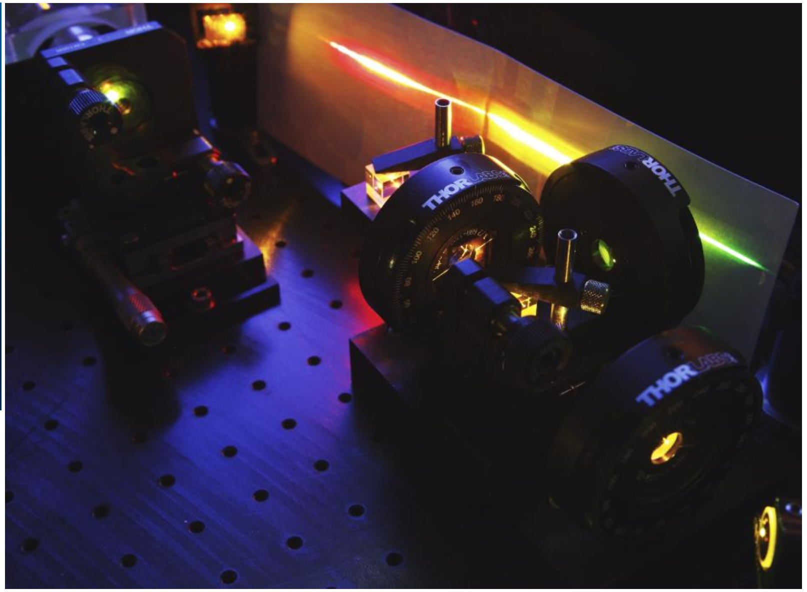
Charge Radii of Light Elements from He-Like Systems

W. Nörtershäuser, P. Imgram, K. König, B. Maaß, P. Müller

TECHNISCHE UNIVERSITÄT DARMSTADT



KOALA

Bundesministerium für Bildung und Forschung

HFHF Helmholtz Forschungsakademie Hessen für FAIR

HESSEN Hessisches Ministerium für Wissenschaft und Kunst

LASER SpHERe

DFG Deutsche Forschungsgemeinschaft

SFB 1245
Atomic Nuclei: From Fundamental Interactions to Structure and Stars

Outline

Introduction

Motivation

Experimental Setup

Results

Summary and Outlook

INTRODUCTION

Isotope Shift in a Nutshell

$$\delta\nu_{\mathrm{IS}}^{AA'} = \nu^{A'} - \nu^{A}$$

$$\delta\nu_{\mathrm{IS}}^{AA'} \approx K_{\mathrm{MS}} \cdot \frac{M_{A'} - M_A}{M_A M_{A'}} + F\,\delta\left\langle r_c^2 \right\rangle^{AA'}$$

$$\delta\left\langle r_c^2 \right\rangle^{AA'} \approx \frac{1}{F}\left[\delta\nu_{\mathrm{IS}}^{AA'} - K_{\mathrm{MS}} \cdot \mu^{A'A}\right]$$

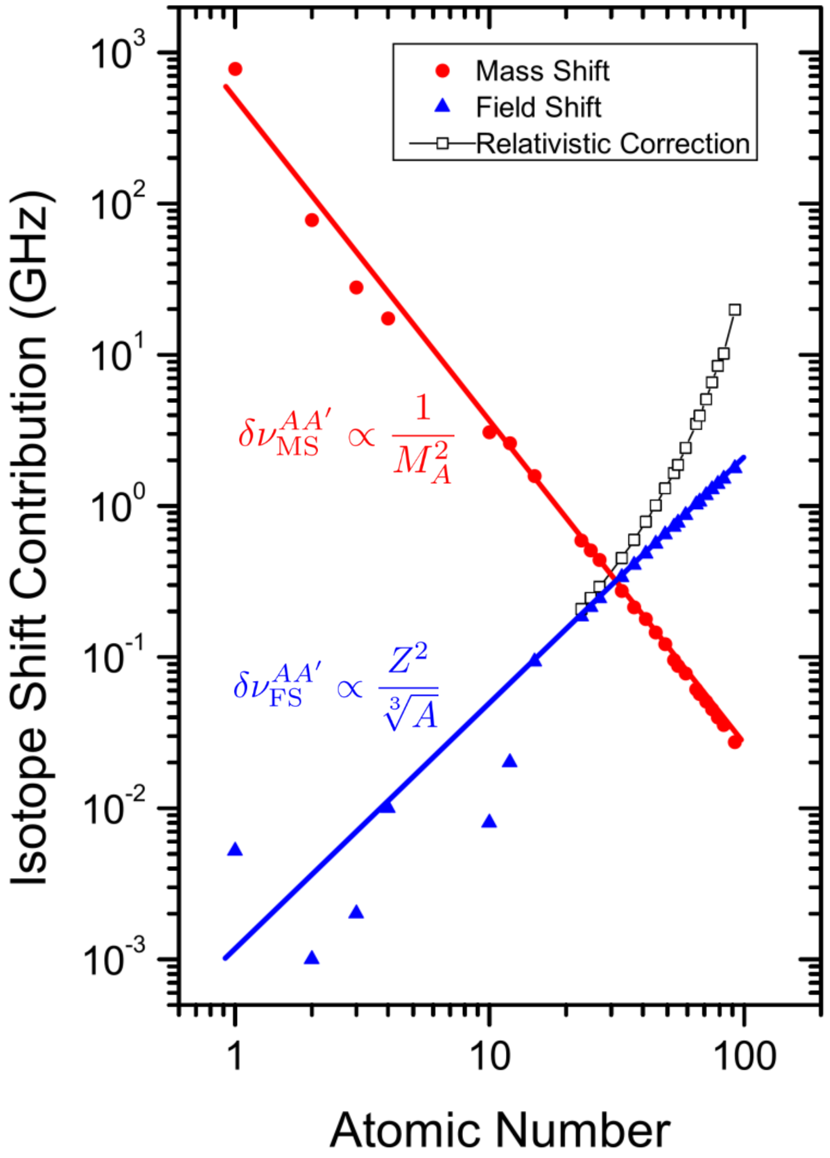
Requires knowledge of F and K_{MS} !

- ab initio NR-QED: up to 5-electron systems
- MCIDF, MBPT, ….
- King plot (requires ≥3 stable isotopes with known R_c)

How to get a radius ?

$$R_c(A) = \sqrt{\underbrace{R_c^2(A_{\mathrm{ref}})}_{} + \delta\langle r_c^2\rangle^{A_{\mathrm{ref}},A}}$$

Reference radius required from a different technique

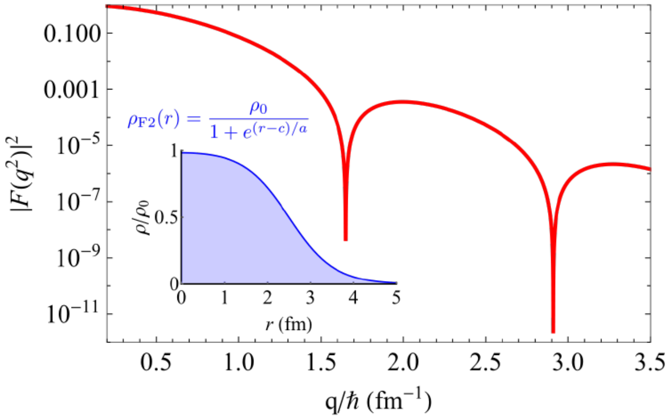


„Absolute“ Nuclear Charge Radii of Stable Isotopes

Elastic electron scattering

Formfactor is the Fourier transform of the charge distribution.

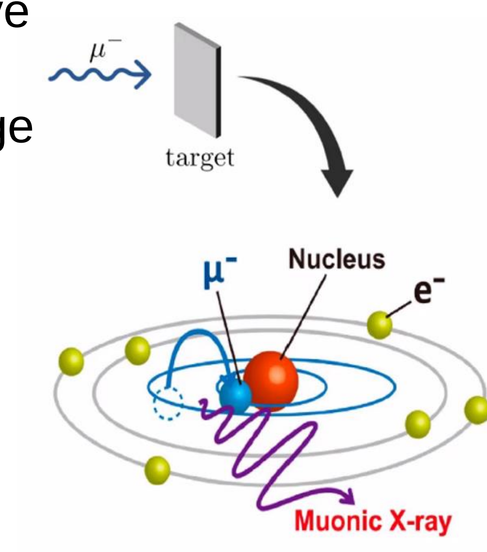
I. Sick, Prog. Part. Nucl. Phys. **47**, 245 (2001)



Muonic atom X-ray spectroscopy

X-ray transition energies have very large finite-nuclear-size (FNS) effects, due to the large overlap of the muon wave function with the nucleus.

E. Borie & G.A. Rinker,
Rev. Mod. Phys. **54**, 67 (1982)
B. Ohayon *et al.*, arXiv:2310.03846v1



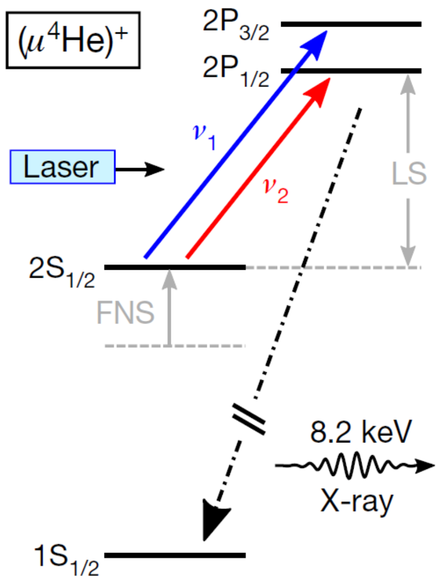
http://www.mdpi.com/2412-382-X/1/1/11/htm

Laser spectroscopy of muonic atoms

Measurement of the 2S-2P splittings in muonic H, D, ^{3,4}He allowed to extract the most accurate mean-square radii radii for these isotopes.

H-like systems ⇒ best for theory

R. Pohl, A. Antognini *et al*, Nature **466**, 213 (2010)
J.J. Krauth *et al.*, Nature **589**, 527 (2021)



proton radius puzzle

Nuclear Charge Radii $\left\langle r_{\mathrm{c}}^{2}\right\rangle = \frac{1}{Z}\int \mathrm{d}^{3}r\ r^{2}\rho_{\mathrm{c}}(\vec{r})$

