Trace anomaly form factors from Lattice QCD

Fangcheng He

Collaborators: B. Wang, G. Wang, J. Liang, T. Draper, K. Liu and Y. Yang

arXiv:2401.0549

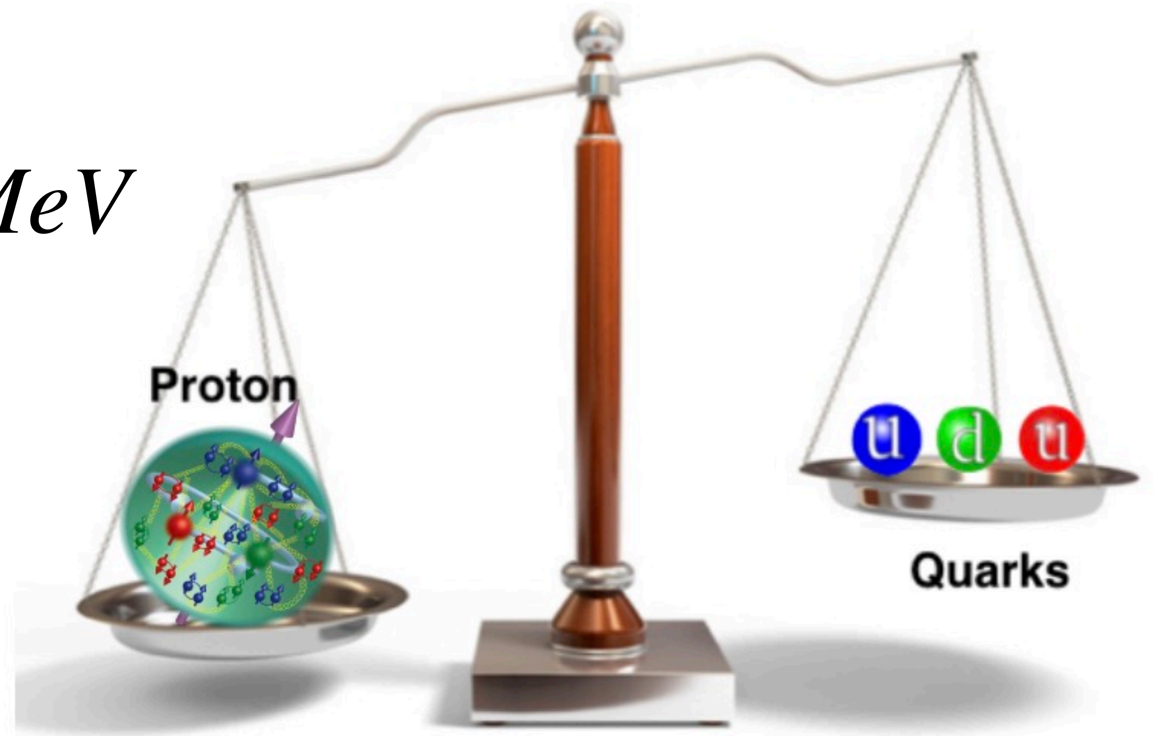
Outline

- Introduction to QCD trace anomaly
- The lattice calculations of gluon trace anomaly
    - Trace anomaly in the forward limit F. He, P. Sun and Y. Yang, PRD104(2021)
    - Gluon trace anomaly FFs Bigeng Wang, F. He, et. al , 2401.0549
- Summary

Proton mass and sigma term

- **Proton mass vs quark mass**

$M_P = 938MeV$



$$m_d = 4.73(12)MeV$$
$$m_u = 2.08(09)MeV$$

- **QCD energy momentum tensor**

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)}\psi + \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}{}_{\alpha},$$

↓ **"Classical Trace"**

$$T^{\mu}_{\mu} = \sum_f m_q \bar{\psi}\psi$$

$$\langle P|T^{\mu\nu}|P\rangle = P^\mu P^\nu/M,$$

↓

$$\langle P|T^{\mu}_{\mu}|P\rangle = M$$

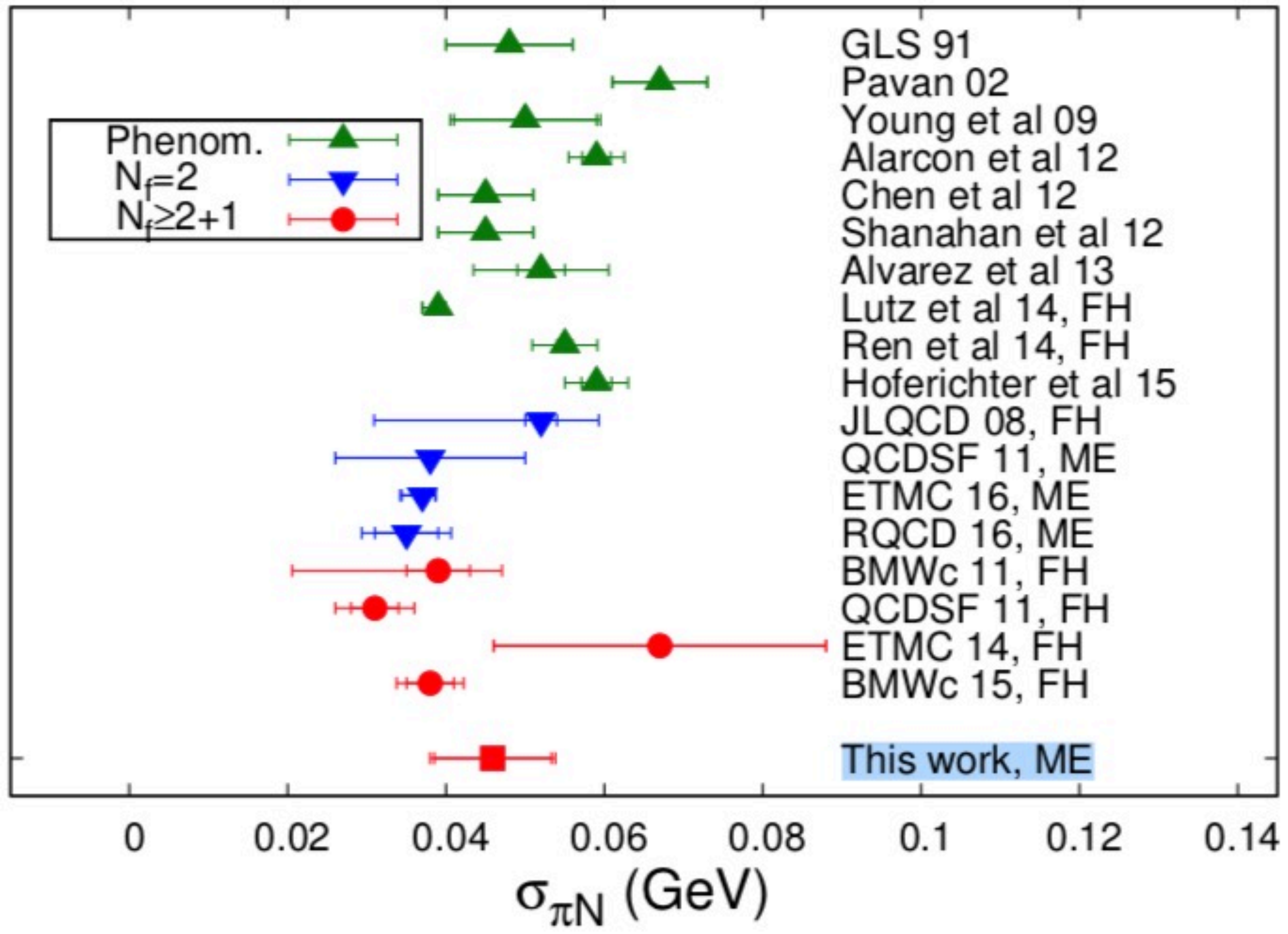
$$M \stackrel{?}{=} \langle P|m_q\bar{\psi}\psi|P\rangle$$

