Multi-particle production in DIS at small x in CGC formalism

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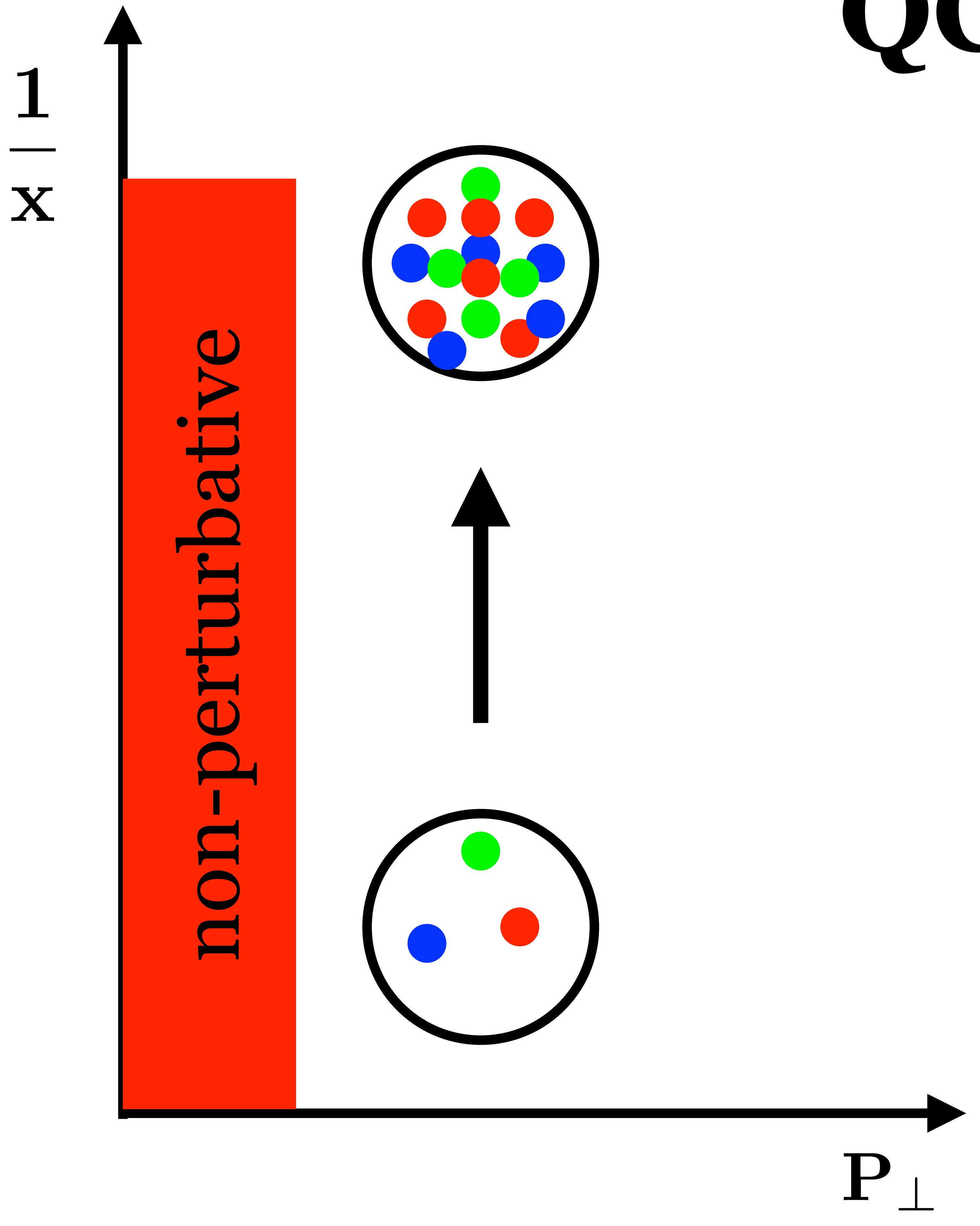
Cold Nuclear Matter Effects: from the LHC to the EIC

Center for Frontiers in Nuclear Science (CFNS), Stony Brook University*, January 13-16, 2025*

Outline

- Gluon saturation at small x
- Next to Leading Order corrections to particle production in DIS
- Sudakov factor in SIDIS
- Is it saturation or just shadowing ?

QCD at small x: gluon saturation



high gluon density: multiple eikonal scatterings

high energy: evolution in x via BK/JIMWLK

p_t broadening

suppression of single inclusive spectra/away side peak

Gluon saturation may be responsible for/contribute to

nuclear shadowing/modification factor, azimuthal correlations

long range rapidity correlations

Connections to spin physics, TMD's,....

$$Q_s^2(x, A, b_\perp) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

$$Q_s^2 \sim 1\,\text{GeV}^2 \text{ at } x \sim 3 \times 10^{-4}$$

$$x \leq 0.01$$

Probing CGC in high energy collisions

nucleus-nucleus collisions: “dense on dense”

significant modeling/QGP

proton-nucleus collisions: “dilute on dense”

DIS: (inclusive/diffractive)

much less modeling

structure functions

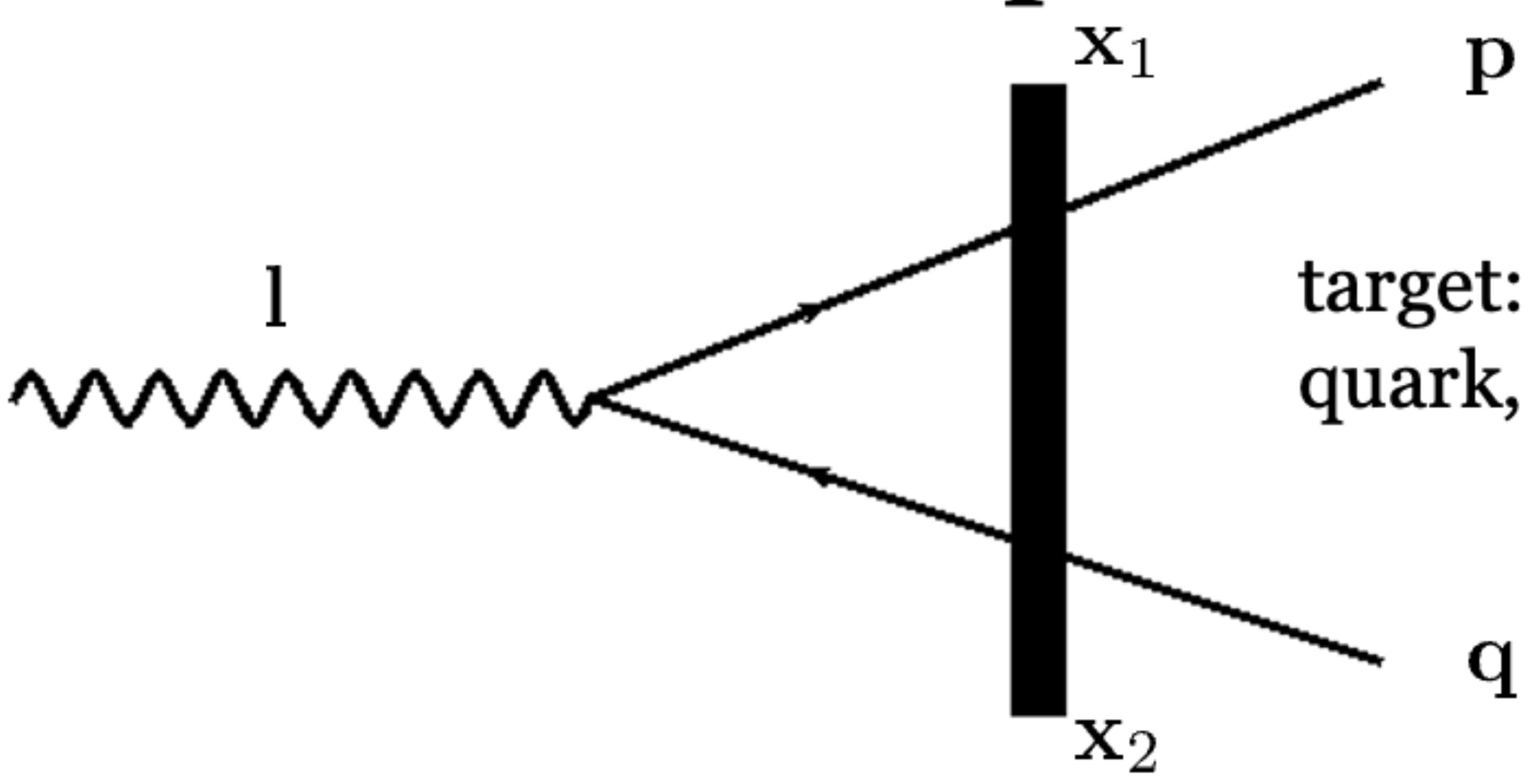
particle production

angular correlations

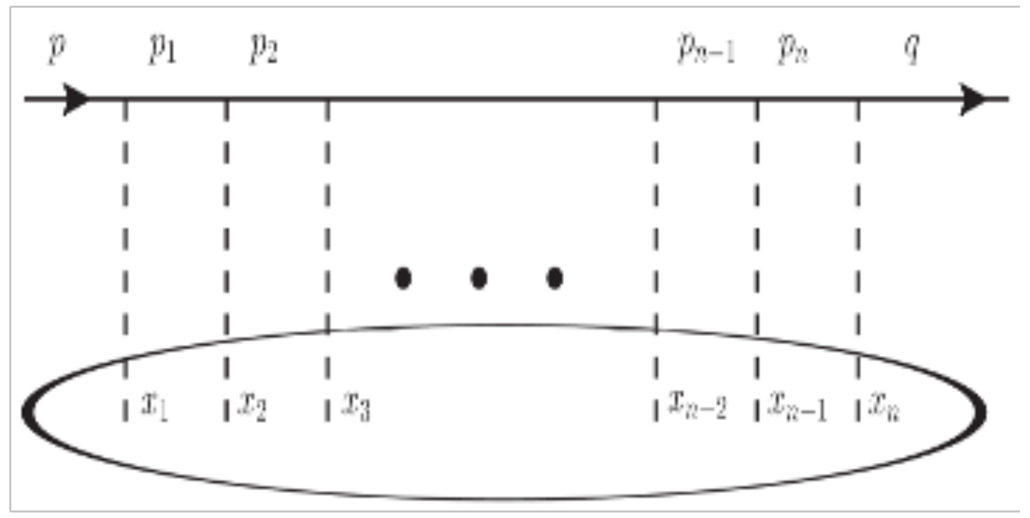
EIC

to start in ~ 10 years

Inclusive dihadron production in forward rapidity: LO



target: a classical color field
quark, antiquark multiply scatter on the target



$$\frac{d\sigma^{\gamma^* A\to q\bar{q}X}}{d^2p\, d^2q\, dy_1\, dy_2} = \frac{e^2Q^2(z_1z_2)^2N_c}{(2\pi)^7}\delta(1-z_1-z_2)$$

$$\int d^8x_\perp e^{ip\cdot(x_1'-x_1)}e^{iq\cdot(x_2'-x_2)}\left[S_{122'1'} - S_{12} - S_{1'2'} + 1\right]$$

with

$$\Big\{4z_1z_2K_0(|x_{12}|Q_1)K_0(|x_{1'2'}|Q_1)+$$

$$(z_1^2+z_2^2)\frac{x_{12}\cdot x_{1'2'}}{|x_{12}||x_{1'2'}|}K_1(|x_{12}|Q_1)K_1(|x_{1'2'}|Q_1)\Big\}$$

dipole $\mathbf{S}_{12} \equiv \frac{1}{N_c}Tr\, V(x_1)\, V^\dagger(x_2)$

$$\mathbf{x}_{12} \equiv \mathbf{x}_1 - \mathbf{x}_2$$

quadrupole

$$\mathrm{S}_{122'1'} \equiv \frac{1}{N_c}Tr\, V(\mathbf{x}_1)\, V^\dagger(\mathbf{x}_2)\, V(\mathbf{x}_{2'})\, V^\dagger(\mathbf{x}_{1'})$$