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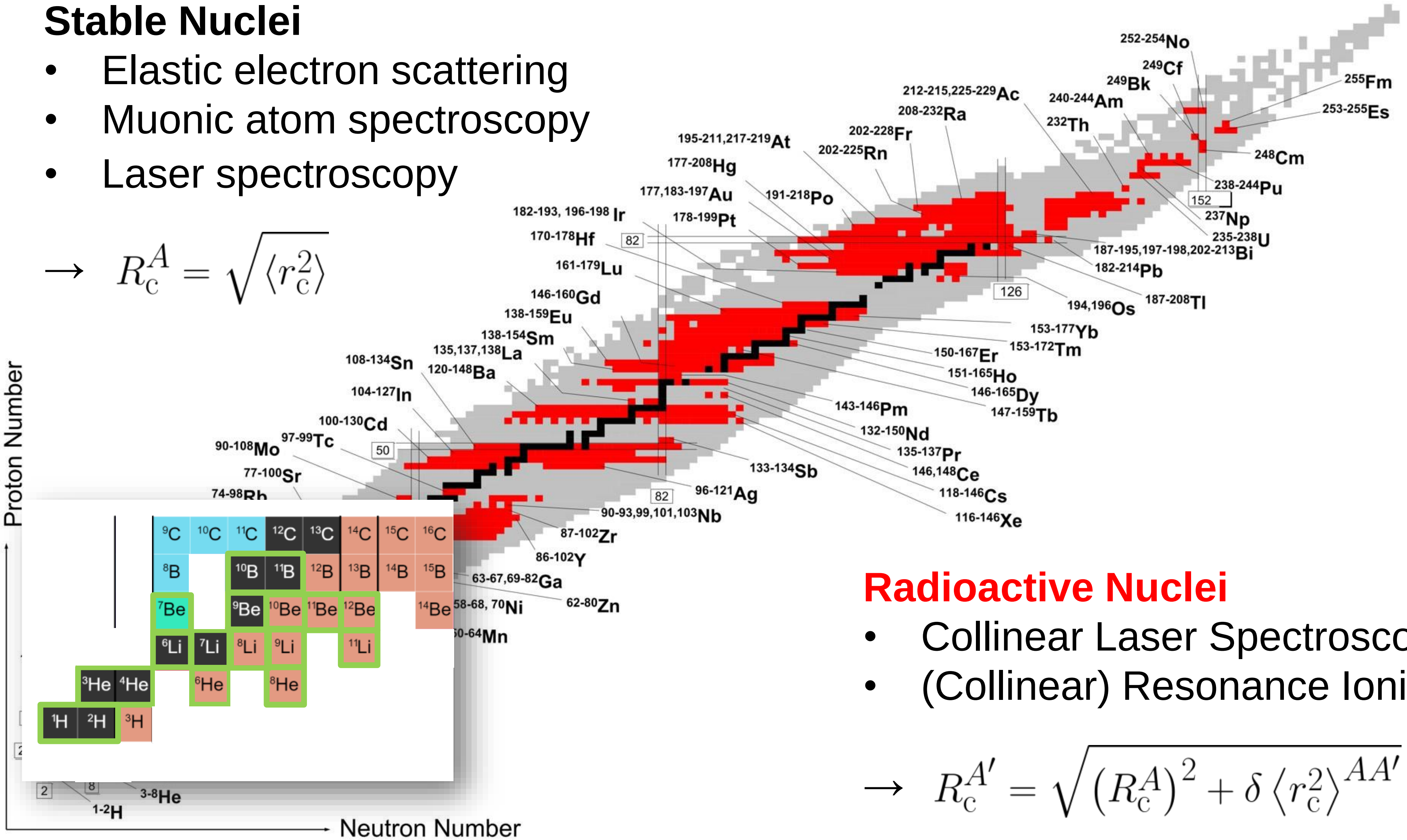
Stable Nuclei

- Elastic electron scattering
- Muonic atom spectroscopy
- Laser spectroscopy

$\rightarrow \quad R_{\mathrm{c}}^{A} = \sqrt{\left\langle r_{\mathrm{c}}^{2}\right\rangle}$

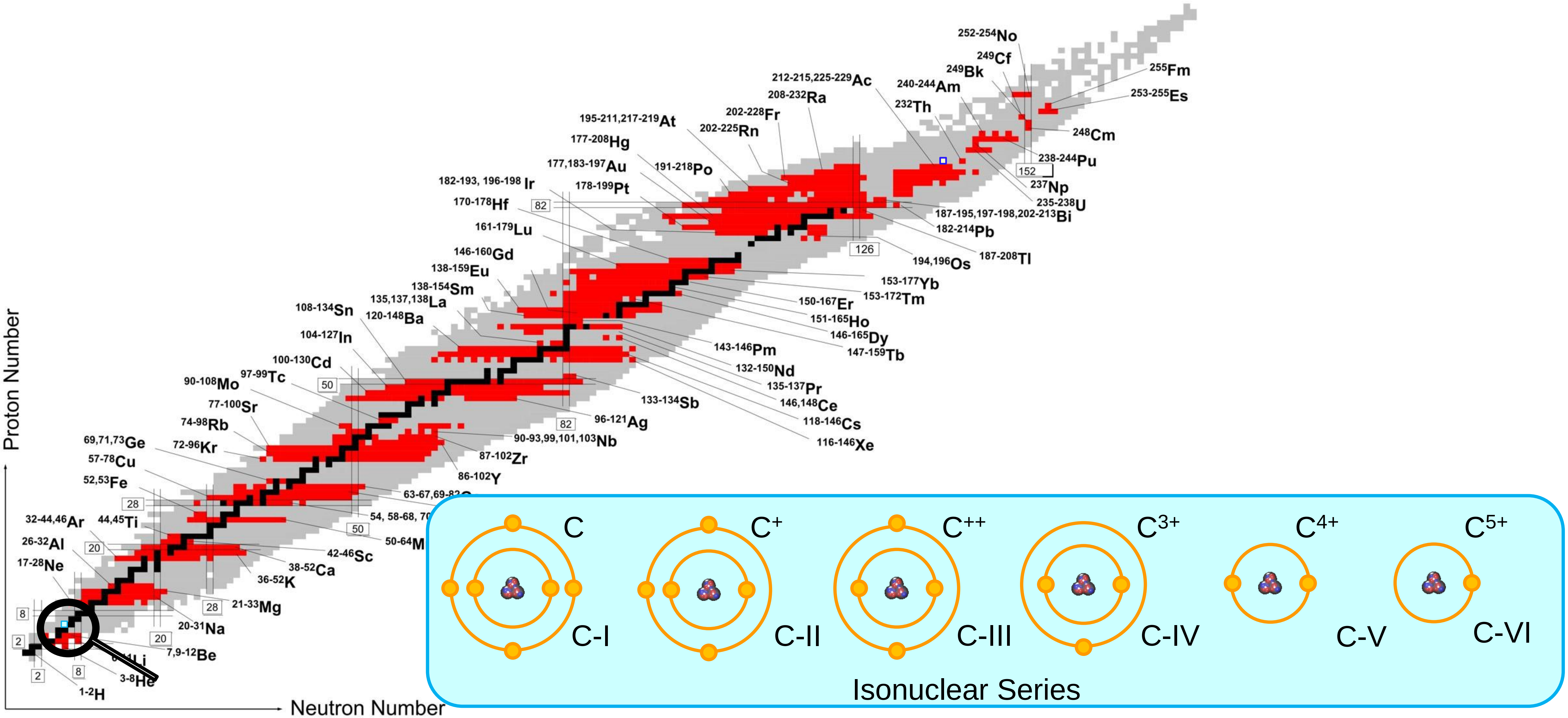


Radioactive Nuclei

- Collinear Laser Spectroscopy
- (Collinear) Resonance Ionization Spectroscopy

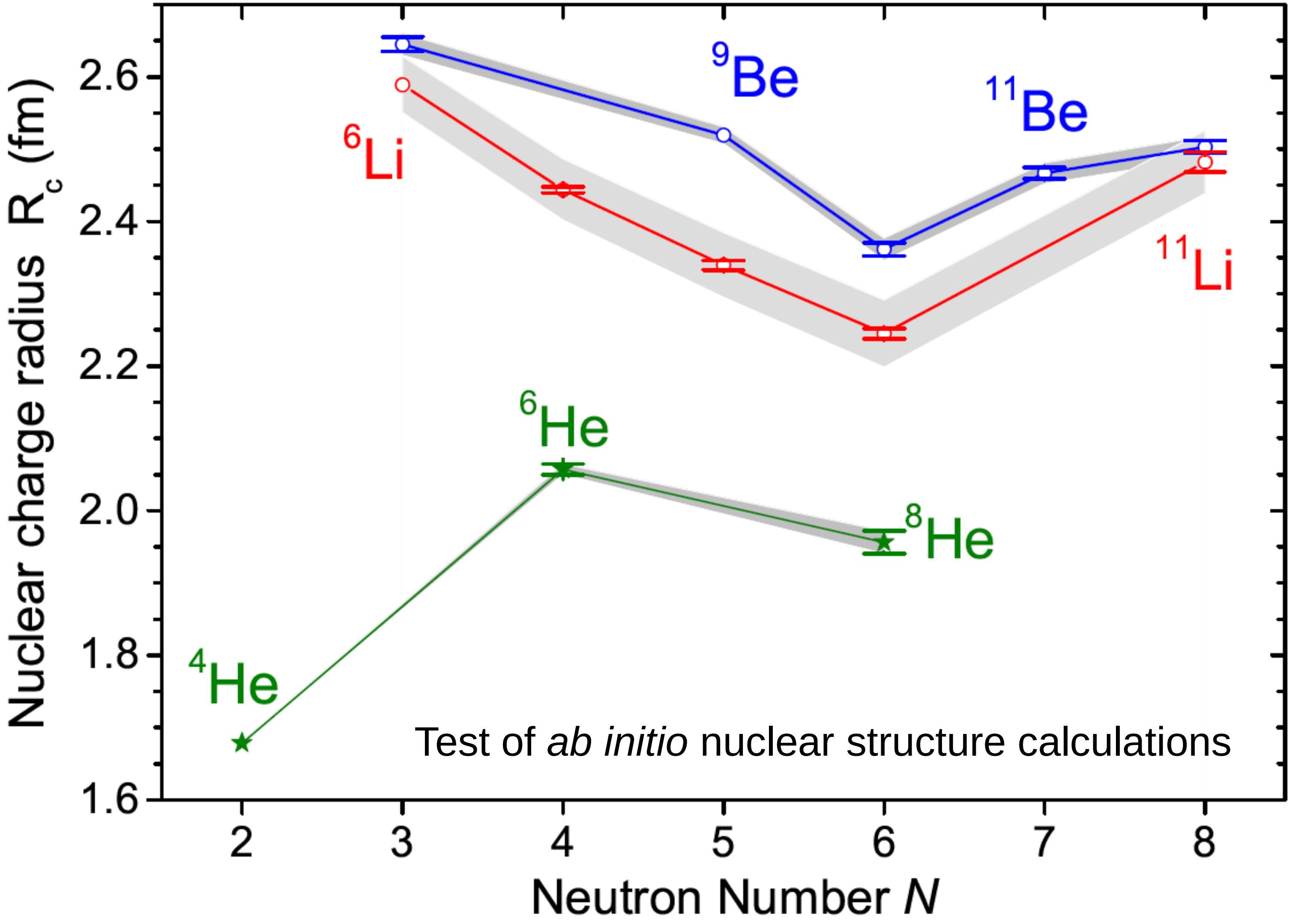
$\rightarrow \quad R_{\mathrm{c}}^{A'} = \sqrt{\left(R_{\mathrm{c}}^{A}\right)^{2} + \delta\left\langle r_{\mathrm{c}}^{2}\right\rangle^{AA'}}$

Additional Dimension



MOTIVATION

Nuclear Radii of the Lightest Isotopes



Error Bars: $\sigma(\delta\nu_{IS})$

A. Krieger *et al.*, PRL **108**, 142501 (2012)

R. Sanchez *et al.*, PRL **96**, 033002 (2006)

P. Müller *et al.*, PRL **99**, 252501 (2008)

Grey Regions: $\sigma(R_c)$

$R_c(^{9}\mathrm{Be}) = 2.519\,(12)\ \mathrm{fm}$

J.A. Jansen *et al.*, Nucl. Phys. A **188**, 337 (1972)

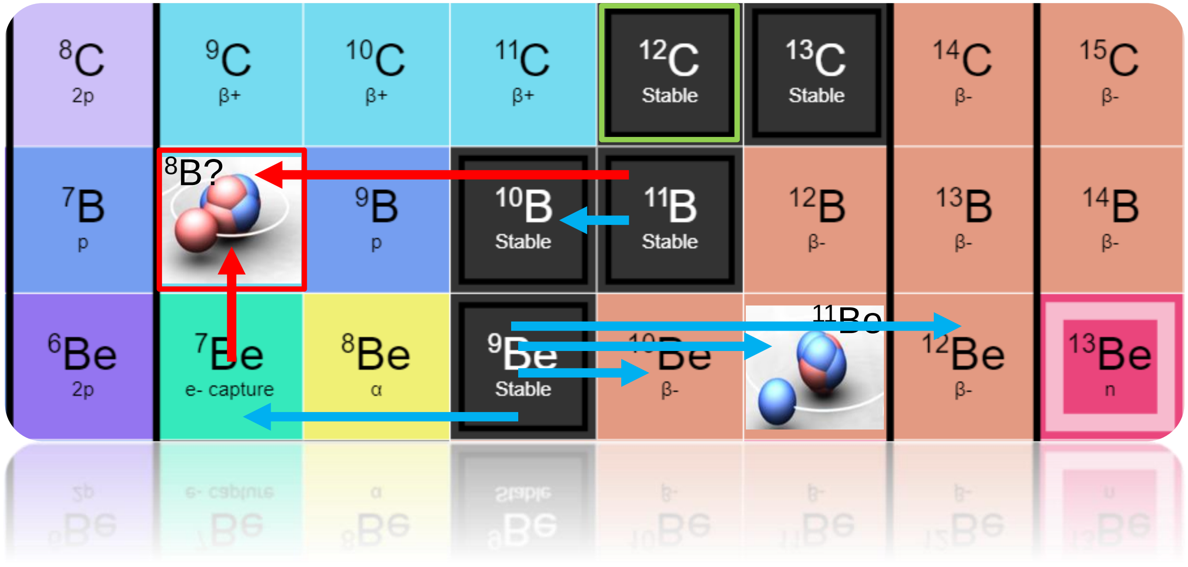
$R_c(^{6}\mathrm{Li}) = 2.589\,(39)\ \mathrm{fm}$

