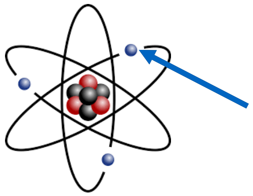
NREC Workshop 2024

# Improved nuclear structure effects in helium and muonic helium atoms


**Simone Salvatore Li Muli**

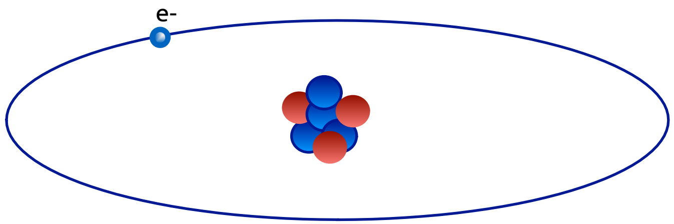
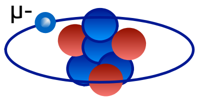
Chalmers University of Technology

# Muonic atoms



Electrons are replaced by a muon

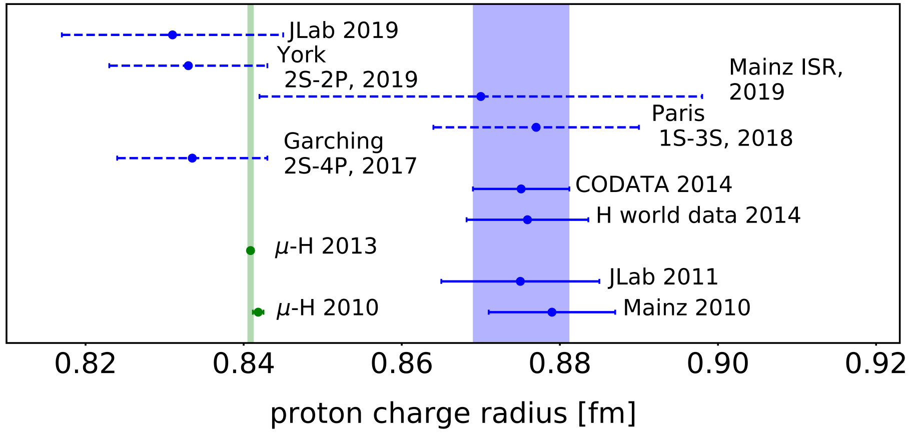
**Hydrogen-like systems**

| Ordinary ions | Muonic atoms |
|---|---|
|   |   |

Muon is more sensitive to the nucleus

**Can be used as precision probes for nuclear physics**

# The proton radius

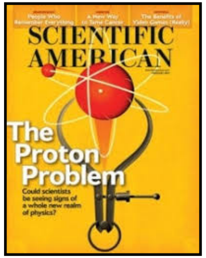



**Muonic Hydrogen**
- Pohl et al., Nature (2010)
- Antognini et al., Science (2013)

**Muonic Deuterium**
- Pohl et al., Science (2016)

**Muonic Helium isotopes**
- Krauth et al., Nature (2021)
- Schuhmann et al., Arxiv (2023)





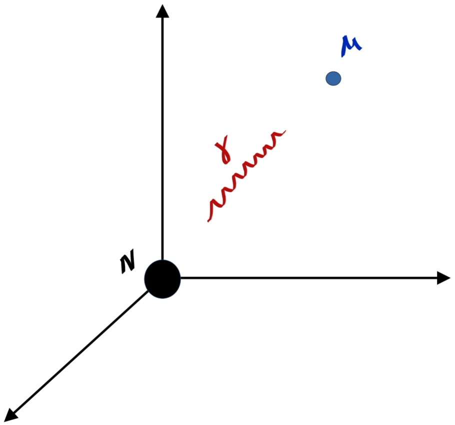
# Lamb shift and charge radii

$$\mathrm{E}_{\mathrm{LS}} = \mathrm{E}_{\mathrm{QED}} + \mathrm{C}r_c^2 + \mathrm{E}_{\mathrm{TPE}} + \ldots$$

# Lamb shift and charge radii

$$\mathrm{E_{LS}} = \mathrm{E_{QED}} + \mathrm{C}r_c^2 + \mathrm{E_{TPE}} + \ldots$$

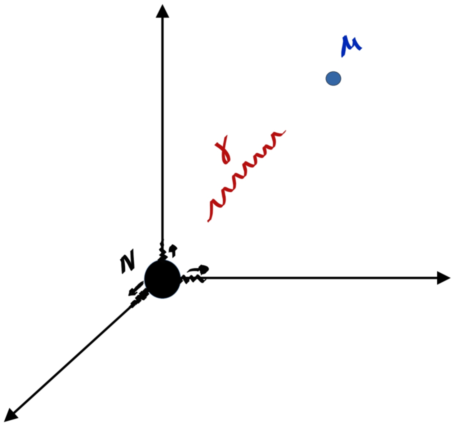
$$\mathrm{E_{QED}} =$$



# Lamb shift and charge radii

$$\mathrm{E_{LS}} = \mathrm{E_{QED}} + \mathrm{C}r_c^2 + \mathrm{E_{TPE}} + \ldots$$

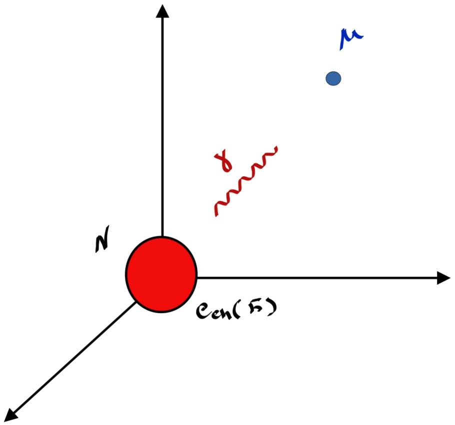
$$\mathrm{E_{QED}} =$$



# Lamb shift and charge radii

$$\mathrm{E}_{\mathrm{LS}} = \mathrm{E}_{\mathrm{QED}} + \mathrm{C}r_c^2 + \mathrm{E}_{\mathrm{TPE}} + \ldots$$

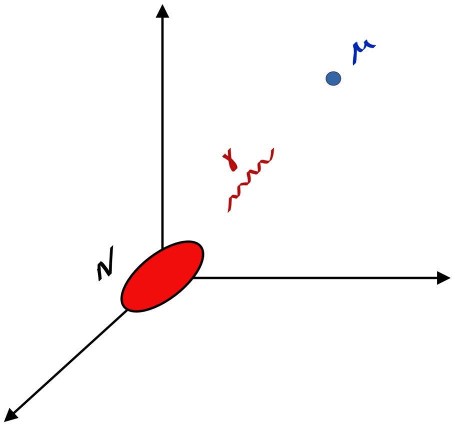
$$\mathrm{C}r_c^2 =$$



# Lamb shift and charge radii

$$\mathrm{E_{LS}} = \mathrm{E_{QED}} + Cr_c^2 + \mathrm{E_{TPE}} + \ldots$$

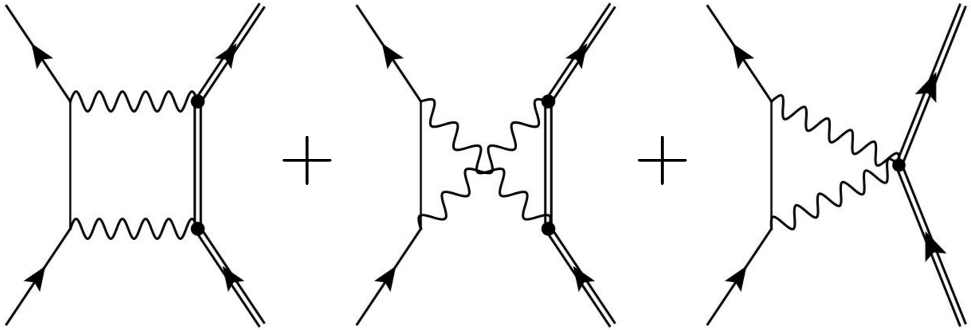
$$\mathrm{E_{TPE}} =$$



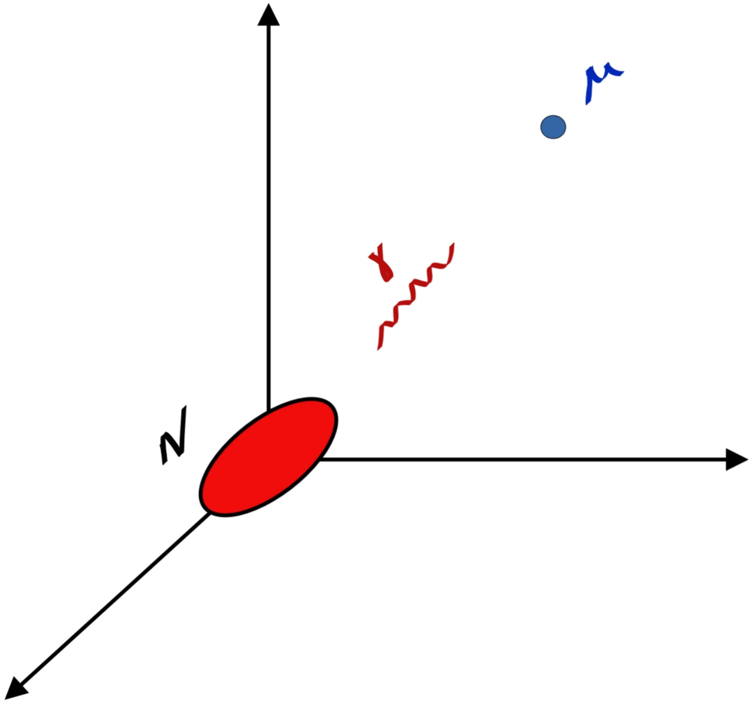
# Lamb shift and charge radii

$$\mathrm{E}_{\mathrm{LS}} = \mathrm{E}_{\mathrm{QED}} + \mathrm{C}r_c^2 + \mathrm{E}_{\mathrm{TPE}} + \ldots$$