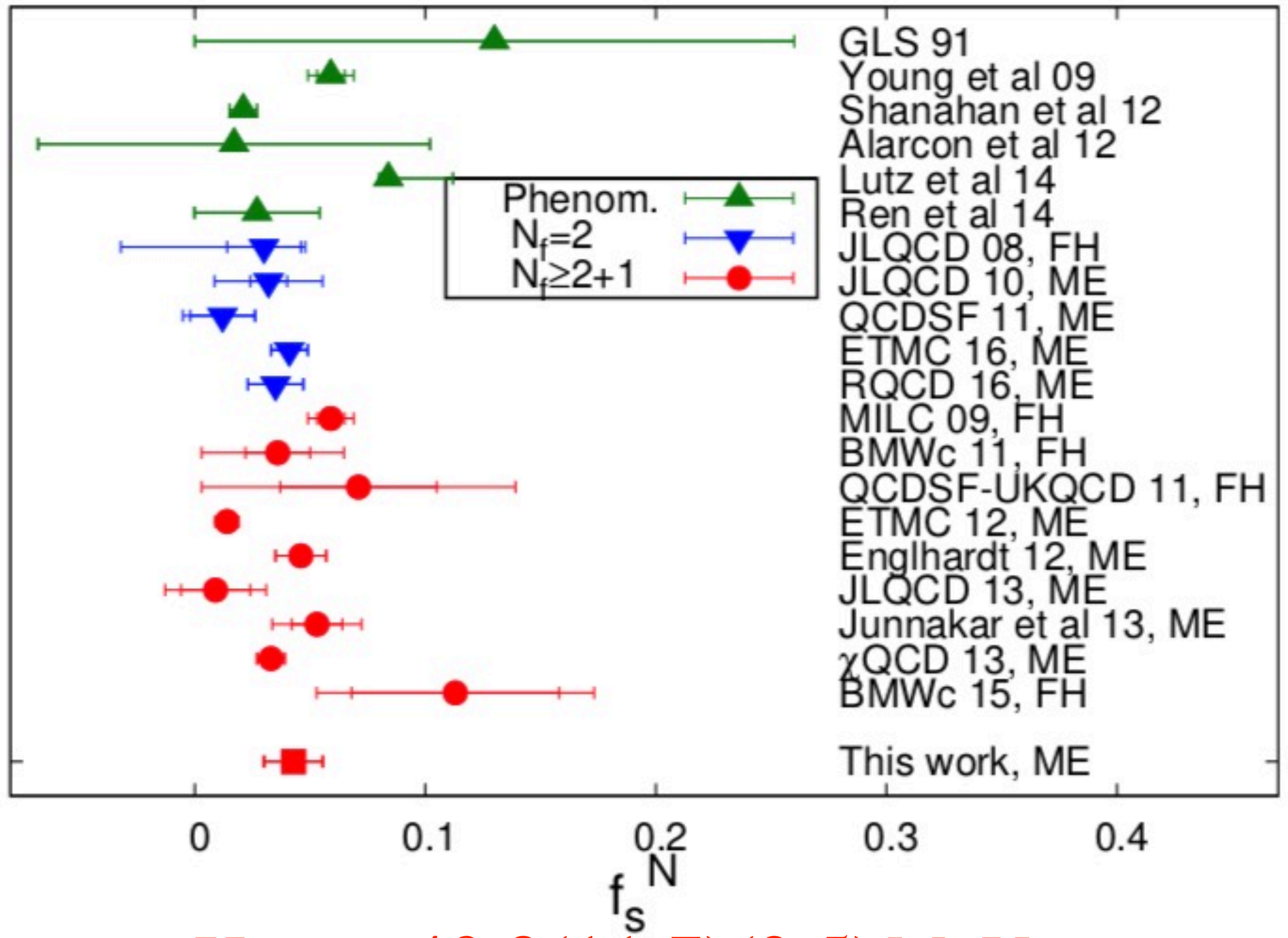
Sigma term

The contribution of sigma term to proton mass

- **Sigma term contributes to proton mass**

Y.B. Yang et,al.(χQCD Collaboration)PRD(2016),054503





$H_{m,u+d} = 45.9(7.4)(2.8)MeV$ $H_{m,s} = 40.2(11.7)(3.5)MeV$

The sigma terms of light quark contribute less than 100MeV to proton mass

Where is the rest mass from?

Scalar transformation and trace anomaly

- **Scale transformation(dilatations):**

$$x \to xe^{\sigma} \qquad \phi(x) \to e^{-D\sigma}\phi(xe^{-\sigma})$$

- **Mass term breaks down the scale symmetry** (Classical case)

J_μ :Norther current

$$\partial_\mu J^\mu = T^\mu_\mu = m_q \bar{q} q$$

- **Scale symmetry is broken when quantum corrections are included**

Peskin and Schroeder, An Introduction to QFT, Chapter 19

$$(T^\mu_\mu)^a = \frac{\beta}{2g}F^2 + \gamma_m m_q \bar{q} q$$

β: QCD beta function

γ_m: Anomalous dimension of quark mass

$$g \to g + \sigma\beta(g)$$

$$m \to m - \sigma m \gamma_m$$

R. J. Crewther, PRL28(1972) 1421
M. S. Chanowitz PLB40(1972) 397
J.Collins et,al. PRD16(1977) 438
N. K. Nielsen, NPB120 (1977) 212

- **Total trace term of QCD EMT**

M. Shifman, et.al., PLB 78 (1978) 443

$$(T^\mu_\mu) = (T^\mu_\mu)^a + m_q \bar{q} q = \frac{\beta_{QCD}}{2g}F^2 + (1+\gamma_m) m_q \bar{q} q = M$$

Outline

- Introduction to QCD trace anomaly
- The lattice calculations of gluon trace anomaly
    - Trace anomaly in the forward limit F. He, P. Sun and Y. Yang, PRD104(2021)
    - Gluon trace anomaly FFs Bigeng Wang, F. He, et. al , 2401.0549
- Summary

Verification of mass sum rule

- **Sum rule:**

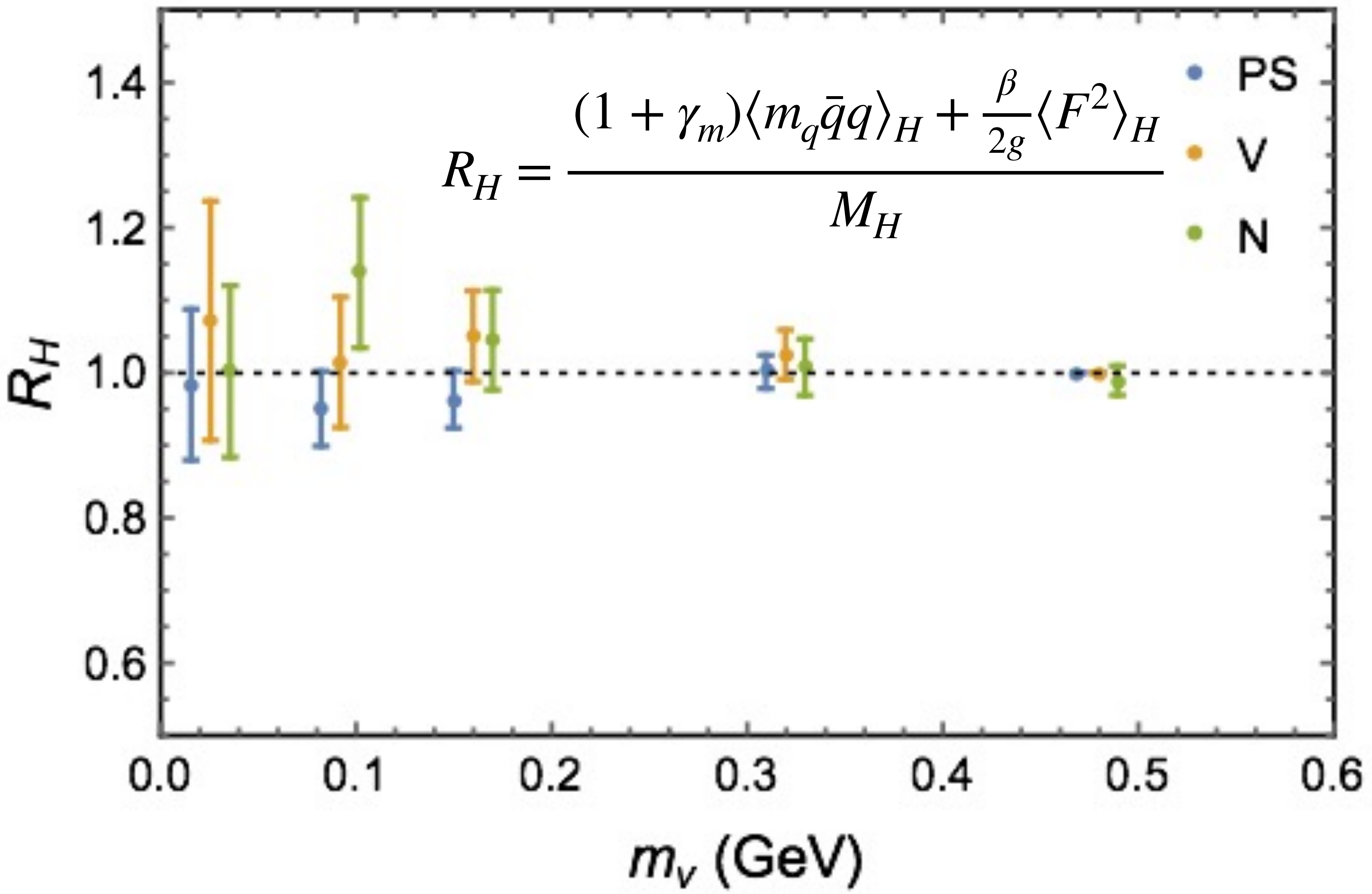
$$M_H = \langle T^\mu_\mu \rangle_H = \langle m_q \bar{q} q \rangle_H + \gamma_m \langle m_q \bar{q} q \rangle_H + \frac{\beta}{2g} \langle F^2 \rangle_H$$

Determination of γ_m and β

$$M_{PS} - (1+\gamma_m)\langle H_m \rangle_{PS} - \frac{\beta}{2g}\langle F^2 \rangle_{PS}|_{m_v=0.5GeV} = 0,$$

$$M_V - (1+\gamma_m)\langle H_m \rangle_V - \frac{\beta}{2g}\langle F^2 \rangle_V|_{m_v=0.5GeV} = 0,$$

$$\gamma_m = 0.38(3) \qquad \frac{\beta}{g^3} = -0.057$$



The ratios R_H are consistent with one within the uncertainty for the different hadron states and different quark mass.