W. Nörtershäuser *et al.*, Phys. Rev. C **84**, 024307 (2011)

$\mathbf{R_c(\alpha) = 1.678\,24(83)\ fm}$

J.J. Krauth *et al.*, Nature **589**, 527 (2021)

The Proton-Halo Nucleus ^{8}B



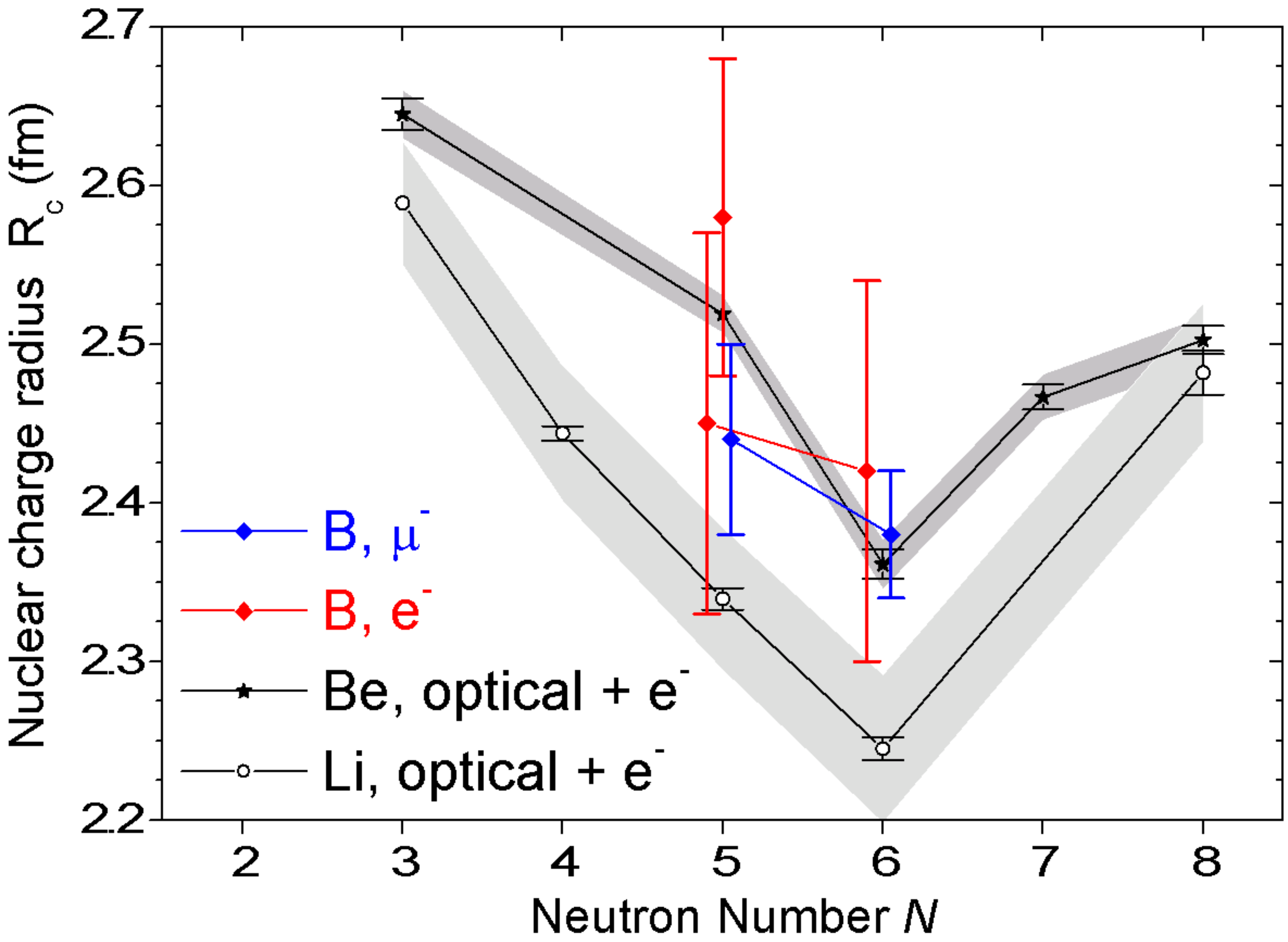
$$\delta\nu_{\mathrm{IS}} - \delta\nu_{\mathrm{MS}}^{\mathrm{Theory}} \propto \delta\langle r_{\mathrm{c}}^2\rangle \longrightarrow R_{\mathrm{c}}(A) = \sqrt{R_{\mathrm{c}}^2(A_{\mathrm{ref}}) + \delta\langle r_{\mathrm{c}}^2\rangle^{A_{\mathrm{ref}},A}}$$

Reference Radii required

„Proton-halo size": $R_{\mathrm{c}}(\mathrm{p_{halo}}) = R_{\mathrm{c}}(^{8}\mathrm{B}) - R_{\mathrm{c}}(^{7}\mathrm{Be})$

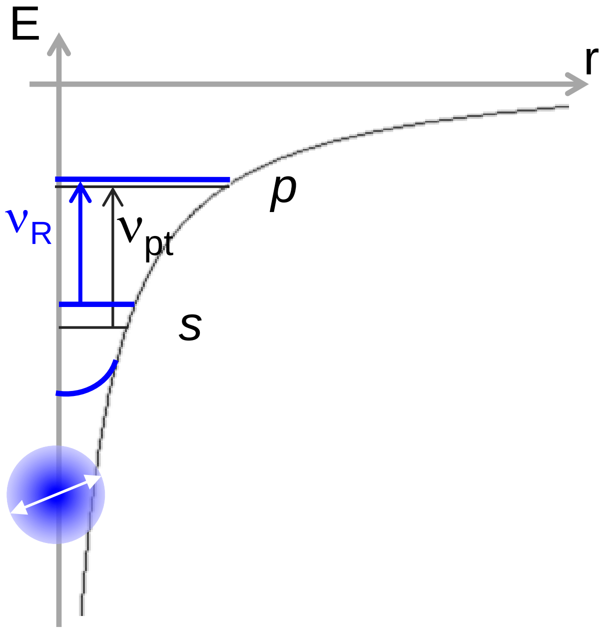
Conclusion: To gain information about the proton halo of ^{8}B, we need reliable reference radii for Be and B **on equal footing** !

„Reference“ Radii of Boron



[1] Stovall, Nucl. Phys. 86, 225 (1966)
[2] Cichocki et al., PRC 51, 2406 (1995)
[3] Schaller *et al.*, Nucl. Phys. A 343, 333 (1980)
[4] Olin *et al.*, Nucl Phys A 360, 426 (1981)

All-Optical Absolute Charge Radii



$$\delta\nu_{FS} = \nu_R - \nu_{pt}$$

$$= -\frac{Ze^2}{6\varepsilon_0}\Delta\left|\Psi_e(0)\right|^2_{i\to f} \times \langle r_c^2 \rangle$$

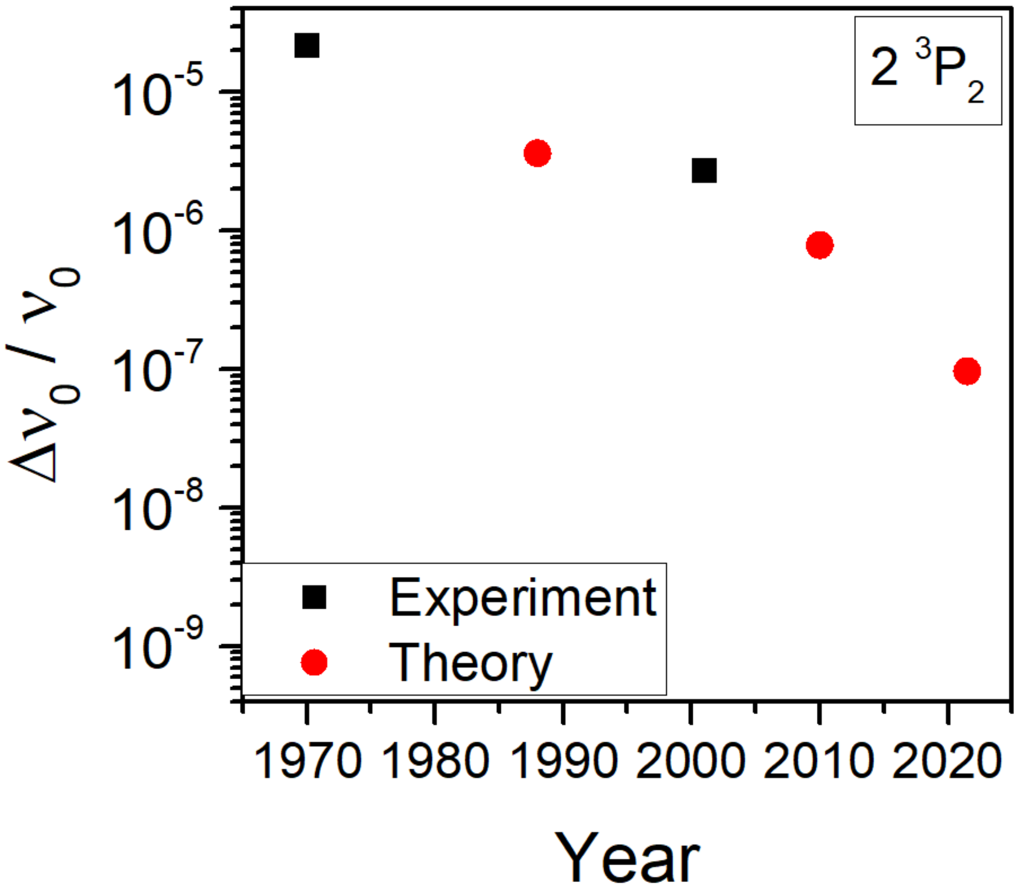
Electronic Factor
(→ Wavefunction)

$$= F_{i\to f}\langle r_c^2 \rangle$$

- Measure **transition frequency** ν_R
- Compare with high precision atomic calculation for a point-like nucleus ν_{pt}
- Difference $\nu_R - \nu_{pt}$ is finite-size effect and **proportional to the ms charge radius**
- So far applied **only for H-like systems**, i.e., H, μH and μHe
- Two-electron system requires elaborate QED calculations, which have been improved considerably

V.A. Yerokhin, V. Patkóš & K. Pachucki, PRA **98**, 032503 (2018)
V. Patkóš, V.A. Yerokhin &K. Pachucki, PRA **103**, 042809 (2021)
V.A. Yerokhin, V. Patkóš & K. Pachucki, PRA **106**, 022815 (2022)

Theory



Exp: S. Ozawa *et al.*, Phys. Scr. **T92**, 195 (2001)
Beam-foil spectroscopy

TABLE X. Comparison of theoretical and experimental $n = 2$ intrashell transition energies, in cm^{-1}.

