Chromostatic Confinement and Hadronic Radii

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Superselection Charges

a charge, Q^S, is Superselected iff Q^S commutes with the Hamiltonian and with all observables, O_i ---

$$[Q^S, H] = 0 \qquad [Q^S, O_i] = 0 \text{ all } i$$

The standard model has 3 superselection

1. fermion number , Q^F
2. baryon number , Q^B
3. electric charge, Q^e

charges, Q^S, carried by quarks

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FOR ANY HADRON

quark radius coincides with charge radii for Q^F, Q^B, Q^S

However color charge and weak charge are not superselected

weak charge mixes with adjoint color charge through topological structures an anomalies (Adler-Bell-Jackiw ----)

we can study this mixing in the framework of spherical symmetry

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The Strong Conjecture

The confinement mechanism for QCD involves a domain wall of topological (CP-odd) charge separating the interior volume of hadrons from an exterior volume.

This conjecture provides a specific starting point for the study of color confinement.

There is a $50K prize for disproving the conjecture

Spherical Symmetry
Dimensional Collapse

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} + \bar{\Psi}(\not{D} - m)\Psi$$

Bars-Witten änsatz for Gauge Connection

$$g A^a_0 = A_0(r,t)\, \hat{r}_a$$

$$g A^a_i = A_1(r,t)\, \rho_{ia} + \frac{a(r,t)}{r} \sin(\omega(r,t))\, \delta^T_{ia} + \frac{a(r,t)\cos(\omega(r,t)) - 1}{r}\, \varepsilon^T_{ia}$$

where ρ_{ia} δ^T_{ia} and ε^T_{ia} carry space indices $i, j, k \ldots$ group indices $a, b, c \ldots$

Spherical Symmetric SU(N)

radial abelian gauge

$E^L(r,t)$ $E^S(r,t)$ $E^A(r,t)$

$B^L(r,t)$ $B^S(r,t)$ $B^A(r,t)$

SU(N)

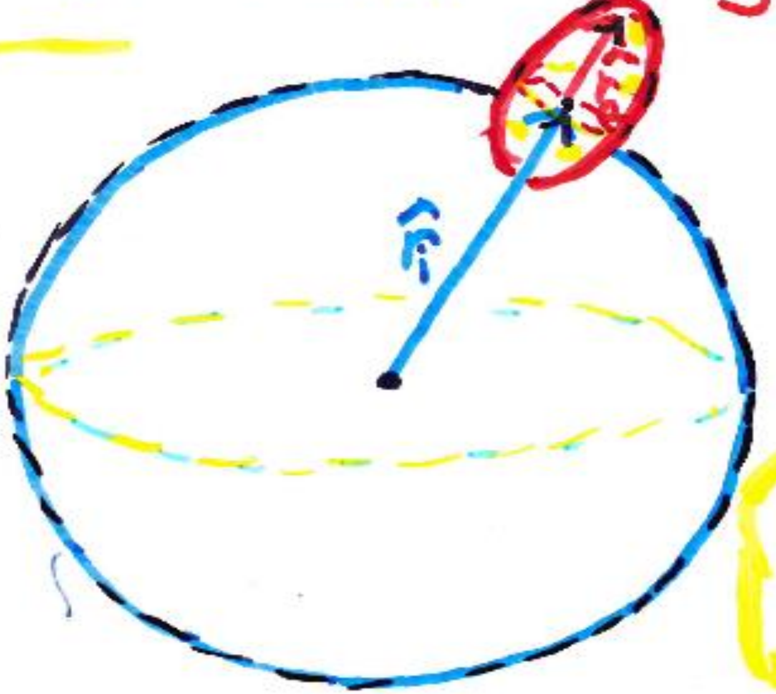
	L	T
SU(2)	1	2
SU(3)	2	6
SU(N)	N-1	N^2-N

Basis vectors

ρ^L_{ia}

$\varepsilon^S_{ia}(\omega(r,t))$

$\varepsilon^A_{ia}(\omega(r,t))$



SO(3)	SU(N)
i j k	a b c

Spherical Symmetric SU(N)

radial abelian gauge

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$B^{L}(r,t)$ $B^{S}(r,t)$ $B^{A}(r,t)$

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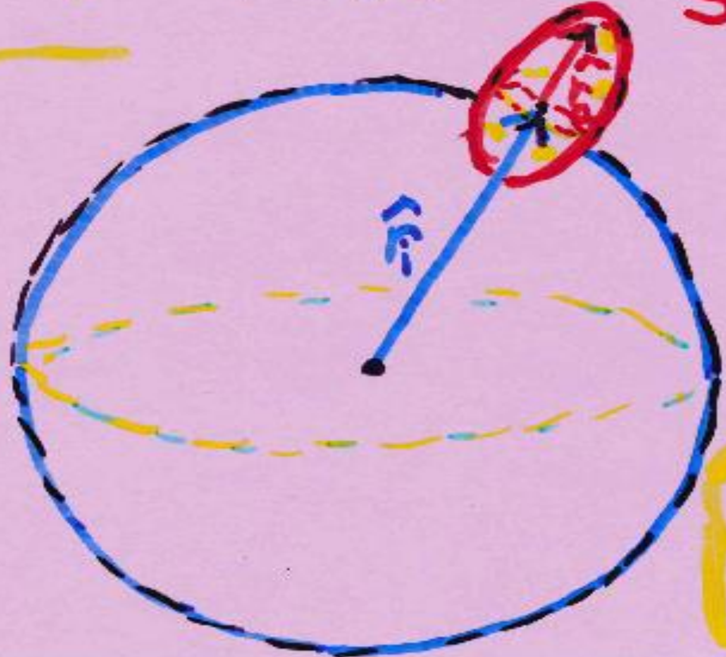


ρ^{L}_{ia}

$\varepsilon^{S}_{ia}(\omega(r,t))$

$\varepsilon^{A}_{ia}(\omega(r,t))$

SO(3) SU(N)

ijk abc

Yang Mills Maxwell $(D^\mu G_{\mu\nu})^a = J_\nu^a$ $(D_\mu {}^*G_{\mu\nu}) = 0$

$$-\frac{\partial}{\partial r}(r^2 E_L) + 2arE_S = J_0(r,t)$$
$$-\frac{\partial}{\partial t}(r^2 E_L) + 2arB_A = J_1(r,t)$$

1+1 dim
Abelian

$$-\frac{\partial}{\partial t}(arE_S) + \frac{\partial}{\partial r}(arB_A) = arj_S(r,t)$$
$$a\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^2}\right) - r^2(E_S^2 - B_A^2) + \frac{a^2(a^2-1)}{r^2} = arj_A(r,t)$$

$$\frac{\partial}{\partial r}(arE_A) - \frac{\partial}{\partial t}(ar\,B_S) = 0$$
$$-E_L + \frac{\partial}{\partial r}(arE_S) + \frac{\partial}{\partial t}(arB_A) = r^2 E_i^a B_i^a(r,t)$$
$$= \partial_\mu K_\mu(r,t)$$

Bianchi Const.

Field Strength Densities

$$E_i^a(r)E_i^a(r) = A_0'^2(r) + \frac{2}{r^2}\left(a(r)A_0(r)\right)^2$$

$$B_i^a(r)B_i^a(r) = \frac{(a^2(r)-1)^2}{r^4} + \frac{2}{r^2}\left[a'(r)^2 + a^2(r)\left(A_1(r) - \omega'(r)\right)^2\right]$$

$$E_i^a(r)B_i^a(r) = \frac{A_0'(r)(a^2(r)-1)}{r^2} + \frac{2}{r^2}A_0(r)a(r)a'(r)$$

gauge covariant

$$\varepsilon^S_{ia}(\omega) = \delta^T_{ia}\cos(\omega(r)) - \varepsilon^T_{ia}\sin(\omega(r))$$

$$\varepsilon^A_{ia}(\omega) = \delta^T_{ia}\sin(\omega(r)) + \varepsilon^T_{ia}\cos(\omega(r))$$