Only dipoles and quadrupoles contribute: DMXY, PRD 83 (2011) 105005

Toward precision CGC: inclusive DIS

NLO BK/JIMWLK evolution equations

Kovner, Lublinsky, Mulian (2013)

Balitsky, Chirilli (2007)

list already outdated! many more papers in the last few months

NLO corrections to structure functions

Beuf, Lappi, Paatelainen (2022)

Beuf (2017)

NLO corrections to SIDIS

Bergabo, JJM (2023, 2024)

Caucal, Ferrand, Salazar (2024)

NLO corrections to dihadron/dijets (+)

Bergabo, JJM (2022, 2023)

Iancu, Mulian (2023)

Caucal, Salazar, Schenke, Stebel, Venugopalan (2023), Caucal, Salazar, Schenke, Venugopalan (2022)

Taels, Altinoluk, Beuf, Marquet (2022), Taels (2023)

Caucal, Salazar, Venugopalan (2021)

Ayala, Hentschinski, JJM, Tejeda-Yeomans (2016,2017),............

Toward precision CGC: exclusive/diffractive DIS

NLO corrections to diffractive structure functions

Beuf, Hanninen, Lappi, Mulian, Mantyssari (2022)

....

NLO corrections to diffractive dihadron/dijets (+)

Boussarie, Grabovsky, Szymanowski, Wallon (2016)

Iancu, Mueller, Triantafyllopoulos (2021, 2022)

Fucilla, Grabovsky, Li, Szymanowski, Wallon (2023)

....

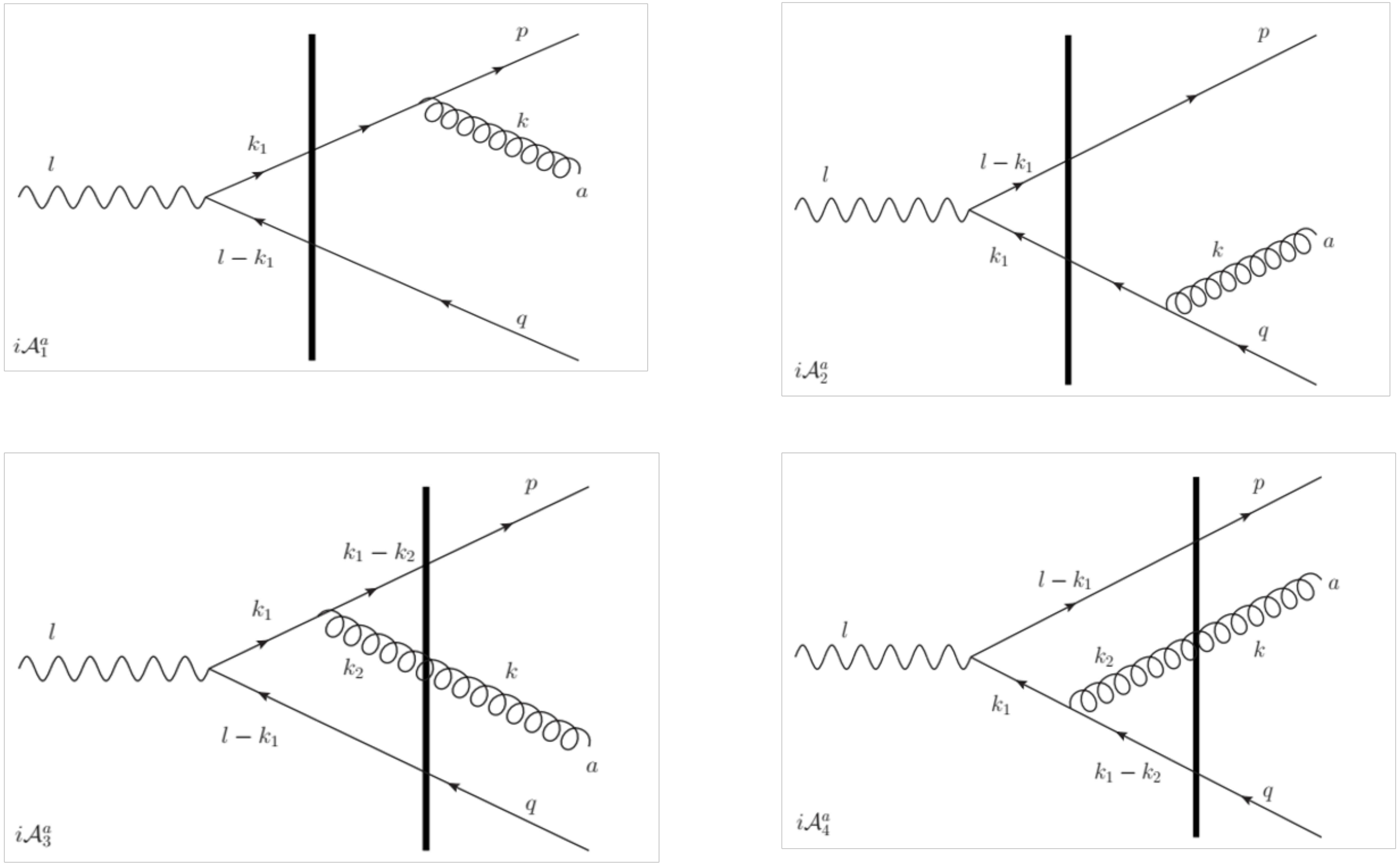
NLO corrections to exclusive light/heavy vector meson production (+)

Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2016)

Mantyssari, Penttala (2021, 2022)

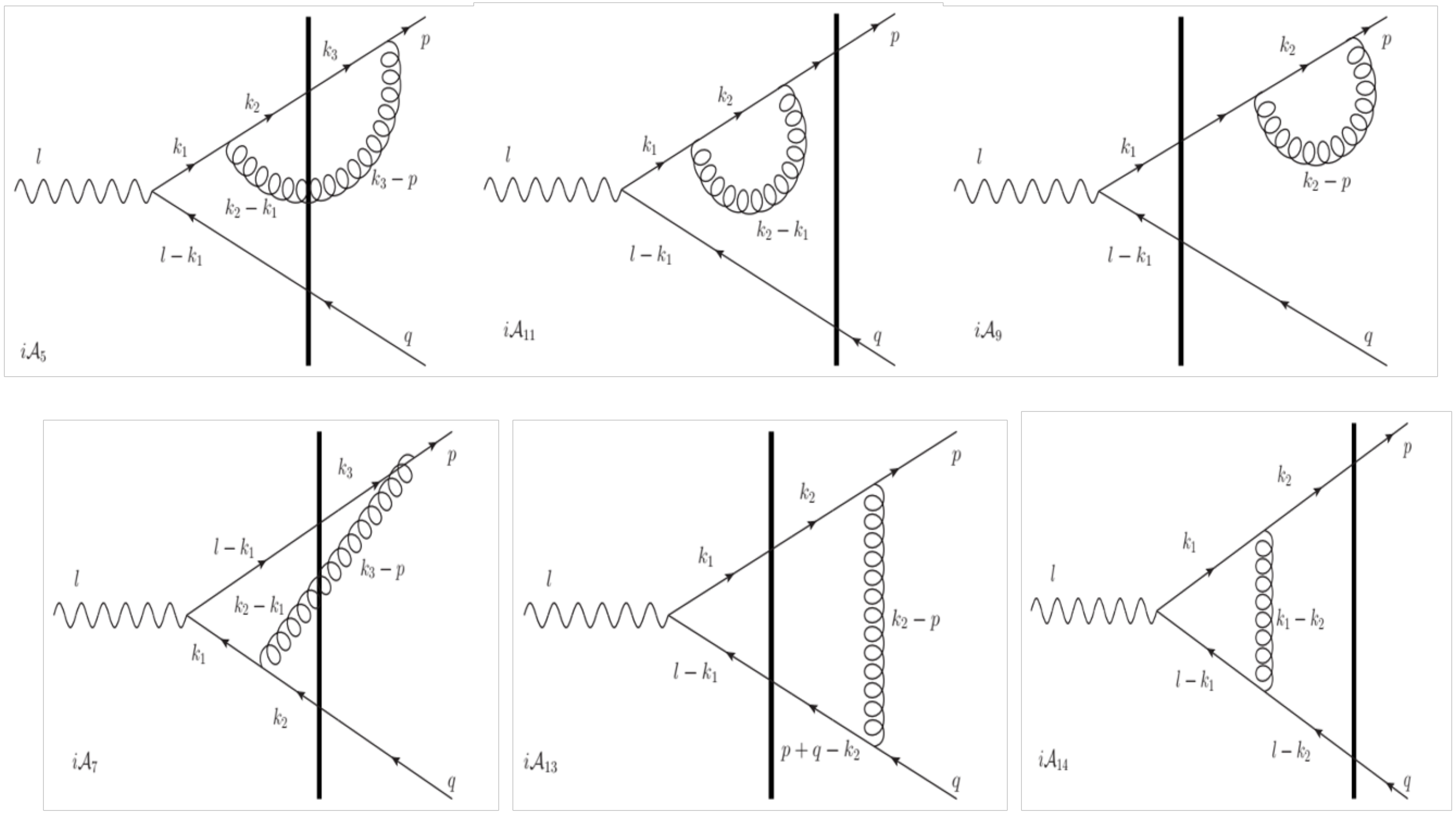
....