Z	Theory	Experiment	Difference	Ref.
$2\,^3S_1 - 2\,^3P_0$				
5	35 393.6211 (49)	35 393.627 (13)	−0.006 (13)	[47]
	35 393.628 (14)[a]			
8	60 978.788 (27)	60 978.44 (52)	0.35 (52)	[48]
	60 978.85 (14)[a]			

V.A. Yerokhin, V. Patkóš & K. Pachucki, PRA **106**, 022815 (2022)

	$1s2s\,^3S_1 \to 1s2p\,^3P_2$	$1s2s\,^3S_1 \to 1s2p\,^3P_1$	$1s2s\,^3S_1 \to 1s2p\,^3P_0$
ΔE_p (eV)	5.45804678(53)	5.4412107(31)	5.4427592(11)
F (μeV/fm^2)	-0.875	-0.875	-0.874
ν_p (GHz)	1319749.83(13)	1315678.89(75)	1316053.32(27)
F (GHz/fm^2)	0.2115	0.2115	0.2113

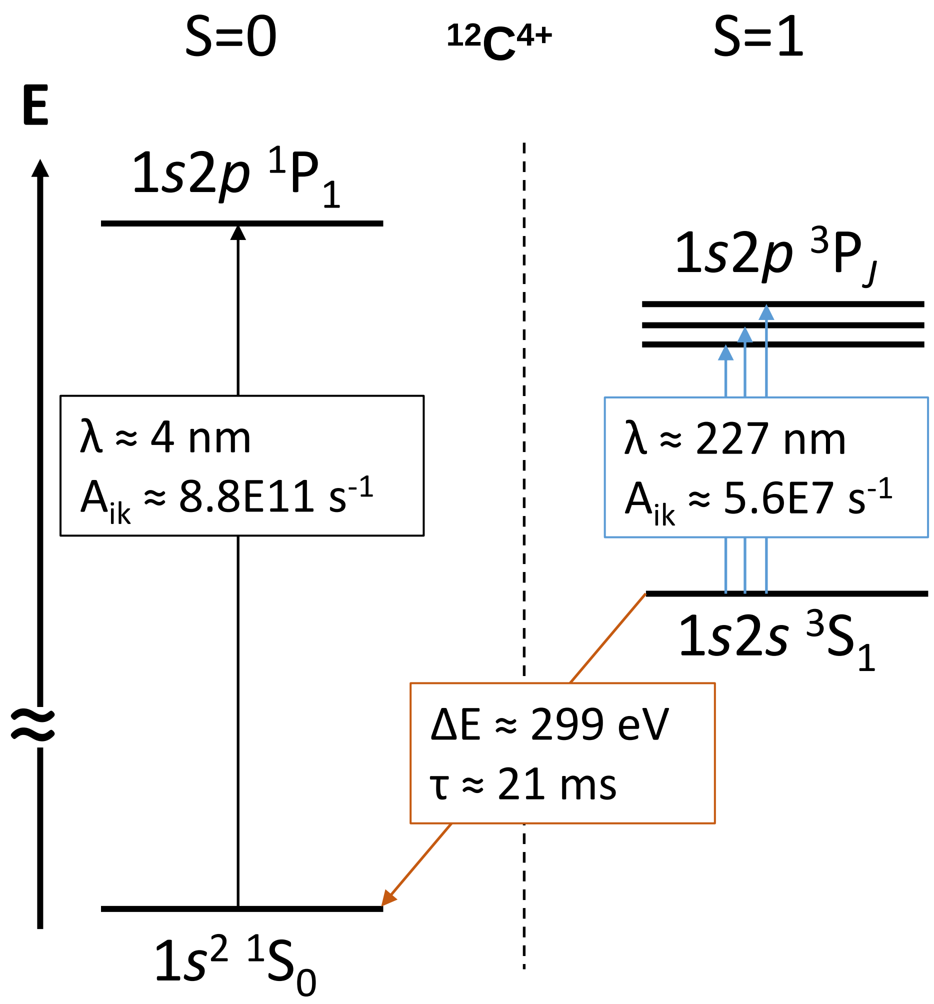
P. Imgram, Dissertation TU Darmstadt (2023)
P. Imgram, PRL**131** 243001 (2023)

$R_c(^{12}C) = 2.471(6)$ fm

I. Sick, PLB 116, 212 (1982)

Required accuracy for $\Delta R_C \leq 0.006$ fm: $\Delta\nu \leq 6$ MHz

The Case of $^{12}C^{4+}$



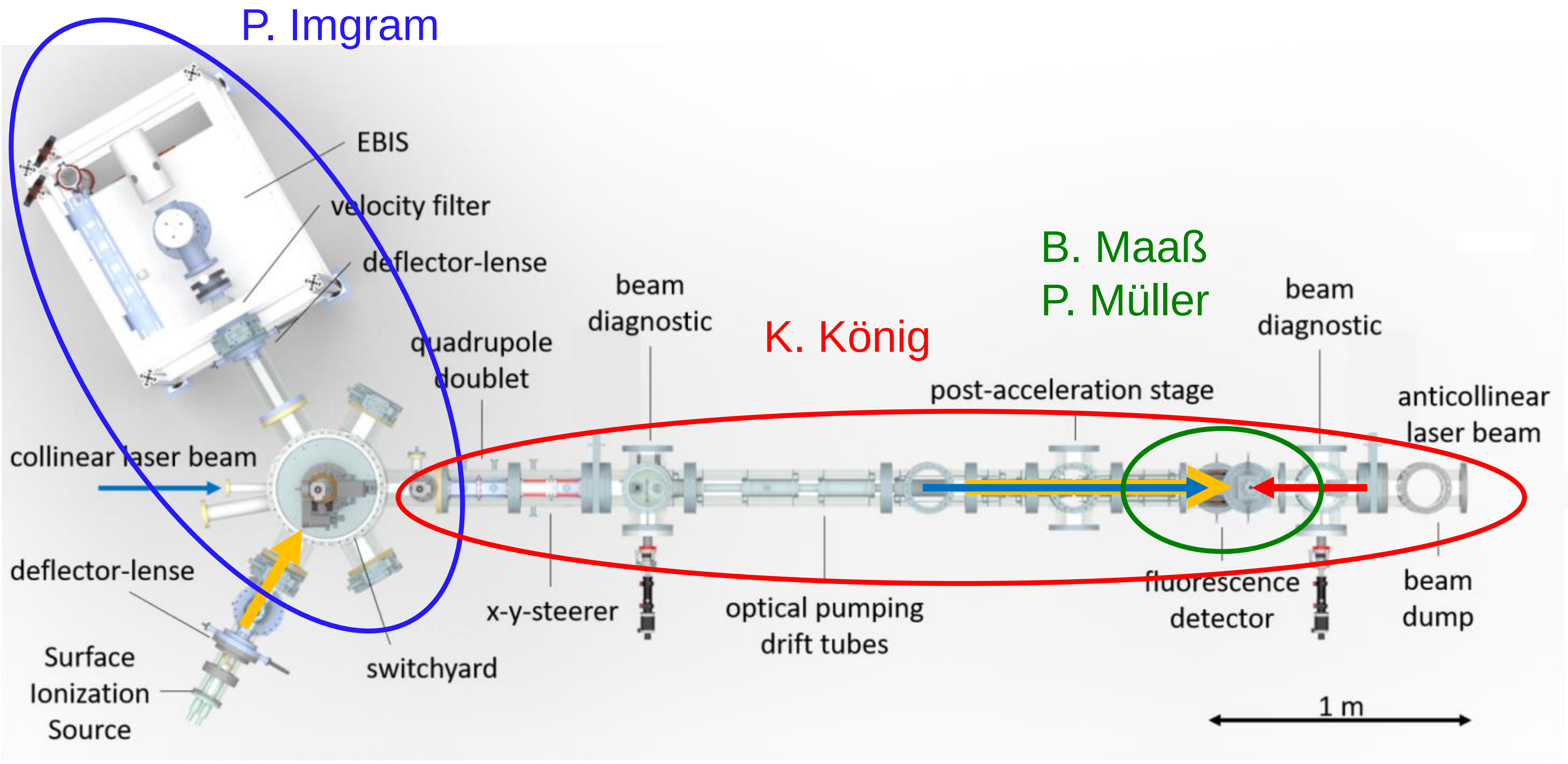
e⁻ scattering: $R_c(^{12}C) = 2.471(6)$ fm
I. Sick, PLB **116**, 212 (1982)

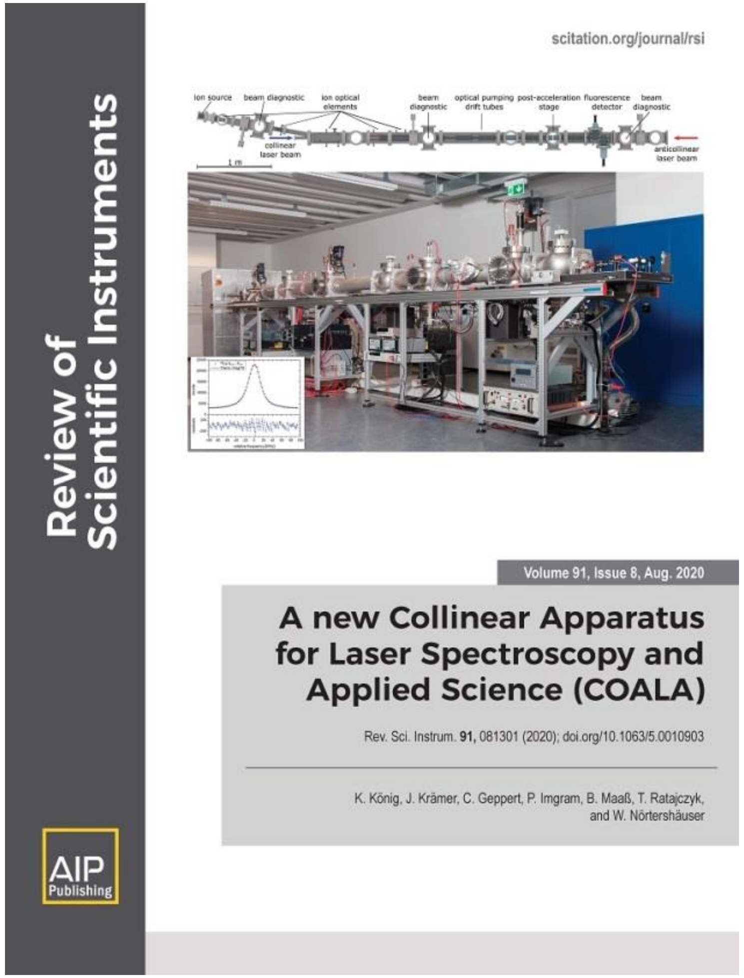
Muonic atoms: $R_c(^{12}C) = 2.4829(19)$ fm
W. Ruckstuhl *et al.*, NPA **430**, 685 (1984)

- Nuclear Charge Radius of ^{12}C well known
  → Test of Theory
- Easy to produce in an EBIS
- $\lambda \approx 227$ nm → Ti:Sa × 4 stabilized to frequency comb
- $I = 0$ → no hyperfine-structure induced level mixing