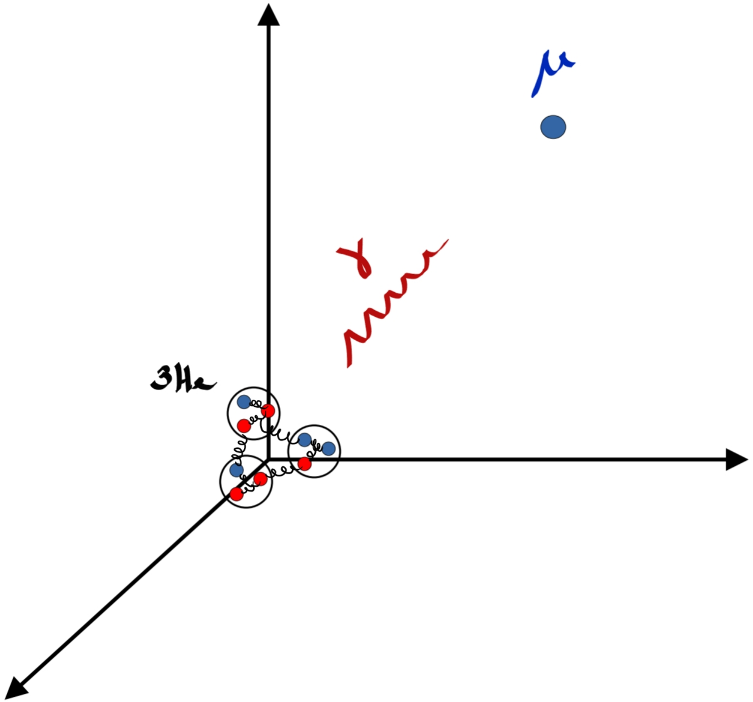
$$\mathrm{E}_{\mathrm{TPE}} =$$



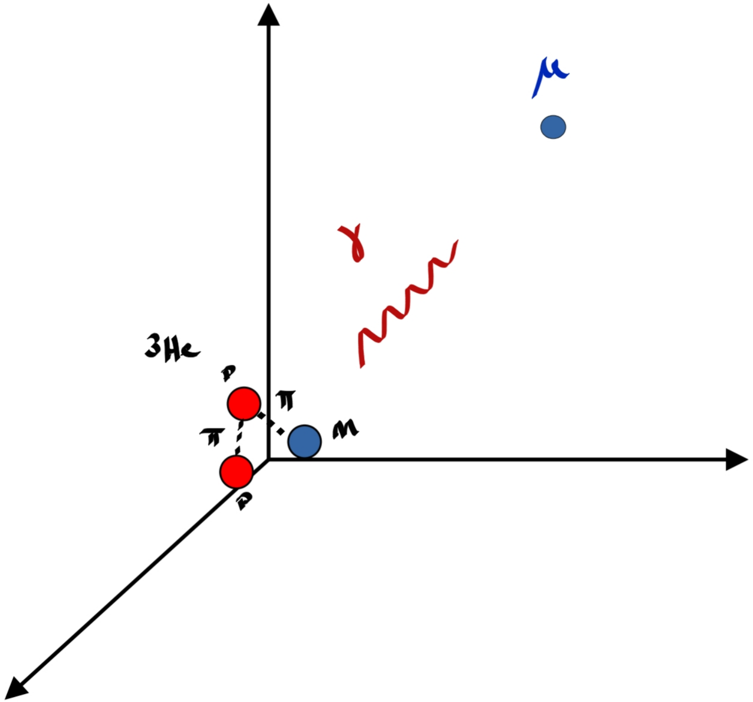
# Ab-initio nuclear theory



# Ab-initio nuclear theory



# Ab-initio nuclear theory



# Ab-initio nuclear theory

- Our strategy is to build models for the operators from first principles

$$\mathrm{H}$$

$$\{\rho_{\mathrm{ch}}, J_i, B_{ij}\}$$

from chiral effective field theory

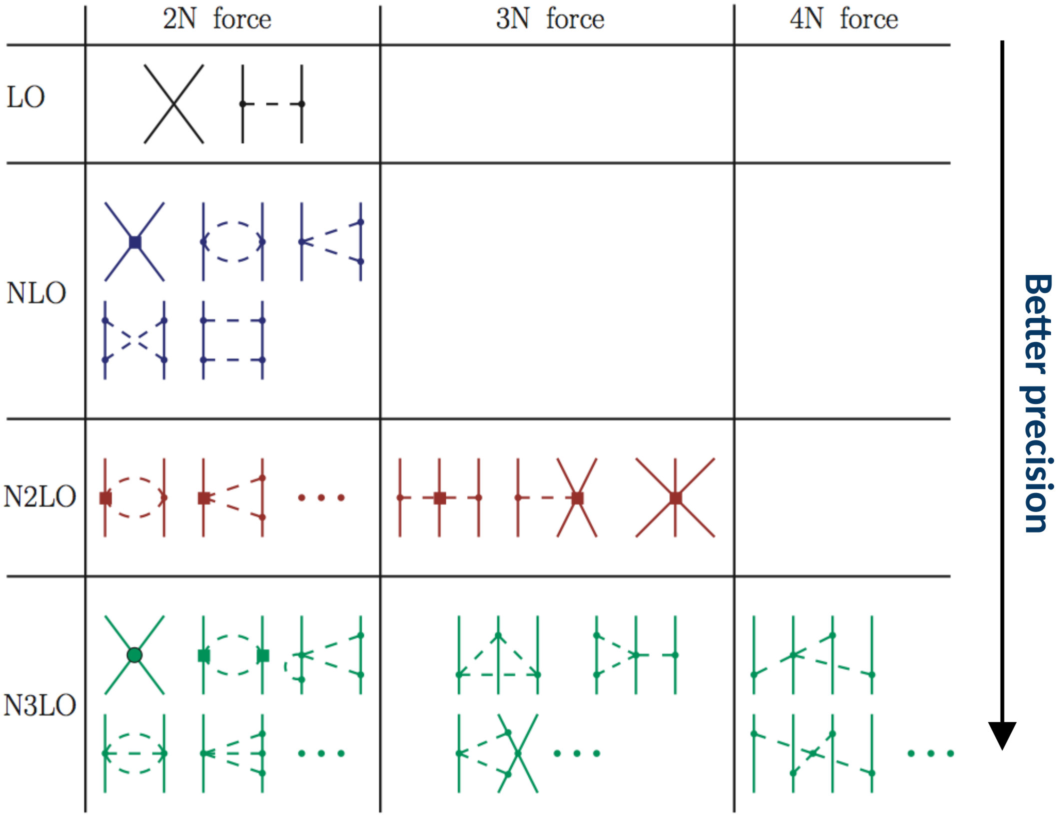
- Solve the many-body time-independent Schrödinger equation of the nucleus

$$\mathrm{H}\,|\mathrm{N}\rangle = \mathrm{E_N}\,|\mathrm{N}\rangle$$

- Calculate the relevant matrix elements with controlled approximations.

