Analytical Evaluation of Lepton-Proton Two-Photon Exchange in Chiral Effective Theory

Eur. Phys. J. A **60**, 69 (2024)

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NREC/PREN/μASTI 2024, Stony Brook University, 6^{th} - 10^{th} May

Motivation: Problems with the Proton's structure

1. **Form factor discrepancy** (since $\sim$ 2008):
   Dramatic contrast in the proton's **Electric** (G_E^p) to **Magnetic** (G_M^p) Sachs form factor ratio measured for $|Q^2| \gtrsim 1\ (\mathrm{GeV/c})^2$ between two well-known techniques:

   Rosenbluth (LT) Separation (traditional$\sim$ 50s) & *Recoil Proton Polarization Transfer* (novel $\sim$ 2000s)

2. **Radius puzzle** (since $\sim$ 2013):
   Stark contrast ($\sim 7\sigma$) in the extracted protons's r.m.s. charge radius measured from 3 types of experimental sources:

   (i) e-p elastic scattering , (ii) H-Lamb-shift spectroscopy , & (iii) μH-Lamb-shift spectroscopy

Elastic ep scattering, in the limit of Born approximation (one photon exchange):

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \left(\frac{E'}{E}\right) \frac{1}{1+\tau} \left({G_E^p}^2(Q^2) + \frac{\tau}{\varepsilon}{G_M^p}^2(Q^2)\right)$$

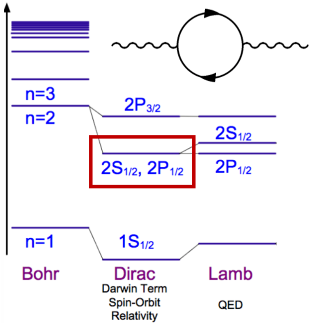
$$-Q^2 = 4EE'\sin^2\frac{\theta}{2} \qquad \tau = \frac{-Q^2}{4M_p^2} \qquad \varepsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1}$$

Taylor expansion of G_E at low Q^2

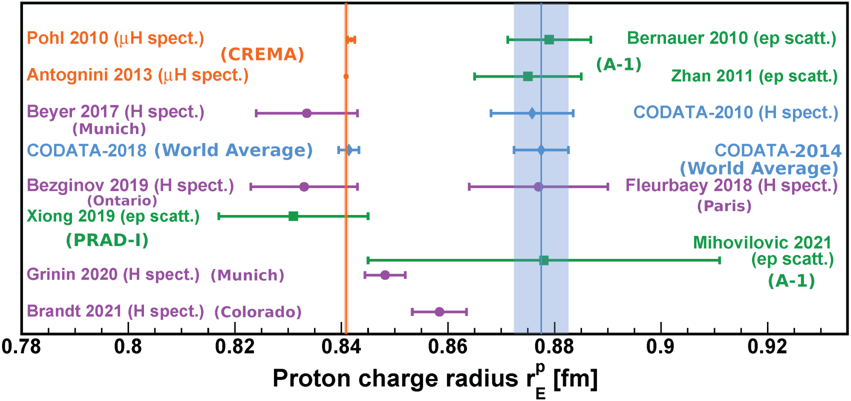
$$G_E^p(Q^2) = 1 + \frac{Q^2}{6}\langle r^2\rangle + \frac{Q^4}{120}\langle r^4\rangle + \ldots$$

Derivative at low Q^2 limit

$$\langle r^2\rangle = +6\left.\frac{dG_E^p(Q^2)}{dQ^2}\right|_{Q^2=0}$$



Resolution: Experimental efforts, e.g. for proton's radius



(courtesy: Gao and Vanderhaeghen 2022)

- **Ongoing:** MUSE (PSI), A1 (MAMI) & GLUEX (JLab)
- **Newly proposed:** COMPASS++/AMBER (CERN), PRAD-II (JLab), MAGIX (MESA), PRES (MAMI), ULQ^2 (Tohoku Univ.), ...

