Deliveries from Effective Field Theories

Antonio Pineda

Universitat Autònoma de Barcelona & IFAE

Workshop on NREC 2024, 09-05-2024

Observables: $ep \to ep$, $\mu p \to \mu p$, $\gamma p \to \gamma p$, atomic physics, muonic atoms, ... at low energies

Scales (and ratios):

$m_p \sim \Lambda_\chi$

$m_\mu \sim m_\pi \sim m_r = \frac{m_\mu m_p}{m_p + m_\mu}$

$m_r \alpha \sim m_e$

$\ldots$

$Q^2 \to 0$

Expansion parameters, ratios between scales, mainly:

$$\frac{m_\pi}{m_p} \sim \frac{m_\mu}{m_p} \sim \frac{1}{9}$$

$$\frac{m_r \alpha}{m_r} \sim \frac{m_r \alpha^2}{m_r \alpha} \sim \alpha \sim \frac{1}{137}$$

Tool: Effective Field Theories $\equiv$ Factorization

Why?: There is a hierarchy of different scales

EFTs are especially useful in these situations.

1) Perturbative calculations much easier and systematic.

2) Nonperturbative information is parameterized in a model independent way.

3) Power counting.

4) Connection with QCD/QED.

Observables: $ep \to ep$, $\mu p \to \mu p$, $\gamma p \to \gamma p$, atomic physics, muonic atoms, ... at low energies

Scales (and ratios):

$m_p \sim \Lambda_\chi$

$m_\mu \sim m_\pi \sim m_r = \frac{m_\mu m_p}{m_p + m_\mu}$

$m_r \alpha \sim m_e$

$\ldots$

$Q^2 \to 0$

Expansion parameters, ratios between scales, mainly:

$$\frac{m_\pi}{m_p} \sim \frac{m_\mu}{m_p} \sim \frac{1}{9}$$

$$\frac{m_r \alpha}{m_r} \sim \frac{m_r \alpha^2}{m_r \alpha} \sim \alpha \sim \frac{1}{137}$$

Tool: Effective Field Theories $\equiv$ Factorization

Why?: There is a hierarchy of different scales

EFTs are especially useful in these situations.

1) Perturbative calculations much easier and systematic.

2) Nonperturbative information is parameterized in a model independent way.

3) Power counting.

4) Connection with QCD/QED.

Observables: $ep \to ep$, $\mu p \to \mu p$, $\gamma p \to \gamma p$, atomic physics, muonic atoms, ... at low energies

Scales (and ratios):

$m_p \sim \Lambda_\chi$

$m_\mu \sim m_\pi \sim m_r = \frac{m_\mu m_p}{m_p + m_\mu}$

$m_r \alpha \sim m_e$

$\dots$

$Q^2 \to 0$

Expansion parameters, ratios between scales, mainly:

$$\frac{m_\pi}{m_p} \sim \frac{m_\mu}{m_p} \sim \frac{1}{9}$$

$$\frac{m_r\alpha}{m_r} \sim \frac{m_r\alpha^2}{m_r\alpha} \sim \alpha \sim \frac{1}{137}$$

Tool: Effective Field Theories $\equiv$ Factorization

Why?: There is a hierarchy of different scales

EFTs are especially useful in these situations.

1) Perturbative calculations much easier and systematic.

2) Nonperturbative information is parameterized in a model independent way.

3) Power counting.

4) Connection with QCD/QED.

Observables: $ep \to ep$, $\mu p \to \mu p$, $\gamma p \to \gamma p$, atomic physics, muonic atoms, ... at low energies

Scales (and ratios):

$m_p \sim \Lambda_\chi$

$m_\mu \sim m_\pi \sim m_r = \frac{m_\mu m_p}{m_p + m_\mu}$

$m_r \alpha \sim m_e$

$\ldots$

$Q^2 \to 0$

Expansion parameters, ratios between scales, mainly:

$$\frac{m_\pi}{m_p} \sim \frac{m_\mu}{m_p} \sim \frac{1}{9}$$

$$\frac{m_r \alpha}{m_r} \sim \frac{m_r \alpha^2}{m_r \alpha} \sim \alpha \sim \frac{1}{137}$$

Tool: Effective Field Theories $\equiv$ Factorization

Why?: There is a hierarchy of different scales

EFTs are especially useful in these situations.

1) Perturbative calculations much easier and systematic.

2) Nonperturbative information is parameterized in a model independent way.

3) Power counting.

4) Connection with QCD/QED.

$$\text{HBET} \longrightarrow E \sim m_\pi$$
$$\text{NRQED} \longrightarrow E \sim m_l\alpha$$
$$\text{pNRQED} \longrightarrow E \sim m_l\alpha^2$$

$$HBET(m_\pi/m_\mu) \to NRQED(m_\mu\alpha) \to pNRQED$$

1) Matching HBET to NRQED. Integrating out the hard scale, $m_\mu \sim m_\pi$
HBET Feynman diagrams $\leftarrow$
2) Matching NRQED to pNRQED. Integrating out the soft scale, $m_\mu v$
Potential = Wilson loops = HQET-like Feynman diagrams $\leftarrow$

Effective Field Theory: Non-relativistic protons, photons and (non-relativistic) electron/muons.

$$\text{HBET} \longrightarrow E \sim m_\pi$$
$$\text{NRQED} \longrightarrow E \sim m_l\alpha$$
$$\text{pNRQED} \longrightarrow E \sim m_l\alpha^2$$

$$HBET(m_\pi/m_\mu) \rightarrow NRQED(m_\mu\alpha) \rightarrow pNRQED$$

1) Matching HBET to NRQED. Integrating out the hard scale, $m_\mu \sim m_\pi$
HBET Feynman diagrams $\leftarrow$
2) Matching NRQED to pNRQED. Integrating out the soft scale, $m_\mu v$
Potential = Wilson loops = HQET-like Feynman diagrams $\leftarrow$

Effective Field Theory: Non-relativistic protons, photons and (non-relativistic) electron/muons.

HBET (m_π)

$$\mathcal{L}_{HBET} = \mathcal{L}_\gamma + \mathcal{L}_l + \mathcal{L}_\pi + \mathcal{L}_{l\pi} + \mathcal{L}_{(N,\Delta)} + \mathcal{L}_{(N,\Delta)l} + \mathcal{L}_{(N,\Delta)\pi} + \mathcal{L}_{(N,\Delta)l\pi},$$

$$\mathcal{L}_\gamma = -\frac{1}{4}F^2 + \frac{d_2}{m_p^2}F_{\mu\nu}D^2F^{\mu\nu} + \cdots$$

$$\mathcal{L}_\pi = \frac{F_\pi^2}{4}\mathrm{Tr}\left[D_\mu U D^\mu U\right] + \cdots \qquad U = u^2 = e^{i\frac{\Pi}{F_\pi}}$$

$$\mathcal{L}_N = N^\dagger(iv^\mu\nabla_\mu + g_A u_\mu S^\mu)N + \cdots + (\Delta) + \cdots - e\frac{c_D}{m_p^2}N_p^\dagger \boldsymbol{\nabla}\cdot\mathbf{E}N_p$$

$$D_\mu = \partial_\mu + ieQA_\mu \qquad \nabla_\mu = \partial_\mu + \Gamma_\mu \qquad u_\mu = iu^\dagger(\nabla_\mu U)u$$

$$\Gamma_\mu = \frac{1}{2}\left\{u^\dagger(\partial_\mu + ieQA_\mu)u + u(\partial_\mu + ieQA_\mu)u^\dagger\right\}$$

$$\mathcal{L}_{N,l} = \frac{1}{m_p^2}\sum_i c_{3,R}^{pl_i}\bar{N}_p\gamma^0 N_p\bar{l}_i\gamma^0 l_i + \frac{1}{m_p^2}\sum_i c_{4,R}^{pl_i}\bar{N}_p\gamma^j N_p\bar{l}_i\gamma_j l_i$$

$$\delta\mathcal{L} = \cdots + \frac{d_2}{m_p^2}F_{\mu\nu}D^2F^{\mu\nu} - e\frac{c_D}{m_p^2}N_p^\dagger\boldsymbol{\nabla}\cdot\mathbf{E}N_p - \frac{c_3}{m_p^2}N_p^\dagger N_p\mu^\dagger\mu + \frac{c_4}{m_p^2}N_p^\dagger\sigma N_p\mu^\dagger\sigma\mu$$