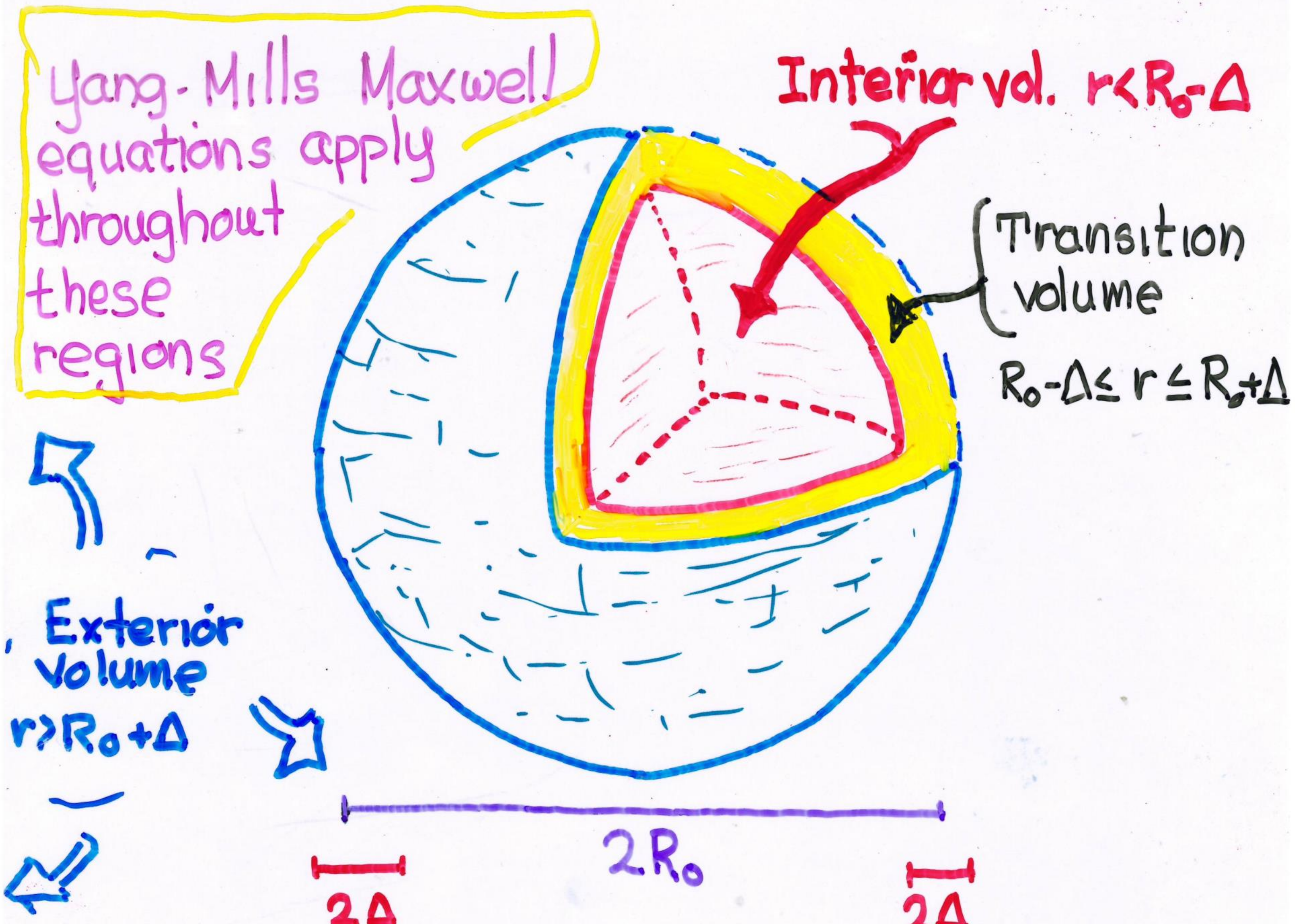
transverse basis tensors

$$gA_i^a(r) = A_1(r)\hat{P}_{ia} + \frac{a(r)}{r}\varepsilon^A_{ia}(\omega(r)) - \varepsilon^A_{ia}\frac{(\omega(r))}{r}$$

transverse gauge connection

$$\varepsilon^S_{ia}(\omega + \pi/2) = -\varepsilon^A_{ia}(\omega) \qquad \varepsilon^A_{ia}(\omega+\pi/2) = \varepsilon^S_{ia}(\omega)$$

$$D_0^{ab}v_b = \varepsilon^{abc}v_bA_0^c(r) \qquad D_i^{ab}\hat{r}_b = \frac{a(r)}{r}\varepsilon^S_{ia}(\omega(r))$$


Yang-Mills Maxwell equations apply throughout these regions
Interior vol. $r < R_0 - \Delta$
Transition volume
$R_0 - \Delta \leq r \leq R_0 + \Delta$
Exterior volume
$r > R_0 + \Delta$
$2R_0$
2Δ
2Δ

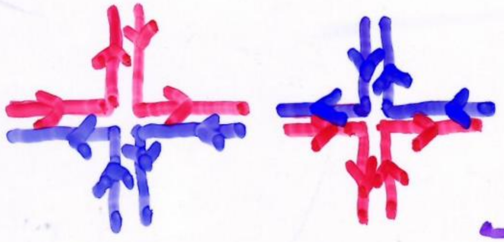
Classification of condensates in SU(2) chromostatics with spherical symmetry

$a(r) = \pm 1$	$E_i^a E_i^a \neq 0$	$B_i^a B_i^a = 0$		color electric
$a(r) = \pm 1$	$E_i^a E_i^a = 0$	$B_i^a B_i^a \neq 0$		color magnetic
$a(r) = \pm 1$	$E_i^a E_i^a \neq 0$	$B_i^a B_i^a \neq 0$		color glass
$a(r) = \pm 1$	$E_i^a E_i^a = 0$	$B_i^a B_i^a = 0$		sterile vacuum
$a(r) = 0$	$E_i^a E_i^a = 0$	$B_L B_L = (\pm)^2 \frac{1}{r^4}$		't Hooft Polyakov
$a(r) = C \neq \pm 1, 0$	$E_i^a E_i^a \neq 0$	$B_i^a B_i^a \neq 0$	$E_i^a B_i^a \neq 0$	topological or dyonic

a domain wall is a region where $a'(r) \neq 0$ that separates other condensates and also carries topological charge

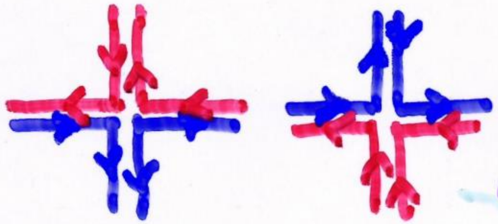
Gauge-Covariant Derivatives have a chiral structure that depends on $a(r,t)$

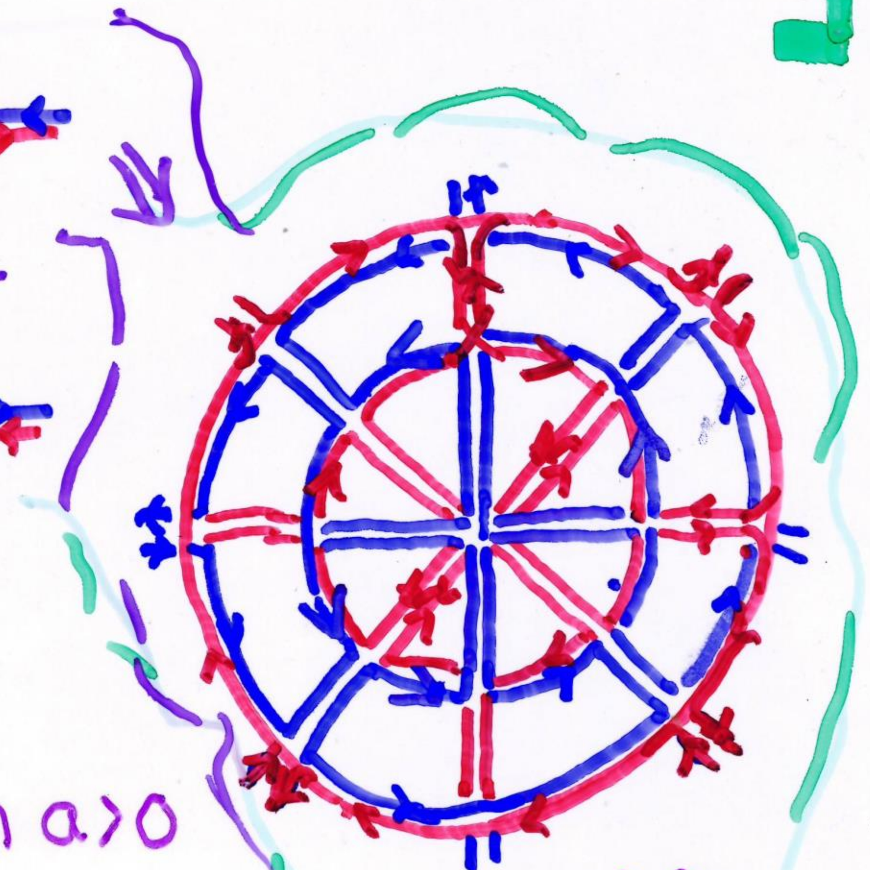
$a(r,t) = +1$



$\hat{r}_a \uparrow$

$a(r,t) = -1$

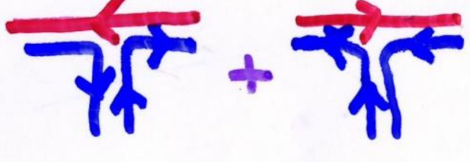




$a > 0$

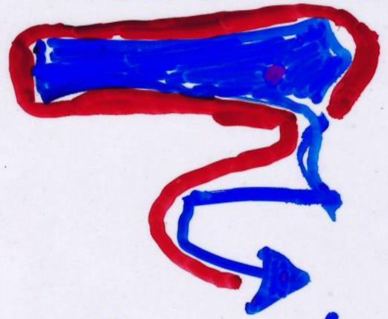
colorflow $E_s B_A \hat{r}_a \hat{r}_i$ directed outward when $a > 0$