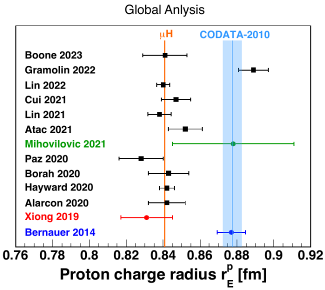
Resolution: Theoretical/Phenomenological Efforts

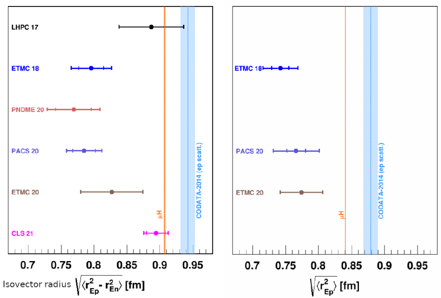
- **Large number of ideas proposed:**

  ↪ Standard Model: QED-inspired hadronic models, Vector Meson Dominance, Constituent Quark models, Generalized Parton Distributions, Dyson-Schwinger QCD, Dispersion Relations & Low-energy EFTs (e.g., NRQED+QCD & χPT)
  ↪ BSM Physics: Lepton non-universality (e.g., scalar and tensor charges @TeVs)

- **Global data fits** using Dispersion Relations analyses consistent with **CREMA 2013** results

- **Lattice QCD** results favour a smaller proton radius scenario





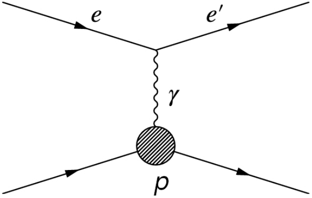
(courtesy: Gao and Vanderhaeghen 2022)

- **REMARKS:** Radius puzzle may be ostensibly resolved, but not quite satisfactory yet, e.g.,

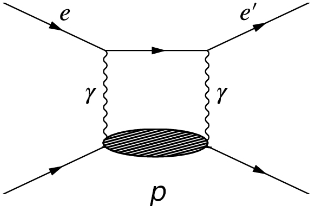
  ↪ Discrepancies existing between A1 (MAMI) and PRAD (JLab) *e*-p scattering results
  ↪ Complete missing information thus far on μ-**p scattering** results
  ↪ Systematic uncertainties from higher-order corrections are still not under control

Low-energy/momentum higher-order corrections to lepton-proton (ℓ-p) Born or "point-like" differential cross section: $\left[\frac{d\sigma^{\text{elastic}}}{d\Omega_\ell}\right]_{\text{Born}}$

1. Two types of higher-order corrections: (i) *Hadronic (QCD)*, and (ii) *Radiative (QED)*
2. Hadronic corrections ($\sim 50-60\%$) dominate over radiative corrections ($\sim 30\%$)
3. *Soft Bremsstrahlung* & *Vacuum Polarization* constitute dominant radiative corrections ($\sim 20\%$)
4. *Vertex* and *Multi-photon Exchange* constitute only minor radiative corrections ($\sim 10\%$)
5. Largest uncertainties ($\sim 1-2\%$) arise from the Two-Photon Exchange (TPE) corrections which resolve bulk of both the existing discrepancies (form factor and radius)



(a) Lepton scattering in the Born approximation.



(b) Two-photon exchange contribution to lepton scattering.

6. At very low-$|Q^2| \lesssim 0.1$ (GeV/c)^2 only elastic proton TPE intermediate state dominantly contributes to elastic cross section with very small inelastic contributions (Δ, N^*, ..., etc. resonances)

MUon Scattering Experiment@PSI (MUSE)

- Simultaneous (unpolarized) scattering of electrons ($e^{\pm}$) and muons ($\mu^{\pm}$) with the protons at unprecedented low-Q^2 range:

Lepton momentum (p_ℓ) in GeV/c	0.115	0.153	0.210
$\lvert Q^2\rvert$ in (GeV/c)^2 for Electron			
Angle $\theta = 20^\circ$	0.0016	0.0028	0.0052
Angle $\theta = 100^\circ$	0.027	0.046	0.082
$\lvert Q^2\rvert$ in (GeV/c)^2 for Muon			
Angle $\theta = 20^\circ$	0.0016	0.0028	0.0052
Angle $\theta = 100^\circ$	0.026	0.045	0.080

- Measure elastic cross-section below 1% precision (target $\sim 0.1\%$ accuracy)
- Allows accurate extraction of the TPE part from *charge asymmetry* measurements

$$\left[\frac{d\sigma_{\rm elastic}(Q^2)}{d\Omega_\ell}\right]^{(\pm)} = \left[\frac{d\sigma_{\rm elastic}(Q^2)}{d\Omega_\ell}\right]_{\rm OPE+Had} \left\{1 \pm \bar{\delta}^{\rm (odd)}_{\rm TPE+Brem.}(Q^2) + \bar{\delta}^{\rm (even)}_{\rm Virt.+Brem.}(Q^2)\right\}_{\rm rad}$$

