Impact of precise nuclear radii on V_{ud} and CKM unitarity tests

Collaborators:
Chien-Yeah Seng
Petr Navratil
Michael Gennari
Mehdi Drissi
Matteo Cadeddu
Nicola Cargioli
Jorge Piekarewicz
Xavi Roca Maza
Hubert Spiesberger
Jens Erler
Ben Ohayon
Bijaya Sahoo

Misha Gorshteyn
Johannes Gutenberg-Universität Mainz

Neutron beta decay review: MG, Seng, Universe 2023, 9(9), 422, arXiv:**2307.01145**
Nuclear beta decay review: MG, Seng, Ann. Rev. Nucl. Part. Sci. 74 (2024) 23-47. arXiv:**2311.00044**

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Outline

Tests of the Standard Model and beyond with Cabibbo unitarity

V_{ud} from superallowed beta decays with radiative corrections

Nuclear radii input in ft-values

Nuclear radii and isospin-symmetry breaking correction δ_C

Nuclear structure in beta decays and μ atoms

Summary & Outlook

Quark Mixing & CKM Unitarity

Muon lifetime —> Fermi constant $G_F = G_\mu = 1.1663788(7) \times 10^{-5} GeV^{-2}$

Cabibbo: mass and flavor eigenstates connected by Cabibbo angle θ_C

$$G_V^{\Delta S=0} = \cos\theta_C G_\mu \qquad G_V^{\Delta S=1} = \sin\theta_C G_\mu$$

$$V_C = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

Kobayashi & Maskawa: 3 flavors + CPV phase

CKM unitarity - measure of completeness of the SM: $VV^\dagger = \mathbf{1}$

Top-row unitarity constraint $V_{ud}{}^2 + V_{us}{}^2 + V_{ub}{}^2 = 1$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

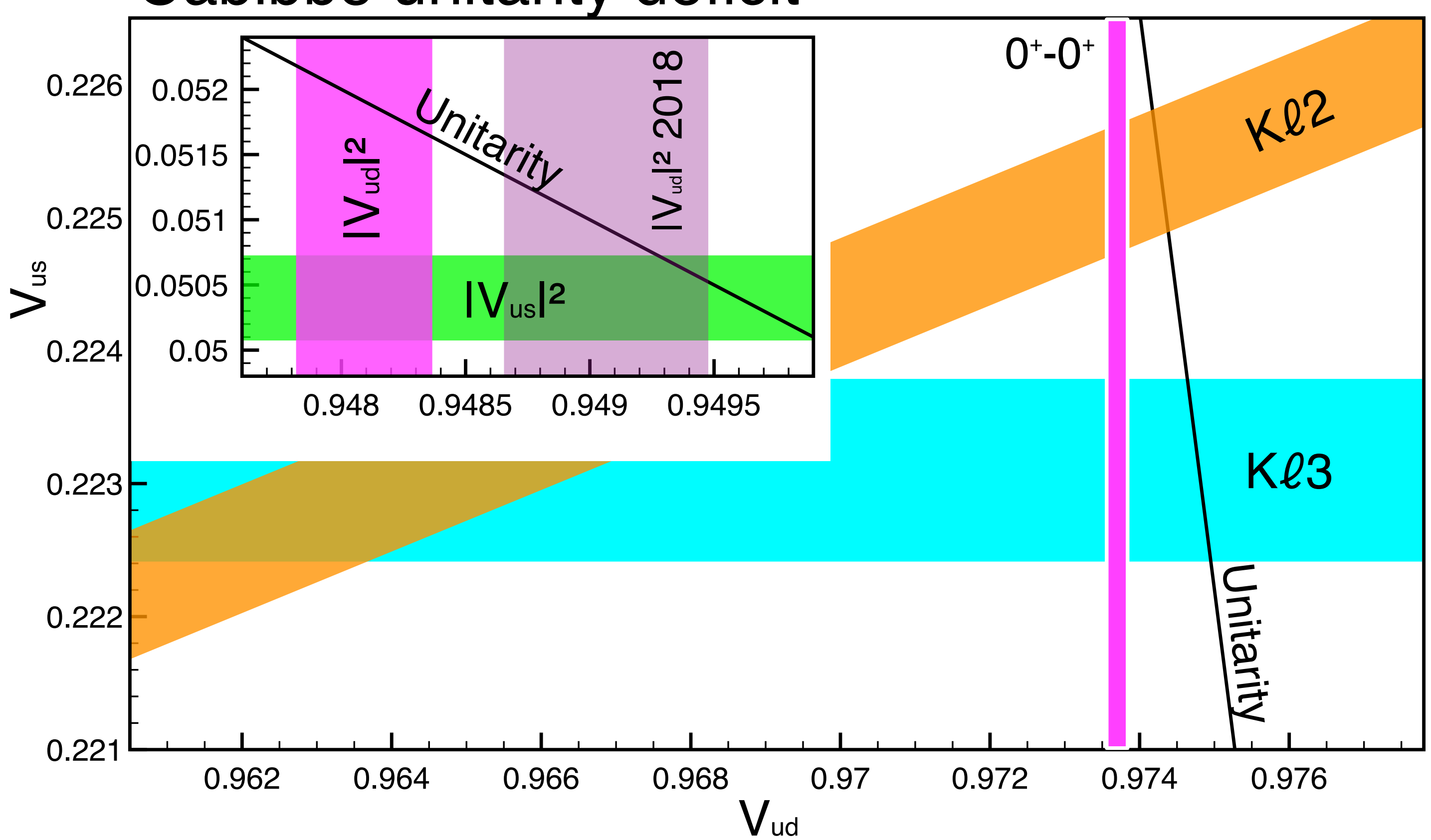
CKM unitarity: precise test of SM!
PDG 2022:

$$\underset{\sim 0.95}{V_{ud}}{}^2 + \underset{\sim 0.05}{V_{us}}{}^2 + \underset{\sim 10^{-5}}{\cancel{V_{ub}}}{}^2 = 0.9985(6)_{V_{ud}}(4)_{V_{us}}$$

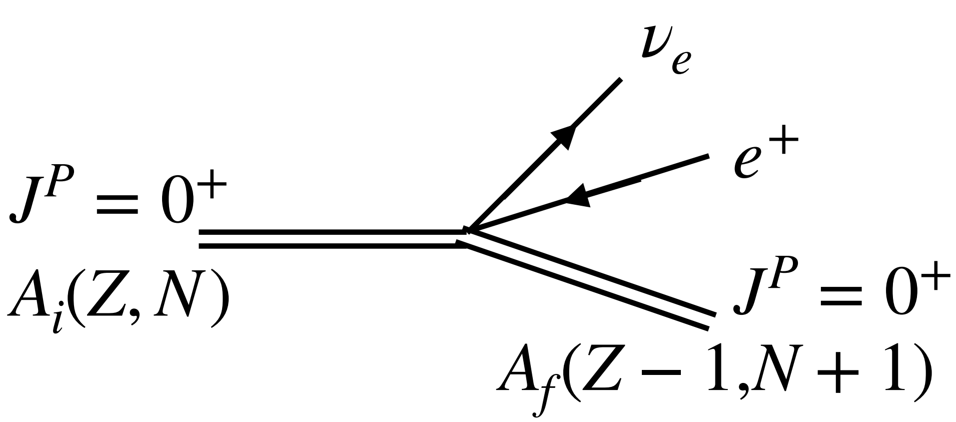
Main news:
V_{ud} from superallowed beta decays: shift downwards
Nuclear theory uncertainty increased

But: Inconsistencies between data sets
(Neutron lifetime and g_A; Kaon decays)

Cabibbo unitarity deficit

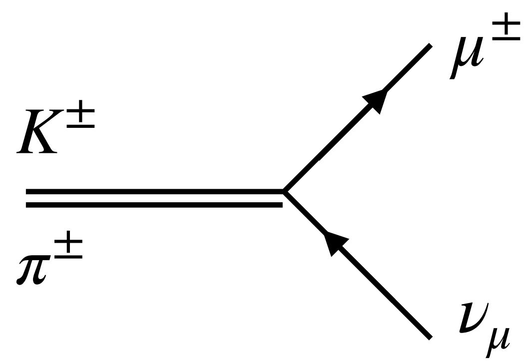


Status of Cabibbo Unitarity



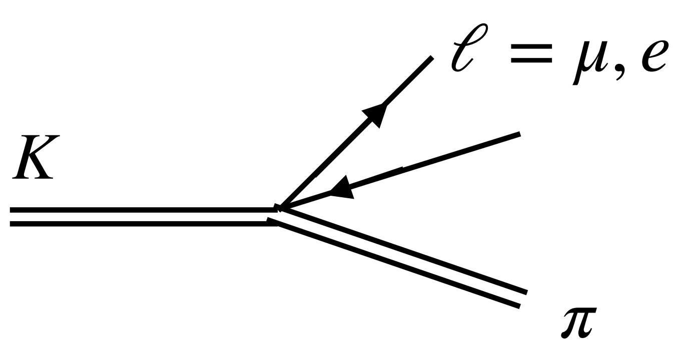
Superallowed nuclear decays: V_{ud}

$$V_{ud} = 0.9737\,(1)_{exp,\,nucl}\,(3)_{NS}\,(1)_{RC}[3]_{total}$$



Ratio of kaon/pion leptonic decay rates: ratio V_{us}/V_{ud}

$$V_{us}/V_{ud} = 0.23108(23)_{\rm exp}(42)_{\rm lat}(16)_{\rm IB} \qquad (51)_{\rm tot} = 0.22\,\%$$



Kaon semileptonic decays: absolute value of V_{us}

$$V_{us} = 0.22330(35)_{\rm exp}(39)_{\rm lat}(8)_{\rm IB} \qquad (53)_{\rm tot} = 0.24\,\%$$

Additional input (lattice, EWRC, nuclear theory & experiment) mandatory to get V_{ud}, V_{us}

Cabibbo Unitarity - 3 anomalies

Assuming experimental AND theoretical inputs correct: Cabibbo anomaly = BSM signal

3 observables: $|V_{us}|^{K\ell 3}$, $|V_{us}/V_{ud}|^{K\mu 2}$, V_{ud}
2 quantities to determine: V_{us}, V_{ud}
▶ **3 ways to test unitarity**

Nuclear beta decays + $K\ell 3$

$$\Delta^{(1)}_{\mathrm{CKM}} = |V_{ud}|^2 + |V_{us}^{K_{\ell 3}}|^2 - 1$$ **= –0.00176(56)   –3.1σ**

Nuclear beta decays + $K\ell 2$

$$\Delta^{(2)}_{\mathrm{CKM}} = |V_{ud}|^2 \left[1 + \left(\left|\frac{V_{us}}{V_{ud}}\right|^{K_{\mu 2}}\right)^2\right] - 1$$ **= –0.00098(58)   –1.7σ**

Only kaon decays

$$\Delta^{(3)}_{\mathrm{CKM}} = |V_{us}^{K_{\ell 3}}|^2 \left[\left(\frac{1}{|V_{us}/V_{ud}|^{K_{\mu 2}}}\right)^2 + 1\right] - 1$$ **= –0.0164(63)   –2.6σ**

Cabibbo Angle Anomaly as BSM Signal

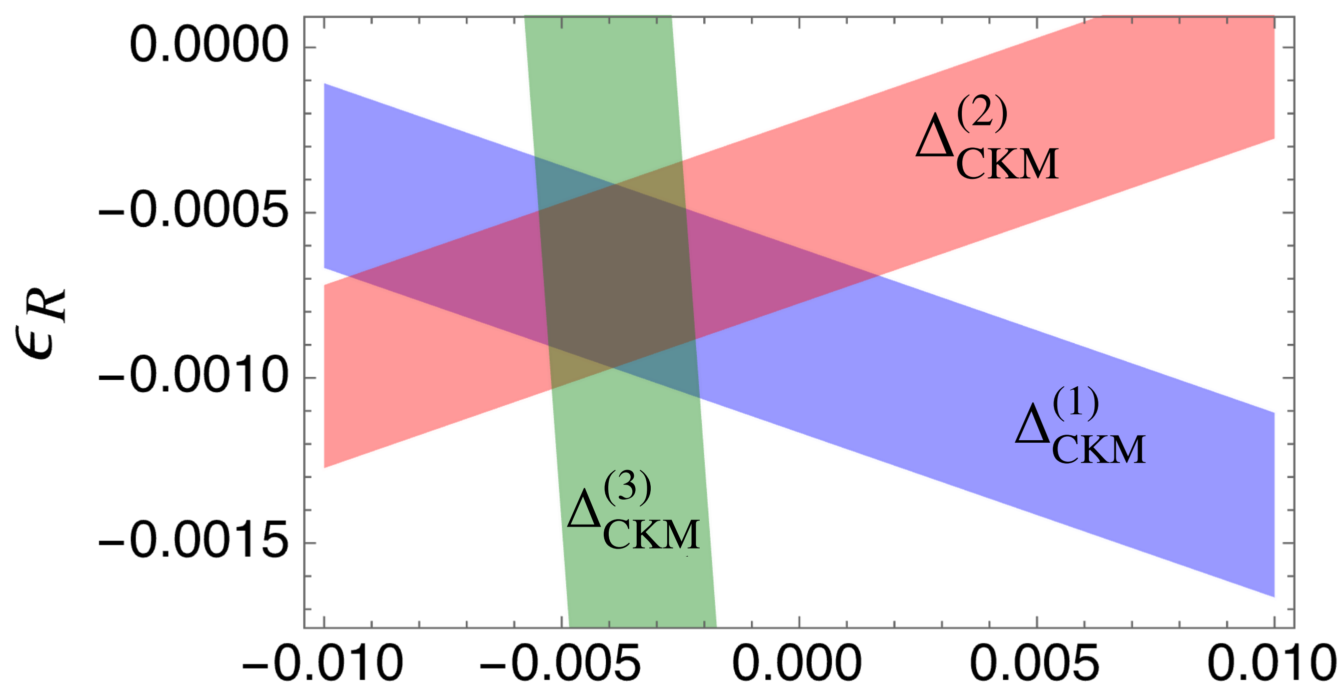
SMEFT scenario: right-handed currents in ud- and us-sectors

$$\mathscr{L}^{\text{SMEFT}}_{ud(s)se\nu} \supset -\sqrt{2}G_F^0 V_{ud}\bar{e}\gamma_\mu \nu_L[\epsilon_R \bar{u}\gamma^\mu(1+\gamma_5)d + \epsilon_R^{(s)}\bar{u}\gamma^\mu(1+\gamma_5)s]$$