HBET (m_π)

$$\mathcal{L}_{HBET} = \mathcal{L}_\gamma + \mathcal{L}_l + \mathcal{L}_\pi + \mathcal{L}_{l\pi} + \mathcal{L}_{(N,\Delta)} + \mathcal{L}_{(N,\Delta)l} + \mathcal{L}_{(N,\Delta)\pi} + \mathcal{L}_{(N,\Delta)l\pi},$$

$$\mathcal{L}_\gamma = -\frac{1}{4}F^2 + \frac{d_2}{m_p^2}F_{\mu\nu}D^2F^{\mu\nu} + \cdots$$

$$\mathcal{L}_\pi = \frac{F_\pi^2}{4}\mathrm{Tr}\left[D_\mu U D^\mu U\right] + \cdots \qquad U = u^2 = e^{i\frac{\Pi}{F_\pi}}$$

$$\mathcal{L}_N = N^\dagger(iv^\mu\nabla_\mu + g_A u_\mu S^\mu)N + \cdots + (\Delta) + \cdots - e\frac{c_D}{m_p^2}N_p^\dagger \boldsymbol{\nabla}\cdot\mathbf{E}N_p$$

$$D_\mu = \partial_\mu + ieQA_\mu \qquad \nabla_\mu = \partial_\mu + \Gamma_\mu \qquad u_\mu = iu^\dagger(\nabla_\mu U)u$$

$$\Gamma_\mu = \frac{1}{2}\left\{u^\dagger(\partial_\mu + ieQA_\mu)u + u(\partial_\mu + ieQA_\mu)u^\dagger\right\}$$

$$\mathcal{L}_{N,l} = \frac{1}{m_p^2}\sum_i c_{3,R}^{pl_i}\bar{N}_p\gamma^0 N_p\bar{l}_i\gamma^0 l_i + \frac{1}{m_p^2}\sum_i c_{4,R}^{pl_i}\bar{N}_p\gamma^j N_p\bar{l}_i\gamma_j l_i$$

$$\delta\mathcal{L} = \cdots + \frac{d_2}{m_p^2}F_{\mu\nu}D^2F^{\mu\nu} - e\frac{c_D}{m_p^2}N_p^\dagger\boldsymbol{\nabla}\cdot\mathbf{E}N_p - \frac{c_3}{m_p^2}N_p^\dagger N_p\mu^\dagger\mu + \frac{c_4}{m_p^2}N_p^\dagger\boldsymbol{\sigma}N_p\mu^\dagger\boldsymbol{\sigma}\mu$$

NRQED

$$iD_0 = i\partial_0 + Z_p e A^0,\ i\mathbf{D} = i\boldsymbol{\nabla} - Z_p e\mathbf{A}$$

$$\mathcal{L}_{\rm NRQED} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + +\frac{d_2}{m_p^2}F_{\mu\nu}D^2F^{\mu\nu}$$

$$+\psi_p^\dagger\left\{iD_0 + \frac{c_k}{2m_p}\mathbf{D}^2 + \frac{c_4}{8m_p^3}\mathbf{D}^4 + \frac{c_F^{(p)}}{2m_p}\boldsymbol{\sigma}\cdot e\mathbf{B} + \frac{c_D^{(p)}}{8m_p^2}(\mathbf{D}\cdot e\mathbf{E} - e\mathbf{E}\cdot\mathbf{D})\right.$$

$$\left. +i\frac{c_S^{(p)}}{8m_p^2}\boldsymbol{\sigma}\cdot(\mathbf{D}\times e\mathbf{E} - e\mathbf{E}\times\mathbf{D}) + c_{A_1}^{(p)}e^2\frac{\mathbf{B}^2-\mathbf{E}^2}{8m_p^3} - c_{A_2}^{(p)}e^2\frac{\mathbf{E}^2}{8m_p^3}\right\}\psi_p$$

$$+(\text{leptons})$$

$$-\frac{c_3^{(pe)}}{m_p m_e}\psi_p^\dagger\psi_p\psi_e^\dagger\psi_e + \frac{c_4^{(pe)}}{m_p m_e}\psi_p^\dagger\boldsymbol{\sigma}\psi_p\psi_e^\dagger\boldsymbol{\sigma}\psi_e + \cdots .$$

Potential Non-Relativistic QED

$$\left.\begin{array}{l}\left(i\partial_0 - \dfrac{\mathbf{p}^2}{2m_r} - \dfrac{\alpha}{r}\right)\psi(\mathbf{r}) = 0 \\ +\text{corrections to the potential} \\ +\text{interaction with ultrasoft photons}\end{array}\right\} \text{potential NRQED} \qquad E \sim mv^2$$

3) Observable: Spectrum or decays

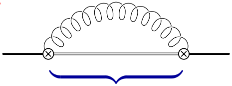
Corrections to the Green Function ($h_s^{(0)} = \mathbf{p}^2/m + V_s^{(0)}$)

$$G_s(E) = P_s \frac{1}{h_s^{(0)} - H_I - E} P_s = G_s^{(0)} + \delta G_s \qquad G_s^{(0)}(E) = \frac{1}{h_s^{(0)} - E}$$

A) Ultrasoft loops (lamb shift-like): $\mathbf{x} \cdot \mathbf{E} \leftarrow$

B) Quantum mechanics perturbation theory$\leftarrow$

δV



$1/(E - V_s^{(0)} - \mathbf{p}^2/m)$

Dictionary (relation Wilson coefficients with low energy constants):
$c_F^{(p)} \to \mu_p$ anomalous magnetic moment (low energy constant)
$c_D \to r_p$ proton radius (quasi low energy constant)
$c_{A_i}^{(p)} \to \alpha_E, \beta_M$ Proton polarizabilities (quasi low energy constant)
$c_{3/4}^{(pe)} \to$ Two-photon exchange ...

Definition of the proton radius

$$\langle p', s|J^\mu|p, s\rangle = \bar{u}(p')\left[F_1(q^2)\gamma^\mu + iF_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m_p}\right]u(p)\,,$$

$$F_i(q^2) = F_i + \frac{q^2}{m_p^2}F_i' + \ldots$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2}F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2).$$

$$r_p^2 = 6\frac{d}{dq^2}G_{E,p}(q^2)|_{q^2=0}$$

Definition of the proton radius

$$\langle p', s|J^\mu|p, s\rangle = \bar{u}(p')\left[F_1(q^2)\gamma^\mu + iF_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m_p}\right]u(p)\,,$$

$$F_i(q^2) = F_i + \frac{q^2}{m_p^2}F_i' + ...$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2}F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2).$$

$$r_p^2 = 6\frac{d}{dq^2}G_{E,p}(q^2)|_{q^2=0}$$

Definition of the proton radius

$$\langle p', s|J^\mu|p, s\rangle = \bar{u}(p')\left[F_1(q^2)\gamma^\mu + iF_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m_p}\right]u(p)\,,$$

$$F_i(q^2) = F_i + \frac{q^2}{m_p^2}F_i' + \ldots$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2}F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2).$$

$$r_p^2 = 6\frac{d}{dq^2}G_{E,p}(q^2)|_{q^2=0}$$

Definition of the proton radius

$$\langle p', s|J^\mu|p, s\rangle = \bar{u}(p')\left[F_1(q^2)\gamma^\mu + iF_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m_p}\right]u(p)\,,$$

$$F_i(q^2) = F_i + \frac{q^2}{m_p^2}F_i' + \ldots$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2}F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2).$$

$$r_p^2 = 6\frac{d}{dq^2}G_{E,p}(q^2)|_{q^2=0}$$

Definition of the proton radius

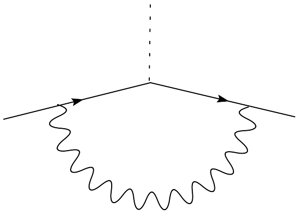
$$\langle p', s|J^\mu|p, s\rangle = \bar{u}(p')\left[F_1(q^2)\gamma^\mu + iF_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m_p}\right]u(p)\,,$$

$$F_i(q^2) = F_i + \frac{q^2}{m_p^2}F_i' + \ldots$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2}F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2).$$

$$r_p^2(\nu) = 6\frac{d}{dq^2}G_{E,p}(q^2)|_{q^2=0}$$

Infrared divergent! $\rightarrow$ Wilson coefficient



Definition of the proton radius

$$\langle p', s|J^\mu|p, s\rangle = \bar{u}(p')\left[F_1(q^2)\gamma^\mu + iF_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m_p}\right]u(p)\,,$$

$$F_i(q^2) = F_i + \frac{q^2}{m_p^2}F_i' + \ldots$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2}F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2).$$

$$r_p^2(\nu) = 6\frac{d}{dq^2}G_{E,p}(q^2)|_{q^2=0} = \frac{3}{4}\frac{1}{m_p^2}\left(c_D^{(p)}(\nu) - 1\right)$$

$$c_D(\nu) = 1 + 2F_2 + 8F_1' = 1 + 8m_p^2\left.\frac{dG_{p,E}(q^2)}{d\,q^2}\right|_{q^2=0},$$