- IR-finite part of TPE to be extracted from cross section data

$$\bar{\delta}_{\rm TPE}(Q^2) = \frac{2\,\mathcal{R}e \sum_{spins} \left(\mathcal{M}^*_{\rm Born}\,\mathcal{M}_{\rm TPE}\right)}{\sum_{spins} \lvert\mathcal{M}_{\rm Born}\rvert^2} - \delta^{\rm (box)}_{\rm IR}(Q^2)$$

Framework: $SU_f(2)$ Heavy Baryon Chiral Perbt. Theory (HBχPT)

- Low-energy Effective Field Theory of QCD + QED
- **Model-independent** with **gauge invariance** naturally incorporated
- Light D.O.F.s, e.g., γ, π^0, $\pi^\pm$, $e^\pm$, $\mu^\pm$, are treated relativistically
- Heavy D.O.F.s like nucleons & baryons, are treated non-relativistically
- **Lepton mass** is fully preserved in calculations, unlike *ultra-relativistic* approximations
- **Power-counting scheme:** Allows systematic control over uncertainties

  ↪ Simultaneous expansion in powers of Q/M_N along with chiral expansion Q/Λ_χ

  $$\text{chiral scale:} \qquad M_N \sim \Lambda_\chi \sim 4\pi f_\pi \sim 1\ \mathrm{GeV/c}$$

  ↪ helps to include dominant proton's **recoil effects** at low-Q^2 MUSE kinematics
- Analysis includes *leading order* (LO) & *next-to-leading order* (NLO) chiral corrections

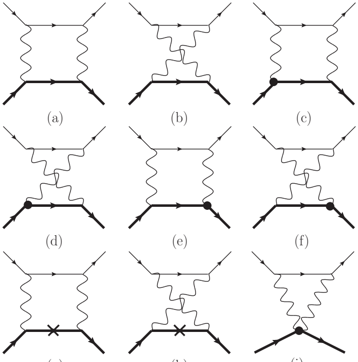
$$\mathcal{L}^{(0)}_{\pi N} = \overline{N}\,(iv\cdot D + ...)\,N \quad , \quad N = (\mathrm{p\ n})^{\mathrm{T}} \quad , \quad \mathrm{v} = (1,\mathbf{0})$$

$$\mathcal{L}^{(1)}_{\pi N} = \overline{N}\left\{\frac{1}{2M_N}(v\cdot D)^2 - \frac{1}{2M_N}D\cdot D + ...\right\}N$$

- **NOTE:** Pion-loops that generate the proton's off-shell form factors will only contribute to TPE@NNLO and hence excluded @NLO accuracy

  ↪ protons effectively behave "point-like" for TPE@NLO

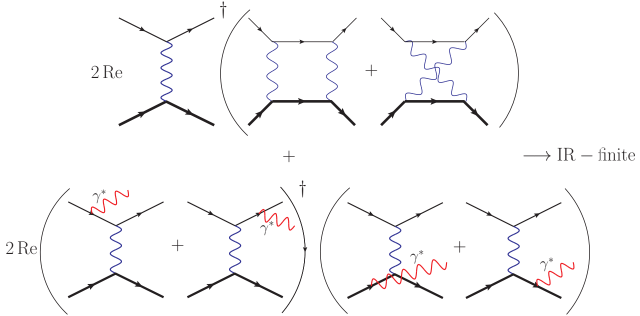
TPE diagrams @LO+NLO in HBχPT



- Contributions to cross section from diagrams (a) & (b) are **IR-divergent**
- Contributions to cross section from diagrams (c) - (i) are all **finite**
- $\mathcal{O}(1/M_N)$ IR-divergences isolated using **Dimensional Regularization**, with $\varepsilon = (4-D)/2 < 0$

  $\hookrightarrow$ IR-divergence drops-out of TPE@LO since $(E_\ell - E'_\ell) \sim (p_\ell - p'_\ell) \sim \mathcal{O}(M_N^{-1})$

$$\delta_{\rm IR}^{(\rm box)}(Q^2) = \frac{\alpha}{\pi}\left\{\frac{1}{\varepsilon} - \gamma_E + \ln\left(\frac{4\pi\mu^2}{m_\ell^2}\right)\right\}\left\{\frac{E_\ell}{p_\ell}\ln\sqrt{\frac{E_\ell + p_\ell}{E_\ell - p_\ell}} - \frac{E'_\ell}{p'_\ell}\ln\sqrt{\frac{E'_\ell + p'_\ell}{E'_\ell - p'_\ell}}\right\} \sim \mathcal{O}\left(\frac{1}{M_N}\right)$$