One loop corrections - real diagrams



3-parton production: Ayala, Hentschinski, JJM, Tejeda-Yeomans
PLB 761 (2016) 229 and NPB 920 (2017) 232

One loop corrections – virtual diagrams



F. Bergabo and JJM, dihadrons, 2207.03606
P. Taels et al., dijets, 2204.11650
P. Caucal et al., dijets, 2108.06347

Divergences

- Ultraviolet

  real corrections are UV finite

  UV divergences cancel among virtual diagrams

- Soft

  soft divergences cancel between real and virtual diagrams

- Collinear

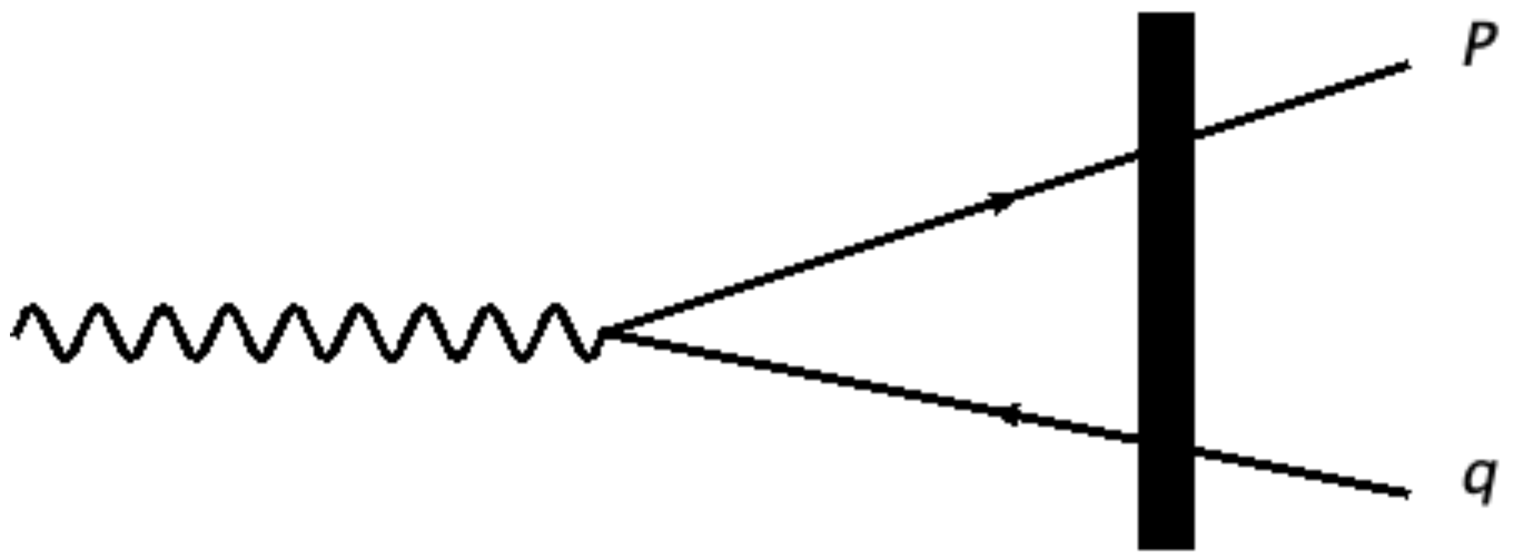
  collinear divergences are absorbed into fragmentation functions

- Rapidity

  Rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

$$\sigma^{\gamma^* A \to h_1 h_2 X} = \sigma_{LO} \otimes \mathrm{JIMWLK} + \sigma_{LO} \otimes D_{h/q}(z_h, \mu^2) \otimes D^{(0)}_{h/q}(z_h) + \sigma^{\text{finite}}_{NLO}$$

Inclusive dihadron production in DIS at small x: back to back limit



Quadrupole

$$\frac{1}{N_c} Tr\, V(\mathbf{x}_1)V^\dagger(\mathbf{x}_2)V(\mathbf{x}_2')V^\dagger(\mathbf{x}_1')$$

$$\mathbf{P}_\perp = \mathbf{p} - \mathbf{q}$$
$$\mathbf{K}_\perp = \mathbf{p} + \mathbf{q}$$
$$\mathbf{K}_\perp \longrightarrow 0$$

for $\mathbf{P}_\perp \sim |\mathbf{p}| \sim |\mathbf{q}|$ one can get large $\log \frac{\mathbf{P}_\perp^2}{\mathbf{K}_\perp^2}$ integrating over radiated gluon

Sudakov double logs in dijets production in DIS at small x:

Taylor expansion of Wilson lines around "center of mass" coordinate ⟶ Weizsacker-Williams field

CGC calculations contain Sudakov double logs, but with the wrong sign! $(+)\left[\log \mathbf{P}_\perp^2 \Delta \mathbf{b}_\perp^2\right]^2$

impose a kinematic constraint on life time of gluon radiation

(the usual strong ordering in + momenta is not sufficient)

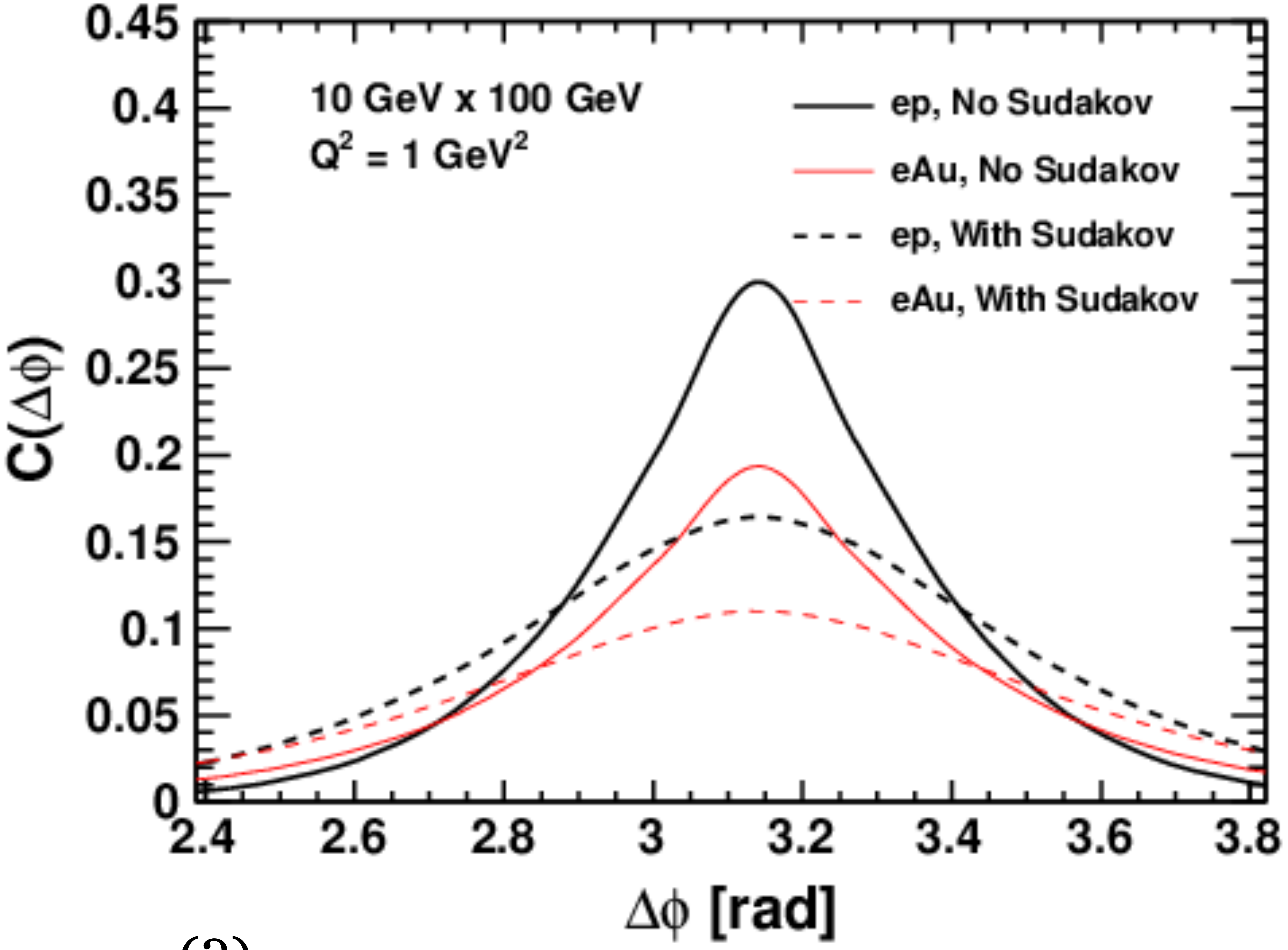
Taels, Altinoluk, Beuf, Marquet, JHEP 10 (2022) 184

Caucal, Salazar, Schenke, Venugopalan, JHEP 11 (2022) 169

……….

Dihadron/dijets at EIC

Zheng, Aschenauer,Lee,Xiao, arXiv:1403.2413



SIDIS a better process (?)

- larger kinematic phase space than dihadrons
- Sudakov effect can be avoided (?)

SIDIS at small x

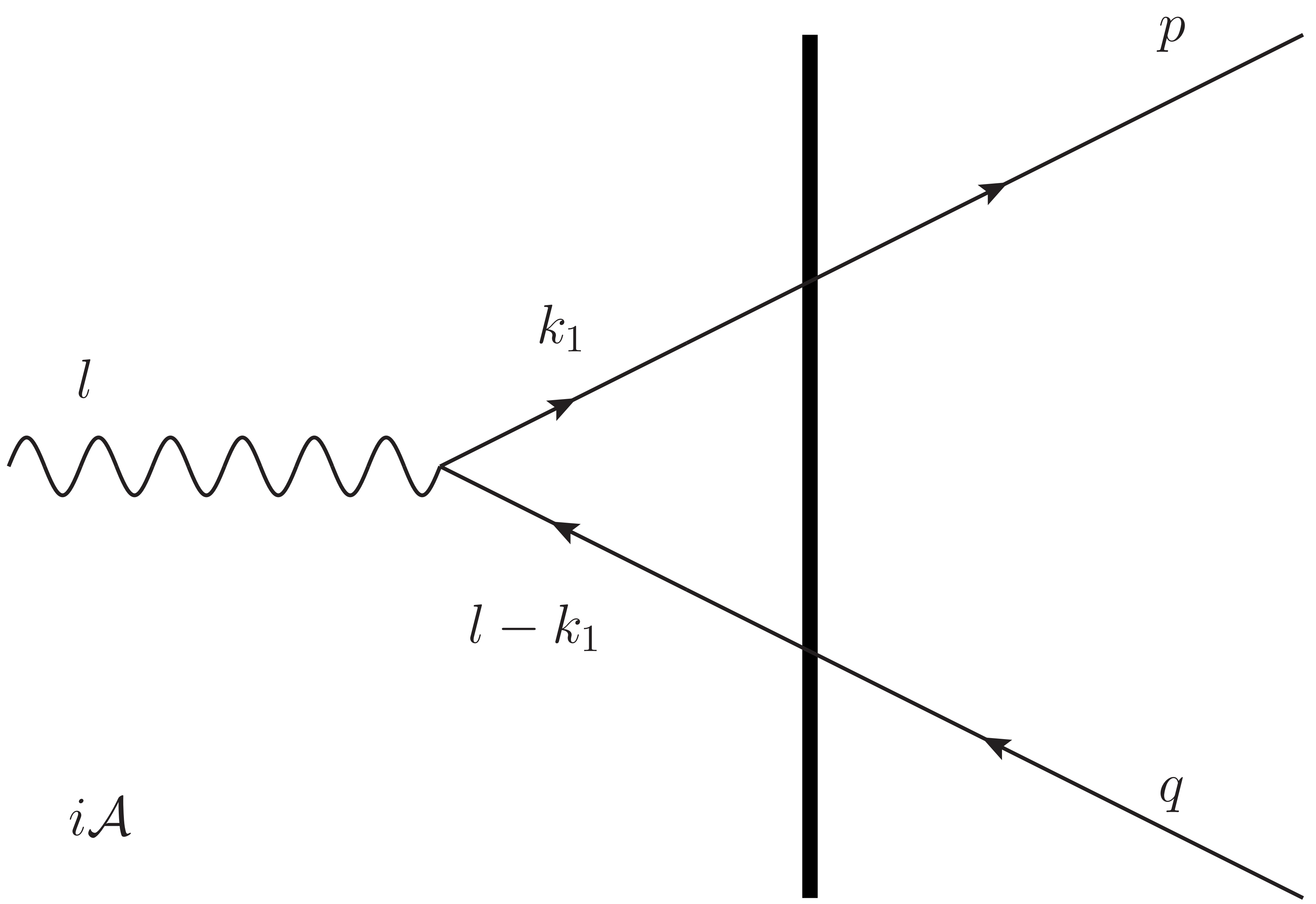
F. Bergabo and JJM, JHEP 01 (2023) 095, PRD109 (2024)

