MAX-PLANCK-INSTITUT FÜR KERNPHYSIK

Challenging Beyond-the-Standard-Model Solutions to the Fine-Structure Anomaly in Heavy Muonic Atoms

(How not to solve the problem)

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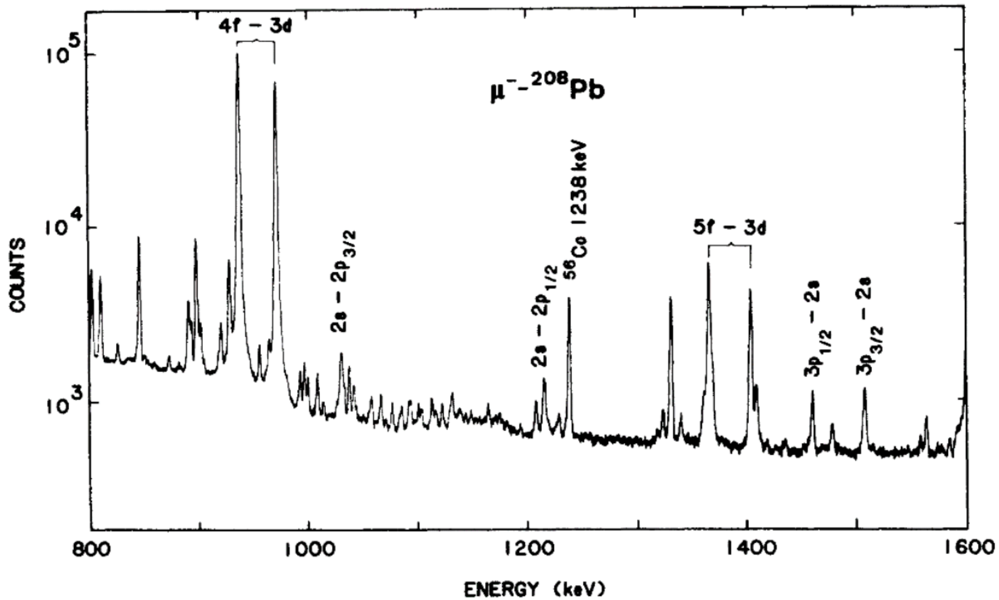
Fine-structure anomaly of heavy muonic atoms