$$\langle \mathrm{N}|\,\rho_{\mathrm{ch}}(\mathbf{x})\,|0\rangle$$

# Nuclear Hamiltonians from ChEFT




- Degrees of freedom
- Symmetries
- Power counting

# Bayesian uncertainty quantification

ChEFT is an expansion in powers of $Q = \frac{m_\pi}{\Lambda_\chi} \sim 0.3$

We assume that a similar expansion holds also for the calculated observables

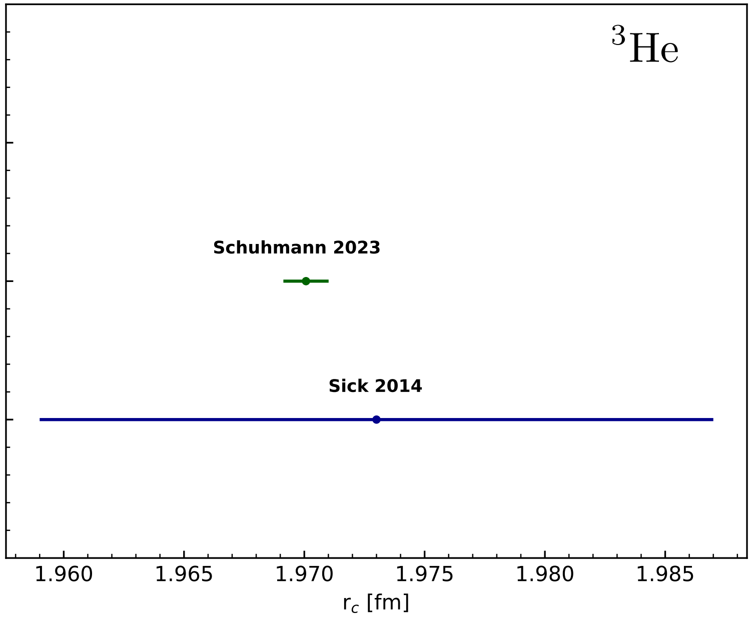
$$X = \sum_{n=0}^{k} D_n + \sum_{n=k+1}^{\infty} D_n$$

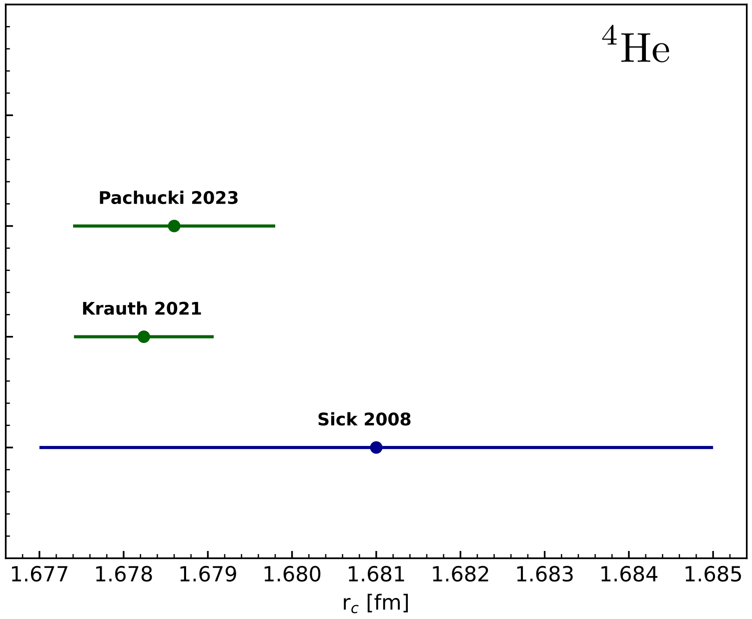
$$= X_{\mathrm{ref}} \left[ \sum_{n=0}^{k} c_n \, Q^n + \sum_{n=k+1}^{\infty} c_n \, Q^n \right]$$

We assume that the expansion coefficients follow the same underlying distribution and use the calculated coefficients to learn about the distribution.

# Helium isotopes charge radii

$$\mathrm{E_{LS}} = \mathrm{E_{QED}} + \mathrm{C}r_c^2 + \mathrm{E_{TPE}} + \mathrm{E_{3PE}} + \ldots$$



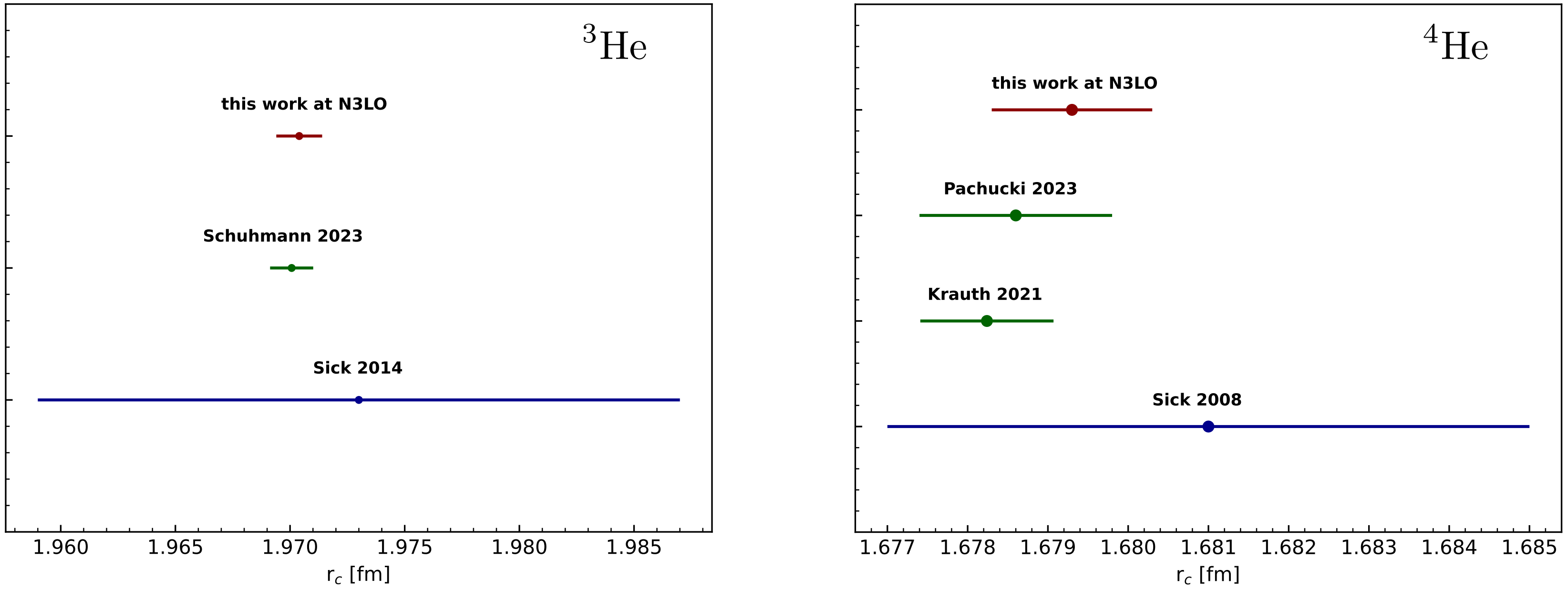



# Helium isotopes charge radii

$$\mathrm{E_{LS}} = \mathrm{E_{QED}} + \mathrm{C}r_c^2 + \mathrm{E_{TPE}} + \mathrm{E_{3PE}} + \ldots$$

Updating this

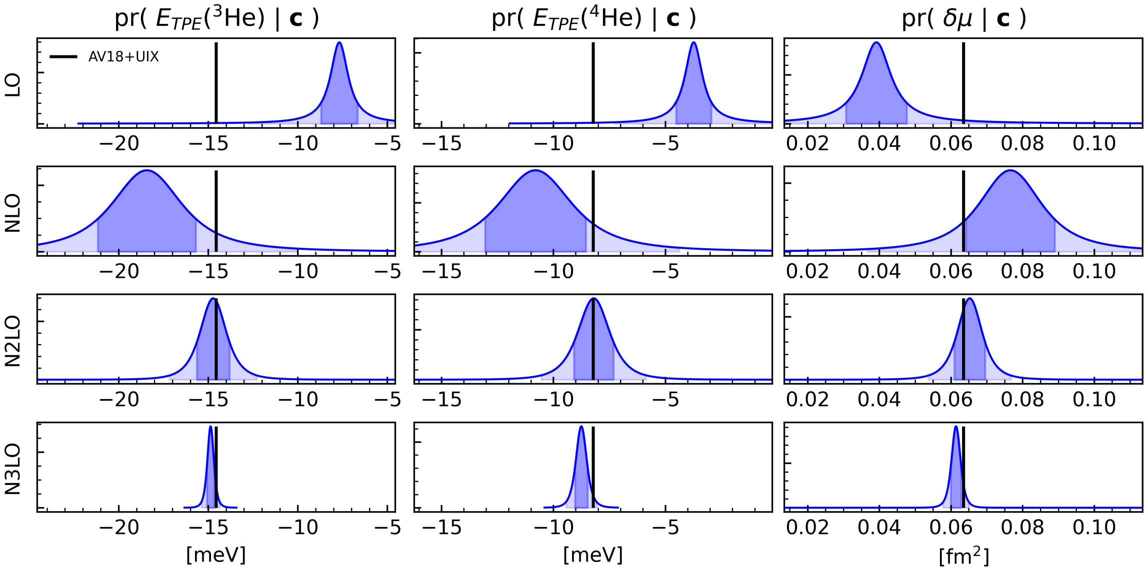
SSLM, Thomas R. Richardson, Sonia Bacca, Arxiv:2401.13424