Caucal, Ferrand, Salazar, JHEP 05 (2024) 110

Forward rapidity

LO:

integrate over final state antiquark



$$\frac{\mathrm{d}\sigma^{\gamma^* p/A \to q(\mathbf{p}, y_1) X}}{\mathrm{d}^2\mathbf{p}\, \mathrm{d}y_1} = \frac{e^2 Q^2 N_c}{(2\pi)^5} \int \mathrm{d}z_2\, \delta(1 - z_1 - z_2)\, (z_1^2\, z_2) \int \mathrm{d}^6\mathbf{x}\, [S_{11'} - S_{12} - S_{1'2} + 1]\, e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}$$
$$\left\{ 4 z_1 z_2 K_0(|\mathbf{x}_{12}| Q_1) K_0(|\mathbf{x}_{1'2}| Q_1) + (z_1^2 + z_2^2)\, \frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\, K_1(|\mathbf{x}_{12}| Q_1) K_1(|\mathbf{x}_{1'2}| Q_1) \right\}$$

NLO corrections to SIDIS: some of the contributions are

$$\frac{\mathrm{d}\sigma^T_{1\times 1}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = \frac{e^2 g^2 Q^2}{(2\pi)^8} \int_0^{1-z_1} \frac{\mathrm{d}z}{z} \frac{(1-z-z_1)(z+z_1)}{z_1} \left[z_1^2 (1-z-z_1)^2 + \left(z_1^2 + (1-z-z_1)^2 \right) (z+z_1)^2 + (z+z_1)^4 \right]$$
$$\int \mathrm{d}^8\mathbf{x}\, K_1(|\mathbf{x}_{12}| Q_{1z}) K_1(|\mathbf{x}_{1'2}| Q_{1z}) N_c C_F \left[S_{11'} - S_{12} - S_{21'} + 1 \right] \frac{\mathbf{x}_{12}\cdot \mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\, \Delta^{(3)}_{11'}\, e^{i\frac{z_1+z}{z_1}\mathbf{p}\cdot\mathbf{x}_{1'1}}$$

$$\frac{\mathrm{d}\sigma^T_{9}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = -\frac{e^2 g^2 Q^2}{2(2\pi)^6} \int_0^{z_1} \frac{\mathrm{d}z}{z} (1-z_1)(z_1^2 + (1-z_1)^2) \left[z_1^2 + (z_1 - z)^2 \right] \int \mathrm{d}^6\mathbf{x}\, K_1(|\mathbf{x}_{12}| Q_1) K_1(|\mathbf{x}_{1'2}| Q_1) e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}$$
$$N_c C_F \left[S_{11'} - S_{12} - S_{21'} + 1 \right] \frac{\mathbf{x}_{12}\cdot \mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|} \int \frac{\mathrm{d}^2\mathbf{x}_3}{(2\pi)^2} \frac{1}{\mathbf{x}_{31}^2}$$

with $\Delta^{(3)}_{ij} = \frac{\mathbf{x}_{3i}\cdot\mathbf{x}_{3j}}{\mathbf{x}_{3i}^2 \mathbf{x}_{3j}^2}$ and

$$Q_1^2 \equiv z_1(1-z_1)Q^2$$
$$Q_{1z}^2 \equiv (1-z-z_1)(z+z_1)Q^2$$

SIDIS at small x: NLO corrections

all quadrupole contributions cancel: dipoles only at leading N_c

cancelation of UV/soft divergences

rapidity/collinear divergences renormalize the dipoles/fragmentation functions

$$\sigma^{\gamma^\star \mathbf{A} \longrightarrow \mathbf{h\, X}} = \sigma_{\mathbf{LO}} \otimes \mathbf{JIMWLK} + \sigma_{\mathbf{LO}} \otimes \mathbf{D_{h/q}}(\mathbf{z_h},\, \mu^2) + \sigma_{\mathbf{NLO}}^{\mathbf{finite}}$$

SIDIS in CGC formalism contains <u>*no*</u> Sudakov double logs <u>*unlike*</u> dihadron production

EIC will have a reasonably large window in $\mathbf{Q^2}$ where $\mathbf{Q^2} >> \mathbf{p^2_{t,h}}$ so that $\alpha_\mathbf{s} \log \left(\frac{\mathbf{Q^2}}{\mathbf{p^2_{t,h}}}\right) \sim \mathbf{1}$

avoid large Sudakov logs: $\mathbf{p^2_{t,h}} \sim \mathbf{Q^2}$ so that $\log \left(\frac{\mathbf{Q^2}}{\mathbf{p^2_{t,h}}}\right) \simeq \mathbf{0}$

SIDIS at small x: including Sudakov double logs

to get Sudakov logs one must introduce a kinematic constraint

needed for self-consistency of evolution equations (avoid negative cross sections)

at small x we require $\mathbf{k^+ < z_f\, l^+}$, this is sufficient at LL accuracy

at NLO accuracy we also need to impose a condition on lifetimes of fluctuations

introduce a cutoff on - component of momenta $\mathbf{k^- > \tilde{z}_f\, l^-}$ with $\mathbf{z_f\, \tilde{z}_f \sim 1}$

Sudakov single logs will be sensitive to the details of this cutoff

SIDIS at small x: including Sudakov double logs

longitudinal factorization $\int_0^{1-z_h} dz = \int_{z_f}^{1-z_h} dz + \int_0^{z_f} dz\,[1-\theta(kin.const.)] + \int_0^{z_f} dz\theta(kin.const.)$

add and subtract the kinematic constraint $\mathbf{\Theta}(\text{kin.const.}) = \mathbf{\Theta}\left(\mathbf{z_f}\frac{\mathbf{k^2}}{\mathbf{Q^2}} - \mathbf{z}\right)$

$$\int_{\mathbf{0}}^{\mathbf{z_f}} \frac{\mathrm{d}\mathbf{z}}{\mathbf{z}}\left[\mathbf{1} - \mathbf{\Theta}\left(\mathbf{z_f}\frac{\mathbf{k^2}}{\mathbf{Q^2}} - \mathbf{z}\right)\right] = \int_{\mathbf{0}}^{\mathbf{z_f}} \frac{\mathrm{d}\mathbf{z}}{\mathbf{z}}\mathbf{\Theta}\left(\mathbf{z} - \mathbf{z_f}\frac{\mathbf{k^2}}{\mathbf{Q^2}}\right) = \mathbf{\Theta}\left(\mathbf{Q^2} - \mathbf{k^2}\right)\ln\left(\frac{\mathbf{Q^2}}{\mathbf{k^2}}\right)$$

and dipoles satisfy constrained JIMWLK evolution

$$\left.\frac{\mathrm{d}\sigma_{LO+NLO}^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = d\sigma_{LO}(z_f)\otimes D_{h/q}(z_h,Q^2) + d\sigma_{NLO-rap-finite} +$$

$$\frac{\pi e^2}{Q^2}\,\frac{D_{h/q}(z_h)}{z_h}\int \frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\,e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'})$$

$$\times\left\{\frac{\alpha_s C_F}{\pi^2}\int^{Q^2}\frac{\mathrm{d}^2\mathbf{k}}{\mathbf{k}^2}\left(e^{i\mathbf{k}\cdot\mathbf{x}_{1'1}} - 1\right)\ln\left(\frac{Q^2}{\mathbf{k}^2}\right)\right\}$$

Sudakov double logs in SIDIS

using

$$\int^{Q^2} \frac{\mathrm{d}^2\mathbf{k}}{\mathbf{k}^2}\left[e^{-i\mathbf{k}\cdot\mathbf{x}_{11'}}-1\right]\ln\left(\frac{Q^2}{\mathbf{k}^2}\right) = 4\pi\int_0^{Q|\mathbf{x}_{11'}|}\frac{\mathrm{d}\tau}{\tau}\left[J_0(\tau)-1\right]\ln\left(\frac{Q|\mathbf{x}_{11'}|}{\tau}\right)$$
$$= -\frac{\pi}{2}\ln^2\left(Q^2\mathbf{x}_{11'}^2/c_0^2\right) + O\left(\frac{1}{\sqrt{Q|\mathbf{x}_{11'}|}}\right)$$

we get

$$\left.\frac{\mathrm{d}\sigma_{LO+NLO}^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = d\sigma_{NLO-rap-finite}$$
$$+\frac{\pi e^2}{Q^2}\,\frac{D_{h/q}(z_h,Q^2)}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\,e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'})\,e^{-S_{sud}(\mathbf{x}_{11'})}$$

with

$$S_{sud}(\mathbf{x}_{11'}) \equiv \frac{\alpha_s C_F}{2\pi}\ln^2\left(Q^2\mathbf{x}_{11'}^2/c_0^2\right)$$