P. Bergem *et al.*, Phys. Rev. C 37, 2821 (1988).

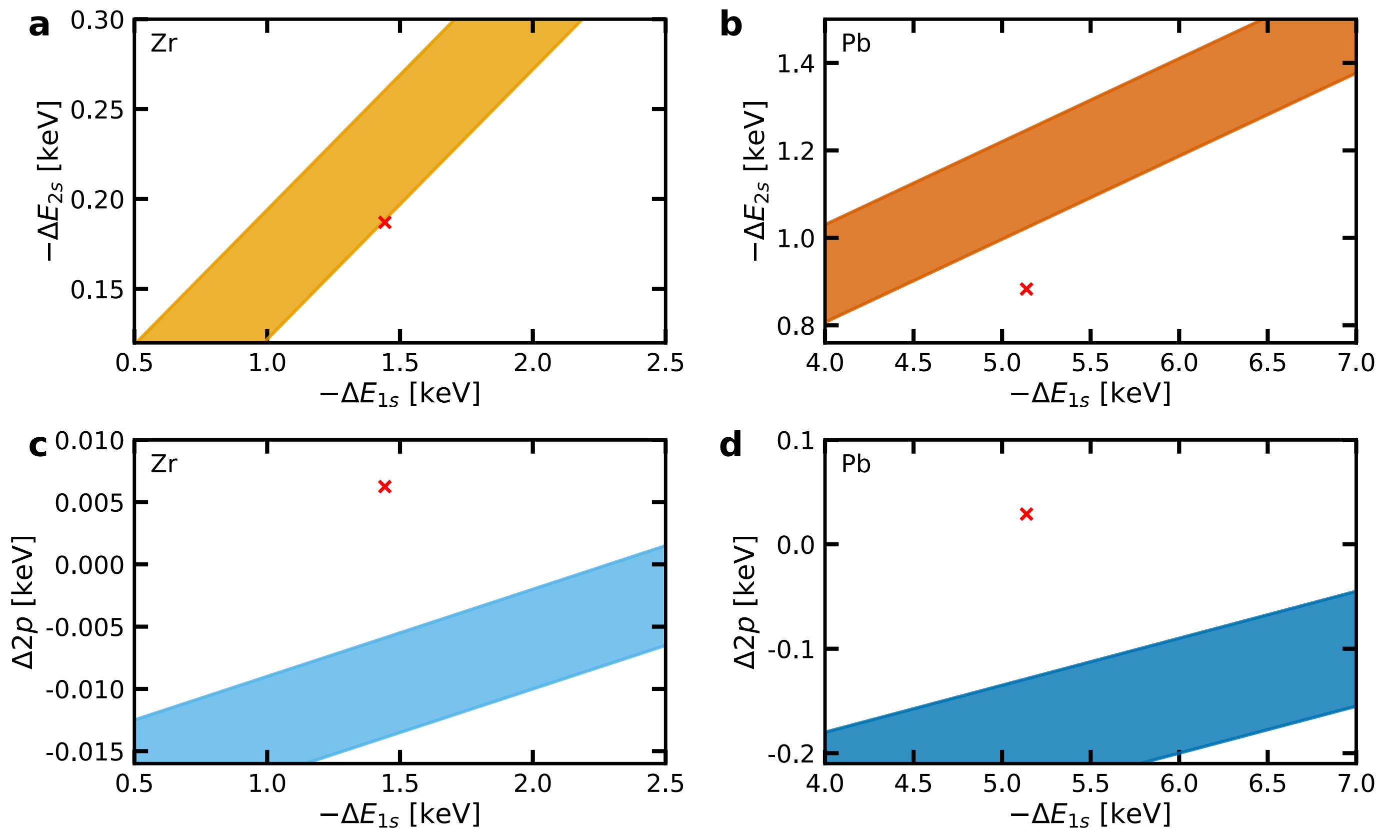
Measurements of muonic ^{90}Zr, ^{120}Sn, and ^{208}Pb reveal a very poor fit to theory.

- Spectral lines are assigned to atomic transitions $\Rightarrow$ $E_a^{\text{exp}}[Z]$
- They are compared with QED calculations $\Rightarrow$ $E_a^{\text{QED}}[Z]$
- Nuclear polarisation (NP) is tough to calculate, use it as free parameter to fit $\Rightarrow$ $\Delta E_a^{\text{NP}}[Z]$

Compare the fitted value to state of the art NP evaluations (Valuev et al. Phys. Rev. Lett. 128 (2022))

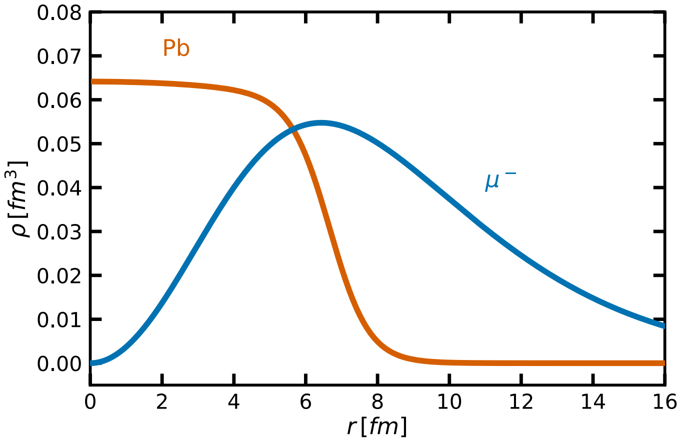
$$E_a^{\text{exp}}[Z] - E_a^{\text{QED}}[Z] \stackrel{?}{=} \Delta E_a^{\text{NP}}[Z]$$

NP correction compared to best fit value reveals discrepancy.



K. A. Beyer et al., arXiv 2306.10889 (2023)

Why Beyond the Standard Model?



The bound muon sits on a much tighter orbit around the nucleus compared to an electron

$$r_\mu = \frac{m_e}{m_\mu} r_e \simeq 0.005\, r_e$$

Fifth forces mediated by new bosons are suppressed by the particles mass

$$V \propto \frac{e^{-mr}}{r}$$

➡ Muonic atoms are affected much stronger by potential new forces than electronic atoms!