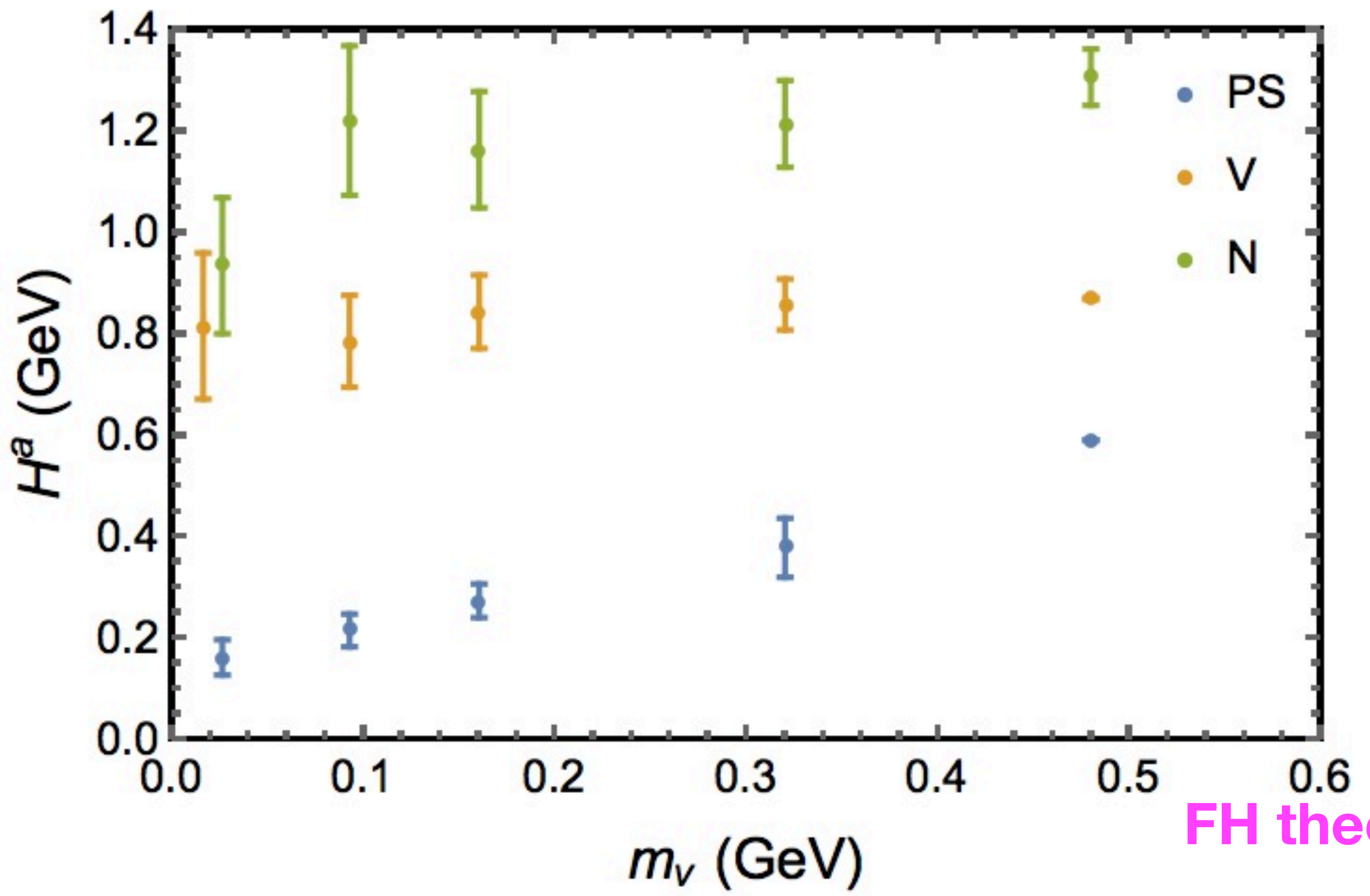
Chiral behavior of trace anomaly in different hadron

F. He, P. Sun and Y. Yang, PRD104(2021)

- **The total trace anomaly in different hadron with different quark (pion) mass**

$$\langle H^a \rangle_H = \gamma_m \langle m_q \bar{q} q \rangle_H + \frac{\beta}{2g} \langle F^2 \rangle_H$$

Total trace anomaly in PS meson have stronger dependence on the quark mass than that in the V meson and nucleon state.



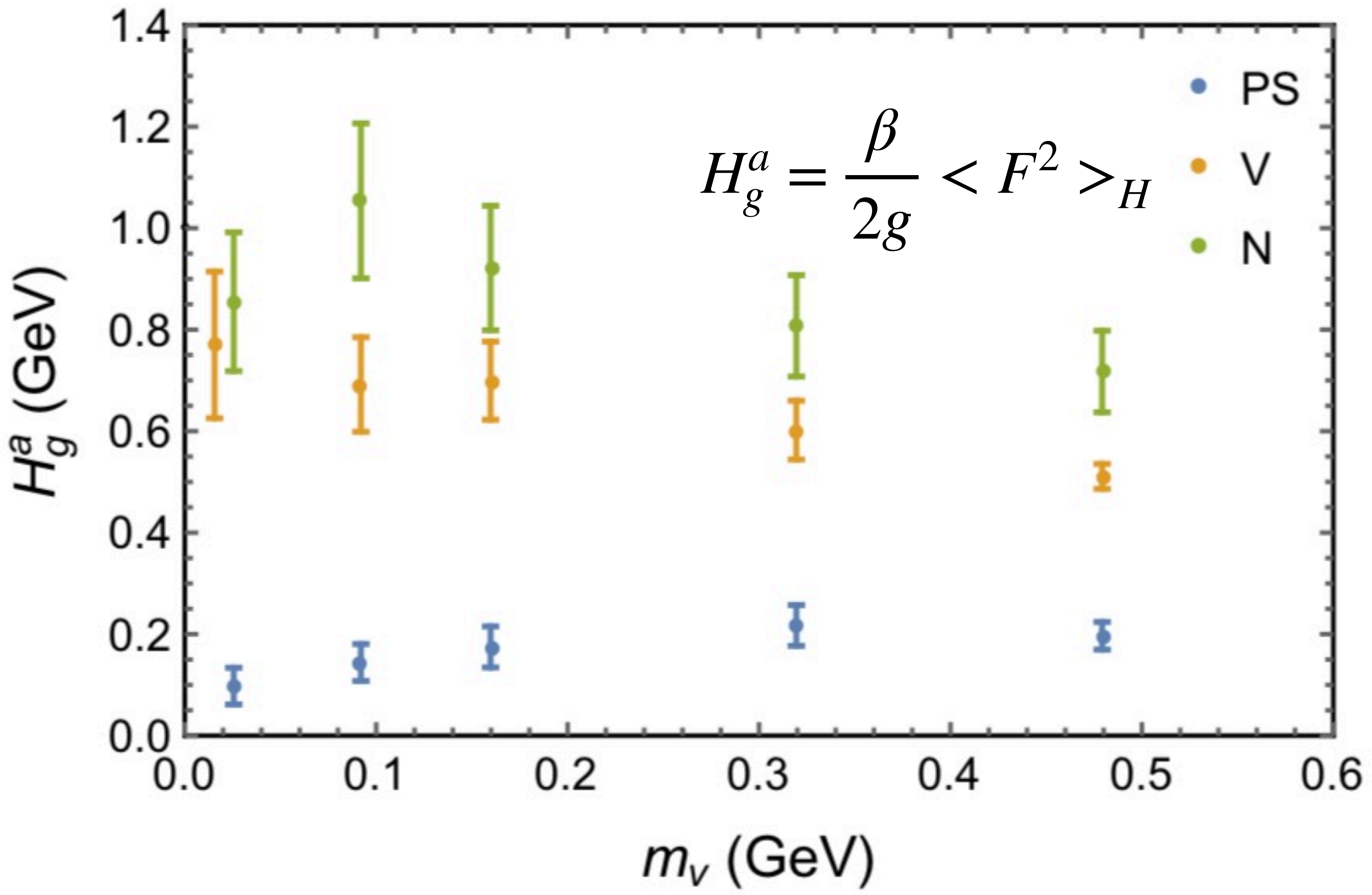
- **Chiral behaviour**

FH theorem: $H^a = M - \frac{m_{\pi,2}}{2} \frac{\partial M}{\partial m_{\pi,2}}$

	PS meson	V meson	Nucleon
Chiral behavior	$M_\pi = m_{\pi,2} + \frac{m_{\pi,2}^3}{32\pi^2 F_\pi^2}$ $H_\pi^a = \frac{m_{\pi,2}}{2} - \frac{m_{\pi,2}^3}{64\pi^2 F_\pi^2}$	$M_\rho = c_0 + c_1 m_{\pi,2} + c_2 m_{\pi,2}^3$ $H_\rho^a = c_0 - \frac{c_2}{2} m_{\pi,2}^3$	$M_N = m_0 - 4c_1 m_{\pi,2}^2 - \frac{3g_A^2 m_{\pi,3}^2}{2(4\pi F_\pi)^2}$ $H_N^a = m_0 + \frac{3g_A^2 m_{\pi,2}^3}{4(4\pi F_\pi)^2}$