$a = 0$ (

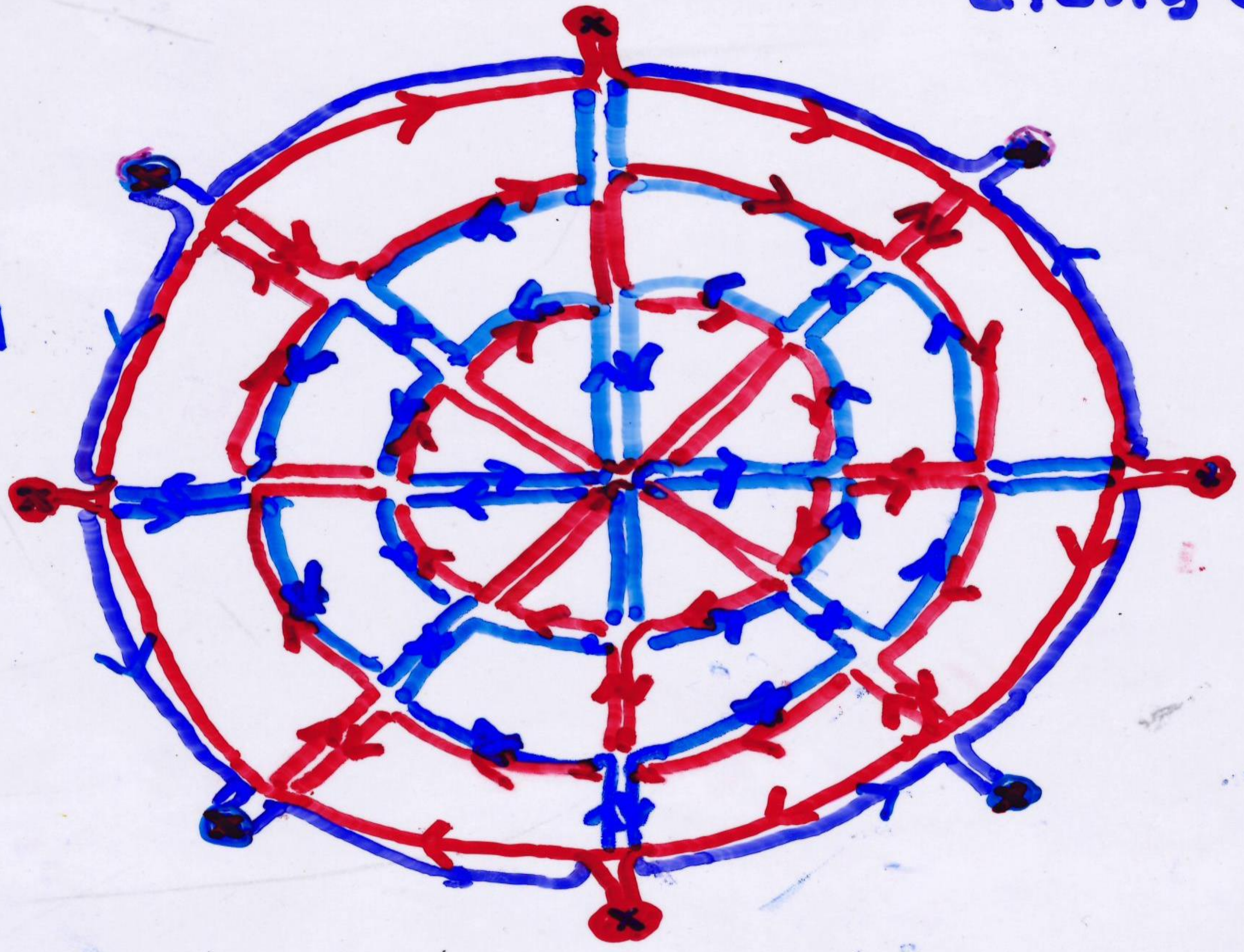


)

Here is another layer with $a(r)=1$ with ends tied off by a distributed SU(2) color source along each diagonal



a standard nontopological sol'n to Yang-Mills Maxwell



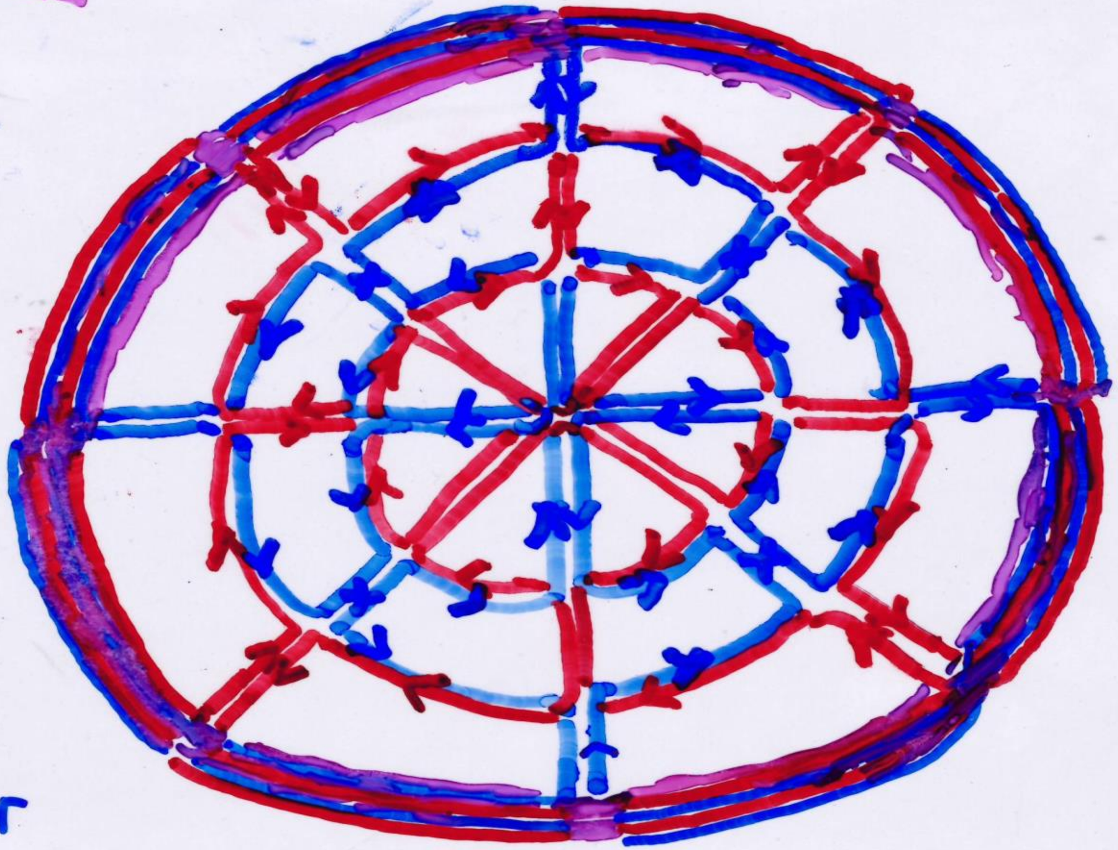
$a=1$

the exterior volume is a sterile vacuum condensate with the same chirality as outer