Quark TMD

Marquet, Xiao, Yuan 0906.1454

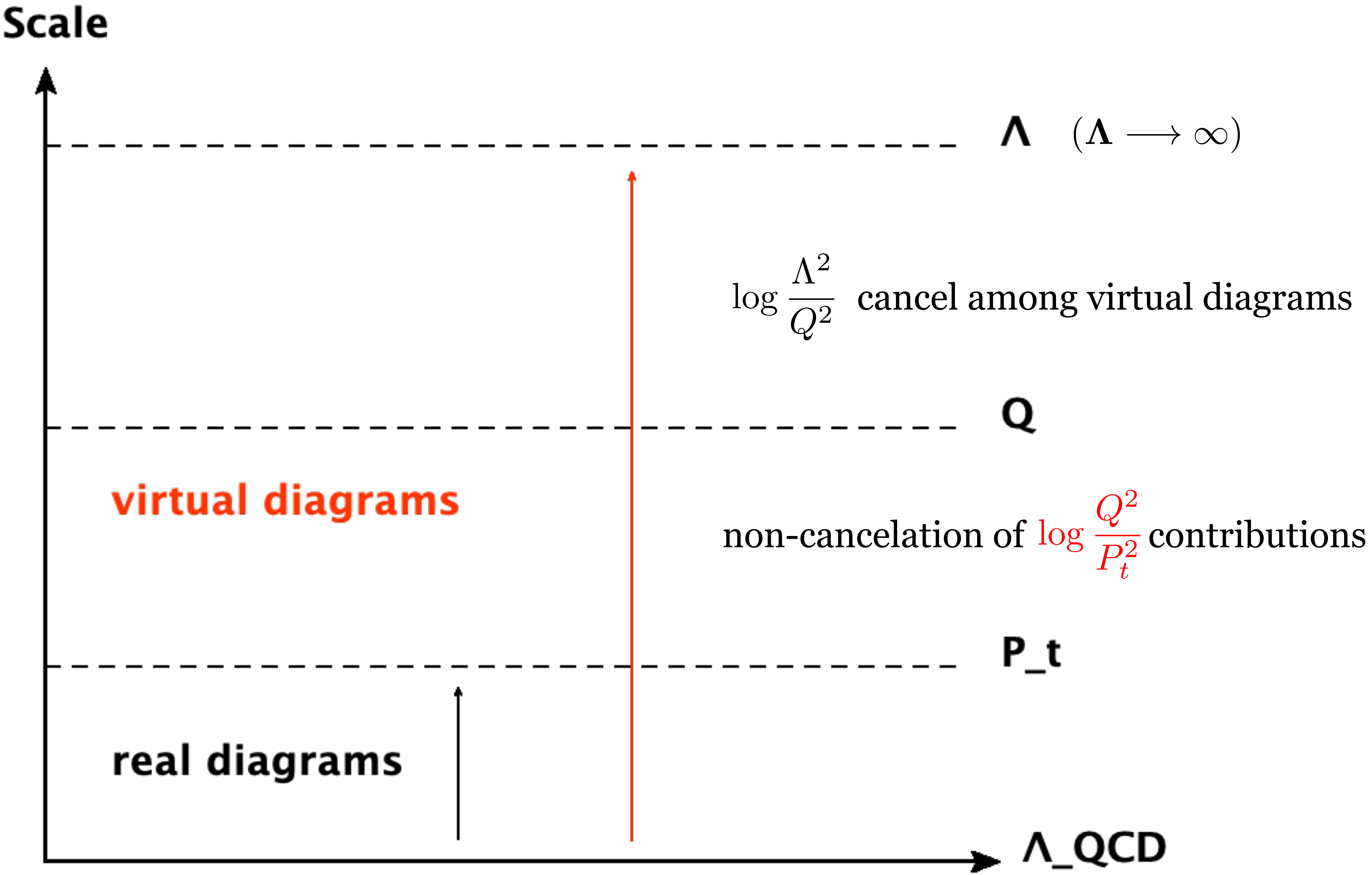
$$xq(x,\mathbf{p}) = \frac{2N_c}{(2\pi)^6}\int \mathrm{d}^6\mathbf{x}\; e^{-i\mathbf{p}\cdot\mathbf{x}_{11'}}\;\; [S_{11'} - S_{12} - S_{1'2} + 1]\,\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\;\int_0^\infty d\bar{Q}^2\bar{Q}^2\, K_1(|\mathbf{x}_{12}|\bar{Q})K_1(|\mathbf{x}_{1'2}|\bar{Q})$$

can be rewritten as

$$xq(x,\mathbf{p}) = \int \mathrm{d}^2\mathbf{k}_g \underbrace{\left(\frac{N_c}{8\pi^4}\frac{\mathbf{k}_g^2}{\alpha_s}\int \mathrm{d}^2\mathbf{x}_1\,\mathrm{d}^2\mathbf{x}_{1'}\; S_{11'}\, e^{-i\mathbf{k}_g\cdot\mathbf{x}_{11'}}\right)}_{\text{dipole gluon TMD}} \underbrace{\left(\frac{1}{4\pi^2}\frac{\alpha_s}{\mathbf{k}_g^2}\int_0^\infty \mathrm{d}\bar{Q}^2\left|\frac{\mathbf{k}_g-\mathbf{p}}{\bar{Q}^2+(\mathbf{k}_g-\mathbf{p})^2}+\frac{\mathbf{p}}{\bar{Q}^2+\mathbf{p}^2}\right|^2\right)}_{\text{momentum dependent splitting function } P_{qg}}$$

dipole gluon TMD

momentum dependent splitting function P_{qg}


Scale
Λ (Λ ⟶ ∞)
$\log \frac{\Lambda^2}{Q^2}$ cancel among virtual diagrams
Q
virtual diagrams
non-cancelation of $\log \frac{Q^2}{P_t^2}$ contributions
P_t
real diagrams
Λ_QCD

Summary (so far!)

Cold matter at high energy

dense hadron/nucleus: gluon saturation, strong color fields - CGC

strong hints from RHIC, LHC,...

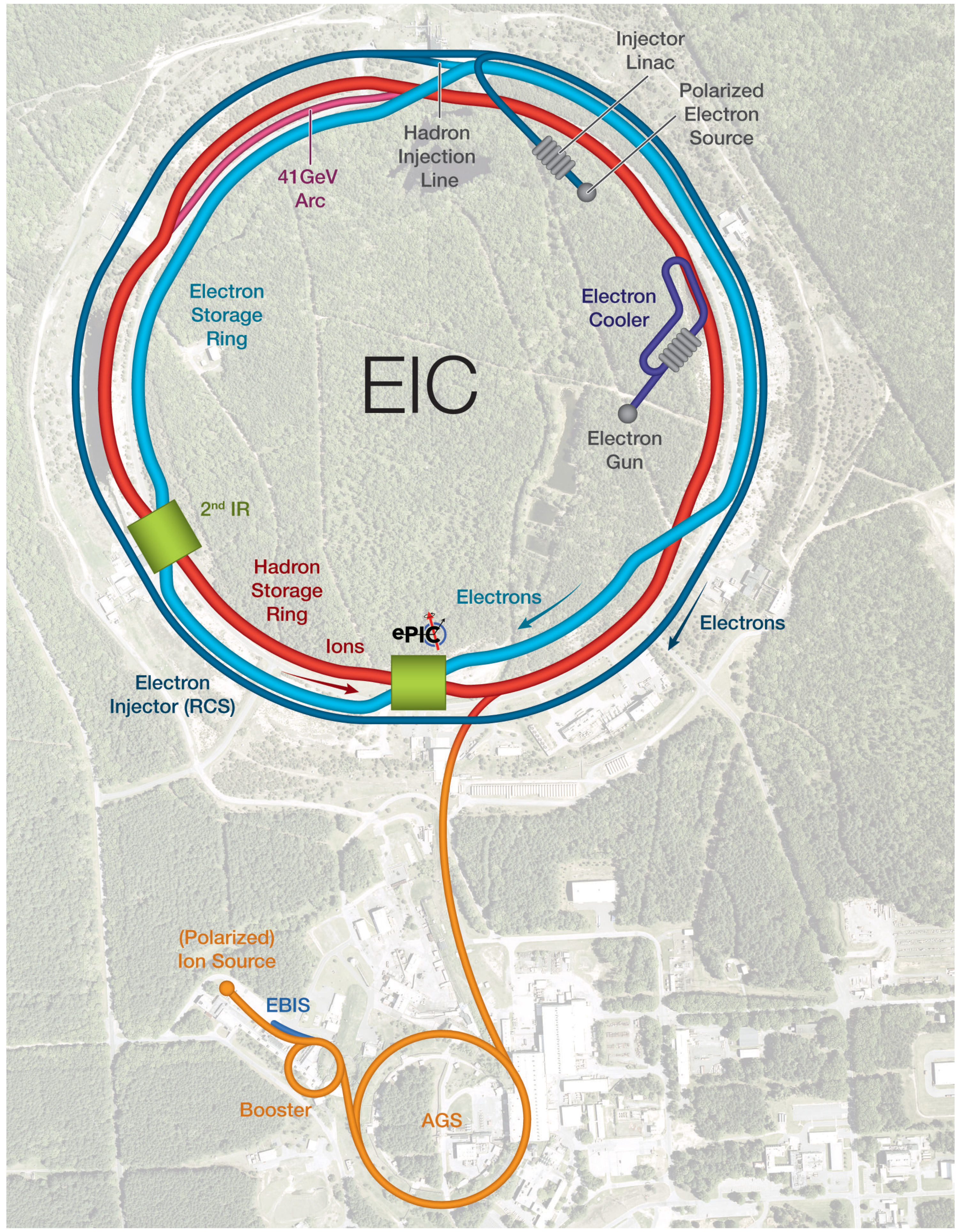
to be probed precisely at EIC

toward precision: NLO, beyond-eikonal corrections, ...

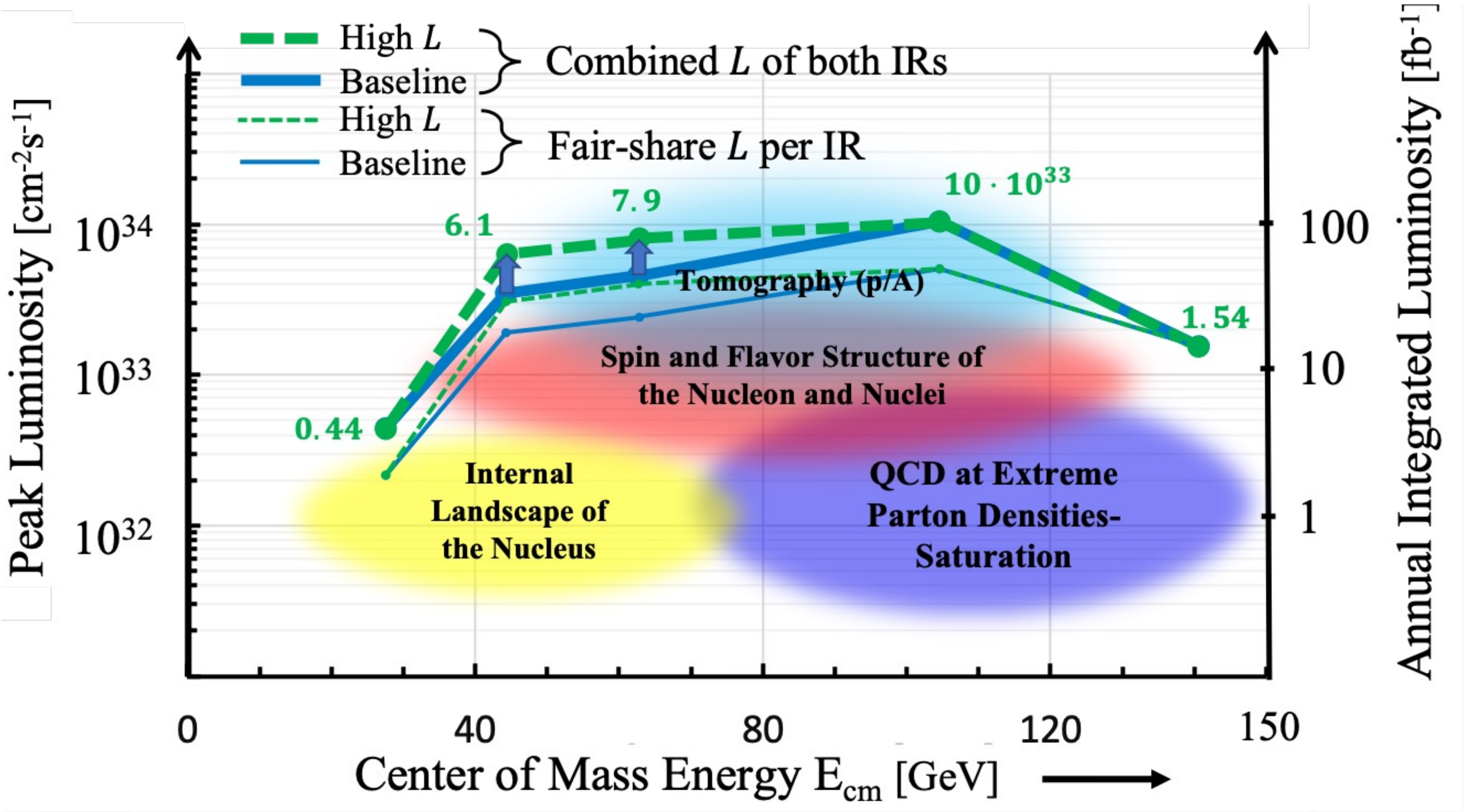
deep connections to TMD, Sudakov physics, ...