Seems natural to ask if the fine-structure anomaly is a result of a fifth force.

New bosons

A new boson (ϕ) which couples to the SM fermions (f) results in an exchange force between the nucleus and the bound muon. The coupling is of the form

$$\mathcal{L} \supset g_X^f \bar{f} X \mathcal{O} f$$

We restrict ourselves to scalars: $X = \phi$ and $\mathcal{O} = id$

Dirac's equation for the muon is now coupled to the electromagnetic and new potential

$$0 = \left(i\gamma^\alpha \partial_\alpha - q_{(\mu)}\gamma^\alpha A_\alpha - m_\mu + g_\mu \phi\right)\mu(x)$$

The latter can be approximated by a static potential sourced by the nucleus

$$\phi(x) = i g_n \int d^4y \int \frac{d^4q}{(2\pi)^4} \frac{i e^{iq.(x-y)}}{q^2 - m_\phi^2} \bar{\mathrm{n}}(y)\mathrm{n}(y)$$

Energy shift from new boson

This potential has the following Yukawa-like form

$$V_\phi(r) = -\alpha_\phi \gamma^0 \int \frac{e^{-m_\phi|\mathbf{r}-\mathbf{R}|}}{|\mathbf{r}-\mathbf{R}|} \rho_n(R) d^3R$$