$a=1$

Here is a sketch showing what a type-1 sol'n could look like

topological $\hat{E}_i$ $\hat{B}_i$ shown in purple



Exterior region

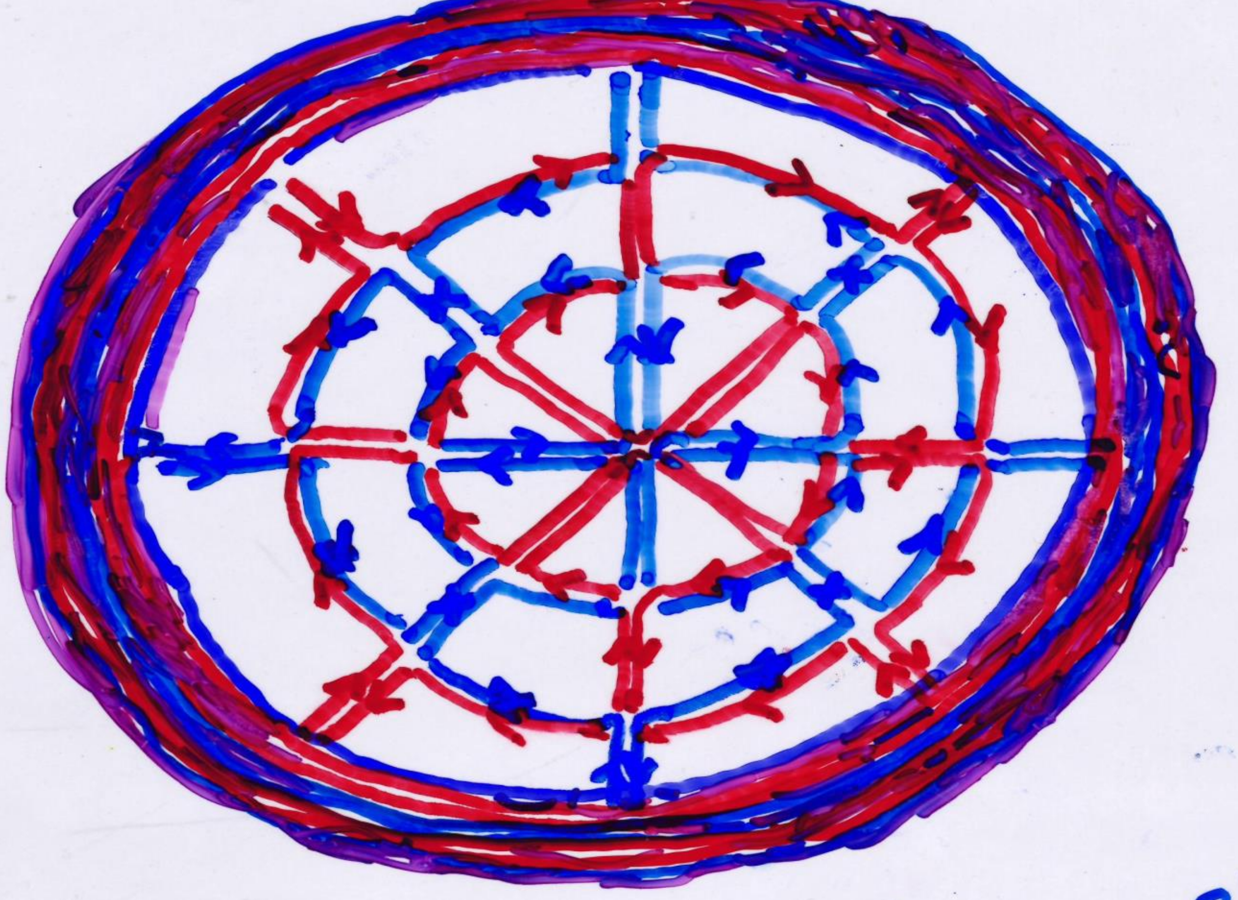
't Hooft Polyakov condensate

$B_L B_L = \frac{\pm 1}{r^4}$

repels al color

$a=0$

Here is a sketch showing what a type-2 soln could look like



requires less topolg charge

exterior region a sterile vacuum condensed with opposite chirality from int

$a = -1$

Derrick's Theorem (1964)

$$\mathcal{L} = -\frac{1}{2}\operatorname{tr} G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}(\partial^{\nu A}\phi)^{\dagger}(\partial_{\nu}^{A}\phi) - V(\phi)$$

$$\mathcal{L}^{\text{static}}_{\text{energy}} = I_G(A) + I_K(A,\phi) + I_V(\phi)$$

$$I_G = \frac{1}{2}\int d^D x \operatorname{tr}(G_{ij}^2)$$

$$I_K = \frac{1}{2}\int d^D x (\partial\phi)(\partial\phi)$$

$$I_V = \int d^D x V(\phi)$$

given a static (soliton) $\bar{A}_j, \bar{\Phi}$

define scaled $f_\lambda(x) = \bar{\Phi}(\lambda x)$ $g_{j\lambda}(x) = \lambda \bar{A}_j(\lambda x)$

$$\mathcal{L}^{\text{static}}_{\lambda} = I_G(g_\lambda) + I_K(g_\lambda, f_\lambda) + I_V(f_\lambda)$$

$$= \lambda^{4-D} I_G(\bar{A}) + \lambda^{2-D} I_K(\bar{A},\bar{\Phi}) + \lambda^{-D} I_V(\bar{\Phi})$$