# Isotope shift of muonic Helium

$$\delta r_c^2 = \left[\frac{E_{LS(^3He)}}{C(^3He)} - \frac{E_{LS(^4He)}}{C(^4He)}\right] + \underbrace{\left[\frac{E_{TPE(^4He)}}{C(^4He)} - \frac{E_{TPE(^3He)}}{C(^3He)}\right]}_{\text{Define the nuclear structure part of this term as } \delta\mu}$$

$$+\left[\frac{E_{3PE(^4He)}}{C(^4He)} - \frac{E_{3PE(^3He)}}{C(^3He)}\right] + \left[\frac{E_{QED(^4He)}}{C(^4He)} - \frac{E_{QED(^3He)}}{C(^3He)}\right] + \ldots$$

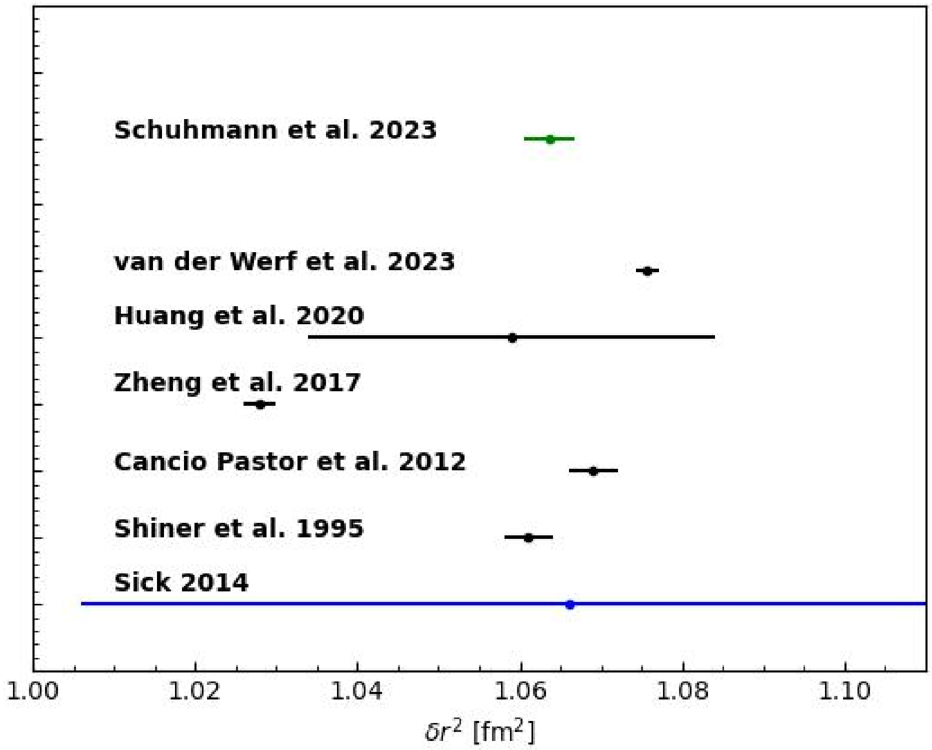


# Exploiting correlations

$$\underbrace{\Delta\left[\mathrm{E_{TPE}}(^4\mathrm{He}) - \mathrm{E_{TPE}}(^3\mathrm{He})\right]}_{\text{Is obtained from the Bayesian analysis}} = \sqrt{\Delta\mathrm{E}^2_{\mathrm{TPE}}(^4\mathrm{He}) + \Delta\mathrm{E}^2_{\mathrm{TPE}}(^3\mathrm{He}) - 2\rho_{12}\underbrace{\Delta\mathrm{E_{TPE}}(^3\mathrm{He})\Delta\mathrm{E_{TPE}}(^4\mathrm{He})}_{\text{Are obtained from the Bayesian analysis}}}$$

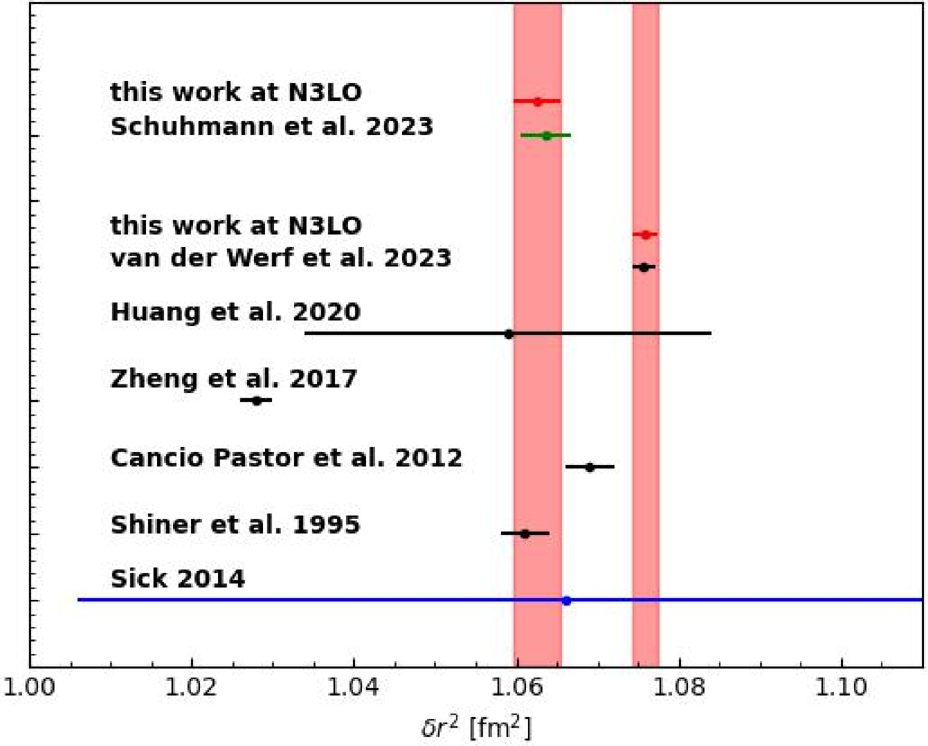
- **We can extract the correlation coefficient**
- $\rho_{12} \approx 0.8$
- **We assume that the remaining nuclear structure effects (3-photon-exchange, etc...) are correlated with the same correlation coefficient.**
- **The inclusion of correlations significantly reduces the uncertainties in the isotope-shift theory.**

# Isotope shift of muonic Helium



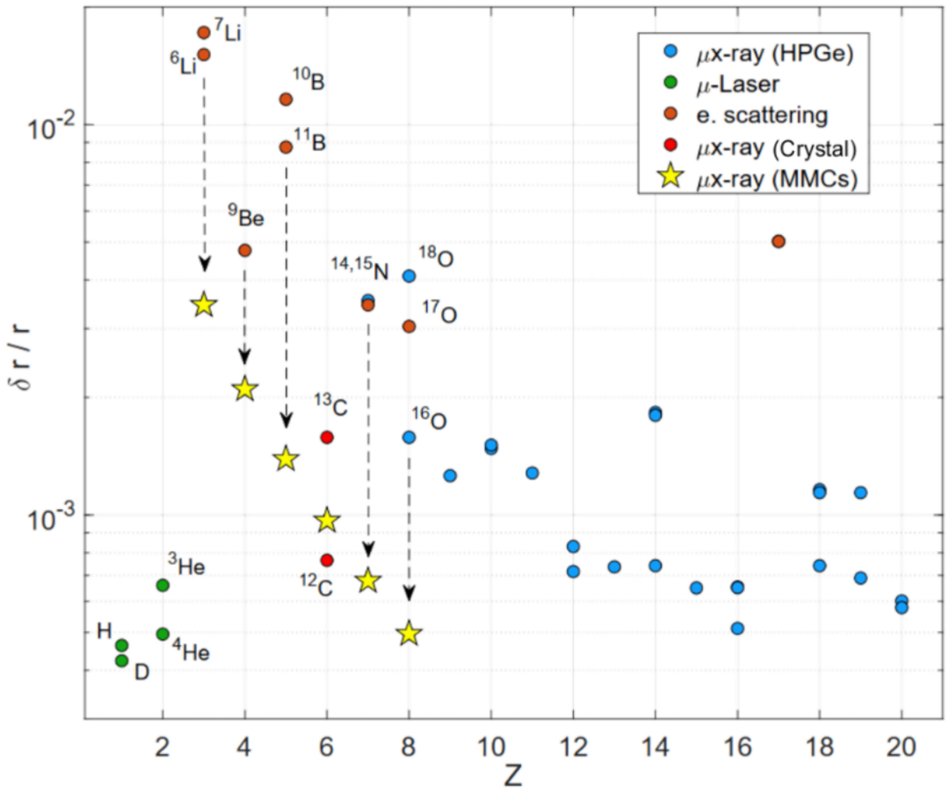
# Isotope shift of muonic Helium

SSLM, Thomas R. Richardson, Sonia Bacca, Arxiv:2401.13424



# The future pipeline

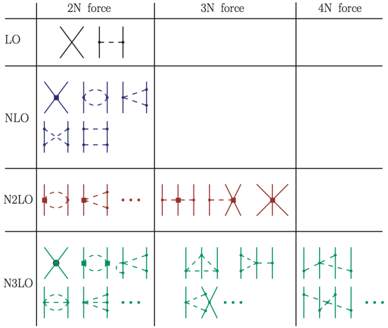
A. Antognini et al, Arxiv:2210.16929



# The future pipeline

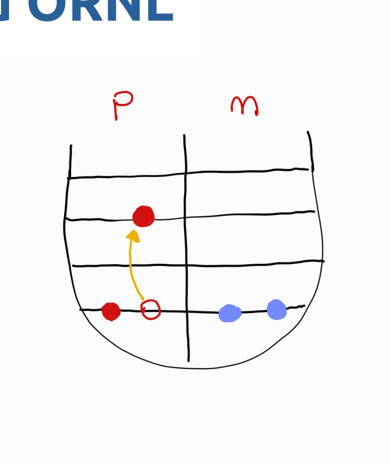
## In collaboration with scientists at Chalmers and ORNL