- **Kinoshita–Lee–Nauenberg (KLN) Theorem** mandated that TPE IR divergence be canceled by ℓ−p **charge-odd interference** soft-bremsstrahlung processes
- $\mathcal{O}(1/M_N)$ IR-divergences arise from Leading chiral order diagrams only

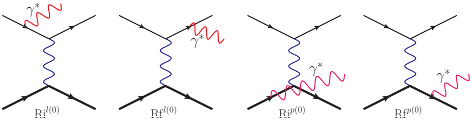
↪ IR-divergence drops-out @LO since $(E_\ell - E'_\ell) \sim (p_\ell - p'_\ell) \sim \mathcal{O}(M_N^{-1})$

$$\delta^{(\text{soft})}_{\text{soft}\gamma^*}(Q^2)\Big|_{\text{IR}} = -\frac{\alpha}{\pi}\left\{\frac{1}{\varepsilon} - \gamma_E + \ln\left(-\frac{4\pi\mu^2}{Q^2}\right)\right\}\left\{\frac{E_\ell}{p_\ell}\ln\sqrt{\frac{E_\ell + p_\ell}{E_\ell - p_\ell}} - \frac{E'_\ell}{p'_\ell}\ln\sqrt{\frac{E'_\ell + p'_\ell}{E'_\ell - p'_\ell}}\right\} \sim \mathcal{O}\left(\frac{1}{M_N}\right)$$

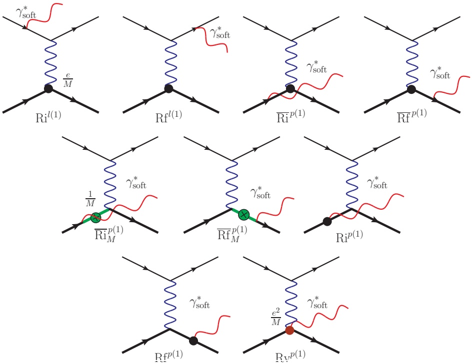
$$\boxed{\bar{\delta}^{(\text{odd})}_{\text{TPE+Brem}}(Q^2) = \delta_{\text{TPE}}(Q^2) + \delta^{(\text{odd})}_{\text{soft}\gamma^*}(Q^2) \longrightarrow \text{IR-finite result}}$$

Charge-odd Soft-photon Bremmstrahlung.: Typical diagrams @LO+NLO

- IR-divergences arise from LO diagrams only:

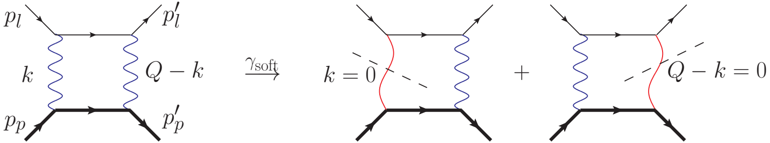


- NLO diagrams lead to IR-finite contributions just like the TPE:



- We do not invoke **Soft Photon Approximations** (SPA) *à la* Maximon and Tjon (2000)

  ↪ One photon is considered **soft** (either $k = 0$ or $k = Q$)
  ↪ Second photon is considered **hard** (either $|(Q-k)^2| \gg 0$ or $|k^2| \gg 0$)



$$\mathcal{M}^{(a)}_{\rm box} = e^4 \int \frac{\mathrm{d}^4 k}{(2\pi)^4 i} \frac{\left[\bar{u}_\ell(p_\ell')\gamma^\mu(\not{p}_\ell - \not{k} + m_\ell)\gamma^\nu u_\ell(p_\ell)\right]\left[\chi_p^\dagger(p_p')v_\mu v_\nu \chi_p(p_p)\right]}{(k^2+i\varepsilon)\left[(Q-k)^2+i\varepsilon\right](k^2-2k\cdot p_\ell+i\varepsilon)(v\cdot k + v\cdot p_p + i\varepsilon)}$$

$$\xrightarrow{\gamma^*_{\rm soft}} -2e^2 E_\ell\, \mathcal{M}_{\rm Born} \int \frac{\mathrm{d}^4 k}{(2\pi)^4 i} \frac{1}{(k^2+i\varepsilon)(k^2-2k\cdot p_\ell+i\varepsilon)(v\cdot k+i\varepsilon)}$$

$$-2e^2 E_\ell'\, \mathcal{M}_{\rm Born} \int \frac{\mathrm{d}^4 k}{(2\pi)^4 i} \frac{1}{\left[(Q-k)^2+i\varepsilon\right](k^2-2k\cdot p_\ell+i\varepsilon)(v\cdot k+i\varepsilon)},$$

where

$$\mathcal{M}_{\rm Born} = -\frac{e^2}{Q^2}\left[\bar{u}_\ell(p_\ell')\gamma^\mu u_\ell(p_\ell)\right]\left[\chi_p^\dagger(p_p')v_\mu \chi_p(p_p)\right]$$

↪ SPA misses kinematic region where both photons are hard ($|k^2| \gg 0$ and $|(k-Q)^2| \gg 0$)
↪ Source of *uncontrolled* systematic errors at low-Q^2 MUSE kinematics

More Comments ...