V_{ud}, V_{us}, V_{ud}/V_{us} overconstrained, can solve

$$\Delta^{(1)}_{\text{CKM}} = 2\epsilon_R + 2\Delta\epsilon_R V_{us}^2,$$
$$\Delta^{(2)}_{\text{CKM}} = 2\epsilon_R - 2\Delta\epsilon_R V_{us}^2,$$
$$\Delta^{(3)}_{\text{CKM}} = 2\epsilon_R + 2\Delta\epsilon_R (2 - V_{us}^2).$$

Cirigliano, Crivellin, Hoferichter, Moulson 2208.11707



$$\Delta\epsilon_R \equiv \epsilon_R^{(s)} - \epsilon_R$$

$$\epsilon_R = -0.69(27) \times 10^{-3}$$
$$\Delta\epsilon_R = -3.9(1.6) \times 10^{-3}$$

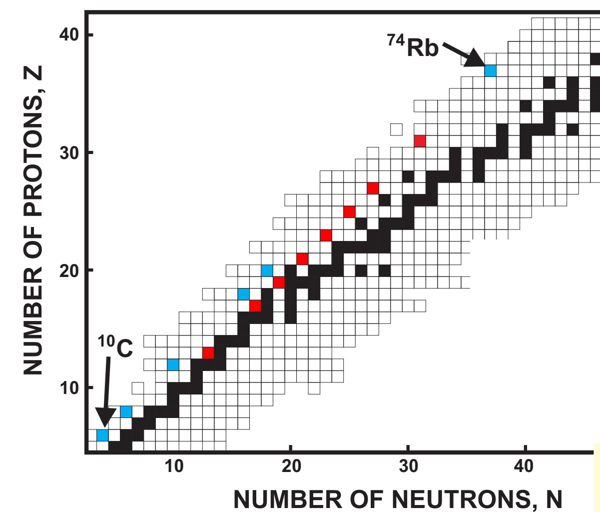
SM ($\epsilon_R = 0, \Delta\epsilon_R = 0$) excluded by 3.1σ

To claim BSM signal: critically check all ingredients
This talk: concentrate on superallowed nuclear decays

V_{ud} from superallowed decays

V_{ud} from superallowed decays

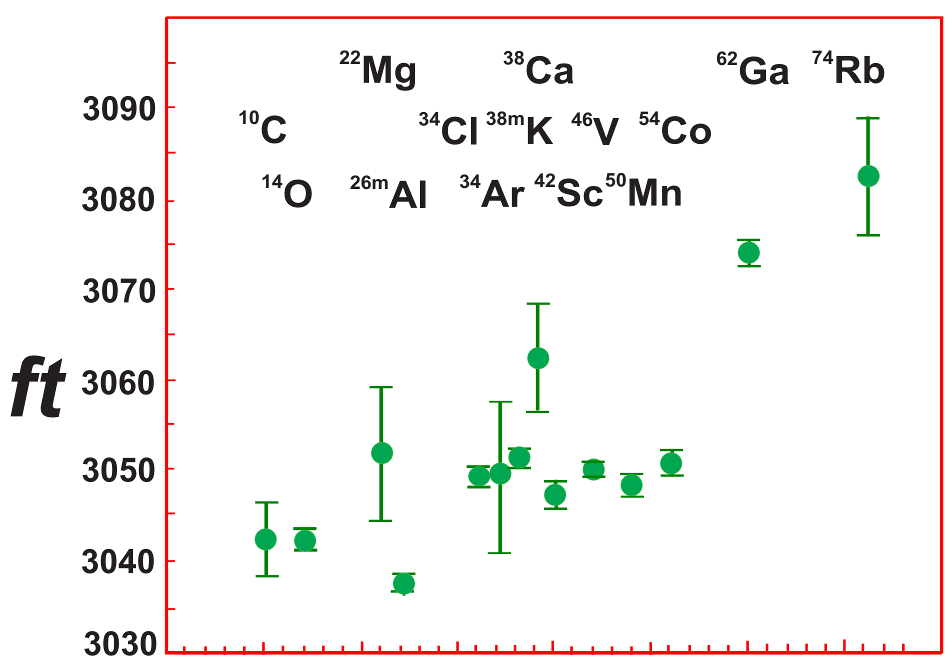
Advantages:
1. Only conserved vector current
2. 15 measured to better than 0.2%
3. 5 measured better than 0.05%
4. Internal consistency as a check
5. SU(2) good —> corrections ~small
6. We know a lot about nuclei



${}^{10}_{6}\mathrm{C} \to {}^{10}_{5}\mathrm{B}$	${}^{26m}_{13}\mathrm{Al} \to {}^{26}_{12}\mathrm{Mg}$
${}^{14}_{8}\mathrm{O} \to {}^{14}_{7}\mathrm{N}$	${}^{34}_{17}\mathrm{Cl} \to {}^{34}_{16}\mathrm{S}$
${}^{18}_{10}\mathrm{Ne} \to {}^{18}_{9}\mathrm{F}$	${}^{38m}_{19}\mathrm{K} \to {}^{38}_{18}\mathrm{Ar}$
${}^{22}_{12}\mathrm{Mg} \to {}^{22}_{11}\mathrm{Na}$	${}^{42}_{21}\mathrm{Sc} \to {}^{42}_{20}\mathrm{Ca}$
${}^{26}_{14}\mathrm{Si} \to {}^{26}_{13}\mathrm{Al}$	${}^{46}_{23}\mathrm{V} \to {}^{46}_{22}\mathrm{Ti}$
${}^{30}_{16}\mathrm{S} \to {}^{30}_{15}\mathrm{P}$	${}^{50}_{25}\mathrm{Mn} \to {}^{50}_{24}\mathrm{Cr}$
${}^{34}_{18}\mathrm{Ar} \to {}^{34}_{17}\mathrm{Cl}$	${}^{54}_{27}\mathrm{Co} \to {}^{54}_{26}\mathrm{Fe}$
${}^{38}_{20}\mathrm{Ca} \to {}^{38}_{19}\mathrm{K}$	${}^{62}_{31}\mathrm{Ga} \to {}^{62}_{30}\mathrm{Zn}$
${}^{42}_{22}\mathrm{Ti} \to {}^{42}_{21}\mathrm{Sc}$	${}^{66}_{33}\mathrm{As} \to {}^{66}_{32}\mathrm{Ge}$
${}^{46}_{24}\mathrm{Cr} \to {}^{46}_{23}\mathrm{V}$	${}^{70}_{35}\mathrm{Br} \to {}^{70}_{34}\mathrm{Se}$
${}^{50}_{26}\mathrm{Fe} \to {}^{50}_{25}\mathrm{Mn}$	${}^{74}_{37}\mathrm{Rb} \to {}^{74}_{36}\mathrm{Kr}$
${}^{54}_{28}\mathrm{Ni} \to {}^{54}_{27}\mathrm{Co}$	

Exp.: **f** - phase space (Q value)

t - partial half-life ($t_{1/2}$, branching ratio)



ft values: same within ~2% but not exactly!

Reason: SU(2) slightly broken

a. RC (e.m. interaction does not conserve isospin)

b. Nuclear WF are not SU(2) symmetric
(proton and neutron distribution not the same)

V_{ud} extraction: Universal RC and Universal Ft

To obtain Vud —> absorb all decay-specific corrections into universal **Ft**

$$\mathrm{ft}(1+\mathrm{RC}+\mathrm{ISB}) = \mathscr{F}\mathrm{t}(1+\Delta_R^V) = \mathrm{ft}(1+\delta_R')(1-\delta_C+\delta_{NS})(1+\Delta_R^V)$$

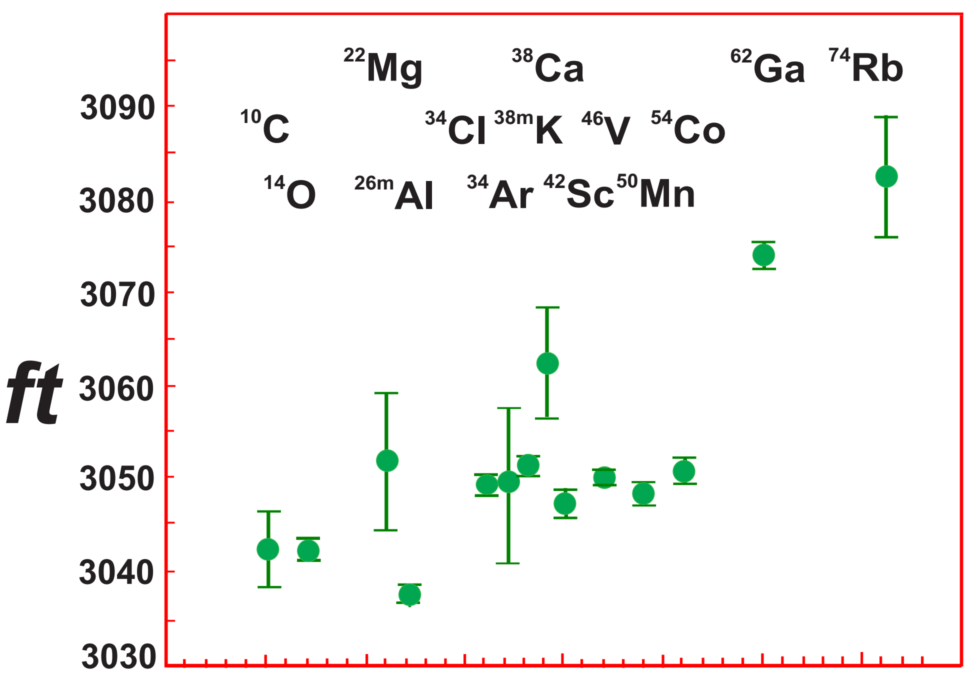
Experiment + Coulomb (theory)

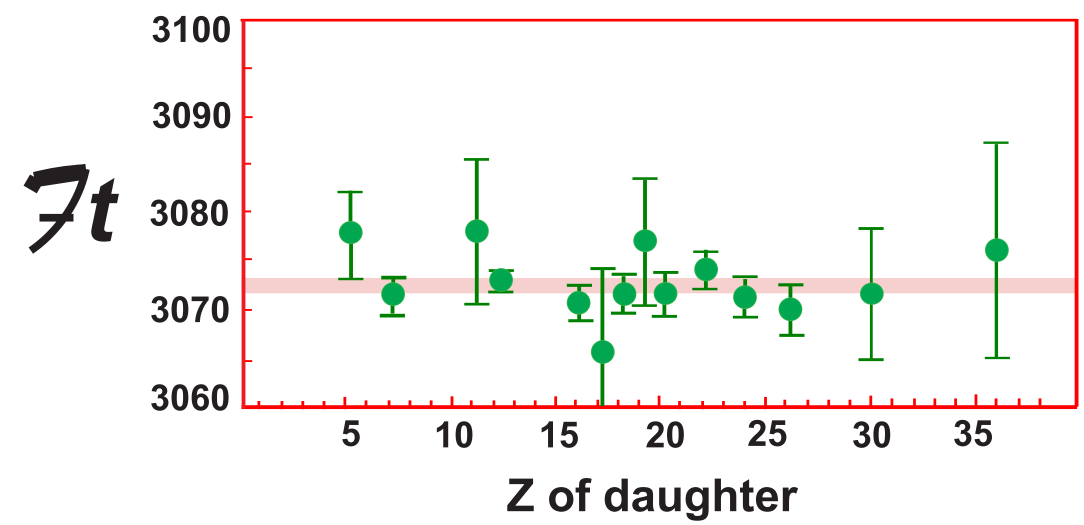
QED (Brems)