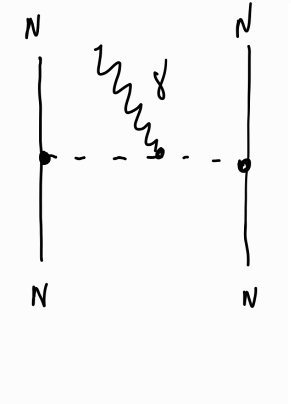
Christian Forssén




Tor Djärv

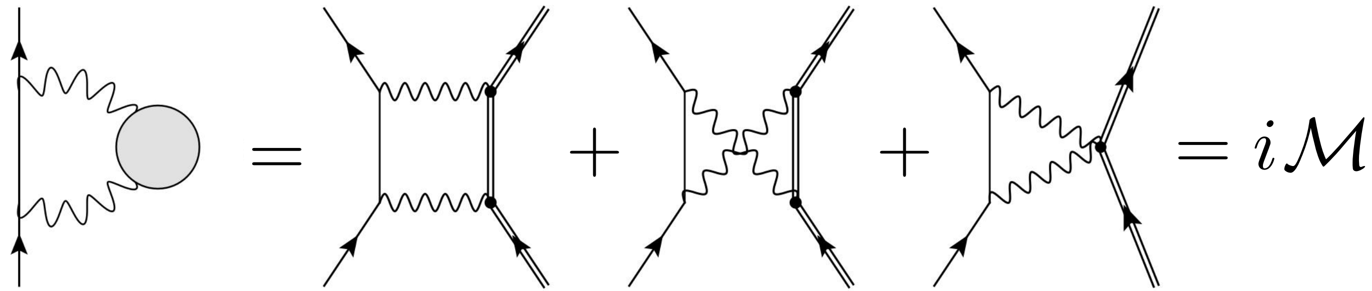
Bijaya Acharya



# Backup

# Evaluation of the TPE term




**The amplitudes define a complex potential that shift the 2S-energy of the bound muon by**

$$\Delta \mathrm{E}_{2\mathrm{S}} = \langle \phi_{2\mathrm{S}} | \, \mathrm{Re}\left[ i\mathcal{M} \right] | \phi_{2\mathrm{S}} \rangle$$

**With $|\phi_{2\mathrm{S}}\rangle$ being the 2S-state of the muon. At our level of precision there are no corrections to 2P-states.**

# Evaluation of the TPE term

$$
\begin{aligned}
\Delta E_{2\mathrm{S}} = -8\alpha^2 m\, \phi_{2\mathrm{S}}^2 \Bigg\{ & \sum_{N\neq 0} \int d^3x\, d^3y\; \langle 0|\, \rho_{\mathrm{ch}}^{\dagger}(\mathbf{y})\, |N\rangle \langle N|\, \rho_{\mathrm{ch}}(\mathbf{x})\, |0\rangle \;\; \mathrm{I}_{\mathrm{N}}(z) \\
& + \sum_{N\neq 0} \int d^3x\, d^3y\, \langle 0|\, J_i^{\dagger}(\mathbf{y})\, |N\rangle \langle N|\, J_j(\mathbf{x})\, |0\rangle \left[\delta_{ij} J_N(z) + z^i z^j \bar{J}_N(z)\right] \\
& + \int d^3x\, d^3y\; B^{ij}(\mathbf{x},\mathbf{y})\, \frac{1}{2}\left[\delta_{ij} K(z) + z_i z_j \bar{K}(z)\right] \Bigg\} .
\end{aligned}
$$

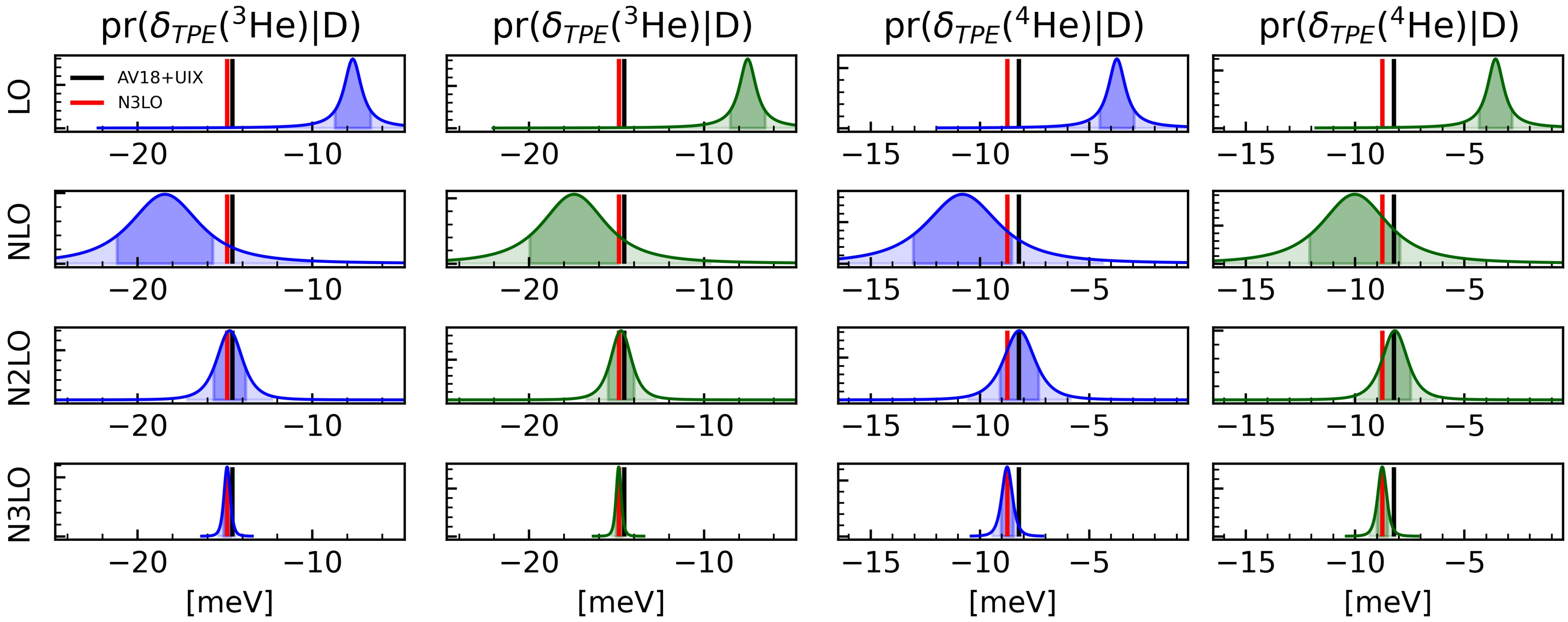
**With $z = |\mathbf{x} - \mathbf{y}|$.**