Electron-Ion Collider (EIC)

EIC Accelerator Design

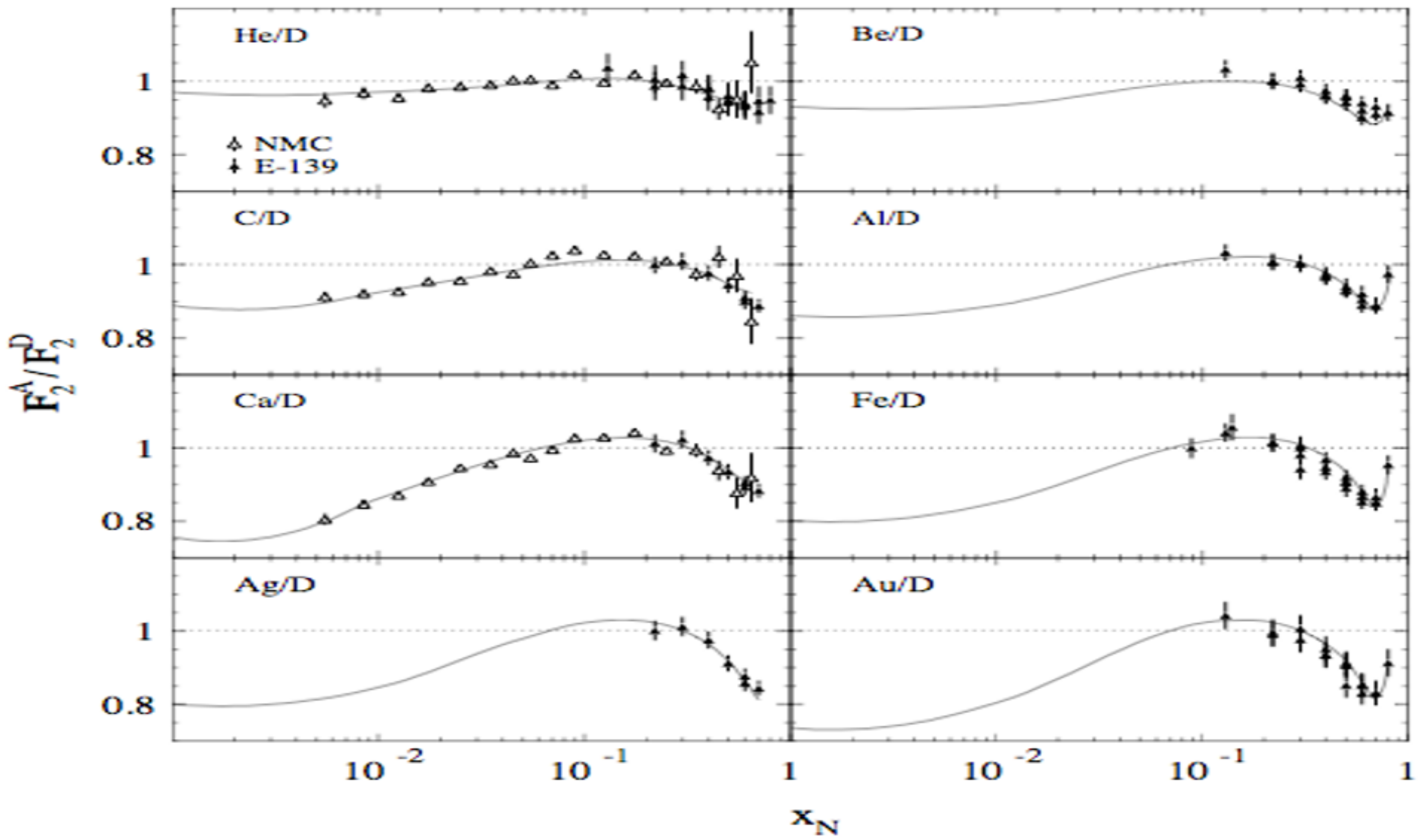


Center of Mass Energies:	20GeV - 140GeV
Luminosity:	10^{33} - 10^{34} $cm^{-2}s^{-1}$ / 10-100fb^{-1} / year
Highly Polarized Beams:	70%
Large Ion Species Range:	p to U
Number of Interaction Regions:	Up to 2!



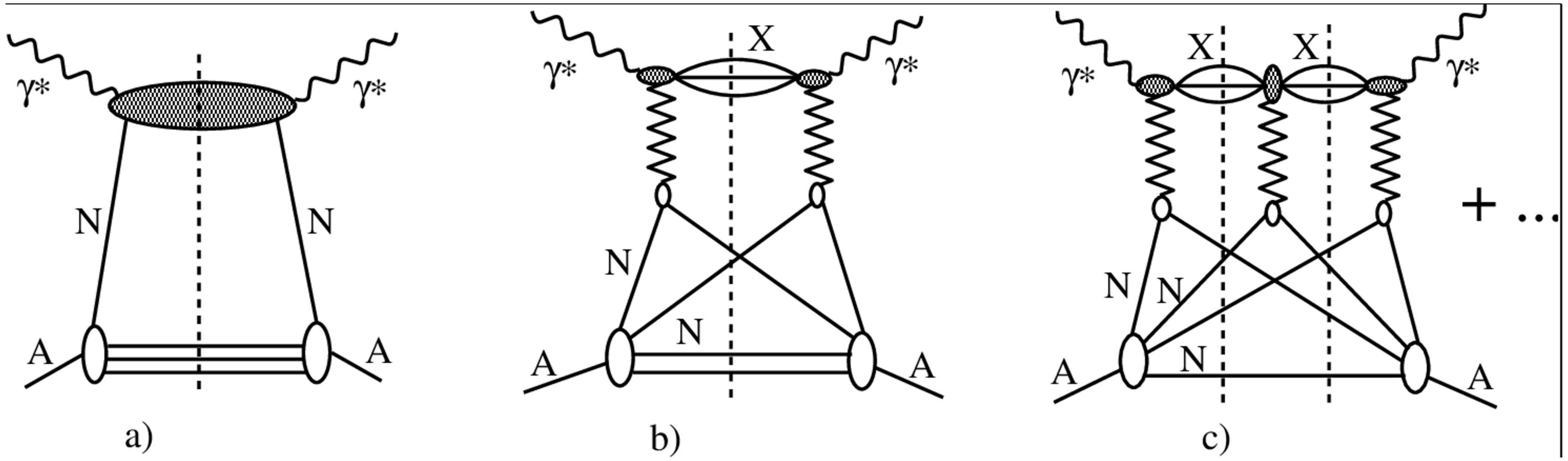
slide courtesy of A. Deshpande

Nuclear modification factor: weak coupling or not?



A non-perturbative (strong coupling) model

Gribov-Glauber model of nuclear shadowing:
multiple hadronic scattering



shadowing as destructive interference between multiple scattering amplitudes
nuclear parton distributions at initial scale Q_0
DGLAP evolution of distribution functions with hard scale Q^2

Frankfurt, Guzey, Strikman, Phys. Rep. 512 (2012) 255

the only way to generate nuclear effects dynamically is via higher twist corrections to collinear factorization and DGLAP evolution of parton distributions

Nuclear modification factor: a weak coupling approach

How much of the observed suppression is due to weakly coupled QCD dynamics?

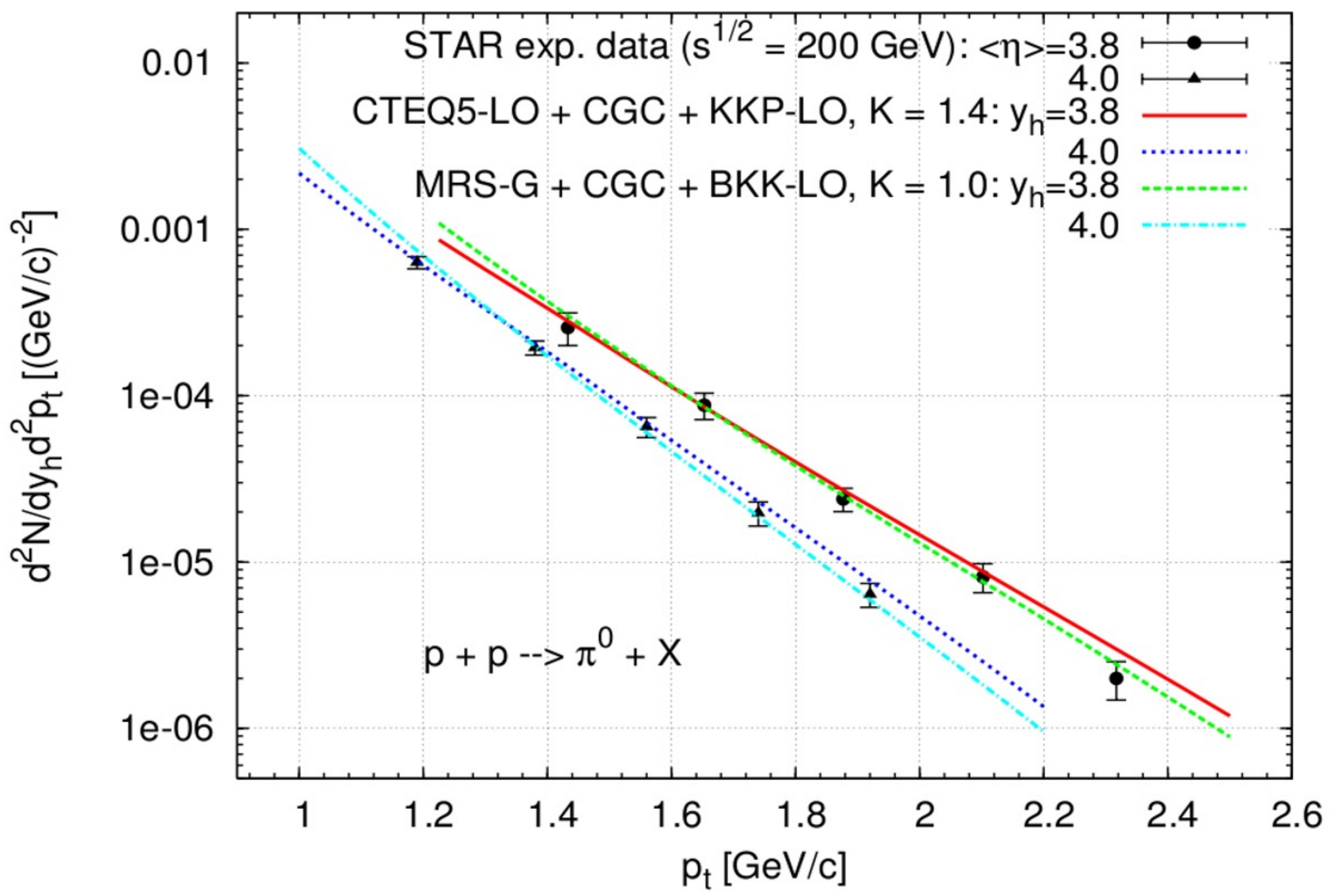
- this contribution must become more and more important at smaller x
  - how do we tell them apart?