Isospin breaking

Nuclear Structure

Universal RC





Average of 15 decays

Hardy, Towner 1972 - 2020

$$V_{ud}^{\ 2} = \frac{2984.43s}{\mathscr{F}t(1+\Delta_R^V)}$$

Pre-2018: $\overline{\mathscr{F}t} = 3072.1 \pm 0.7\ s$

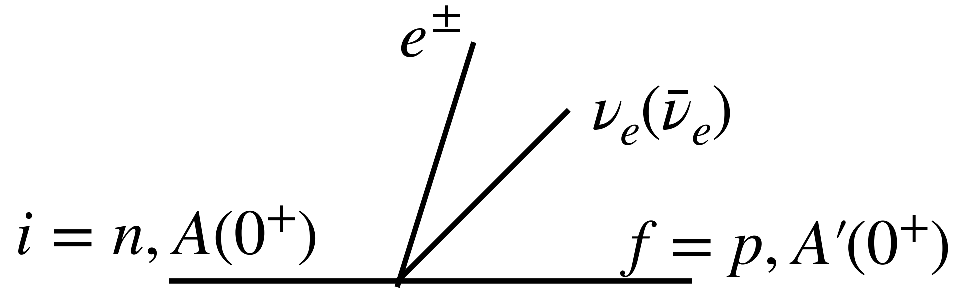
PDG 2022: $\overline{\mathscr{F}t} = 3072 \pm 2\ s$

$$V_{ud}^{0^+-0^+} = 0.9737\,(1)_{exp,\,nucl}\,(3)_{NS}\,(1)_{RC}[3]_{total}$$

RC to beta decay: overall setup

RC to beta decay: overall setup

Tree-level amplitude



$\sim V_{ud}$

Radiative corrections to tree-level amplitude $\sim \alpha/2\pi \approx 10^{-3}$

Precision goal for V_{ud} extraction 1×10^{-4}

Electron carries away energy E < Q-value of a decay

E-dep RC: $\frac{\alpha}{2\pi}\left(\frac{E}{\Lambda}, \ln\frac{E}{\Lambda}, \ldots\right)$

Energy scales Λ

Λ

Weak boson scale
$M_Z, M_W \sim 90\,\mathrm{GeV}$

Hadronic scale
$\Lambda_{\mathrm{had}} = 300\,\mathrm{MeV}$

Universal

Nuclear scale
$\Lambda_{\mathrm{nuc}} = 10 - 30\,\mathrm{MeV}$

Nuclear structure dependent
(QCD)

Decay Q-value (endpoint energy)
$Q_{if} = M_i - M_f = 1 - 10\,\mathrm{MeV}$

Nucleus-specific

Electron mass
$m_e \approx 0.5\,\mathrm{MeV}$

Nuclear structure independent
(QED)

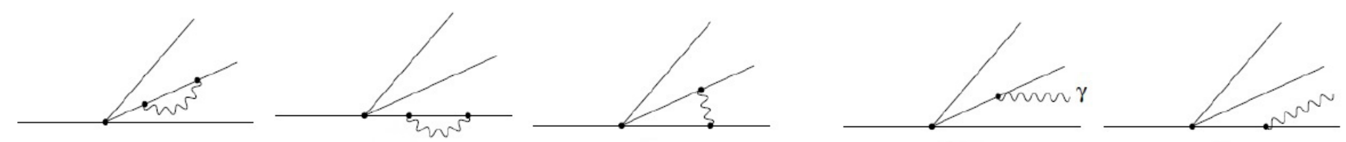
RC to beta decay: overall setup

Generically: only IR and UV extremes feature large logarithms!
Works by Sirlin (1930-2022) and collaborators: all large logs under control

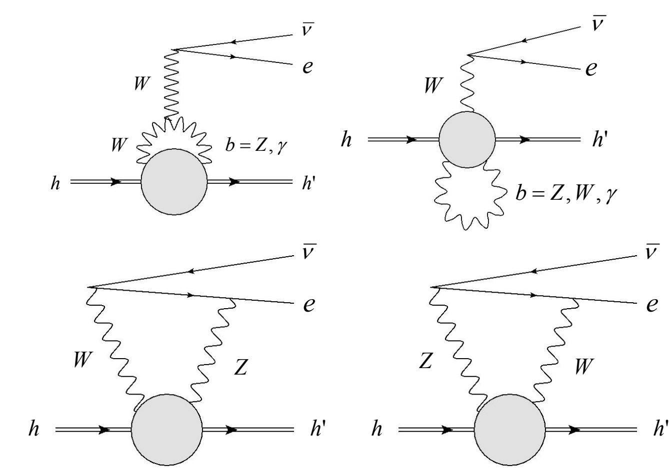
IR: Fermi function + Sirlin function

Fermi function: resummation of $(Z\alpha)^n$ —> Dirac - Coulomb problem

Sirlin function (outer correction):
All IR-div. pieces beyond Coulomb distortion



UV: large EW logs + pQCD corrections

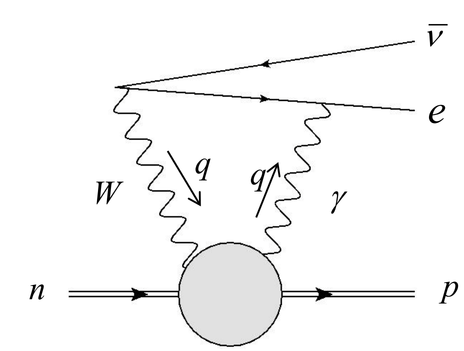


Inner RC:
energy- and model-independent

W,Z - loops
UV structure of SM

γW-box: sensitive to all scales

New method for computing EW boxes: dispersion theory
Combine exp. data with pQCD, lattice, EFT, ab-initio nuclear



RC to β decay - scale separation: history

Fermi function (pure Coulomb + nuclear size & recoil + atomic) —> phase-space **f**

Fermi, Behrens-Bühring, Wilkinson…

Soft Bremsstrahlung: universal Sirlin's function + nucleus specific corrections —> δ'_R

All IR-sensitive pieces: recent review

Hayen et al RMP 2018

UV-sensitive RC on free neutron Δ_R^V: Sirlin, Marciano, Czarnecki 1967 - 2006

$$G_V^2 = V_{ud}^{\;2}\left[1 + \frac{\alpha}{2\pi}\left\{3\ln\frac{M_Z}{M_p} + \ln\frac{M_Z}{M_W} + \tilde{a}_g\right\} + \delta_{\rm QED}^{HO} + 2\,\Box_{\gamma W}\right] \equiv [1+\Delta_R^V]$$

All scales assumed to be perfectly separated!

Isospin breaking (non-RC): Coulomb repulsion b. protons —> δ_C

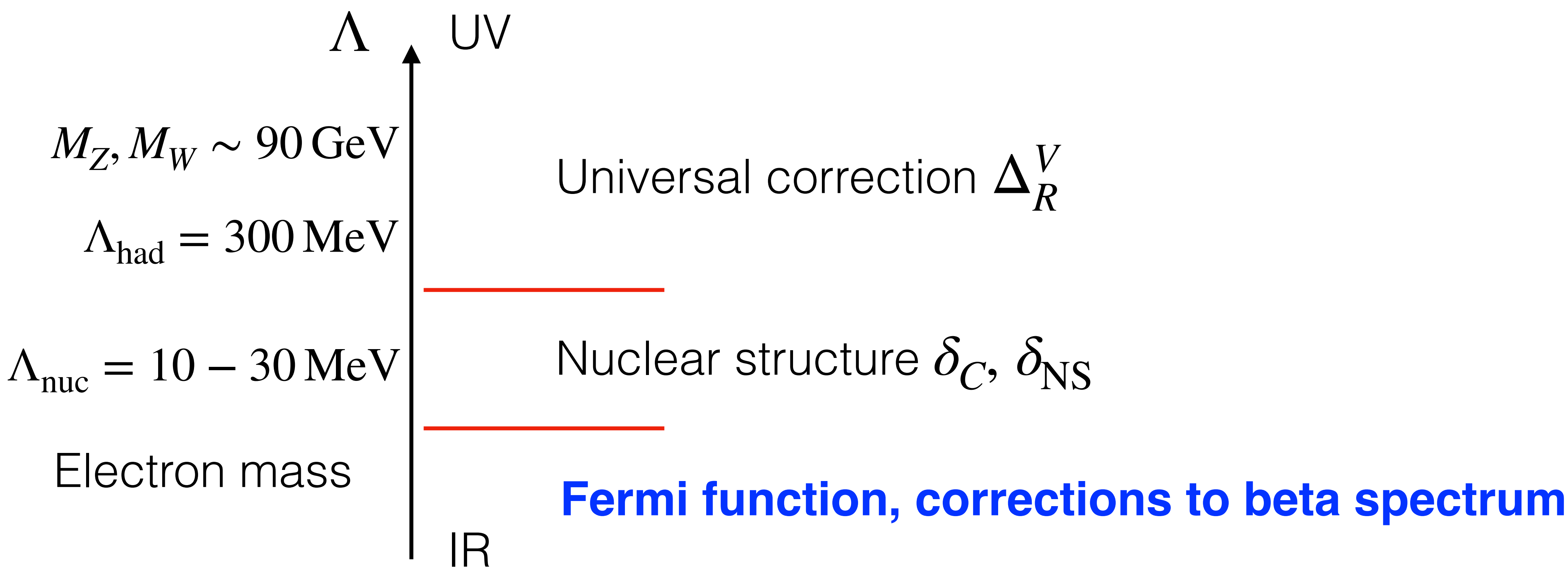
MacDonald 1958

Nuclear structure δ_{NS} —> only since1990

Jaus, Rasche 1990

Hardy, Towner 1992-2020

$$ft(1 + \mathrm{RC} + \mathrm{ISB}) = \mathscr{F}\mathrm{t}(1 + \Delta_\mathrm{R}^\mathrm{V}) = \mathrm{ft}(1 + \delta'_\mathrm{R})(1 - \delta_\mathrm{C} + \delta_\mathrm{NS})(1 + \Delta_\mathrm{R}^\mathrm{V})$$



Long-Range QED Corrections to Beta Spectrum and ft-values

QED: Corrections to Decay Spectrum

$$f = m_e^{-5} \int_{m_e}^{E_0} dE_e \; \boxed{\vec{p}_e \;\; E_e (E_0 - E_e)^2} \, F(E_e) C(E_e) Q(E_e) R(E_e) r(E_e)$$

Unperturbed beta spectrum

Fermi function: e+ in Coulomb field of daughter nucleus

Recoil correction

Shape factor: spatial distribution of decay

Atomic screening and overlap corrections

Coulomb distortion numerically large: escapes the usual scaling α/π
Fermi function $F_0 \sim Z\alpha\pi/\beta$ (coherent effect, Sommerfeld and π^2 enhancement)

Since Fermi fn is of order 1 —> even small corrections should be assessed.
A myriad of corrections introduced/estimated by different people in past 9 decades!

$$N(W)dW = \frac{G_V^2 V_{ud}^2}{2\pi^3} \; F_0(Z,W)\; L_0(Z,W)\; U(Z,W)\; D_{\mathrm{FS}}(Z,W,\beta_2)\; R(W,W_0)\; R_N(W,W_0,M)$$
$$\times \; Q(Z,W)\; S(Z,W)\; X(Z,W)\; r(Z,W)\; C(Z,W)\; D_C(Z,W,\beta_2)\; pW(W_0-W)^2\; dW$$

Unified method of calculation (matching between them is well-defined)
numerical solution of Dirac equation with inputs from nuclear theory and experiment

Nuclear Structure Inputs in ft

$$f = m_e^{-5} \int_{m_e}^{E_0} dE_e \; \vec{p}_e \; E_e (E_0 - E_e)^2 F(E_e) C(E_e) Q(E_e) R(E_e) r(E_e)$$

QED

Fermi Fn: daughter **nuclear charge form factor** $F_{Ch}(q^2)$

Shape factor: **nuclear weak CC transition FF** $F_{CW}(q^2)$

Charge form factors: combination of e-scattering, X-ray/laser/optical atom spectroscopy
Slope of the charge FF at origin: nuclear charge radius
Not all radii are known —> have to be guessed (theory)

Charged-current weak transition form factors: only accessible with the decay itself (tough);
Historically estimated in nuclear shell model with 1B current (Wilkinson; Hardy & Towner; …)
Typical result: very similar to charge FF

New development:
use isospin symmetry and known charge radii to predict the weak transition radius!