Standard definition (corresponds to the experimental number):

$$r_p^2 = \frac{3}{4}\frac{1}{m_p^2}\left(c_D(\nu) - c_{D,point-like}(\nu)\right)$$

$$c_{D,point-like} = 1 + \frac{\alpha}{\pi}\left(\frac{4}{3}\ln\frac{m_p^2}{\nu^2}\right)$$

Definition of the proton radius

$$\langle p', s|J^\mu|p, s\rangle = \bar{u}(p') \left[F_1(q^2)\gamma^\mu + iF_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m_p}\right] u(p)\,,$$

$$F_i(q^2) = F_i + \frac{q^2}{m_p^2}F_i' + \ldots$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2}F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2).$$

$$r_p^2(\nu) = 6\frac{d}{dq^2}G_{E,p}(q^2)|_{q^2=0} = \frac{3}{4}\frac{1}{m_p^2}\left(c_D^{(p)}(\nu) - 1\right)$$

$$c_D(\nu) = 1 + 2F_2 + 8F_1' = 1 + 8m_p^2\left.\frac{dG_{p,E}(q^2)}{d\,q^2}\right|_{q^2=0}\,,$$

Standard definition (corresponds to the experimental number):

$$r_p^2 = \frac{3}{4}\frac{1}{m_p^2}\left(c_D(\nu) - c_{D,point-like}(\nu)\right)$$

$$c_{D,point-like} = 1 + \frac{\alpha}{\pi}\left(\frac{4}{3}\ln\frac{m_p^2}{\nu^2}\right)$$

Definition of the proton radius

$$\langle p', s | J^\mu | p, s \rangle = \bar{u}(p') \left[F_1(q^2)\gamma^\mu + iF_2(q^2)\frac{\sigma^{\mu\nu} q_\nu}{2m_p} \right] u(p) \,,$$

$$F_i(q^2) = F_i + \frac{q^2}{m_p^2} F_i' + \ldots$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2} F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2).$$

$$r_p^2(\nu) = 6 \frac{d}{dq^2} G_{E,p}(q^2)|_{q^2=0} = \frac{3}{4} \frac{1}{m_p^2} \left(c_D^{(p)}(\nu) - 1 \right)$$

$$c_D(\nu) = 1 + 2F_2 + 8F_1' = 1 + 8m_p^2 \left. \frac{dG_{p,E}(q^2)}{d\,q^2} \right|_{q^2=0} \,,$$

Standard definition (corresponds to the experimental number):

$$r_p^2 = \frac{3}{4} \frac{1}{m_p^2} \left(c_D(\nu) - c_{D,point-like}(\nu) \right)$$

$$c_{D,point-like} = 1 + \frac{\alpha}{\pi} \left(\frac{4}{3} \ln \frac{m_p^2}{\nu^2} \right)$$

Definition of the neutron radius

$$\langle p', s|J^\mu|p, s\rangle = \bar{u}(p')\left[F_1(q^2)\gamma^\mu + iF_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m_p}\right]u(p)\,,$$

$$F_i(q^2) = F_i + \frac{q^2}{m_p^2}F_i' + \ldots$$

Definition of the neutron radius

$$\langle p', s|J^\mu|p, s\rangle = \bar{u}(p')\left[F_1(q^2)\gamma^\mu + iF_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m_p}\right]u(p)$$

$$F_i(q^2) = 0 + \frac{q^2}{m_p^2}F_i' + \ldots$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2}F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2).$$

$$r_n^2 = 6\frac{d}{dq^2}G_{n,E}(q^2)|_{q^2=0} = \frac{3}{4}\frac{1}{m_p^2}c_D^{(n)}$$

$$c_D = 0 + 2F_2 + 8F_1' = 0 + 8m_n^2\left.\frac{dG_{n,E}(q^2)}{d\,q^2}\right|_{q^2=0}$$

Standard definition (corresponds to the experimental number):

$$r_n^2 = \frac{3}{4}\frac{1}{m_n^2}c_D$$

Neutron-lepton scattering length = REAL low energy constant

$$b_{nl} = \frac{1}{4m_n}\left(\alpha c_D - \frac{2}{\pi}c^{nl}_{3,NR}\right) \sim D_d^{(n)had}$$

It is not proportional to the radius

Definition of the neutron radius

$$\langle p', s|J^\mu|p, s\rangle = \bar{u}(p')\left[F_1(q^2)\gamma^\mu + iF_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m_p}\right]u(p)$$

$$F_i(q^2) = 0 + \frac{q^2}{m_p^2}F_i' + \dots$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2}F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2).$$

$$r_n^2 = 6\frac{d}{dq^2}G_{n,E}(q^2)|_{q^2=0} = \frac{3}{4}\frac{1}{m_p^2}c_D^{(n)}$$

$$c_D = 0 + 2F_2 + 8F_1' = 0 + 8m_n^2\left.\frac{dG_{n,E}(q^2)}{d\,q^2}\right|_{q^2=0}$$

Standard definition (corresponds to the experimental number):

$$r_n^2 = \frac{3}{4}\frac{1}{m_n^2}c_D$$

Neutron-lepton scattering length = REAL low energy constant

$$b_{nl} = \frac{1}{4m_n}\left(\alpha c_D - \frac{2}{\pi}c_{3,NR}^{nl}\right) \sim D_d^{(n)had}$$

It is not proportional to the radius

Hadronic corrections

$$\frac{\delta V^{(2)}(r)}{m_\mu^2} \to \frac{1}{m_p^2} D_d^{had.} \delta^3(\mathbf{r}) \to \Delta E \sim \frac{1}{m_p^2} D_d^{had.} (m_r \alpha)^3 \frac{\delta_{l0}}{n^3}$$

$$D_d^{(p\mu)} = -c_3^{p\mu} - 16\pi\alpha d_2 + \frac{\pi\alpha}{2} c_D^{(p)}$$

$$\frac{\delta V^{(2)}(r)}{m_\mu^2} \to \frac{1}{m_p^2} D_d^{had.} (\mathbf{S}_1 + \mathbf{S}_2)^2 \delta^3(\mathbf{r})$$

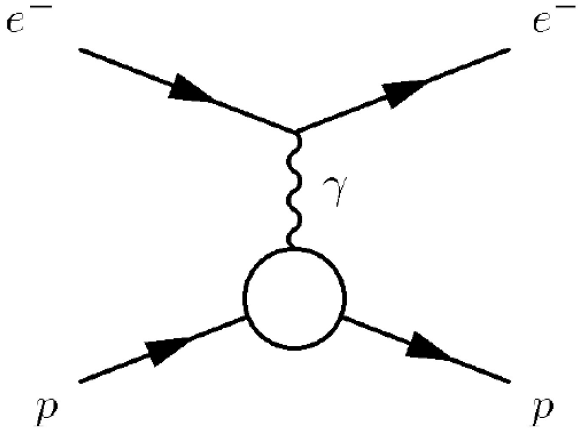
$$D_s^{had.} = 2 c_4^{p\mu}$$

c_3, c_4, d_2, $c_D^{(p)}$, ... matching coefficients of NRQED.

$$\delta\mathcal{L} = \cdots + \frac{d_2}{m_p^2} F_{\mu\nu} D^2 F^{\mu\nu} - e\frac{c_D}{m_p^2} N_p^\dagger \boldsymbol{\nabla}\cdot\mathbf{E} N_p - \frac{c_3^{p\mu}}{m_p^2} N_p^\dagger N_p \mu^\dagger \mu + \frac{c_4}{m_p^2} N_p^\dagger \boldsymbol{\sigma} N_p \mu^\dagger \boldsymbol{\sigma} \mu$$

Field theory concept of minimal basis!!