CGC (saturation) dynamically generates shadowing

- it has both leading twist (LT) and higher twist (HT) shadowing
  - small x resummation is the only known way to generate LT shadowing in weak coupling QCD
    - how do non-eikonal (finite x) corrections affect small x evolution?
    - partially coherent cold matter energy loss ?
    - how important are non-perturbative effects (large dipoles, ...)?

Thank you!

Particle production: kinematics

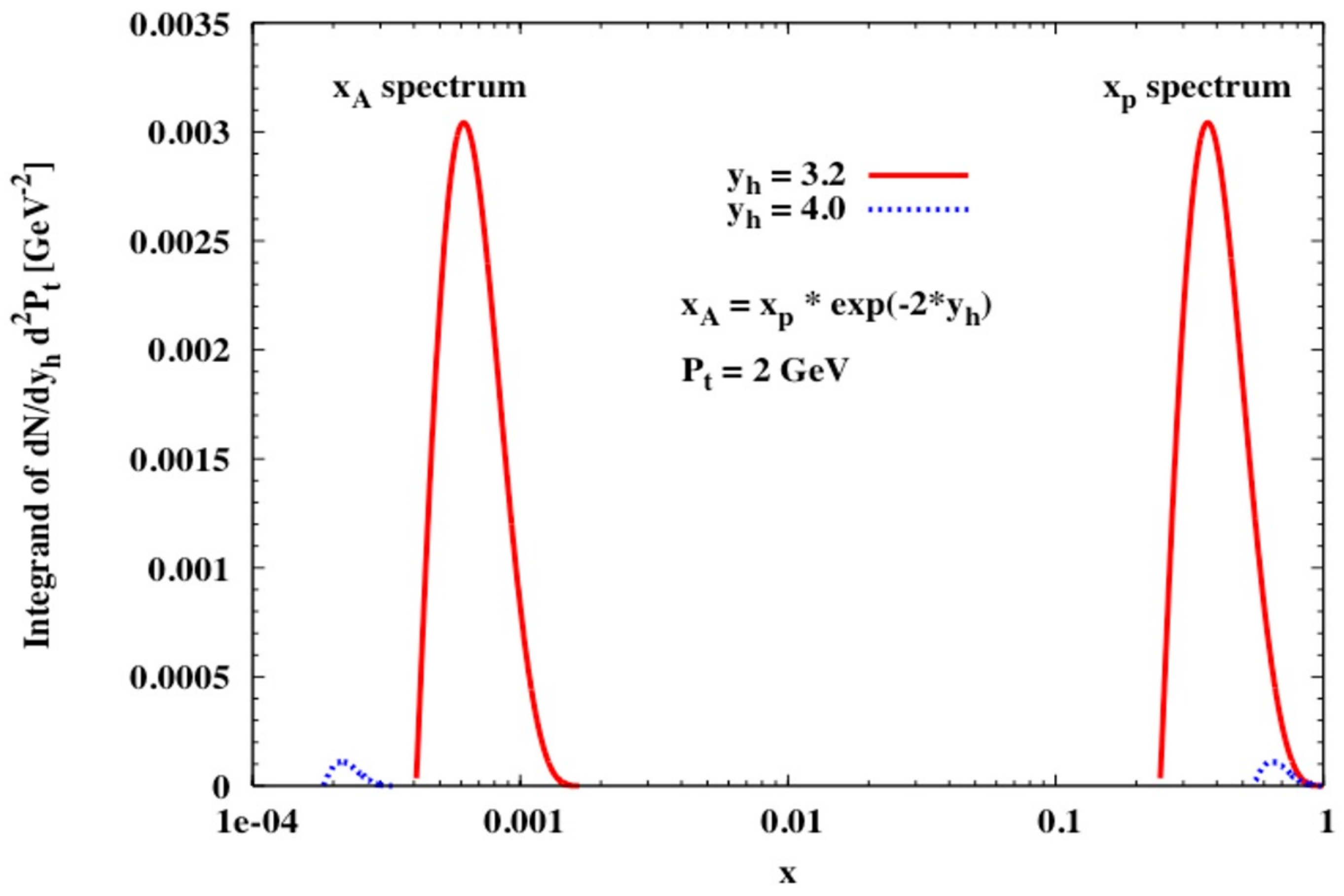


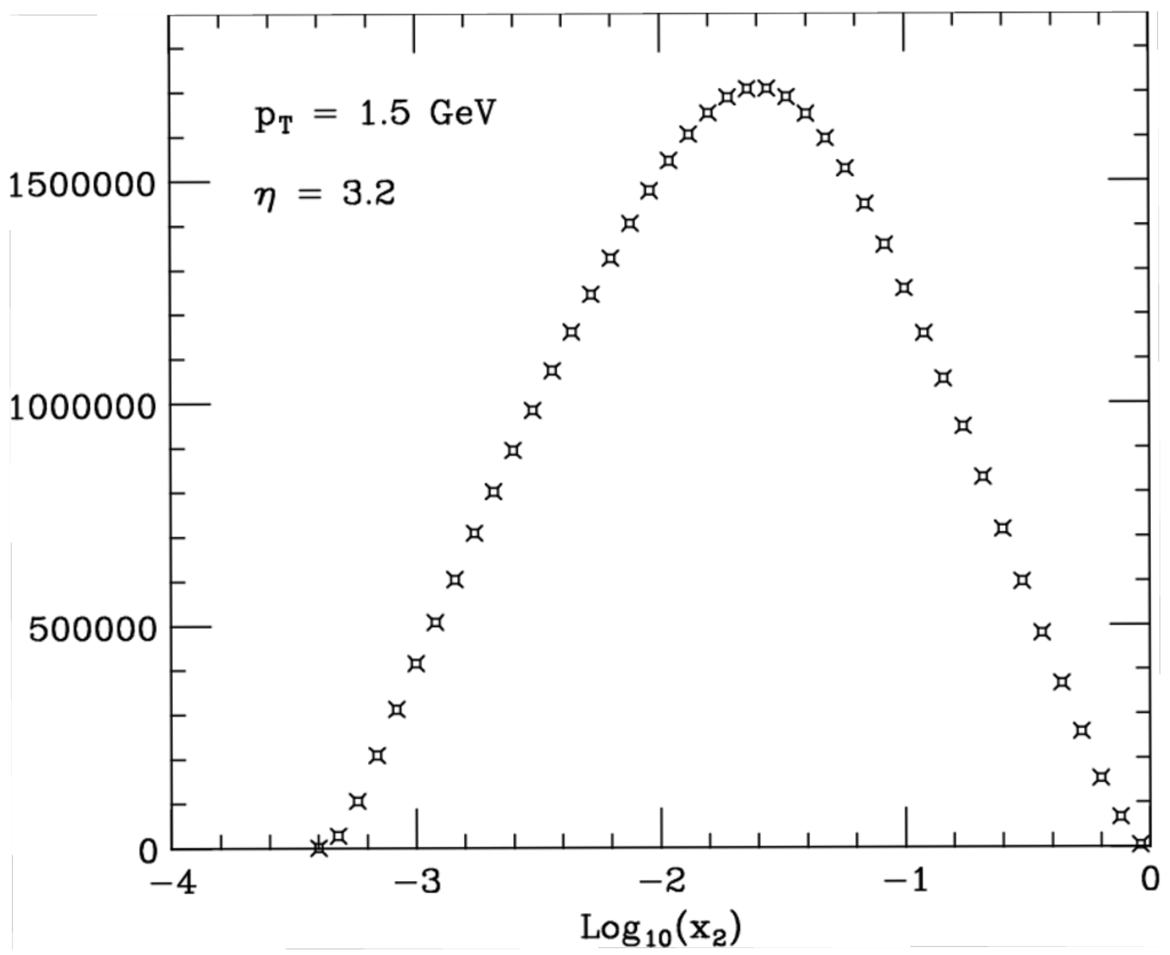
CGC

vs

collinear factorization

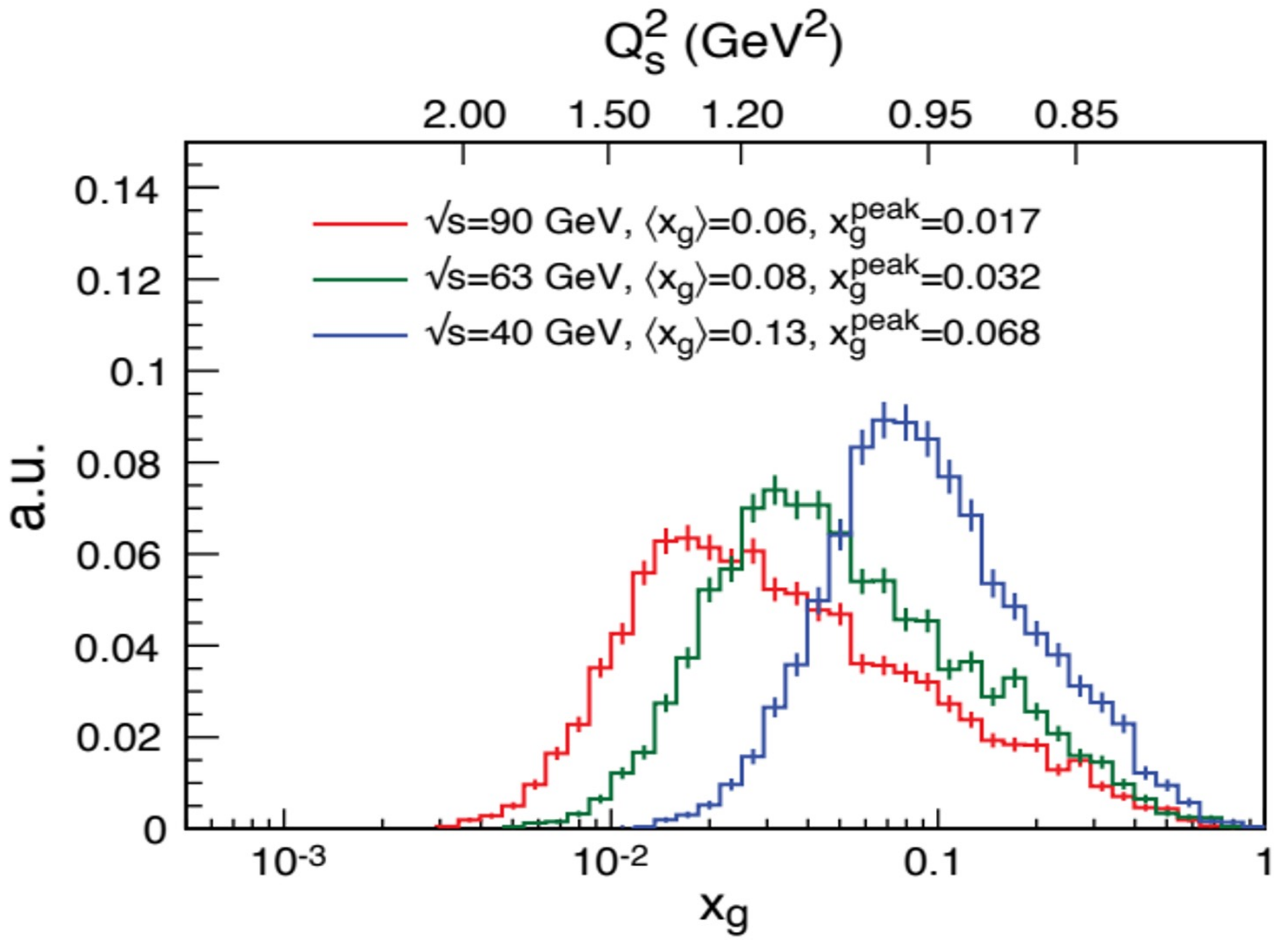
GSV, PLB603 (2004) 173-183





EIC

kinematics of inclusive dihadron production



Aschenauer et al. arXiv:1708.01527

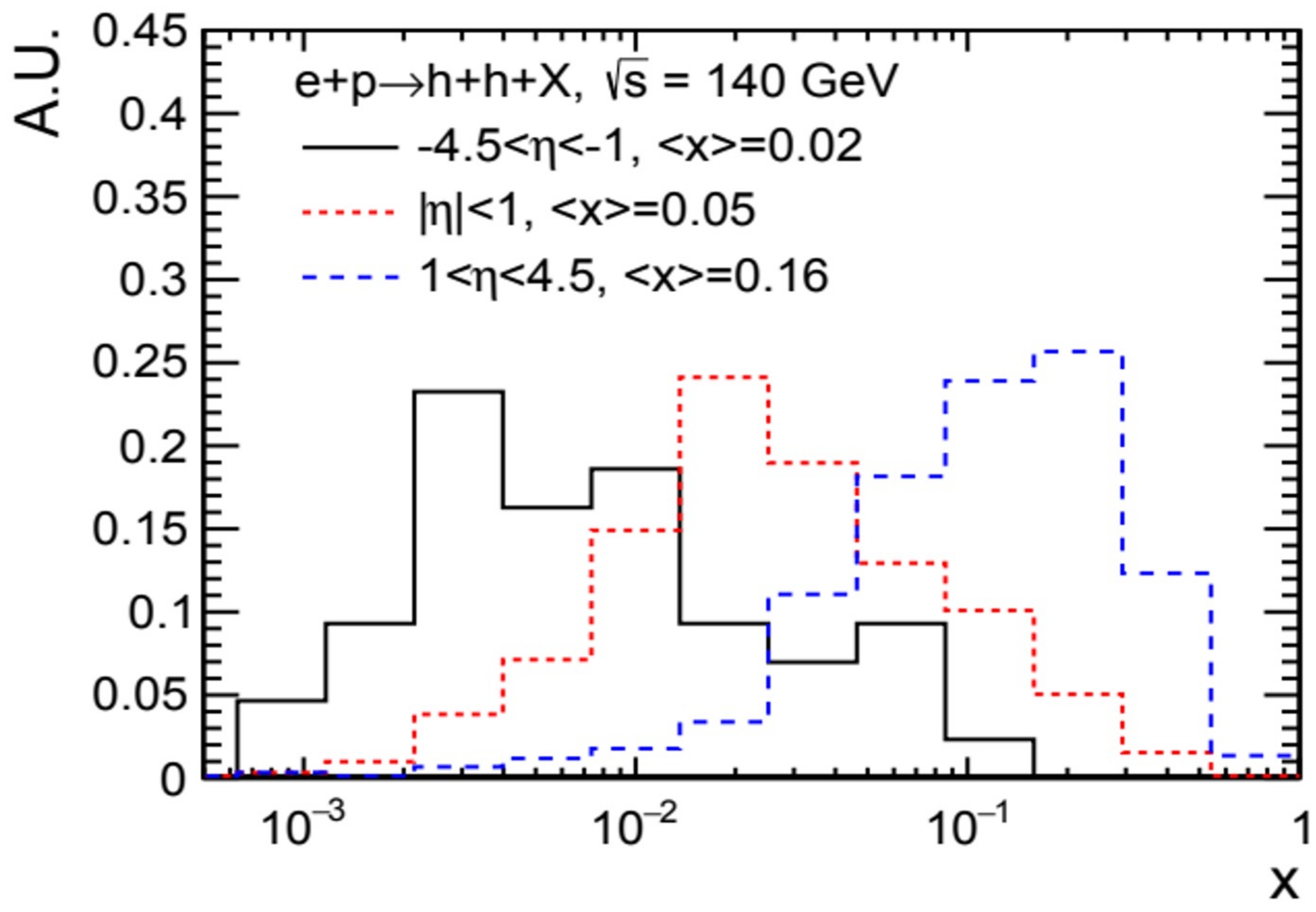