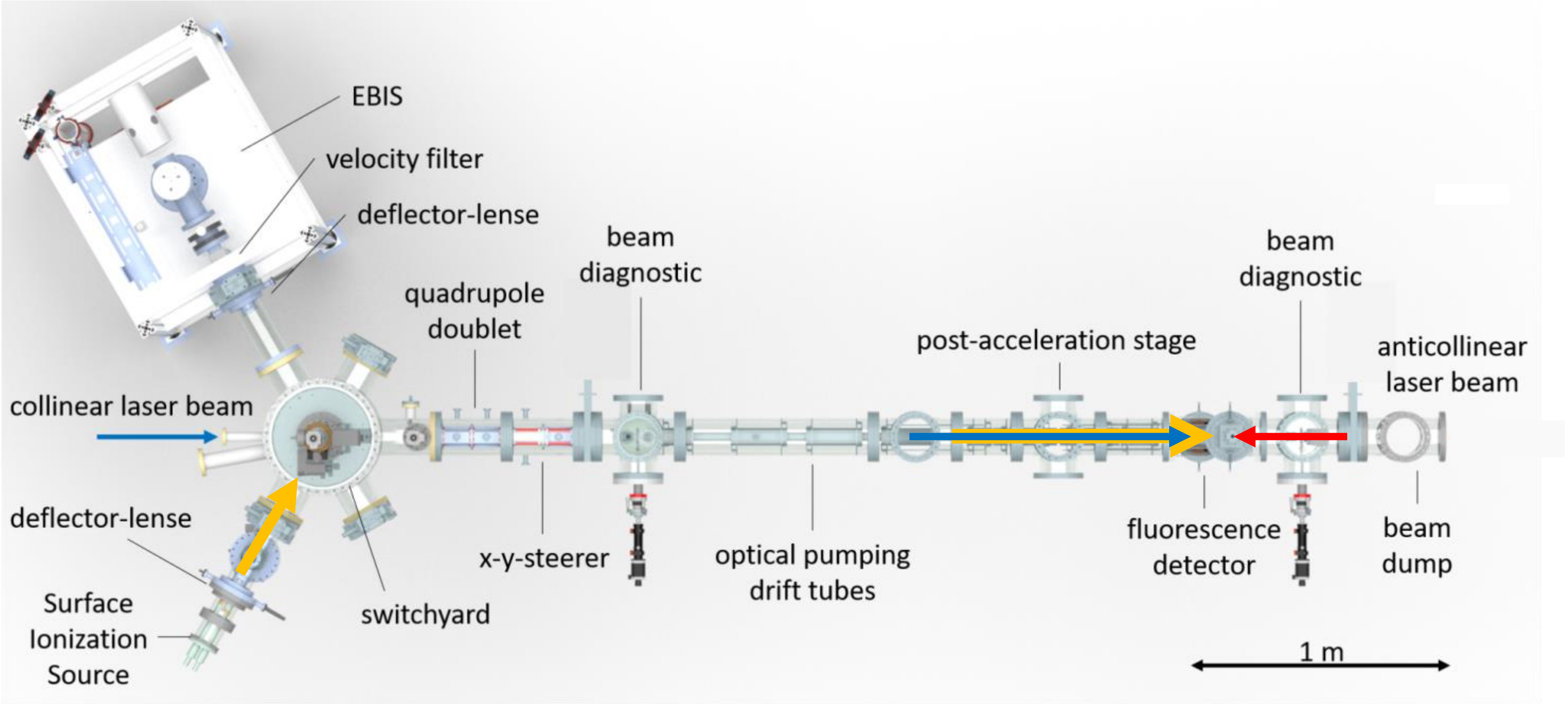
EXPERIMENTAL SETUP

COALA = Collinear Apparatus for Laser Spectroscopy and Applied Physics





Measurement Scheme



$$\nu_c = \nu_0 \gamma (1 + \beta)$$
$$\nu_a = \nu_0 \gamma (1 - \beta)$$

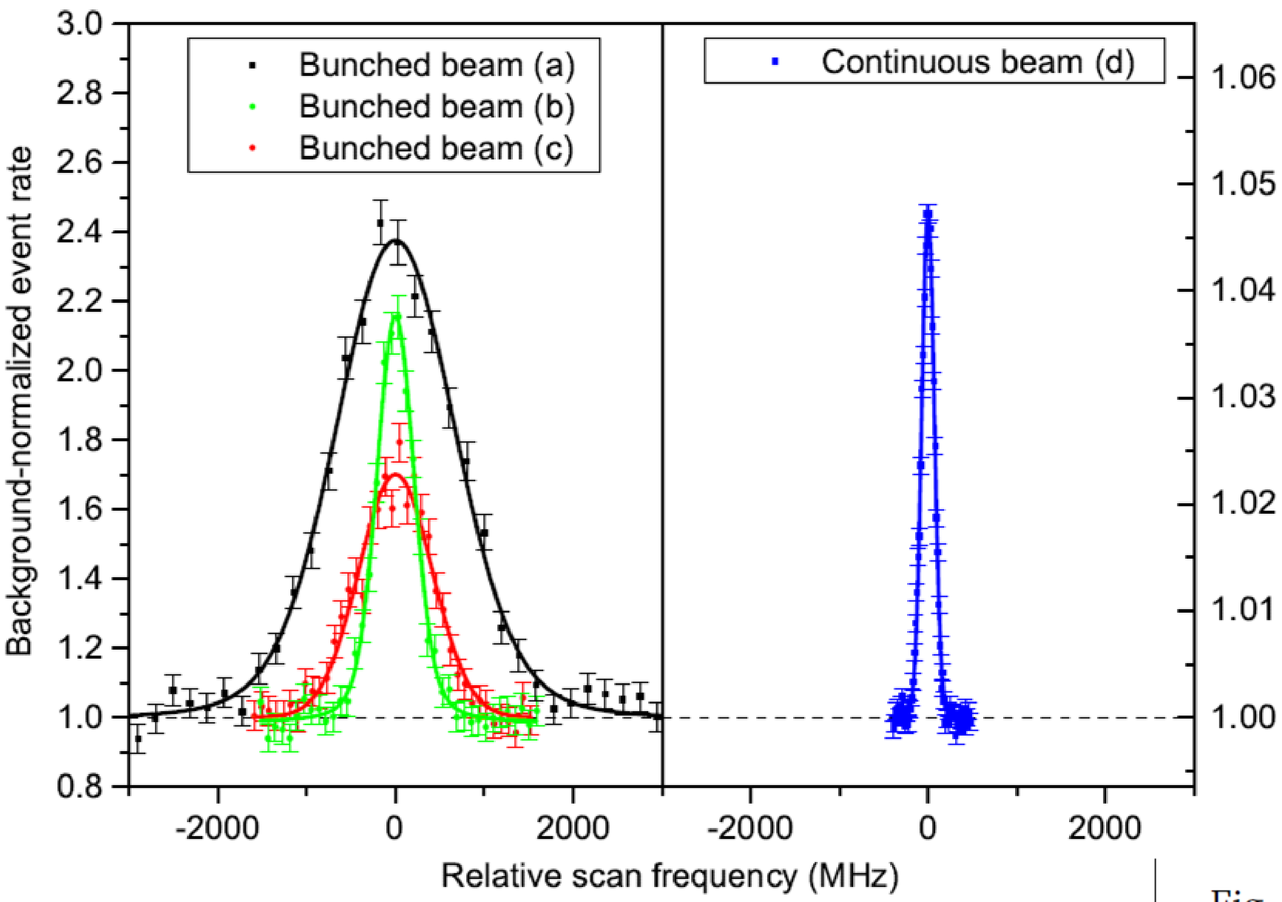
$$\nu_c \cdot \nu_a = \nu_0^2 \gamma^2 \cdot (1 + \beta)(1 - \beta) = \nu_0^2$$

→ Measurement of ν_0 instead of $\delta\nu^{A',A}$ removes uncertainties caused by the inaccurate knowledge of β (and γ).

LASER SPECTROSCOPY

OPTIMIZATION AND RESULTS

Comparison of Pulsed and Continuous Extraction

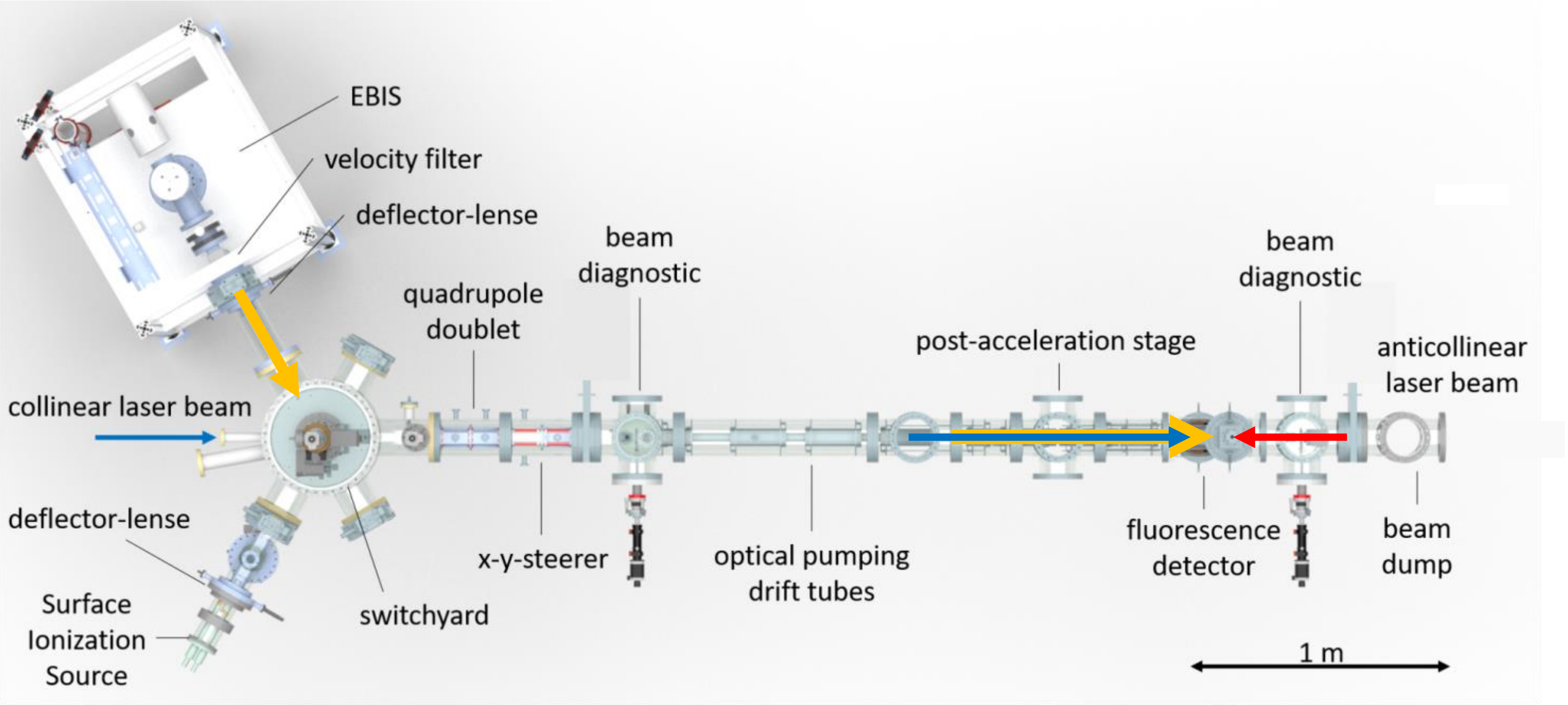


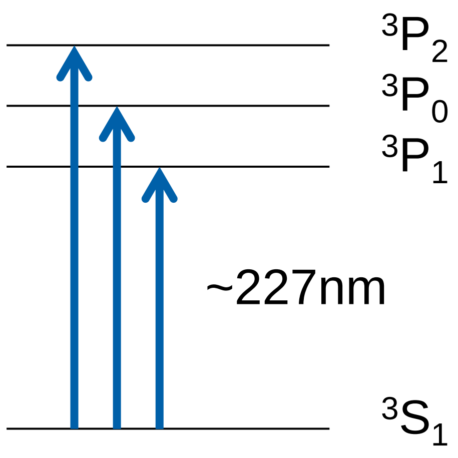
continuous extraction (leaky mode):

- good signal/noise ratio (similar to pulsed)
- smallest linewidth (gain factor 3)
- smallest statistical uncertainty for line centre

	Fig.	I_e / mA	ΔU_{trap} / V	p_{gas} / mbar	t_{breed} / ms	SNR	FWHM / GHz	$\Delta\nu_{center}$ / MHz
(a)	5.1/5.2	80	71	$6 \cdot 10^{-8}$	15	58	1.59(4)	16.6
(b)		25	26	$6 \cdot 10^{-8}$	15	51	0.50(1)	5.7
(c)	5.2	25	26	$2 \cdot 10^{-8}$	15	17	0.97(4)	17.8
(d)		80	171	$6 \cdot 10^{-8}$	leaky	59	0.168(2)	0.7