**– The five structure functions are known by calculating the leptonic part of the Feynman diagrams.**

**– The nuclear matrix elements must be calculated numerically from Nuclear Theory.**

# TPE corrections in muonic Helium

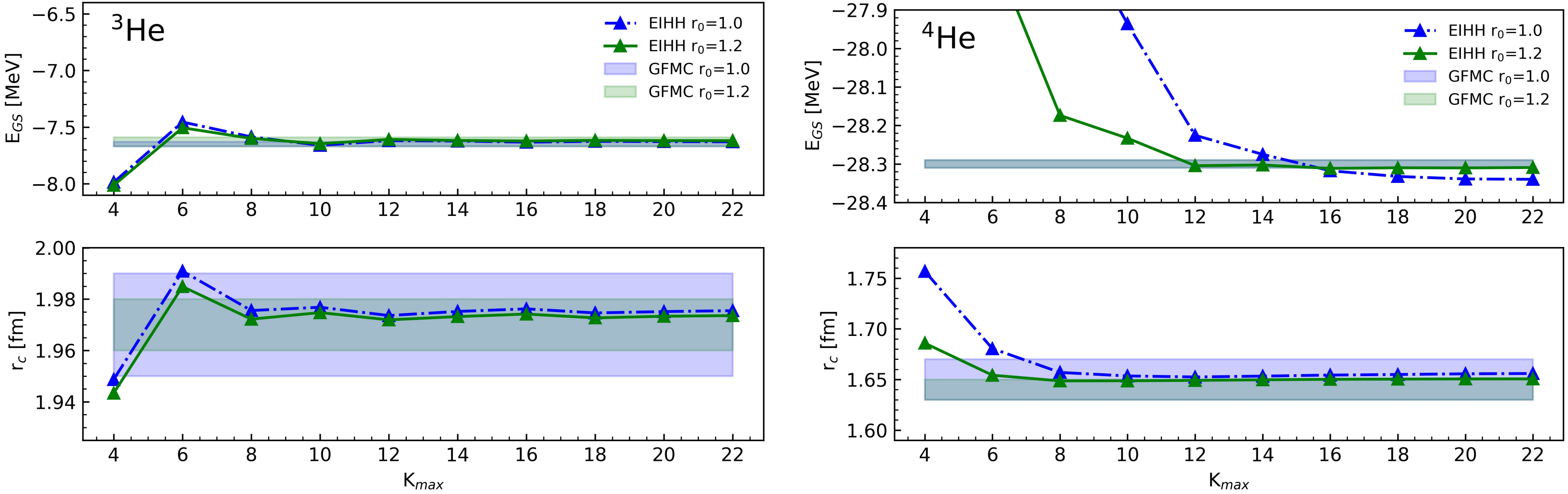
**S.S.LM, et al. In preparation for 2023**



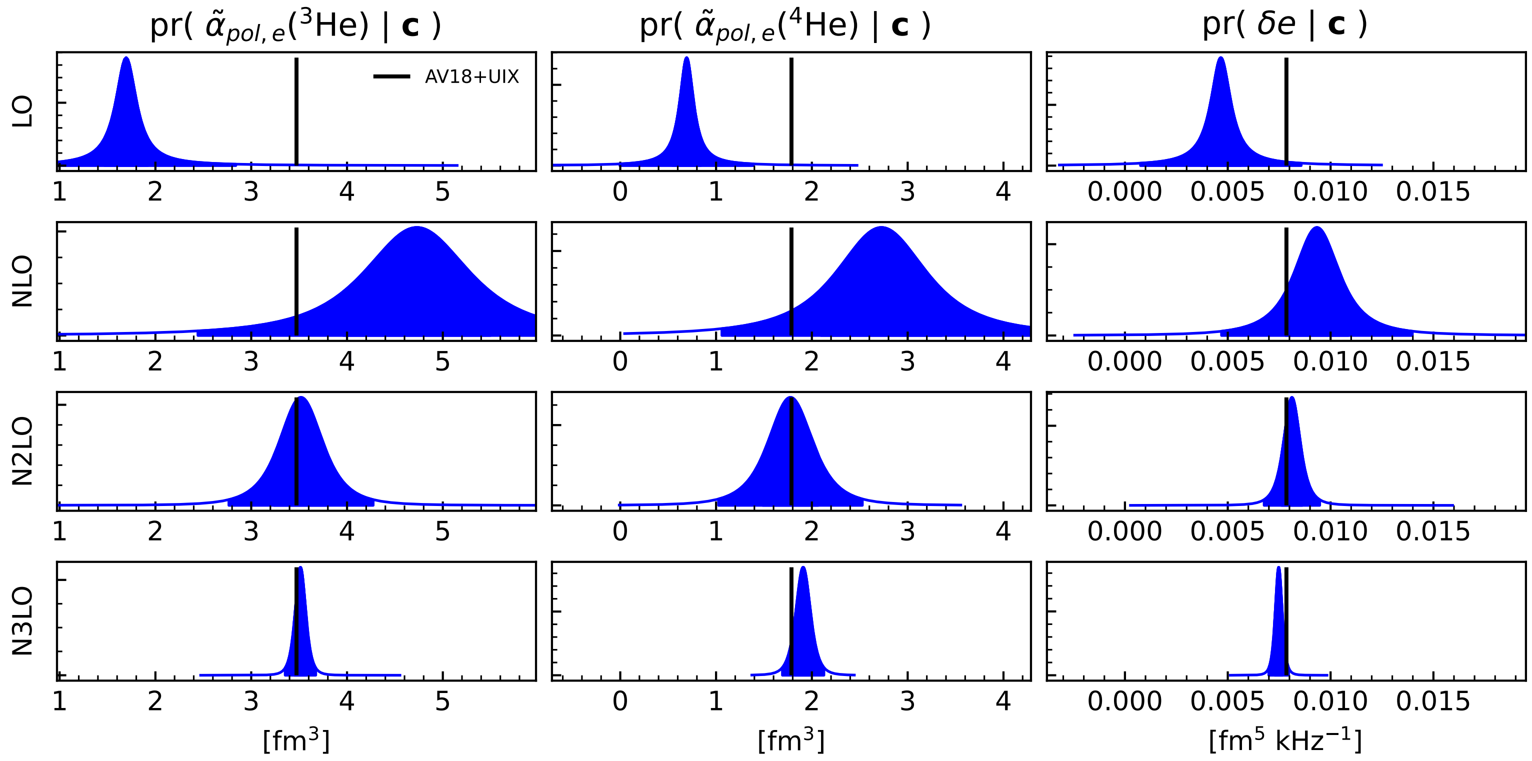

**C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)**

# Benchmark tests

S.S. LM, S. Bacca , N. Barnea, Front. Phys. 9, 671869 (2021)



# Benchmark tests



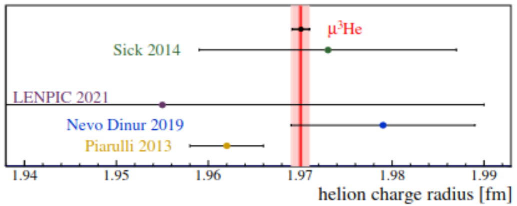
# A matter of precision

$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED+NR}} + \delta^{(4)}_{\mathrm{FS}} \times r_c^2 + \delta^{(5)}_{\mathrm{TPE}} + \delta^{(6)}_{\mathrm{3PE}} + \dots$$

**For the muonic Helium-3 ion**

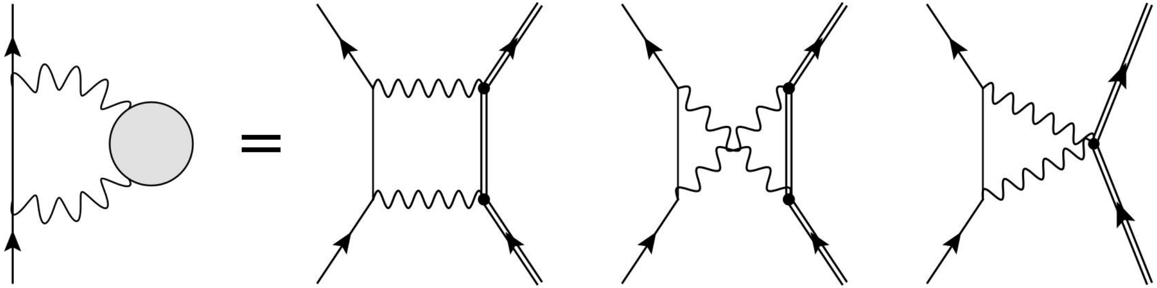
$$\begin{aligned} \delta_{\mathrm{QED+NR}} &= +1,644.348(8)\ \mathrm{meV} \\ \delta^{(4)}_{\mathrm{FS}} &= -103.383\ \mathrm{meV\ fm}^{-2} \\ \delta^{(5)}_{\mathrm{TPE}} &= +15.499(378)\ \mathrm{meV} \\ \delta^{(6)}_{\mathrm{3PE}} &= -0.214(214)\ \mathrm{meV} \\ \delta^{(5)}_{\mathrm{HO}} &= -0.667(25)\ \mathrm{meV} \end{aligned}$$