electron-proton scattering at $\mathcal{O}(\alpha)$



electron-proton scattering at $\mathcal{O}(\alpha)$

laboratory frame: $p = (M, 0)$, $k = (E, \mathbf{k})$, $p' = (E'_p, \mathbf{k} - \mathbf{k}')$, $k' = (E', \mathbf{k}')$, and the lepton scattering angle is $\theta_{\rm lab}$.

$$\left(\frac{d\sigma_{1\gamma}}{d\Omega}\right)_{\rm point-like} \equiv \frac{2M\alpha^2}{Q^2}\frac{|\mathbf{k}'|}{|\mathbf{k}|}\frac{\varepsilon}{1-\varepsilon_{\rm T}}\frac{1+\frac{\tau_p}{\varepsilon}}{M+E-E'\frac{|\mathbf{k}|}{|\mathbf{k}'|}\cos\theta_{\rm lab}},$$

with kinematic variables

$$\varepsilon_{\rm T} = \frac{\nu^2 - M^4\tau_p(1+\tau_p)(1+2\varepsilon_0)}{\nu^2 + M^4\tau_p(1+\tau_p)(1-2\varepsilon_0)}, \qquad \varepsilon = \varepsilon_{\rm T} + \varepsilon_0(1-\varepsilon_{\rm T}), \qquad \varepsilon_0 = \frac{2m_\mu^2}{Q^2},$$

$$\left(\frac{d\sigma_{1\gamma}}{d\Omega}\right)_{\rm Sachs} = \frac{2M\alpha^2}{Q^2}\frac{|\mathbf{k}'|}{|\mathbf{k}|}\frac{\varepsilon}{1-\varepsilon_{\rm T}}\frac{G_E^2+\frac{\tau_p}{\varepsilon}G_M^2}{M+E-E'\frac{|\mathbf{k}|}{|\mathbf{k}'|}\cos\theta_{\rm lab}}.$$

To reproduce the point-like particle result, one has to make the replacement $G_E \to 1$ and $G_M \to 1$.

Nonrelativistic proton and lepton (Peset, Tomalak, Pineda)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm measured} = Z^2\left(\frac{d\sigma_{1\gamma}}{d\Omega}\right)_{\rm point-like} + \frac{d\sigma_{\rm Mott}}{d\Omega}\left[\delta^{(p)}_{\rm soft} + Z^2\left(\delta^{(\mu)}_{\rm soft}+\delta_{\rm VP}\right) + Z^3\left(\delta^{(p\mu)}_{\rm soft}+\delta_{\rm TPE}\right)\right.$$
$$\left.+\mathcal{O}(\alpha^2)+\mathcal{O}(\tau^2)\right].$$

$$\delta^{(p)}_{\rm soft} = \tau_p\left[\beta^2\left(c_F^{(p)\,2}-Z^2\right) - Z\left(c_D^{(p)\overline{\rm MS}}(\nu)-Z\right) + \frac{4}{3}\frac{Z^4\alpha}{\pi}\left(2\ln\frac{2\Delta E}{\nu}-\frac{5}{3}\right)\right],$$

$$\delta_{\rm VP} = 32\tau_\mu\left[d_2^{(\mu)} + \frac{m_\mu^2}{M^2}d_2 + \frac{m_\mu^2}{m_\tau^2}d_2^{(\tau)}\right]$$

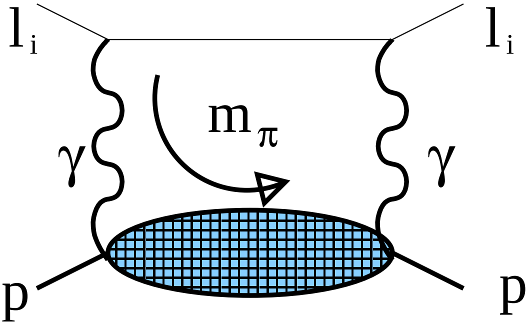
$$\delta^{\rm point-like}_{\rm TPE} = \delta_{\rm pot} + \delta_{\rm soft} + \delta^{\rm point-like}_{\rm hard}.$$

$$d_s(\nu) = -\frac{Z^2\alpha^2}{m_{l_i}^2-M^2}\left[m_{l_i}^2\left(\ln\frac{M^2}{\nu^2}+\frac{1}{3}\right) - M^2\left(\ln\frac{m_{l_i}^2}{\nu^2}+\frac{1}{3}\right)\right],$$

$$\delta^{\rm point-like}_{\rm hard} \longrightarrow \delta_{\rm hard} = -\frac{Q^2}{2Mm_\mu}\left[\frac{d_s(\nu)}{\pi\alpha} - \frac{m_\mu}{M}\frac{c_3^{\rm had}}{\pi\alpha}\right].$$

TWO-PHOTON EXCHANGE Spin-independent (c_3) correction

$c_3^{p\mu} =$



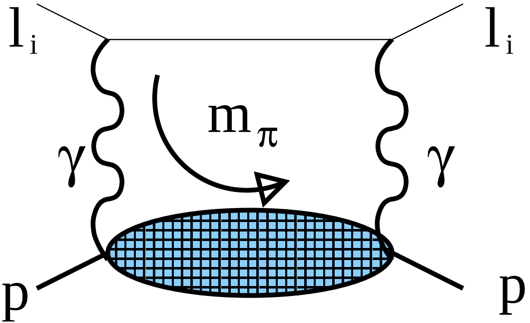
$$T^{\mu\nu} = i\int d^4x\, e^{iq\cdot x}\langle p,s|TJ^\mu(x)J^\nu(0)|p,s\rangle\,,$$

$$T^{\mu\nu} = \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)S_1(\rho,q^2)+\frac{1}{m_p^2}\left(p^\mu-\frac{m_p\rho}{q^2}q^\mu\right)\left(p^\nu-\frac{m_p\rho}{q^2}q^\nu\right)S_2(\rho,q^2)$$

$S_1 =??\qquad S_2 =??$

TWO-PHOTON EXCHANGE Spin-independent (c_3) correction

$c_3^{p\mu} =$

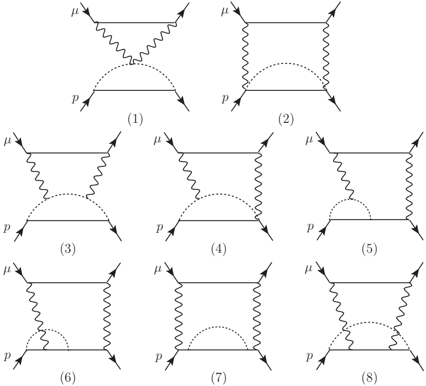


$$T^{\mu\nu} = i\int d^4x\, e^{iq\cdot x}\langle p,s|TJ^\mu(x)J^\nu(0)|p,s\rangle\,,$$

$$T^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) S_1(\rho,q^2) + \frac{1}{m_p^2}\left(p^\mu - \frac{m_p\rho}{q^2}q^\mu\right)\left(p^\nu - \frac{m_p\rho}{q^2}q^\nu\right) S_2(\rho,q^2)$$

$$S_1 = ?? \qquad S_2 = ??$$

TWO-PHOTON EXCHANGE Spin-independent (c_3) correction



m_μ extra suppression+χPT (Model independent)

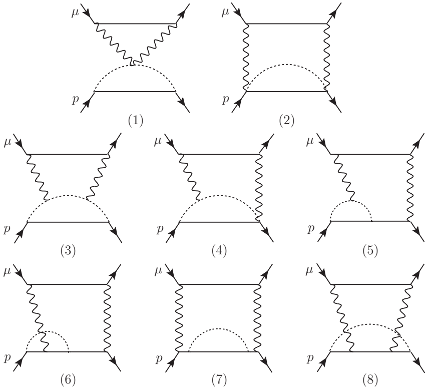
Power-like chiral enhanced ($\rightarrow \chi$PT can predict the leading order!)

$$c_3^{p\mu} \sim \alpha^2 \frac{m_\mu}{m_\pi} + \mathcal{O}\left(\alpha^2 \frac{m_\mu}{\Lambda_{QCD}}\right) \qquad \delta E \sim \mathcal{O}(m_\mu \alpha^5 \times \frac{m_\mu^2}{\Lambda_\chi^2} \times \frac{m_\mu}{m_\pi})$$

Error ($\Delta = M_\Delta - M_p \sim 300$ MeV): LO $\times \frac{m_\pi}{\Delta} \simeq$ LO $\times \frac{1}{2}$