this is stationary at $\lambda=1$ if

$$0 = (D-4)I_G(\bar{A}) + (D-2)I_K(\bar{A},\bar{\Phi}) + D I_V(\bar{\Phi})$$

This allows soliton solutions in $D=2$ and $D=3$

Only solution $D=4$ soliton in pure gauge sector

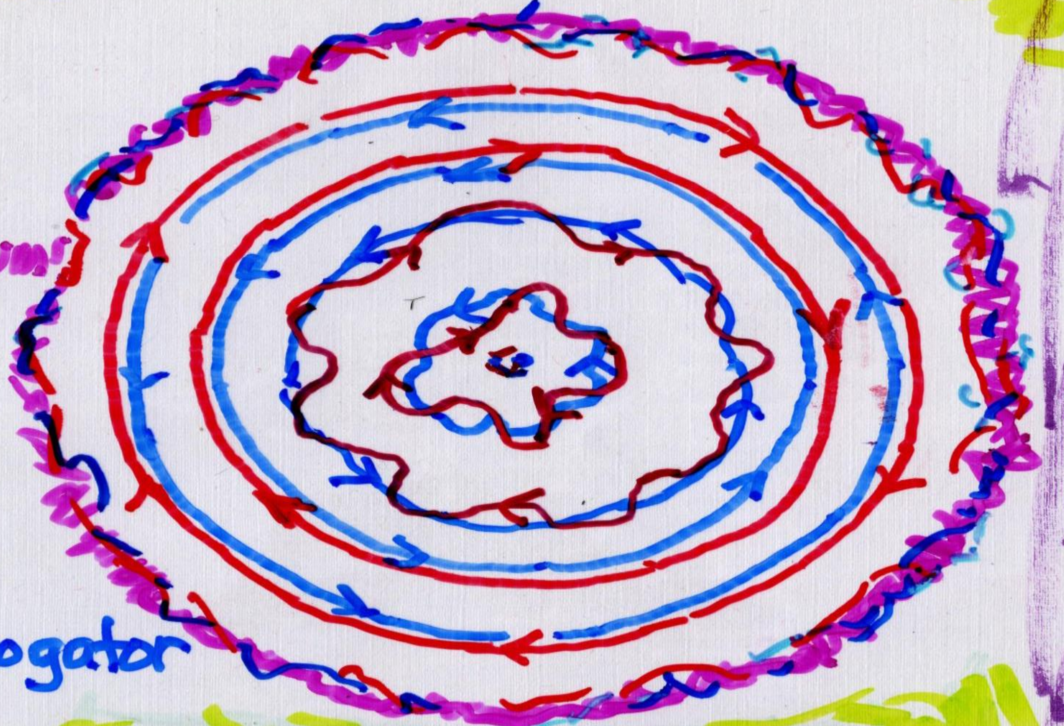
no scalars and I_G scale invariant

Leinweber & Collaborators

dynamical fermions increas density of color vortices in vacuum

pure gauge 3277 ± 156

full qcd 5923 ± 259



Improves fit for confining potential & Landau gauge propogator

$^3P_0$ $q\bar{q}$ pairs $J^{PC} = 0^{++}$

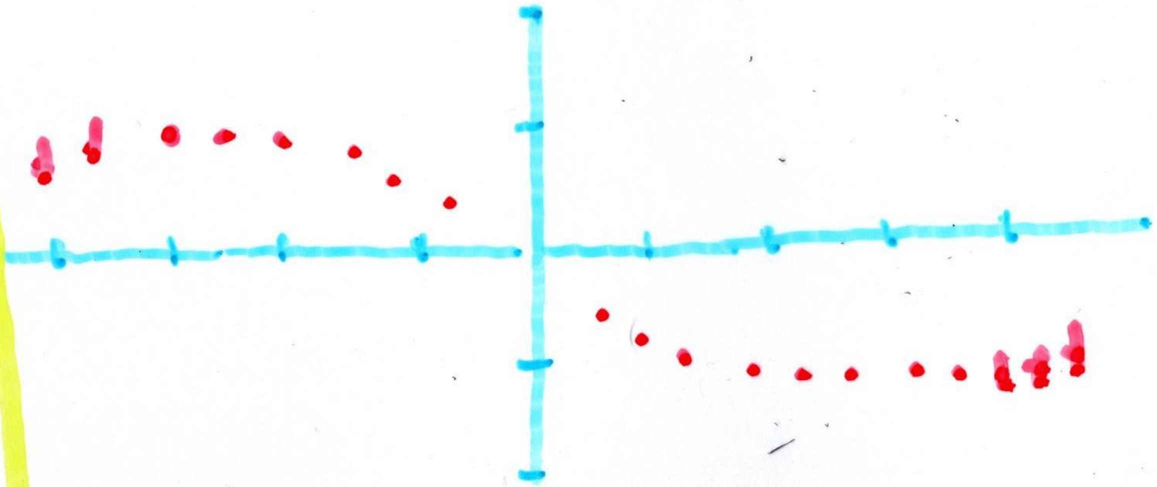
Chromostatics

Interior Volume of hadron one chiral adiabatic vortex $J^{PC} = 0^{++}$

M. Engelhardt, B. Musch, J Negele, A. Schäfer

Strong Evidence of large-scale

Chiral Structure



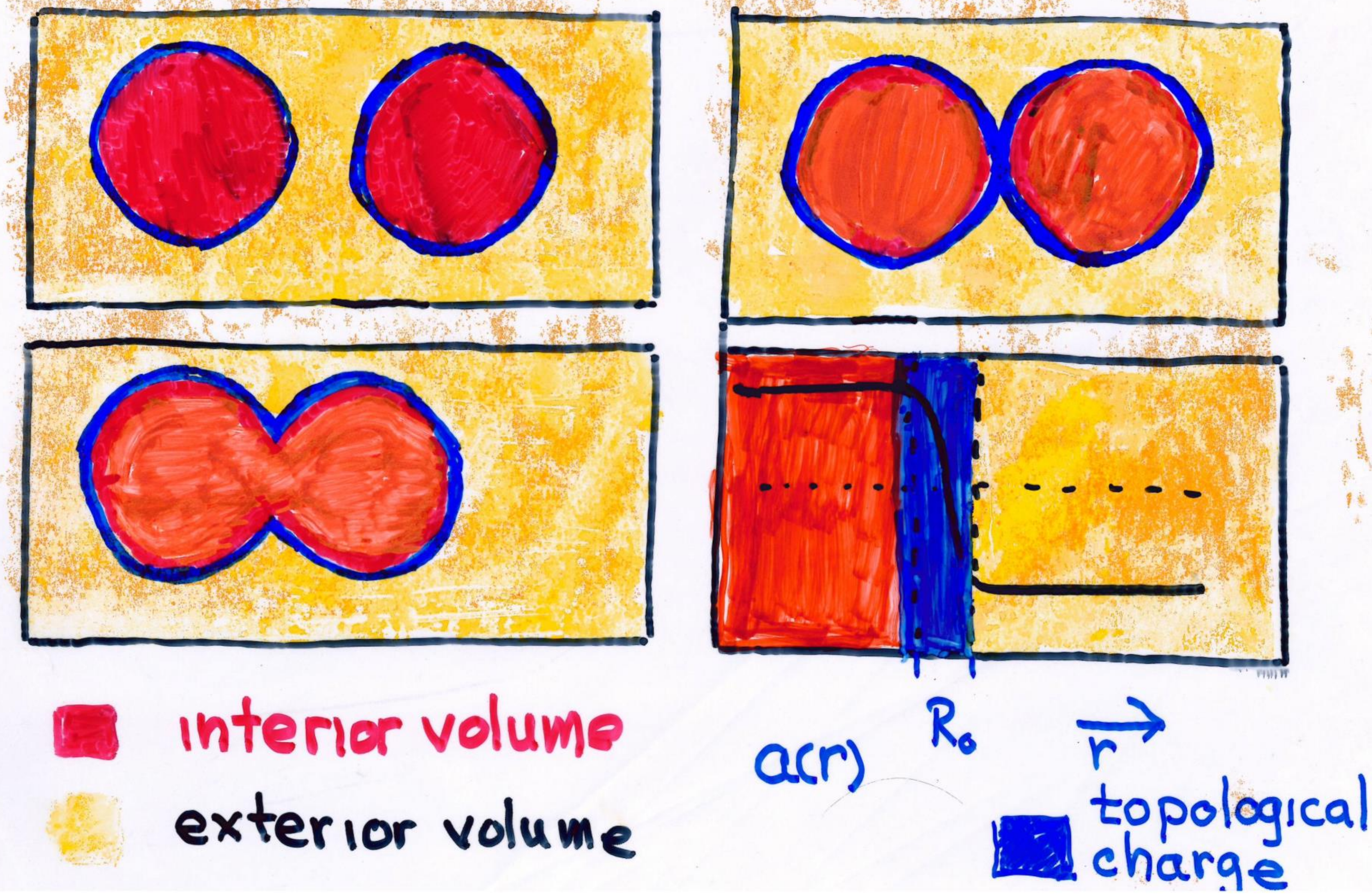
u-quark Boer-Mulders spin-directed momentum shif in a pion

$\langle 2\Sigma \vec{L}\cdot\vec{S}\rangle = -0.65 \pm 0.10$

M. Engelhardt
QCD Evolution 2021

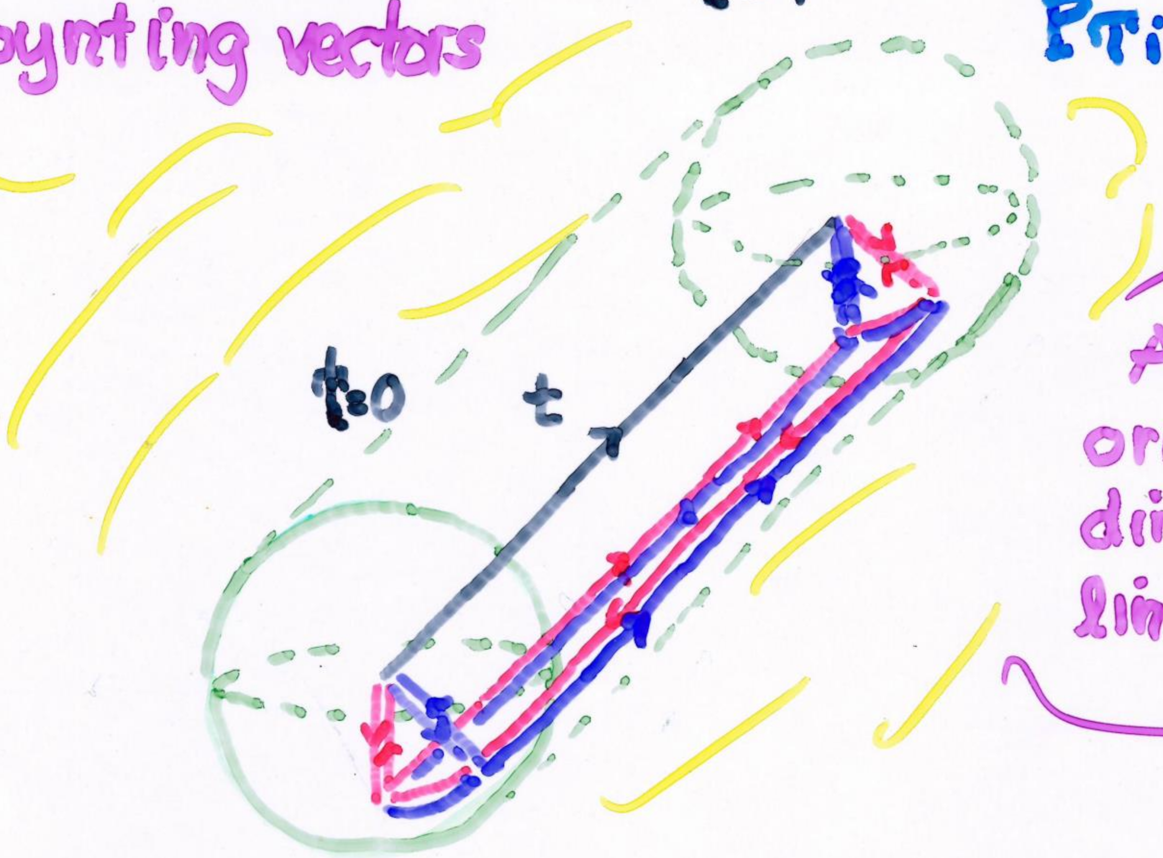
$^3P_0$ $\bar{q}q$ pairs

Domain Zones



Adiabatic Color-flow Poynting vectors

$$P_{L\,ia} = E_S B_A R_{ia}$$
$$P_{T\,ia} = -E_L B_A \varepsilon^S_{ia} + E_L E_S \varepsilon^T_{ia}$$



Away from the origin time-directed Wilson lines carry charge

Adjoint Wilson loops with $A_0(r) = 0$

Adiabatic Evolution Confined Condensate

Spherical SU(3) YM Dirac Eqn. in 1+1 dimensions

$\gamma_0^{(2)} = r\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\gamma_1^{(2)} = r\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\gamma_5^{(2)} = \frac{1}{r^2}\gamma_0^{(2)}\gamma_1^{(2)}$

2-dim spinors

$R = \begin{pmatrix} R^- \\ -R^+ \end{pmatrix}$ $L = \begin{pmatrix} L^+ \\ L^- \end{pmatrix}$ $\Phi^{(2)} = a(r,t)\exp\{i\omega(r,t)\gamma_5^{(2)}\}$