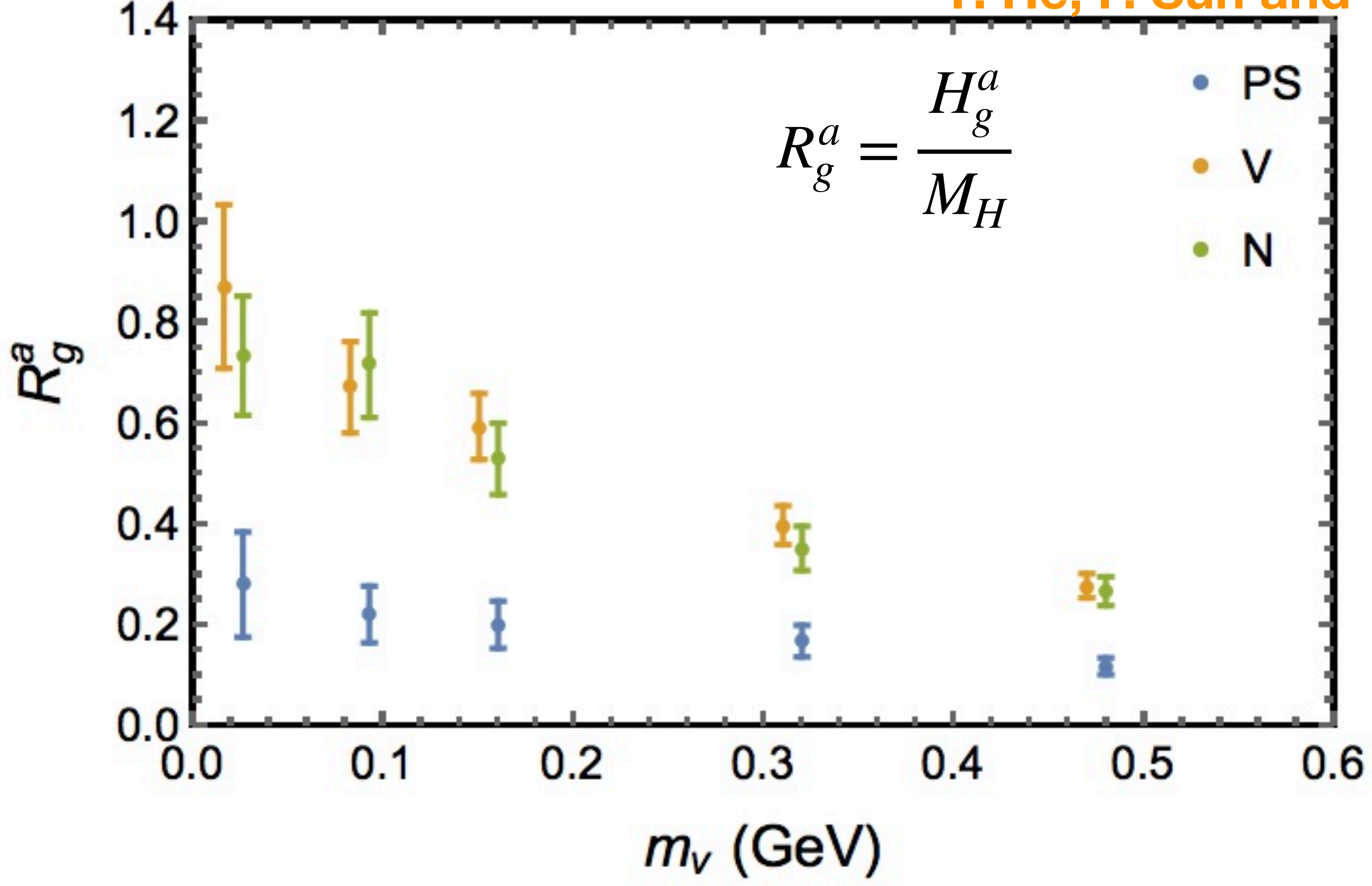
Gluon trace anomaly

The gluon trace anomaly



The ratio of gluon trace anomaly to hadron mass

F. He, P. Sun and Y. Yang, PRD104(2021)



1. **The gluon trace anomaly in the pseudoscalar meson is always much smaller than that in the other hadrons, especially around the chiral limit.**
2. **The ratio of gluon trace anomaly to total hadron mass in pseudoscalar meson is also smaller.**

Outline

- **Introduction to QCD trace anomaly**
- **The lattice calculations of gluon trace anomaly**
  - **Trace anomaly in the forward limit** F. He, P. Sun and Y. Yang, PRD104(2021)
  - **Gluon trace anomaly FFs** Bigeng Wang, F. He, et. al , 2401.0549
- **Summary**

Pion and nucleon gravitational form factors

- **The definition of gravitational form factors**

X. Ji, PRL78 (1997) 610

Nucleon GFFs

$$\langle N(p)'|T^{\mu\nu}_{g,q}|N(p)\rangle = \bar{u}(p')\left[A^N_{g,q}(t)\gamma^{(\mu}P^{\nu)} + B^N_{g,q}\frac{i\,P^{(\mu}\sigma^{\nu)\alpha}}{2m_N} + D^N_{g,q}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4m_N} + \bar{C}^N_{g,q}m_N g^{\mu\nu}\right]u(p)$$

Sum rule for spin $J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right]$

Pion GFFs

$$\langle \pi(p)'|T^{\mu\nu}_{g,q}|\pi(p)\rangle = 2A^\pi_{g,q}(t)P^\mu P^\nu + \frac{1}{2}D^\pi_{g,q}(t)\left(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right) + 2m_\pi^2\bar{C}^\pi_{g,q}g^{\mu\nu}$$

- **Scalar gravitational form factors** $\bar{C}_q + \bar{C}_g = 0$

$$\langle N(p)'|T^{\mu\mu}_g|N(p)\rangle = \bar{u}(p')u(p)G^N_g(t) \qquad G^N_g(t) = A^N_g(t) + B^N_g(t)\left(1 - \frac{\Delta^2}{4m_N^2}\right) + D^N_g(t)\frac{3\Delta^2}{4m_N^2} + 4m_N\bar{C}^N_g(t)$$

$$\langle \pi(p)'|T^{\mu\mu}_g|\pi(p)\rangle = G^\pi_g(t) \qquad G^\pi_g(t) = m_\pi^2 A^\pi_g(t) - \frac{\Delta^2}{2}\left(A^\pi_g(t) + 3D^\pi_g(t)\right) + 8m_\pi^2\bar{C}^\pi_g(t)$$

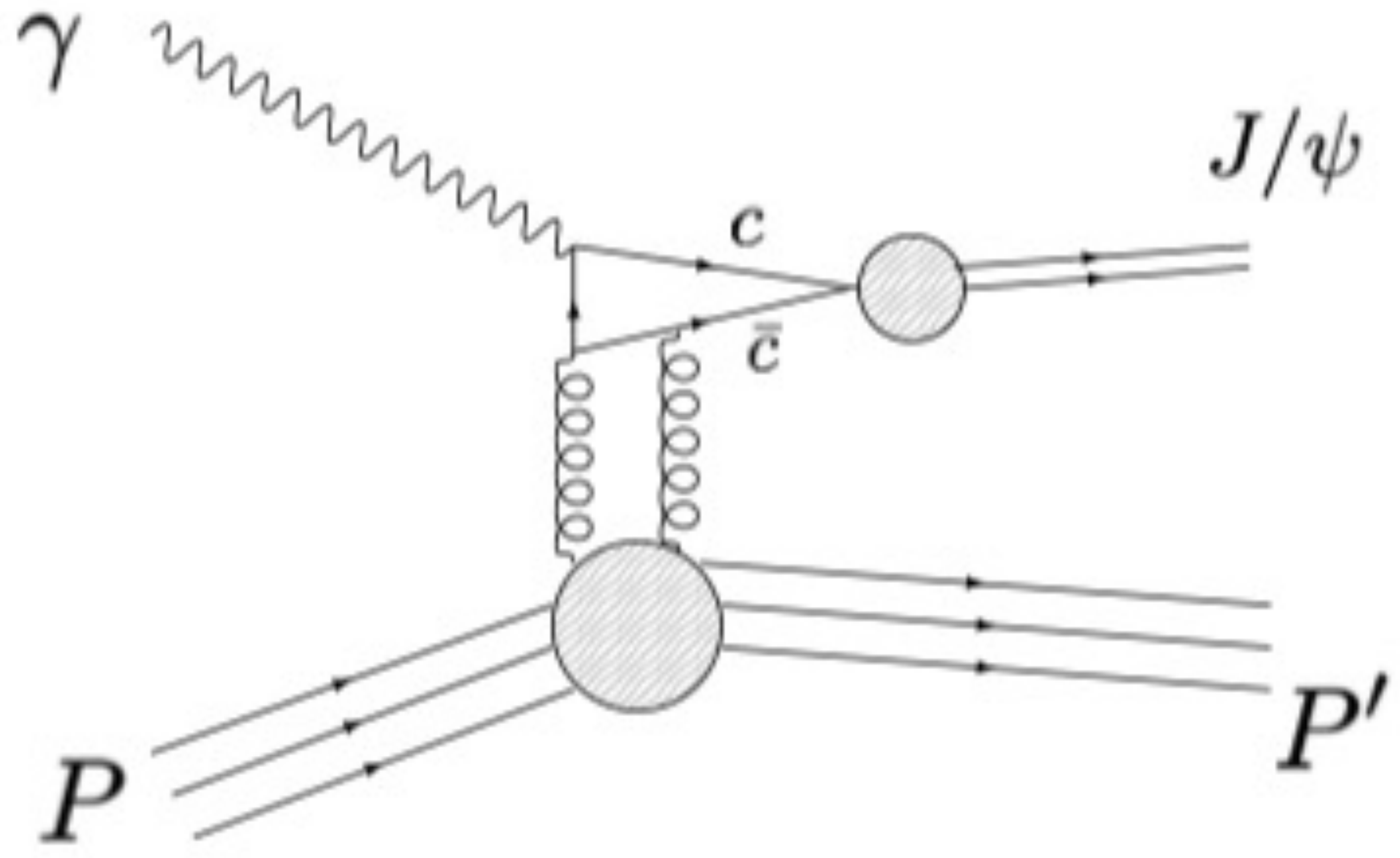
Experimental measures about gluon trace anomaly FF