Isospin symmetry + Charge Radii in 0^+ isotriplet

$0^+, T=1, T_z=-1$ — $R_{\mathrm{Ch},-1}$ $\xrightarrow{R_{\mathrm{CW}}}$ $0^+, T=1, T_z=0$ — $R_{\mathrm{Ch},0}$ $\xrightarrow{R_{\mathrm{CW}}}$ $0^+, T=1, T_z=1$ — $R_{\mathrm{Ch},1}$

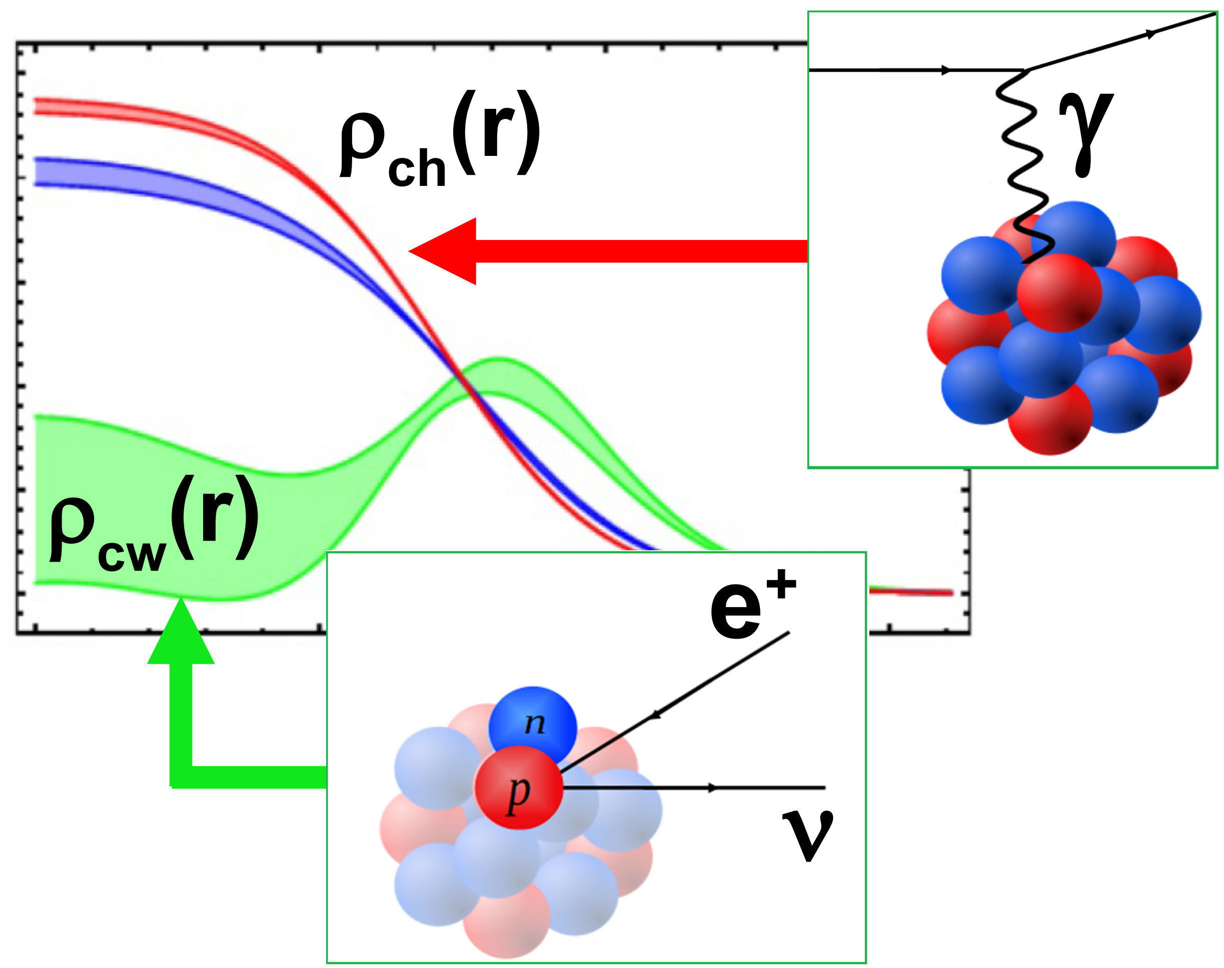
CY Seng, 2212.02681

Charged-Current weak current: pure isovector
Electromagnetic current isovector + isoscalar

Remove isoscalar part:
Relate weak <—> charge radii

$$
\begin{aligned}
R^2_{\mathrm{CW}} &= R^2_{\mathrm{Ch},1} + Z_0(R^2_{\mathrm{Ch},0} - R^2_{\mathrm{Ch},1}) \\
&= R^2_{\mathrm{Ch},1} + \frac{Z_{-1}}{2}(R^2_{\mathrm{Ch},-1} - R^2_{\mathrm{Ch},1})
\end{aligned}
$$

Large factors ~Z multiply small differences



Photon probes the entire nuclear charge
Only the outer protons can decay: all neutron states in the core occupied

Isospin symmetry + Charge Radii in $T=1, O^{+}$isotriplet

A	$\langle r^2_{\mathrm{ch},-1}\rangle^{1/2}$ (fm)	$\langle r^2_{\mathrm{ch},0}\rangle^{1/2}$ (fm)	$\langle r^2_{\mathrm{ch},1}\rangle^{1/2}$ (fm)	$\langle r^2_{\mathrm{cw}}\rangle^{1/2}$ (fm)
10	${}^{10}_{6}$C	${}^{10}_{5}$B(ex)	${}^{10}_{4}$Be: 2.3550(170)[a]	N/A
14	${}^{14}_{8}$O	${}^{14}_{7}$N(ex)	${}^{14}_{6}$C: 2.5025(87)[a]	N/A
18	${}^{18}_{10}$Ne: 2.9714(76)[a]	${}^{18}_{9}$F(ex)	${}^{18}_{8}$O: 2.7726(56)[a]	3.661(72)
22	${}^{22}_{12}$Mg: 3.0691(89)[b]	${}^{22}_{11}$Na(ex)	${}^{22}_{10}$Ne: 2.9525(40)[a]	3.596(99)
26	${}^{26}_{14}$Si	${}^{26m}_{13}$Al: 3.130(15)[f]	${}^{26}_{12}$Mg: 3.0337(18)[a]	4.11(15)
30	${}^{30}_{16}$S	${}^{30}_{15}$P(ex)	${}^{30}_{14}$Si: 3.1336(40)[a]	N/A
34	${}^{34}_{18}$Ar: 3.3654(40)[a]	${}^{34}_{17}$Cl	${}^{34}_{16}$S: 3.2847(21)[a]	3.954(68)
38	${}^{38}_{20}$Ca: 3.467(1)[c]	${}^{38m}_{19}$K: 3.437(4)[d]	${}^{38}_{18}$Ar: 3.4028(19)[a]	3.999(35)
42	${}^{42}_{22}$Ti	${}^{42}_{21}$Sc: 3.5702(238)[a]	${}^{42}_{20}$Ca: 3.5081(21)[a]	4.64(39)
46	${}^{46}_{24}$Cr	${}^{46}_{23}$V	${}^{46}_{22}$Ti: 3.6070(22)[a]	N/A
50	${}^{50}_{26}$Fe	${}^{50}_{25}$Mn: 3.7120(196)[a]	${}^{50}_{24}$Cr: 3.6588(65)[a]	4.82(39)
54	${}^{54}_{28}$Ni: 3.738(4)[e]	${}^{54}_{27}$Co	${}^{54}_{26}$Fe: 3.6933(19)[a]	4.28(11)
62	${}^{62}_{32}$Ge	${}^{62}_{31}$Ga	${}^{62}_{30}$Zn: 3.9031(69)[b]	N/A
66	${}^{66}_{34}$Se	${}^{66}_{33}$As	${}^{66}_{32}$Ge	N/A
70	${}^{70}_{36}$Kr	${}^{70}_{35}$Br	${}^{70}_{34}$Se	N/A
74	${}^{74}_{38}$Sr	${}^{74}_{37}$Rb: 4.1935(172)[b]	${}^{74}_{36}$Kr: 4.1870(41)[a]	4.42(62)

Seng, 2212.02681
MG, Seng 2311.16755

Weak radii differ significantly from R_{ch}
Shape factor—> Fermi Fn —> ft

Transition	$(ft)_{\mathrm{HT}}$ (s)	$(ft)_{\mathrm{new}}(s)$
^{18}Ne$\to{}^{18}$F	2912 ± 79	2912 ± 80
^{22}Mg$\to{}^{22}$Na	3051.1 ± 6.9	3050.4 ± 6.8
^{26}Si$\to{}^{26m}$Al	3052.2 ± 5.6	3050.7 ± 5.6
^{34}Ar$\to{}^{34}$Cl	3058.0 ± 2.8	3057.1 ± 2.8
^{38}Ca$\to{}^{38m}$K	3062.8 ± 6.0	3062.2 ± 5.9
^{42}Ti$\to{}^{42}$Sc	3090 ± 88	3085 ± 86
^{50}Fe$\to{}^{50}$Mn	3099 ± 71	3098 ± 72
^{54}Ni$\to{}^{54}$Co	3062 ± 50	3063 ± 49
^{26m}Al$\to{}^{26}$Mg	3037.61 ± 0.67	3036.5 ± 1.0
^{34}Cl$\to{}^{34}$S	$3049.43^{+0.95}_{-0.88}$	3048.0 ± 1.1
^{38m}K$\to{}^{38}$Ar	3051.45 ± 0.92	3050.5 ± 1.1
^{42}Sc$\to{}^{42}$Ca	3047.7 ± 1.2	3045.0 ± 2.7
^{50}Mn$\to{}^{50}$Cr	3048.4 ± 1.2	3046.1 ± 3.6
^{54}Co$\to{}^{54}$Fe	$3050.8^{+1.4}_{-1.1}$	$3051.3^{+1.7}_{-1.4}$
^{74}Rb$\to{}^{74}$Kr	3082.8 ± 6.5	3086 ± 11

New ft vs estimates by Hardy and Towner

Relative shift downwards of 0.01-0.1%
Non-negligible given the precision goal 0.01%

More -and more precise- charge radii necessary!
Working closely with exp. (PSI, FRIB, ISOLDE)

Isospin symmetry + Charge Radii in $T=1, O^{+}$isotriplet

Above treatment assumes isospin symmetry — but we know that it is slightly broken!
Why isospin symmetry assumption is good enough?

Shape factor and finite size effects are ~small corrections to Fermi function
1-2% ISB effect on top of a RC may be assumed negligible (but needs to be tested)

Test requires that all 3 nuclear radii in the isotriplet are known;
Currently only the case for A=38 system

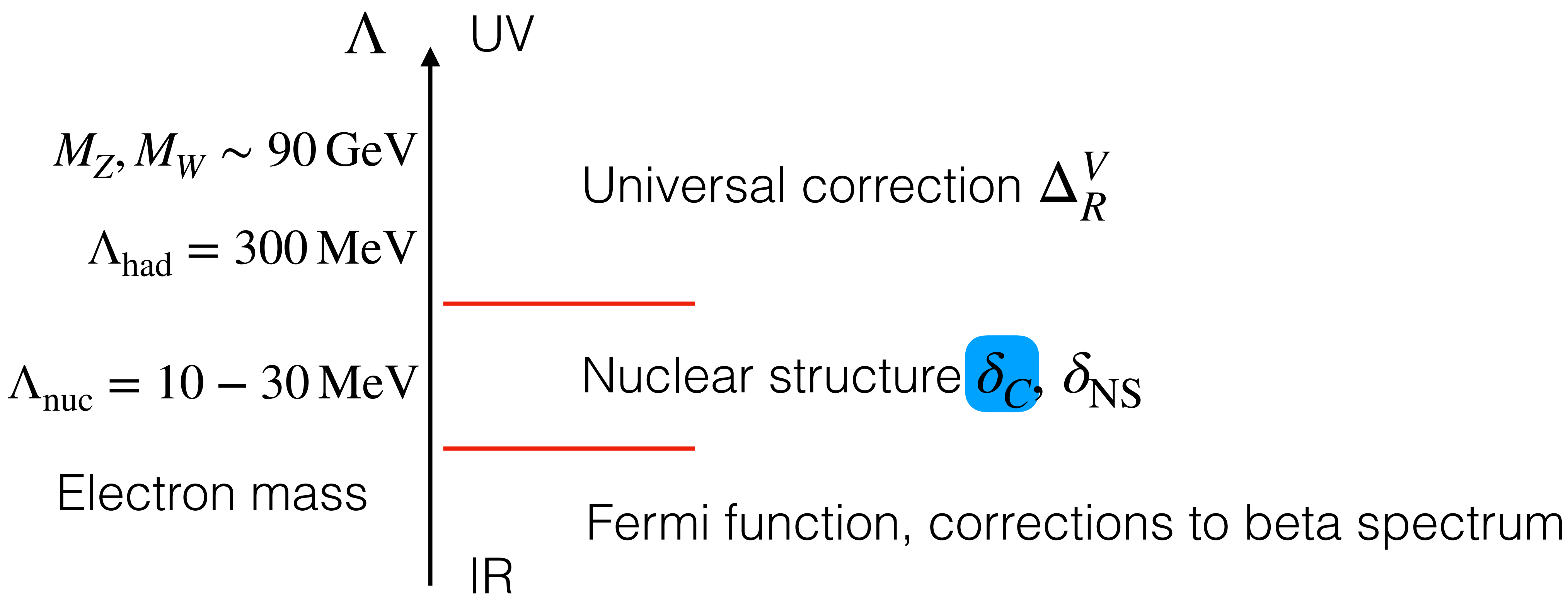
26	${}^{26}_{14}\mathrm{Si}$	${}^{26m}_{13}\mathrm{Al}$: $3.130(15)^f$	${}^{26}_{12}\mathrm{Mg}$: $3.0337(18)^a$	4.11(15)
30	${}^{30}_{16}\mathrm{S}$	${}^{30}_{15}\mathrm{P(ex)}$	${}^{30}_{14}\mathrm{Si}$: $3.1336(40)^a$	N/A
34	${}^{34}_{18}\mathrm{Ar}$: $3.3654(40)^a$	${}^{34}_{17}\mathrm{Cl}$	${}^{34}_{16}\mathrm{S}$: $3.2847(21)^a$	3.954(68)
38	${}^{38}_{20}\mathrm{Ca}$: $3.467(1)^c$	${}^{38m}_{19}\mathrm{K}$: $3.437(4)^d$	${}^{38}_{18}\mathrm{Ar}$: $3.4028(19)^a$	3.999(35)
42	${}^{42}_{22}\mathrm{Ti}$	${}^{42}_{21}\mathrm{Sc}$: $3.5702(238)^a$	${}^{42}_{20}\mathrm{Ca}$: $3.5081(21)^a$	4.64(39)
46	${}^{46}\mathrm{Cr}$	${}^{46}\mathrm{V}$	${}^{46}\mathrm{Ti}$: $3.6070(22)^a$	N/A