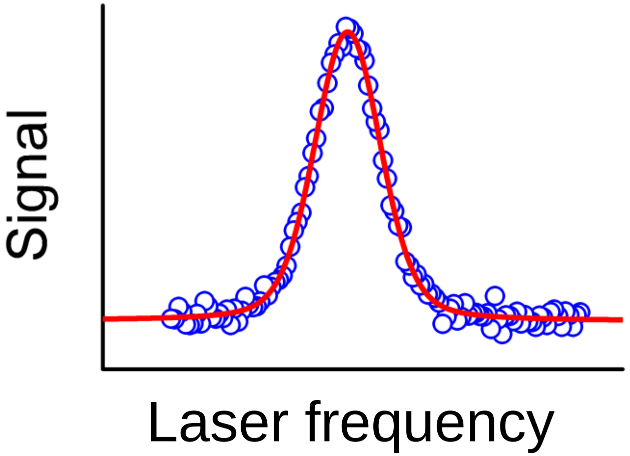
Laser spectroscopy of He-like ions





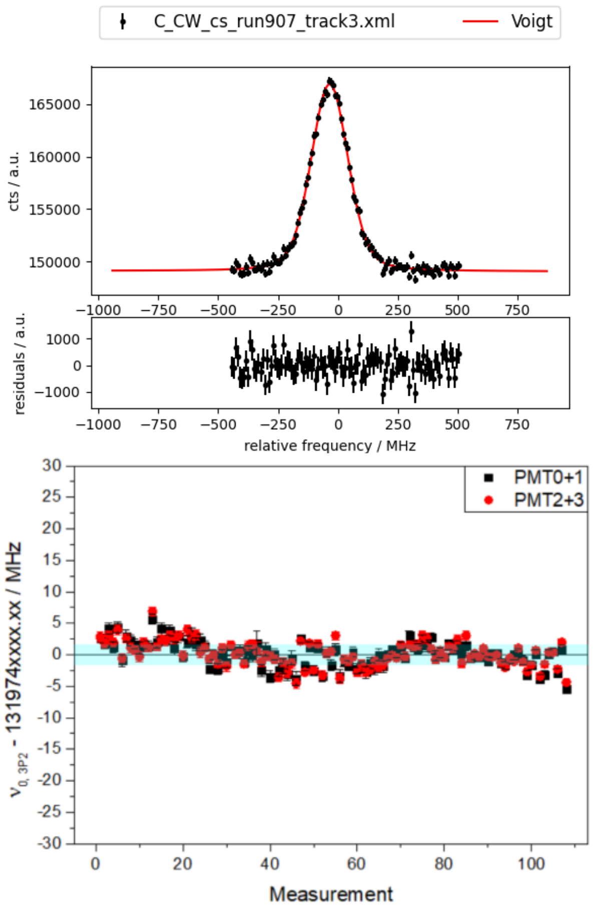
$$\nu_c = \nu_0 \gamma (1+\beta)$$
$$\nu_a = \nu_0 \gamma (1-\beta)$$