- IR-divergent TPE loops involving "heavy" NR propagators are difficult to evaluate analytically using existing software packages (e.g., CalcHEP, Feyncalc, Reduce, etc.)

$$i\Delta_p^{(\mathrm{NR})}(p_p) = \left[\frac{i}{v\cdot p_p + i\varepsilon}\right]_{\mathrm{LO}} + \frac{i}{2M_N}\left[1 - \frac{p_p^2}{(v\cdot p_p + i\varepsilon)^2}\right]_{\mathrm{NLO}} \; ; \; v = (1, \vec{0})$$

- Tensor Feynman loop-integrals are reduced into scaler *master integrals* using the well-known **Passarino-Veltmen (PV)** technique

- Complicated 4-point scalar loop-integrals are expressed in terms of simpler 2,3-point loop-integrals using a combination of **Integration-by-parts (IBP)** & **Partial fractions**

$$I^-(p_\ell, \omega | n_1, n_2, n_3, n_4) = \frac{1}{i}\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{(k^2 + i\varepsilon)^{n_1}\,[(k-Q)^2 + i\varepsilon]^{n_2}\,(k^2 - 2k\cdot p_\ell + i\varepsilon)^{n_3}\,(v\cdot k + \omega + i\varepsilon)^{n_4}}$$

$$I^+(p'_\ell, \omega | n_1, n_2, n_3, n_4) = \frac{1}{i}\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{(k^2 + i\varepsilon)^{n_1}\,[(k-Q)^2 + i\varepsilon]^{n_2}\,(k^2 + 2k\cdot p'_\ell + i\varepsilon)^{n_3}\,(v\cdot k + \omega + i\varepsilon)^{n_4}}$$

$$Z^-(\Delta, i\sqrt{-Q^2}/2, m_\ell, \omega) = \frac{1}{i}\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{\left[(k+\Delta)^2 - \frac{1}{4}Q^2 + i\varepsilon\right]\,(k^2 - m_\ell^2 + i\varepsilon)\,(v\cdot k + \omega + i\varepsilon)}$$

$$Z^+(\Delta', i\sqrt{-Q^2}/2, m_\ell, \omega) = \frac{1}{i}\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{\left[(k+\Delta')^2 - \frac{1}{4}Q^2 + i\varepsilon\right]\,(k^2 - m_\ell^2 + i\varepsilon)\,(v\cdot k + \omega + i\varepsilon)}$$

$$I_1^{-\mu}(p_\ell, \omega | n_1, n_2, n_3, n_4) = \frac{1}{i}\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{k^\mu}{(k^2 + i\varepsilon)^{n_1}\,[(k-Q)^2 + i\varepsilon]^{n_2}\,(k^2 - 2k\cdot p_\ell + i\varepsilon)^{n_3}\,(v\cdot k + \omega + i\varepsilon)^{n_4}}$$

$$I_1^{+\mu}(p'_\ell, \omega | n_1, n_2, n_3, n_4) = \frac{1}{i}\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{k^\mu}{(k^2 + i\varepsilon)^{n_1}\,[(k-Q)^2 + i\varepsilon]^{n_2}\,(k^2 + 2k\cdot p'_\ell + i\varepsilon)^{n_3}\,(v\cdot k + \omega + i\varepsilon)^{n_4}}$$

Example: Leading chiral order TPE corrections

- From **Box diagram (a)**:

$$\delta^{(a)}_{\rm box}(Q^2) = -8\pi\alpha\left[\frac{Q^2}{Q^2+4E_\ell E'_\ell}\right]\mathcal{R}e\Big\{E'_\ell I^-(p_\ell,0|0,1,1,1)+E_\ell\, I^-_{\rm IR}(p_\ell,0|1,0,1,1)-(E_\ell+E'_\ell)I^-(p_\ell,0|1,1,0,1)$$
$$-(Q^2+8E_\ell E'_\ell)I^-(p_\ell,0|1,1,1,0)+\left(\frac{Q^2E_\ell+8E_\ell^2E'_\ell}{Q^2}\right)\Big[I^-(p_\ell,0|0,1,1,1)$$
$$+I^-_{\rm IR}(p_\ell,0|1,0,1,1)-2Z^-(\Delta,i\sqrt{-Q^2}/2,m_\ell,E_\ell)\Big]\Big\}$$