$$r_c = 1.97007(12)_{\mathrm{ex}}(93)_{\mathrm{th}}\ \mathrm{fm}$$




K. Schuhmann et. al. Arxiv 2305.11679 (2023)

# Evaluation of the NS effects



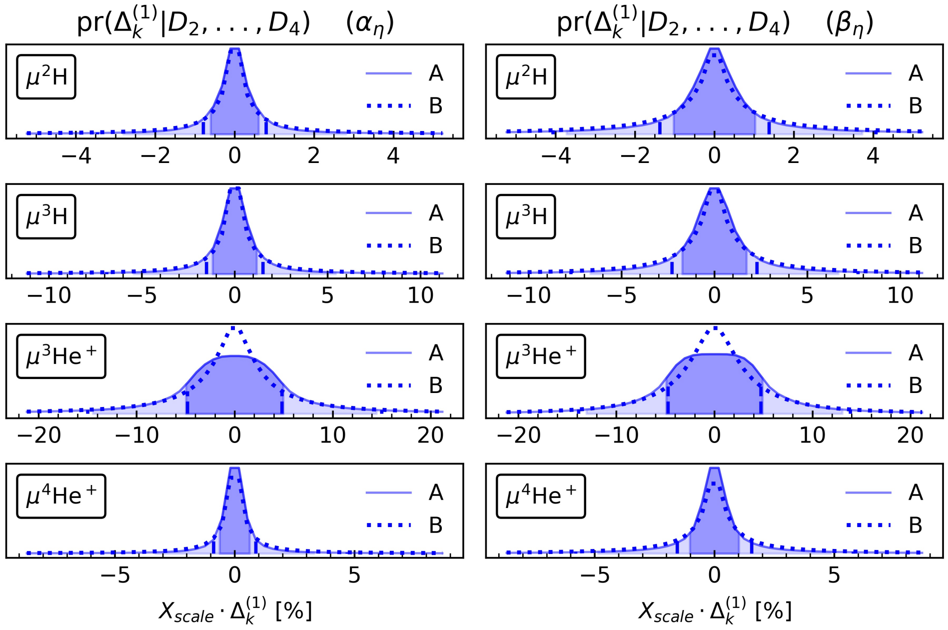
$$\Delta E_{nl} = -8\alpha^2 m\, \phi_{nL}^2 \int \frac{d^3q}{4\pi} \left\{ \sum_{N\neq 0} \left|\langle N|\, \rho_{\mathrm{ch}}(\mathbf{q})\, |0\rangle\right|^2 \frac{(2E_q + \omega_N)}{q^4 E_q\left[(E_q+\omega_N)^2 - m^2\right]} \right.$$

$$+ \sum_{N\neq 0} \left|\langle 0|\, \mathbf{J}_\perp(\mathbf{q})\, |N\rangle\right|^2 \left[ \frac{q^2}{4m^2} \frac{(2E_q+\omega_N)}{q^4 E_q\left[(E_q+\omega_N)^2 - m^2\right]} - \frac{1}{4m^2q^3} \frac{\omega_N + 2q}{(\omega_N + q)^2} \right]$$

$$\left. + B_\perp(\mathbf{q}) \frac{1}{8m^2q^2} \left( \frac{1}{q} - \frac{1}{E_q} \right) \right\}$$

# eta-expansion uncertainty

$$\Delta E^{\rm NR}_{nl} = -8\alpha^2\ \phi^2_{nl} \sum_{N\neq 0} \int d^3x\ d^3y\ \langle 0|\, \rho^\dagger_{\rm ch}(\mathbf{y})\,|N\rangle \langle N|\, \rho_{\rm ch}(\mathbf{x})\,|0\rangle\ \ {\rm I}_{\rm NR}(z)$$
$$= -8\alpha^2\ \phi^2_{nl} \sum_{N\neq 0} \int d^3x\ d^3y\ \langle 0|\, \rho^\dagger_{\rm ch}(\mathbf{y})\,|N\rangle \langle N|\, \rho_{\rm ch}(\mathbf{x})\,|0\rangle\ \ \left({\rm I}^{(2)}_{\rm NR}(z) + {\rm I}^{(3)}_{\rm NR}(z) + {\rm I}^{(4)}_{\rm NR}(z) + ..\right)$$




S.S.LM, et al. 2022 J. Phys. G: Nucl. Part. Phys. 49 105101

| | $\mu^2$H | $\mu^3$H | $\mu^3$He$^+$ | $\mu^4$He$^+$ |
|---|---|---|---|---|
| **C. Ji, et al. (2018)** | 0.4% | 1.3% | 1.1% | 0.8% |
| **This work** | 0.8% | 1.5% | 4.8% | 0.9% |

# Uncertainty budget in TPE

| | $\mu^3He^+$ | | | $\mu^4He^+$ | | |
|---|---|---|---|---|---|---|
| | $\delta_{pol}$ [%] | $\delta_{Zem}$ [%] | $\delta_{TPE}$ [%] | $\delta_{pol}$ [%] | $\delta_{Zem}$ [%] | $\delta_{TPE}$ [%] |
| Numerical **[1]** | 0.4 | 0.1 | 0.1 | 0.4 | 0.3 | 0.4 |
| Numerical | 0.1 | 0.2 | 0.1 | 0.4 | 0.3 | 0.2 |
| Nuclear model **[1]** | 0.7 | 1.8 | 1.5 | 3.9 | 4.6 | 4.4 |
| Nuclear model (N2LO) | 4.8 | 6.9 | 6.2 | 14.5 | 9.4 | 11 |
| Nuclear model (N3LO) | 1.6 | 1.6 | 1.4 | 4.1 | 2.8 | 3 |
| $\eta$-expansion **[1]** | 1.1 | – | 0.3 | 0.8 | – | 0.2 |
| $\eta$-expansion | 4.8 | – | 1.4 | 0.9 | – | 0.2 |
| $Z\alpha$ **[1]** | 1.5 | 0.0 | 0.4 | 1.5 | 0.0 | 0.4 |
| $Z\alpha$ | – | – | – | – | – | – |
| ISB **[1]** | 1.8 | 0.2 | 0.5 | 2.2 | 0.5 | 0.5 |
| Nucleon-size **[1]** | 1.2 | 1.3 | 0.9 | 2.7 | 2.0 | 1.2 |
| Relativistic **[1]** | 0.4 | – | 0.1 | 0.1 | – | 0.0 |
| Coulomb **[1]** | 3.0 | – | 0.9 | 0.4 | – | 0.1 |
| Total **[1]** | 4.2 | 2.2 | 2.1 | 5.5 | 5.1 | 4.6 |
| Total (N2LO) | 7.6 | 6.9 | 6.5 | 10.6 | 9.6 | 10.9 |
| Total (N3LO) | 6.3 | 2.1 | 2.4 | 5.3 | 3.5 | 3.5 |

**S.S.LM, et al. In preparation for 2023**

**[1] C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)**

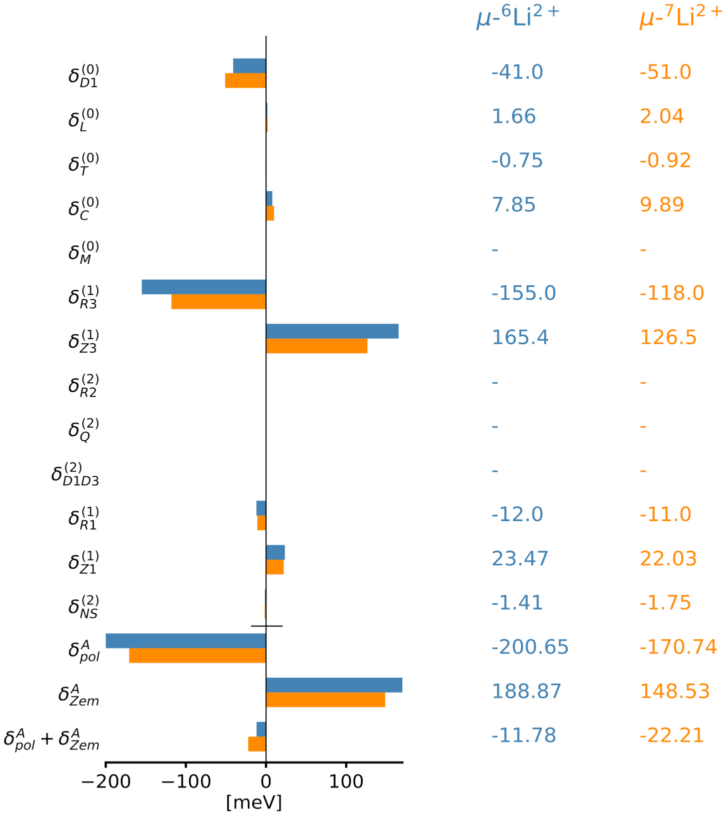
# Preliminary works on Li atoms

$$\delta_{\mathrm{TPE}} = \delta^{(0)}_{\mathrm{D1}} + \delta^{(0)}_{\mathrm{C}} + \delta^{(1)}_{\mathrm{Z1}} + \delta^{(1)}_{\mathrm{Z3}} + \delta^{(2)}_{\mathrm{NS}} + \delta^{(2)}_{\mathrm{Q}} + \ldots$$

| $\delta_{\mathrm{TPE}}$ | Ref. **(1) (meV)** | Ref. **(2) (meV)** |
|---|---|---|
| **$\mu$-$^6Li^{2+}$** | -11.8(3) | -15(4) |
| **$\mu$-$^7Li^{2+}$** | -22.2(4) | -21(4) |