$\alpha_\phi \equiv \frac{g_\mu g_n}{4\pi}$

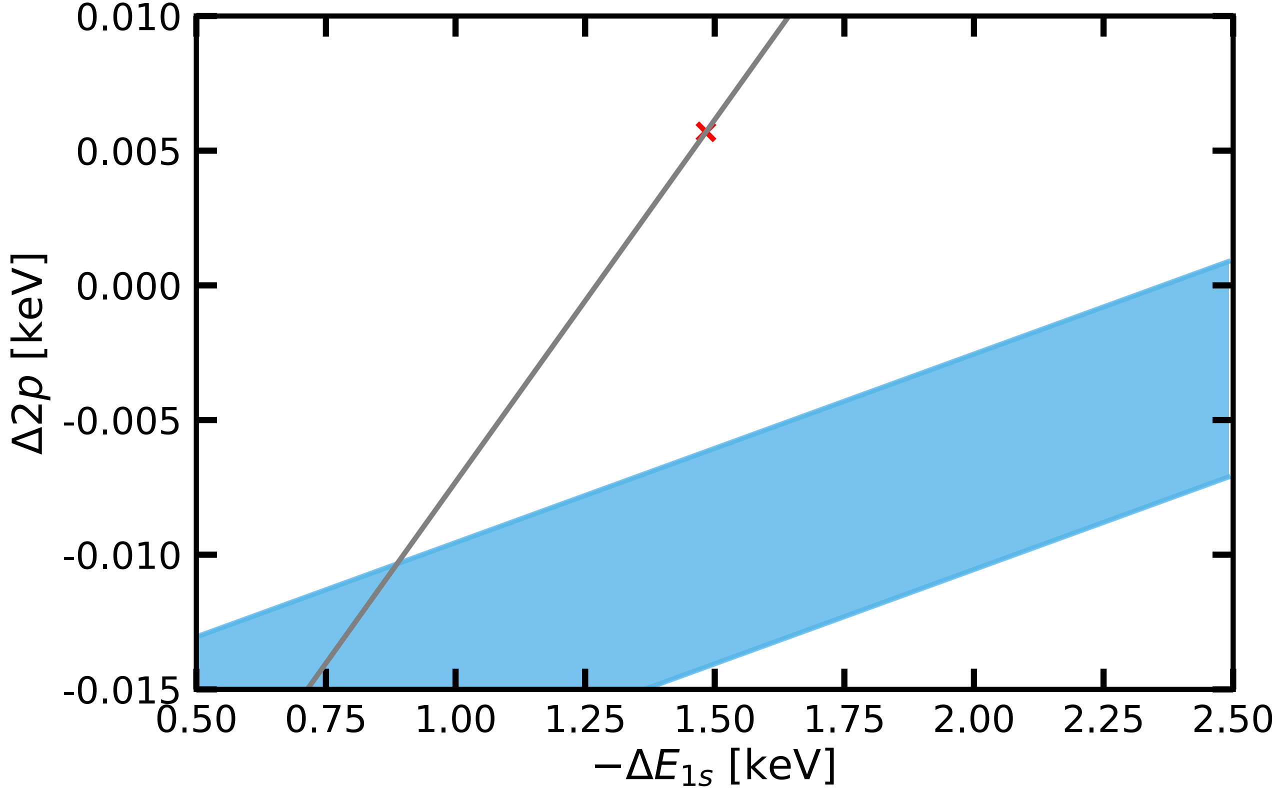
The resulting energy shift is

$$\Delta E_a = \langle a|V_\phi(r)|a\rangle$$

with the muon wavefunction the solution to Dirac's equation in a central Coulomb potential

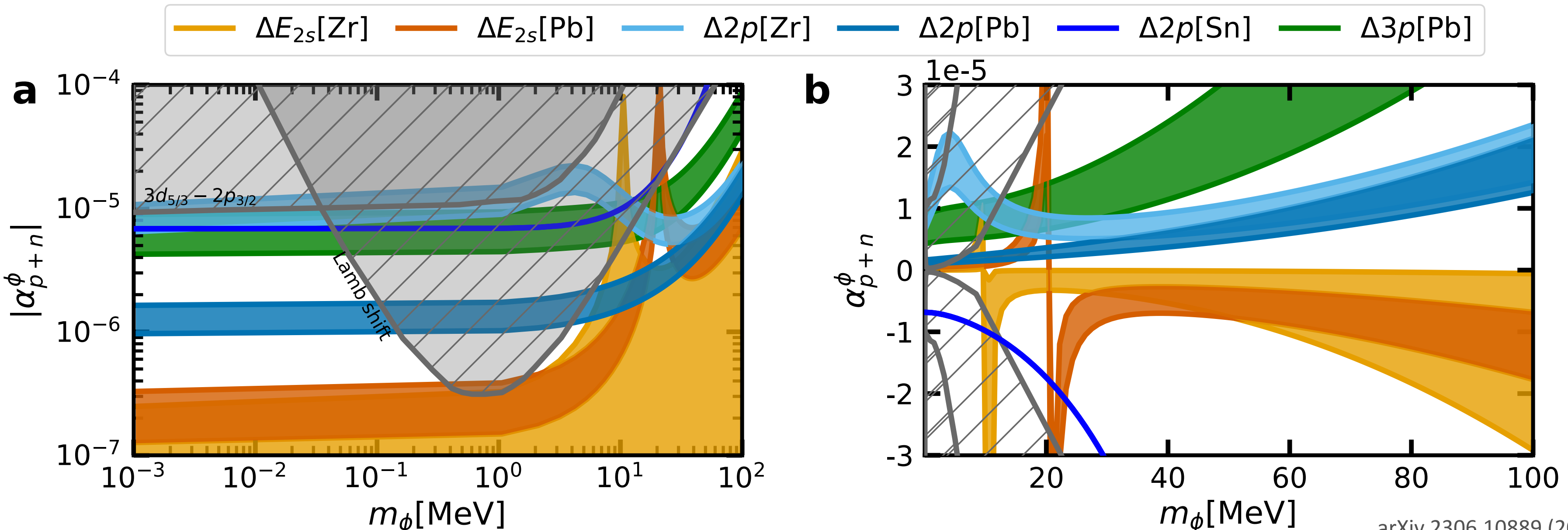
$$(\boldsymbol{\alpha}\cdot\mathbf{p} + \beta m_\mu + V_C(r))\,|a\rangle = E_a\,|a\rangle$$

Adding the BSM contribution moves the theory prediction :



So, can we make the fit work?

Use the two free parameters, coupling strength and boson mass, to fit each state individually.



arXiv 2306.10889 (2023)

Then look for overlap to identify potential parameter space to remedy the anomaly.