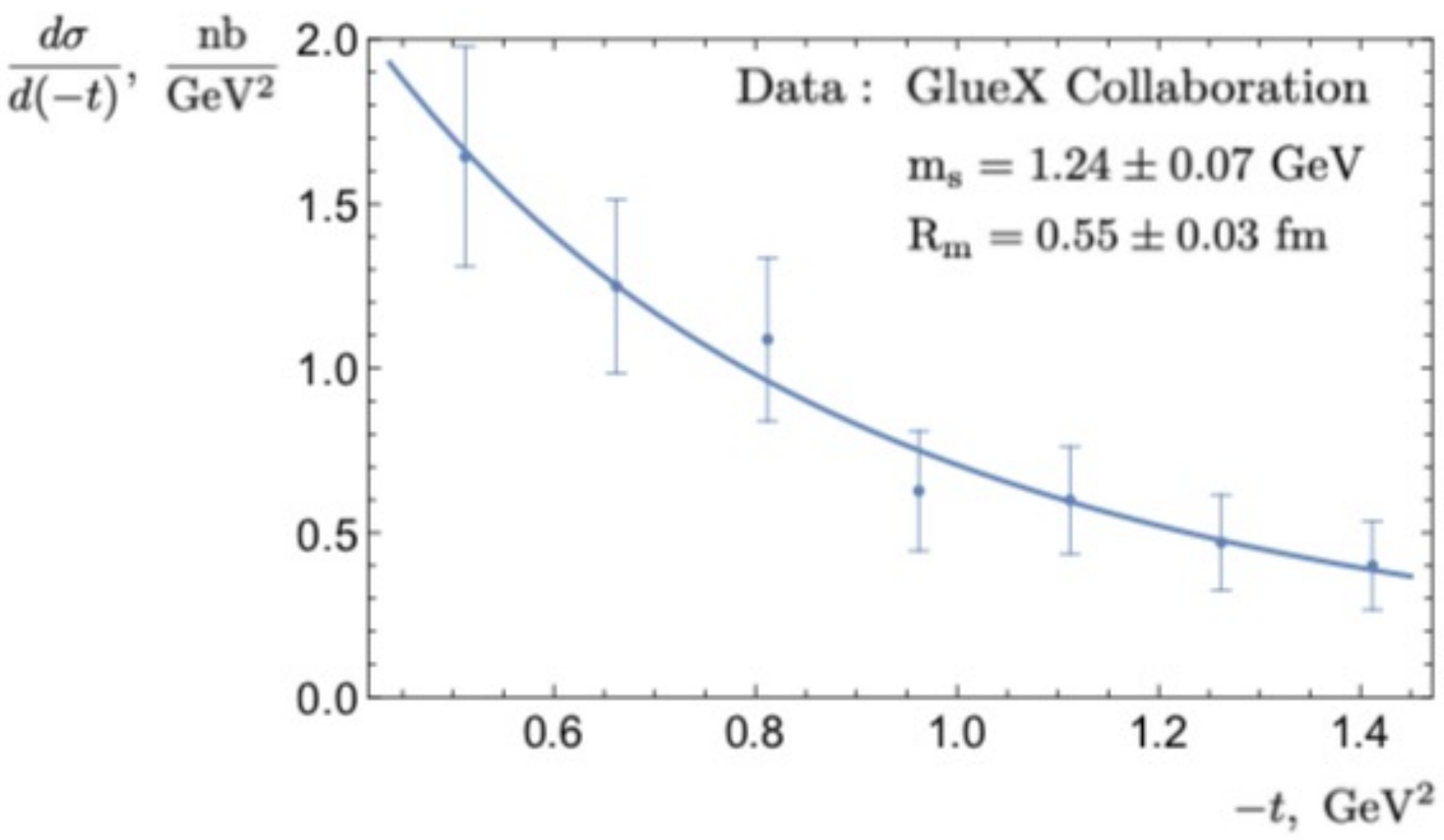
- **Detect gluon GFFs through measuring the differential cross section of J/psi photo-production process**

V. Novikov, M. A. Shifman,Z.Phys.C 8(1981)
D. Kharzeev, H. Satz, A. Syamtomov, G. Zinovjev, EPJC9
D. Kharzeev, PRD 104(2021)5

$$\mathcal{M}_{\gamma P\to\psi P}(t) = -Qe\ c_2\ \frac{16\pi^2 M}{b}\ \langle P'|T|P\rangle.$$

GlueX Collaboration, PRL123 (2019)

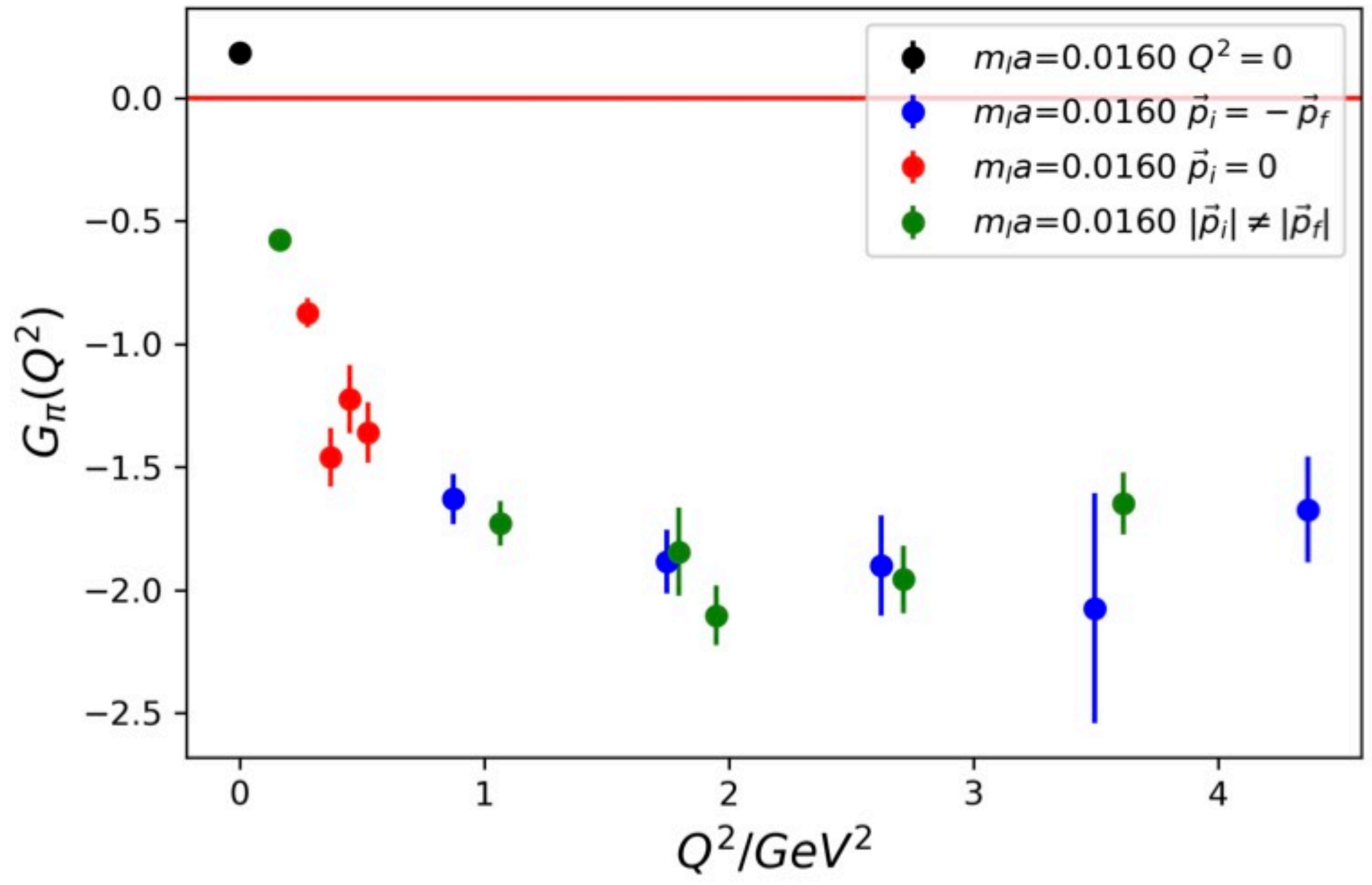




Gluon trace anomaly form factor of pion

Lattice result

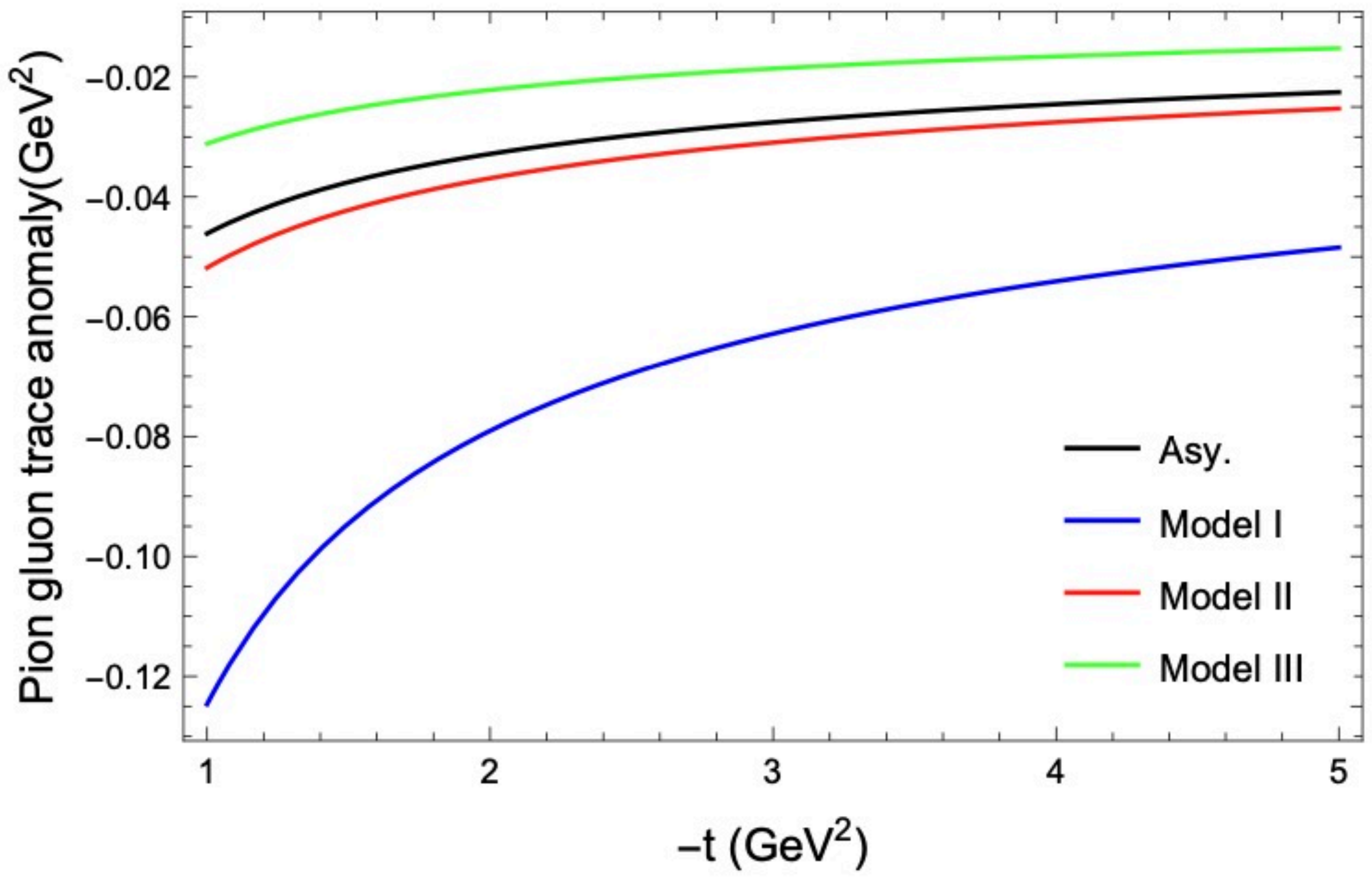
Bigeng Wang, F. He, et. al , 2401.0549



ChPT prediction $G^g_\pi \approx G^g_a \approx \frac{1}{2} - \frac{Q^2}{2m_\pi^2}$