Fig. courtesy of Xiaoxuan Chu

transition region: from large x to small x

toward unifying small and large x (multiple scattering)

JJM, 1708.07533, 1809.04625, 1912.08878

scattering from small x modes of the target field $\mathbf{A^- = n^- \; S}$ involves only

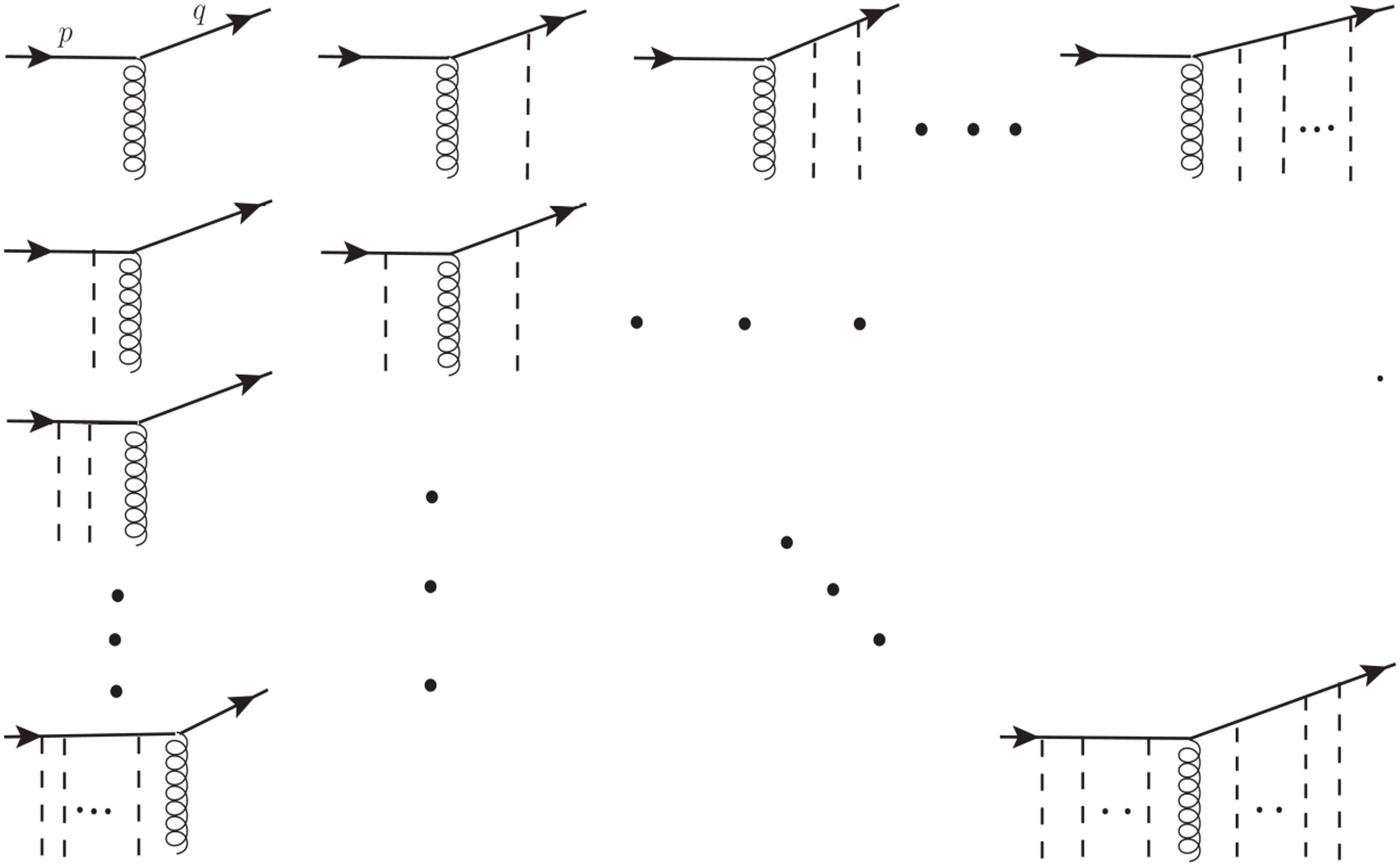
small transverse momenta exchange (small angle deflection)

$$
\begin{aligned}
p^\mu &= (p^+ \sim \sqrt{s},\, p^- = 0, p_t = 0) \\
S &= S(p^+ \sim 0, p^-/P^- \ll 1, p_t)
\end{aligned}
$$

allow hard scattering by including one all x field during which there is large momenta exchanged and quark can get deflected by a large angle.

$A^\mu_a(x^+, x^-, x_t)$

include eikonal multiple scattering before and after (along a different direction) the hard scattering


p
q