- From **Cross-box diagram (b)**:

$$\delta^{(b)}_{\rm xbox}(Q^2) = -8\pi\alpha\left[\frac{Q^2}{Q^2+4E_\ell E'_\ell}\right]\mathcal{R}e\Big\{E_\ell I^+(p'_\ell,0|0,1,1,1)+E'_\ell\, I^+_{\rm IR}(p'_\ell,0|1,0,1,1)-(E_\ell+E'_\ell)I^+(p'_\ell,0|1,1,0,1)$$
$$+(Q^2+8E_\ell E'_\ell)I^+(p'_\ell,0|1,1,1,0)+\left(\frac{Q^2E_\ell+8E_\ell^2E'_\ell}{Q^2}\right)\Big[I^+(p'_\ell,0|0,1,1,1)$$
$$+I^+_{\rm IR}(p'_\ell,0|1,0,1,1)-2Z^+(\Delta',i\sqrt{-Q^2}/2,m_\ell,-E'_\ell)\Big]\Big\}$$

- Large LO cancellations in the sum with exception: $I^-(p,0|1,1,0,1)\equiv I^+(p',0|1,1,0,1)=I(Q|1,1,0,1)$

$$I^-(p_\ell,0|0,1,1,1)+I^+(p'_\ell,0|0,1,1,1)\sim\mathcal{O}(M_N^{-1})\quad\text{finite terms}$$
$$I^-_{\rm IR}(p_\ell,0|1,0,1,1)+I^+_{\rm IR}(p'_\ell,0|1,0,1,1)\sim\mathcal{O}(M_N^{-1})\quad\text{IR-div terms}$$
$$I^-(p_\ell,0|1,1,1,0)+I^+(p'_\ell,0|1,1,1,0)\sim\mathcal{O}(M_N^{-1})\quad\text{finite terms}$$
$$Z^-(\Delta,i\sqrt{-Q^2}/2,m_\ell,E_\ell)+Z^+(\Delta',i\sqrt{-Q^2}/2,m_\ell,-E'_\ell)\sim\mathcal{O}(M_N^{-1})\quad\text{finite terms}$$

- **@Leading order** [i.e., $\mathcal{O}(\alpha M_N^0)$]: Results is IR-finite ($E'_\ell = E_\ell$ & $p'_\ell = p_\ell$ strictly):

$$\delta^{(a+b)}_{\rm TPE}(Q^2)\Big|_{\rm LO} = \frac{2\mathcal{R}e \sum_{spins}\left(\mathcal{M}^*_{\rm Born}\,\mathcal{M}^{(a+b)}_{\rm TPE}\right)}{\sum_{spins}|\mathcal{M}_{\rm Born}|^2}\Bigg|_{E'_\ell=E_\ell\ \&\ p'_\ell=p_\ell}$$

$$= 32\pi\alpha E_\ell\left[\frac{Q^2}{Q^2+4E_\ell^2}\right]\mathcal{R}e\left[I(Q|1,1,0,1)\right]_{\rm LO}$$

$$= \pi\alpha\frac{\sqrt{-Q^2}}{2E_\ell}\left[\frac{1}{1+\frac{Q^2}{4E_\ell^2}}\right]$$

$\hookrightarrow$ Close resemblance to the well-known McKinley-Feshbach (1948) potential scattering result

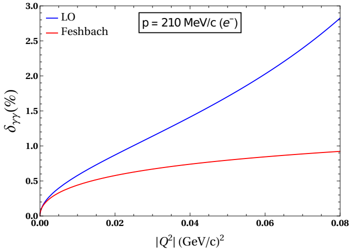
$$\delta_{\rm FM}(Q^2) = \pi\alpha\frac{\sqrt{-Q^2}}{2E_\ell}\left[\frac{1-\frac{\sqrt{-Q^2}}{2p_\ell}}{1+\frac{Q^2}{4E_\ell^2}}\right]$$