$$\nu_c \cdot \nu_a = \nu_0^2 \gamma^2 \cdot (1+\beta)(1-\beta) = \nu_0^2$$

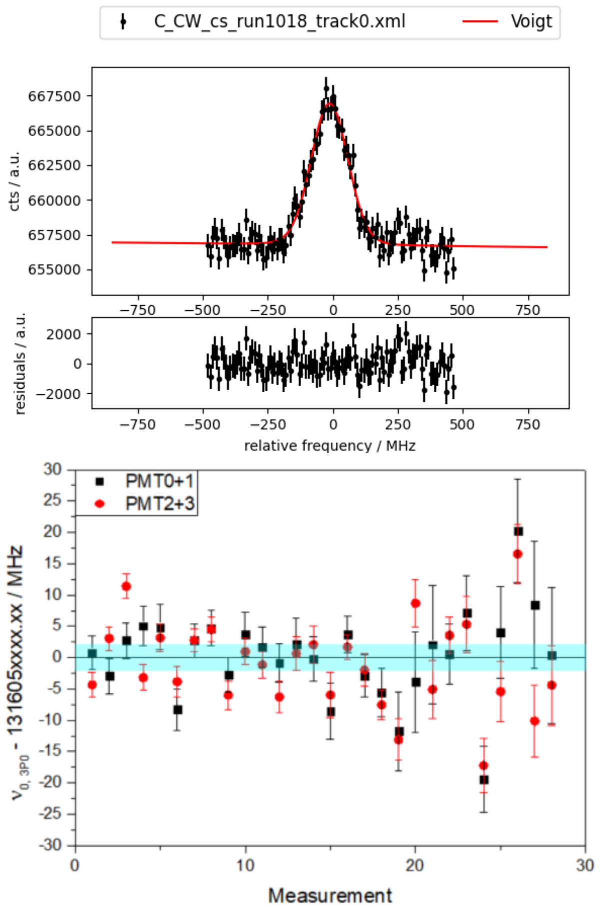


Laser spectroscopy results for $^{12}C^{4+}$

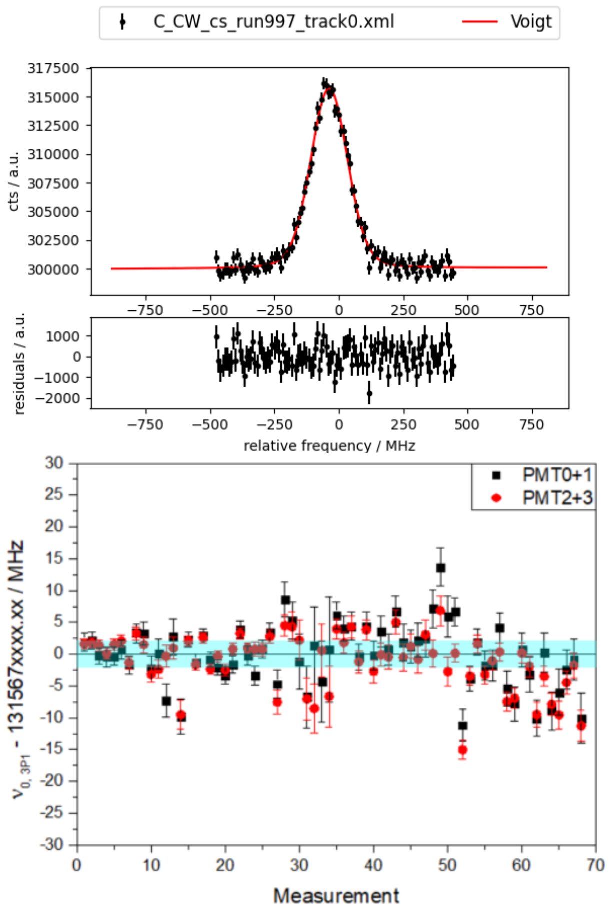
$^3P_2$

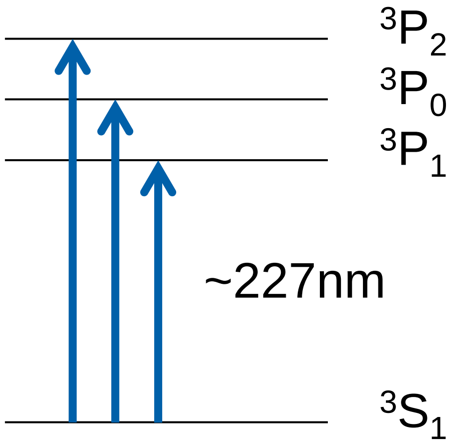


$^3P_0$

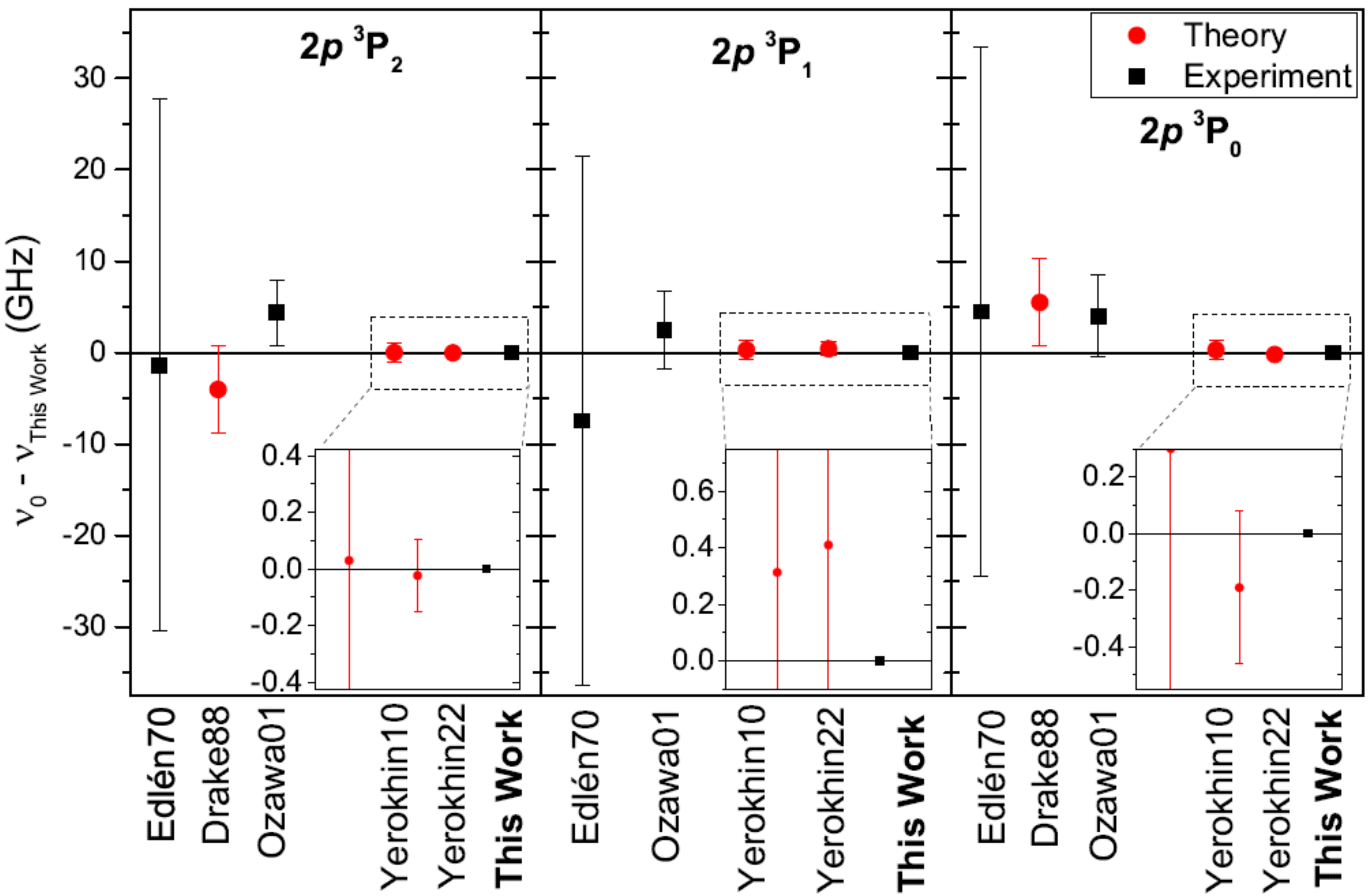


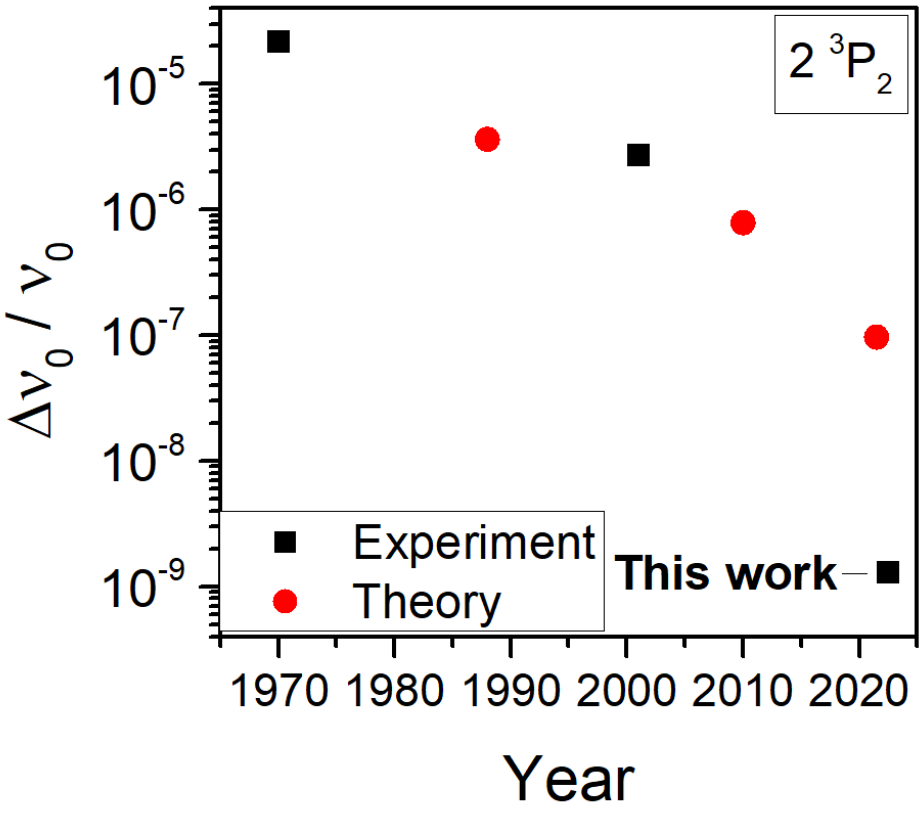
$^3P_1$





Transition Frequencies: Comparison to Literature

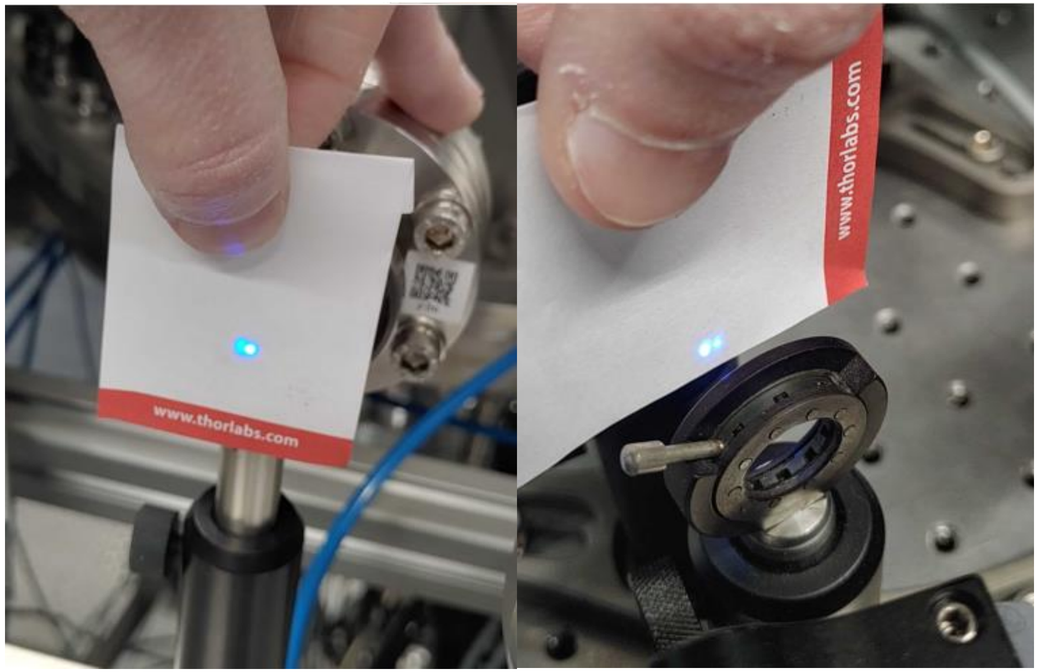




Uncertainty Budget and Fine Structure Splitting

TABLE IV. All investigated systematic uncertainties in MHz. The main systematic uncertainty originates from the alignment of the two laser beams.

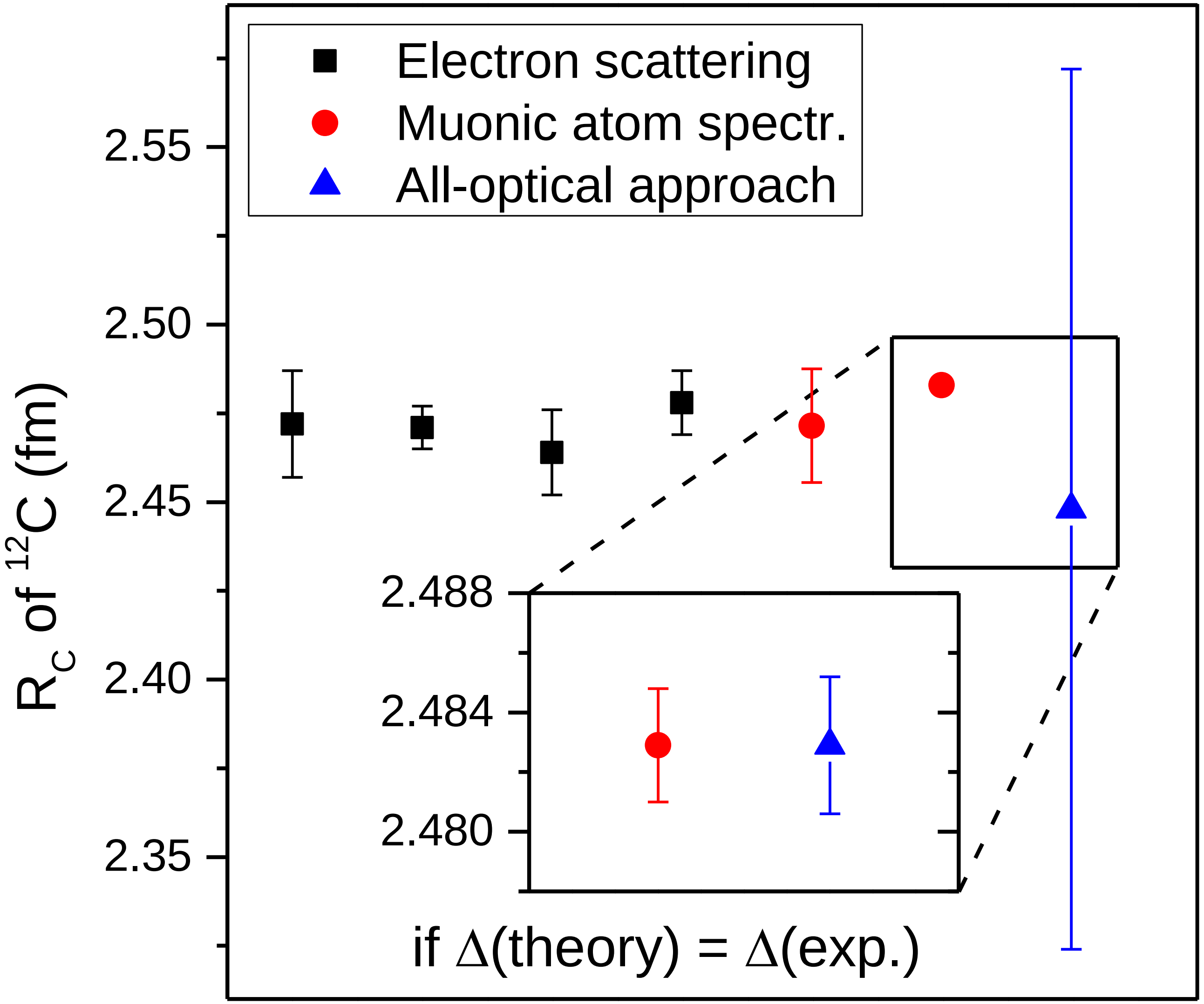
Uncertainty	
Line shape	<0.01
Start potential	0.01
Beam alignment	1.7
Photon recoil	<0.1
Scan voltage	<0.07
Laser polarization	0
Total	1.7



(J, J')	This work	Theory [52]	$m\alpha^{8+}$ contr.
$(0, 1)$	$-375\,026.5\,(2.5)$	$-374\,996.3\,(48.0)$	$-30.2\,(2.5)$
$(0, 2)$	$3\,696\,352.1\,(2.5)$	$3\,696\,343.5\,(10.0)$	$8.6\,(2.5)$

Relatively large contribution of higher orders
⇒ may guide further QED calculations

First all-optical nuclear charge radius of ^{12}C



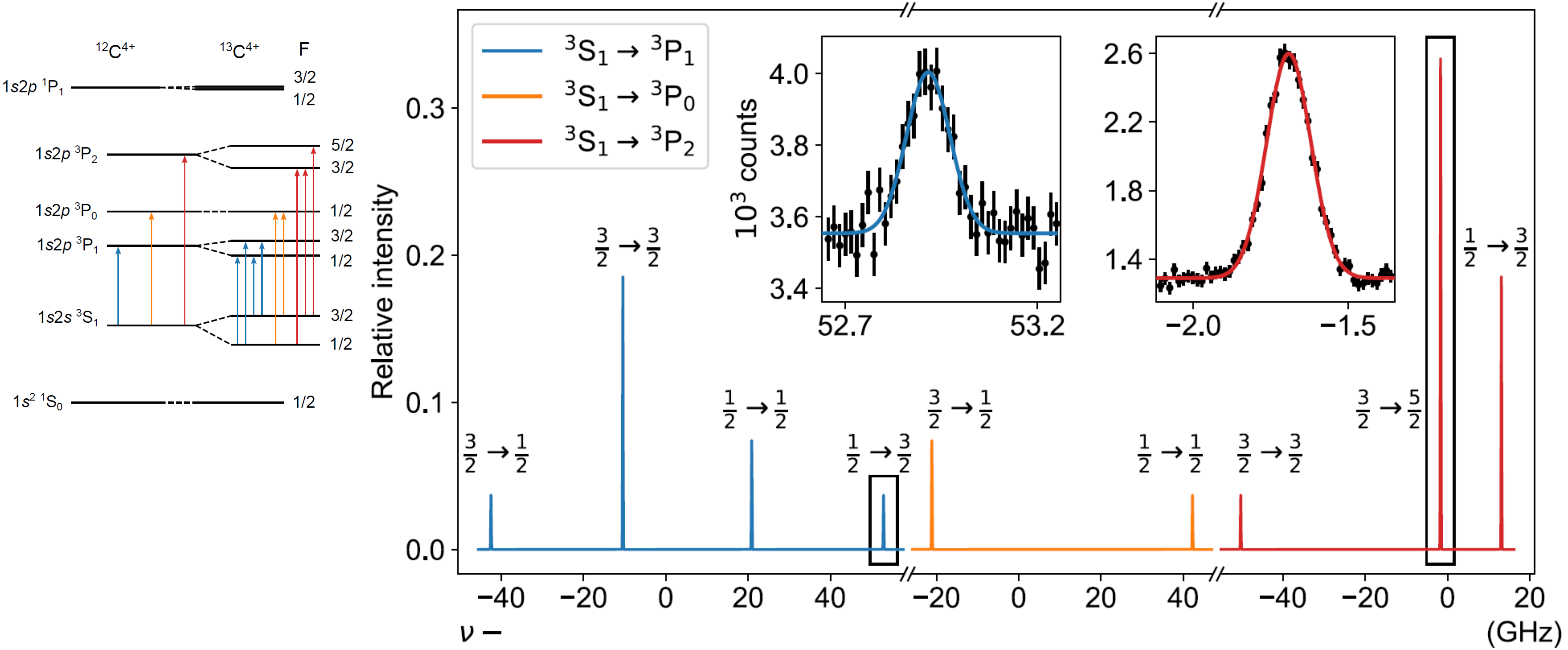
→ more work needed for competitive all-optical R_C