ISB-sensitive combination (more on that later)

$$\Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2\right) - Z_0 R_{p,0}^2 = 0 \quad \text{if isospin symmetry exact}$$

$$\frac{1}{2}\left(20 \times 3.467(1)^2 + 18 \times 3.4028(19)^2\right) - 19 \times 3.437(4)^2 = -0.00(12)(14)(52)$$

Improvement of K-38m radius necessary! (Plans at TRIUMF on IS K-38m, K-37?)



Isospin breaking in nuclear WF: δ_C

Tree-level effect — ISB “large”

Isospin symmetry breaking in superallowed β-decay

Tree-level Fermi matrix element

$$M_F = \langle f \;\; \tau^+ \;\; i \rangle$$

τ^+ — Isospin operator

$i\rangle, \;\; f\rangle$ — members of T=1 isotriplet

If isospin symmetry were exact, $M_F \to M_0 = \sqrt{2}$

Isospin symmetry is broken in nuclear states
(e.g. Coulomb, nucleon mass difference, …)

In presence of isospin symmetry breaking (ISB):

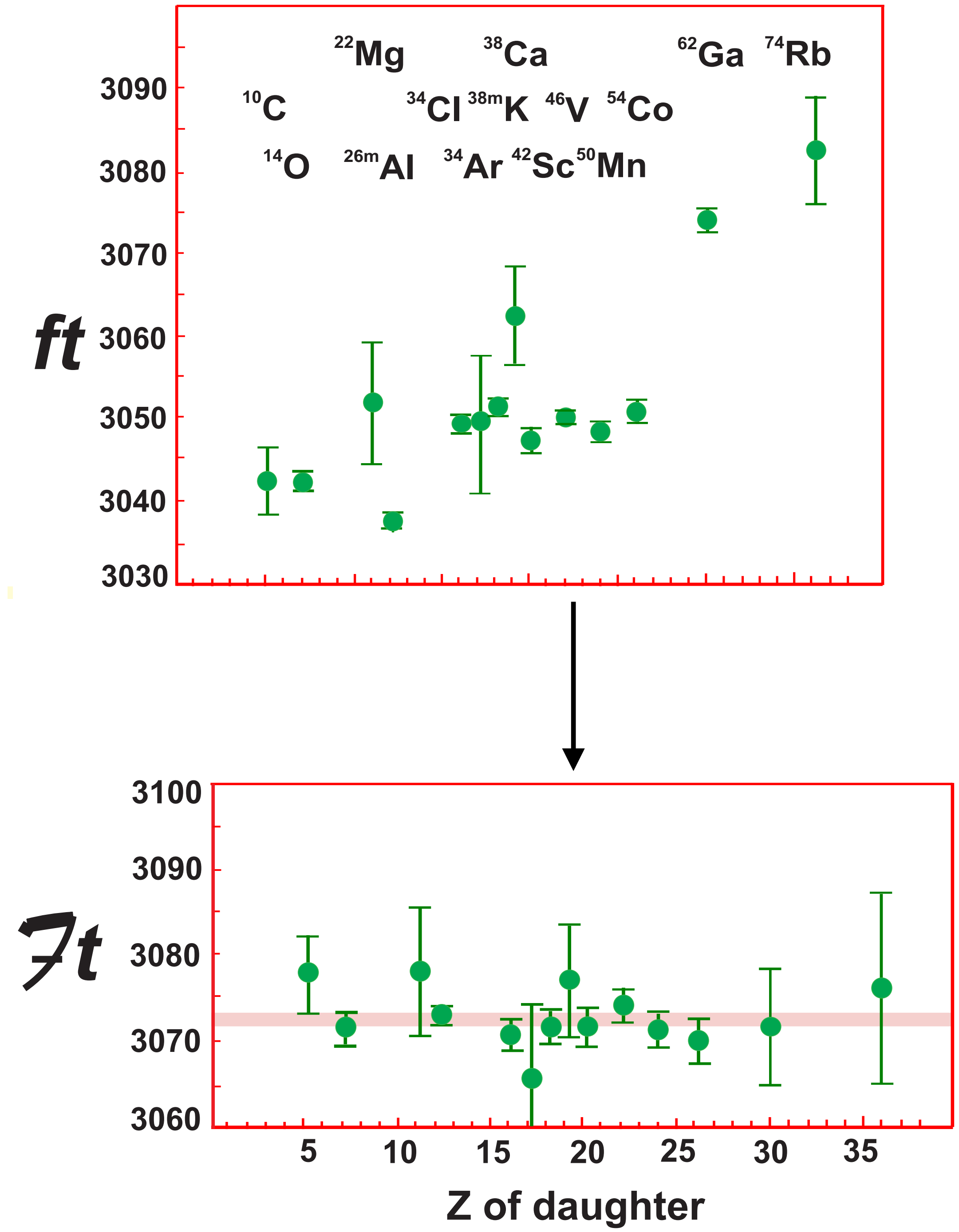
$$M_F^{\;2} = \; M_0^{\;2}(1 - \delta_C)$$

MacDonald 1958

ISB correction almost singlehandedly aligns ft-values!

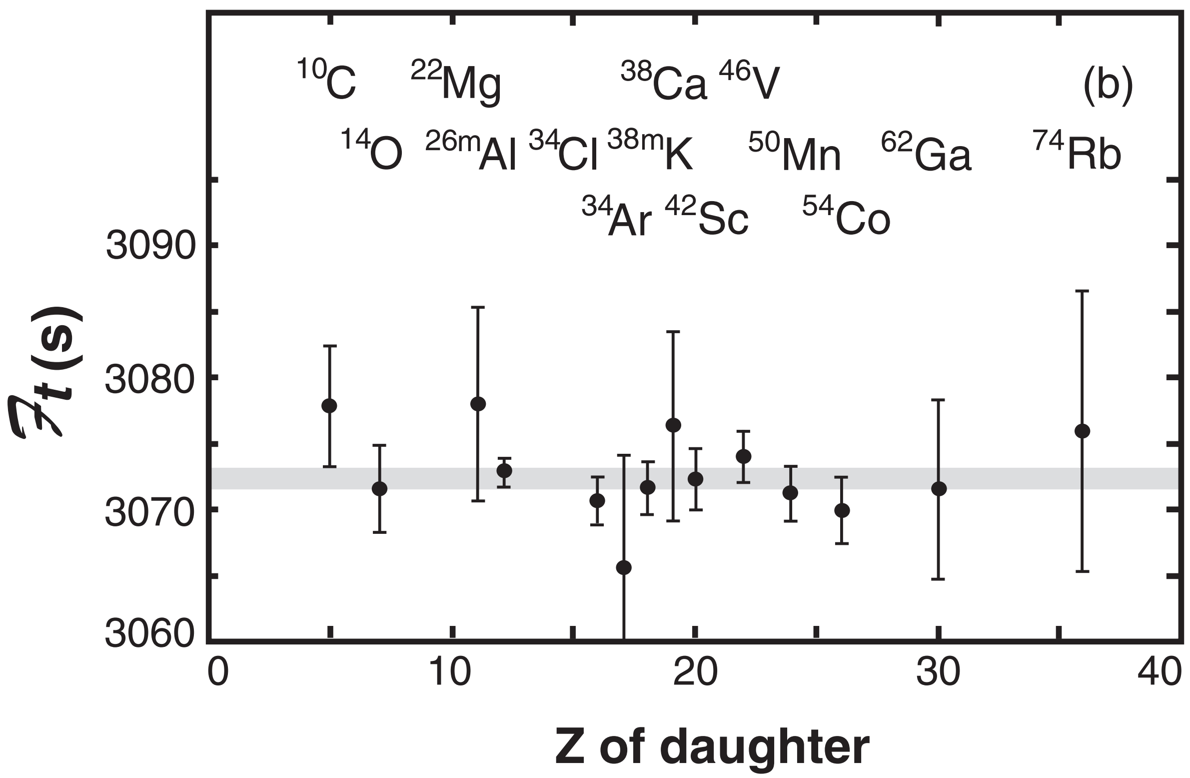
$$\delta_C \sim 0.17\% - 1.6\%!$$

Crucial for V_{ud} extraction



J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501

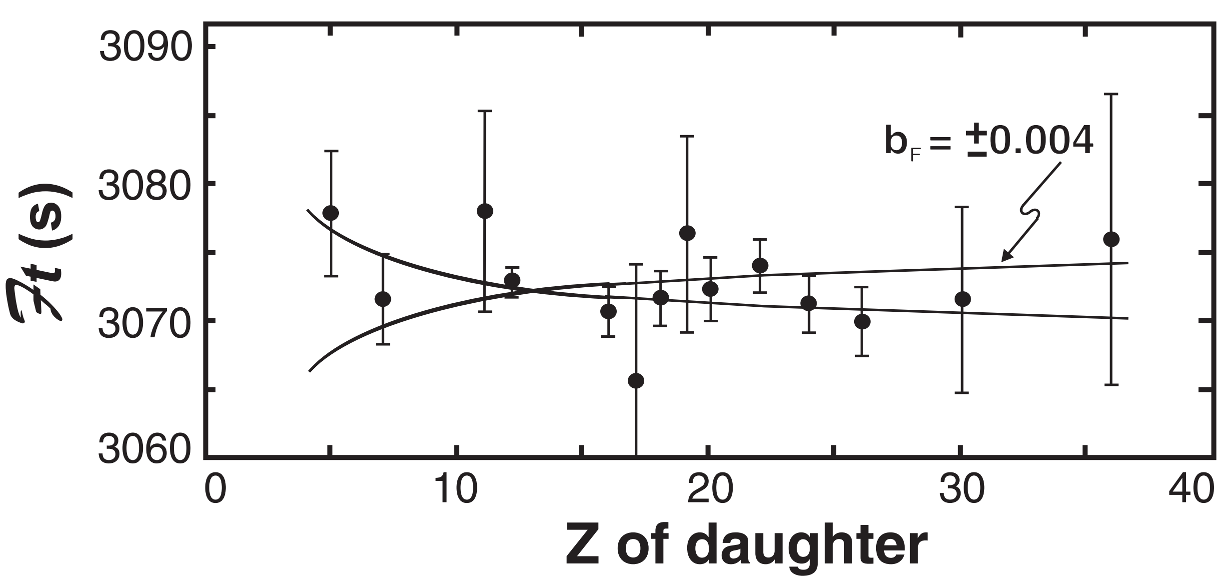
Nuclear Corrections vs. scalar BSM



Once all corrections are included:
CVC —> Ft constant

Fit to 14 transitions:
Ft constant within 0.02%

Hardy, Towner 2020



If BSM scalar currents present: Fierz interference b_F

$$\mathcal{F}t^{SM} \rightarrow \mathcal{F}t^{SM}\left(1 + b_F \frac{m_e}{\langle E_e \rangle}\right)$$

$Q_{EC} \uparrow$ with Z —> effect of $b_F \downarrow$ with Z
Introduces nonlinearity in the Ft plot