$\rightarrow c_3^{p\mu} = \alpha^2 \frac{m_\mu}{m_\pi} 47.2(23.6)$

TWO-PHOTON EXCHANGE Spin-independent (c_3) correction

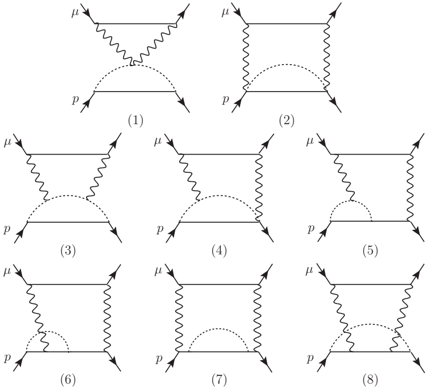


m_μ extra suppression+χPT (Model independent)
Power-like chiral enhanced ($\rightarrow$ χPT can predict the leading order!)

$$c_3^{p\mu} \sim \alpha^2 \frac{m_\mu}{m_\pi} + \mathcal{O}\left(\alpha^2 \frac{m_\mu}{\Lambda_{QCD}}\right) \qquad \delta E \sim \mathcal{O}(m_\mu \alpha^5 \times \frac{m_\mu^2}{\Lambda_\chi^2} \times \frac{m_\mu}{m_\pi})$$

Error ($\Delta = M_\Delta - M_p \sim 300$ MeV): LO $\times \frac{m_\pi}{\Delta} \simeq$ LO $\times \frac{1}{2}$
$\rightarrow c_3^{p\mu} = \alpha^2 \frac{m_\mu}{m_\pi} 47.2(23.6)$

TWO-PHOTON EXCHANGE Spin-independent (c_3) correction



m_μ extra suppression+χPT (Model independent)
Power-like chiral enhanced ($\to \chi$PT can predict the leading order!)

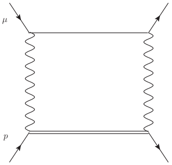
$$c_3^{p\mu} \sim \alpha^2 \frac{m_\mu}{m_\pi} + \mathcal{O}\left(\alpha^2 \frac{m_\mu}{\Lambda_{QCD}}\right) \qquad \delta E \sim \mathcal{O}(m_\mu \alpha^5 \times \frac{m_\mu^2}{\Lambda_\chi^2} \times \frac{m_\mu}{m_\pi})$$

Error ($\Delta = M_\Delta - M_p \sim 300$ MeV): LO $\times \frac{m_\pi}{\Delta} \simeq$ LO $\times \frac{1}{2}$
$\to c_3^{p\mu} = \alpha^2 \frac{m_\mu}{m_\pi} 47.2(23.6)$

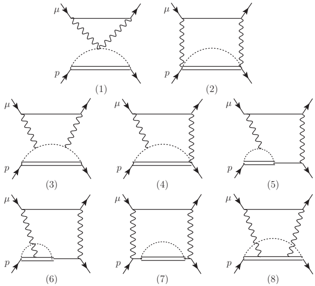
Large N_c. Including the Δ particle

Error:

$$\frac{m_\mu}{\Delta} \sim N_c \frac{m_\mu}{\Lambda_{QCD}} \to N_c \frac{m_\mu}{\Lambda_{QCD}} \sim \frac{1}{3}$$



+



$$c_3^{p\mu} \sim \alpha^2 \frac{m_\mu}{m_\pi}\left[1 + \# \frac{m_\pi}{\Delta} + \cdots\right] + \mathcal{O}\left(\alpha^2 \frac{m_\mu}{\Lambda_{QCD}}\right) = \alpha^2 \frac{m_\mu}{m_\pi}\begin{cases} 47.2(23.6) & (\pi), \\ 56.7(20.6) & (\pi + \Delta), \end{cases}$$

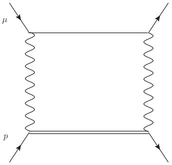
$$\Delta E_{\rm TPE} = 28.59(\pi) + 5.86(\pi\&\Delta) = 34.4(12.5)\mu{\rm eV} \quad ({\rm Peset\&Pineda})\,.$$

(Model dependent+DR: $\Delta E_{\rm TPE} = 33(2)\mu{\rm eV}$ (Birse-McGovern))

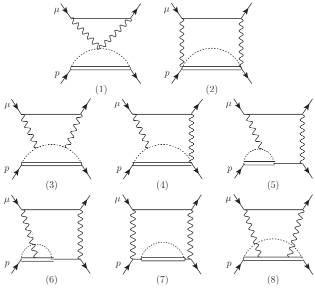
Large N_c. Including the Δ particle

Error:

$$\frac{m_\mu}{\Delta} \sim N_c \frac{m_\mu}{\Lambda_{QCD}} \to N_c \frac{m_\mu}{\Lambda_{QCD}} \sim \frac{1}{3}$$



$+$



$$c_3^{p\mu} \sim \alpha^2 \frac{m_\mu}{m_\pi}\left[1 + \# \frac{m_\pi}{\Delta} + \cdots\right] + \mathcal{O}\left(\alpha^2 \frac{m_\mu}{\Lambda_{QCD}}\right) = \alpha^2 \frac{m_\mu}{m_\pi}\begin{cases} 47.2(23.6) & (\pi), \\ 56.7(20.6) & (\pi + \Delta), \end{cases}$$

$$\Delta E_{\rm TPE} = 28.59(\pi) + 5.86(\pi \& \Delta) = 34.4(12.5)\mu{\rm eV} \quad ({\rm Peset\&Pineda})\,.$$

(Model dependent+DR: $\Delta E_{\rm TPE} = 33(2)\mu{\rm eV}$ (Birse-McGovern))

$$\Delta E_{\rm TPE} \sim m_\mu \alpha^5 \times \frac{m_\mu^2}{(4\pi F_\pi)^2} \times \frac{m_\mu}{m_\pi} \sum_{n=0}^{\infty} c_n (N_c \sqrt{m_q})^n$$

$$\frac{\#}{\sqrt{m_q}} + ? + ?\sqrt{m_q} + \cdots$$

plus large N_c

$$\frac{\#}{\sqrt{m_q}} + \left[\# N_c + ? + \frac{?}{N_c} + \cdots\right] + \left[\# N_c^2 + ?N_c + ? + \cdots\right]\sqrt{m_q} + \cdots$$

? $\rightarrow$ Size of the counterterm in HBET

TWO-PHOTON EXCHANGE Spin-dependent (c_4) corrections

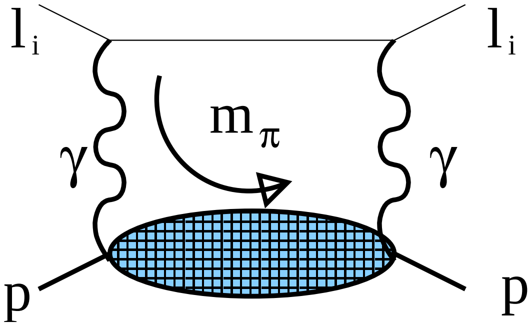


Figure: *Symbolic representation (plus permutations) of the spin-dependent correction.*

$$c_4^{pl} = -\frac{ig^4}{3}\int \frac{d^Dk}{(2\pi)^D}\frac{1}{k^2}\frac{1}{k^4-4m_l^2k_0^2}\left\{A_1(k_0,k^2)(k_0^2+2k^2)+3k^2\frac{k_0}{m_p}A_2(k_0,k^2)\right\}$$

Drell-Sullivan(67)

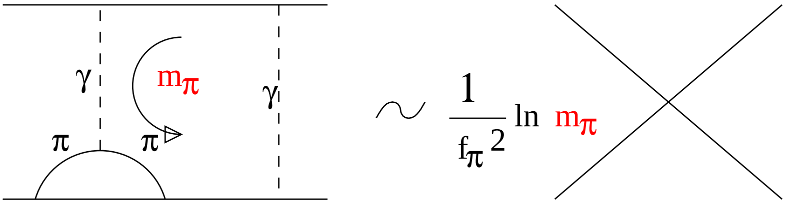
$$T^{\mu\nu} = i\int d^4x\, e^{iq\cdot x}\langle p,s|TJ^\mu(x)J^\nu(0)|p,s\rangle\,,$$

