and much higher error bars

What is left behind? An elephant in the room

- RMS: high importance
- From muonic spectra
- High accuracy: 0.02% for lead

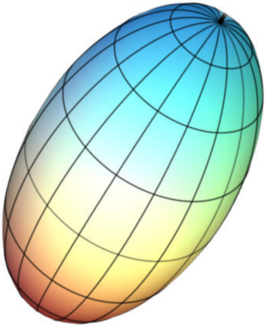
- Fine-structure anomaly (NP)
- Poor fit $\chi^2/\mathrm{DF} = 187$
- Estimation for theory
- How much can we trust it?...



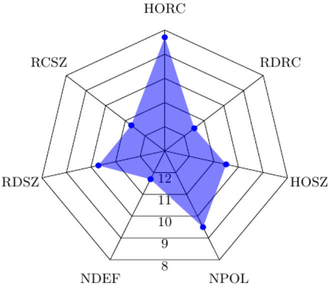
NSO

Current challenges

- NP for deformed nuclei



- NP for atoms for the physics beyond the Standard Model

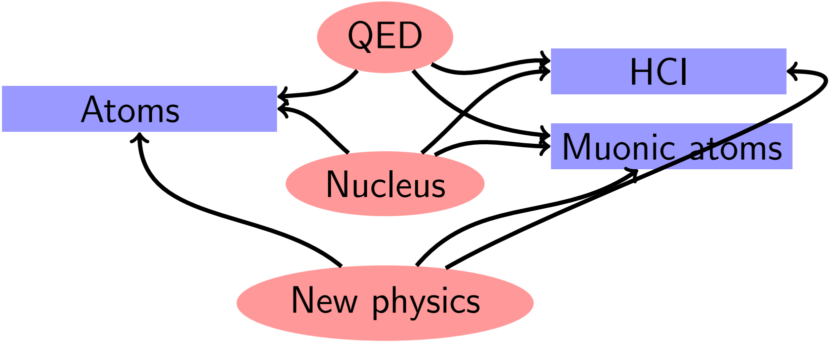


- Search for simple models: one order of magnitude discrepancy

Challenging muons in muonic atoms

- Asocial far from e^-, always H-like
- Unhealthy lifetime $0.1 - 2.2\ \mu$s
- Destructive $\mu^- + p \to n + \nu_\mu$
- Non-cooperative passive spectroscopy
  $5g - 4f - 3d - 2p - 1s$
- Demanding complicated QED
- Unreliable Every muonic atom is different
- Co-dependent highly sensitive to nuclear structure: dynamical structure/splitting $|\mu\rangle \otimes |N\rangle \to |\mu N\rangle$
- And now to the bad part... nuclear polarization: includes the complete muon and nuclear spectra
- Why don't we ignore it? Give best probes of the short-ranged interactions

Summary



1 I. A. Valuev and NSO, PRA **109**, 042811 (2024)

2 V. A. Yerokhin and NSO, PRA **108**, 052824 (2023)

3 NSO, PRR **4**, L042040 (2022)

4 I. A. Valuev, G. Coló, X. Roca-Maza, C. H. Keitel, and NSO, PRL **128**, 203001 (2022) A. Antognini *et al.* PRC **101**, 054313 (2020)

5 N. Michel, NSO, and C. H. Keitel, PRA **96**, 032510 (2017)

6 N. Michel and NSO, PRA **99**, 042501 (2019)