PQCD prediction

X. Tong, J. Ma and F. Yuan , *JHEP* 10 (2022) 046

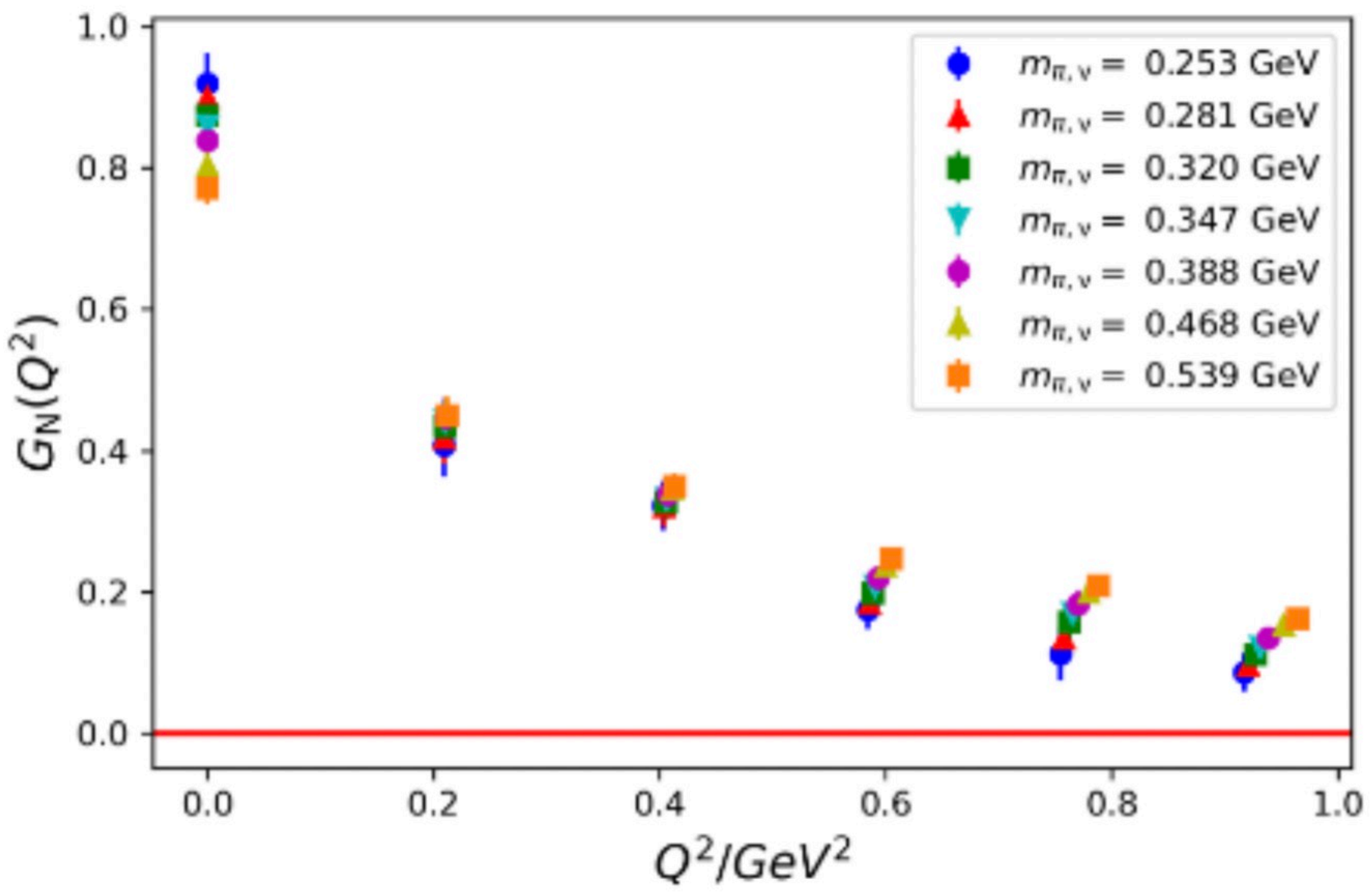


The Sign change also has been observed in perturbative calculation

Gluon trace anomaly form factor of proton

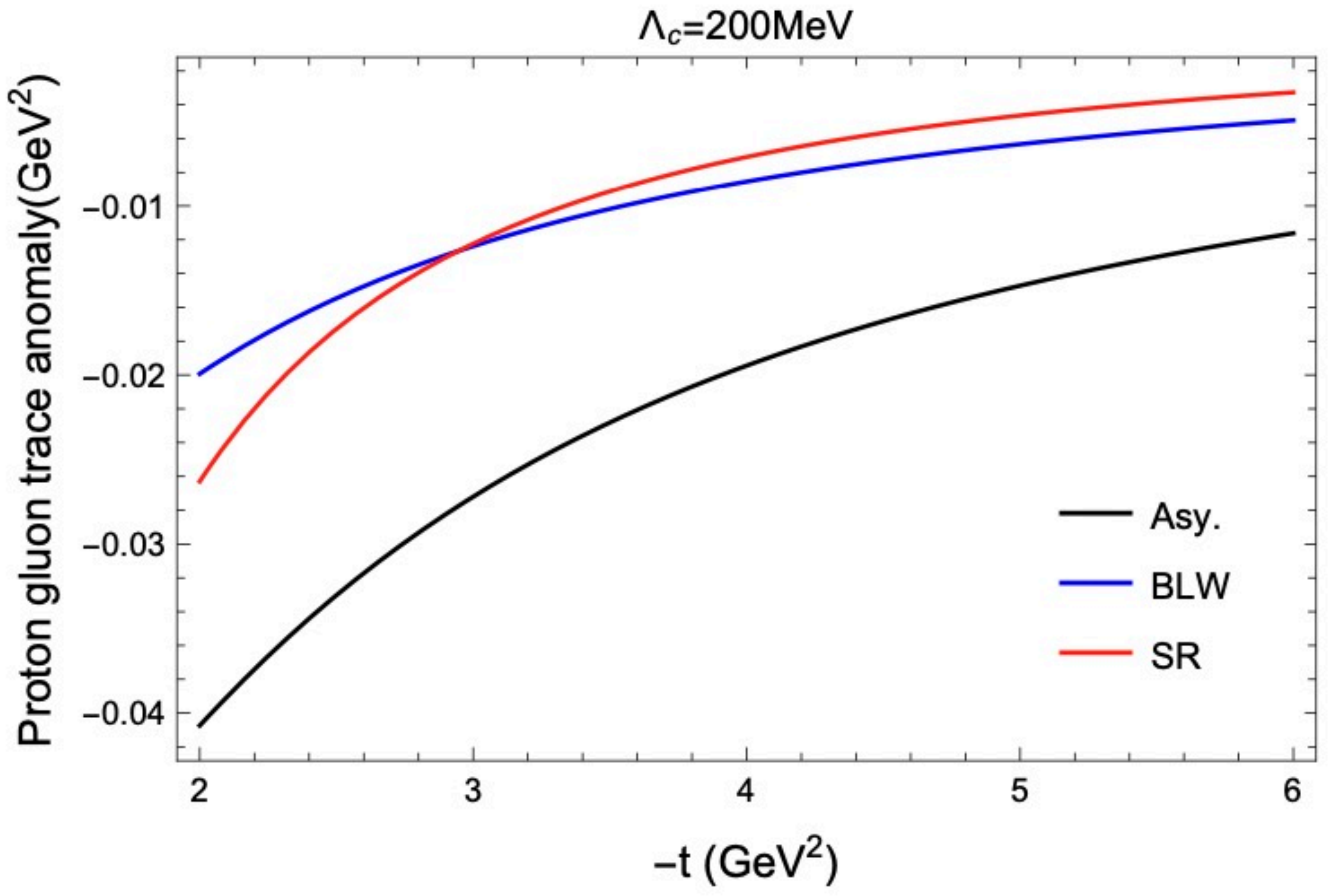
Lattice result

Bigeng Wang, F. He, et. al , 2401.0549



PQCD prediction

X. Tong, J. Ma and F. Yuan , *JHEP* 10 (2022) 046

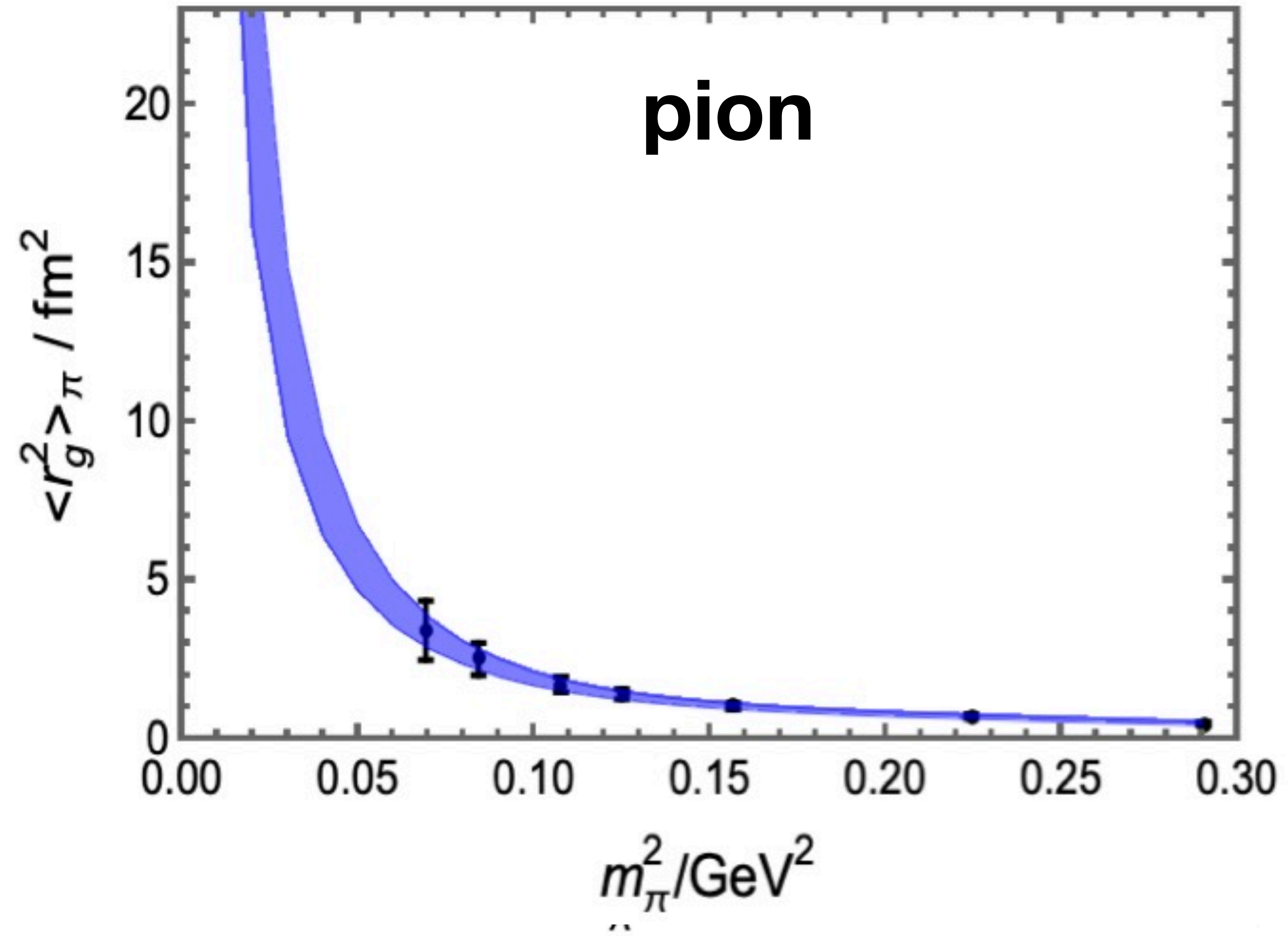


Need to verify the sign change at large t on Lattice

Mass radius

Scalar mass radius $\langle r_g^2 \rangle = -6\frac{dG_g(Q^2)}{dQ^2}|_{Q=0}$