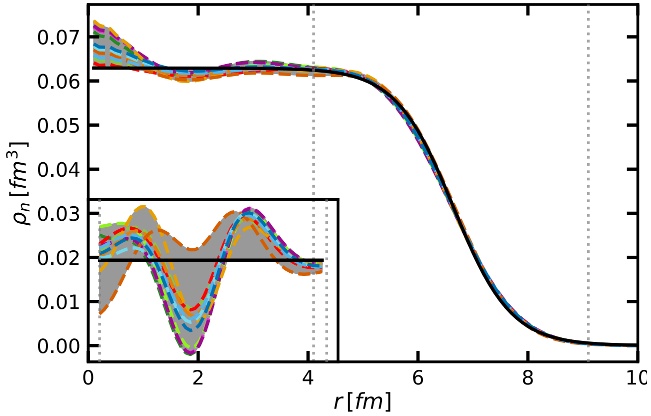
➡ There is none.

Conclusions

- Solving the fine-structure anomaly in heavy muonic atoms through a new boson is disfavoured.

➡ Clearly there is a need to re-evaluate and re-run the experiments, and new ideas are needed!

I would assume that the nucleus will play a role in solving this puzzle.



RMS Charge Radius - BSM

For heavy muonic atoms the finite-nuclear-size correction to the spectrum is very large.
To first order changing the radius results in a change in spectrum

For simplicity we assume the nucleus is a solid sphere of radius R_0. This is just for illustration.

$$\Delta E_a^{\delta r_{\mathrm{rms}}}[\delta R_0] = \frac{3}{2} Z\alpha \frac{\delta R_0}{R_0} \int_0^{R_0} \left(G_a^2(r) + F_a^2(r)\right) \left(\frac{r^2}{R_0^2} - 1\right) \frac{r^2}{R_0} dr + \mathcal{O}(\delta^2)$$

The energy shift resulting from a new boson we calculated before

$$\Delta E_a^{\phi}[\alpha_f^{\phi}, m_\phi] = \frac{-3N_f \alpha_f^{\phi}}{(m_\phi R_f)^3} \left[\int_0^{R_f} \mathcal{C}_2 \left(G_a^2(r) - F_a^2(r)\right) r^2 dr + \int_{R_f}^{\infty} \mathcal{C}_1 \left(G_a^2(r) - F_a^2(r)\right) r^2 dr\right]$$

$$\mathcal{C}_2 \equiv m_\phi - e^{-m_\phi R_f} \left(1 + m_\phi R_f\right) \frac{\sinh(m_\phi r)}{r} \qquad \mathcal{C}_1 \equiv \frac{e^{-m_\phi r}}{r} \left[m_\phi R_f \cosh(m_\phi R_f) - \sinh(m_\phi R_f)\right]$$

Is there a limit to the accuracy to which we can extract the rms charge radius because of potential BSM physics?