**(1)** S.L., A.Poggialini, S.Bacca, SciPost Phys. Proc. 3, 028 (2020)

**(2)** Drake et al, Phys. Rev. A **32**, 713 (1985)



# Priors

$$X = \sum_{n=0}^{k} D_n + \sum_{n=k+1}^{\infty} D_n$$

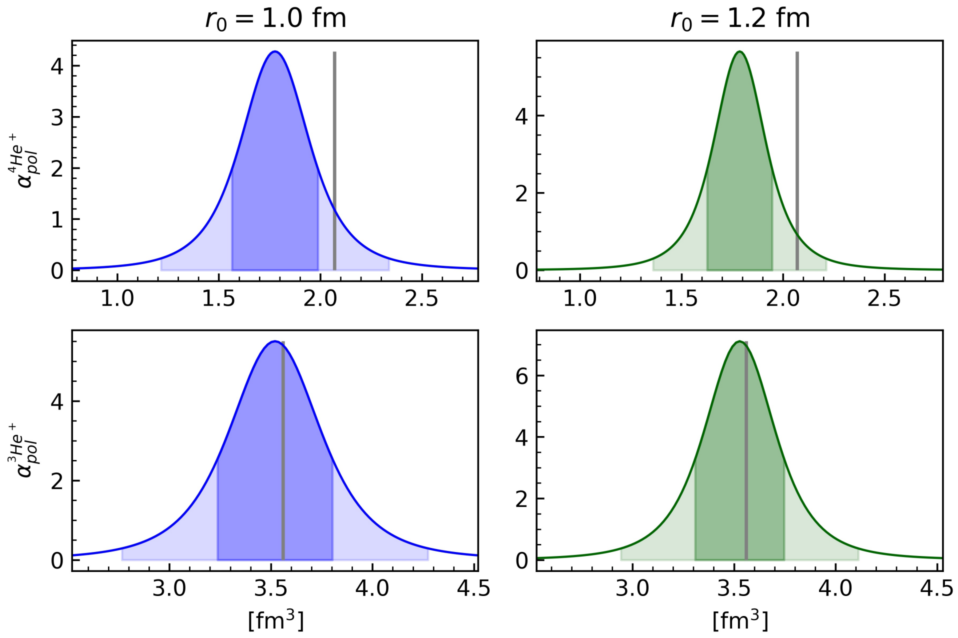
$$= X_{\text{ref}} \left[ \sum_{n=0}^{k} c_n \; Q^n + \sum_{n=k+1}^{\infty} c_n \; Q^n \right]$$

| Priors | $\mathrm{pr}(\eta)$ |
|---|---|
| $\alpha_\eta$ | $\frac{1}{\lambda}\ \exp\left(\frac{\eta}{\lambda}\right)$ |
| $\beta_\eta$ | $\beta(a,b)$ |

| Priors | $\mathrm{pr}(c_i \vert \bar{c})$ | $\mathrm{pr}(\bar{c})$ |
|---|---|---|
| A | $\frac{1}{2\bar{c}}\theta(\bar{c} - \vert c_i \vert)$ | $\frac{1}{\ln(\bar{c}_>/\bar{c}_<)\bar{c}}\theta(\bar{c} - \bar{c}_<)\theta(\bar{c}_> - \bar{c})$ |
| B | $\frac{1}{\sqrt{2\pi}\bar{c}}\exp\left(-\frac{c_i^2}{2\bar{c}^2}\right)$ | $\frac{1}{\ln(\bar{c}_>/\bar{c}_<)\bar{c}}\theta(\bar{c} - \bar{c}_<)\theta(\bar{c}_> - \bar{c})$ |

# NS effects in 3He+ and 4He+

$$\delta^{(5)}_{\mathrm{pol}} = -m\alpha^4 \left\langle \sum_a \delta^3(r_a) \right\rangle (m^3 \alpha_{\mathrm{pol}})$$



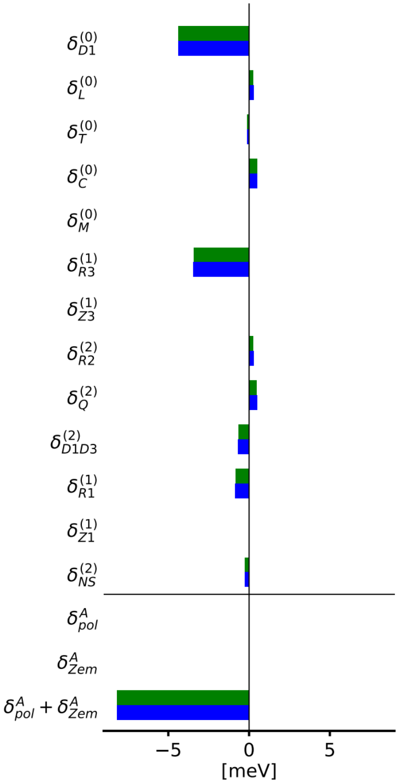

— [1] K. Pachucki, A.M. Moro Phys.Rev.A 75,032521(2007)

S.S.LM, et al. In preparation for 2022

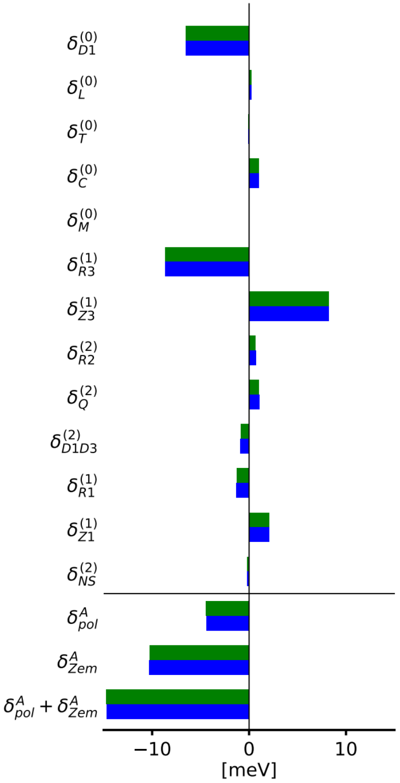
| 1S-2S | $^3\mathrm{He}^+$ | $^4\mathrm{He}^+$ |
|---|---|---|
| **[1]** | 48(5)kHz | 28(3)kHz |
| **This work** | 48(6)kHz | 24(4)kHz |

# NS corrections in μ4He+

S.S.LM, et al. In preparation for 2022




| | $r_0 = 1.2$ | $r_0 = 1.0$ |
|---|---|---|
| $\delta^{(0)}_{D1}$ | -4.386 | -4.373 |
| $\delta^{(0)}_{L}$ | 0.272 | 0.287 |
| $\delta^{(0)}_{T}$ | -0.124 | -0.125 |
| $\delta^{(0)}_{C}$ | 0.517 | 0.514 |
| $\delta^{(0)}_{M}$ | 0.011 | 0.011 |
| $\delta^{(1)}_{R3}$ | -3.422 | -3.477 |
| $\delta^{(1)}_{Z3}$ | - | - |
| $\delta^{(2)}_{R2}$ | 0.267 | 0.285 |
| $\delta^{(2)}_{Q}$ | 0.484 | 0.505 |
| $\delta^{(2)}_{D1D3}$ | -0.668 | -0.69 |
| $\delta^{(1)}_{R1}$ | -0.846 | -0.856 |
| $\delta^{(1)}_{Z1}$ | - | - |
| $\delta^{(2)}_{NS}$ | -0.272 | -0.277 |
| $\delta^{A}_{pol}$ | - | - |
| $\delta^{A}_{Zem}$ | - | - |
| $\delta^{A}_{pol} + \delta^{A}_{Zem}$ | -8.174 | -8.196 |




| | $r_0 = 1.2$ | $r_0 = 1.0$ |
|---|---|---|
| $\delta^{(0)}_{D1}$ | -6.562 | -6.552 |
| $\delta^{(0)}_{L}$ | 0.232 | 0.232 |
| $\delta^{(0)}_{T}$ | -0.104 | -0.104 |
| $\delta^{(0)}_{C}$ | 1.018 | 1.015 |
| $\delta^{(0)}_{M}$ | 0.008 | 0.008 |
| $\delta^{(1)}_{R3}$ | -8.674 | -8.691 |
| $\delta^{(1)}_{Z3}$ | 8.223 | 8.255 |
| $\delta^{(2)}_{R2}$ | 0.67 | 0.696 |
| $\delta^{(2)}_{Q}$ | 1.043 | 1.062 |
| $\delta^{(2)}_{D1D3}$ | -0.877 | -0.899 |
| $\delta^{(1)}_{R1}$ | -1.297 | -1.302 |
| $\delta^{(1)}_{Z1}$ | 2.054 | 2.057 |
| $\delta^{(2)}_{NS}$ | -0.191 | -0.193 |
| $\delta^{A}_{pol}$ | -4.458 | -4.411 |
| $\delta^{A}_{Zem}$ | -10.277 | -10.312 |
| $\delta^{A}_{pol} + \delta^{A}_{Zem}$ | -14.735 | -14.723 |