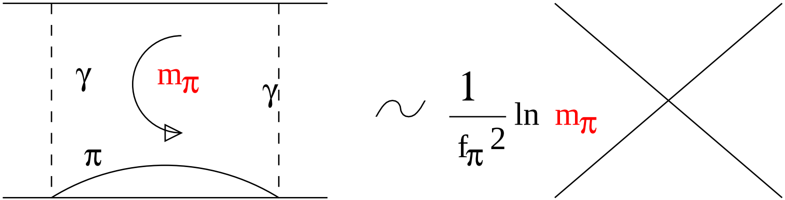
which has the following structure ($\rho = q\cdot p/m$):

$$\begin{aligned}
T^{\mu\nu} \;=\;& \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)S_1(\rho,q^2)\\
&+\frac{1}{m_p^2}\left(p^\mu-\frac{m_p\rho}{q^2}q^\mu\right)\left(p^\nu-\frac{m_p\rho}{q^2}q^\nu\right)S_2(\rho,q^2)\\
&-\frac{i}{m_p}\,\epsilon^{\mu\nu\rho\sigma}q_\rho s_\sigma A_1(\rho,q^2)\\
&-\frac{i}{m_p^3}\,\epsilon^{\mu\nu\rho\sigma}q_\rho\left((m_p\rho)s_\sigma-(q\cdot s)p_\sigma\right)A_2(\rho,q^2)
\end{aligned}$$

A_1, A_2 (χPT): Ji-Osborne; Peset-Pineda

Leading chiral logs to the hyperfine splitting





$$\delta V = 2\frac{c_4}{m_p^2}\mathbf{S}^2\delta^{(3)}(\mathbf{r})\,.$$

$$\delta E_{HF} \sim \mathcal{O}(m_\mu \alpha^5 \times \frac{m_\mu^2}{\Lambda_\chi^2} \times \ln m_\pi)$$

The leading chiral logs can be determined for Hydrogen and muonic hydrogen hyperfine splitting (Pineda).

$$\begin{aligned} c_4^{pl_i} &\simeq \left(1-\frac{\mu_p^2}{4}\right)\alpha^2 \ln \frac{m_{l_i}^2}{\nu^2} + \frac{b_{1,F}^2}{18}\alpha^2 \ln \frac{\Delta^2}{\nu^2} \\ &+ \frac{m_p^2}{(4\pi F_0)^2}\alpha^2 \frac{2}{3}\left(\frac{2}{3}+\frac{7}{2\pi^2}\right)\pi^2 g_A^2 \ln \frac{m_\pi^2}{\nu^2} \\ &+ \frac{m_p^2}{(4\pi F_0)^2}\alpha^2 \frac{8}{27}\left(\frac{5}{3}-\frac{7}{\pi^2}\right)\pi^2 g_{\pi N\Delta}^2 \ln \frac{\Delta^2}{\nu^2} \\ &\overset{(N_c\to\infty)}{\simeq} \alpha^2 \ln \frac{m_l^2}{\nu^2} + \frac{m_p^2}{(4\pi F_0)^2}\alpha^2 \pi^2 g_A^2 \ln \frac{m_\pi^2}{\nu^2} \,. \end{aligned}$$

$$E_{\rm HF} = 4\frac{c_4^{pl_i}}{m_p^2}\frac{1}{\pi}(\mu_{l_i p}\alpha)^3 \sim m_{l_i}\alpha^5 \frac{m_{l_i}^2}{m_p^2} \times (\ln m_q, \ln \Delta, \ln m_{l_i}) \,.$$

$$c_4^{pl_i} = c_{4,\rm R}^{pl_i} + c_{4,\rm point-like}^{pl_i} + c_{4,\rm Born}^{pl_i} + c_{4,\rm pol}^{pl_i} + \mathcal{O}(\alpha^3) \,.$$

$$\delta E_{HF} \sim \mathcal{O}(m_\mu \alpha^5 \times \frac{m_\mu^2}{\Lambda_\chi^2} \times \ln m_\pi)$$

The leading chiral logs can be determined for Hydrogen and muonic hydrogen hyperfine splitting (Pineda).

$$\begin{aligned}
c_4^{pl_i} \quad &\simeq \quad \left(1-\frac{\mu_p^2}{4}\right)\alpha^2 \ln\frac{m_{l_i}^2}{\nu^2} + \frac{b_{1,F}^2}{18}\alpha^2 \ln\frac{\Delta^2}{\nu^2} \\
&\quad + \frac{m_p^2}{(4\pi F_0)^2}\alpha^2 \frac{2}{3}\left(\frac{2}{3}+\frac{7}{2\pi^2}\right)\pi^2 g_A^2 \ln\frac{m_\pi^2}{\nu^2} \\
&\quad + \frac{m_p^2}{(4\pi F_0)^2}\alpha^2 \frac{8}{27}\left(\frac{5}{3}-\frac{7}{\pi^2}\right)\pi^2 g_{\pi N\Delta}^2 \ln\frac{\Delta^2}{\nu^2} \\
&\overset{(N_c\to\infty)}{\simeq} \quad \alpha^2 \ln\frac{m_l^2}{\nu^2} + \frac{m_p^2}{(4\pi F_0)^2}\alpha^2 \pi^2 g_A^2 \ln\frac{m_\pi^2}{\nu^2}\,.
\end{aligned}$$

$$E_{\rm HF} = 4\frac{c_4^{pl_i}}{m_p^2}\frac{1}{\pi}(\mu_{l_i p}\alpha)^3 \sim m_{l_i}\alpha^5 \frac{m_{l_i}^2}{m_p^2} \times (\ln m_q, \ln\Delta, \ln m_{l_i})\,.$$

$$c_4^{pl_i} = c_{4,\rm R}^{pl_i} + c_{4,\rm point-like}^{pl_i} + c_{4,\rm Born}^{pl_i} + c_{4,\rm pol}^{pl_i} + \mathcal{O}(\alpha^3)\,.$$

Hyperfine: Hydrogen and muonic hydrogen

Experiment:

$$E^{\rm exp}_{\rm hyd,HF}(1S) = 1420.405751768(1)\ {\rm MHz}\,,$$

$$E^{\rm exp}_{\mu p,\rm HF}(2S) = 22.8089(51)\ {\rm meV}\,.$$

Theory:

$$\frac{\delta V^{(2)}(r)}{m_\mu^2} \rightarrow \frac{1}{m_p^2} D_d^{had.} (\mathbf{S}_1 + \mathbf{S}_2)^2 \delta^3(\mathbf{r})$$

$$D_s^{had.} = 2c_4$$

Fixing c_4^{pe}.Hydrogen. By fixing the scale $\nu = m_\rho$ we obtain the following number for the total sum in the SU(2) case:

$$E_{\rm HF,logarithms}(m_\rho) = -0.031\ {\rm MHz}\,,$$

which accounts for approximately 2/3 of the difference between theory (pure QED) and experiment.

$$E_{\rm HF}(QED) - E_{\rm HF}(exp) = -0.046\ {\rm MHz}.$$

What is left gives the expected size of the counterterm. Experimentally what we have is $c^{pe}_{4,NR} = -48.69(3)\alpha^2$ and $c^{pe}_{4,R}(m_\rho) \simeq c^{p}_{4,R}(m_\rho) \simeq -16\alpha^2$.

Hyperfine: Hydrogen and muonic hydrogen

Experiment:

$$E^{\rm exp}_{\rm hyd,HF}(1S) = 1420.405751768(1)\ {\rm MHz}\,,$$

$$E^{\rm exp}_{\mu p,\rm HF}(2S) = 22.8089(51)\ {\rm meV}\,.$$

Theory:

$$\frac{\delta V^{(2)}(r)}{m_\mu^2} \to \frac{1}{m_p^2} D_d^{had.} (\mathbf{S}_1 + \mathbf{S}_2)^2 \delta^3(\mathbf{r})$$

$$D_s^{had.} = 2c_4$$

Fixing c_4^{pe}.Hydrogen. By fixing the scale $\nu = m_\rho$ we obtain the following number for the total sum in the SU(2) case:

$$E_{\rm HF,logarithms}(m_\rho) = -0.031\ {\rm MHz}\,,$$

which accounts for approximately 2/3 of the difference between theory (pure QED) and experiment.

$$E_{\rm HF}(QED) - E_{\rm HF}(exp) = -0.046\ {\rm MHz.}$$

What is left gives the expected size of the counterterm. Experimentally what we have is $c_{4,NR}^{pe} = -48.69(3)\alpha^2$ and $c_{4,R}^{pe}(m_\rho) \simeq c_{4,R}^{p}(m_\rho) \simeq -16\alpha^2$.