If **theory** gets **better by two orders of magnitude,** we are **competitive** with the most precise **muonic atom results.**

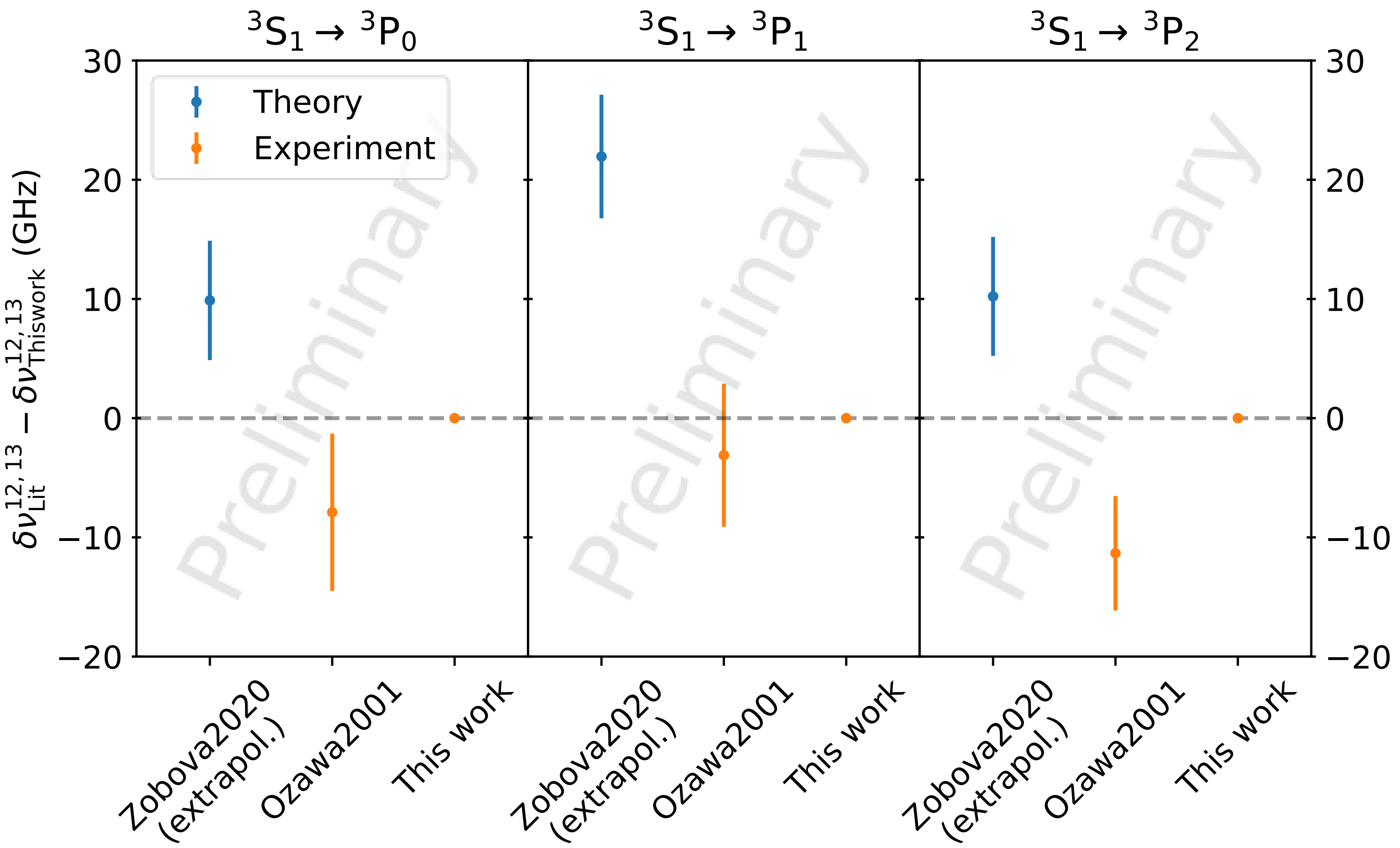
→ may resolve the discrepancies between muonic and elastic electron scattering results

First high-precision laser spectroscopy in C isotope chain
→ starting point for regular isotope shift measurements to extract $\delta\langle r^2\rangle$ **of ^{13}C and ^{14}C**

Measurements in ^{13}C



Comparison with Results and Predictions



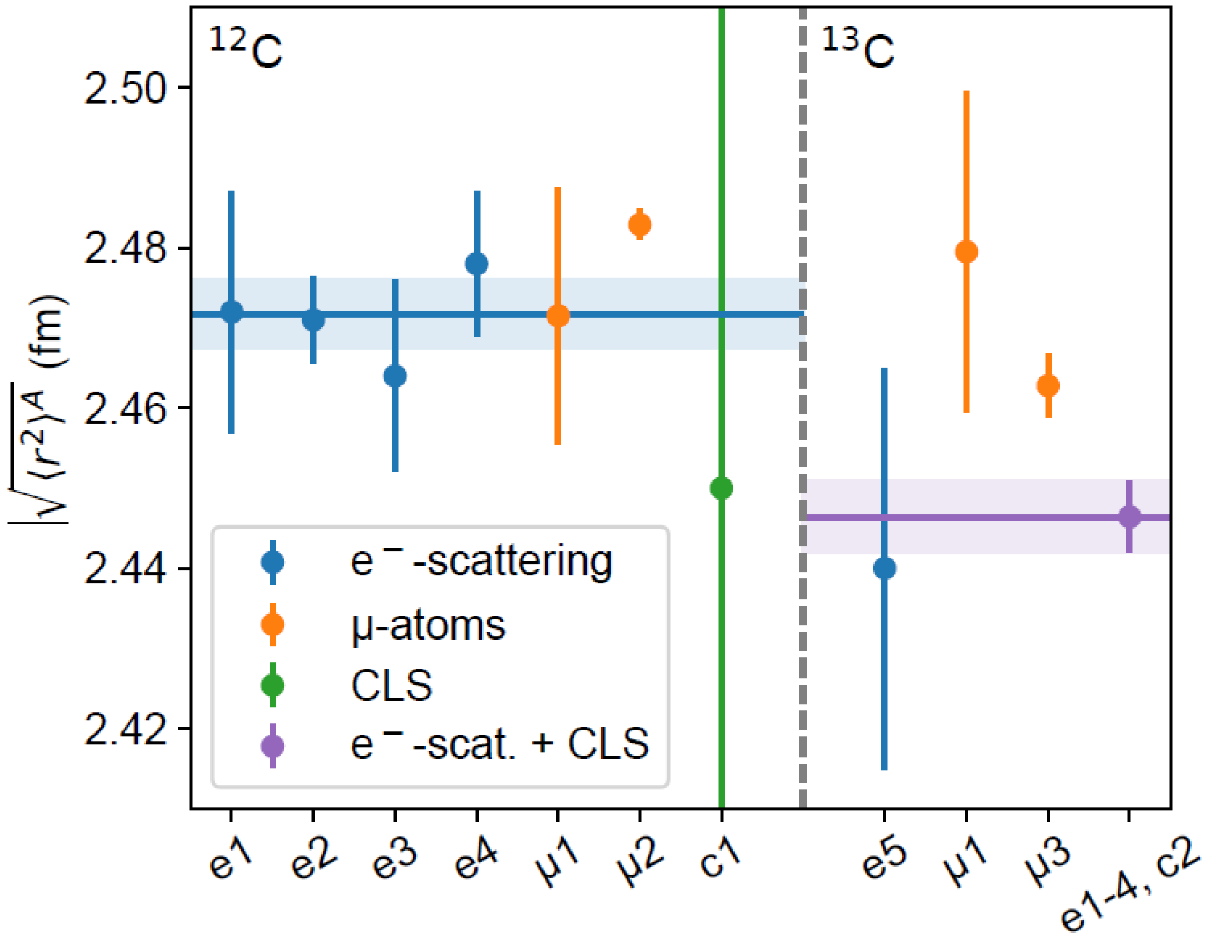
Determination of the isotope shifts with < 3 MHz uncertainty.

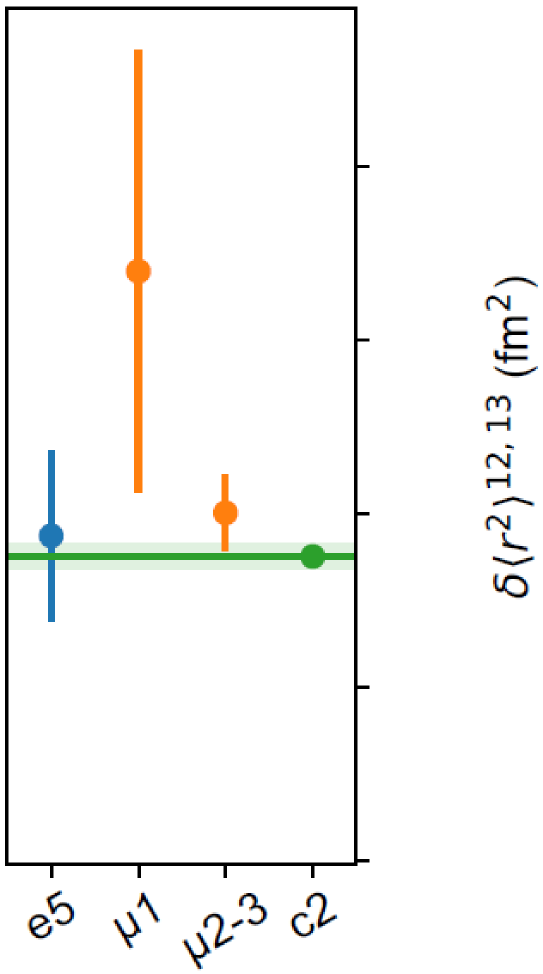
Extract $\boldsymbol{\delta\langle r^2\rangle}$ with 0.009 fm² (~6 %) uncertainty

Calculation of the specific mass shift required

$$H^A_{\mathrm{SMS}} = \frac{1}{M_A}\sum_{i<j}\mathbf{p}_i \cdot \mathbf{p}_j$$

Summary: Nuclear charge radii of $^{12,13}C$





- **Improved** previous experiments by **>3 orders** of magnitude
- **All-optical** nuclear charge radius of ^{12}C → more work needed for competitive all-optical R_C
- **First** high-precision laser spectroscopy in C isotope chain → $\boldsymbol{\delta\langle r^2\rangle}$ **in C chain**

- **Ab initio nuclear-structure calculatons of ^{12,13}C** are carried out in the groups of A. Schwenk and R. Roth

What Next with He-like Ions?

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Atomic structure theory
(NR-QED calculations)

- Remaining uncertainty originating from **neglected higher order terms** ($m\alpha^{8+}$)
  → Measured fine-structure splitting in $^{12}C^{4+}$ will **benchmark new calculations**

- This will **improve** the extraction of **α** from fine-structure measurements in He & (hopefully) as well the all-optical nuclear charge radius determination

Laser spectroscopy of He-like ions

- Measurement of $^{13}C^{4+}$ already completed
  → Analysis completed, Publication in prep.

- $^{14}C^{4+}$ is planned for first half of 2024

- $^{10,11}B^{3+}$ and $^{10,11}B^{2+}$ in 2024

- $^{9}Be^{2+}$ in > 2024
  → Feeding of Be^{+} into EBIS needs further development

LoI at ISOLDE/CERN accepted (INTC-I-265)

possible physics cases: ^{7}Be, $^{12,13,14}B$, $^{10,11,15,16}C$

Thank you!

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DFG Deutsche Forschungsgemeinschaft

SFB 1245

Federal Ministry of Education and Research

HFHF Helmholtz Forschungsakademie Hessen für FAIR

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research