Bispinors with fund. rep of SU(3)

Dirac Spinors → 2 Weyl Spinors

$$(\gamma_\ell^{(2)} D^\ell + \gamma_5^{(2)}\Phi^{(2)})R = mL$$

$$(\gamma^{(2)} D^\ell + \gamma_5^{(2)}\Phi^{(2)})L = -mR$$

Spherical SU(3) YM Dirac Eqn. in 1+1 dimensions

$$\gamma_0^{(2)} = r\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma_1^{(2)} = r\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \gamma_5^{(2)} = \frac{1}{r^2}\gamma_0^{(2)}\gamma_1^{(2)}$$

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Dirac Spinors → 2 Weyl spinors

$$(\gamma_\ell^{(2)} D^\ell + \gamma_5^{(2)} \Phi^{(2)})\, R = mL$$

$$(\gamma^{(2)} D^\ell + \gamma_5^{(2)} \Phi^{(2)})\, L = -mR$$

Field Strength Densities

$$E_i^a(r)E_i^a(r) = A_0'^2(r) + \frac{2}{r^2}(a(r)A_0(r))^2$$

$$B_i^a(r)B_i^a(r) = \frac{(a^2(r)-1)^2}{r^4} + \frac{2}{r^2}\left[a'(r)^2 + a^2(r)(A_1(r)-\omega'(r))^2\right]$$

$$E_i^a(r)B_i^a(r) = \frac{A_0'(r)(a^2(r)-1)}{r^2} + \frac{2}{r^2}A_0(r)a(r)a'(r)$$

gauge covariant

$$\varepsilon^S_{ia}(\omega) = \delta^T_{ia}\cos(\omega(r)) - \varepsilon^T_{ia}\sin(\omega(r))$$

$$\varepsilon^A_{ia}(\omega) = \delta^T_{ia}\sin(\omega(r)) + \varepsilon^T_{ia}\cos(\omega(r))$$

transverse basis tensors

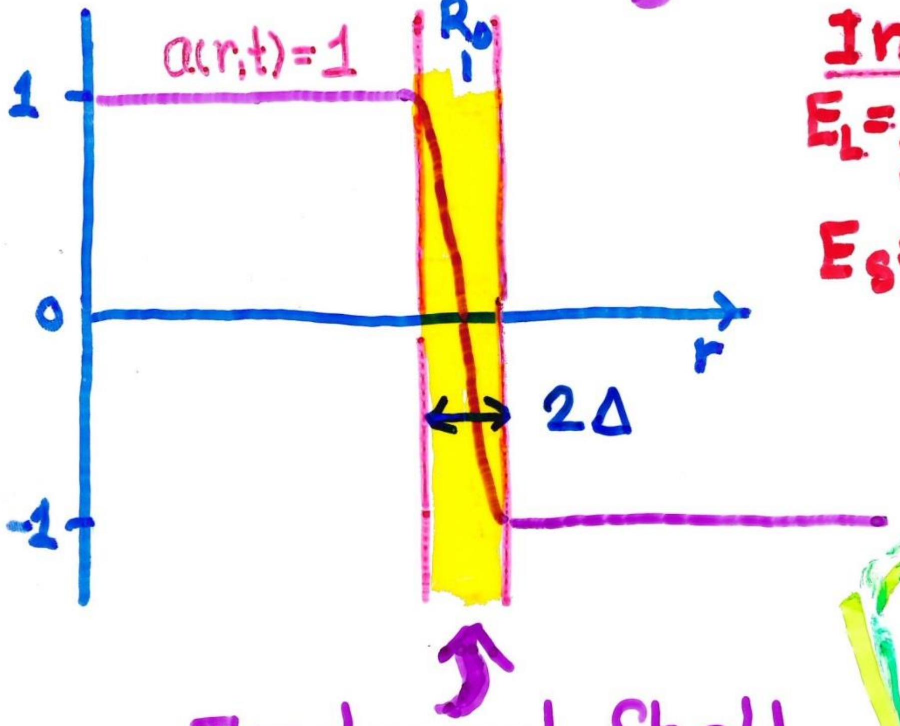
$$gA_i^a(r) = A_1(r)\,P_{ia} + \frac{a(r)}{r}\varepsilon^A_{ia}(\omega(r)) - \varepsilon^A_{ia}\frac{(\omega(r))}{r}$$

transverse gauge connection

$$\varepsilon^S_{ia}(\omega+\pi/2) = -\varepsilon^A_{ia}(\omega) \qquad \varepsilon^A_{ia}(\omega+\pi/2) = \varepsilon^S_{ia}(\omega)$$

$$D_0^{ab}v_b = \varepsilon^{abc}v_bA_0^c(r) \qquad D_i^{ab}\hat{r}_b = \frac{a(r)}{r}\varepsilon^S_{ia}(\omega(r))$$

Confining Boundary Conditions



Interior $r<R_0-\Delta$ $a(r,t)=1$

$E_L = \frac{\partial A_0}{\partial r} - \frac{\partial A_1}{\partial t}$ $B_L = E_A = B_S = 0$