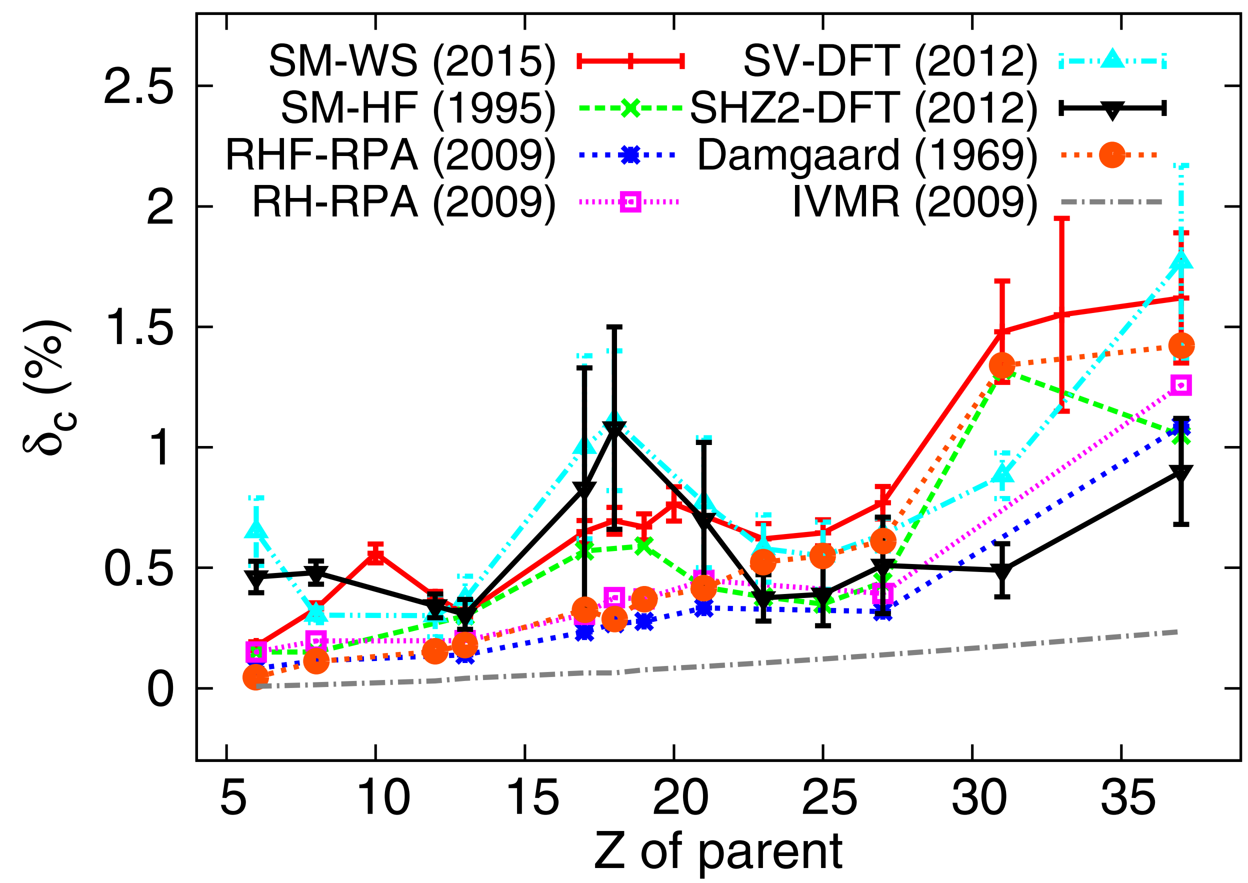
$b_F = -0.0028(26)$ ~ consistent with 0

Nuclear model dependence of δ_C

J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501

			RPA				
	SM-WS	SM-HF	PKO1	DD-ME2	PC-F1	IVMR[a]	DFT
$T_z = -1$							
^{10}C	0.175	0.225	0.082	0.150	0.109	0.147	0.650
^{14}O	0.330	0.310	0.114	0.197	0.150		0.303
^{22}Mg	0.380	0.260					0.301
^{34}Ar	0.695	0.540	0.268	0.376	0.379		
^{38}Ca	0.765	0.620	0.313	0.441	0.347		
$T_z = 0$							
^{26m}Al	0.310	0.440	0.139	0.198	0.159		0.370
^{34}Cl	0.650	0.695	0.234	0.307	0.316		
^{38m}K	0.670	0.745	0.278	0.371	0.294	0.434	
^{42}Sc	0.665	0.640	0.333	0.448	0.345		0.770
^{46}V	0.620	0.600					0.580
^{50}Mn	0.645	0.610					0.550
^{54}Co	0.770	0.685	0.319	0.393	0.339		0.638
^{62}Ga	1.475	1.205					0.882
^{74}Rb	1.615	1.405	1.088	1.258	0.668		1.770
χ^2/ν	1.4	6.4	4.9	3.7	6.1		4.3[b]

L. Xayavong, N.A. Smirnova, Phys.Rev. C 97 (2018), 024324



HT: χ^2 as criterion to prefer SM-WS; V_{ud} and limits on BSM strongly depend on nuclear model

Nuclear community embarked on ab-initio δ_C calculations (NCSM, GFMC, CC, IMSRG)
Especially interesting for light nuclei accessible to different techniques!

Constraints on δ_C from nuclear radii

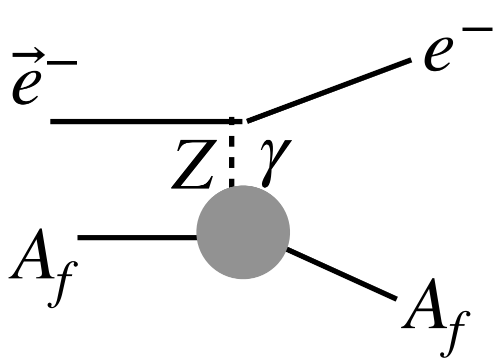
ISB-sensitive combinations of radii can be constructed **Seng, MG 2208.03037; 2304.03800; 2212.02681**

$$\Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2\right) - Z_0 R_{p,0}^2$$

$\Delta M_B^{(1)} = 0$ used for ft-value in isospin limit

$$\Delta M_A^{(1)} \equiv -\langle r_{\rm CW}^2\rangle + \left(\frac{N_1}{2}\langle r_{n,1}^2\rangle - \frac{Z_1}{2}\langle r_{p,1}^2\rangle\right)$$

Neutron radius: measurable with PV e- scattering!



$$A^{PV} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\frac{Q_W}{Z}\frac{F_{NW}(Q^2)}{F_{Ch}(Q^2)}$$

$R_{\rm NW} \approx R_n$

Z-boson couples to neutrons, photon - to protons;
PV asymmetry at low Q^2 sensitive to the difference $\langle r_{n,1}^2\rangle - \langle r_{p,1}^2\rangle$ - neutron skin
Extensive studies in neutron rich nuclei (PREX, CREX) —> input to physics of neutron stars

Upcoming exp. program at Mainz (MREX)
Neutron skins of stable daughters (e.g. Mg-26, Ca-42, Fe-54)
PV asymmetry on C-12 for a sub-% measurement of R_n

N. Cargioli, M. Cadeddu, MG,
J. Piekarewicz, X. Roca Maza,
H. Spiesberger — *in preparation*

Impact of precise nuclear radii on Ft and V_{ud}

Recent measurement at ISOLDE

Plattner et al, arXiv: 2310.15291

Wilfried's talk

Isotope shift in aluminum 27-26m
$3s^2 3p\ ^2P_{3/2}$ —> $3s^2 4s\ ^2S_{1/2}$ transition

$$\delta\nu^{27,26m} = F\delta\langle r^2\rangle^{27,26m} + M\frac{m_{26m} - m_{27}}{m_{27}(m_{26m} + m_e)}$$

Input from atomic theory: F,M
Reference radius Al-27 from μ atoms

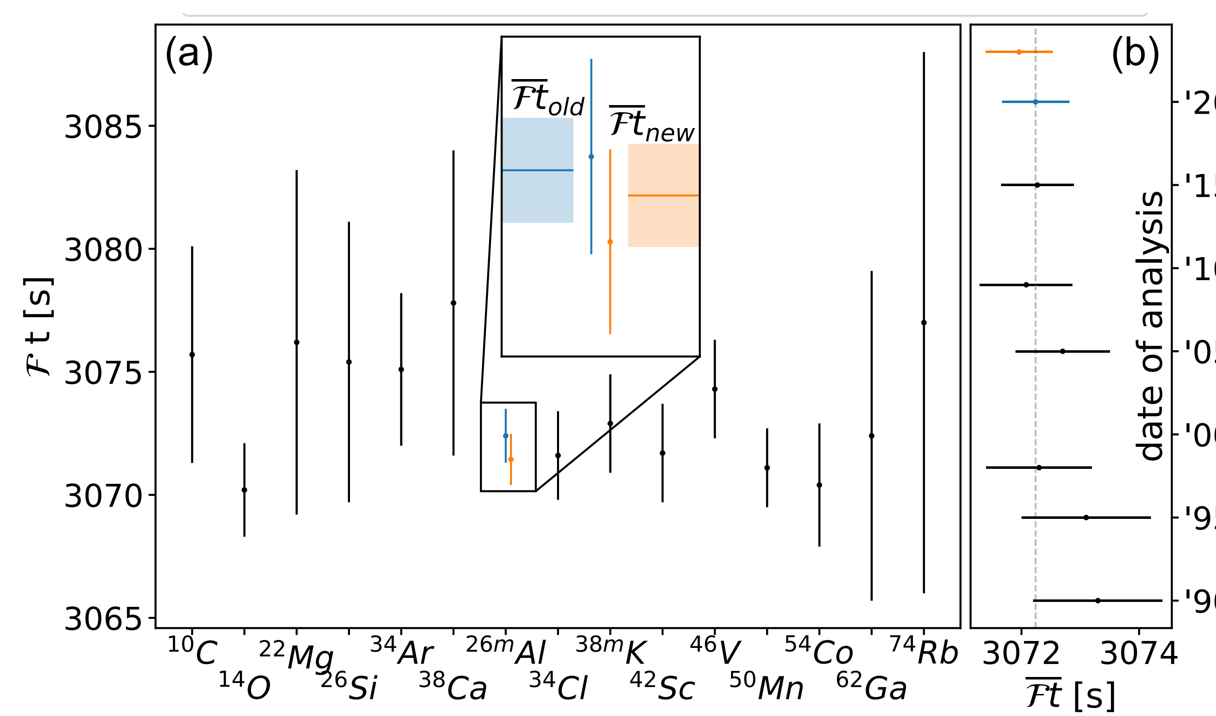
$$R_c(^{26m}\mathrm{Al}) \equiv \langle r^2\rangle^{1/2}_{26m} = \sqrt{R_c(^{27}\mathrm{Al})^2 + \delta\langle r^2\rangle^{27,26m}}$$

$$\delta\nu^{27,26m} = 377.5(3.4)\ \mathrm{MHz} \longrightarrow R_c(^{26m}\mathrm{Al}) = 3.130(15)\ \mathrm{fm}$$

Previously guessed $R_c(^{26m}Al) = 3.040(20)\,\mathrm{fm}$

Charge radii serve as input parameters for corrections supreallowed beta decay rates

Quantity	Previous value	This Letter
R_c	3.040(20) fm [27]	3.130(15) fm
δ_{C2}	0.280(15)% [10]	0.310(14)%
$\mathcal{F}t(^{26m}Al)$	3072.4(1.1) s [10]	3071.4(1.0) s
$\overline{\mathcal{F}t}$	3072.24(1.85) s [10]	3071.96(1.85) s
Δ_{CKM}	$152(70)\times 10^{-5}$ [7]	$144(70)\times 10^{-5}$



Al-26m the most precisely measured transition!
Direct impact on Ft and Vud extraction

Impact of precise nuclear radii on Ft and V_{ud}

IS measurement + QED & Nuclear Theory —> Radius

$$\delta\nu^{27,26m} = F\delta\langle r^2\rangle^{27,26m} + M\frac{m_{26m}-m_{27}}{m_{27}(m_{26m}+m_e)}$$

Radius + QED & Nuclear Theory —> Ft-value

$$R_c(^{26m}\mathrm{Al}) \equiv \langle r^2\rangle^{1/2}_{26m} = \sqrt{R_c(^{27}\mathrm{Al})^2 + \delta\langle r^2\rangle^{27,26m}}$$

Many-body QED to calculate F, M factors —>
Theory calculation up to date?
All uncertainties under control?