The proton radius in *ep* scattering from χPT

Hessels, Horbatsch, Pineda

$$G_E(Q^2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} Q^{2n} \langle r^{2n} \rangle$$

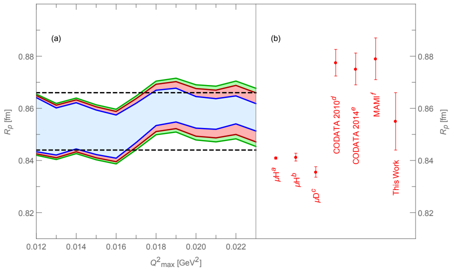
- Extrapolation from $|\mathbf{q}| \sim 100$ MeV to $|\mathbf{q}| = 0$
- dependence on the fitting functions: normalization factors, full data set ...

Higher moments diverge in the chiral limit

$$\langle r^{2k} \rangle \sim m_\pi^{2-2k}$$

Extrapolation controlled by χPT (at low Q^2): $r_p \sim 0.855$.

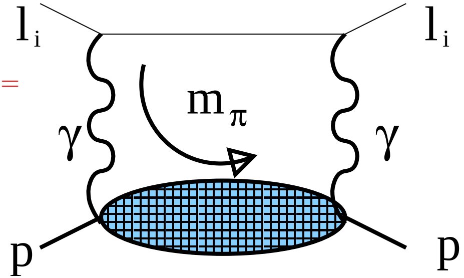
Bigger values for the moments produce larger values of r_p.



Relating hydrogen and muonic hydrogen in the UV (hyperfine)

$$c_{4,\mathrm{TPE}}^{p\mu} = c_{4,\mathrm{TPE}}^{pe} + [c_{4,\mathrm{TPE}}^{p\mu} - c_{4,\mathrm{TPE}}^{pe}](\chi PT) + \mathcal{O}(\alpha)\,.$$

$c_3^{(p\mu)/(pe)} =$



$$[c_{4,\mathrm{TPE}}^{p\mu} - c_{4,\mathrm{TPE}}^{pe}](\chi PT) = 3.68(72) \qquad \text{(Peset, Pineda)}$$

c_4^{pe} fixed from hydrogen $\rightarrow c_4^{p\mu}$

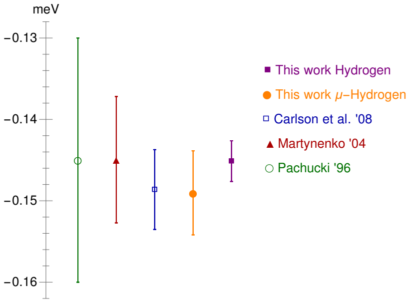


Figure: Two-photon exchange contribution to the hyperfine splitting of the 2S muonic hydrogen. Peset-Pineda

Variation of this idea has later been applied using DR (Tomalak). Error $\sim 1/2$.

(Non) Asymptotic Freedom

Hydrogen:

$$\tilde{V}^{(0)} \equiv -4\pi Z_\mu Z_p \alpha_V(k) \frac{1}{\mathbf{k}^2} = -4\pi Z_e Z_p \alpha \frac{1}{\mathbf{k}^2} \longrightarrow \beta(\alpha) = \nu \frac{d}{d\nu} \alpha_V = 0 \longrightarrow \Delta E_L = 0$$

Muonic atoms ($|\mathbf{k}| \gg \mathbf{m_e}$):

$$\tilde{V}^{(0)} \equiv -4\pi Z_\mu Z_p \alpha_V(k) \frac{1}{\mathbf{k}^2},$$

$$\alpha_V(k) = \alpha \left(1 + \frac{\alpha}{4\pi}\beta_0^{QED} \ln \frac{m_e^2}{\mathbf{k}^2} + \mathcal{O}(\alpha^2)\right) \qquad \beta_0^{QED} = -\frac{4}{3} T_F < 0$$

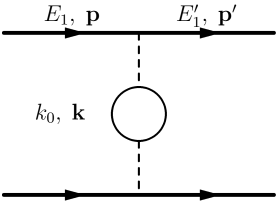


Figure: *Leading correction to the Coulomb potential due to the electron vacuum polarization.* $\mathbf{k} = \mathbf{p} - \mathbf{p}'$ *and* $k_0 = E_1 - E_1'$.

$$\beta(\alpha) = -2\alpha \left\{\beta_0^{QED}\frac{\alpha}{4\pi} + \mathcal{O}(\alpha)\right\} \longrightarrow E(1P) - E(2S) \propto -\beta_0^{QED} m_\mu \alpha^3 > 0$$

Non-Asymptotically Free theory!

Muonic atoms ($|\mathbf{k}| \gg m_e$):

$$\tilde{V}^{(0)} \equiv -4\pi Z_\mu Z_p \alpha_V(k) \frac{1}{\mathbf{k}^2},$$

$$\alpha_V(k) = \alpha \left(1 + \frac{\alpha}{4\pi}\beta_0^{QED} \ln \frac{m_e^2}{\mathbf{k}^2} + \mathcal{O}(\alpha^2)\right) \qquad \beta_0^{QED} = -\frac{4}{3}T_F < 0$$

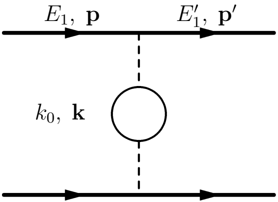


Figure: *Leading correction to the Coulomb potential due to the electron vacuum polarization.* $\mathbf{k} = \mathbf{p} - \mathbf{p}'$ *and* $k_0 = E_1 - E_1'$.

$$\beta(\alpha) = -2\alpha \left\{\beta_0^{QED}\frac{\alpha}{4\pi} + \mathcal{O}(\alpha)\right\} \longrightarrow E(1P) - E(2S) \propto -\beta_0^{QED} m_\mu \alpha^3 > 0$$

Non-Asymptotically Free theory!

Muonic atoms ($|\mathbf{k}| \gg \mathbf{m_e}$):

$$\tilde{V}^{(0)} \equiv -4\pi Z_\mu Z_p \alpha_V(k) \frac{1}{\mathbf{k}^2},$$

$$\alpha_V(k) = \alpha \left(1 + \frac{\alpha}{4\pi}\beta_0^{QED} \ln \frac{m_e^2}{\mathbf{k}^2} + \mathcal{O}(\alpha^2)\right) \qquad \beta_0^{QED} = -\frac{4}{3} T_F < 0$$

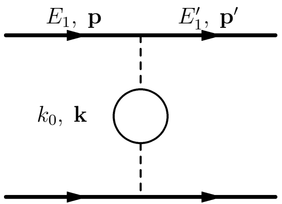


Figure: *Leading correction to the Coulomb potential due to the electron vacuum polarization.* $\mathbf{k} = \mathbf{p} - \mathbf{p}'$ *and* $k_0 = E_1 - E_1'$.

$$\beta(\alpha) = -2\alpha \left\{\beta_0^{QED} \frac{\alpha}{4\pi} + \mathcal{O}(\alpha)\right\} \longrightarrow E(1P) - E(2S) \propto -\beta_0^{QED} m_\mu \alpha^3 > 0$$

Non-Asymptotically Free theory!

Heavy Quarkonium ($|\mathbf{k}| \gg \lambda_{\mathbf{QCD}}$):

$$\tilde{V}^{(0)} \equiv -4\pi Z_\mu Z_p \alpha_V(k) \frac{1}{\mathbf{k}^2},$$

$$\alpha_V(k) = \alpha \left(1 + \frac{\alpha}{4\pi}\beta_0^{QCD} \ln\frac{\nu^2}{\mathbf{k}^2} + \mathcal{O}(\alpha^2)\right) \qquad \beta_0^{QCD} = \frac{11}{3}C_A - \frac{4}{3}T_F n_l > 0$$

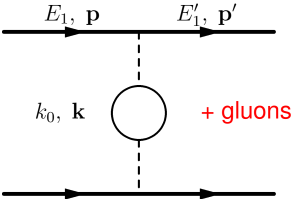


Figure: *Leading correction to the Coulomb potential due to the electron vacuum polarization plus gluons.* $\mathbf{k} = \mathbf{p} - \mathbf{p}'$ *and* $k_0 = E_1 - E_1'$.

$$\beta(\alpha) = -2\alpha\left\{\beta_0^{QCD}\frac{\alpha}{4\pi} + \mathcal{O}(\alpha)\right\} \longrightarrow E(1P) - E(2S) \propto -\beta_0^{QCD} m_Q \alpha^3 < 0$$

Asymptotically Free theory!