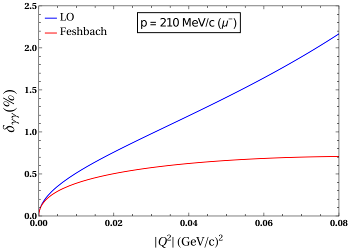
- **@Next-to-leading order** [i.e., $\mathcal{O}(\alpha M_N^{-1})$]: Result is IR-divergent, arising only from diagrams (a) & (b)

$$\delta^{(a+b)}_{\rm TPE}(Q^2)\Big|_{\rm NLO} = -\frac{\alpha Q^2}{2\pi M_N E_\ell \beta_\ell^2}\left[\frac{1}{\epsilon}-\gamma_E+\ln\left(\frac{4\pi\mu^2}{m_\ell^2}\right)\right]\left\{1+\left(\beta_\ell-\frac{1}{\beta_\ell}\right)\ln\sqrt{\frac{1+\beta_\ell}{1-\beta_\ell}}\right\} + \text{IR-finite terms}$$

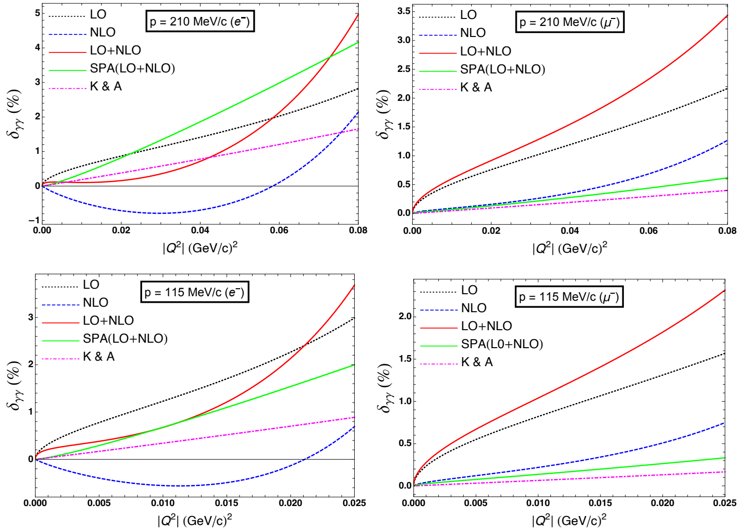
$$\delta^{(c+d+e+f+g+h+i)}_{\rm TPE}(Q^2)\Big|_{\rm NLO} = \text{IR-finite terms}$$

LO Results relevant to MUSE kinematic range: e-p and μ-p cases





Full NLO results relevant to MUSE kinematic range: e-p and μ-p cases



K & A: QED-inspired hadronic model calculation by Koshchii & Afanasev (2017)

$\hookrightarrow$ diagrammatic approach, with elastic proton TPE intermediate state, but invoking SPA

Outlook: Moving forward ...

- The virtual TPE $\delta_{\rm TPE}(Q^2) \rightarrow$ IR-divergent & "unphysical" by themselves
- Rather (TPE + Soft-Bremsstrahlung)$_{\text{charge-odd}}$ combination $\bar{\delta}^{(\rm odd)}_{\rm TPE+Brem.}(Q^2)$ is "physical":

$$\bar{\delta}^{(\rm odd)}_{\rm TPE+Brem}(Q^2) = \delta_{\rm TPE}(Q^2) + \delta^{(\rm odd)}_{{\rm soft}\,\gamma^*}(Q^2) \rightarrow \text{IR-finite}$$

- **Total cross section** including NNLO chiral corrections in HBχPT Talukdar et al. (2021):

$$\left[\frac{d\sigma_{\rm elastic}(Q^2)}{d\Omega_\ell}\right]^{(\pm)} = \left[\frac{d\sigma_{\rm elastic}(Q^2)}{d\Omega_\ell}\right]_{\rm LO}\left\{1 + \delta^{(\rm even)}_{\chi}(Q^2) + \pm\,\bar{\delta}^{(\rm odd)}_{\rm TPE+Brem.}(Q^2) + \bar{\delta}^{(\rm even)}_{\rm Virt.+Brem.}(Q^2)\right\}$$

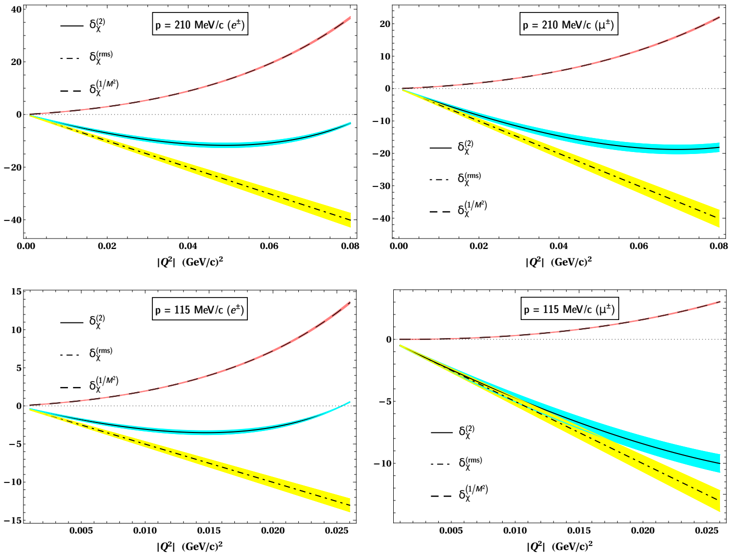
- **OPE cross section** including NNLO chiral corrections:

$$\left[\frac{d\sigma_{\rm elastic}(Q^2)}{d\Omega_\ell}\right]_{\rm OPE+Had} = \left[\frac{d\sigma_{\rm elastic}(Q^2)}{d\Omega_\ell}\right]_{\rm LO}\left\{1 + \delta^{(\rm even)}_{\chi}(Q^2)\right\}$$

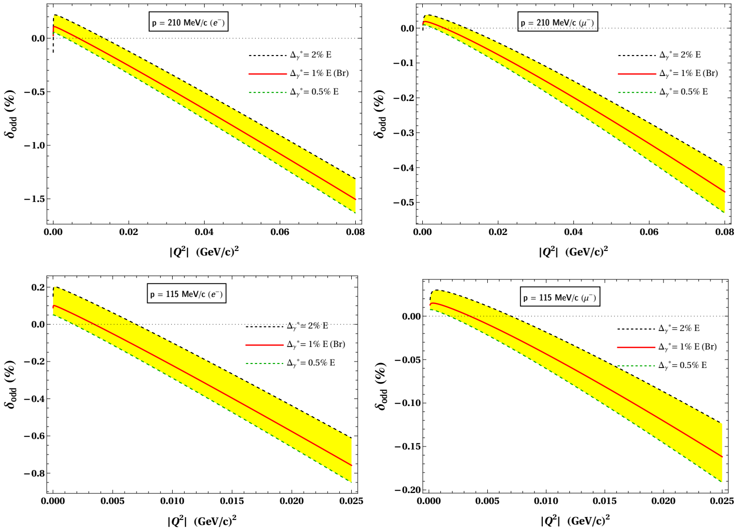
- **Charge Asymmetry:**

$$\mathcal{A} \approx \frac{\left[\frac{d\sigma_{\rm elastic}(Q^2)}{d\Omega_\ell}\right]^{(+)} - \left[\frac{d\sigma_{\rm elastic}(Q^2)}{d\Omega_\ell}\right]^{(-)}}{2\left[\frac{d\sigma_{\rm elastic}(Q^2)}{d\Omega_\ell}\right]_{\rm OPE+Had}} = \frac{\bar{\delta}^{(\rm odd)}_{\rm TPE+Brem}(Q^2)}{1 + \delta^{(\rm even)}_{\chi}(Q^2)}$$

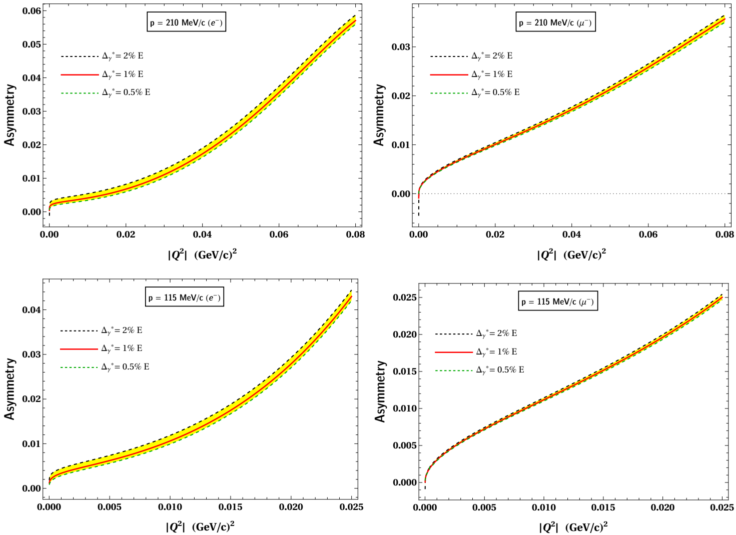
NNLO Chiral corrections in HBχPT Talukdar et al. (2021)



NLO Charge-odd Soft-bremsstrahlung (Preliminary)



NLO Charge Asymmetry (Preliminary)



THANK YOU