Bigeng Wang, F. He, et. al , 2401.0549

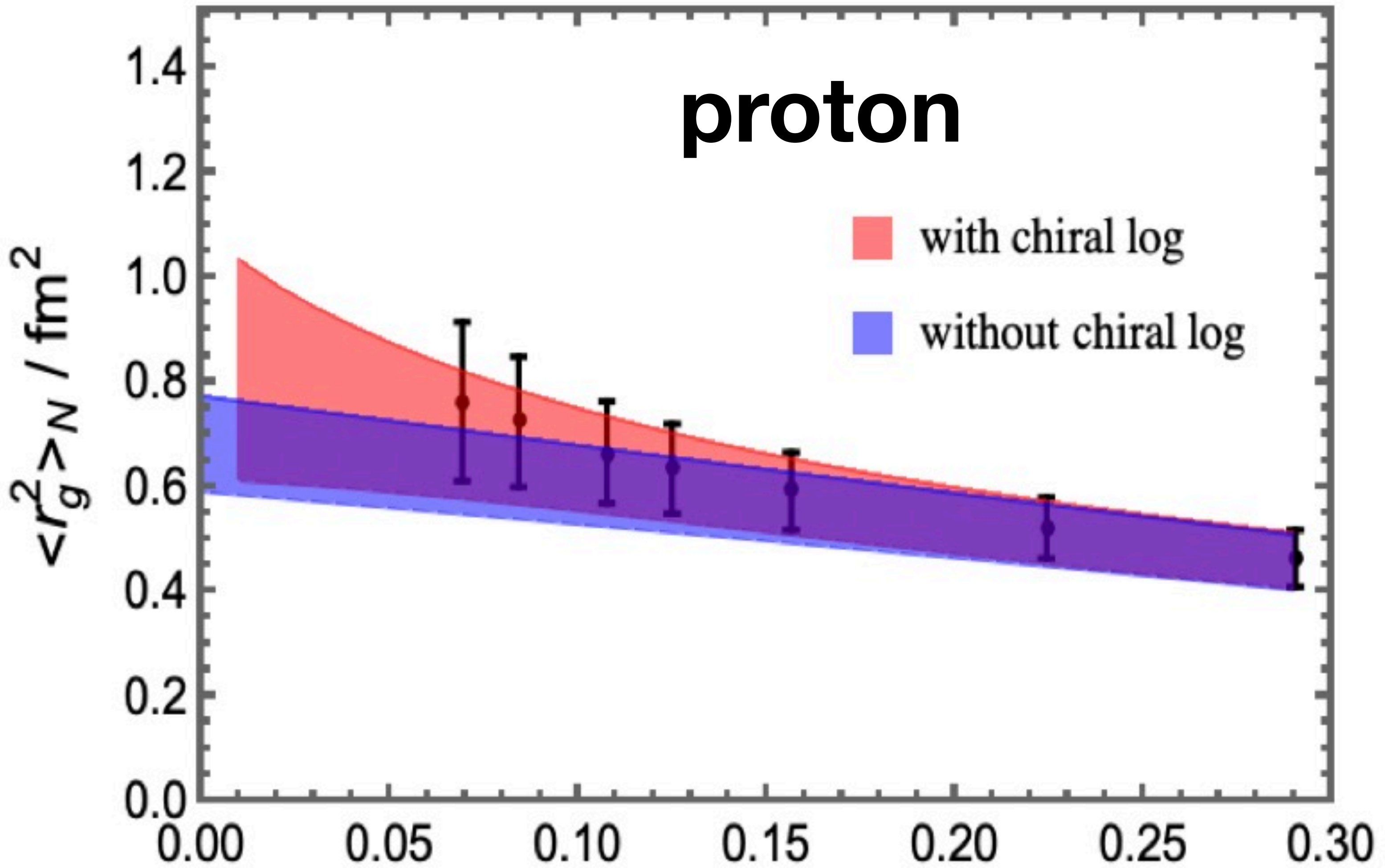


$$\langle r_g^2\rangle_\pi = a_\pi/m_\pi^2 + b_\pi + c_\pi \log\left(\frac{m_\pi^2}{m_{\pi,\mathrm{phy}}^2}\right) + d_\pi m_\pi^2,$$

$$\langle r_g^2\rangle_\pi^{\mathrm{phy}}=21.5(5.2)(11.7)\ \mathrm{fm}^2$$

ChPT prediction

$$\langle r^2\rangle_{\mathrm{m}}^{\mathrm{ChPT}}(\pi) = \frac{3}{m_\pi^2}.$$



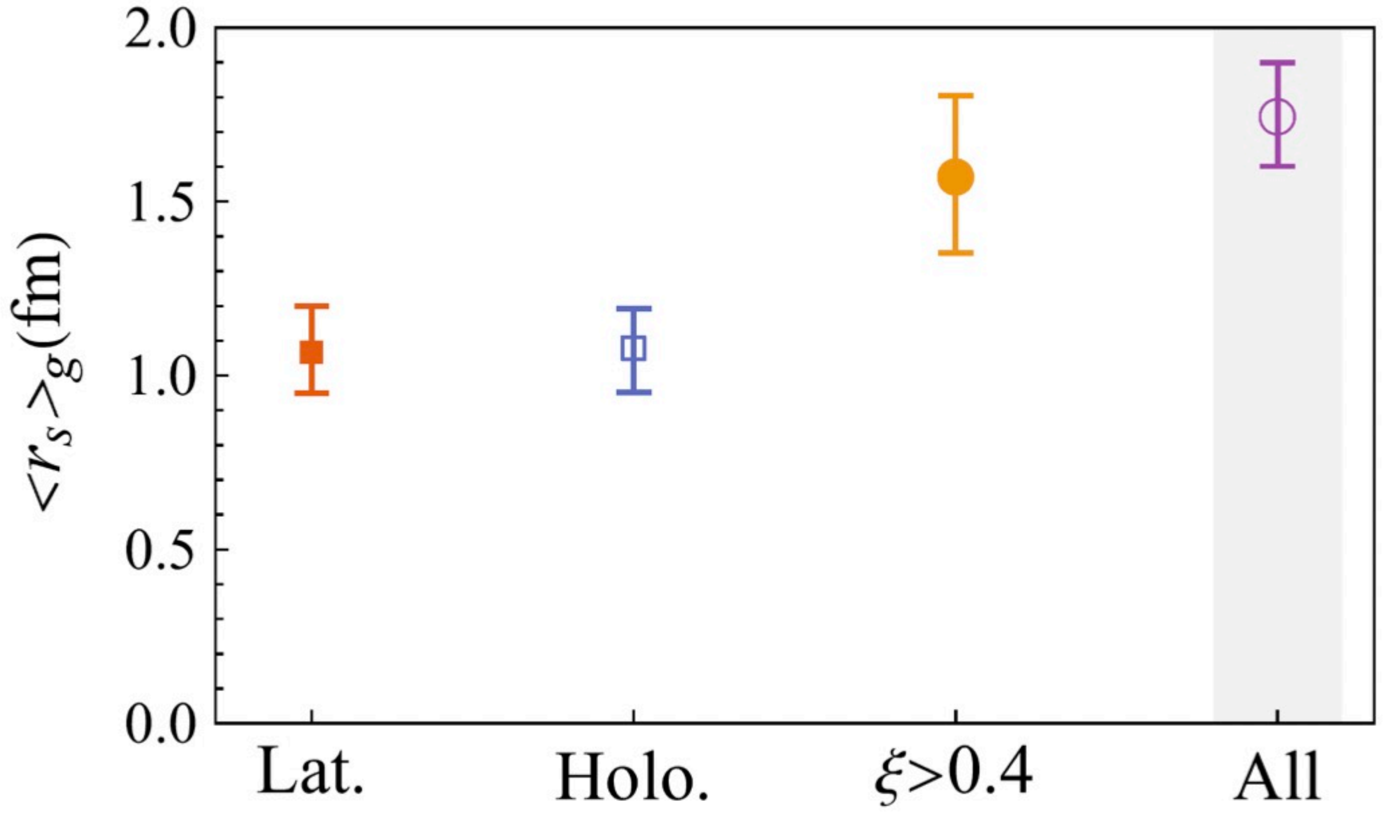
$$\langle r_g^2\rangle_N = a_\mathrm{N} + b_\mathrm{N}\ m_\pi^2 + c_\mathrm{N}\ m_\pi^2 \log\left(\frac{m_\pi^2}{m_{\pi,\mathrm{phy}}^2}\right)$$