RMS Charge Radius - BSM

Begin by comparing the level of uncertainty which each adds to the spectrum

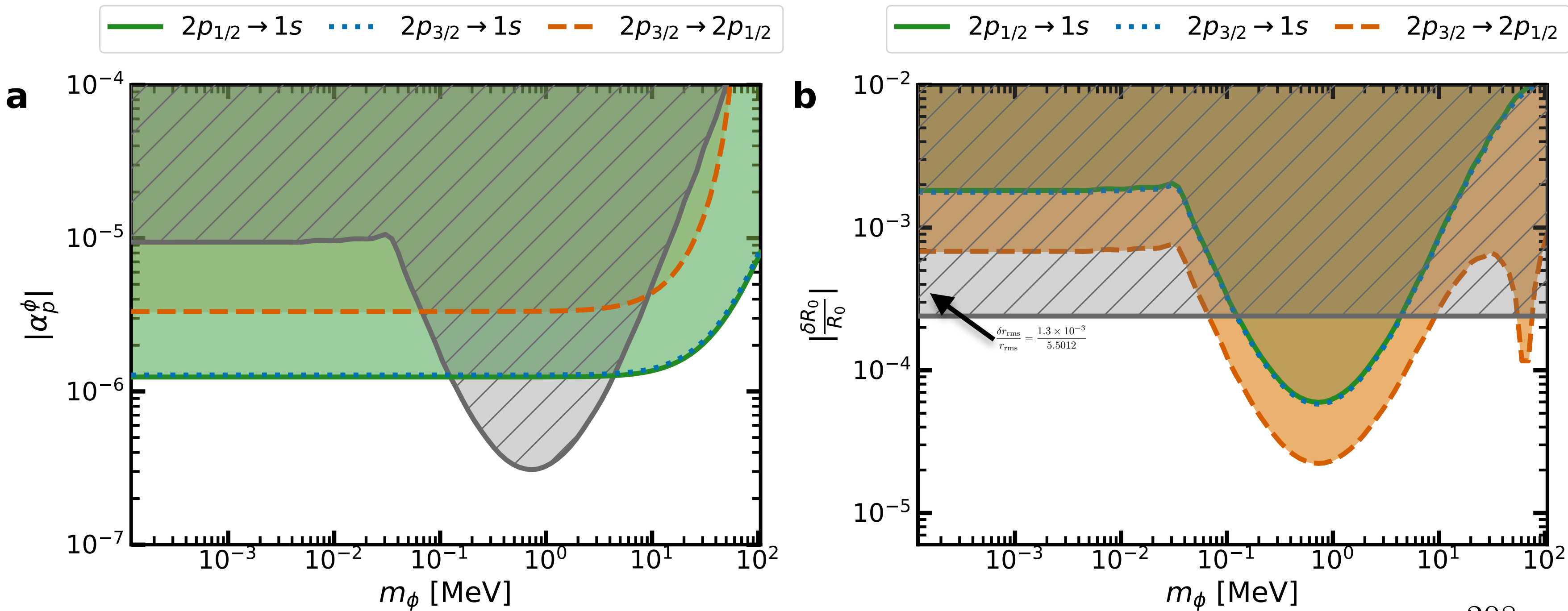
Predicted transition energy

Energy change due to BSM physics

$$\mathcal{E}^{\text{exp}}_{b\to a} - E^{\text{theo}}_{b\to a}[R_0] = \Delta E^{\delta r_{\text{rms}}}_{b\to a}[\delta R_0] + \Delta E^{\phi}_{b\to a}[\alpha_\phi, m_\phi]$$

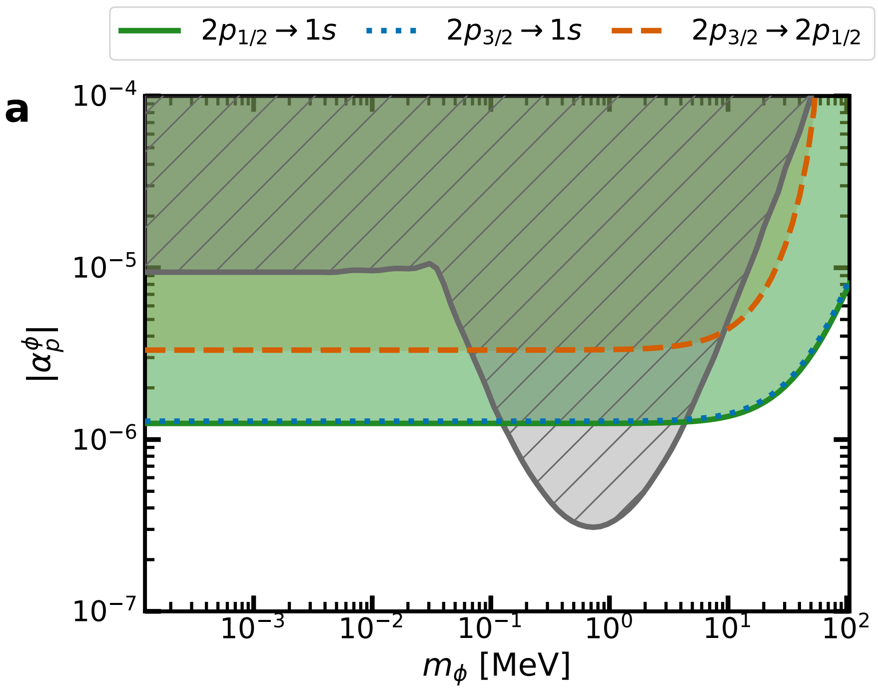
Measured transition energy

Energy change due to radius uncertainty



$\mu - {}^{208}\text{Pb}$

RMS Charge Radius - BSM



For each individual transition:

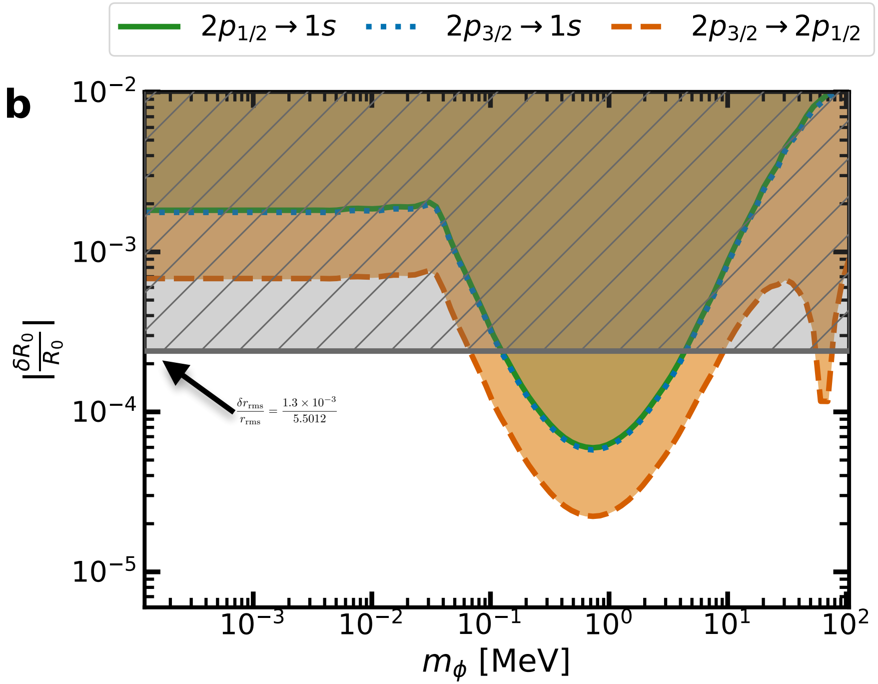
- The effect of a new boson on the spectrum is balanced against the uncertainty stemming from the rms charge radius.

$$\alpha_f^\phi = N_f^{-1} \frac{\Delta E_{b\to a}^{\delta r_{\mathrm{rms}}}}{\Delta E_{b\to a}^{\phi}[m_\phi]} \left(\frac{\mathcal{E}_{b\to a}^{\mathrm{exp}} - E_{b\to a}^{\mathrm{theo}}[R_0]}{\Delta E_{b\to a}^{\delta r_{\mathrm{rms}}}} - \frac{\delta R_0}{R_0} \right)$$

- New Physics would be detectable unless the coupling is smaller than the previous muon specific bounds.

→ Of course we can't claim these bounds as long as the fine-structure anomaly persists.

RMS Charge Radius - BSM



For each individual transition:

- What is the effective change in the rms charge radius which would absorb the effect of new Physics?

$$\frac{\delta R_0}{R_0} = \frac{\Delta E^{\phi}_{b\to a}[m_\phi]}{\Delta E^{\delta r_{\rm rms}}_{b\to a}} \left(\frac{\mathcal{E}^{\rm exp}_{b\to a} - E^{\rm theo}_{b\to a}[R_0]}{\Delta E^{\phi}_{b\to a}[m_\phi]} - N_f \alpha^{\phi}_{f} \right)$$

- The effective change in the rms charge radius is larger than the quoted error on the tabulated value.
  - “Systematic error?”

➡ Of course in reality we use multiple transitions.

Conclusions

- The fine-structure anomaly in muonic Pb, Sn, and Zr can not be solved with a new fifth force stemming from a new interaction boson.
- The level of accuracy to which the spectrum is known is comparable to current exclusion bounds for BSM physics.
  - Maybe fitting the spectrum with a fifth force could put competitive bounds on such forces coupling muons to nucleons.

➡ If the extraction of the rms charge radius relies on a fit to the atomic spectrum, can we always distinguish between rms charge radius and a fifth force?