$E_S = (K_1 - A_0)/r$ $B_A = (K_0 + A_1)/r$

$J^a_\mu(r) \neq 0$ $J^{PC} = 0^{++}$

$\langle |E_i^a E_i^a| \rangle \geq 0$ $\langle |B_i^a B_i^a| \rangle \geq 0$

Topological Shell

$J^{PC} = 0^{-+}$ CP odd

Topological Charges $\langle |E_i^a B_i^a| \rangle \neq 0$

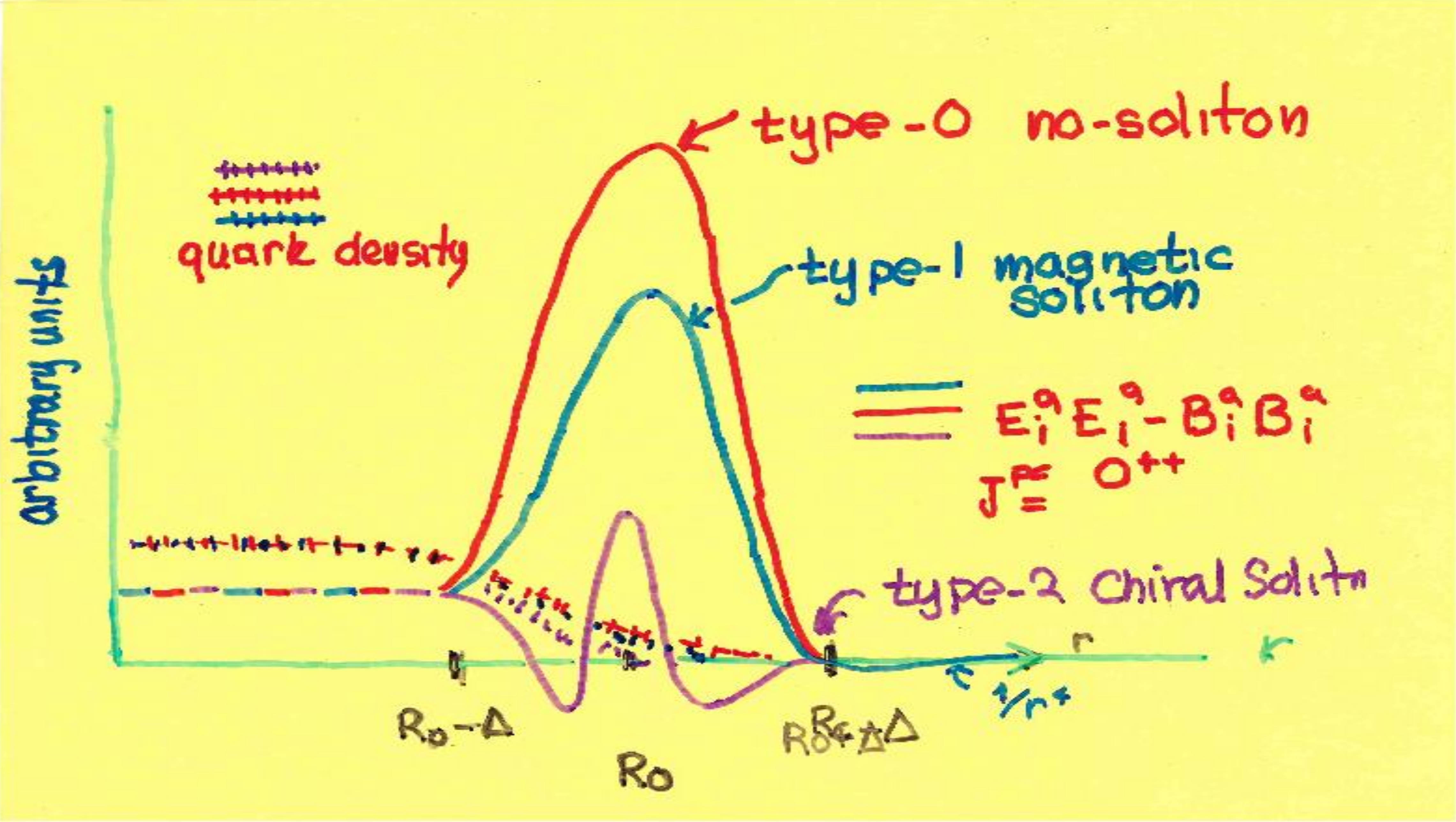
Dyonic Charges at $r=R_0$

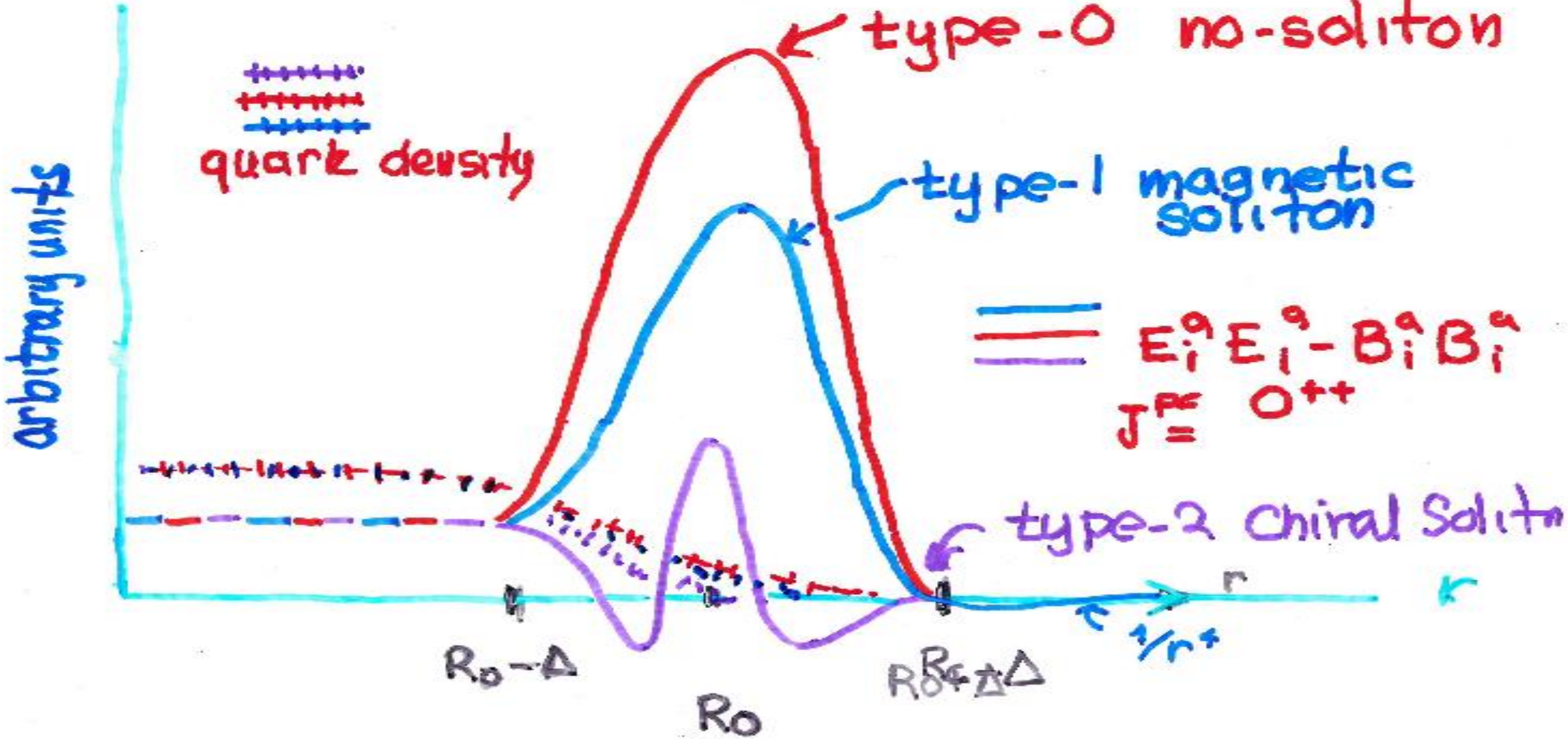
External "Vacuum" $r > R_0+\Delta$

$a(r,t)=-1$ $\cos\omega(r,t)=-1$ $A_0=A_1=0$

$E_i^a=0$ $B_i^a=0$

$J^{PC} = 0^{++}$


type-0 no-soliton
quark density
type-1 magnetic soliton
$E_i^a E_i^a - B_i^a B_i^a$
$J^{PC} = 0^{++}$
arbitrary units
type-2 Chiral Solitn
r
$1/r^4$
$R_0 - \Delta$
R_0
$R_0 + \Delta$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R_0
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r
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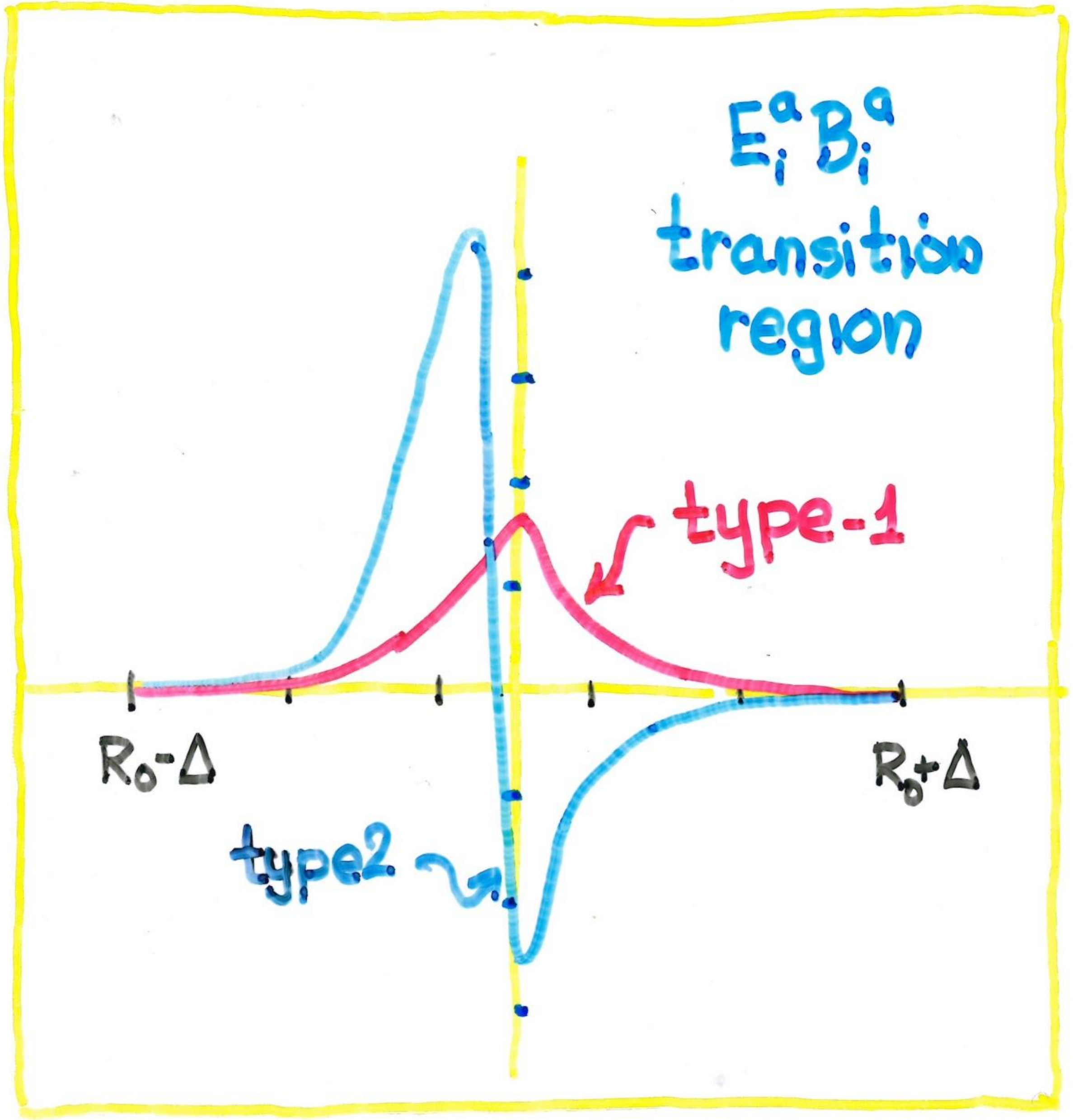