$$\langle r_g^2\rangle_N = a_\mathrm{N} + b_\mathrm{N}\ m_\pi^2$$

$$\sqrt{\langle r_g^2\rangle_N^{\mathrm{phy}}}=0.89(10)(07)\ \mathrm{fm}$$

Phenomenological extraction

Y. Guo, X. Ji, Y, Liu and J. Yang, PRD 108 (2023)



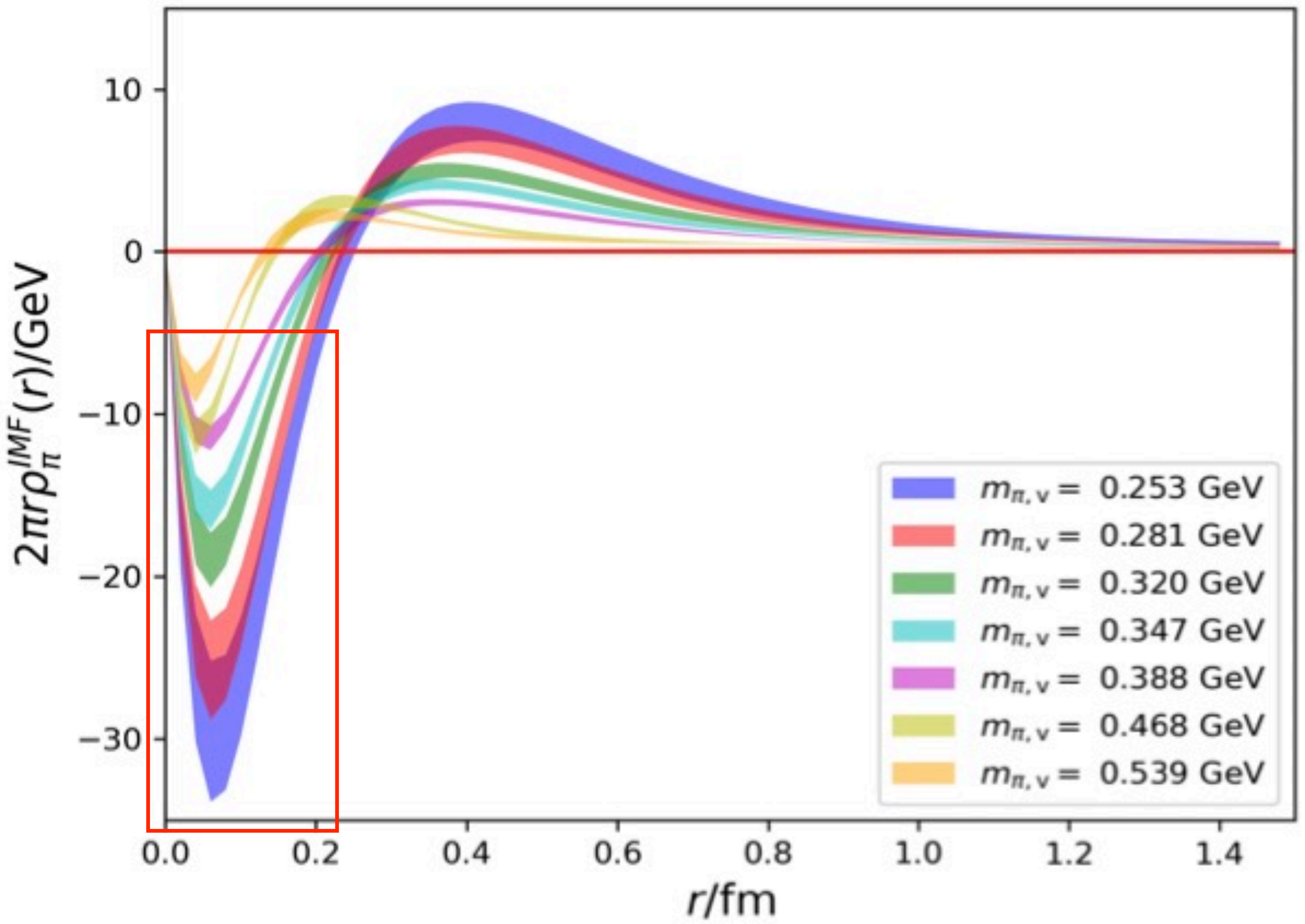
Spatial distribution of gluon trace anomaly

- **2D density in the infinite momentum-frame (IMF)**

Bigeng Wang, F. He, et. al , 2401.0549

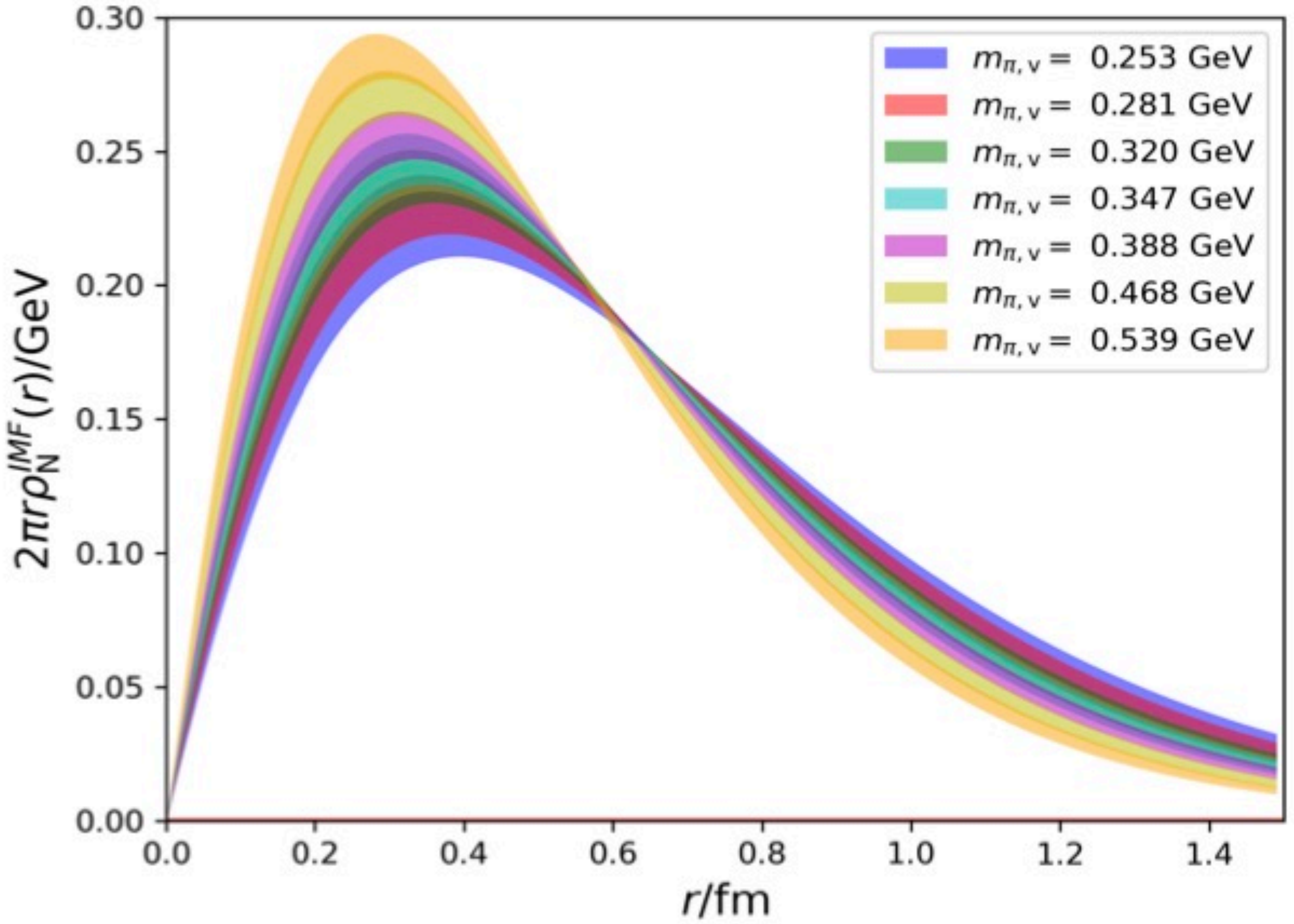
$$\rho_{\mathrm{H}}^{\mathrm{IMF}}(\mathbf{r}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{r}_\perp}\, \tilde{G}_{\mathrm{H}}(Q^2)\Big|_{\mathbf{P}\cdot\mathbf{\Delta}=0}^{P_z\to\infty},$$

pion



Distribution is negative in the centre region of pion

proton



Summary

- **We calculate the contribution of trace anomaly to the hadron mass and find the gluon trace anomaly contributes to the most of proton mass. The contribution to pion mass tends to be zero in the chiral limit.**

- **The gluon trace anomaly form factor for pion becomes negative at small transfer momentum. The spatial distribution in pion is negative in the centre region.**