	$\delta\langle r^2\rangle^{27,26m}$	$R_c(^{26m}\mathrm{Al})$
Refs. [10, 11]		3.040(20)
Ref. [9]	0.429(45)(76)	3.130(15)
This work	0.443(44)(19)	3.132(10)

QED uncertainty

Recently revisited **Skripnikov et al, 2404.13369**

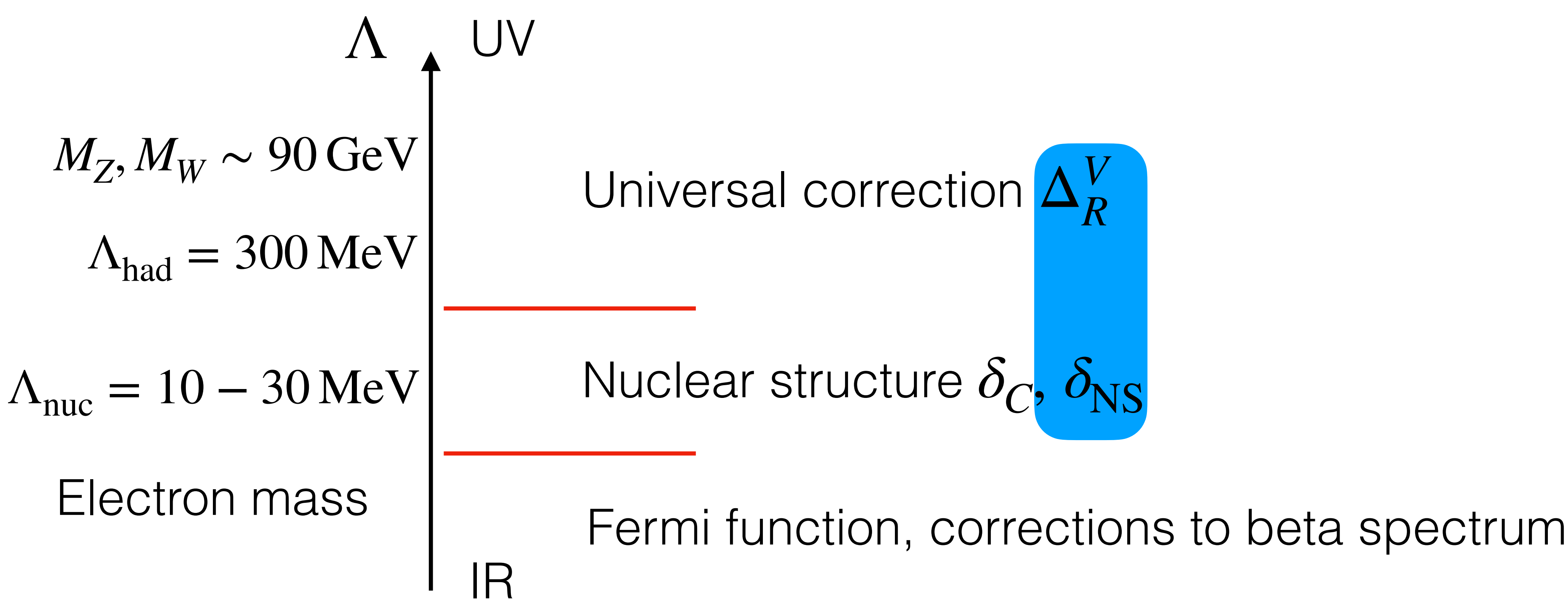
Charge radius of Al-27 from **Fricke, Heilig Nuclear Charge Radii compilation 2004**

A	E_{exp} [keV]	E_{theo} [keV]	$Npol$ [keV]	c [fm]	$<r^2>^{1/2}_{model}$ [fm]	α [1/fm]	k	C_z [10^{-3}fm/eV]	$R^{\mu}_{k\alpha}$ [fm]
27	346.828(2)	346.827	0.040	3.0554(4)	3.063	0.0419	2.0573	–0.196	3.9354(4;24)

Npol = 40(12) eV stem from 1970’s (review by **Borie, Rinker RMP 54 (1982) 67-118**)
May be underestimated (neglected nucle**on** polarizability ~ 17 eV)

New value of radius affects ft-value via the shape factor — additional $\sim 1\sigma$ shift!

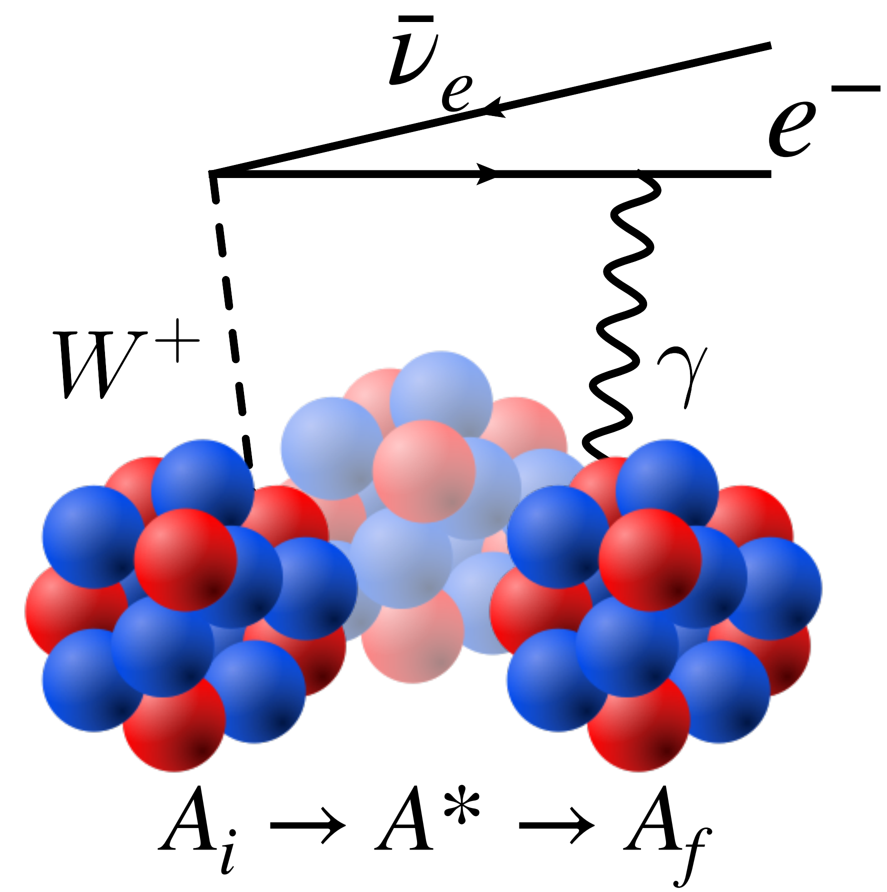
Dedicated paper addressing all ingredients is in preparation (MG, B. Ohayon, B. Sahoo, C-Y Seng)
Stay tuned!



Unified Formalism for Δ_R^V and δ_{NS}

Dispersion Theory of the γW-box

(Generalized) nucleon/ar polarizabilities and Impact on V_{ud}, radii and BSM searches



Virtual W or photon may polarize the nucleon/nucleus

Short-range: perturbative QCD (model-independent)

Long-range: hadronic and nuclear **polarizabilities**

Fundamental properties of nuclei/nucleon (one-boson exchange) related to details of strong interaction (depending on precision)

Polarizabilities related to the entire excitation spectrum

Universal RC from dispersion relations

Interference γW structure functions $\quad \mathrm{Im} T^{\mu\nu}_{\gamma W} = \ldots + \frac{i\varepsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(pq)} F_3^{\gamma W}(x, Q^2)$

After some algebra (isospin decomposition, loop integration)

$$\Box^{b,\mathrm{e}}_{\gamma W}(E_e) = \frac{\alpha}{\pi} \int_0^\infty dQ^2 \frac{M_W^2}{M_W^2 + Q^2} \int_{\nu_{\mathrm{thr}}}^\infty \frac{d\nu'}{\nu'} \frac{\nu' + 2\sqrt{\nu'^2 + Q^2}}{(\nu' + \sqrt{\nu'^2 + Q^2})^2} \frac{F_{3,-}(\nu', Q^2)}{M f_+(0)} + \mathcal{O}(E_e^2)$$

$$\Box^{b,\mathrm{o}}_{\gamma W}(E_e) = \frac{2\alpha E_e}{3\pi} \int_0^\infty dQ^2 \int_{\nu_{\mathrm{thr}}}^\infty \frac{d\nu'}{\nu'} \frac{\nu' + 3\sqrt{\nu'^2 + Q^2}}{(\nu' + \sqrt{\nu'^2 + Q^2})^3} \frac{F_{3,+}(\nu', Q^2)}{M f_+(0)} + \mathcal{O}(E_e^3)$$

2-fold integral: depending on Q^2 different physical pictures dominate

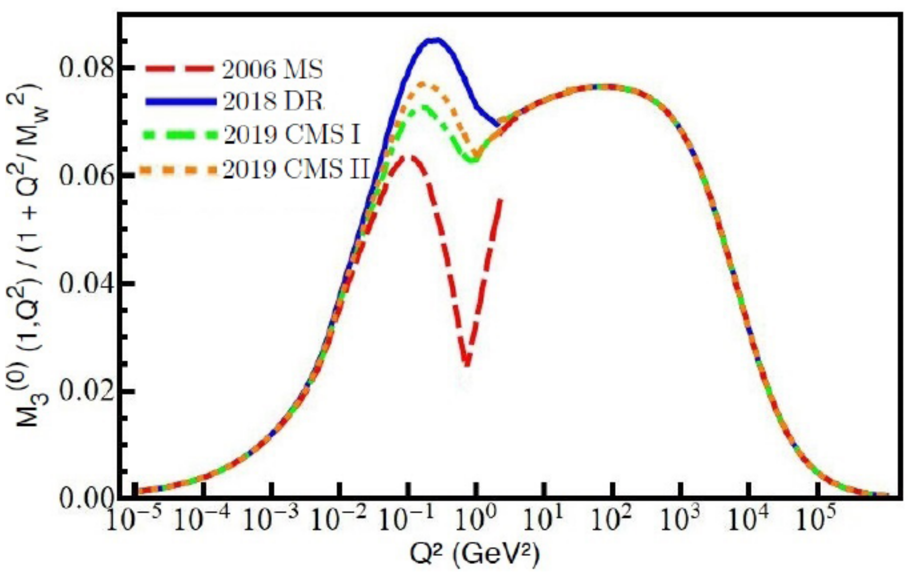
Explicit energy dependence quantifiable (earlier was neglected)

Input into dispersion integral - $\nu/\bar{\nu}$ data

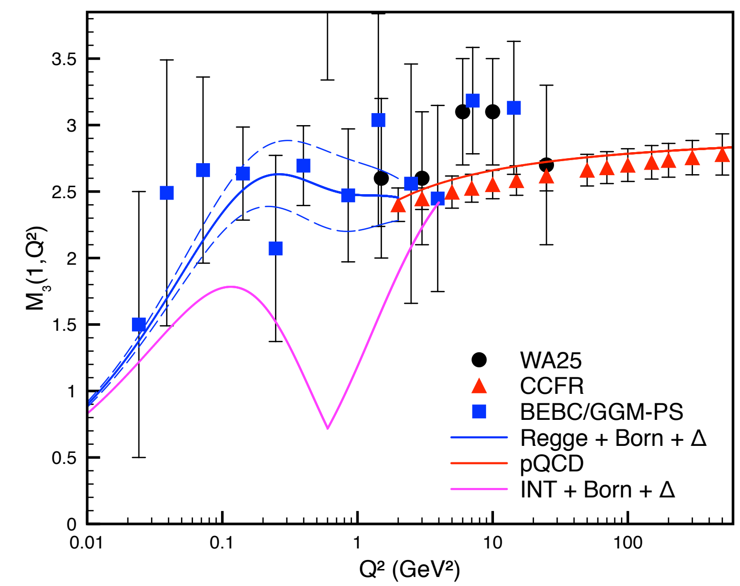
Mixed CC-NC γW SF (no data) <—> Purely CC WW SF (inclusive neutrino data)

Isospin symmetry: vector-isoscalar current related to vector-isovector current

Free neutron γW box



Neutrino scattering data



Marciano, Sirlin 2006: $\Delta_R^V = 0.02361(38)$ —> $V_{ud} = 0.97420(10)_{Ft}(18)_{RC}$

DR (Seng et al. 2018): $\Delta_R^V = 0.02467(22)$ —> $V_{ud} = 0.97370(10)_{Ft}(10)_{RC}$