Heavy Quarkonium ($|\mathbf{k}| \gg \lambda_{\text{QCD}}$):

$$\tilde{V}^{(0)} \equiv -4\pi Z_\mu Z_p \alpha_V(k) \frac{1}{\mathbf{k}^2},$$

$$\alpha_V(k) = \alpha \left(1 + \frac{\alpha}{4\pi}\beta_0^{QCD} \ln \frac{\nu^2}{\mathbf{k}^2} + \mathcal{O}(\alpha^2)\right) \qquad \beta_0^{QCD} = \frac{11}{3} C_A - \frac{4}{3} T_F n_l > 0$$

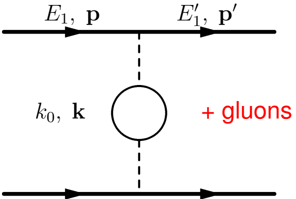


Figure: *Leading correction to the Coulomb potential due to the electron vacuum polarization plus gluons.* $\mathbf{k} = \mathbf{p} - \mathbf{p}'$ *and* $k_0 = E_1 - E_1'$.

$$\beta(\alpha) = -2\alpha \left\{\beta_0^{QCD} \frac{\alpha}{4\pi} + \mathcal{O}(\alpha)\right\} \longrightarrow E(1P) - E(2S) \propto -\beta_0^{QCD} m_Q \alpha^3 < 0$$

Asymptotically Free theory!

Heavy Quarkonium ($|\mathbf{k}| \gg \lambda_{\mathbf{QCD}}$):

$$\tilde{V}^{(0)} \equiv -4\pi Z_\mu Z_p \alpha_V(k) \frac{1}{\mathbf{k}^2},$$

$$\alpha_V(k) = \alpha \left(1 + \frac{\alpha}{4\pi}\beta_0^{QCD} \ln \frac{\nu^2}{\mathbf{k}^2} + \mathcal{O}(\alpha^2)\right) \qquad \beta_0^{QCD} = \frac{11}{3}C_A - \frac{4}{3}T_F n_l > 0$$

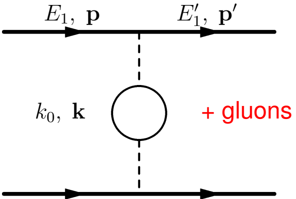


Figure: *Leading correction to the Coulomb potential due to the electron vacuum polarization plus gluons.* $\mathbf{k} = \mathbf{p} - \mathbf{p}'$ *and* $k_0 = E_1 - E_1'$.

$$\beta(\alpha) = -2\alpha \left\{\beta_0^{QCD} \frac{\alpha}{4\pi} + \mathcal{O}(\alpha)\right\} \longrightarrow E(1P) - E(2S) \propto -\beta_0^{QCD} m_Q \alpha^3 < 0$$

Asymptotically Free theory!

DELIVERIES

Effective Field Theories provide with a model independent, efficient and systematic (Power counting) approach to the dynamics of Non-relativistic systems and a unified framework to determine the nonperturbative effects.

Rigorous connection between Quantum Field Theories (Wilson coefficients) and a quantum-mechanical formulation of the non-relativistic systems (potentials).

Definitions. Low energy constants/Wilson coefficients. Example: The proton radius is a Wilson coefficient of the effective theory

Minimal basis. Combination of Wilson coefficients that appear in observables.

Universality. Same Wilson coefficients in different observables.

m_q and N_c. Analytic control of the QCD dynamics: non-analytic m_q and N_c dependence under control.

Wilson coefficients in the UV. Control in the relation between different scales/physical systems. Example: Hyperfine splitting of Hydrogen and muonic hydrogen.

DELIVERIES

Effective Field Theories provide with a model independent, efficient and systematic (Power counting) approach to the dynamics of Non-relativistic systems and a unified framework to determine the nonperturbative effects.

Rigorous connection between Quantum Field Theories (Wilson coefficients) and a quantum-mechanical formulation of the non-relativistic systems (potentials).

Definitions. Low energy constants/Wilson coefficients. Example: The proton radius is a Wilson coefficient of the effective theory

Minimal basis. Combination of Wilson coefficients that appear in observables.

Universality. Same Wilson coefficients in different observables.

m_q and N_c. Analytic control of the QCD dynamics: non-analytic m_q and N_c dependence under control.

Wilson coefficients in the UV. Control in the relation between different scales/physical systems. Example: Hyperfine splitting of Hydrogen and muonic hydrogen.

DELIVERIES

Effective Field Theories provide with a model independent, efficient and systematic (Power counting) approach to the dynamics of Non-relativistic systems and a unified framework to determine the nonperturbative effects.

Rigorous connection between Quantum Field Theories (Wilson coefficients) and a quantum-mechanical formulation of the non-relativistic systems (potentials).

Definitions. Low energy constants/Wilson coefficients. Example: The proton radius is a Wilson coefficient of the effective theory

Minimal basis. Combination of Wilson coefficients that appear in observables.

Universality. Same Wilson coefficients in different observables.

m_q and N_c. Analytic control of the QCD dynamics: non-analytic m_q and N_c dependence under control.

Wilson coefficients in the UV. Control in the relation between different scales/physical systems. Example: Hyperfine splitting of Hydrogen and muonic hydrogen.

DELIVERIES

Effective Field Theories provide with a model independent, efficient and systematic (Power counting) approach to the dynamics of Non-relativistic systems and a unified framework to determine the nonperturbative effects.

Rigorous connection between Quantum Field Theories (Wilson coefficients) and a quantum-mechanical formulation of the non-relativistic systems (potentials).

Definitions. Low energy constants/Wilson coefficients. Example: The proton radius is a Wilson coefficient of the effective theory

Minimal basis. Combination of Wilson coefficients that appear in observables.

Universality. Same Wilson coefficients in different observables.

m_q and N_c. Analytic control of the QCD dynamics: non-analytic m_q and N_c dependence under control.

Wilson coefficients in the UV. Control in the relation between different scales/physical systems. Example: Hyperfine splitting of Hydrogen and muonic hydrogen.

DELIVERIES

Effective Field Theories provide with a model independent, efficient and systematic (Power counting) approach to the dynamics of Non-relativistic systems and a unified framework to determine the nonperturbative effects.

Rigorous connection between Quantum Field Theories (Wilson coefficients) and a quantum-mechanical formulation of the non-relativistic systems (potentials).

Definitions. Low energy constants/Wilson coefficients. Example: The proton radius is a Wilson coefficient of the effective theory

Minimal basis. Combination of Wilson coefficients that appear in observables.

Universality. Same Wilson coefficients in different observables.

m_q and N_c. Analytic control of the QCD dynamics: non-analytic m_q and N_c dependence under control.

Wilson coefficients in the UV. Control in the relation between different scales/physical systems. Example: Hyperfine splitting of Hydrogen and muonic hydrogen.

DELIVERIES

Effective Field Theories provide with a model independent, efficient and systematic (Power counting) approach to the dynamics of Non-relativistic systems and a unified framework to determine the nonperturbative effects.

Rigorous connection between Quantum Field Theories (Wilson coefficients) and a quantum-mechanical formulation of the non-relativistic systems (potentials).

Definitions. Low energy constants/Wilson coefficients. Example: The proton radius is a Wilson coefficient of the effective theory

Minimal basis. Combination of Wilson coefficients that appear in observables.

Universality. Same Wilson coefficients in different observables.

m_q and N_c. Analytic control of the QCD dynamics: non-analytic m_q and N_c dependence under control.

Wilson coefficients in the UV. Control in the relation between different scales/physical systems. Example: Hyperfine splitting of Hydrogen and muonic hydrogen.

DELIVERIES

Effective Field Theories provide with a model independent, efficient and systematic (Power counting) approach to the dynamics of Non-relativistic systems and a unified framework to determine the nonperturbative effects.

Rigorous connection between Quantum Field Theories (Wilson coefficients) and a quantum-mechanical formulation of the non-relativistic systems (potentials).

Definitions. Low energy constants/Wilson coefficients. Example: The proton radius is a Wilson coefficient of the effective theory

Minimal basis. Combination of Wilson coefficients that appear in observables.

Universality. Same Wilson coefficients in different observables.

m_q and N_c. Analytic control of the QCD dynamics: non-analytic m_q and N_c dependence under control.

Wilson coefficients in the UV. Control in the relation between different scales/physical systems. Example: Hyperfine splitting of Hydrogen and muonic hydrogen.