CP-odd
condensate

Surface Effects in non-Abelian flux ruptures

Jet fragmentation

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IV. Constructive Field Theory

a. jaffe

A topological stable solution to the classical (Yang-Mills Maxwell) eq'ns provides robust scaffolding

for understanding hadron structure

I. Clay Mathematics Institute Millenium Prize Problems ($1 G)

(www.claymath.org/millenium)

Arthur Jaffe, Edward Witten "Quantum Yang Mills"

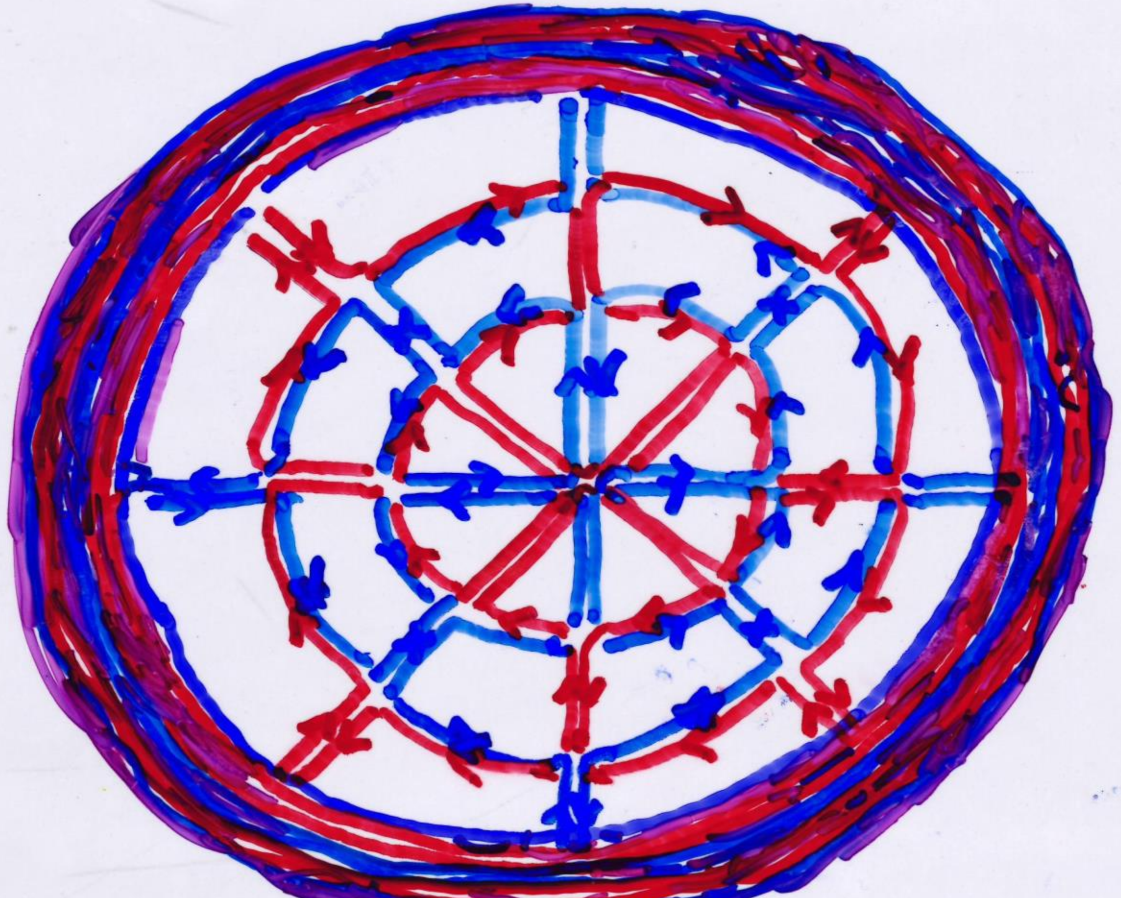
Demonstrate the existence of a QFT in 4-dim Minkowski space with a non-Abelian compact gauge group G displaying

1. mass gap Δ
2. confinement of quarks & gluons
3. chiral symmetry breaking

$(G = SU(3)) \Rightarrow$ QCD a fully consistent QFT

Backups

Here is a sketch showing what a type-2 soln could look like



requires less topolg charge

exterior region a sterile vacuum condensed with opposite chirality from int

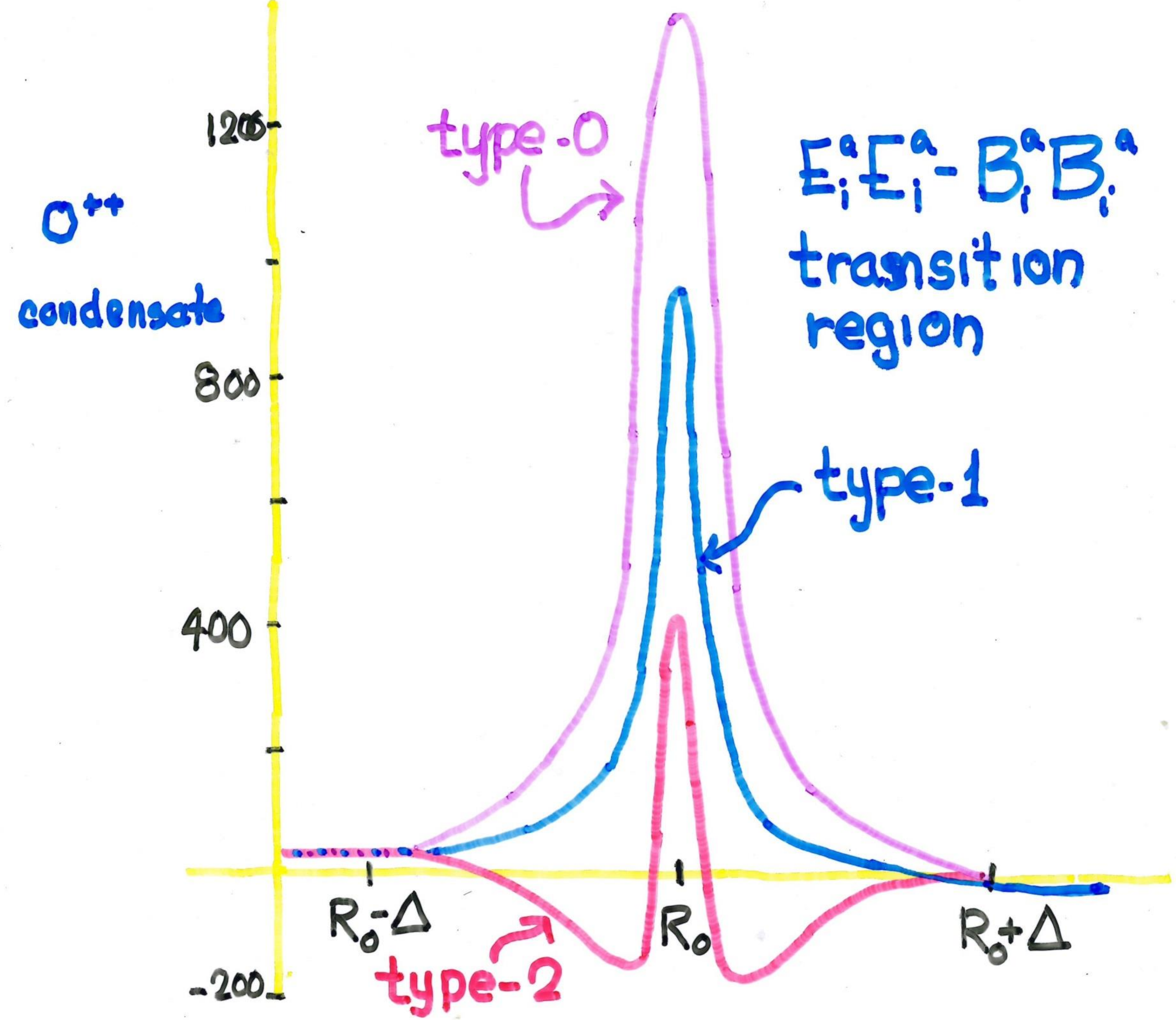
$a = -1$

CHROMOSTATICS and Perturbative Evolution

Applying the Yang-Mills Maxwell equations to study the topological structure of confined systems

Topological condensates and soft-gluon theorems

dennis sivers - Portland Physics Ins
& U. Michigan



O^{++} condensate
1200
800
400
-200
type-0
$E_i^a E_i^a - B_i^a B_i^a$
trasnsition region
type-1
type-2
$R_0 - \Delta$
R_0
$R_0 + \Delta$

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transverse basis tensors

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$$D_0^{ab}v_b = \varepsilon^{abc}v_b A_0^c(r) \qquad D_i^{ab}\hat{r}_b = \frac{a(r)}{r}\varepsilon^S_{ia}(\omega(r))$$