Since then: confirmed by lattice QCD

$$\Delta_R^V = 0.02477(24)_{\mathrm{LQCD}\pi+\mathrm{pheno}}$$

Seng, MG, Feng, Jin, 2003.11264

$$\Delta_R^V = 0.02439(19)_{\mathrm{LQCDn}}$$

Ma, Feng, MG et al 2308.16755

Shift upwards by 3σ + reduction of uncertainty by factor 2

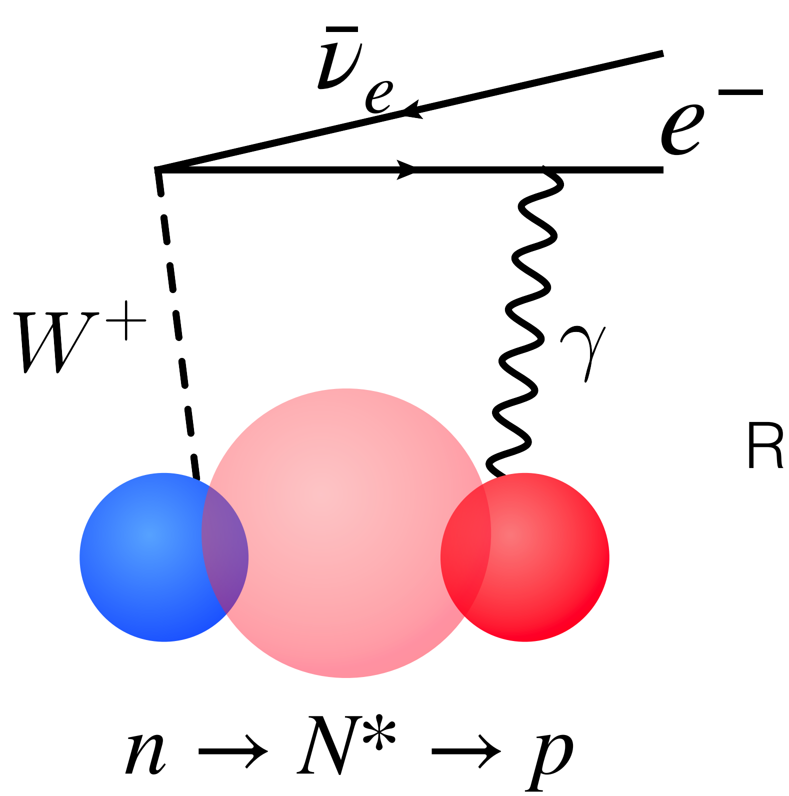
δ_{NS} from dispersion relations

Same formulas for free neutron and nuclei;

$$\Box^{b,\mathrm{e}}_{\gamma W}(E_e) = \frac{\alpha}{\pi}\int_0^\infty dQ^2 \frac{M_W^2}{M_W^2+Q^2}\int_{\nu_{\mathrm{thr}}}^\infty \frac{d\nu'}{\nu'}\frac{\nu'+2\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^2}\frac{F_{3,-}(\nu',Q^2)}{Mf_+(0)} + \mathcal{O}(E_e^2)$$

$$\Box^{b,\mathrm{o}}_{\gamma W}(E_e) = \frac{2\alpha E_e}{3\pi}\int_0^\infty dQ^2 \int_{\nu_{\mathrm{thr}}}^\infty \frac{d\nu'}{\nu'}\frac{\nu'+3\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^3}\frac{F_{3,+}(\nu',Q^2)}{Mf_+(0)} + \mathcal{O}(E_e^3)$$

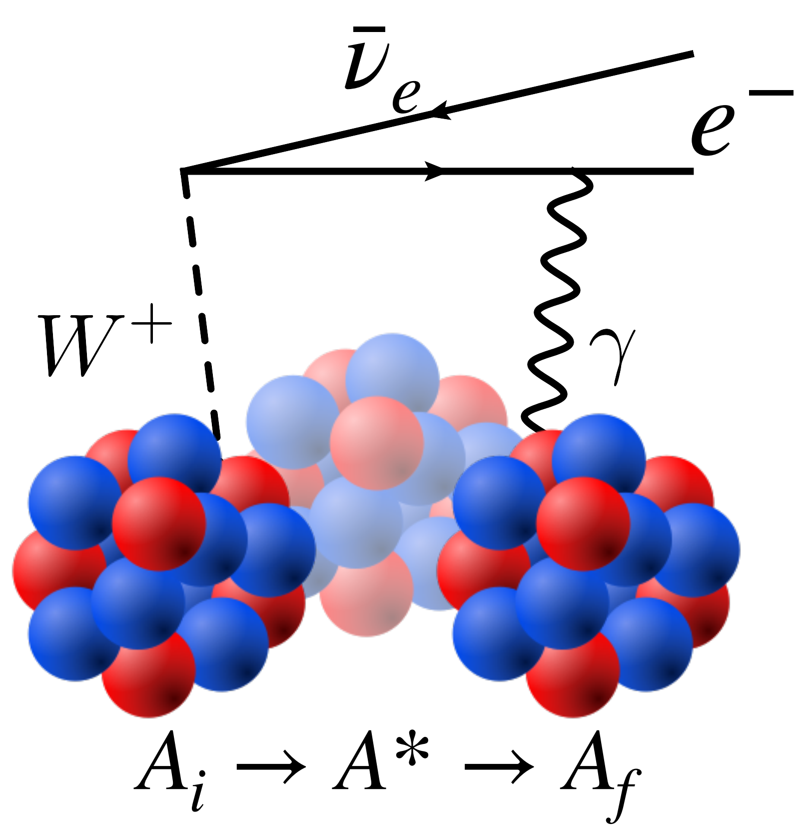
NS correction reflects extraction of the free box $\quad \delta_{\mathrm{NS}} = 2[\,\Box^{\mathrm{VA,\,nucl}}_{\gamma W} - \Box^{\mathrm{VA,\,free\,n}}_{\gamma W}\,]$



Differences due to:

Richer excitation spectrum in nuclei

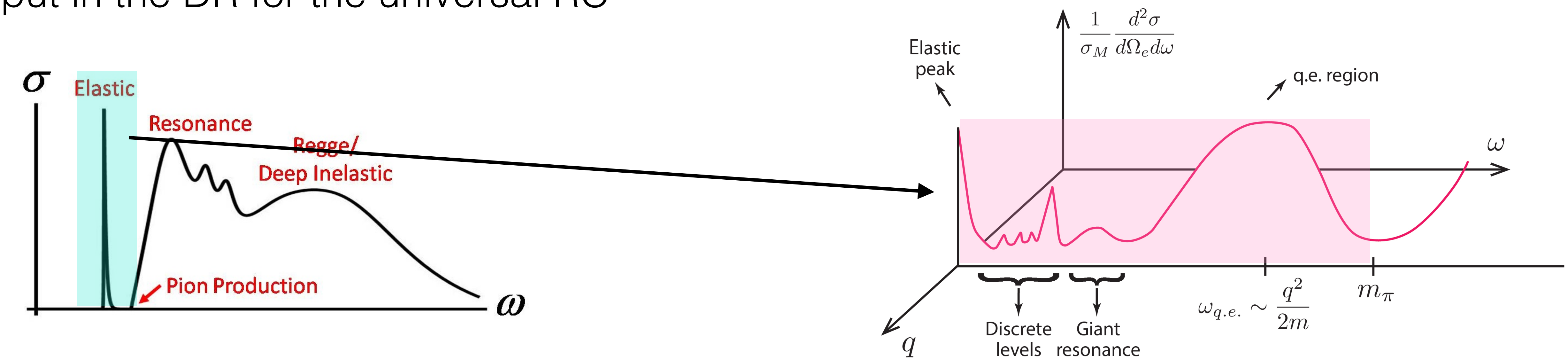
Different quantum numbers
(spin, isospin)



δ_{NS} from dispersion relations

Input in the DR for the universal RC

Input in the DR for the RC on a nucleus



δ^A_{NS} from DR with energy dependence

C-Y Seng, MG, M J Ramsey-Musolf 1812.03352
MG 1812.04229; Seng, MG 2211.10214

$$\delta^A_{NS} = \frac{2\alpha}{\pi NM}\int_0^{\text{few GeV}^2} dQ^2 \int_{\nu_{thr}}^{\nu_\pi} \frac{d\nu}{\nu}\left[\frac{\nu+2q}{(\nu+q)^2}\left(F_3^{(0)\,QE} - F_3^{(0),\,B}\right) + \frac{2\langle E\rangle}{3}\frac{\nu+3q}{(\nu+q)^3}F_3^{(-)\,QE}\right]$$

HT value 2018:

$\mathscr{F}t = 3072.1(7)s$

Old estimate: $\delta\mathscr{F}t = -(1.8 \pm 0.4)s \quad + \quad (0 \pm 0)s$

New estimate: $\delta\mathscr{F}t = -(3.5 \pm 1.0)s \quad + \quad (1.6 \pm 0.5)s$

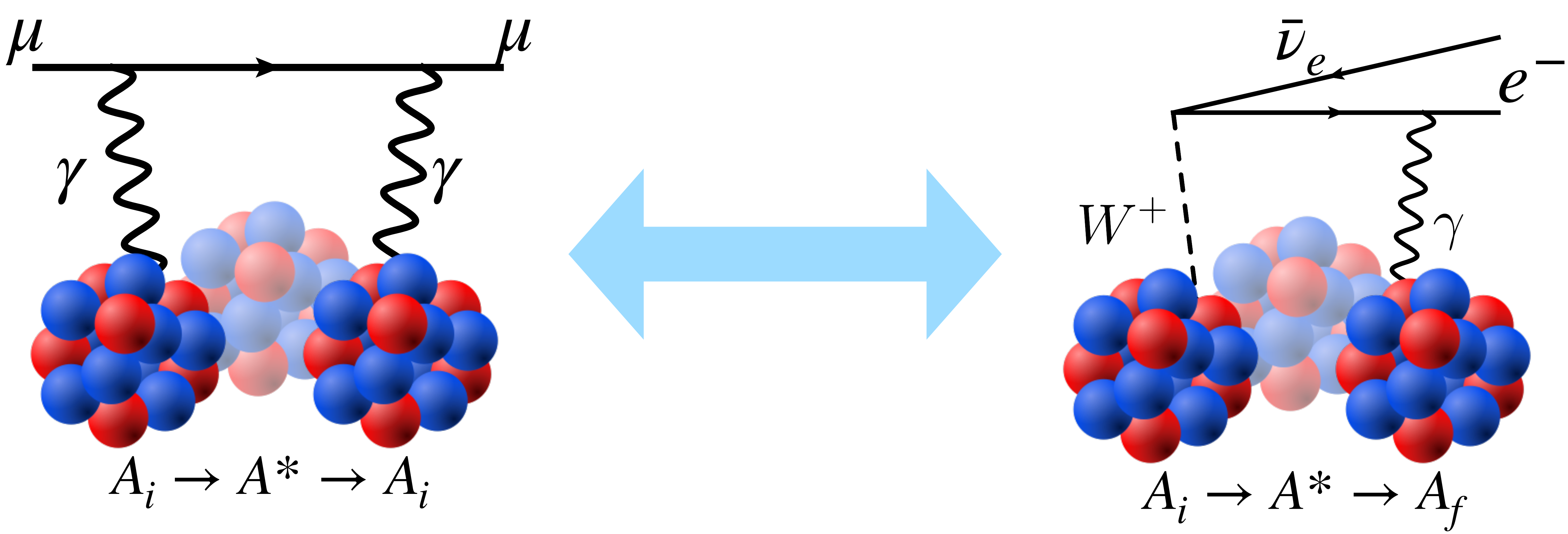
Nuclear structure uncertainty tripled! $\mathscr{F}t = (3072 \pm 2)s$

δ_{NS} and nuclear polarization in μ atoms

Only a warm-up calculation — complete ab-initio δ_{NS} necessary!

Effort on light systems C-10, O-14 with modern ab-initio methods initiated
No-Core Shell Model **with Michael Gennari, Petr Navratil, Mehdi Drissy**
Green's Function MC **with Garrett King, Saori Pastore**
Coupled Clusters **with Sonia Bacca, Asia Sobczyk, Gaute Hagen**

Great similarity with nuclear polarization contribution in muonic atoms:
Currently major limitation for extracting precise nuclear radii!
Coordinated effort necessary



Summary & Outlook

Great improvement in theory of SM rad. corr. in past 5 years

Great improvement in free-n; bright future UCNτ, τSPECT, BL3
Meson decays: future PIONEER, Belle II, NA62, K0T0

Barely any exp. improvement in $0^+ - 0^+$ nuclear decays
Nuclear uncertainties critically scrutinized - community effort
More precise nuclear radii necessary

Cabibbo anomaly interpretable in terms of BSM (RH and scalar)

However: to constrain BSM need to define what is the anomaly.
Uncertainties, uncertainties, uncertainties!

International workshop on **E**lectroweak **P**recision **I**nterse**C**tions

EPIC 2024

September 22-27 2024, Cala Serena Beach Resort (Geremeas)

Bring together different communities:

Particle, Nuclear, Atomic, Neutrino, Astro, GW

Study existing synergies & elaborate new ones!

Nuclear radii is one of focuses

1-day pre-workshop school for PhD students

Apply and have your students apply!

https://epic2024.ca.infn.it/

International workshop on **E**lectroweak **P**recision **I**nterse**C**tions

EPIC 2024

September 22-27 2024, Cala Serena Beach Resort (Geremeas)

EPIC 2024

Electroweak Physics Intersections

22-27 Sept 2024 **CalaSerena, Geremeas IT**

EPIC 2024 is the first workshop dedicated to **precision electroweak physics**, with focus on:

- **Precision tests of the Standard Model and beyond with atomic nuclei**
- **Lepton- and neutrino-nucleus interactions**
- **Nuclear matter across energy scales and multi-messenger astronomy**

PRE-WORKSHOP SCHOOL

- One-day lectures on precision physics with atoms, neutrino physics, and nuclear EoS in the multimessenger era.
- Dedicated poster session for students at the workshop with teaser-talk event.

SCIENTIFIC PROGRAM COMMITTEE

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